State change modal method for numerical simulation of dynamic processes in a nuclear reactor

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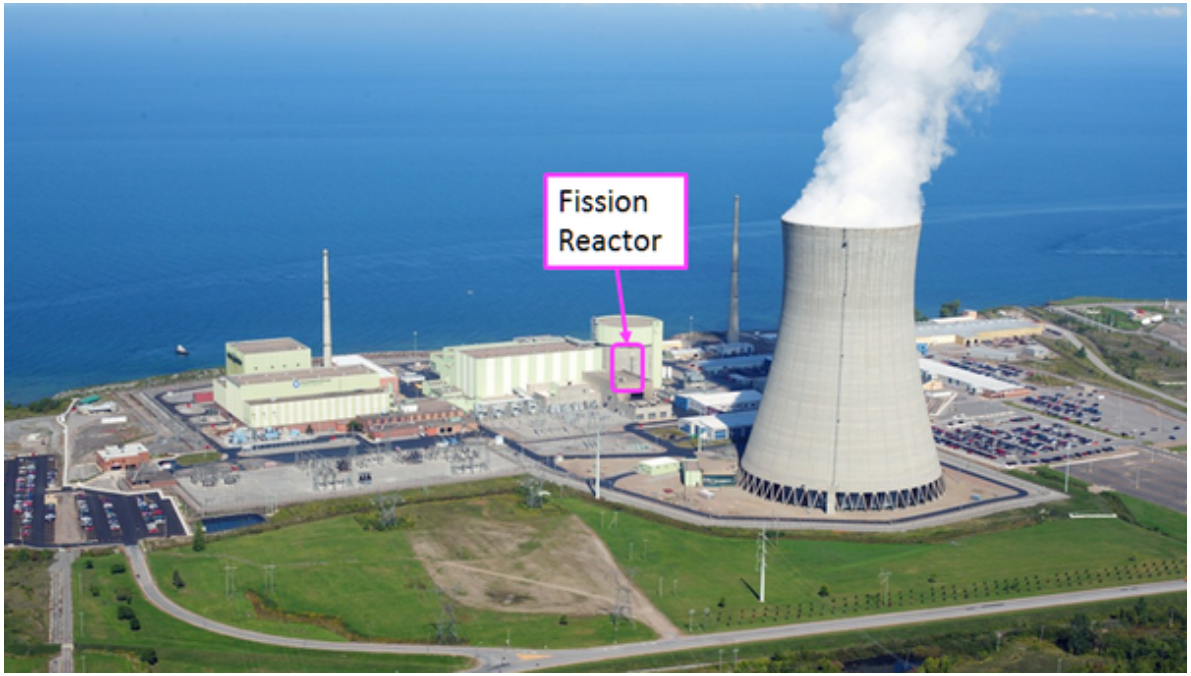
Introduction



- neutron-transport equation (time, energy, spatial and angular variables)
- simplified forms, multigroup diffusion approximation

- spectral problems, dynamic method (two layer, Runge-Kutta, Rosenbroek)
- quasistatic method, modal approximation (Stacey 1969)
- we formulate a general strategy for the approximate solution of nonstationary problems of neutron diffusion in nuclear reactors, which is oriented to fast real-time calculations using the State Change Modal (SCM) method.

Problem statement

Multigroup diffusion approximation

$$\frac{1}{v_g}\frac{\partial \phi_g}{\partial t} - \nabla \cdot D_g \nabla \phi_g + \Sigma_{rg}\phi_g - \sum_{g \neq g'=1}^{G} \Sigma_{s,g' \to g}\phi_{g'} =$$

$$= (1-\beta)\chi_g \sum_{g'=1}^{G} \nu \Sigma_{fg'}\phi_{g'} + \widetilde{\chi}_g \sum_{m=1}^{M} \lambda_m c_m, \quad g = 1, 2, ..., G.$$

