Literature Review

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1 Data Compression

1.1 Introduction

Data compression or source coding is the process of encoding information using fewer bits. There are two classes of data compression; lossless and lossy. Lossless data compression does not lose information in the process of encoding. Rather, it eliminates statistical redundancy. On the other hand, lossy data compression partially discards information such that the original data cannot be fully reconstructed. For this reason, lossy compression is also known as irreversible compression.

Data compression relies on the framework of information theory established fundamentally by Claude E. Shannon in "A Mathematical Theory of Communication" [1]. Information theory studies the transmission of digital information through communication systems. The communication system Shannon theorises is composed of five parts: the information source, transmitter, channel, receiver and destination. Depending on the information transmitted through the system, it is classified into three main categories; discrete, continuous and mixed.

In the discrete case, the information source can be represented by a stochastic process

$$\{X_t : t \in \mathbb{N}^+\}$$

with a discrete state space or alphabet of symbols S. Governed by this mathematical model, the information source produces a sequence of symbols from its alphabet which is received by the transmitter. If the probability of the transmitter observing the next symbol depends only on the current symbol and is conditionally independent of previous symbols, the process satisfies the Markov property. In this case, the information source can be represented by a Markov chain with a finite state space $S_1, S_2, \ldots, S_n$ and a

set of transition probabilities $p_i(j)$ defined as the probability that the system will transition from state S_i to S_j.

The entropy rate, H, of an information source is a measure of how much information it 'produces' per symbol on average. Or rather, how uncertain the transmitter is of the next symbol on average. For a discrete random variable X with possible symbols $s_1, s_2 \ldots, s_n$; the entropy is given by

$$H(X) = -\sum_{i=1}^{n} p_i \log_2 p_i$$

where p_i is the probability of observing the i-th symbol [1]. The units here are in bits. For an information source represented as a stochastic process, the entropy rate is given by

$$H(X) = \lim_{n \to \infty} \frac{1}{n} H(X_1, X_2, \ldots, X_n)$$

when the limit exists, where $H(X_1, X_2, \ldots, X_n)$ is the joint entropy of the first n members of the process [2]. The limit is known to exist for a stationary process where the joint probability distribution is invariant to shifts in time. For example, for a stationary Markov chain the entropy rate simplifies to

$$H(X) = -\sum_{i,j} \mu_i p_i(j) \log_2 p_i(j)$$

where μ is the stationary distribution of the chain.

A binary source code, C, of a random variable X is a mapping from its possible outcomes Ω to a set of finite length strings of symbols from $\{0, 1\}$ known as codewords. A code is described as a prefix code or instantaneous code when no codeword is a prefix of another. In such a case, any encoded string can be conveniently decoded without reference to future codewords. For uniquely decodable codes, the expected length of a codeword cannot be smaller than the entropy of X. That is,

$$\sum_{i} p_i l_i \geq H(X)$$

where $l_i = |C(s_i)|$ is the length of the i-th symbol's codeword. With equality possible if and only if the probability distribution is dyadic. That is, when $\forall i\ \exists n \in \mathbb{N}_0$ such that $p_i = 2^{-n}$. Similarly, the entropy rate also defines a lower limit on the expected number of bits per symbol for a source encoding of an information source represented as a stochastic process.

1.2 Entropy coding

Many lossless data compression methods have been proposed since Shannon's formative paper in 1948. Entropy coding is one such scheme that does not depend on the data's characteristics. It typically encodes each fixed-width symbol with a variable-length binary prefix code as described above. Fundamentally, it attempts to achieve a bit rate as close as possible to the entropy rate of the information source.

Huffman coding is a well-known entropy coding technique which minimises the expected length of prefix codes [3]. It solves the following optimisation problem:

$$\underset{l_i}{\text{minimise}} \sum_i p_i l_i$$
$$\text{subject to} \sum_i 2^{-l_i} \leq 1$$

where $l_i \in \mathbb{N}^+$. The inequality constraint is known as the Kraft-McMillan inequality and defines a necessary condition for the existence of a uniquely decodable binary code [4]. The Huffman coding algorithm is a greedy algorithm which constructs a binary tree from the bottom-up. Recursively, it takes the two symbols with the lowest probabilities and constructs a new node with these two as its children. This new node is considered to be a new symbol with the combined probability of its children. Once the tree is constructed, the edges to the left are labelled as 0 and those to the right as 1. A hash table mapping symbols to binary codewords is then constructed by traversing the tree from the root to each leaf node and concatenating the edge labels along the path taken. The time complexity for this algorithm is $O(n \log n)$ or $O(n)$ if the probabilities are already sorted, where n is the number of symbols [5].

