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In the name of Allah, Most Gracious, Most Merciful

CSE 4303
Data Structure

Topic: Introduction to data structures, Complexity Time-Space Tradeoff

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What is Data Structure?

Data: Simply values or sets of values, raw facts or figure without any specific meaning.

Data Structure: The logical or mathematical model of a particular organization of data.

- ➢ Can store data

  Example: Integers, Strings, Floats, … …
- ➢ Can answer some questions about the stored data

  Example: What is the smallest value not greater than x?
- ➢ Can add or remove data

  Example: add the element x after y, remove values less than x.

Why Study Data Structure?

Applications of Data Structure:

- Computer file system (Data structure maps file names onto hard drive sectors)
- Google and other search engines (Data structure maps keywords on web pages containing those keywords)
- What is the longest common subsequence of two DNA can be found?
- Geographic systems (Data structure find data relevant to the current view/location)
- Finding large Prime Numbers
- Block chain (Linked list)
- Google Map (Finding shortest distances in terms of distance and time)
- Data Compression (Huffman's encoding)
- Natural Language Processing (Strings)
- … … … … …

Many problems are solved efficiently just using the right data structure …

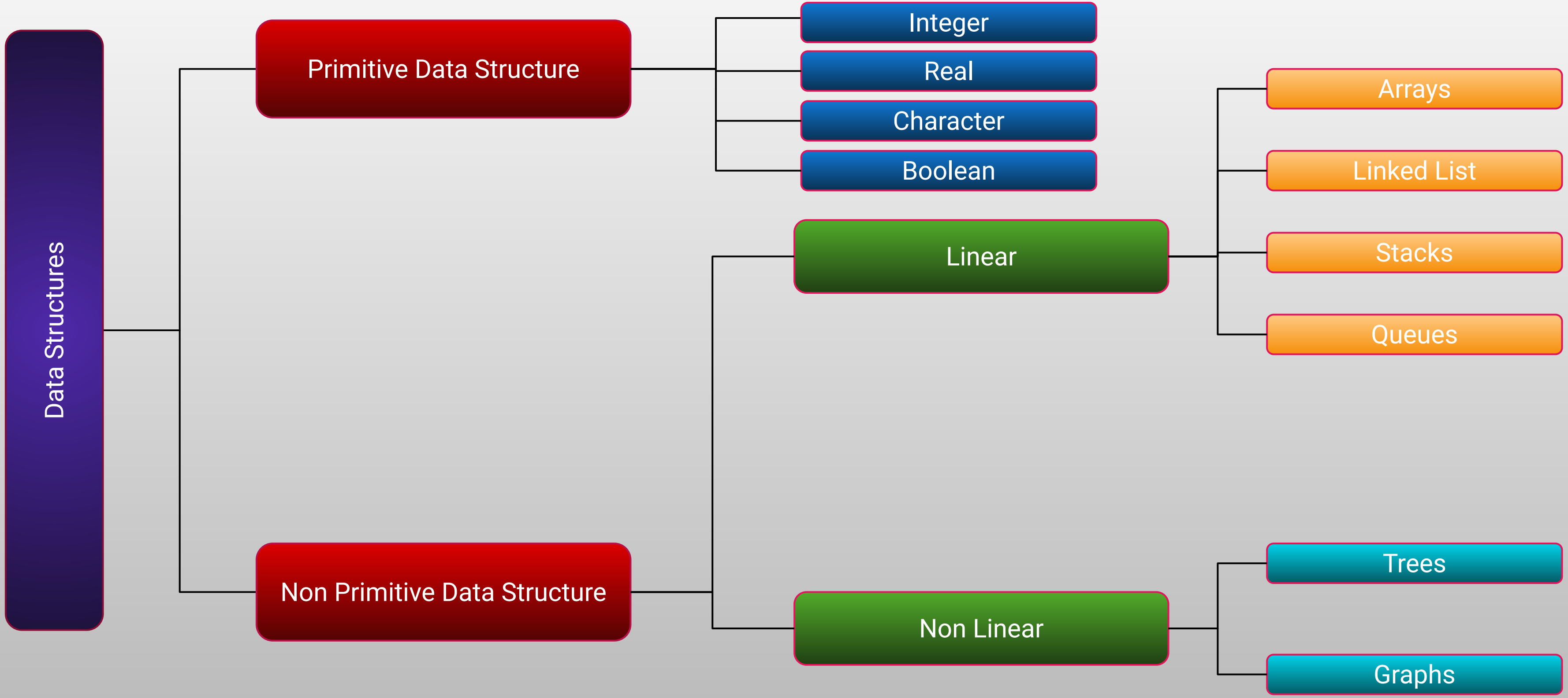
How do We Study Data Structures?

- What does the data structure represents?
  Computer file system (data structure maps file names onto hard drive track and sectors)

- What are the operations does it supports?
  - Reading: looking something up at a particular spot within the data structure.
  - Searching: looking for a particular value within a data structure.
  - Inserting: adding a new value to the data structure.
  - Deleting: removing a value from the data structure.
  - Sorting: rearranging element in some logical order.
  - Merging: Combining records of two different sorted files into one sorted files.

- What kind of performance does it have?
  - How long does each operation take? (Time complexity)
  - How much space does it use? (Memory complexity)

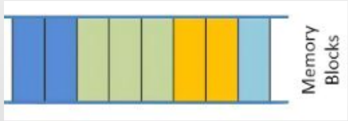
Classification of Data Structure



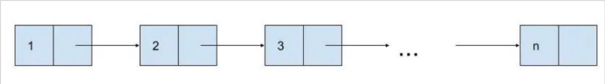
Memory Allocation

Memory allocation can be classified into followings:

- Contiguous
  - Example: arrays



- Linked
  - Example: linked lists

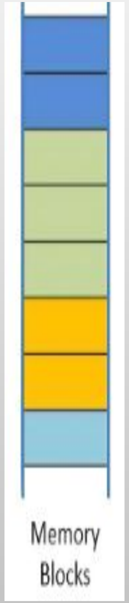


- Indexed
  - Example: array of pointers.