The density of sources of delayed neutrons is described as follows:

$$\frac{\partial c_m}{\partial t} + \lambda_m c_m = \beta_m \sum_{g=1}^{G} \nu \Sigma_{fg}\phi_g, \quad m = 1, 2, ..., M,$$

where β_m is is a fraction of delayed neutrons of m type, and

$$\beta = \sum_{m=1}^{M} \beta_m.$$

Initial and boundary conditions

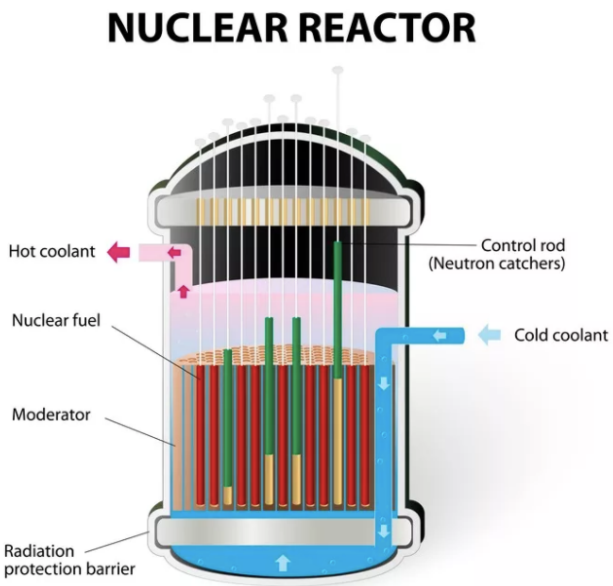
The albedo-type conditions are set at the boundary $\partial\Omega$ of the area Ω:

$$D_g \frac{\partial \phi_g}{\partial n} + \gamma_g \phi_g = 0, \qquad g = 1, 2, ..., G,$$

where n is outer normal to the boundary $\partial\Omega$.

We consider boundary problem with the initial condition:

$$\phi_g(\boldsymbol{x}, 0) = \phi_g^0(\boldsymbol{x}), \quad g = 1, 2, ..., G,$$
$$c_m(\boldsymbol{x}, 0) = c_m^0(\boldsymbol{x}), \quad m = 1, 2, ..., M.$$



Operator formulation

Let's write the boundary problem in operator form. Define vectors $\phi = \{\phi_1, \phi_2, ..., \phi_G\}$, $\boldsymbol{c} = \{c_1, c_2, ..., c_M\}$ and matrix:

$$V = (v_{gg'}), \quad v_{gg'} = \delta_{gg'} v_g^{-1}, \qquad D = (d_{gg'}), \quad d_{gg'} = -\delta_{gg'} \nabla \cdot D_g \nabla,$$
$$S = (s_{gg'}), \quad s_{gg'} = \delta_{gg'} \Sigma_g - \Sigma_{s,g' \to g}, \qquad R = (r_{gg'}), \quad r_{gg'} = (1-\beta)\chi_g \nu \Sigma_{fg'},$$
$$B = (b_{gm}), \quad b_{gm} = \widetilde{\chi}_g \lambda_m, \qquad \Lambda = (\lambda_{mm'}), \quad \lambda_{mm'} = \lambda_m \delta_{mm'},$$
$$Q = (q_{mg}), \quad q_{mg} = \beta_m \nu \Sigma_{fg}, \qquad g, g' = 1, 2, ..., G, \quad m, m' = 1, 2, ..., M.$$

Using the set definitions, we get boundary problem in operator formulation:

$$V \frac{d\phi}{dt} + (D + S)\phi = R\phi + B\boldsymbol{c},$$
$$\frac{d\boldsymbol{c}}{dt} + \Lambda \boldsymbol{c} = Q\phi.$$

The Cauchy problem is solved under the following initial conditions:

$$\phi(0) = \phi^0, \quad \boldsymbol{c}(0) = \boldsymbol{c}^0,$$

where $\phi^0 = \{\phi_1^0, \phi_2^0, ..., \phi_G^0\}, \quad \boldsymbol{c}^0 = \{c_1^0, c_2^0, ..., c_M^0\}$.

Finite element method

Let $H^1(\Omega)$ – sobolev space, $v \in H^1$: v^2 and $|\nabla v|^2$ have a finite integral in Ω. For $\boldsymbol{v} = \{v_1, v_2, ..., v_d\}$ define $V^d = [H^1(\Omega)]^d$. For test functions use notations $\boldsymbol{\xi} = \{\xi_1, \xi_2, ..., \xi_G\}$, $\boldsymbol{\zeta} = \{\zeta_1, \zeta_2, ..., \zeta_M\}$.
In variation formulation we are looking for $\boldsymbol{\phi} \in V^D$, $\boldsymbol{c} \in V^M$ such that:

$$\int_\Omega \left(V \frac{\boldsymbol{\phi}^{n+1} - \boldsymbol{\phi}^n}{\tau} + S\boldsymbol{\phi}^{n+1} \right) \boldsymbol{\xi} d\boldsymbol{x} + \int_\Omega \sum_{g=1}^{G} D_G \nabla \phi_g^{n+1} \nabla \xi_g d\boldsymbol{x} +$$

$$+ \int_{\partial\Omega} \sum_{g=1}^{G} \gamma_g \phi_g^{n+1} \xi_g d\boldsymbol{x} = \int_\Omega R\boldsymbol{\phi}^{n+1} \boldsymbol{\xi} d\boldsymbol{x} + \int_\Omega B\boldsymbol{c}^{n+1} \boldsymbol{\xi} d\boldsymbol{x},$$

