Validation of the Aharonov-Bohm effect through measuring total phase difference in a SQUID

Hasif Ahmed[1]

[1]*Physics Department, Lawrence University, 711 E Boldt Way, Appleton, WI 54911*

May 17, 2024

This study investigates the Aharonov-Bohm effect in Superconducting Quantum Interference Devices (SQUIDs) by measuring the total phase difference around the superconducting loop. The total phase difference observed within the superconducting loop of the SQUID represents the influence of the magnetic vector potential ($\mathbf{A}$), illustrating how electromagnetic environments, even in the absence of actual magnetic field ($\mathbf{B}$), can affect the behavior of Cooper pairs. Utilizing a SQUID that expels magnetic fields via the Meissner effect, we focus on the influence of the magnetic vector potential in environments devoid of magnetic field. By supplying an external microwave frequency, we propose a novel technique to measure the fundamental ratio e/h with high precision, without the need for extreme physical conditions such as very high magnetic fields or ultra-low temperatures. Our findings, within 95.6 % of theoretical prediction in total phase difference calculations, not only confirm the Aharonov-Bohm effect within SQUIDs but also highlight their potential in advancing quantum computing technologies. Future efforts will aim to enhance measurement precision and explore different SQUID configurations.

Introduction

The SQUID (Superconducting Quantum Interference Device) consists of a superconducting loop that can effectively expel magnetic fields from its interior due to the Meissner effect [1] where superconductors expel all magnetic fields, a phenomenon that occurs below a critical temperature and field strength. SQUID is used widely for exposition of high-temperature superconductivity in undergraduate laboratory courses and also for its functionality as a magnetometer [2]. However, it would be an understatement to claim that SQUID's usefulness is limited to these mentioned purposes. In this paper, we demonstrate how using a simple setup of SQUID along with supply of liquid nitrogen and a source of microwave frequency generator, we can measure a fundamental ratio e/h and consequently validate the Aharonov-Bohm effect by measuring the total phase difference around the superconducting loop.

In 1959, Aharonov and Bohm noticed that quantum mechanics requires measurable effects due to the vector potential ($\mathbf{A}$) in regions of space where the magnetic and electric fields vanish. Aharonov-Bohm's insight was that $\mathbf{A}$ could be finite in a region where $\mathbf{B}$ vanishes. This adds a geometric phase change to the dynamical phase change of a particle's trajectory. They verified this claim by observing interference between electron waves travelling on either side of a solenoid [1] [3]. For our purpose, within the superconducting material of SQUID, the magnetic field $\mathbf{B}$ is nearly zero due to Meissner effect (Fig. 1) but $\mathbf{A}$ is non-zero (Fig. 1). The superconducting loop consists of non-conducting barriers between two superconductors which are called Josephson junctions. The barrier is thin enough ($\approx$ 10 nm) for quantum tunneling of electron Cooper pairs across it, allowing current to flow around the loop [4]. When a supercurrent flows through such a junction, it exhibits quantum interference effects that are sensitive to the phase difference of the wavefunction representing Cooper pairs on either side of the junction. We argue that the quantum interference effects exhibited in SQUID align with the predictions of the Aharonov-Bohm effect.

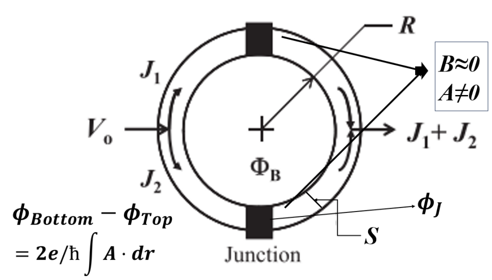

Figure 1: Josephson parallel-junction superconducting quantum interference device (SQUID), showing two junctions, upper and lower superconducting loops into which current is injected by a potential V_0 and recombined as interference current density $J = J_1 + J_2$, and a magnetic flux Φ_B through a center region of radius R but not inside the superconducting loops. ϕ_{Top}, ϕ_{Bottom} represent the phase change across the loop's top and bottom arms. $\phi_{Bottom} - \phi_{Top}$ is the total phase difference across the loop with surface area S (see Eq. 1 and 2). ϕ_j represents the phase change across each junction [4].

