Modelling dynamic processes in a nuclear reactor by state change modal method

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Multigroup diffusion approximation

Instantaneous neutrons:

$$\frac{1}{v_g}\frac{\partial \phi_g}{\partial t} - \nabla \cdot D_g \nabla \phi_g + \Sigma_{rg}\phi_g - \sum_{g \neq g'=1}^{G} \Sigma_{s,g' \to g}\phi_{g'} =$$

$$= (1-\beta)\chi_g \sum_{g'=1}^{G} \nu \Sigma_{fg'}\phi_{g'} + \widetilde{\chi}_g \sum_{m=1}^{M} \lambda_m c_m, \quad g = 1, 2, ..., G.$$

Delayed neutrons:

$$\frac{\partial c_m}{\partial t} + \lambda_m c_m = \beta_m \sum_{g=1}^{G} \nu \Sigma_{fg}\phi_g, \quad m = 1, 2, ..., M,$$

where β_m is is a fraction of delayed neutrons of m type, and

$$\beta = \sum_{m=1}^{M} \beta_m.$$

Initial and boundary conditions

The albedo-type conditions are set at the boundary $\partial\Omega$ of the area Ω:

$$D_g \frac{\partial \phi_g}{\partial n} + \gamma_g \phi_g = 0, \qquad g = 1, 2, ..., G.$$

We consider boundary problem with albedo boundary condition and the initial condition:

$$\phi_g(\boldsymbol{x}, 0) = \phi_g^0(\boldsymbol{x}), \quad g = 1, 2, ..., G, \quad c_m(\boldsymbol{x}, 0) = c_m^0(\boldsymbol{x}), \quad m = 1, 2, ..., M.$$

Operator formulation

Define vectors $\phi = \{\phi_1, \phi_2, ..., \phi_G\}$, $\boldsymbol{c} = \{c_1, c_2, ..., c_M\}$ and matrix:

$$V = (v_{gg'}), \quad v_{gg'} = \delta_{gg'} v_g^{-1}, \qquad D = (d_{gg'}), \quad d_{gg'} = -\delta_{gg'} \nabla \cdot D_g \nabla,$$
$$S = (s_{gg'}), \quad s_{gg'} = \delta_{gg'} \Sigma_g - \Sigma_{s,g' \to g}, \qquad R = (r_{gg'}), \quad r_{gg'} = (1-\beta)\chi_g \nu \Sigma_{fg'},$$
$$B = (b_{gm}), \quad b_{gm} = \widetilde{\chi}_g \lambda_m, \qquad \Lambda = (\lambda_{mm'}), \quad \lambda_{mm'} = \lambda_m \delta_{mm'},$$
$$Q = (q_{mg}), \quad q_{mg} = \beta_m \nu \Sigma_{fg}, \qquad g, g' = 1, 2, ..., G, \quad m, m' = 1, 2, ..., M.$$

Using the set definitions, we get boundary problem in operator formulation:

$$V \frac{d\phi}{dt} + (D + S)\phi = R\phi + B\boldsymbol{c},$$
$$\frac{d\boldsymbol{c}}{dt} + \Lambda \boldsymbol{c} = Q\phi.$$

The Cauchy problem is solved under initial conditions:

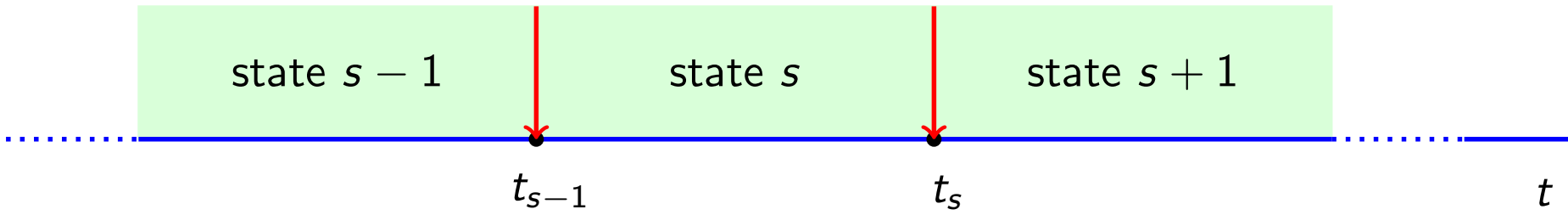
$$\phi(0) = \phi^0, \quad \boldsymbol{c}(0) = \boldsymbol{c}^0,$$

where

$L = (l_{gg'}), \quad l_{gg'} = \delta_{gg'} \gamma_g, \quad \phi^0 = \{\phi_1^0, \phi_2^0, ..., \phi_G^0\}, \quad \boldsymbol{c}^0 = \{c_1^0, c_2^0, ..., c_M^0\}.$

State change scheme

The state of the reactor is characterized by the constant coefficients of the system of multigroup diffusion equations.



Dynamic processes in a nuclear reactor can be considered as a change of states. At a certain time $t = t_s,\ s = 1, 2, ...$ an instantaneous change of state occurs. The state s is defined by the parameters in equations:

$$V(t) = V(t_s), \quad D(t) = D(t_s), \quad S(t) = S(t_s), \quad R(t) = R(t_s), \quad B(t) = B(t_s)$$

$$\Lambda(t) = \Lambda(t_s), \quad Q(t) = Q(t_s), \quad t_{s-1} < t \leq t_s, \quad s = 1, 2, ...$$

Time scale processes

The initial condition includes two components $\boldsymbol{u}_h^s(\boldsymbol{x}) = (\phi_h^s(\boldsymbol{x}), \boldsymbol{c}_h^s(\boldsymbol{x}))$. Dynamic behaviour of these components is due to different time-scale processes. Delayed neutrons source determines **slow processes**, when $\boldsymbol{c}(\boldsymbol{x}, t)$ changes slightly with the reactor state change. In contrast, neutron flux $\phi(\boldsymbol{x}, t)$ determines **fast processes** when the reactor state changes. By virtue of this separation of dynamic processes, we model the slow phase of the dynamics of the reactor with modal approximation and orientate ourselves on the approximate prediction of the initial state for delayed neutrons, only the function $\boldsymbol{c}_h^s(\boldsymbol{x})$ is approximated. The approximation $\phi_h^s(\boldsymbol{x})$ is not of interest to us; we do not model a fast phase of the state change.

