Algorithm of Time Step $\boxed{\text{Selection}}$ for Numerical Solution of Boundary Value Problem for Parabolic Equations

Лучше везде в тексте Selection заменить на Evaluation.

Abstract. ~~In this work automatic time step evaluation algorithm for~~ solving the boundary value problem for parabolic equations is proposed. The solution is obtained using evaluation approach and fully implicit schemes. The time step evaluation formulas ~~are derivation fulfilled based on the estimation of the approximation error~~ at next time step. ~~Calculation results are obtained for model problem that demonstrate reliability of the proposed algorithm for time step control.~~

In this work the algorithm of automatic time step evaluation for…

… are derived on the basis of the approximation error estimation…

Reliability of the proposed algorithm is demonstrated using the model problem calculations.

INTRODUCTION

For the second order parabolic equations unconditionally stable schemes are constructed on the basis of implicit approximations [1]. In computational practice two-layer schemes are mostly ~~used. For example three-layer and~~ multi-layer schemes are not so often used. The problem of the time step control is relatively well developed for the Cauchy problem for differential equations systems [2, 3, 4]. The basic approach ~~based on~~ using additional calculations at a new time step to estimate the approximate solution. The time step is estimated using the theoretical asymptotic dependences of the accuracy on time step [5].

…used; three-layer and…

…is based on…

Additional calculations for estimating the error of the approximate solution can be carried out in different ways. The best-known strategy ~~connected with the calculation of the problem solution on a separate time intervals using the given step (the first solution) and with a step two times smaller (the second solution)~~. The noted ways ~~of selecting~~ the time step are related t~~o the class of a posteriori accuracy estimation methods.~~ The decision ~~as to suitable the time step or the re-calculation~~ is accepted only after the calculation is completed.

…is connected the calculation at separate time interval using the given time step (the first solution) and the half step (the second solution).

…to evaluate…

… to methods of a posteriori accuracy estimation.

…to change or not the time step…

In this paper, we consider a priori selection of the time step ~~in t~~he approximate solution of boundary value problems for parabolic equations. ~~The results of calculations for a model parabolic boundary value problem that~~ demonstrate the reliability of the proposed time stepping algorithm ~~are presented~~.

…to obtain…

The calculation results for the model problem for parabolic equations…

Then we can obtain a difference analogue of the estimate

$$\|y^{n+1}\| \leq \|u^0\| + \sum_{k=0}^{n} \tau^{k+1} \|f^{k+1}\| \tag{4}$$

for problem (3). For the error of the approximate solution $z^n = y^n - u^n$ we have

~~Where the~~ approximation error ψ^{n+1} is defined as

$$\psi^{n+1} = f^{n+1} - \frac{u^{n+1} - u^n}{\tau^{n+1}} - A^{n+1} u^{n+1}. \tag{5}$$

The…

Checking the error we can focus on the total error $\delta\tau^{n+1}$ in interval $t^n < t < t^{n+1}$. Then from (6) we obtain $\|z^{n+1}\| \leq \delta t^{n+1}$. The error ~~accumulates~~ and increases linearly. The solution is obtained using the unconditionally stable implicit scheme. The major part of computational capability are related with this scheme. The step control is performed using the solution produced by explicit scheme. The algorithm stability is not violated and determined by the implicit scheme properties. The error accumulation from the time layer t^n to the new layer t^{n+1} is defined as

…is accumulated…

Consider the basic algorithm of time step selection. We select the time step based on the analysis ~~of the previous step solutions.~~ The predicted time step is determined as following

$$\widetilde{\tau}^{n+1} = \gamma \tau^n, \tag{9}$$

…of the solution at the previous step.

where γ is numerical parameter. The default value of γ is 1.25 or 1.5. Using the explicit scheme we can obtain a solution $\widetilde{y}^{n+1}$ at time $\widetilde{t}^{n+1} = t^n + \widetilde{\tau}^{n+1}$. The calculation is performed only at single time step. Therefore the possible

…is a…

CALCULATED FORMULAS

We present the calculated formulas for the time step control. The predictive solution $\widetilde{y}^{n+1}$ is defined from

Лучше заменить на calculation.