In this paper, we demonstrate a novel approach to quantitatively measure the mentioned phase difference by measuring the fundamental ratio e/h using our SQUID. The fundamental ratio is instrumental to measuring the total phase difference as elucidated in the theory section of this paper (Eq. 7). Our approach to experimentally measure this ratio requires supply of microwave frequency. Through supplying frequency to our SQUID apparatus, we can induce Shapiro steps in our V-I measurement of the SQUID. The Shapiro steps allow us to compute the e/h ratio in a noninvasive manner (Fig. 2).

In this paper we derive the relations that tie the e/h ratio with the total phase difference in the superconducting loop. We also discuss Josephson relations and the theory underlying Shapiro steps that is instrumental to understanding our experiment.

Theory

BCS theory describes supercurrent to be flow of Cooper pairs rather than merely electrons [1]. Cooper pairs are bosons and form the condensate that carries the supercurrent in SQUID which consists of high-temperature superconducting material (YBCO). The wavefunction of a Cooper pair in SQUID is described by ψ. ψ, at a given point, in the superconductor of SQUID is unique, so it follows that the change in phase round a closed path entirely within a superconducting loop is $2n\pi$, where n is an integer [1].

Using the Ginzburg and Landau approach, [1] derives the relation:

$$0 = \frac{2e}{\hbar}\oint \mathbf{A}\cdot d\mathbf{r} + \phi_{Top} - \phi_{Bottom} \tag{1}$$

Using the definition of magnetic vector potential $\mathbf{A}$ and $\mathbf{B}$, we know:

$$\oint \mathbf{A}\cdot d\mathbf{r} = \int \mathbf{B}\cdot d\mathbf{S} = \Phi_B \tag{2}$$

where Φ_B describes the magnetic flux in SQUID (Fig. 1). Combining Eq. 1 and Eq. 2 we get:

$$\Delta\phi = \phi_{Bottom} - \phi_{Top} = \frac{2e\Phi_B}{\hbar} \tag{3}$$

Aharonov-Bohm effect predicts the phase difference across each Josephson junction to be $\phi_j = e\Phi_B/\hbar$ [1]. So the Aharonov-Bohm effect prediction for the total phase difference around the superconducting loop would be:

$$\Delta\phi_{AB} = 2\phi_j = \frac{2e\Phi_B}{\hbar} \tag{4}$$

Aharonov-Bohm phase change predicts the flux quantization relation to be [1]:

$$\Phi_B = \frac{nh}{2e} \tag{5}$$

It follows that the Aharonov-Bohm effect's theoretical prediction (Eq. 4) aligns perfectly with the Ginzburg and Landau prediction (Eq. 3). However, the purpose of the paper is to propose an experimental approach to verify this prediction. As mentioned before, this requires us to determine the e/h ratio. For that purpose, we need to delve into the interaction of our SQUID with supplied external microwave frequency.

When exposed to microwave frequency, our SQUID output voltage is expected to exhibit V-I characteristics as shown below [2]:

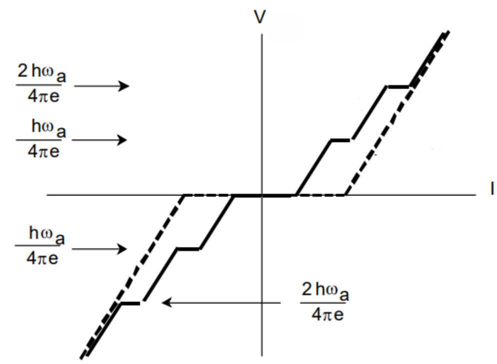

Figure 2: The constant voltage steps (i.e. discontinuous shift in the DC current) in the V-I curve are called Shapiro steps [2]. These Shapiro steps appear when $V_0 = nh\omega_a/4\pi e$ with n an integer. Here, ω_a represents the oscillation frequency of the current flowing across Josephson junctions [2].

According to [2], the voltage change per step is given by:

$$\Delta V_0 = \frac{(n+1)h\omega_a}{4\pi e} - \frac{nh\omega_a}{4\pi e} = \frac{h\nu_a}{2e} \tag{6}$$

[2] then derives the following relation:

$$\frac{e}{h} = \frac{\nu_a}{2V_0} \tag{7}$$

where ν_a represents the externally applied frequency and V_0 represents the voltage step value of Shapiro Steps (See Fig. 2).