Contiguous Memory Allocation

An array stores n objects in a single contiguous space of memory.

- ➔ Can directly access any point randomly. Random access is possible.
- ➔ Unfortunately, if more memory is required, a request for new memory usually requires copying all information into the new memory.
- ➔ In general, you cannot request for the operating system to allocate to you the next n memory locations

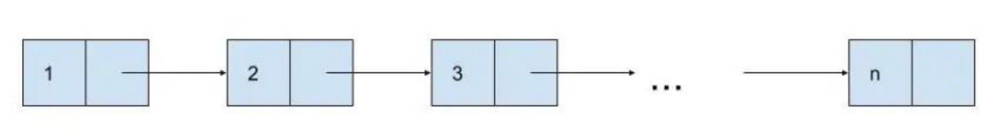


Linked Memory Allocation

Linked storage such as a linked list associates two pieces of data with each item being stored:

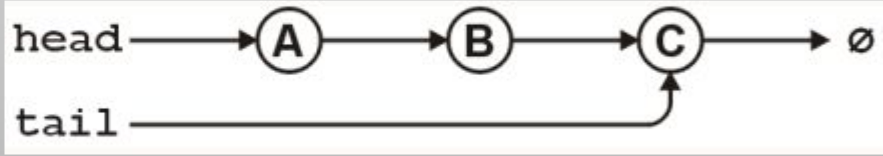
- ➔ The object itself, and
- ➔ A reference to the next item

- ➔ Random access to any data apart from the beginning is not possible since the address of a particular data is only stored to its previous data.



The actual linked list class must store two pointers

- ➔ A head and tail:
  - ◆ Node *head;
  - ◆ Node *tail;

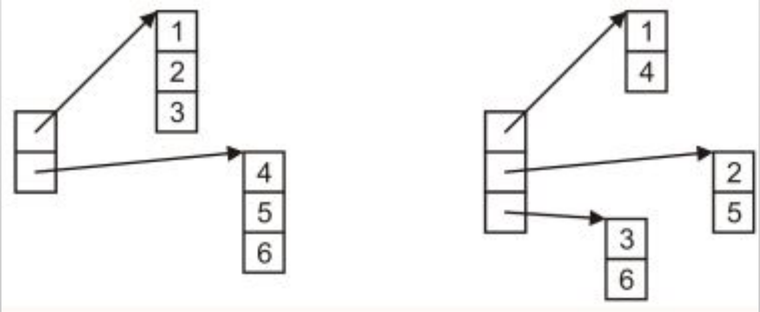


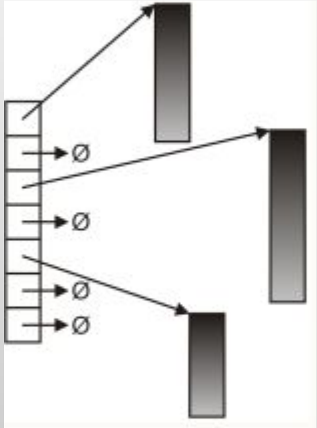
Indexed Memory Allocation

With indexed allocation, an array of pointers (possibly NULL) link to a sequence of allocated memory locations.

Matrices can be implemented using indexed allocation:

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$$



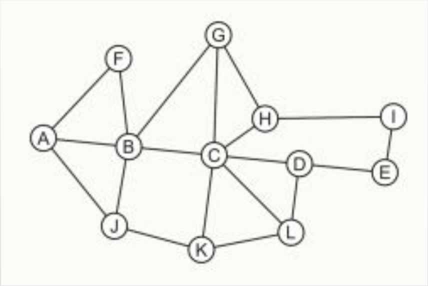


Other Memory Allocations

We will look at some varieties or hybrids of these memory allocations including:

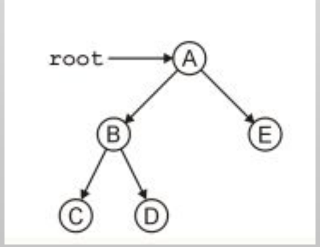
- ➔ Trees
- ➔ Graphs

Arbitrary relations among the objects in a container

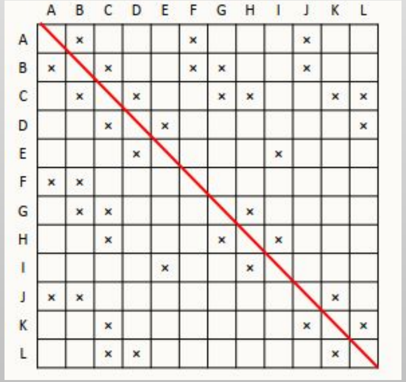


Graph

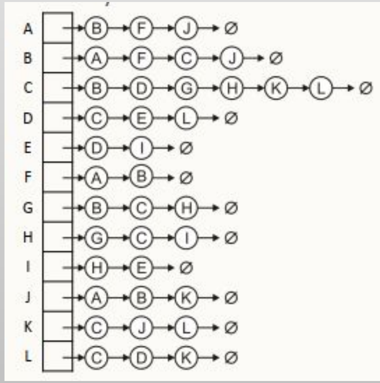
A rooted tree is similar to a linked list but with multiple next pointers



Tree



adjacency matrix



adjacency list

Complexity, Time-space tradeoff

- A function that estimates the running time/space with respect to the input size.
- Less time and space requirement is a blessing!
- Deals with large input size.
- Tradeoff: Increased amount of space to store data can sometimes reduce time requirement (or vice-versa).

Why Do We Care?

solution#1

```
for i=2 to n-1
   if i divides n
        n is not a prime
```

$(n-2)$ divisions in worst case

solution#2

for i=2 to $\sqrt{n}$
   if i divides n
        n is not a prime

$(\sqrt{n}-1)$ divisions in worst case

Complexity, Time-space tradeoff

Assuming 1 ms to perform a division		
	<u>Solution #1</u>	<u>Solution#2</u>
n=11	9 ms	~2 ms
n=101	99 ms	~9 ms
n=1000003 =10^6+3	~10^6 ms =1000 sec =16.66min	~10^3 ms = 1sec
n=10^10	10^10 ms =10^7sec =115 days	~10^5 ms = 100sec = 1.66 mins

Complexity, Time-space tradeoff

Two functions plotted in this graph:

$f(x) = x$ (red)

$f(x) = \sqrt{x}$ (blue)

Blue function is a bit costly in the beginning, but cheaper as x increases.