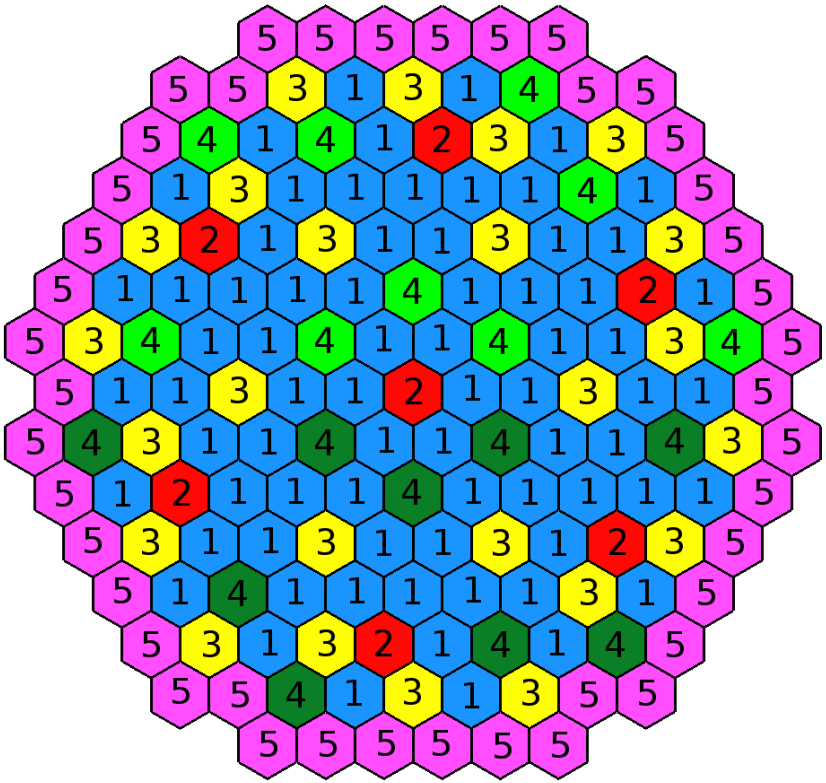
Computation scheme

Off-line calculation. Calculation of the coefficients of the mathematical model of the multigroup diffusion approximation for the isolated reactor states, which is performed in advance. The status passport also includes calculated dominant eigenvalues and eigenfunctions of the α-eigenvalue problem. These data can be supplemented by dominant eigenvalues and eigenvalues of the conjugate eigenvalue problem.

On-line calculation. Real-time modeling is carried out on the basis of the modal solution of the problem. The coefficients in the representation are calculated from the initial condition. The solution for other time intervals is determined according to modal approximation.

VVER-1000 benchmark

The dynamics of the VVER-1000 reactor during the transition from the supercritical mode to the subcritical mode



- two-dimensional and two-group approximation
- one groups of delayed neutrons
- the number of triangles per cassette κ varies from 6 to 96
- order of finite elements p varies from 1 to 3
- two types of perturbation

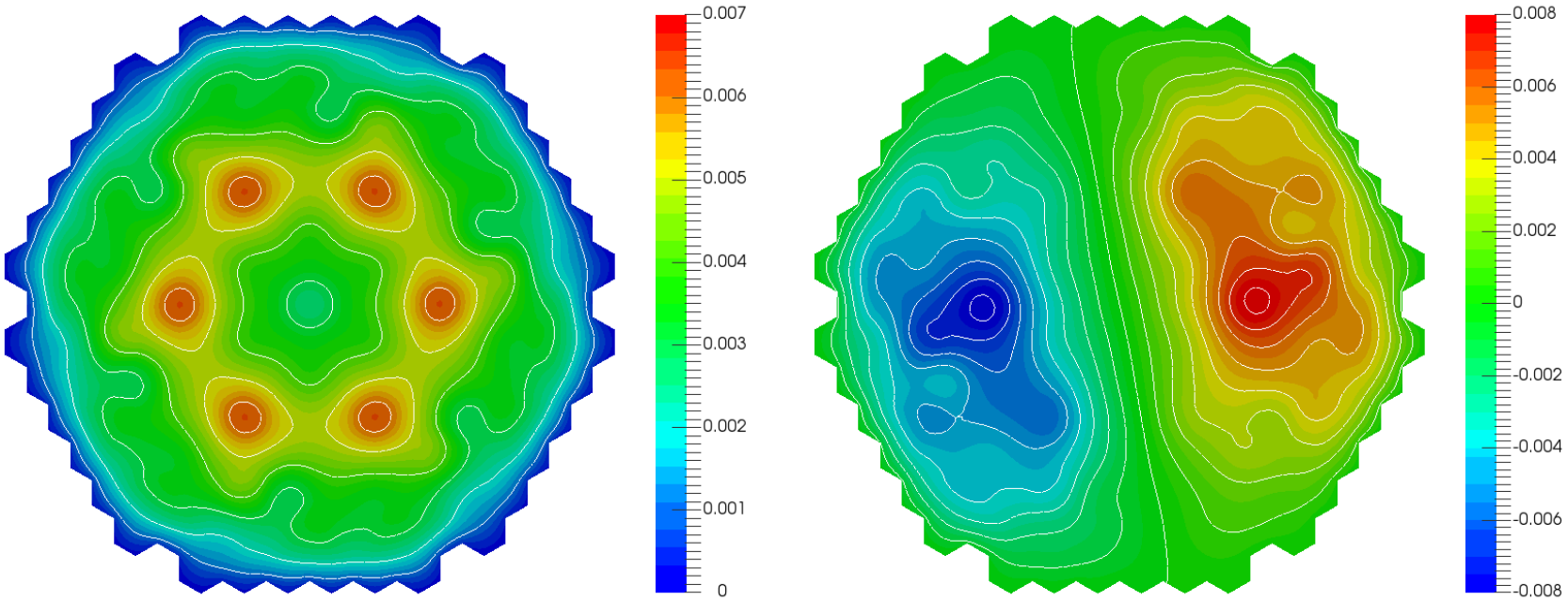


Figure: The eigenfunction $\varphi_1^{(1)}$ (left) and real part of eigenfunctions $\varphi_1^{(2)}$, $\varphi_1^{(3)}$ (right).

