International conference

Multiscale and high-performance

For multiphysical problems

Numerical Calculation of Spectral Problems in SP_3 Approximation by FEM

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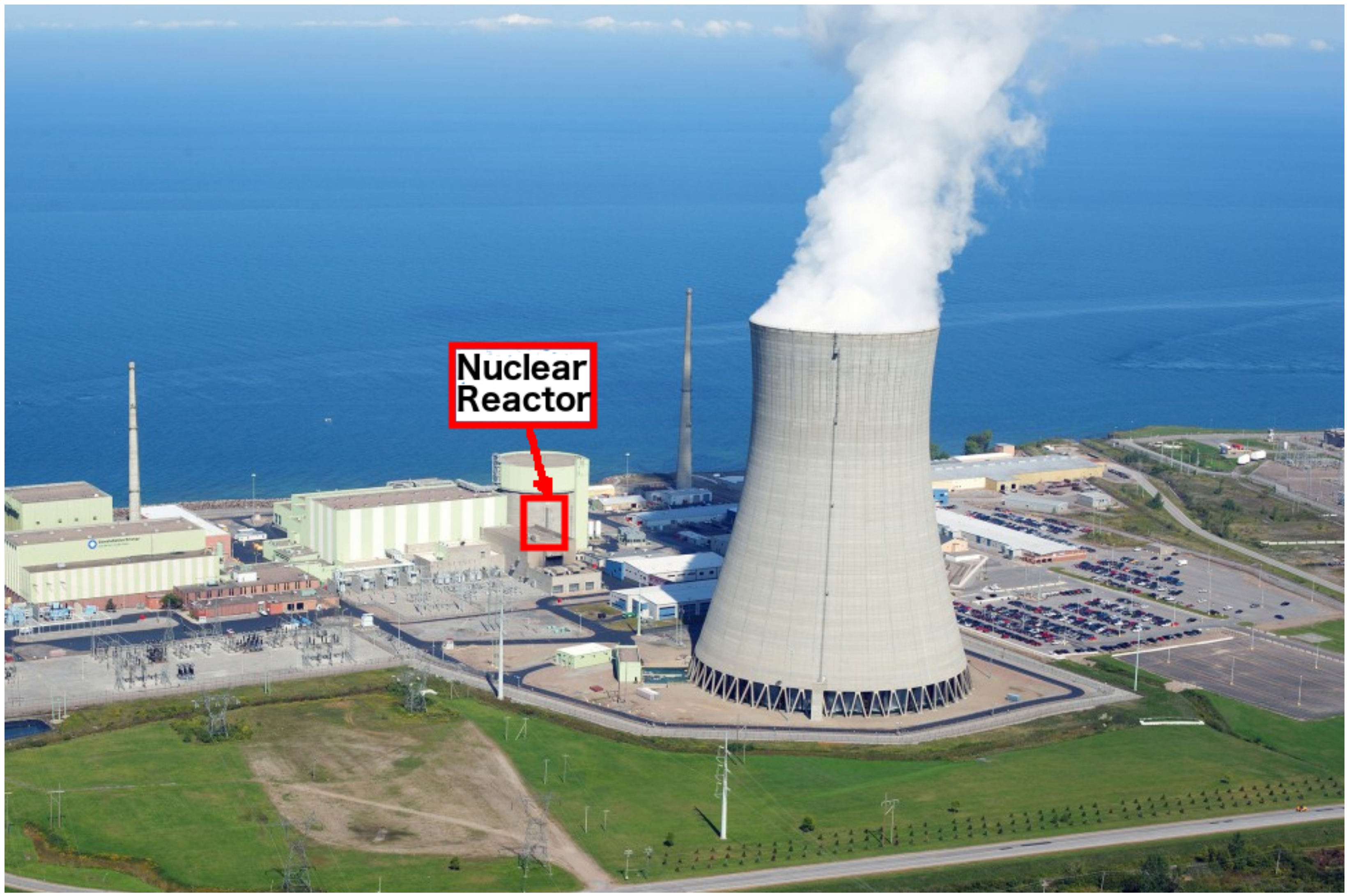
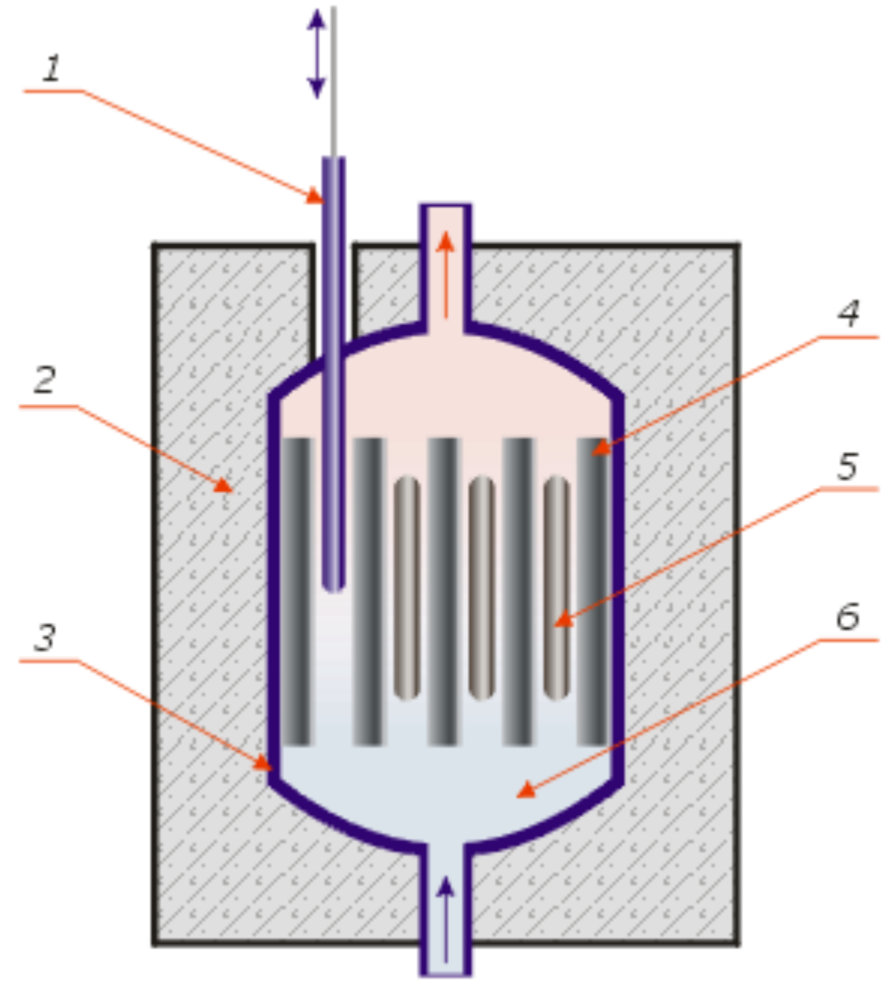
Motivation