Time Complexity Analysis

Measures how fast the time requirement of a program grows when the input size increases.

Running time of program may depend on:

- ➔ Single vs multi processor
- ➔ Read/write speed of memory
- ➔ 32-bit or 64-bit
- ➔ Size of input

For time complexity analysis, we are only interested in (size of input)

- Takes same amount of time regardless of input size
- Constant time algorithm
- Time Complexity O(1)

```
Sum(a,b) {
  return a+b
}
```

Let's think about this function

Time requirement: ~ 2 time-units (1 unit for addition, 1 unit for return statement)

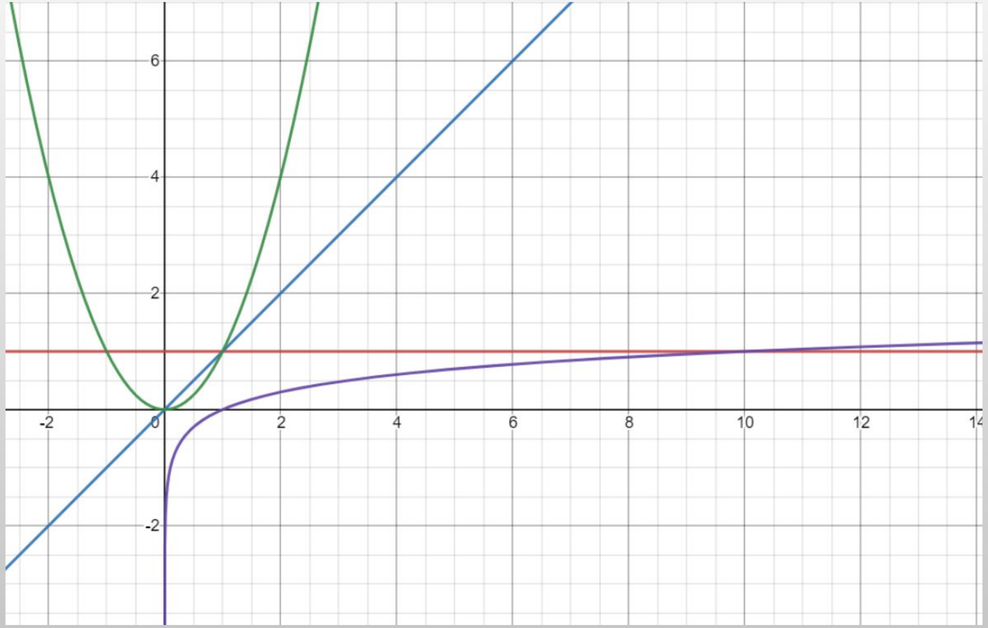
Time Complexity Analysis

	# times	Cost unit
1. `sumOfList (A, n) {`		
2. `total=0`	1	1 (c1)
3. `for i=0 to n-1`	n+1	2 (c2)
4. `total = total + A[i]`	n	2 (c3)
5. `return total`	1	1 (c4)
6. `}`		

Comments
<u>In line 3:</u> - Executes n+1 times. One extra checking for breaking condition. - c2: 1 unit for increment, 1 unit for assignment. <u>In line 4:</u> - 1 unit for addition, 1 for assignment.

- $T(n) = 1 + 2(n+1) + 2n + 1 = 4n + 4$
- In other words, $\mathrm{T(n)} = c_1 + c_2(n+1) + c_3 n + c_4 = \boldsymbol{cn + c'}$
  - here $(c = c_2 + c_3\,, \& \; c' = c_1 + c_3 + c_4)$
- Don't care much about value of $c \; or \; c'$, focus on the rate of growth.
- Here the growth is linear. Termed as ***O(n)***, AKA 'Big-oh of n' AKA 'Order of n' .

Some Growth Functions



$\boldsymbol{f(x) = 1}$ (red),

$\boldsymbol{f(x) = x}$ (blue),

$\boldsymbol{f(x) = x^2}$ (green),

$\boldsymbol{f(x) = log\, x}$ (purple)

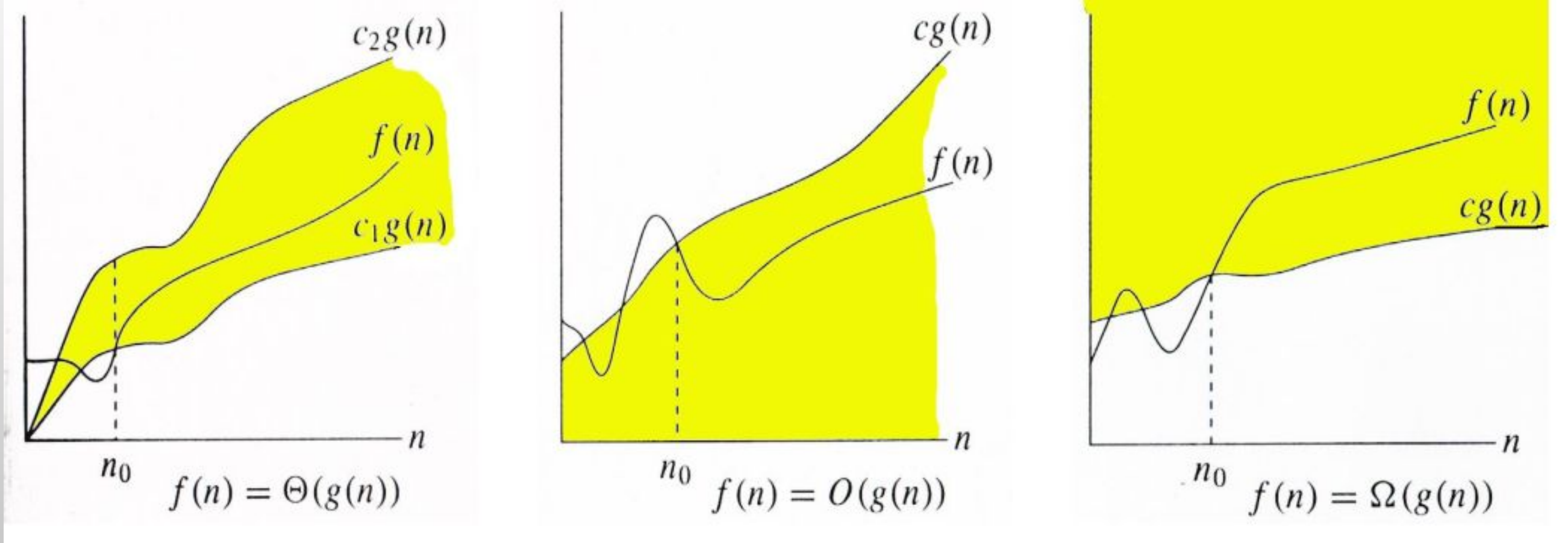
Check the growth of function as values in x axis grows!