Equation 7 is an expression of the AC Josephson effect. The wonderful thing about this effect is that we have now established a non-invasive strategy to measure the e/h ratio without the need for supplying high magnetic field conditions. This sets us up to describe the apparatus where we explain the source of applied frequency.

Experiment

Our SQUID [2] uses YBCO, which exhibits superconductivity at comparatively higher temperature (77 K) compared to the existing low temperature superconductors in the 1980s. In our research, we use the SQUID as the basis of our apparatus to perform experiments that help us determine the e/h factor using well-established AC Josephson relations.

In order to observe superconductive properties through our SQUID, we need to supply liquid N_2 which has a critical temperature $T_c = 77$ K. The supercurrent flowing through SQUID can be observed through an oscilloscope as we have discussed in appendix A.

Figure 1 shows a schematic diagram of the SQUID used in our experiments. This configuration depicts a basic SQUID loop with two Josephson junctions, essential for the device's capability to sense magnetic flux variations. The supplied voltage V across the junctions, together with the bias current I_{bias}, establish the conditions for the AC Josephson effect observation. This effect is essential for measuring the ratio e/h via the supercurrent's modulation in the apparatus. To ensure that we have minimized the effect of any external magnetic field, we have our SQUID apparatus protected with mu-metal shield which has high magnetic permeability. Also due to well-known Meissner effect, the superconducting materials when cooled below their critical temperature, expel all internal magnetic fields including earth's magnetic field creating an effective shielding environment.

To measure the value of e/h factor, we apply MW Frequency of value 44 GHz. Our apparatus appears as [2]:

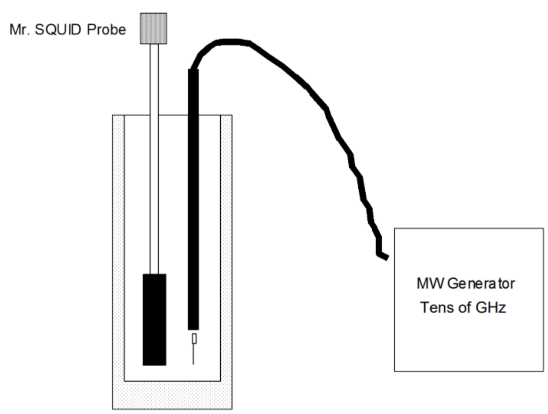

Figure 3: Schematic of our SQUID apparatus while we are supplying Microwave Frequency using a MW Generator.

By applying the external frequency while our SQUID and therefore Josephson junctions are in liquid N_2 bath, we want to observe the V-I characteristics of the junctions.

Results

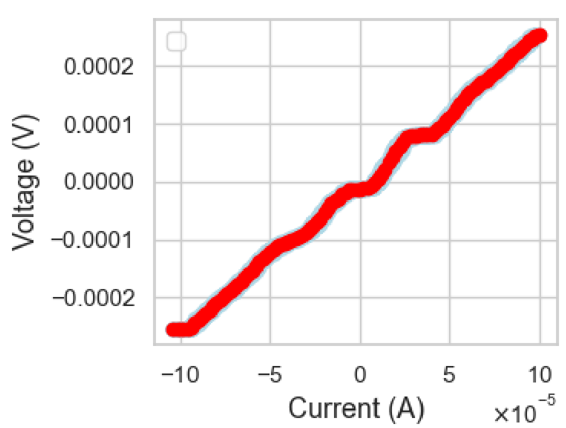

Figure 4: V-I characteristics of our Josephson junctions under applied MW Frequency. This measurement exhibits behavior predicted by AC Josephson effect [2] and Fig. 2.

The graph, similar to Fig. 2, demonstrates Shapiro steps, which appear as horizontal plateaus in the V-I curve when the junction is exposed to 44 GHz microwave radiation. The 'hump' or sharp increase in voltage observed after certain current thresholds indicates the transition out of the superconductive state, as the device begins to exhibit a voltage response to the applied current.

It is noteworthy that we did not observe the 'humps' as distinctly as predicted by the theoretical model in Fig. 2. We attribute this discrepancy to limitations in our measurement capabilities. The data shown in Fig. 4 was collected by manually adjusting the microwave output cable within our SQUID apparatus. This 'brute-force' data collection method restricted our ability to capture clean and distinct Shapiro steps, as seen in theoretical predictions. Despite these challenges, our experimental setup allowed us to observe Shapiro steps with a calculated standard deviation of 3 μV.