Nuclear power plant



Active zone

1 - Control rods
2,3 - Protection systems
4 - Moderator
5 - Fuel
6 - Coolant

- Neutron flux
- Neutron transport equation
    - time, energy, spatial and angular variables (7 unknowns)

Practical calculation

- Diffusion approximation
- SP_3 approximation

Equations

Diffusion approximation

ϕ_g - neutron flux

$$\frac{1}{v_g}\frac{\partial \phi_g}{\partial t} - \nabla \cdot D_g \nabla \phi_g + \Sigma_{rg}\phi_g = (1-\beta)\chi_g S_n + S_{s,g} + \widetilde{\chi}_g S_d, \quad G = 1,2,\ldots,G,$$

$$S_n = \sum_{g'=1}^{G} \nu\Sigma_{f,g'}\phi_{g'}, \quad S_{s,g} = \sum_{g\neq g'=1}^{G} \Sigma_{s,g'\to g}\phi_{g'}, \quad S_d = \sum_{m=1}^{M} \lambda_m c_m .$$

SP3 approximation

$\phi_{0,g} = \phi_g + 2\phi_{2,g}$

- pseudo 0th moment of angular flux
- second moment of angular flux

$$\frac{1}{v_g}\frac{\partial \phi_{0,g}}{\partial t} - \frac{2}{v_g}\frac{\partial \phi_{2,g}}{\partial t} - \nabla \cdot D_{0,g}\nabla \phi_{0,g} + \Sigma_{r,g}\phi_{0,g} - 2\Sigma_{r,g}\phi_{2,g} = (1-\beta)\chi_{n,g}S_n + S_{s,g} + \chi_{d,g}S_d,$$

$$-\frac{2}{v_g}\frac{\partial \phi_{0,g}}{\partial t} + \frac{9}{v_g}\frac{\partial \phi_{2,g}}{\partial t} - \nabla \cdot D_{2,g}\nabla \phi_{2,g} + (5\Sigma_{t,g} + 4\Sigma_{r,g})\phi_{2,g} - 2\Sigma_{r,g}\phi_{0,g} = -2(1-\beta)\chi_{n,g}S_n - 2S_{s,g} - 2\chi_{d,g}S_d .$$

Delayed neutron source

$$\frac{\partial c_m}{\partial t} + \lambda_m c_m = \beta_m S_n, \quad m = 1,2,\ldots,M .$$

Boundary conditions

Albedo type condition

Diffusion approximation

$$D_g \frac{\partial \phi_g}{\partial n} + \gamma_g \phi_g = 0, \qquad g = 1,2,\dots,G,$$

where n *is* outer normal to the boundary.

Marshak type condition

SP3 approximation

$$\begin{bmatrix} J_{0,g}(\mathbf{x}) \\ J_{2,g}(\mathbf{x}) \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{3}{8} \\ -\frac{3}{8} & \frac{21}{8} \end{bmatrix} \begin{bmatrix} \phi_{0,g}(\mathbf{x}) \\ \phi_{2,g}(\mathbf{x}) \end{bmatrix}, \quad J_{i,g}(\mathbf{x}) = -D_{i,g} \nabla \phi_{i,g}(\mathbf{x}), \quad i = 0,2.$$

Initial conditions

$$\phi_g(\mathbf{x},0) = \phi_g^0(\mathbf{x}), \quad g = 1,2,\dots,G, \quad c_m(\mathbf{x},0) = c_m^0(\mathbf{x}), \quad m = 1,2,\dots,M.$$

Operator notation

Diffusion approximation

$$\mathbf{u} = \{\phi_1, \phi_2, \ldots, \phi_G\}, \quad \mathbf{c} = \{c_1, c_2, \ldots, c_M\}$$

$$V\frac{\partial \mathbf{u}}{\partial t} + A\mathbf{u} = (1-\beta)F\mathbf{u} + E\mathbf{c}, \qquad \frac{\partial \mathbf{c}}{\partial t} + \Lambda\mathbf{c} = Q\mathbf{u}\,.$$