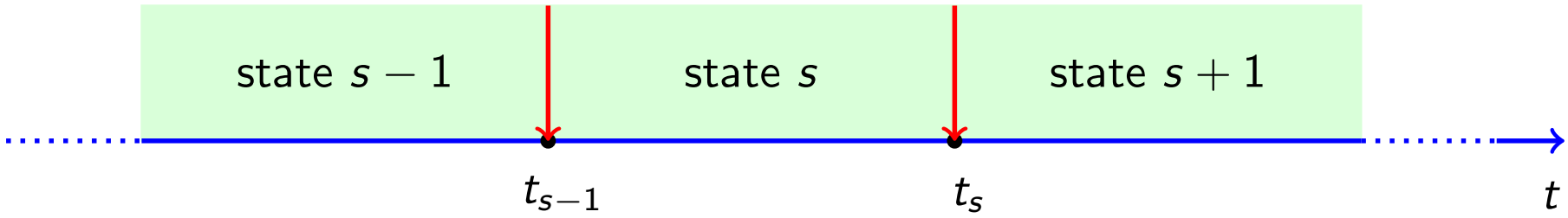
$$\int_\Omega \boldsymbol{c}^{n+1} \boldsymbol{\zeta} d\boldsymbol{x} = \int_\Omega \widetilde{\Lambda} \boldsymbol{c}^n \boldsymbol{\zeta} d\boldsymbol{x} + \int_\Omega \tau Q \boldsymbol{\phi}^{n+1} \boldsymbol{\zeta} d\boldsymbol{x}$$

for all $\boldsymbol{\xi} \in V^D$, $\boldsymbol{\zeta} \in V^M$.
Let introduce a discrete function spaces $V_h^D \subset V^D$, $V_h^M \subset V^M$ and define discrete variational problem.

The standard Lagrangian finite elements of degree p = 1, 2 and 3 are used.

State change scheme

The state of the reactor is characterized by the constant coefficients of the system of multigroup diffusion equations.



Dynamic processes in a nuclear reactor can be considered as a change of states. At a certain time $t = t_s,\ s = 1, 2, ...$ an instantaneous change of state occurs. The state s is defined by the parameters in equations:

$$V(t) = V(t_s), \quad D(t) = D(t_s), \quad S(t) = S(t_s), \quad R(t) = R(t_s), \quad B(t) = B(t_s)$$

$$\Lambda(t) = \Lambda(t_s), \quad Q(t) = Q(t_s), \quad t_{s-1} < t \leq t_s, \quad s = 1, 2, ...$$

Modal approximation

An approximate description of the non-stationary process at a separate stage is based on modal approximation. An approximate solution is sought in the form of decomposition in eigenfunctions of time and α-eigenvalue problem.

Let's $\boldsymbol{u} = \{\phi, \boldsymbol{c}\}, \quad \phi(t_{s-1}) = \phi^s, \quad \boldsymbol{c}(t_{s-1}) = \boldsymbol{c}^s$. In a separate stage s the following system is considered

$$\boldsymbol{B}\frac{d\boldsymbol{u}}{dt} + \boldsymbol{A}\boldsymbol{u} = 0, \quad t_{s-1} < t \leq t_s,$$

with constants

$$\boldsymbol{A} = \begin{pmatrix} D(t_s) + S(t_s) - R(t_s) & -B(t_s) \\ -Q(t_s) & \Lambda(t_s) \end{pmatrix}, \quad \boldsymbol{B} = \begin{pmatrix} V(t_s) & 0 \\ 0 & I \end{pmatrix}.$$

Supplemented by the corresponding initial condition

$$\boldsymbol{u}(t_{s-1}) = \boldsymbol{u}^s.$$

The main feature of the problems we are considering is that the matrices $\boldsymbol{A}_h$ and $\boldsymbol{B}_h$ are real and asymmetric.