Asymptotic Analysis

- Asymptotic Analysis is the big idea that helps to analyze algorithms.
- In Asymptotic Analysis, we evaluate the performance of an algorithm in terms of input size (we don't measure the actual running time).
- Define mathematical bound of how the time (or space) taken by an algorithm increases with the input size.
- An algorithm that is asymptotically more efficient will be the best choice for all but very small inputs.

[Generally, the term 'asymptotic' means approaching but never connecting with a line or curve.]

Asymptotic Analysis



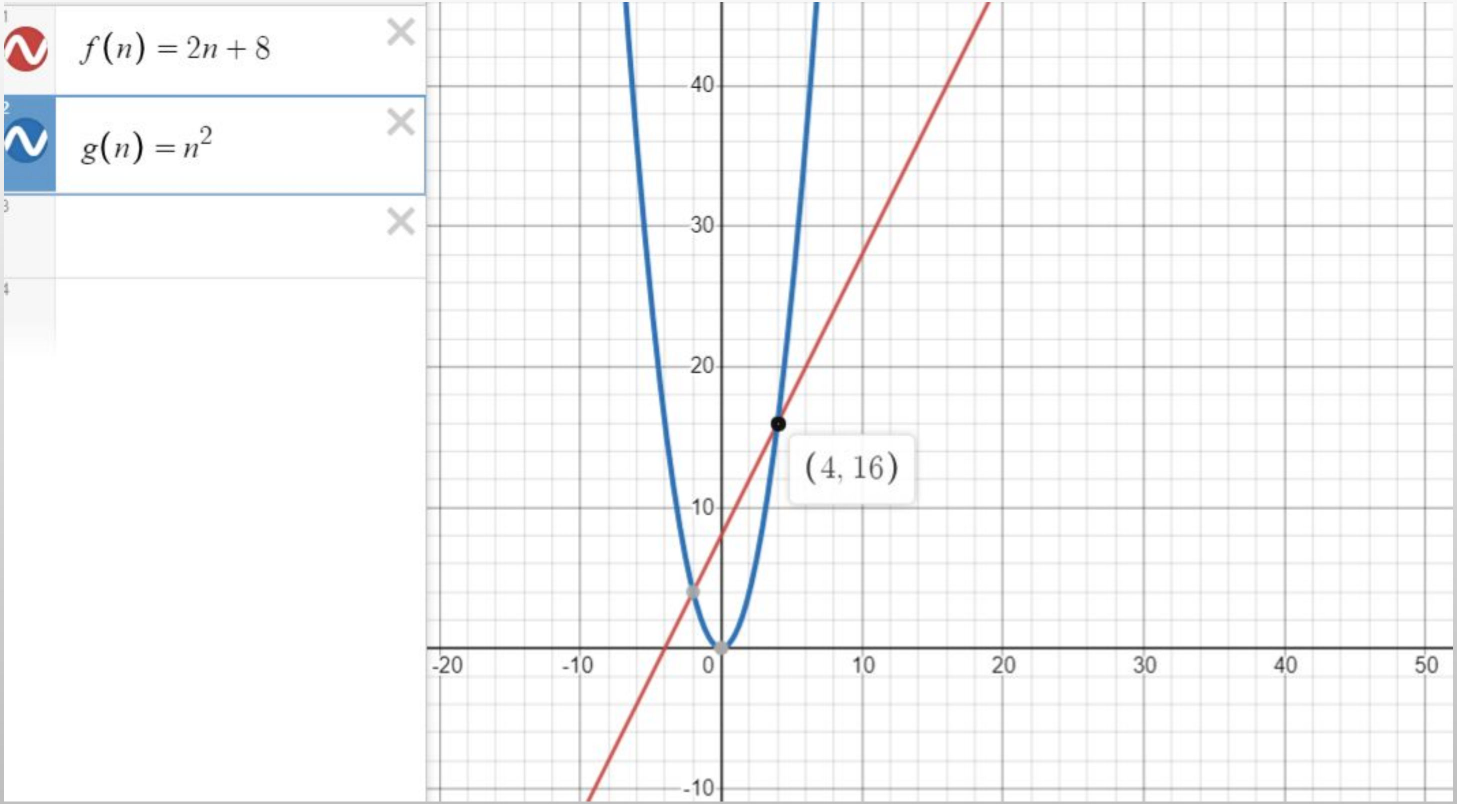
The Big 'O'

- The $'O'$ Notation
  - A function $f(n) = O\big(g(n)\big)$ if there exists n_0 and c such that $f(n) < cg(n)$
  - Whenever, $n > n_0$
    - O (pronounced big-oh) is the formal method of expressing *upper bound* of an algorithm's running time.
    - Measures the *longest amount of time it could possibly take*.
    - $g(n)$ is an *asymptotic upper bound* for $f(n)$.