SP_3 approximation

$$\mathbf{u_1} = \{\phi_{0,1}, \phi_{0,2}, \cdots, \phi_{0,G}\}, \quad \mathbf{u_2} = \{\phi_{2,1}, \phi_{2,2}, \cdots, \phi_{2,G}\}\,.$$

$$V(\frac{\partial \mathbf{u_1}}{\partial t} - 2\frac{\partial \mathbf{u_2}}{\partial t}) + A_1\mathbf{u_1} + B\mathbf{u_2} = (1-\beta)F(\mathbf{u_1} - 2\mathbf{u_2}) + E\mathbf{c},$$

$$V(-2\frac{\partial \mathbf{u_1}}{\partial t} + 9\frac{\partial \mathbf{u_2}}{\partial t}) + A_2\mathbf{u_2} + B\mathbf{u_1} = -2(1-\beta)F(\mathbf{u_1} - 2\mathbf{u_2}) - 2E\mathbf{c},$$

$$\frac{\partial \mathbf{c}}{\partial t} + \Lambda\mathbf{c} = Q(\mathbf{u_1} - 2\mathbf{u_2})\,.$$

Operator are responsible: right - generation, left - loss

Spectral problems

Diffusion approximation | **SP_3 approximation**

Lambda-

$$A\mathbf{y} = \lambda^{(k)} F\mathbf{y} \qquad\qquad L\varphi = \lambda^{(k)} M\varphi$$

$k_{eff} = 1/\lambda_1^{(k)}$ - the effective multiplication factor

Alpha-

$$A\mathbf{y} - (1-\beta)F\mathbf{y} - E\mathbf{s} = \lambda^{(\alpha)} V\mathbf{y} \qquad\qquad L\varphi - (1-\beta)M\varphi - I\mathbf{s} = \lambda^{(\alpha)} W\varphi,$$

$$\Lambda\mathbf{s} - Q\mathbf{y} = \lambda^{(\alpha)}\mathbf{y} \qquad\qquad \Lambda\mathbf{s} - R\varphi = \lambda^{(\alpha)}\mathbf{s}\,.$$

$\alpha = \lambda_1^{(\alpha)}$ - the period eigenvalue

$$\varphi = \{\mathbf{y_1}, \mathbf{y_2}\}, \quad L = \begin{pmatrix} A_1 & B \\ B & A_2 \end{pmatrix}, \quad M = \begin{pmatrix} F & -2F \\ -2F & 4F \end{pmatrix}, \quad I = \begin{pmatrix} E \\ -2E \end{pmatrix}, \quad R = \begin{pmatrix} Q & -2Q \end{pmatrix}, \quad W = \begin{pmatrix} V & -2V \\ -2V & 9V \end{pmatrix}$$