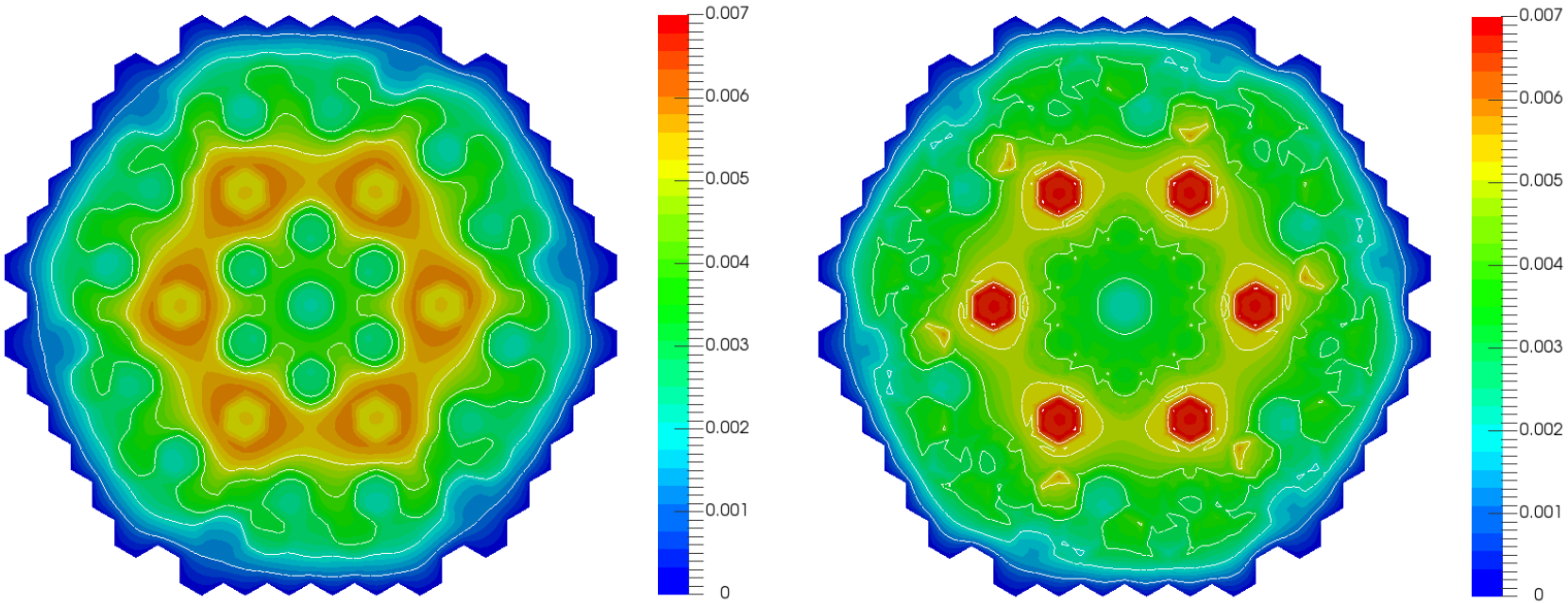


Figure: The eigenfunction $\varphi_2^{(1)}$ (left) and the eigenfunctions $s^{(1)}$ (right).