The Big ‘O’

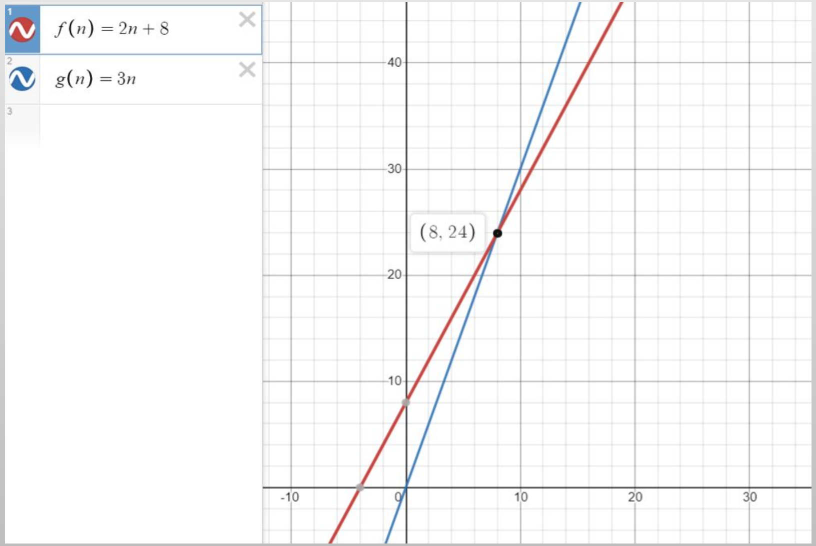
- Example of $'O'$ notation:
  - Suppose, $f(n) = 2n + 8$ and $g(n) = n^2$
  - Can we find a constant n_0, so that $2n + 8 \leq n^2$?
  - $n_0 = 4$ works here!
  - For any number n greater than 4, this will still work. Since we are trying to generalize this for large values of n
    - $f(n)$ is bounded by $g(n)$ and will always be less. (here $c = 1$ is good enough.)
    - Conclusion, $f(n) = O\big(g(n)\big),\ for\ all\ n > 4$
    - Thus here, $f(n) = O(n^2)$

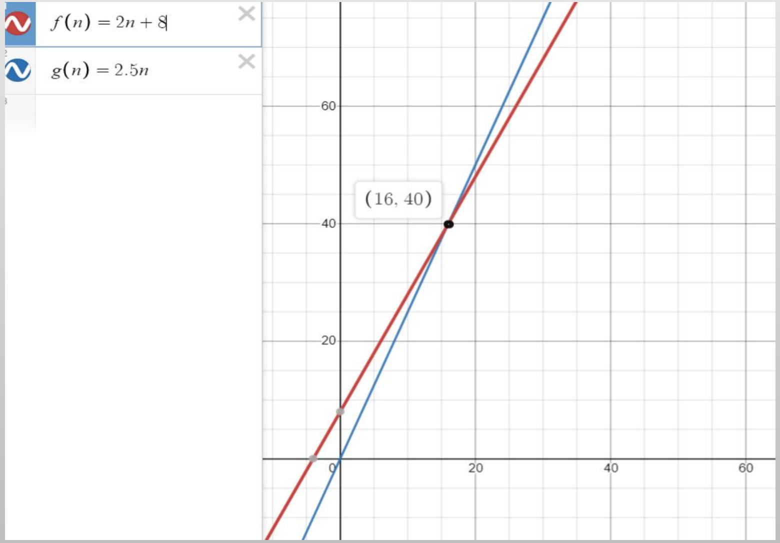
The Big 'O'



The Big ‘O’

- Can we bound $f(n) = 2n + 8$ using $g(n) = n$? (meaning, can $f(n) = O(n)$ be true?)
  - Yes! Pick the value of $'c'$ carefully!
  - $if\ c = 3,\ f(n) = O(n)\ for\ all\ n \geq 8$
  - We can also define, $if\ c = 2.5,\ f(n) = O(n)\ for\ all\ n \geq 16$



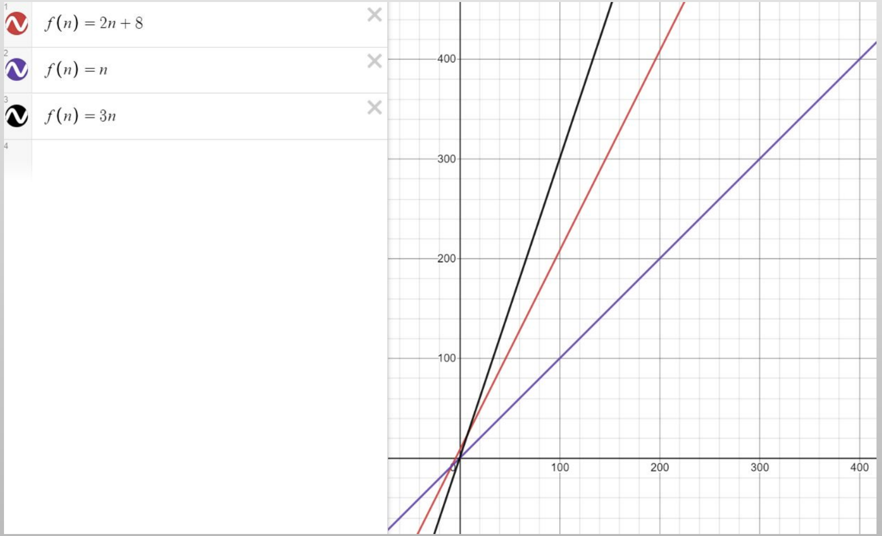


The Big 'Ω'

- Big-Omega Notation:
  - A function $f(n) = \Omega(g(n))$ if there exists n_0 and c such that $f(n) > cg(n)$
  - Whenever $n > n_0$:
    - Almost same definition as Big-Omega, except that $'f(n) > cg(n)'$
    - This makes $g(n)$ a *__lower bound__* function, instead of a upper bound function.
    - $g(n)$ is an *__asymptotic lower bound__* for $f(n)$
    - Describes the *__best that can happen__* for a given data size.