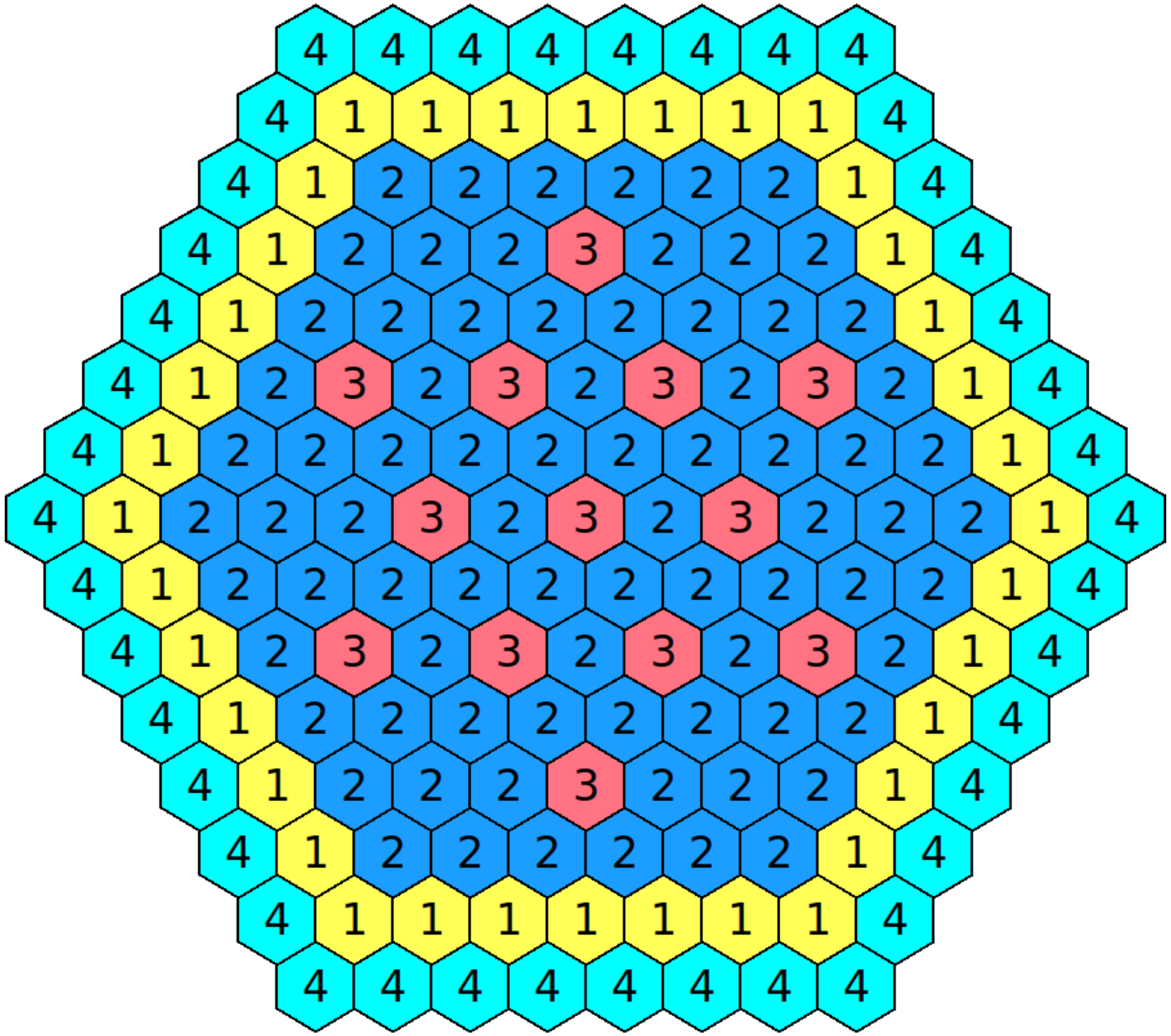
Software

FEniCS PROJECT

SLEPc

python™

IAEA-2D with reflector



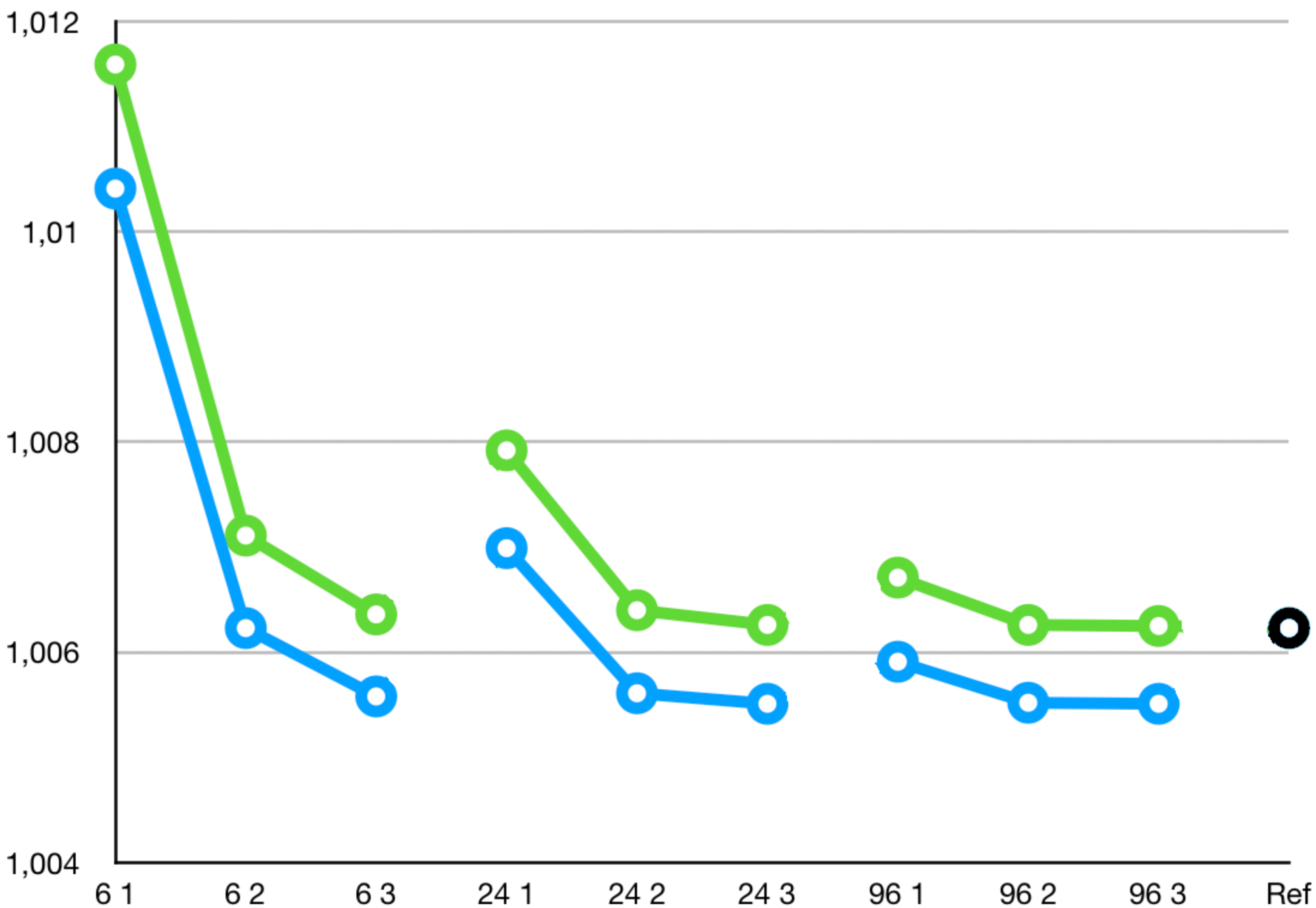
2 groups of prompt (G=2) and
1 group of delayed (M=1) neutrons

Lambda- and alpha- spectral problems

Varied:
n - number of triangles per assembly
p - order of finite element

Solution of lambda modes spectral problem

Fig: The effective multiplication factor



green - SP_3 model
blue - diffusion model

Ref: MCNP4C code (Bahabadi M.H. at el 2016 Annals of Nuclear Energy **98** 74-80)

Solution of lambda modes spectral problem

Table: The effective multiplication factor

n	p	k_{dif}	Δ_{dif}	δ_{dif}	t	k_{sp_3}	Δ_{sp_3}	δ_{sp_3}	t
6	1	1.01041	418	14.34	0.01	1.01159	536	14.14	0.02
	2	1.00623	0	1.95	0.04	1.00711	88	2.19	0.14
	3	1.00558	65	0.70	0.10	1.00636	13	0.35	0.47
24	1	1.00699	76	4.82	0.03	1.00792	169	4.96	0.13
	2	1.00561	62	0.77	0.21	1.00640	17	0.42	0.96
	3	1.00551	72	0.70	0.56	1.00626	3	0.17	2.97
96	1	1.00591	32	1.47	0.19	1.00671	48	1.42	0.85
	2	1.00552	71	0.73	1.14	1.00626	3	0.18	6.37
	3	1.00551	72	0.72	3.49	1.00625	2	0.18	20.34
Ref.		1.00623				1.00623			