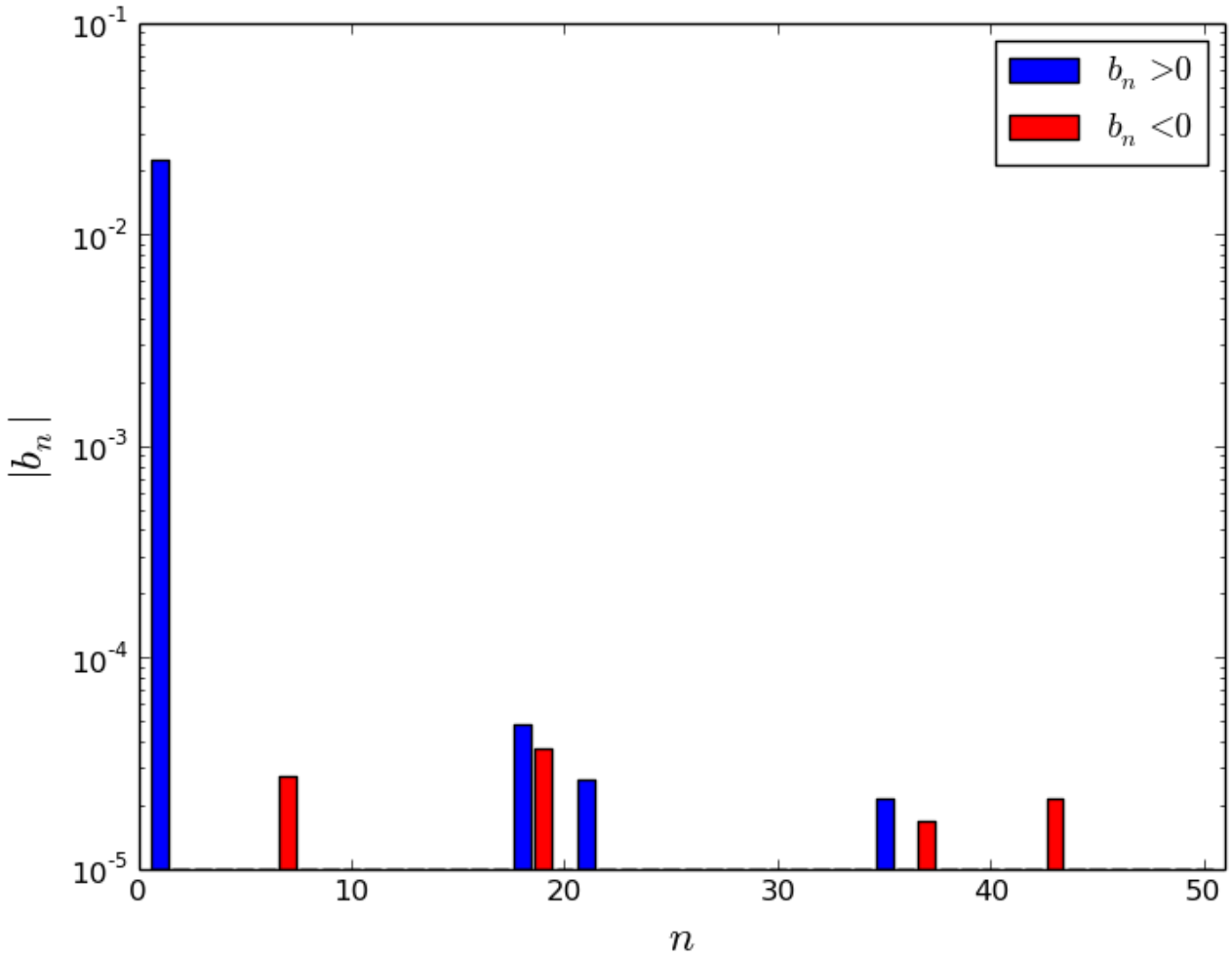


Figure: Approximate solution coefficients

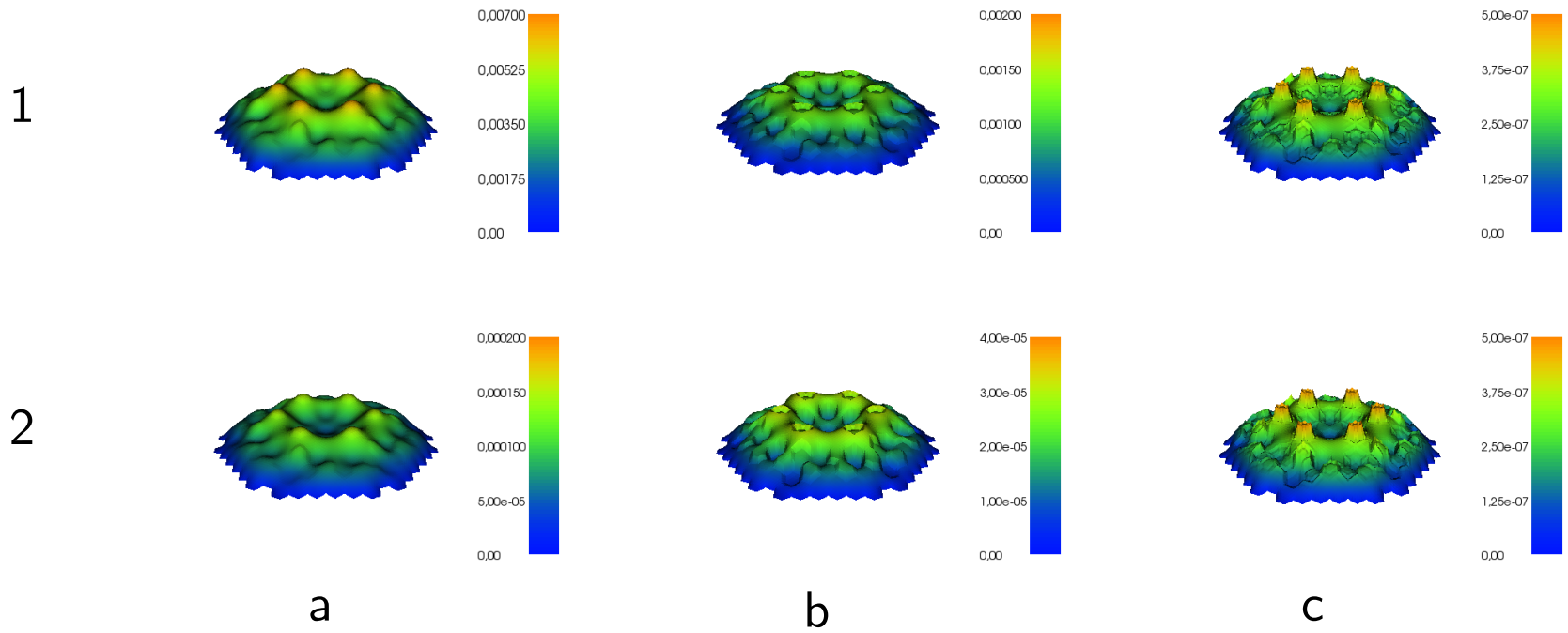


Figure: Function $\boldsymbol{u}(\boldsymbol{x}, 0)$ (string 1) and function $\boldsymbol{u}_N(\boldsymbol{x}, 0)$ (ctring 2): a — neutron flux of group 1, b — neutron flux of group 2, c — delayed neutrons source.

The asymmetric perturbation

Consider a more complex transition to a subcritical state. The subcritical stage will be characterized by a different increase in the coefficient Σ_2 for material 4 in the diffusion constants in the upper and lower half of the reactor cross-section. Now let the reactor dynamics corresponds to the following transformation

$$\Sigma_2 \longrightarrow \begin{cases} 1.1\Sigma_2, & \text{material 4 (top part)}, \\ 1.2\Sigma_2, & \text{material 4 (bottom part)}. \end{cases}$$

All eigenvalues for this reactor state are real.