However, perhaps a more appropriate measure of time complexity is the number of passes over an input string of length m. In this case, Huffman coding takes $O(m)$ time as it requires one pass if the probability distribution is already known. Otherwise, the probability distribution can be determined by a preliminary pass over the input. Or instead, an adaptive Huffman coding algorithm such as the Faller-Gallager-Knuth (FGK) or Vitter algorithm can be used [6, 7]. In the latter cases, the hash table or Huffman tree must also be encoded in the output.

Among all source codes when the probability distribution of the alphabet is known beforehand, Huffman coding is indeed optimal. However, once these restrictions are relaxed, other techniques prove to be more compressible. In particular, Huffman coding tends to be inefficient on small alphabets such

as the binary numbers $\{0, 1\}$ wherein no compression is possible without an alphabet extension [8]. Similarly, if the probability distribution is not dyadic, other non–source-coding techniques are more optimal.

Arithmetic coding is one such entropy coding technique which encodes the input string as a fraction in the range $[0, 1)$ [8]. The starting interval $[0, 1)$ is divided into sub-intervals corresponding to each symbol in the alphabet with the length of each sub-interval equal to its symbol's expected probability of occurrence. Each time a symbol is read from the input string its sub-interval is chosen and divided as before. Recursively, this process is repeated until the final symbol is read, at which point any fraction in the final interval can be returned. Arithmetic coding overcomes the weaknesses of Huffman coding described above and requires the same number of passes over the input. However, in practice it is more difficult to implement and face issues of computational accuracy. Range coding is an equivalent technique with a slightly different implementation [9].

1.3 Dictionary coding

Dictionary coding is another lossless compression scheme which searches in the input data for strings stored in a dictionary and substitutes matches with a reference to their location in the dictionary. There are two main classes of dictionary coders; ones that use a static predetermined dictionary and others which dynamically update their dictionary.

Byte-pair encoding falls under the second class and recursively replaces the most frequently occurring pair of adjacent bytes with a new byte not found in the input data [10]. Despite being so simple, Gage claims it is on par with the Lempel, Ziv and Welch method [11]. It is however quite slow at compression, requiring many passes of the data, but takes only one pass for decompression.

Lempel-Ziv coding is another dictionary coding technique which is widely used in practice and has many variants. LZ77 is the first such algorithm to be introduced in the literature and replaces repeated occurrences of substrings with references to their original location in the input [12]. It maintains a fixed-width sliding window represented as a circular buffer in order to keep track of the most recent data whilst keeping the algorithm's complexity under control. Each match is encoded by the substring's length and the distance to its original occurrence. LZ78 is very similar but instead constructs an explicit dictionary of substrings and replaces repeated occurrences with references to their location in the dictionary [13]. Both variants require one pass over the input string and have been proven to be asymptotically optimal as the length of the string grows to infinity.

1.4 Other encodings

There are many other lossless data compression schemes described in the literature. Run-length encoding is one such method which encodes substrings consisting of repeated consecutive symbols, known as *runs*, with their repeated symbol and length. First employed in 1967 for transmission of analogue television signals, run-length encoding proves beneficial when there are many runs, especially of long length [14]. Unfortunately, nanopore signal data does not contain many such runs and would likely encode poorly under this method.

Burrows-Wheeler transform is a data compression preprocessing step used to rearrange data in order to increase its number of runs [15]. It is easily reversible, such that the original untransformed data can be obtained from the Burrows-Wheeler transformed data. It has been used to great effect in bioinformatics to compress to basecalled genomic data in the form of FASTQ files [16].