Ref: MCNP4C code (Bahabadi M.H. at el 2016 Annals of Nuclear Energy **98** 74-80)

$$P = a \int_{\Omega} \sum_{i=1}^{G} \Sigma_{f,g} \phi_g d\mathbf{x}$$

Solution of lambda modes spectral problem

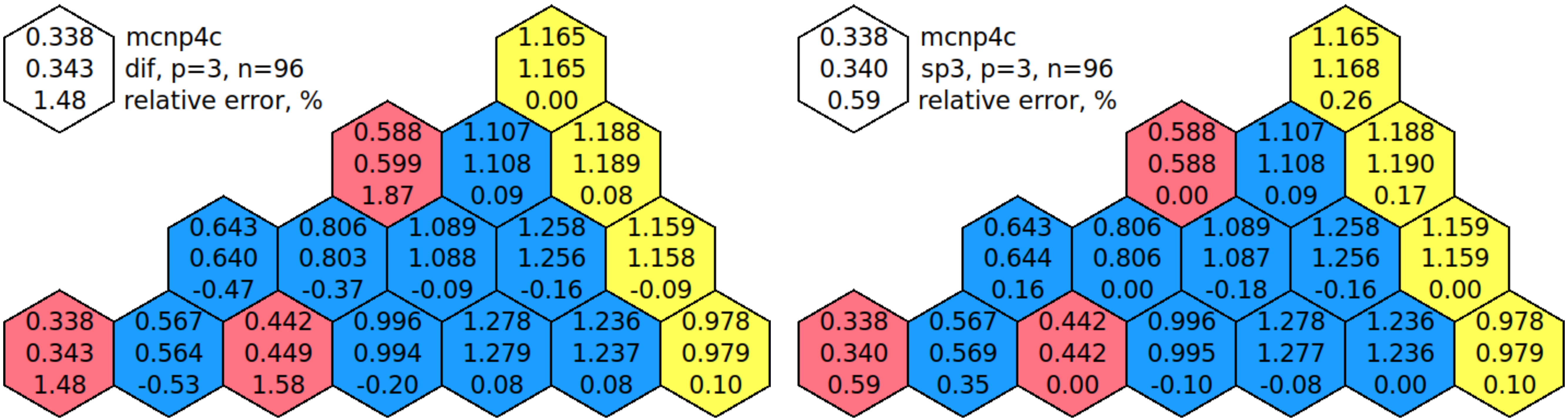


Fig: Power and error distributions using diffusion (left) and SP_3 (right) models

Solution of lambda modes spectral problem

Table: The first 10 eigenvalues for p=3, n=96

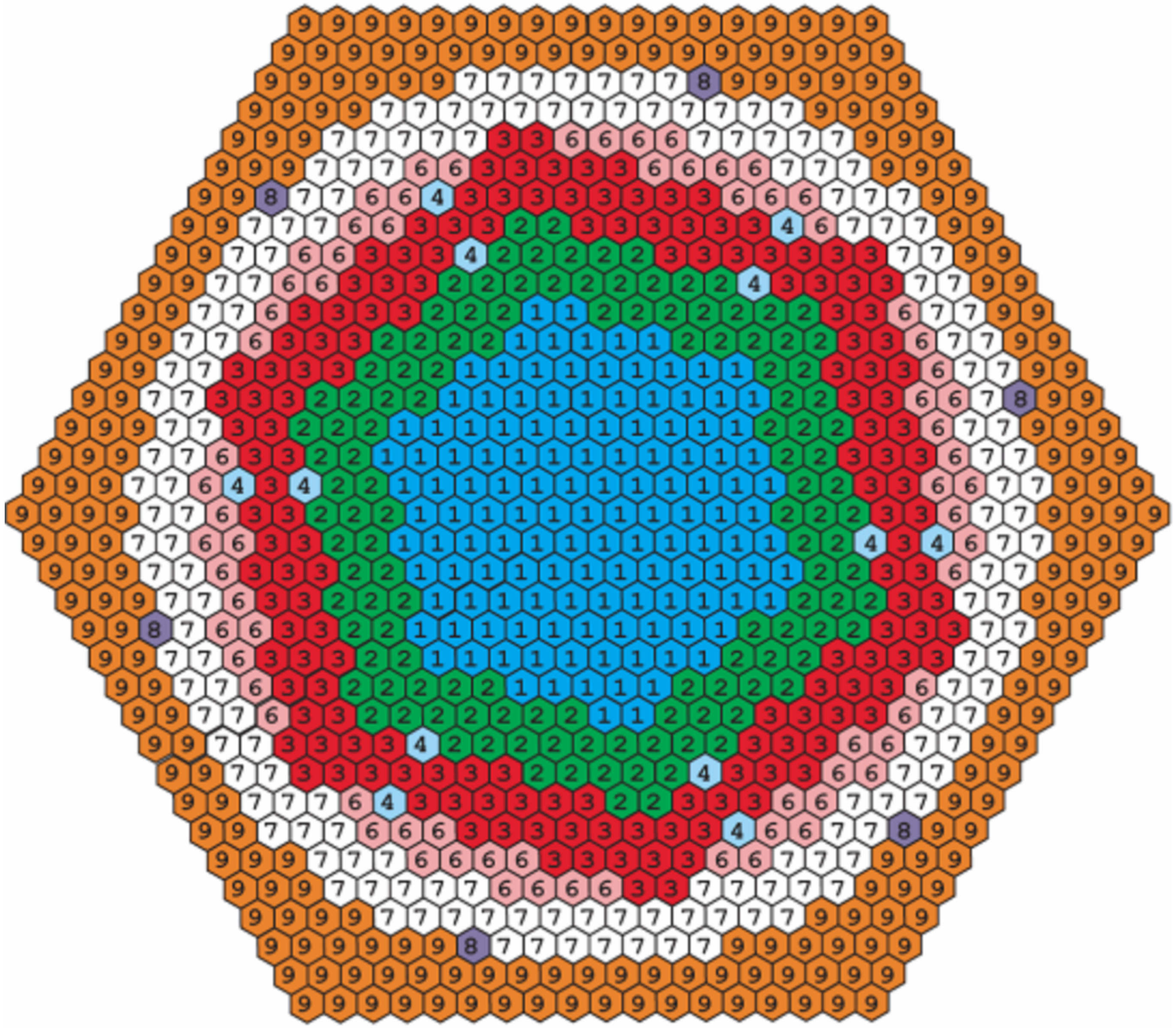
i	Diffusion	SP_3
1	1.005509678491	1.006244515310
2	0.996489969484	0.997253707156
3	0.996489969416	0.997253707121
4	0.976790579591	0.977758816863
5	0.976790579352	0.977758816817
6	0.958683528959	0.959895076934
7	0.928979605001	0.930969293068
8	0.924186320247	0.925931354406
9	0.904788471277	0.907348873881
10	0.904788471274	0.907348873799