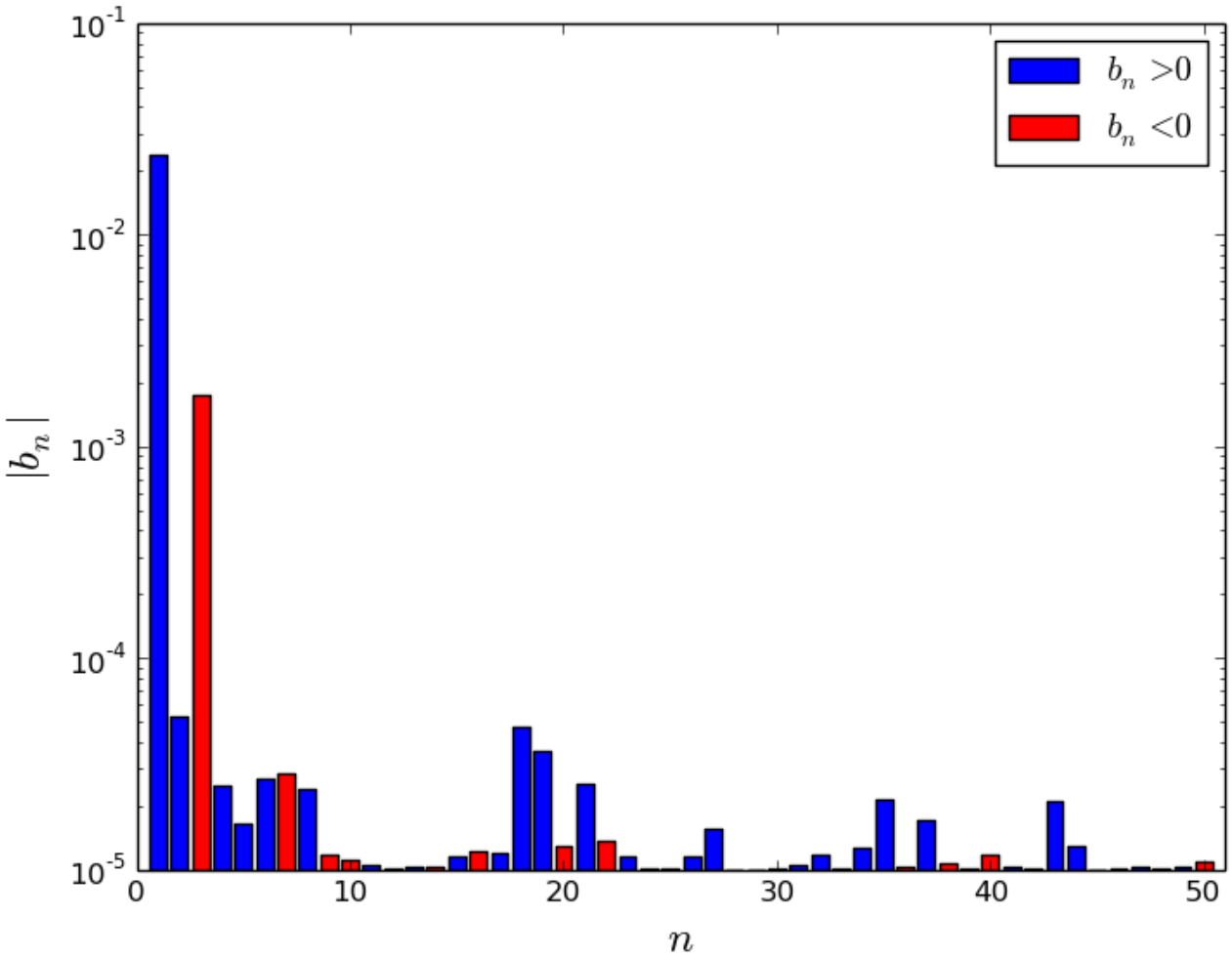


Figure: Approximate solution coefficients

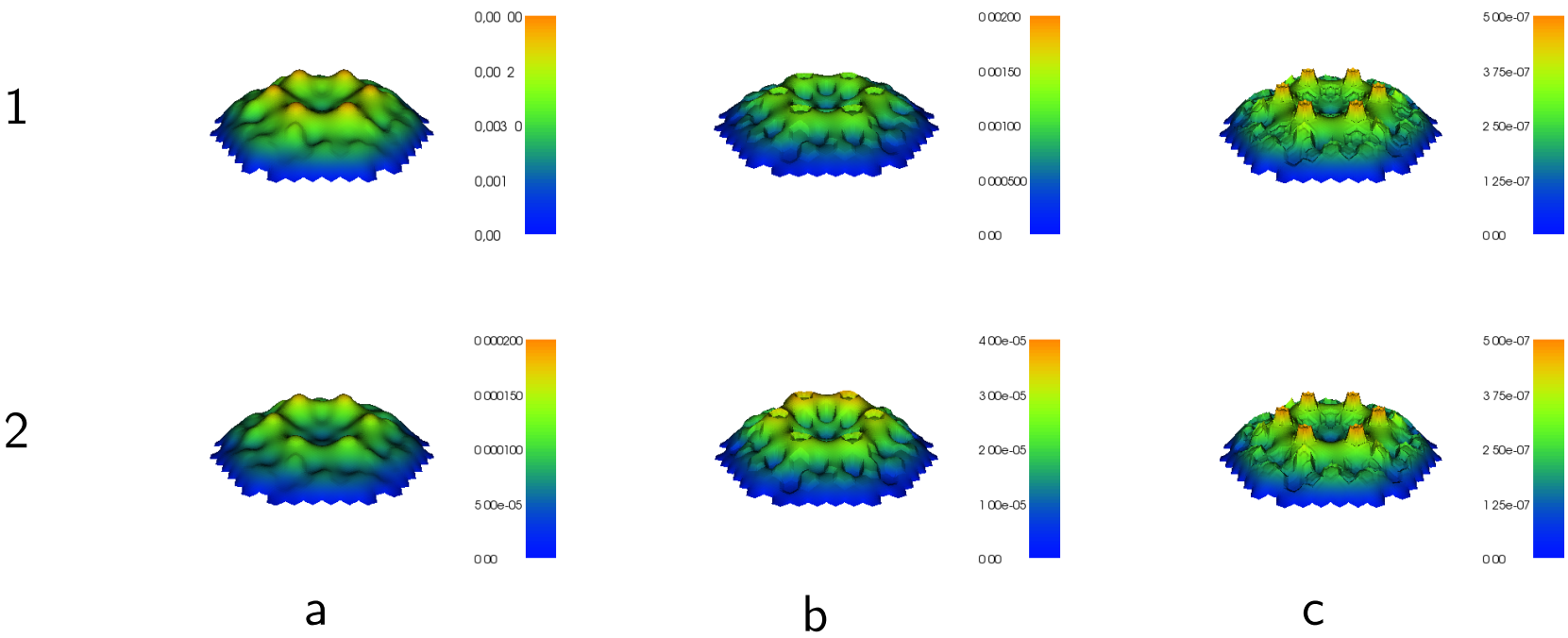
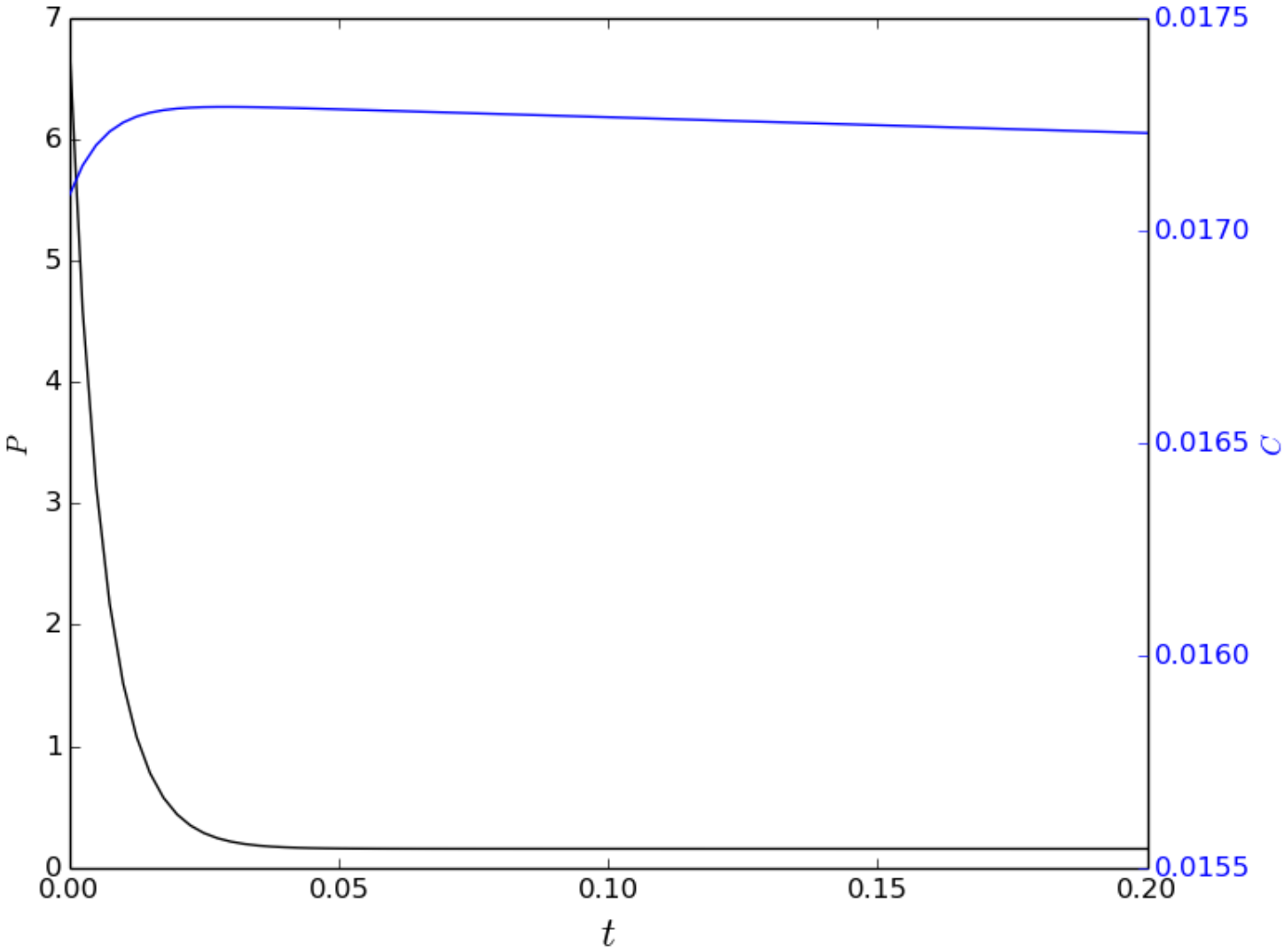
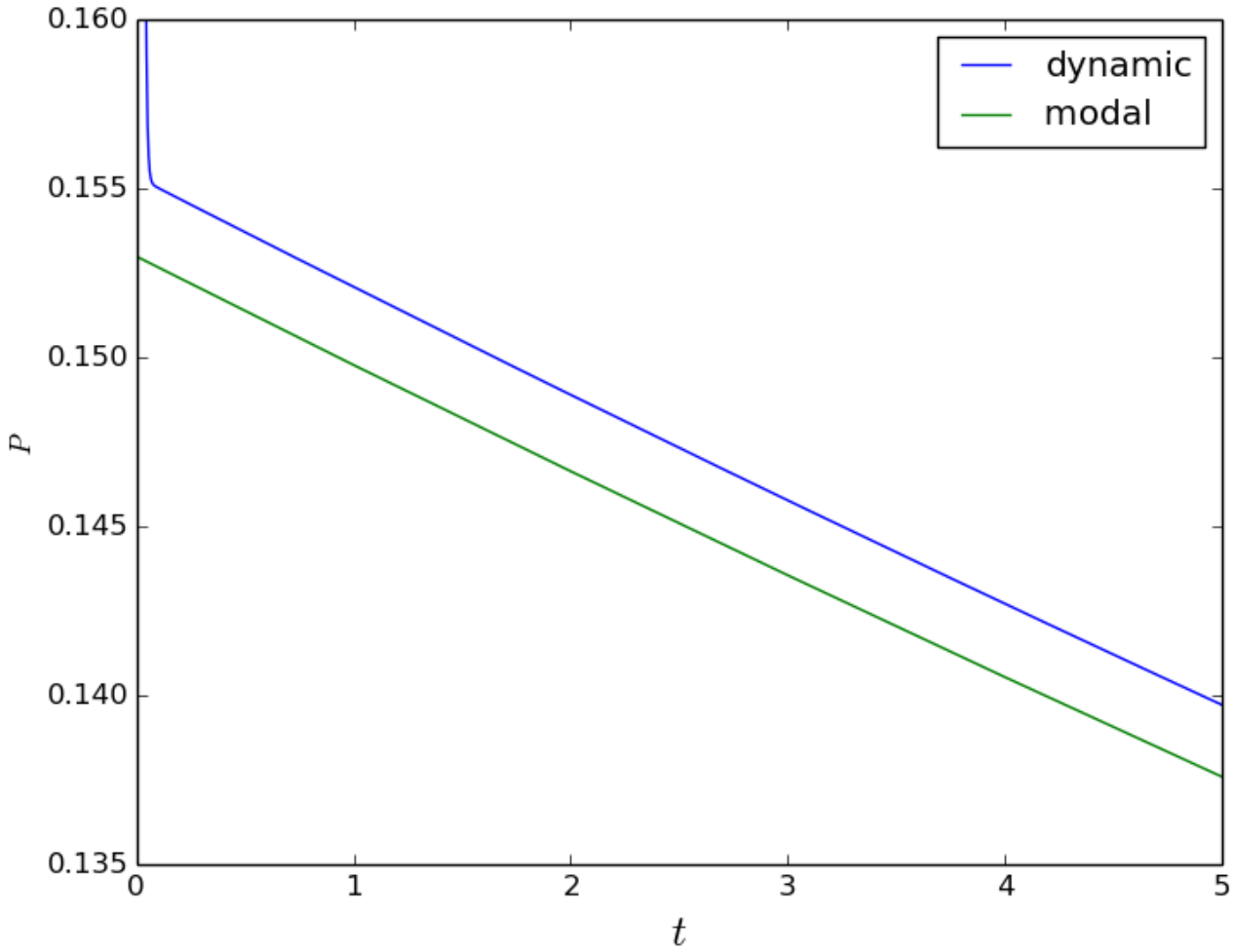


Figure: Function $\boldsymbol{u}(\boldsymbol{x}, 0)$ (string 1) and function $\boldsymbol{u}_N(\boldsymbol{x}, 0)$ (string 2) for asymmetric perturbation: a — neutron flux of group 1, b — neutron flux of group 2, c — delayed neutrons source.