Modal approximation

The modal approximation corresponds to the representation of the approximate solution ($\boldsymbol{u}_h \approx \boldsymbol{u}_N$) of problem in the following form

$$\boldsymbol{u}_N(\boldsymbol{x}, t) = \sum_{n=1}^{N} a_n(t)\boldsymbol{w}_n(\boldsymbol{x}),$$

where N is the number of dominant eigenvalues of the spectral problem, $\boldsymbol{w}_n(\boldsymbol{x})$ — corresponding eigenfunctions.
Let us define eigenfunctions and eigenvalues as the solution of the α-eigenvalue problem:

$$\boldsymbol{A}_h\boldsymbol{v} = \lambda \boldsymbol{B}_h\boldsymbol{v}.$$

Then we obtain

$$a_n(t)\boldsymbol{w}_n(\boldsymbol{x}) = b_n \mathrm{Re}\big(\exp(-\lambda_n(t - t_{s-1}))\boldsymbol{v}_n(\boldsymbol{x})\big),$$
$$a_{n+1}(t)\boldsymbol{w}_{n+1}(\boldsymbol{x}) = b_{n+1} \mathrm{Im}\big(\exp(-\lambda_n(t - t_{s-1}))\boldsymbol{v}_n(\boldsymbol{x})\big).$$

A special attention should be paid to define the coefficients $a_n(t_{s-1}) = b_n,\ n = 1, 2, ..., N$. For this, the initial condition is involved. For example, in the case of real eigenvalues, we have

$$\boldsymbol{u}_h^s(\boldsymbol{x}) = \sum_{n=1}^{N_h} b_n \boldsymbol{v}_n(\boldsymbol{x}).$$

Time scale processes

The initial condition includes two components

$$\boldsymbol{u}_h^s(\boldsymbol{x}) = (\phi_h^s(\boldsymbol{x}), \boldsymbol{c}_h^s(\boldsymbol{x})).$$

Dynamic behaviour of these components is due to different time-scale processes.

Delayed neutrons source determines **slow processes**, when $\boldsymbol{c}(\boldsymbol{x}, t)$ changes slightly with the reactor state change. In contrast, neutron flux $\phi(\boldsymbol{x}, t)$ determines **fast processes** when the reactor state changes.

By virtue of this separation of dynamic processes, we model the slow phase of the dynamics of the reactor with modal approximation and orientate ourselves on the approximate prediction of the initial state for delayed neutrons, only the function $\boldsymbol{c}_h^s(\boldsymbol{x})$ is approximated. The approximation $\phi_h^s(\boldsymbol{x})$ is not of interest to us, we do not model a fast phase of the state change.

Adjoint spectral problem

Consider the adjoint spectral problem

$$\boldsymbol{A}_h^T \widetilde{\boldsymbol{v}} = \lambda \boldsymbol{B}_h^T \widetilde{\boldsymbol{v}}.$$

The eigenfunctions of problems are orthogonal in the sense of the equality

$$(\boldsymbol{B}_h \boldsymbol{v}_n, \widetilde{\boldsymbol{v}}_m) = 0, \quad m \neq n, \quad m, n = 1, 2, ..., N_h.$$

In view of this, one can obtain

$$b_n = \frac{1}{(\boldsymbol{B}_h \boldsymbol{v}_n, \widetilde{\boldsymbol{v}}_n)} (\boldsymbol{u}_h^s, \boldsymbol{B}_h \widetilde{\boldsymbol{v}}_n), \quad n = 1, 2, ..., N_h.$$

In the approximate solution of problem only the first N coefficients b_n are used:

$$\boldsymbol{c}_h^s(\boldsymbol{x}) \approx \sum_{n=1}^{N} b_n \boldsymbol{c}_n(\boldsymbol{x}),$$

where $\boldsymbol{v}_n(\boldsymbol{x}) = (\phi_n(\boldsymbol{x}), \boldsymbol{c}_n(\boldsymbol{x}))$. In this case, the spectral problems are solved for N dominant eigenvalues.

Computation scheme

Off-line calculation. Calculation of the coefficients of the mathematical model of the multigroup diffusion approximation for the isolated reactor states, which is performed in advance. The status passport also includes calculated dominant eigenvalues and eigenfunctions of the α-eigenvalue problem. These data can be supplemented by dominant eigenvalues and eigenvalues of the conjugate eigenvalue problem.

On-line calculation. Real-time modeling is carried out on the basis of the modal solution of the problem. The coefficients in the representation are calculated from the initial condition. The solution for other time intervals is determined according to modal approximation.