Solution of alpha modes spectral problem

Table: The first 10 eigenvalues for p=3, n=96

i	Diffusion	SP_3
1	−0.418414021	−1.337480417
2	0.028108057	0.023799162
3	0.028108075	0.023804256
4	0.062814035	0.062218273
5	0.062814041	0.062220971
6	0.069514636	0.069228497
7	0.073730817	0.073541211
8	0.074126208	0.073987939
9	0.075346220	0.075210662
10	0.076266017	0.076175218

HWR test problem



2 groups of prompt (G=2) neutrons

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Varied:

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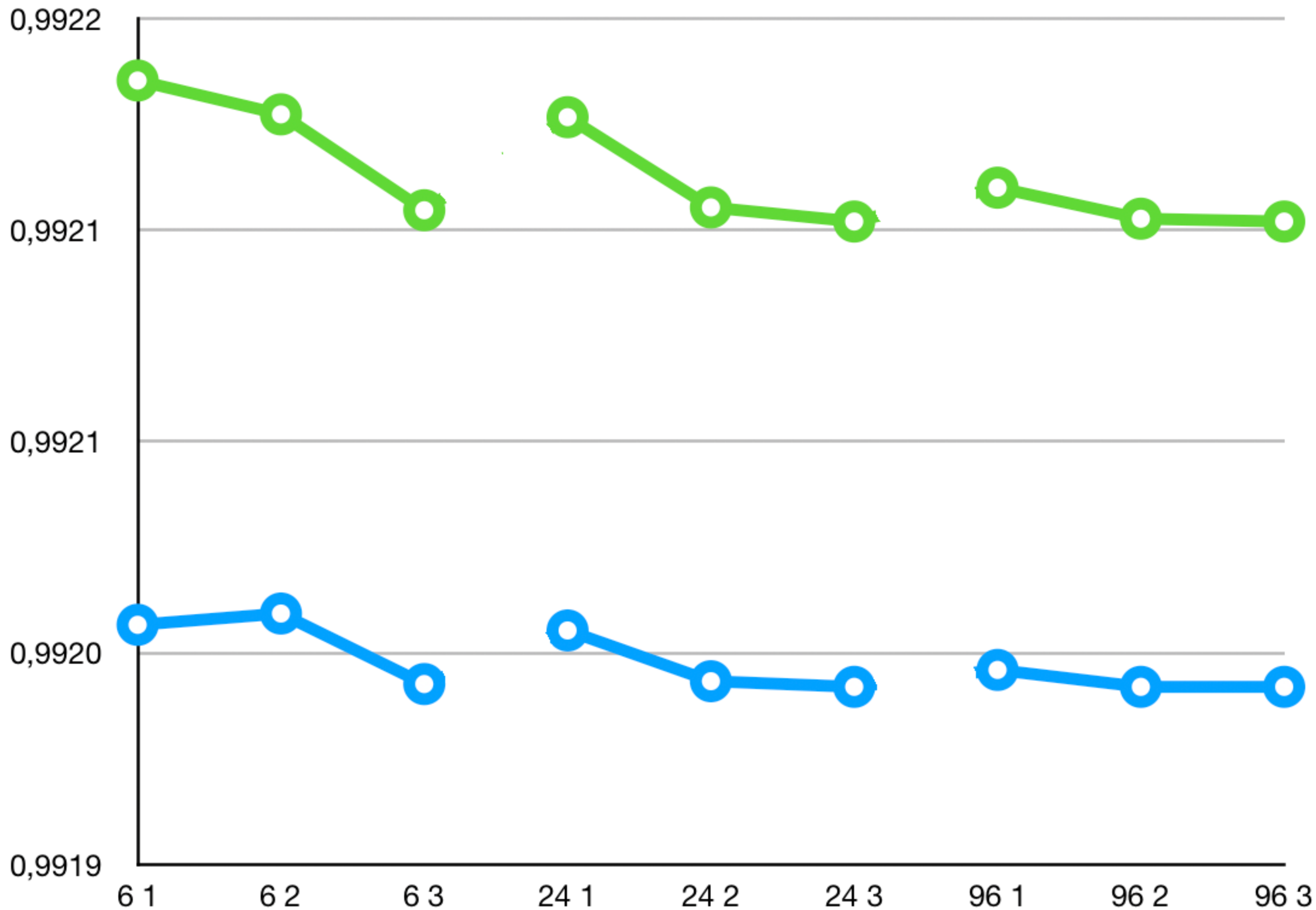
Solution of lambda modes spectral problem