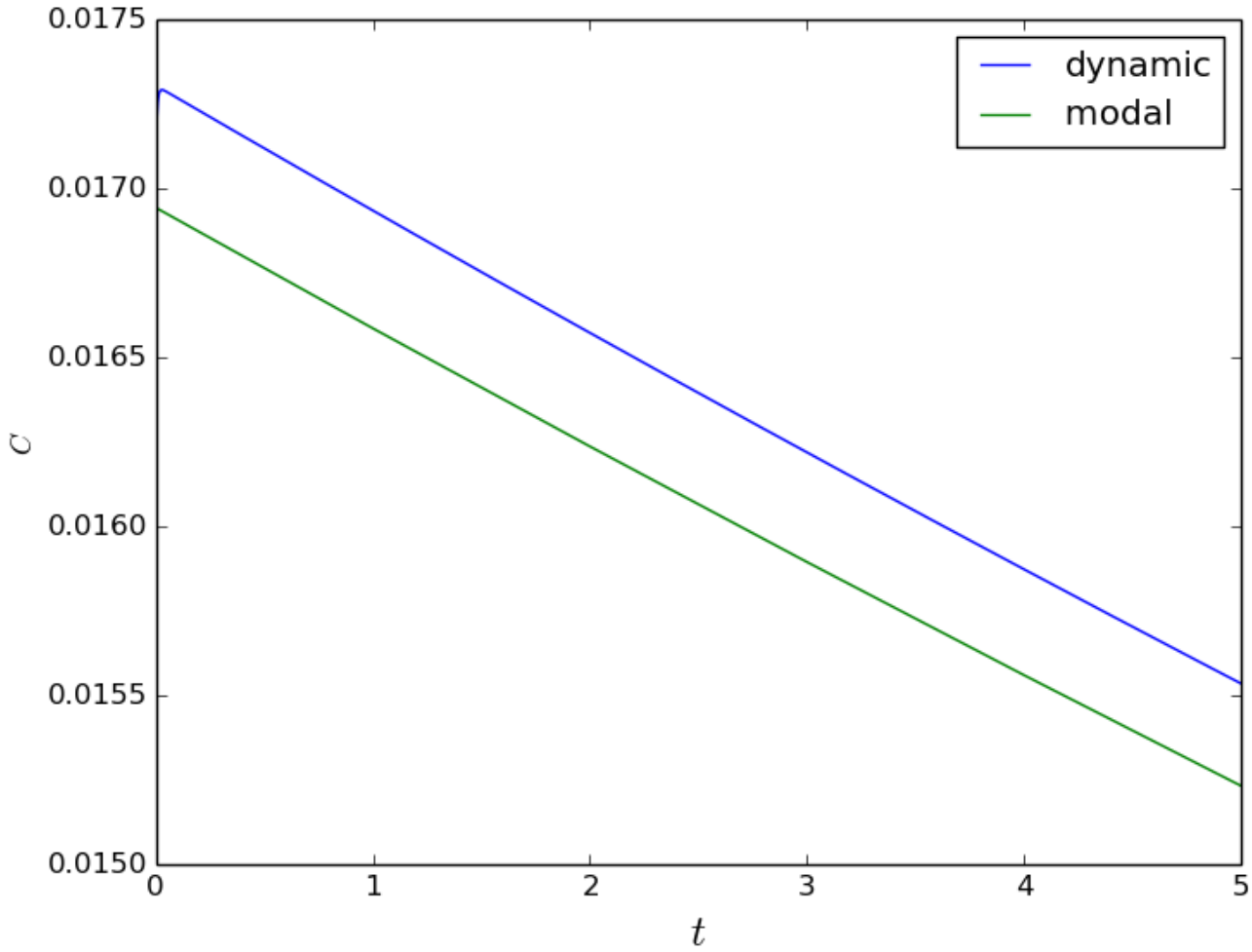
Fast stage of reactor state: neutronic power



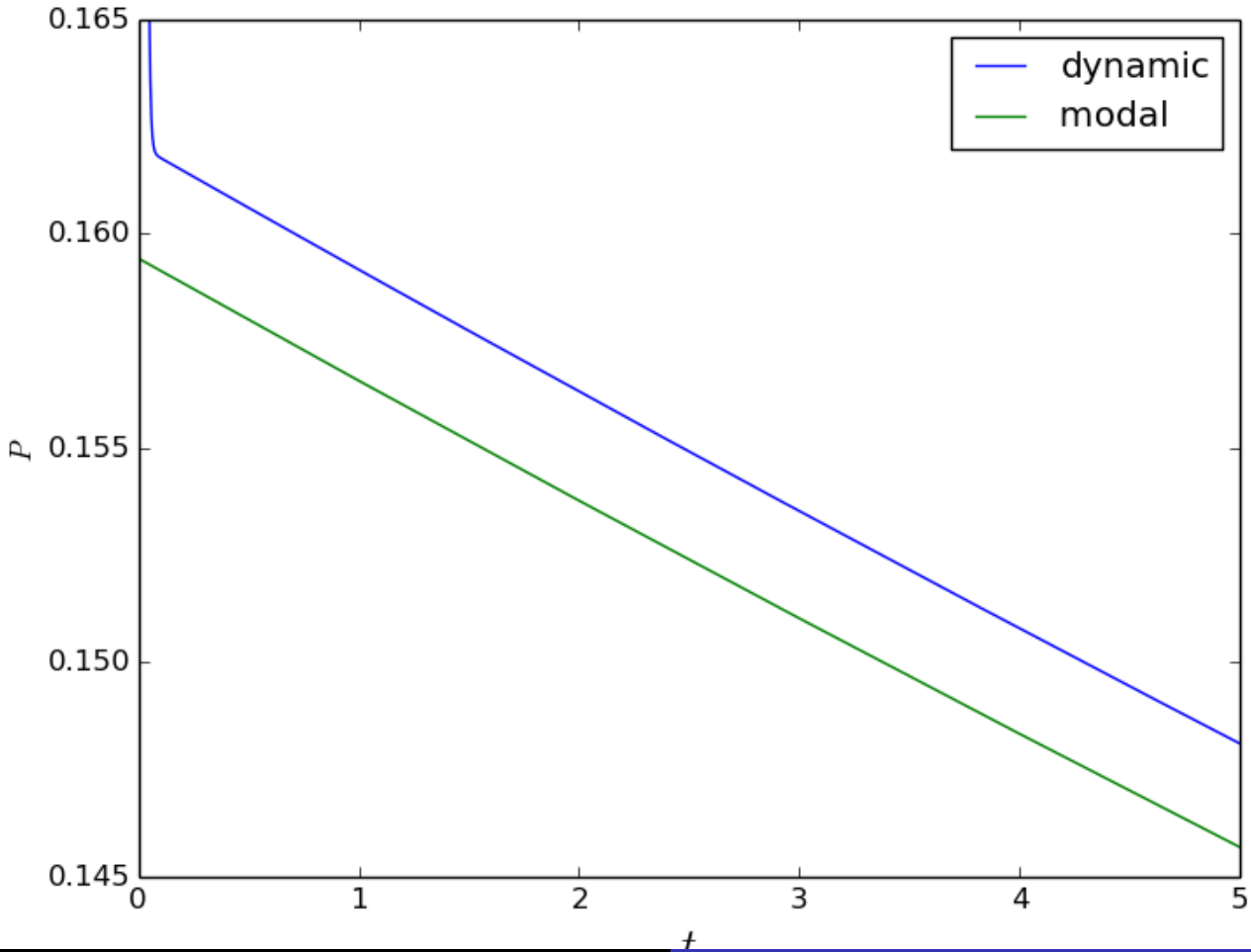
Slow stage of reactor state: neutronic power