Hardware

CPU:

- 160 nodes (1920 cores)
- 23.5 TFLOPS (peak), 20.21 TFLOPS (Linpack)

GPU:

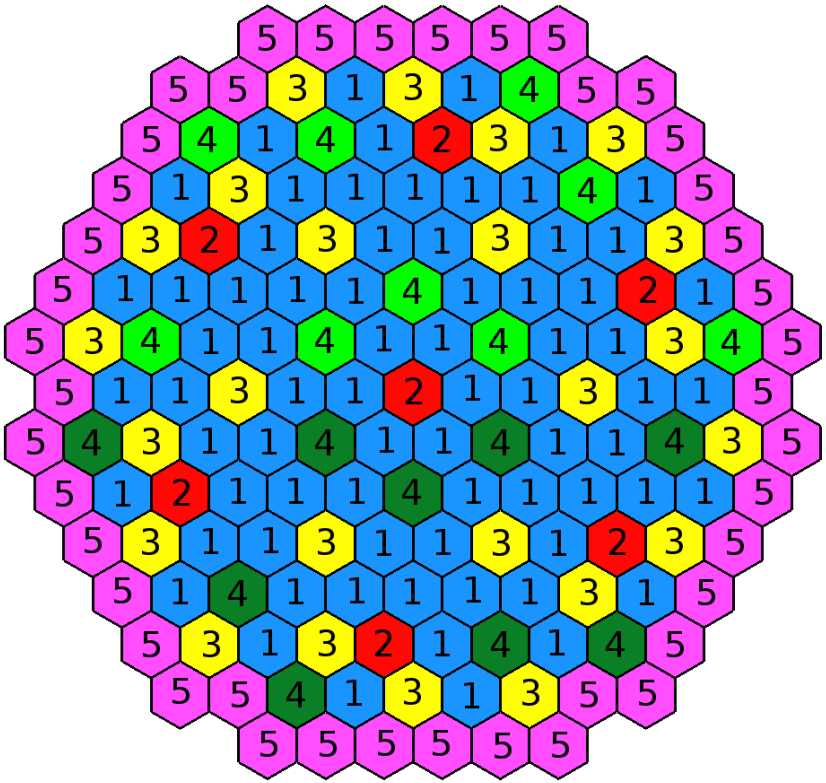
- 15 nodes (45 gpu)
- 22.5 TFLOPS (peak), 11.12 TFLOPS (Linpack)

Software

Geometry and mesh	 Gmsh 3.0.6
Eigenvalue	 SLEPc 3.8
FEM library	 FEniCS 2017.1
Programming	 Python 2.7
Visualization	 ParaView 5.4

VVER-1000 benchmark

The dynamics of the VVER-1000 reactor during the transition from the supercritical mode to the subcritical mode



- two-dimensional
- two group instantaneous neutrons
- one or six group of delayed neutrons
- triangles per cassette κ varies from 6 to 96
- order of finite elements p varies from 1 to 3
- two types of perturbation

Supercritical state: α-eigenvalue problem

Table: Eigenvalues $\alpha_n = \lambda_n^{(\alpha)}$, $n = 1, 2, ..., 5$

p	κ	α_1	α_2, α_3	α_4, α_5
	6	-0.22557	0.04241 $\mp$ 3.08808e-06i	0.06588 $\mp$ 4.80449e-07i
1	24	-0.82690	0.03777 $\mp$ 5.37884e-06i	0.06489 $\mp$ 1.37315e-06i
	96	-1.74998	0.03619 $\mp$ 5.69002e-06i	0.06456 $\mp$ 1.40299e-06i
	6	-2.10154	0.03592 $\mp$ 4.96474e-06i	0.06452 $\mp$ 1.21320e-06i
2	24	-2.46601	0.03562 $\mp$ 5.78277e-06i	0.06445 $\mp$ 1.40897e-06i
	96	-2.50375	0.03559 $\mp$ 5.80693e-06i	0.06444 $\mp$ 1.41324e-06i
	6	-2.47975	0.03561 $\mp$ 5.83718e-06i	0.06445 $\mp$ 1.41869e-06i
3	24	-2.50294	0.03559 $\mp$ 5.80783e-06i	0.06444 $\mp$ 1.41341e-06i
	96	-2.51280	0.03558 $\mp$ 5.80954e-06i	0.06444 $\mp$ 1.41362e-06i

