

HW4

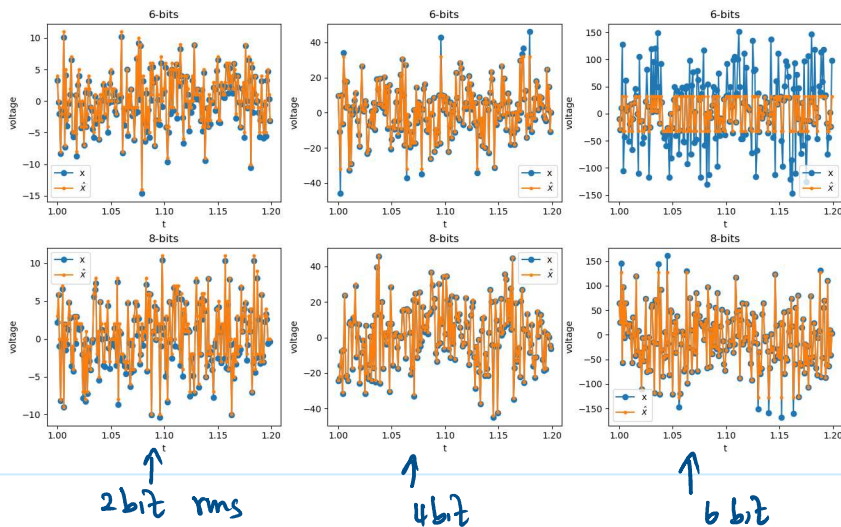
Thursday, February 16, 2023 8:56 AM

Q2

- Simulate the effects of sampling
- quantize with 6/8 bits.
 - Input gaussian noise

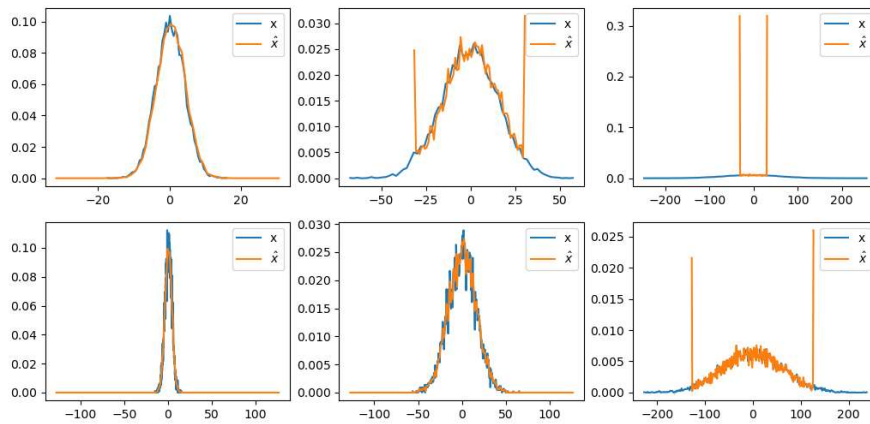
$$RMS = \sqrt{\frac{\sum x_i^2}{n}}$$

RMS to 2 bits $\rightarrow 2^2 = 4$ levels



Notes on selecting 6/8 bits.

- for the case 6 bit rms, 8 bit is needed. if 6 bit is chosen the signal is clipping as shown in the top right plot



← 6-bit ADC

← 8-bit ADC

Noise: \uparrow 2 bits \uparrow 4 bits \uparrow 8 bits \uparrow

the shift of 0.5 is due to the convention
for book use where if total no of
bins are even then use

and $n \in [m, m+1)$ as bins.
 $\hat{n} = m + 1/2$

- for the original signal using auto bins to show the full histogram

Note on the clipping I see. its odd to see a high
peak only at the minimum value. → Fixed this.

Clipping percentage

	ADC bits	
	6	8
2	0	0
4	5	0
6	62.4	4.2

\uparrow
this is expected

Correct samples

a. Not clear what this exactly means.
out of how many points set to max and min
or actually in that range

	ADC bits	
	6	8
2	—	—
4	14.76%	—
6	1.76%	3.67%

Some don't show clipping when the RMS is low compared to the total number of ADC bits used.