The Big 'θ'

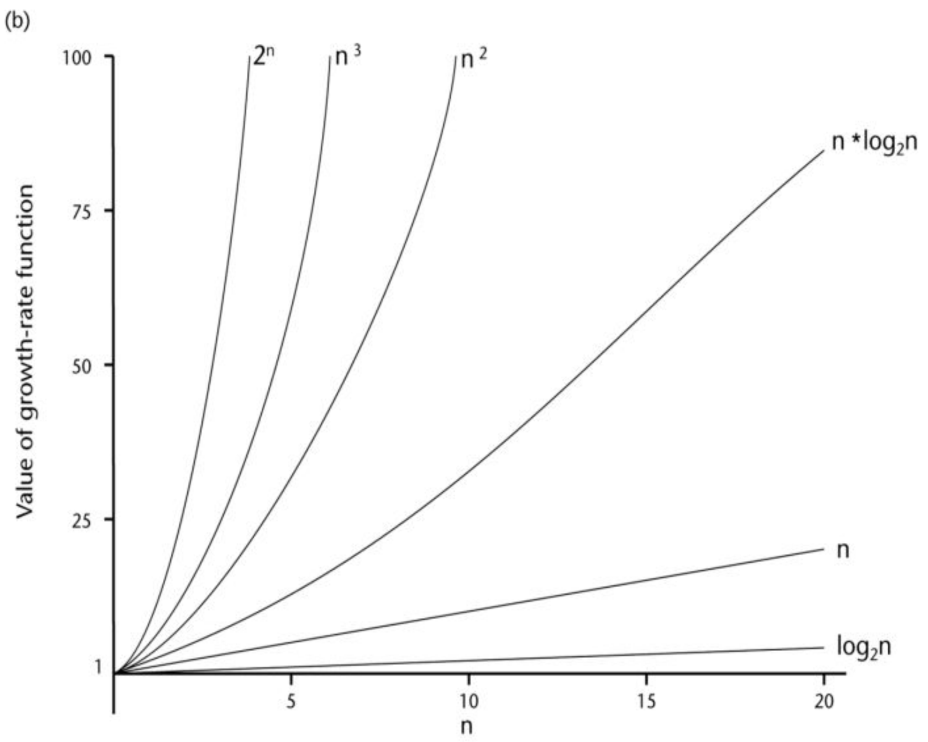
- <u>Big-Theta Notation</u>:
  - A function $f(n) = \Theta(g(n))$ if and only if $f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$
  - $f(n)$ is *<u>bounded both from the top and bottom</u>* ***<u>by the same function</u>*** $\boldsymbol{g(n)}$.
  - Thus, $g(n)$ is an *<u>asymptotic tight bound</u>* for $f(n)$
  - Tight bounds are obtained from asymptotic upper and lower bounds.
  - $3n + 3$ is:
    - $O(n)$ (let's say for $c = 4$)
    - $\Omega(n)$ (let's say for $c = 1$)
    - So it can be written as $\Theta(n)$
  - $3n + 3$ is
    - $O(n^2)$ $(for\ all\ n \geq 4)$
    - ~~$\Omega(n^2)$~~ (only true for $n = 1,2,3$)
    - So it **can not** be written as ~~$\Theta(n^2)$~~



Most Common Growth Functions

The most common classes are given names:

$\Theta(1)$	constant
$\Theta(\ln(n))$	logarithmic
$\Theta(n)$	linear
$\Theta(n \ln(n))$	"n log n"
$\Theta(n^2)$	quadratic
$\Theta(n^3)$	cubic
$2^n, e^n, 4^n, \ldots$	exponential



Growth Rate of Functions

- If an algorithm takes *1 second to run with problem size = 8*, what is the time requirement (approximately) for with the problem size = 16 ?

Running Time on Operators

- Each *machine instruction* can be executed in a fixed number of cycles,
  - We assume each *operation* requires a fixed number of cycles.
- Time required for any operator is $\theta(1)$.

Operation type	Symbols
Retrieving/ storing variables from memory	
Variable assignment	`=`
Integer operations	`+ - * / % ++ --`
Bitwise operations	`& | ^ ~`
Relational operations	`== != < <= > >=`
Logical operations	`&& || !`
Memory allocation and deallocation	`new delete`

Running Time on Block of operations

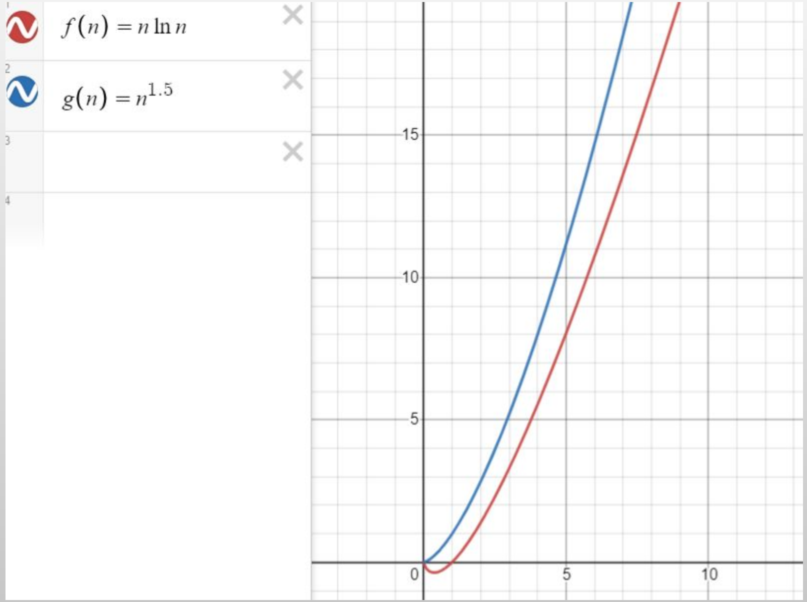
- If each operation runs in $\theta(1)$ time, any fixed number of operations also run in $\theta(1)$ time.

```
// swap variables a and b
int temp = a;
a = b;
b = temp;
```

```
// update a sequence of values
++index;
prev_modulus = modulus;
modulus = next_modulus;
next_modulus = modulus_table[index];
```