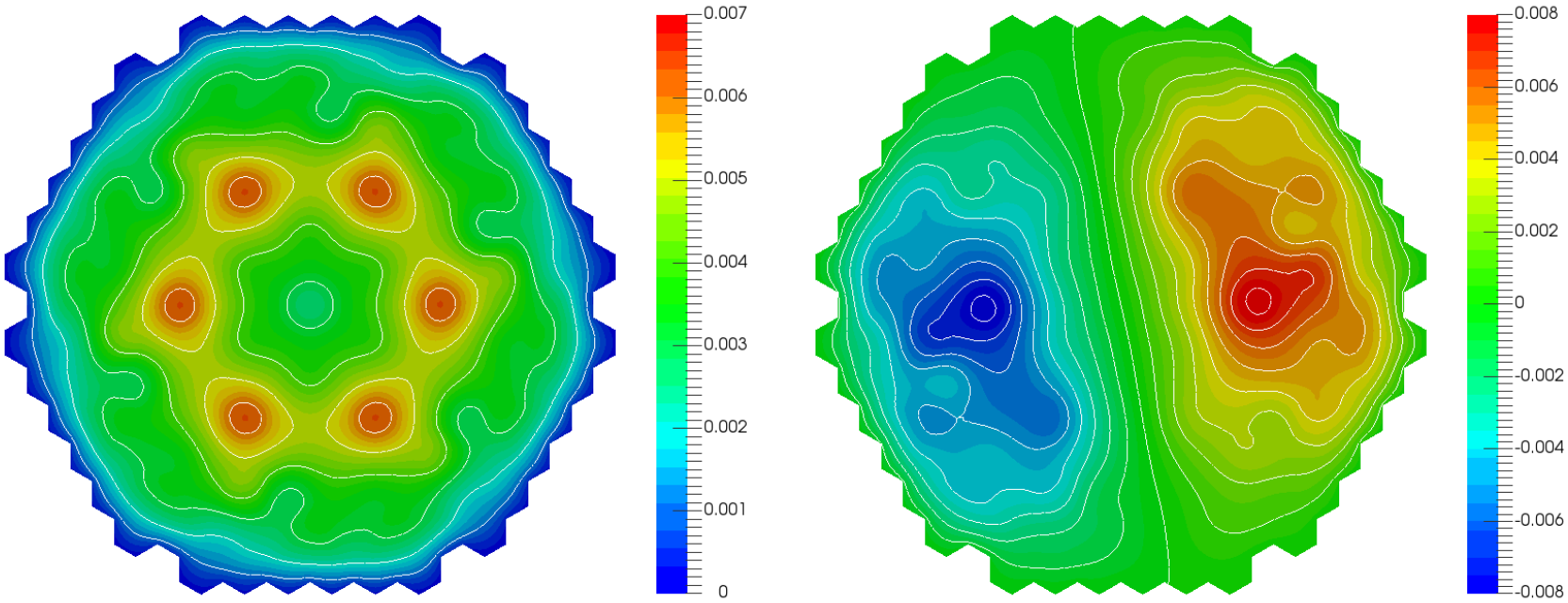


Figure: The eigenfunction $\varphi_1^{(1)}$ (left) and real part of eigenfunctions $\varphi_1^{(2)}$, $\varphi_1^{(3)}$ (right).