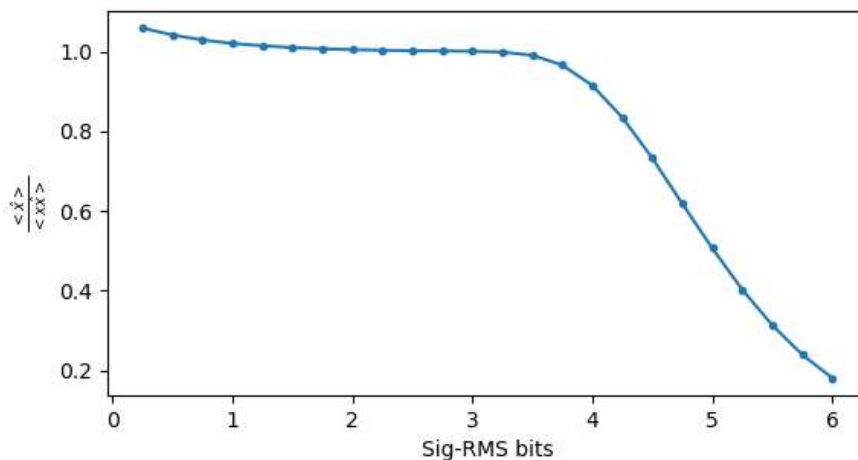
Variation of $\frac{\langle \hat{x} \rangle}{\langle \hat{x}' \rangle} \rightarrow$ with rms of input signal.

$$\text{bits } \frac{1}{4} = 2^{-2}$$

$$\frac{1}{2} = 2^{-1}$$

$$1 = 2^0 + 2^{-2} \quad 1 + \frac{1}{4}$$

$$2 = 2^1$$



Q1 ADC Theory (using the book version)
assume 6 bits.

Increase of variance from
2 bits \rightarrow 5 bits.

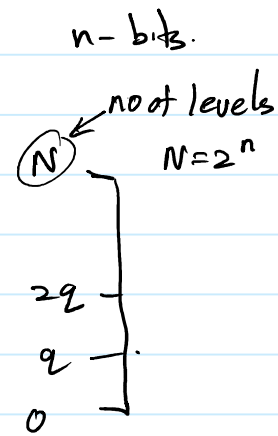
using the form given in the book
for even number of levels (N)

$$= \frac{2/\pi \left(1/2 + \sum_{m=1}^{N-1} e^{-m^2 \epsilon^2 / 2} \right)^2}{(N-1/2) - 2 \sum_{m=1}^{N-1} m \operatorname{erf}\left(\frac{m\epsilon}{\sqrt{2}}\right)}$$

ϵ - spacing between the levels.

from the formula we calculate

$$\frac{\langle \hat{x} \rangle}{\langle \hat{x} \hat{x} \rangle} = \frac{(N-1/2)^2 - 2 \sum_{n=1}^{N-1} n \operatorname{erf}\left(\frac{nq}{\sqrt{2}}\right)}{2/\pi \left(\sum_{n=1}^{N-1} e^{-n^2 q^2 / 2} + 1/2 \right)^2}$$



In the derivation we did we assumed the rms of the incoming signal will be one.
and $q=1$ (level spacing)

To modify the rms to given number of bits we need to stretch that out.