Each Shapiro step exhibits a voltage difference of approximately $V_0 = 95\mu$V, consistent with the steps, suggesting a well-defined relationship between the applied frequency and the induced voltages due to the AC Josephson effect. Plugging this value of our measurement of V_0 into Eq. 7, we obtain a value for e/h of 2.316×10^{14} Hz/V.

After considering the uncertainty calculation (appendix B), we find a value for e/h of $(2.316 \pm 0.156) \times 10^{14}$ Hz/V. The official value for the e/h ratio is 2.4179671×10^{14} Hz/V. Our measured value is consistent with the official value within the uncertainty, as the difference between our measured value and the official value is within the range of our measurement uncertainty.

According to Eq. 3 and Eq. 4, multiplying e/h by 4π (since $\hbar = h/2\pi$) and Φ_B should give us the value for the total phase difference around the superconducting loop. Φ_B here is one flux quantum measured to be 2.07×10^{-15} Wb. The experimental measurement of the total phase difference then comes to $\Delta\phi = 6.0088$ radians.

The theoretical Aharonov-Bohm effect prediction for this total phase difference is:

$$\Delta\phi_{AB} = \frac{2e}{\hbar}\Phi_B = \frac{2e}{h} \cdot 2\pi \cdot \frac{h}{2e} = 2\pi \text{ radians}$$

Using the flux quantization relation (Eq. 5) where $n = 1$, we compare our experimental total phase difference value with the theoretical prediction. Our measured total phase difference of 6.0088 radians is consistent with the theoretical prediction of 2π radians within our experimental uncertainty. Specifically, the ratio $\frac{6.0088}{2\pi} \approx 0.956$ suggests that our measurement aligns with the expected value when considering potential sources of error, such as thermal noise and minor discrepancies in data collection discussed in the next section.

Discussion

The experimental findings of Shapiro steps and the quantified e/h ratio confirm the theoretical predictions based on the Josephson frequency relations and further validate the quantum mechanical behavior of superconducting circuits under the influence of electromagnetic potentials. The non-invasive nature of our technique provides a strategic way to measure the e/h ratio without the need for extreme physical conditions. The relatively high level of accuracy using such a simple measurement is the reason the AC Josephson Effect is used by the National Institute of Science and Technology to set the standard for the "official" value of the volt [2].

As an extension, this study successfully demonstrated the Aharonov-Bohm effect in a superconducting quantum interference device (SQUID) by observing the phase shifts of Cooper pairs due to a non-zero magnetic vector potential, despite the absence of a magnetic field.

The experimental results align closely with the theoretical model, particularly the total phase difference calculations using the Aharonov-Bohm effect. The observed total phase difference around the superconducting loop was within 95.6% of the predicted value. This minor discrepancy could be attributed to experimental limitations such as thermal noise, precision in the measurement of the magnetic flux.

Information about the phase measurement is instrumental to maximizing supercurrents flowing in SQUIDs and devices that perform based on similar principles. This information can prove to be helpful in maximizing supercurrents within high-temperature superconductors used within quantum computers. This could have implications for quantum computing, where the manipulation of quantum states through electromagnetic potentials rather than direct magnetic field interactions could lead to more efficient quantum gates and circuits. The reader is advised to refer to [5] to see an example of geometric phase detection experiments in Josephson devices that tell us further about these phases' relation with the design of gates for quantum computation.

While the experiments were largely successful, several challenges remain. If the reader looks at the scope picture of the Shapiro steps in Fig. 4, the steps are not perfectly flat and show some rounding. This might be due to small fluctuations in the microwave generator frequency or they might also be due to thermal noise in the SQUID. Although 77 K may seem cold to us, it is rather "hot" compared to the superconducting transition temperature of our SQUID (77K/ $T_c \approx 0.85$). So, there is some uncertainty in the placement of the steps. [2]

Future research will focus on overcoming the current measurement limitations to achieve more distinct observations of Shapiro steps. Key steps include:

1. **Enhanced Measurement Techniques**: We will utilize advanced measurement equipment with higher precision and sensitivity to reduce the standard deviation in voltage measurements.