Stream VByte is a specialised codec for compressing 32-bit unsigned integers [17]. It stores each integer using a variable number of bytes (1 to 4) depending on its size. For an integer in the interval $[2^{n-1}, 2^{n+7})$ only n bytes are required to represent it. The number of bytes used for each integer is stored in an array of control bytes which preface the actual data. 2-bit words are used to store the number of bytes used for each integer with 00, 01, 10 and 11 corresponding to 1, 2, 3 and 4 bytes respectively. This codec is currently incorporated in the state-of-the-art approach to compressing nanopore signal data known as *VBZ* [18]. The approach consists of

1. taking the integer differences between successive nanopore data points;

2. converting the signed differences to unsigned integers;

3. applying Stream VByte with 0 to 3 bytes (rather than the canonical 1 to 4); and

4. compressing using the Zstandard format [19].

This requires at least four passes over the input data depending on how many passes Zstandard performs.

2 Nanopore Signal Data

The primary interest of bioinformatics is to understand biological data. This is typically achieved through the development of software tools. Nanopore

sequencing is one approach employed in bioinformatics to determining the primary structure of biopolymers such as DNA and RNA. It involves recording the ionic current as a biopolymer passes through a pore of nanometer size with a voltage applied to the membrane. The resulting current signal is then used to determine the structure of the biopolymer. Oxford Nanopore is the leading producer of nanopore sequencing machines. The signal recorded by such machines is digitised and given as a sequence of 16-bit signed integers. This sequence, hereafter referred to as *nanopore signal data*, is the focus of this thesis.

There have been many studies investigating genomic data compression [20]. Few, however, have successfully focussed on losslessly compressing the signal data produced directly from nanopore sequencing. One tool called Picopore does not produce any novel insights, but rather, increases the level of gzip compression to the highest possible [21]. VBZ, described in section 1.4, is the current state-of-the-art encoding introduced by Oxford Nanopore in 2019.

Rather interestingly, there has been some recent research effects to investigate lossy compression of nanopore signal data. Chandak et. al. found that lossy time-series compressors (LFZip and SZ) tend to have a very small impact on the downstream analysis of nanopore data [22, 23]. In particular, after reducing the input size by 35-50%, basecalling and consensus accuracy is reduced by less than 0.2% and 0.002% respectively. However, they seem to overlook the prospect of a more efficient lossless compression technique for nanopore signal data:

> "obtaining further improvements in lossless compression (to VBZ) is challenging due to the inherently noisy nature of the current measurements" [22].

For this reason and perhaps due to the novel nature of nanopore sequencing, little has been further attempted in the literature to losslessly compressing nanopore signal data.

Fortunately, signal data similar in form to nanopore signal data is commonly recorded and extensively researched. Some examples include electrograms of the brain (EEG) and heart (ECG); seismic and radio waves; telemetry data from astronomy; and sonar signals. MCDRC is a deep learning approach to losslessly compressing sensor signal data based on a recurrent neural network architecture known as a multi-channel recurrent unit [24]. Its results are quite promising, outperforming existing techniques such as BSC, PAQ and CMIX.

A timeline of the major events in the data compression and nanopore sequencing literature is presented in Table 1.

Table 1: A timeline of the major events in the literature.

1948	Shannon publishes "A Mathematical Theory of Communication" [1]
1952	Huffman publishes a method for finding optimal prefix codes [3]
1967	Run-length encoding first described [14]
1976	Pasco and Rissanen develop arithmetic coding independently [8]
1977	Lempel and Ziv publish LZ77 [12]
1978	Lempel and Ziv improve upon LZ77 with LZ78 [13]
1979	Martin proposes range coding [9]
1984	Welch publishes LZW based on LZ78 [11]
1987	Vitter improves upon the adaptive Huffman coding algorithm FGK [7]
1994	Burrows and Wheeler publish the Burrows-Wheeler transform [15]; and Gage submits byte-pair encoding as a C article [10]
2014	Oxford Nanopore release the first portable nanopore sequencing device
2017	A tool for reducing nanopore data, Picopore, is released [21]
2018	Lemire publishes Stream VByte [17]
2019	VBZ is first released
2020	Lossy compression of nanopore signal data is investigated [22]
2021	MCDRC is published [24]

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