$$\Rightarrow q = 2^Q \quad Q - \text{no of bits in the rms.}$$

In the derivation

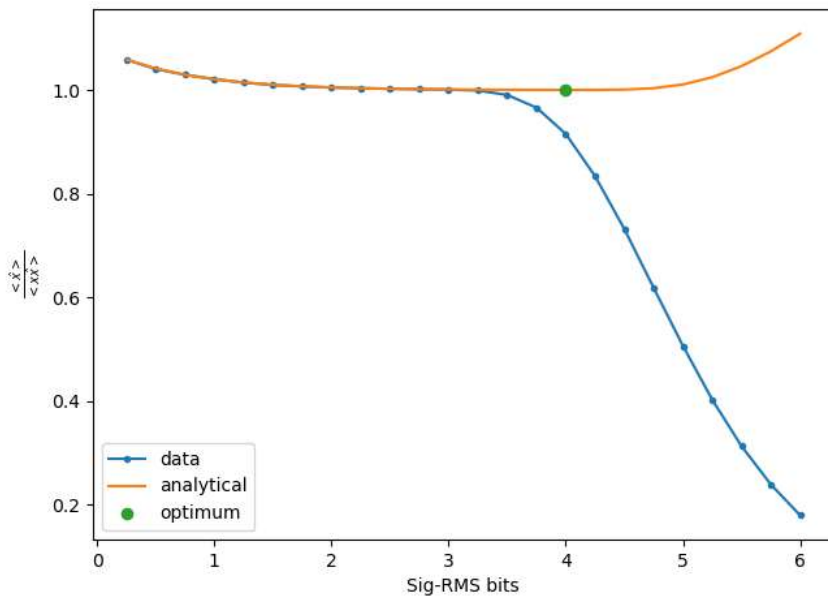
$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad \begin{matrix} \mu=0 \\ \sigma=1 \end{matrix} \quad \begin{matrix} 1 = 2 \\ 0 = -2 \end{matrix}$$

$$= \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

to change the incoming signal now

I can change q to match the scale

$$q = \frac{1}{2^{k-1}}$$



Code used for the activity: [Git/HW4](#)