Fig: The effective multiplication factor



green - SP_3 model
blue - diffusion model

Solution of lambda modes spectral problem

Table: The first 10 eigenvalues for p=3, n=96

i	diffusion	SP_3
1	0.991963	0.992128
2	0.983594 + 1.1645e-05i	0.983793 + 1.2072e-05i
3	0.983594 − 1.1645e-05i	0.983793 − 1.2072e-05i
4	0.964240 + 2.1564e-05i	0.964523 + 2.2337e-05i
5	0.964240 − 2.1564e-05i	0.964523 − 2.2337e-05i
6	0.943290	0.943733
7	0.923872	0.924257
8	0.918657	0.918798
9	0.895682 + 3.5570e-05i	0.896317 + 3.6750e-05i
10	0.895682 − 3.5570e-05i	0.896317 + 3.6750e-05i

Solution of alpha modes spectral problem

Table: The first 10 eigenvalues for p=3, n=96

i	Diffusion	SP_3
1	42.263	41.380
2	$84.867 - 0.06130i$	$83.821 - 0.06358i$
3	$84.867 + 0.06130i$	$83.821 + 0.06358i$
4	$182.914 - 0.11367i$	$181.471 - 0.11805i$
5	$182.914 + 0.11367i$	$181.471 + 0.11805i$
6	293.017	290.940
7	371.528	369.374
8	$515.465 - 0.16397i$	$512.337 - 0.17197i$
9	$515.465 + 0.16397i$	$512.337 + 0.17197i$
10	518.670	517.975