2. **Automated Data Collection**: We will implement an automated data collection system to replace the manual adjustment of the microwave output cable, ensuring consistent and repeatable measurements.

Additionally, we plan to scale these experiments to more complex quantum circuits, specifically exploring the impact of different geometries on the Aharonov-Bohm effect in superconductors. We will also investigate alternative materials for the Josephson junctions and their superconducting properties under different external conditions. This will provide deeper insights into the scalability and robustness of quantum interference devices.

Acknowledgements

This experiment was funded by Lawrence University Physics Department and we are grateful to them for their institutional support. We would like to acknowledge the advisory of Professor Douglas S. Martin throughout this project. We are also thankful to LeRoy Frahm for his help regarding electronics and data collection throughout the project.

Appendix A: Superconductivity in our SQUID Apparatus

When cooled below the critical temperature of YBCO- that is the core material used in our SQUID Apparatus, we expect to see superconductive behavior. The way to confirm that our device exhibits superconducting as expected is to notice its V-I characteristics:

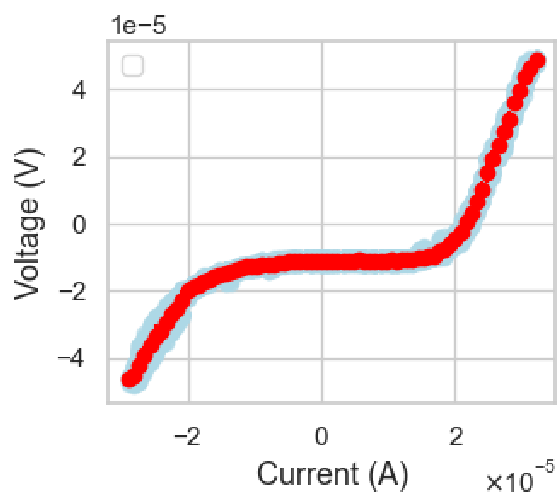

Figure 5: V-I characteristics in our SQUID: the light blue regions represent all the voltage measurements with respect to particular current values. The red curve represents the mean voltage value computed at particular current values. The horizontal plateau observed here in our red curve is the superconducting region.

The superconductive region can be identified by the section of the plot where the voltage remains zero or near-zero over a range of currents. This indicates that the SQUID is in a superconducting state, allowing current to flow without electrical resistance. The data points (in light blue) in Fig. 5 suggest the raw measurements, whereas the red dots with error bars indicate the mean voltage at each current level, with the bars representing the standard deviation. The 'hump' or the sharp increase in voltage after certain current thresholds suggests the transition out of the superconductive state, as the device begins to exhibit a voltage response to the applied current. This is where the supercurrent, or the maximum current that can flow without voltage (thus without resistance), can be measured.

Appendix B: Error Calculation

The uncertainty for our e/h ratio measurement is calculated as:

$$\sigma_{e/h} = \left| \frac{\nu_a}{V_0^2} \sigma_V \right| = 1.56 \times 10^{13} \text{ Hz/Volt} \quad (8)$$

Here, σ_V is the standard deviation from our data demonstrated in Fig. 4 which we measured to be 3 microvolts.

References

[1] Ian Kenyon. "Quantum 20/20". In: *Quantum 20/20*. Oxford University PressOxford, Sept. 2019, pp. 231–281. DOI: 10.1093/oso/9780198808350.002.0003.

[2] Robin Cantor and Star Cryoelectronics. *Mr. SQUID ® User's Guide*. Tech. rep. 1996.

[3] Y. Aharonov and D. Bohm. "Significance of Electromagnetic Potentials in the Quantum Theory". In: *Physical Review* 115.3 (Aug. 1959), pp. 485–491. ISSN: 0031-899X. DOI: 10.1103/PhysRev.115.485.

[4] Keith J. Kasunic. "Magnetic Aharonov-Bohm effects and the quantum phase shift: A heuristic interpretation". In: *American Journal of Physics* 87.9 (Sept. 2019), pp. 745–751. ISSN: 0002-9505. DOI: 10.1119/1.5115499.

[5] Giuseppe Falci et al. "Detection of geometric phases in superconducting nanocircuits". In: *Nature* 407.6802 (Sept. 2000), pp. 355–358. ISSN: 0028-0836. DOI: 10.1038/35030052.