1) $m(a+bX) = \frac{1}{N}\sum_{i=1}^{N}(\underbrace{a}_{\text{constant}}) + \frac{1}{N}\sum_{i=1}^{N}(\underbrace{b}_{\text{constant}}x_i)$

$$= a + b\frac{1}{N}\sum_{i=1}^{N} x_i = a + b\,m(x) \checkmark$$

2) $cov(X, a+bY) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))((a+by_i) - m(a+bY))$

$$= \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))(\cancel{a} + by_i - \cancel{a} - b\,m(y))$$

$$= \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))(b)(y_i - m(y)) = b \cdot \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))(y_i - m(y))$$

$$= b \cdot cov(X, y) \checkmark$$

3) $cov(X, x) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))(x_i - m(X)) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))^2 = s^2 \checkmark$

$$cov(a+bX, a+bX) = \frac{1}{N}\sum_{i=1}^{N}((a+bx_i) - m(a+bX))((a+bx_i) - m(a+bX))$$

$$= \frac{1}{N}\sum_{i=1}^{N}(\cancel{a} + bx_i - \cancel{a} - b\,m(x))(\cancel{a} + bx_i - \cancel{a} - b\,m(x))$$

$$= \frac{1}{N}\sum_{i=1}^{N} b(x_i - m(x))\,b(x_i - m(x))$$

$$= b^2 \cdot \frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))^2 = b^2 cov(x, x) \checkmark$$

4) since the transformation is non-decreasing, the order of the values stays the same through the transformation. This means a non-decreasing transformation of the median is the median of the transformed variable This also applies for quantiles, as order is preserved. This does NOT apply for the IQR and range because the transformation can alter the spread of values while still being non-decreasing.

5) $m(g(x)) = g(m(x))$ only for linear transformations.