Solution of alpha modes spectral problem

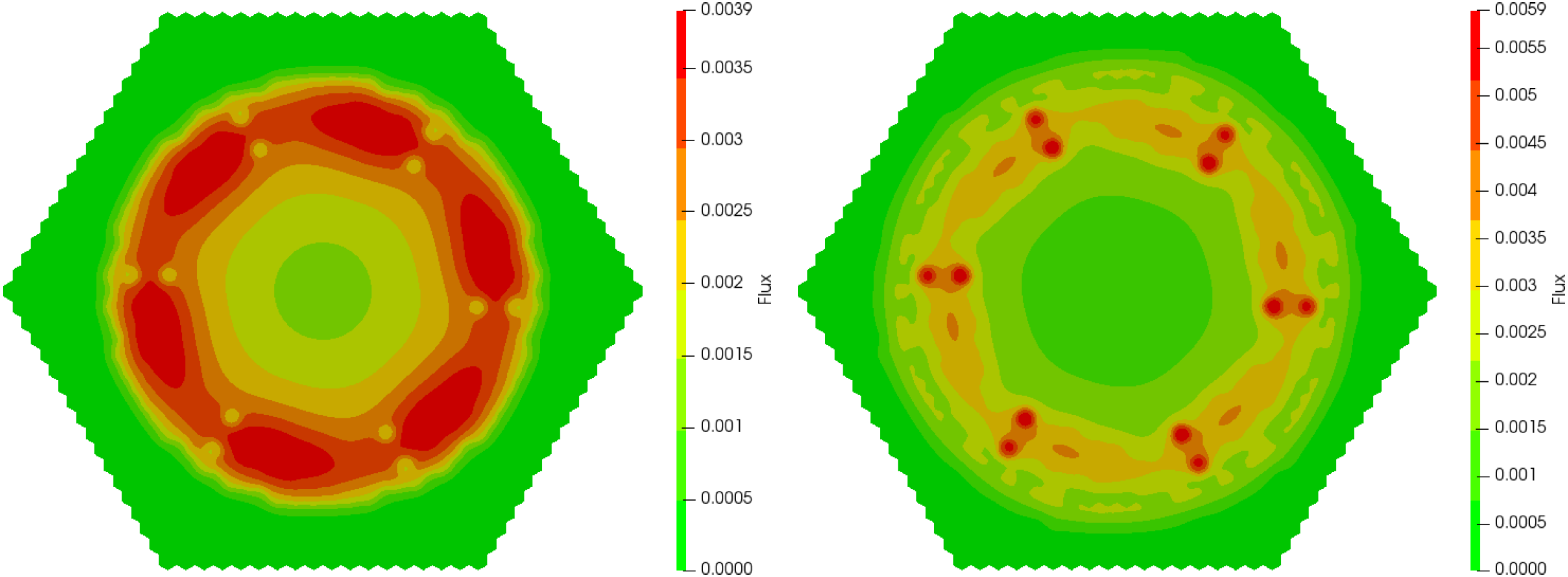


Fig: The eigenfunctions $\phi_1^{(1)}, \phi_2^{(1)}$

Solution of alpha modes spectral problem

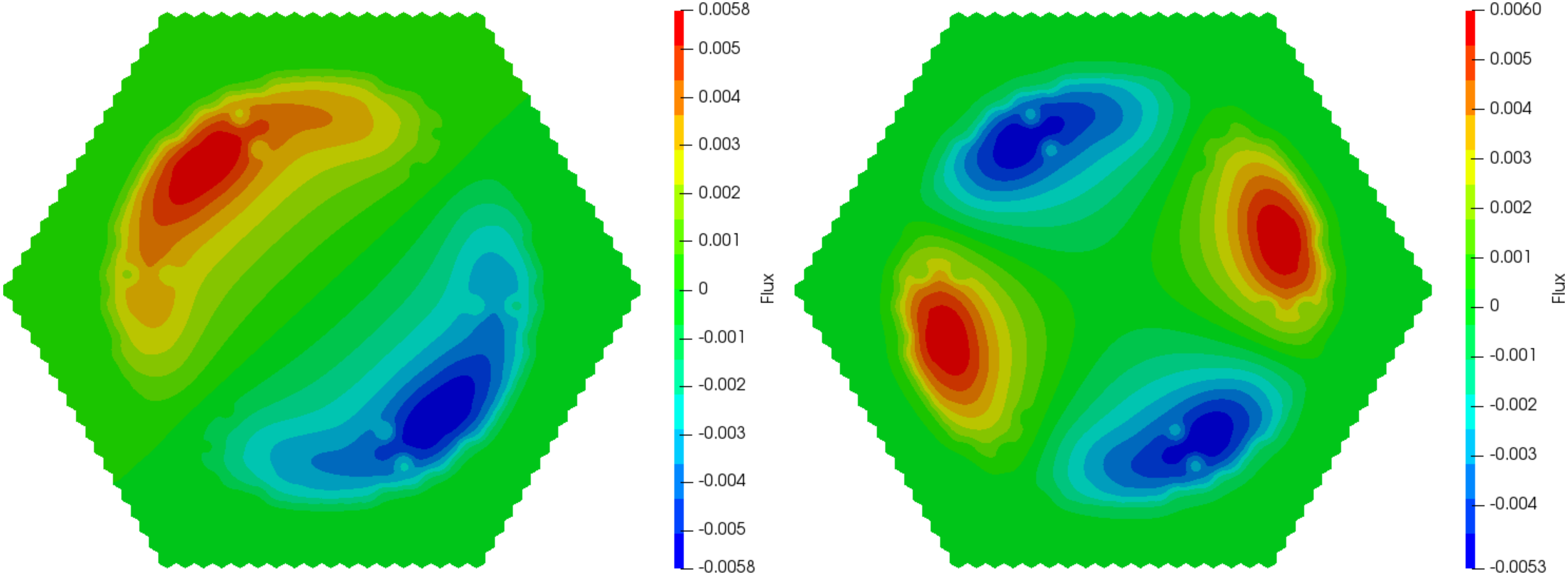


Fig: Real part of eigenfunctions $\phi_1^{(2)}, \phi_2^{(3)}$ and $\phi_1^{(4)}, \phi_2^{(5)}$

Solution of alpha modes spectral problem

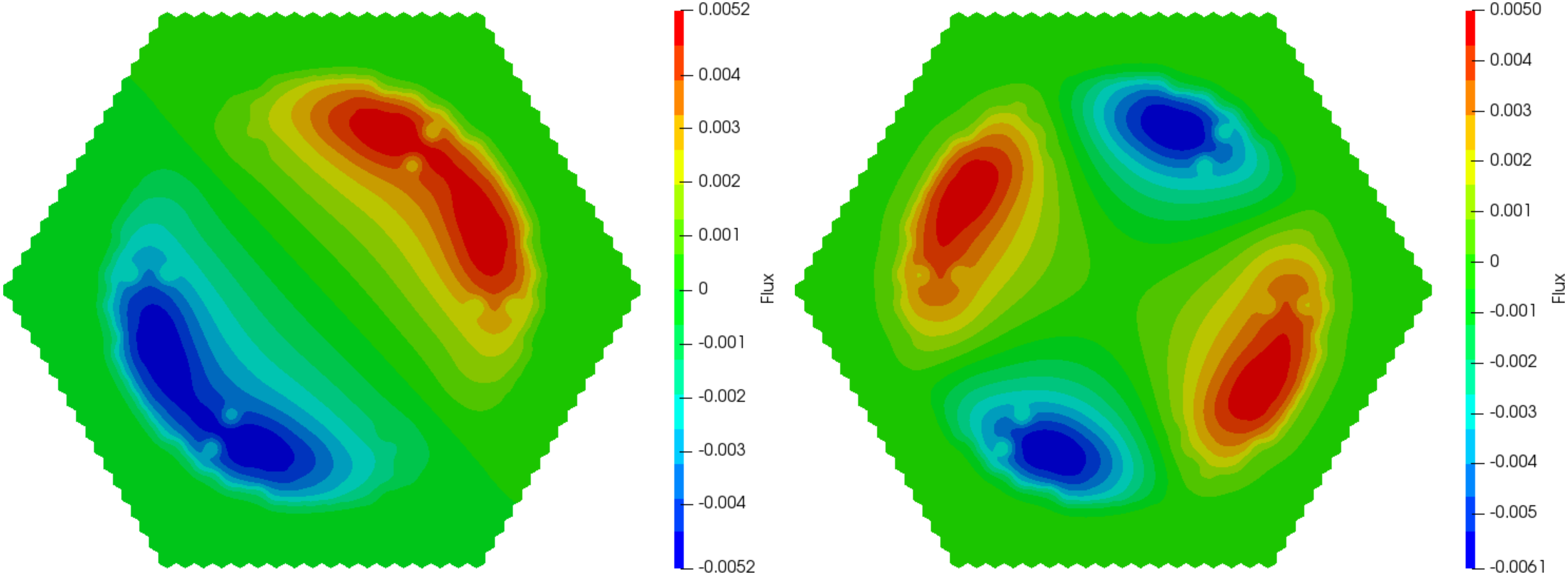


Fig: imaginary part of eigenfunctions $\phi_1^{(2)}, -\phi_2^{(3)}$ and $\phi_1^{(4)}, -\phi_2^{(5)}$

Conclusion

- Compared the spectral parameters and non-stationary solutions, calculated by both the diffusion and SP_3 options using the FEM.
- Solution of the lambda- and alpha- spectral problems has been tested for the IAEA-2D with reflector and HWR reactor benchmark test.
- Of particular interest is the problem associated with appearance of complex eigenvalues and eigenfunctions. It was found that this tendency occurs for both the diffusion and SP_3 solutions of the HWR reactor test.

Thank you for your attention!