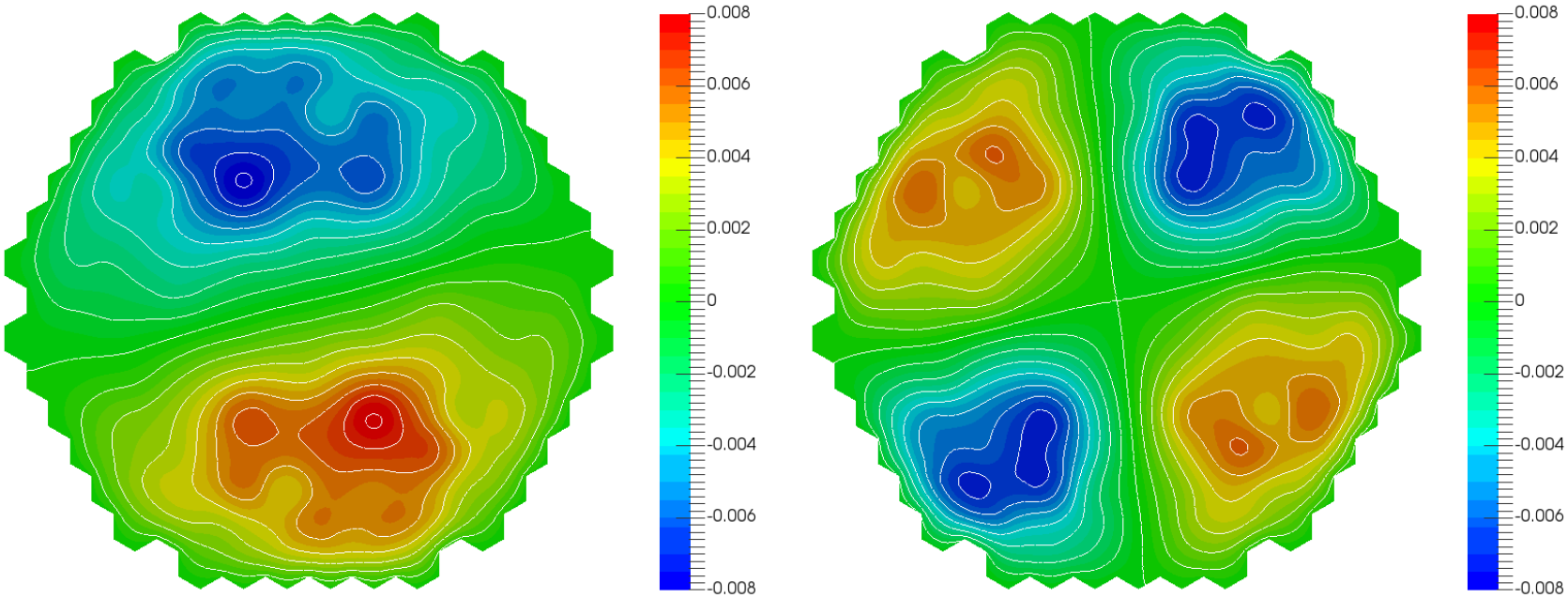


Figure: Imaginary part of eigenfunctions $\varphi_1^{(2)}$, $-\varphi_1^{(3)}$ (left) and real part of eigenfunctions $\varphi_1^{(4)}$, $\varphi_1^{(5)}$ (right).

The symmetric perturbation

In the supercritical mode, due to the sufficiently large magnitude of the main eigenvalue, the regular regime of the reactor is rapidly developing, where

$$\boldsymbol{u}(\boldsymbol{x}, t) \approx a_1 \exp(-\alpha_1 t)\boldsymbol{v}_1^0(\boldsymbol{x}).$$

Here $\boldsymbol{v}_1^0(\boldsymbol{x})$ is the first mode of the supercritical state. We consider the problem with the transition from this supercritical state at $t_0 = 0$ to the subcritical state.
The subcritical stage is characterized by a 15% increase in the coefficient Σ_2 for material 4 in the VVER-1000 test diffusion constants. Thus, the dynamics of the reactor is as follows:

$$\Sigma_2 \longrightarrow 1.15\Sigma_2 \quad (\text{material } 4).$$

The initial state is characterized by specifying the initial conditions at $t_0 = 0$ as

$$\boldsymbol{u}(\boldsymbol{x}, 0) = \boldsymbol{v}_1^0(\boldsymbol{x}).$$

Adjoint spectral problem

Table: Eigenvalues $\alpha_n = \lambda_n^{(\alpha)}$, $n = 1, 2, ..., 10$ for direct and adjoint problems

n	direct	adjoint
1	-2.51280117966	-2.51280117972
2,3	0.0355815000364 $\mp$ 5.809545e-06	0.0355815000365 $\mp$ 5.809544e-06
4,5	0.0644427013767 $\mp$ 1.413622e-06	0.0644427013767 $\mp$ 1.413622e-06
6	0.0702618501639	0.0702618501639
7	0.0714652882224	0.0714652882164
8	0.0726456060606	0.0726456060606
9,10	0.0734708921578 $\mp$ 4.023323e-08	0.0734708921578 $\mp$ 4.023321e-08