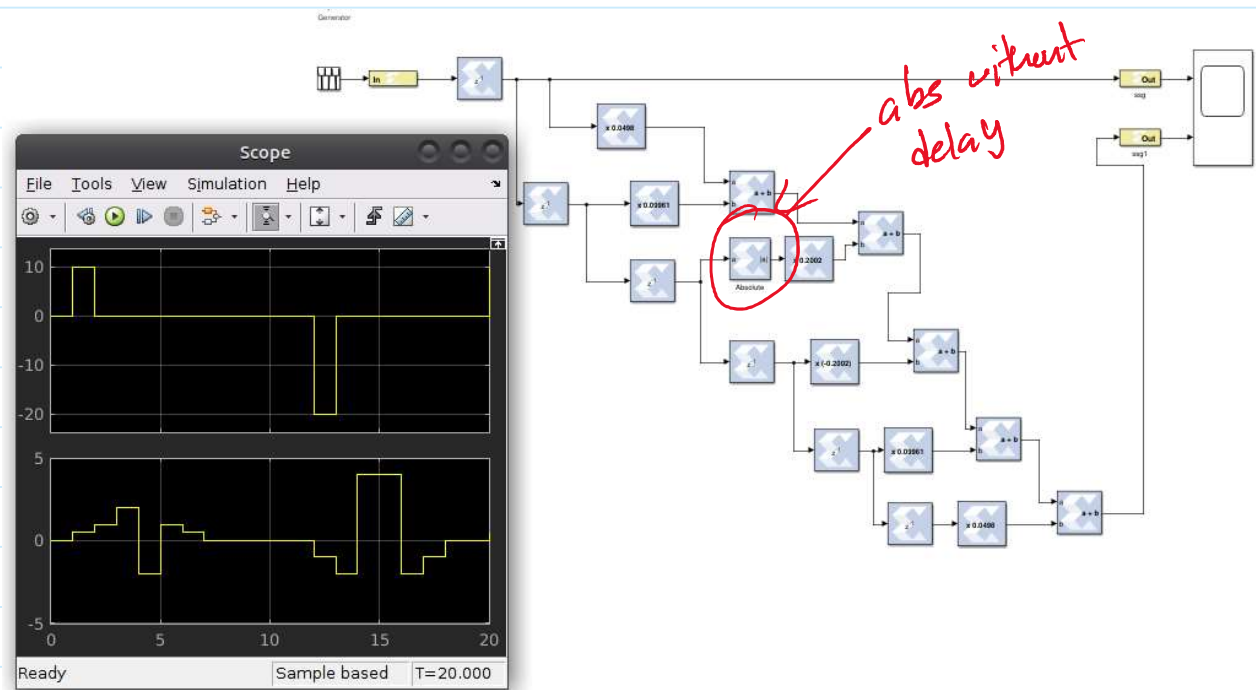
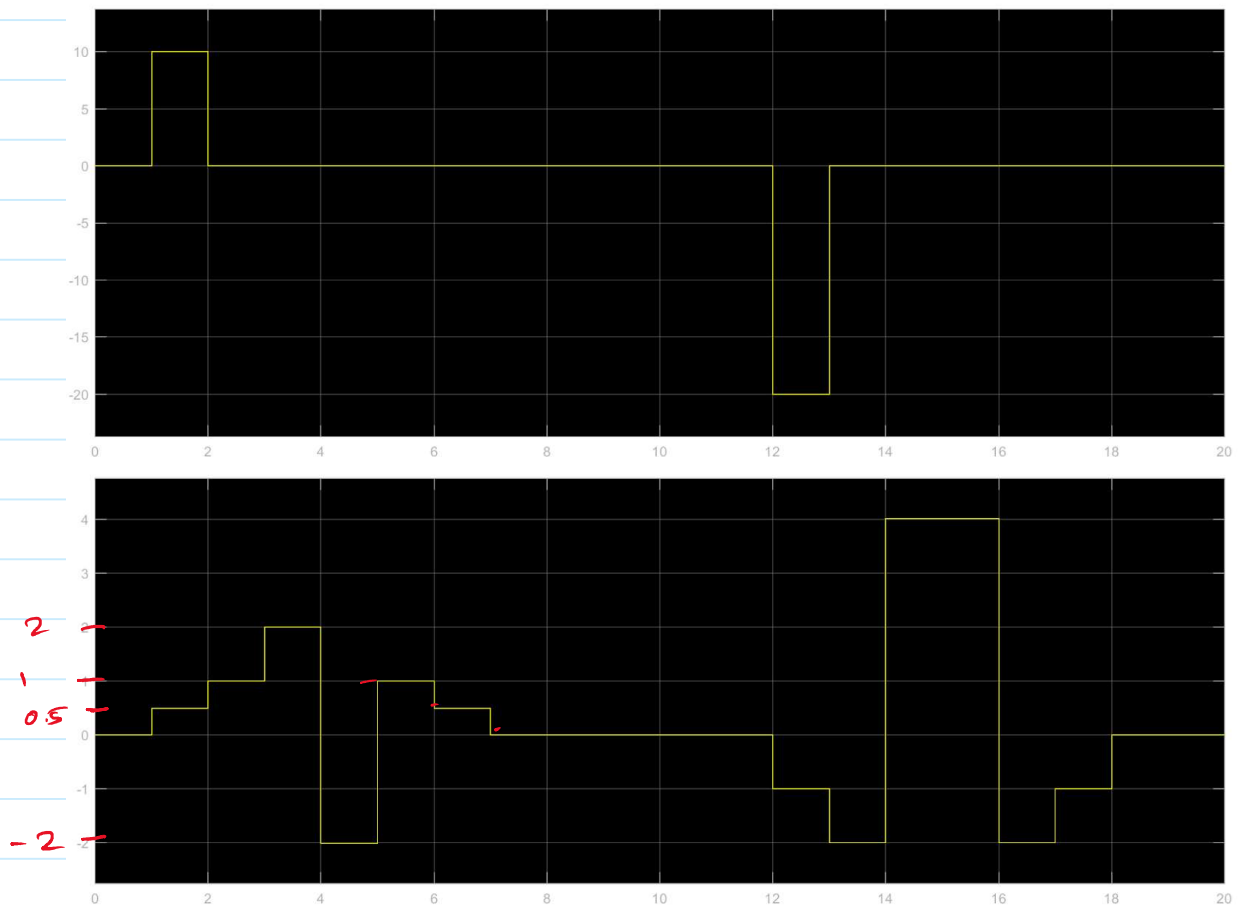
the simulated vs the calculated value matches well for up to 3 rms bits. as it increases beyond that it deviates. The likely reason for this is the assumption we made $\hat{x} = x - \alpha \hat{x}$ is not valid.

If you are using a $\sin()$ signal the optimal signal would be a signal with $V_{\max} = 6$ bits.
No clipping in such a case.

Q3

- Basic project given works well:

- figuring out the transformation. for the given question



Non linear response.