In accordance with (5), the approximation error is calculated from the exact solution for two time moments: ~~in our case,~~ for t^n and $\widetilde{t}^{n+1}$. To estimate the error we take $u(t^n)$ instead of y^n. An exact solution at a new time step $u(\widetilde{t}^{n+1})$, is matched by an approximate solution $u(\widetilde{t}^{n+1})$, which is obtained by the explicit scheme. By virtue of this, we set

~~Close formulas for choosing a time step~~ were obtained earlier in the papers [5, 6] on the basis of an estimate of the change in the approximate solution. ~~That methods was obtained through~~ two auxiliary steps with a predictable time step. The first step ~~forward is done~~ according to the explicit scheme, the second one – according to the explicit scheme backwards. Here we rely on a simpler and more general procedure for estimating the approximation error at the predicted ~~step in time~~ according to the approximate solution ~~that was obtained through perform one step ahead using an explicit scheme~~.

Similar formalas for the time step evaluation…

This method uses

…is performed..

…time step…

…by explicit scheme.

TEST PROBLEM

In this section, ~~numerical example is presented~~ to illustrate the efficiency of the algorithm. We solve the boundary-value problem for a one-dimensional parabolic equation

…we present a numerical test…

In this paper, we consider the problems of time discretization and therefore the spatial grid ~~didn't vary.~~ The accuracy of the approximate solution was estimated from the reference solution that we use as a numerical solution on a sufficiently detailed grid in time ($\tau = 1 \cdot 10^{-7}$). The error is estimated in the norm of $L_2(\omega)$ ($\varepsilon_2 = \|\cdot\|$) or $L_\infty(\omega)$ ($\varepsilon_\infty = \|\cdot\|_\infty$), and

…does not change.

First, we ~~provide data on the error~~ of the approximate solution when using uniform grids on time. Figure 1 shows the dependence of the accuracy in the norm of $L_2(\omega)$ (on left) and in $L_\infty(\omega)$ (on right). The above data demonstrate ~~a losing of accuracy~~ at the beginning of the calculation process and ~~increased level of error~~ at the point $t = 0.05$ (the discontinuity of the right-hand side of $f(t)$) and at the point $t = 0.075$ (the discontinuity of the lowest coefficient of the equation $p(t)$). A successful strategy for selecting a time step should capture the noted features: a calculation with a small time step in the right-hand neighborhood $t = 0$ and in the neighborhood $t = 0.05$, $t = 0.075$.

…estimate the error…

…the error decreasing…

…the error increasing…

- не понятно, может лучше использовать …in the vicinity of…

The ~~level of approximation error~~ is determined taking into account that $T = 0.1$ and the approximate solution is determined on a unit interval in x. Amplitude of the solution has order 1. Figure 2 shows the time steps when using the norm $L_2(\omega)$ for estimating the approximation error and the accuracy obtained in this case when specifying a different level of approximation error. For the time step increase parameter put $\gamma = 1.5$. The calculation begins with a small step and gradually increases. In the neighborhood of $t = 0.05$ ~~is a transition to an~~ essentially smaller step ~~in time occurs~~ and a time step is also reset in the neighborhood of $t = 0.075$. The accuracy of the approximate solution increases significantly at small times. In fact there is no loss of accuracy in the neighborhood of the discontinuity of the right-hand side and the coefficient of the equation. Similar data were obtained using the $L_\infty(\omega)$ norm to estimate the level of approximation error. Thus, the time stepping algorithm works by selecting different norms for estimating the level of approximation error.

…approximation error estimate…

- не понятно, может лучше разбить на несколько коротких связанных предложений

…and then it…

…we see a transition to…

the level of approximation error - может просто the approximation error

Among the basic parameters of the proposed algorithm we can note γ that is associated with the selection of the predicted step. ~~As the data in figure 3 show that the effect of the time increment parameter~~ is insignificant. Special attention should be paid ~~to the study of~~ the effect of the initial conditions – the parameter σ. A typical situation is the presence of a boundary layer which requires the use of a ~~shallow~~ step at small times. For $\sigma = 1$, we have a more favorable situation with an error for small t (see figure 4). The irregular grid and the accuracy of the approximate solution are shown in Fig. 5. In this case the error in the initial part of the process is small that is well worked out by the algorithm time step selection. The influence of other parameters of the problem (λ and χ) is not so significant.

As shown in figure 3, the effect of increasing of γ …

…on…

…fine…

- нужно разбить на два коротких предложения

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