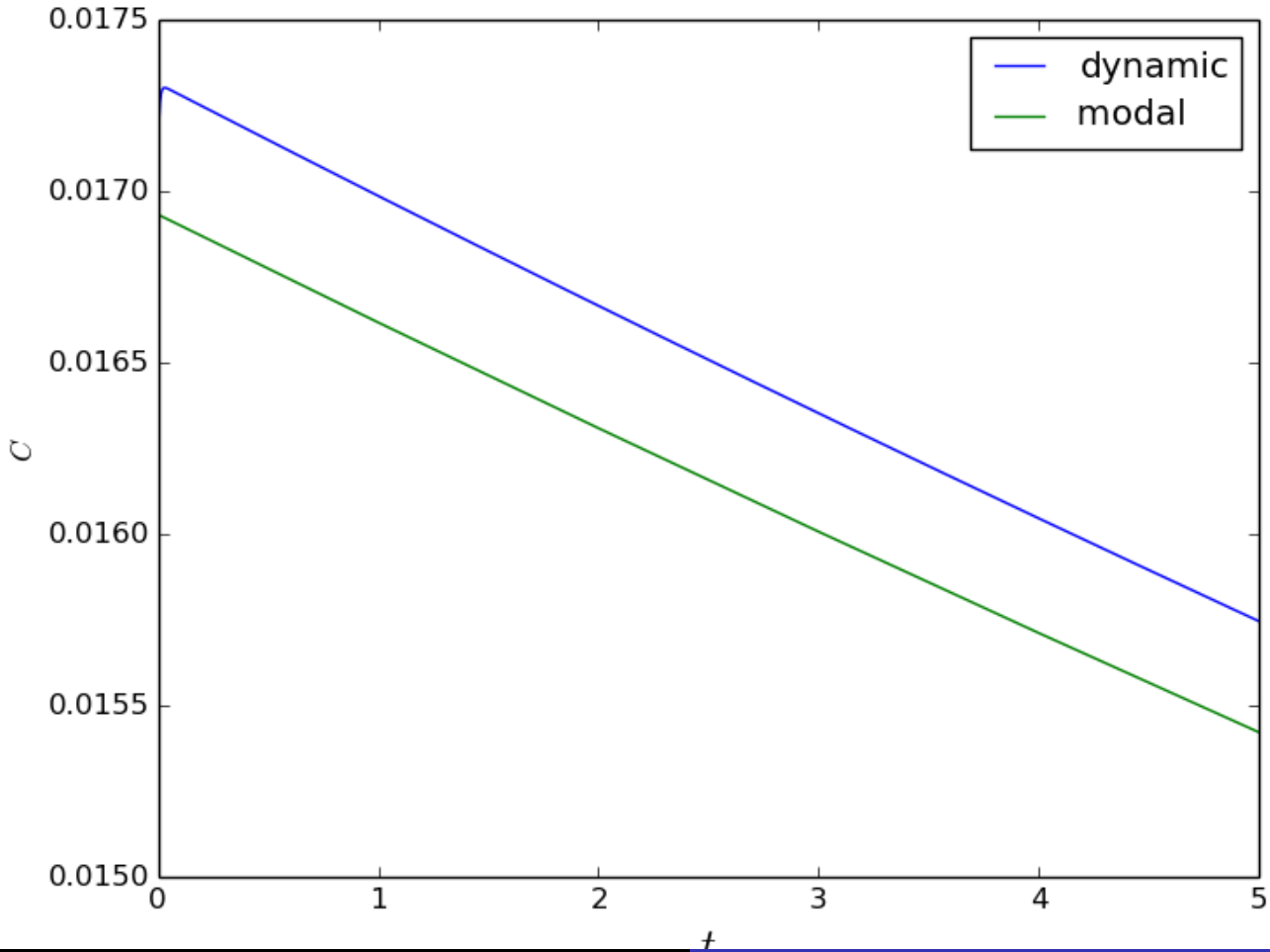
Slow stage of reactor state: delayed neutrons sourse



Slow stage of reactor state for the asymmetric perturbation: neutronic power



Slow stage of reactor state for the asymmetric perturbation: delayed neutrons sourse



Conclusion

- The problem of simulation of reactor dynamic processes is considered on the basis of multigroup neutron diffusion equations accounting for delayed neutrons. The modal approximation is used.
- In the developed SCM method a fast phase and slow phase is allocated
- Offline calculation, online calculation
- Finite Element Method, FEniCS, SLEPc
- The modeling of the reactor state change as a transfer from one state to another state, symmetric and assymetric perturbation
- Comparison of the calculational results obtained by using two methods, one based on modal approximation and another based on the full dynamics calculation

Thank you for your attention!