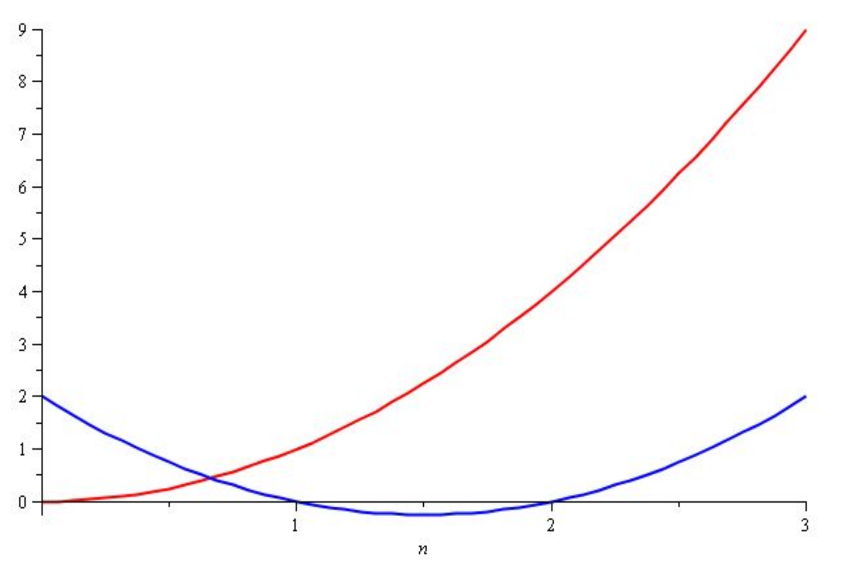
Running Time on Block of Sequence

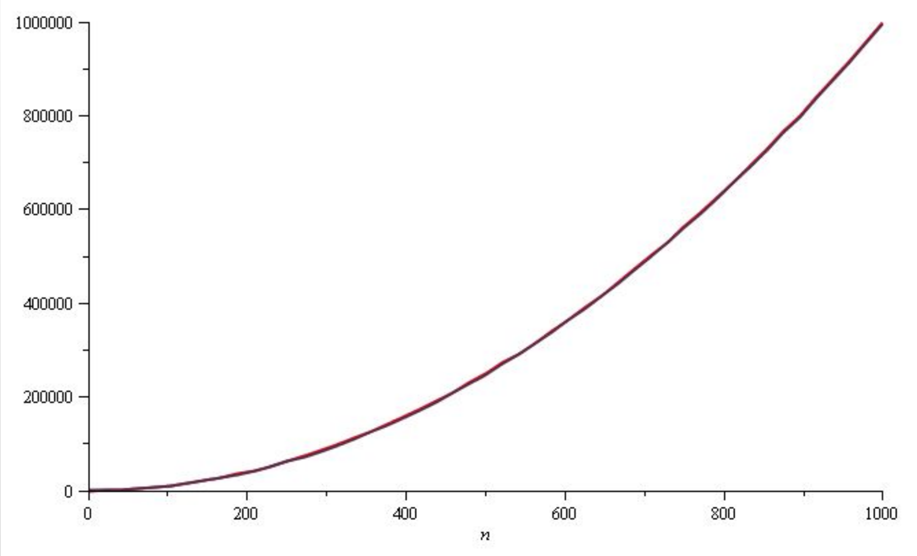
- Other examples include:
  - Run three blocks of code which are $\Theta(1), \Theta(n^2)$ *and* $\Theta(n)$
    - Total runtime $\Theta(1 + n^2 + n) = \Theta(n^2)$
  - Run two blocks of code which are $\Theta(nlogn)$ *and* $\Theta(n^{1.5})$
    - Total runtime $\Theta(nlogn + n^{1.5}) = \Theta(n^{1.5})$
  - While considering a sum, *take the dominant term.*



Quadratic Growth

$f(n) = n^2$ $g(n) = n^2 - 3n + 2$

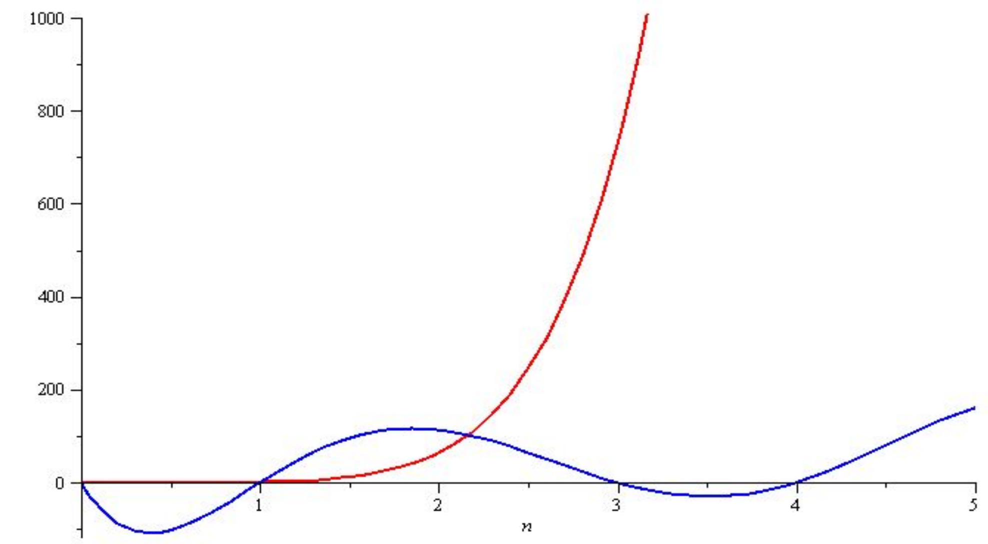


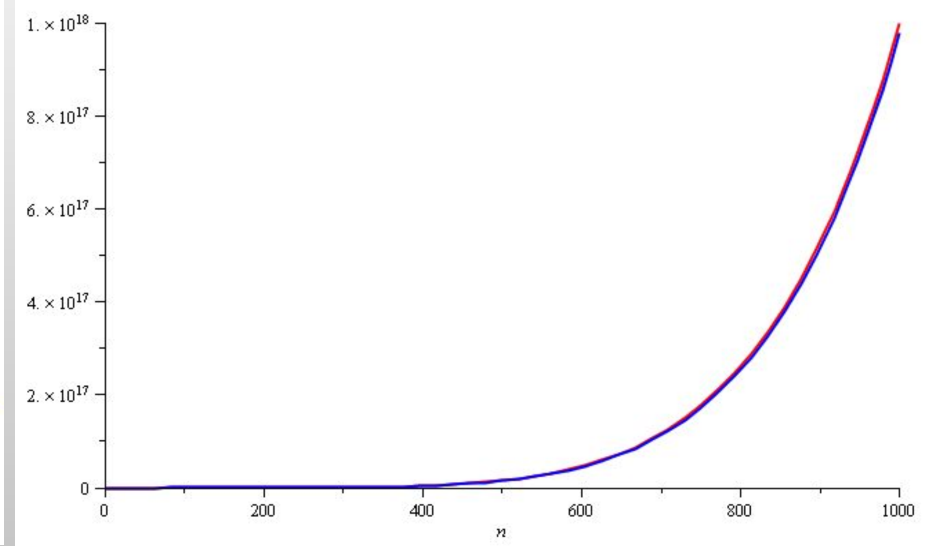


- Close to *n* = 0, they look very different
- Yet on the range *n* = [0, 1000], they are (relatively) indistinguishable.

Polynomial Growth

$f(n) = n^6$ $\quad\quad$ $g(n) = n^6 - 23n^5 + 19n^4 - 729n^3 + 126n^2 - 648n$





- Around $n = 0$, they look very different
- Still, around $n = 1000$, the relative difference is less than 3%

Running Time on Control Statement

Next we will look at the following control statements

- These are statements which *<u>potentially alter the execution of instructions</u>*
  - Conditional statements
    - `if, switch`
  - Condition-controlled loops
    - `for, while, do-while`

`if (condition) {`
`    // true body`
`}`
`else {`
`    // false body`
`}`
	Runtime of a conditional statement =
	The runtime of the condition (the test) +
	The runtime of the *<u>body</u>*

Condition Controlled Loops

```
for (int i=0; i<N; ++i) {
    // …
}
```

```
int i=0;                 // initialization
while ( i<N ) {          // condition
    // …
    ++i;                 // increment
}
```

- The initialization, condition-checking and increment statements are usually $\theta(1)$
- But repetitive condition checking increases overall cost !
- Assuming there are no break or return statements in the loop, the runtime is $\Omega(n)$

Condition Controlled Loops

```
/**
 * Cubic maximum contiguous subsequence sum algorithm.
 */
int maxSubSum1( const vector<int> & a )
{
    int maxSum = 0;

    for( int i = 0; i < a.size( ); ++i )
        for( int j = i; j < a.size( ); ++j )
        {
            int thisSum = 0;

            for( int k = i; k <= j; ++k )
                thisSum += a[ k ];

            if( thisSum > maxSum )
                maxSum = thisSum;
        }

    return maxSum;
}
```

$$\sum_{k=i}^{j} 1 = j - i + 1$$

Most Inner Loop

$$\sum_{j=i}^{N-1} (j - i + 1) = \frac{(N - i + 1)(N - i)}{2}$$

Middle Loop

$$\begin{aligned}
\sum_{i=0}^{N-1} \frac{(N-i+1)(N-i)}{2} &= \sum_{i=1}^{N} \frac{(N-i+1)(N-i+2)}{2} \\
&= \frac{1}{2}\sum_{i=1}^{N} i^2 - \left(N + \frac{3}{2}\right)\sum_{i=1}^{N} i + \frac{1}{2}(N^2 + 3N + 2)\sum_{i=1}^{N} 1 \\
&= \frac{1}{2}\frac{N(N+1)(2N+1)}{6} - \left(N + \frac{3}{2}\right)\frac{N(N+1)}{2} + \frac{N^2 + 3N + 2}{2}N \\
&= \frac{N^3 + 3N^2 + 2N}{6}
\end{aligned}$$

Outer Loop

Controlled Statements

<pre>switch (i) {
    case 1: /* do stuff */ break;
    case 2: /* do other stuff */ break;
    case 3: /* do even more stuff*/ break;
    case 4: /* tired yet? */ break;
    case 5: /* do stuff */ break;
    default: /* do default stuff */
}</pre>
	<pre>if (i==1){ /* do stuff */ }
	else if (i==2){ /* do other stuff */ }
	else if (i==3){ /* do even more stuff*/ }
	else if (i==4){ /* tired yet? */ }
	else if (i==5){ /* do stuff */ }
	else { /* do default stuff */}</pre>

- Switch statements appear to be nested if statements.
- Thus a switch statement would appear to run in $O(\#cases)$ time. (assuming each block takes $O(1)$ time to execute.

Cases

- As well as determining the runtime of an algorithm, if the data may not be deterministic, we may be interested in:
  - Best-case runtime
  - Average-case runtime
  - Worst-case runtime
- In many cases, they will be significantly different
- Example:
  - Searching a list linearly is simple enough

Cases

Searching a list linearly is simple enough.

We will count the number of comparisons

- Best case:
  - The first element is the one we're looking for: O(1)
- Worst case:
  - The last element is the one we're looking for, or it is not in the list: O(n)
- Average case?
  - We need some information about the list...

Assume the case we are looking for is in the list and equally likely distributed.
If the list is of size n, then there is a 1/n chance of it being in the ith location :

If summing the probabilities

$$\frac{1}{n}\sum_{i=1}^{n} i = \frac{1}{n}\frac{n(n+1)}{2} = \frac{n+1}{2}$$

So its O(n)

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