Table: Scalar product $(\phi_1^{(n)}, \phi_1^{(m)})$, $n, m = 1, 2, ..., 5$

$n \backslash m$	1	2	3	4	5
1	1.0e-00	1.3e-08	2.2e-08	-3.8e-08	9.8e-09
2	1.3e-08	1.0e-00	-1.6e-08	-1.6e-08	1.4e-08
3	2.2e-08	-1.6e-08	1.0e-00	-9.8e-09	-1.1e-08
4	-3.8e-08	-1.6e-08	-9.8e-09	1.0e-00	-3.9e-10
5	9.8e-09	1.4e-08	-1.1e-08	-3.9e-10	1.0e-00
6	-1.8e-09	4.1e-08	-1.8e-08	-1.1e-08	2.9e-09
7	1.0e-02	1.2e-09	1.1e-08	1.4e-08	-1.6e-08
8	-3.2e-09	-2.0e-07	-3.3e-08	4.0e-09	-1.9e-08
9	-2.2e-08	-3.1e-03	-7.5e-03	3.0e-09	6.3e-09
10	1.6e-09	7.5e-03	-3.1e-03	-1.1e-08	6.3e-09

Modal approximation

The spectral problems under consideration are characterized by small imaginary parts of the eigenvalues. Therefore, we can expect that the eigenfunctions of problem are close to orthogonal.
For an approximate solution, we use the following formulation:

$$\boldsymbol{u}_N(\boldsymbol{x}, t) = \sum_{n=1}^{N} b_n \exp(-\mathrm{Re}\,\alpha_n\, t)\boldsymbol{v}_n(\boldsymbol{x}),$$

where the coefficients b_n, $n = 1, 2, ..., N$ are calculated according to the given initial condition.

Coefficients for symmetric perturbation

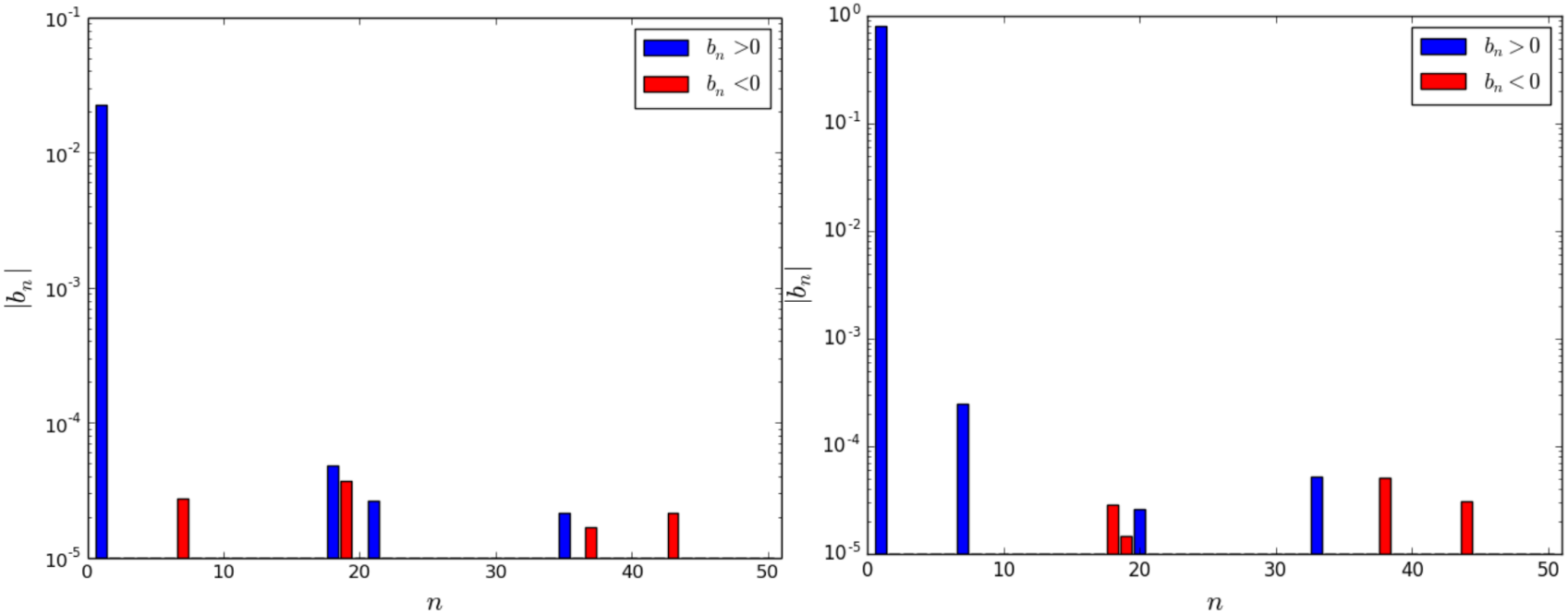


Figure: Approximate solution coefficients t=0 and t=10

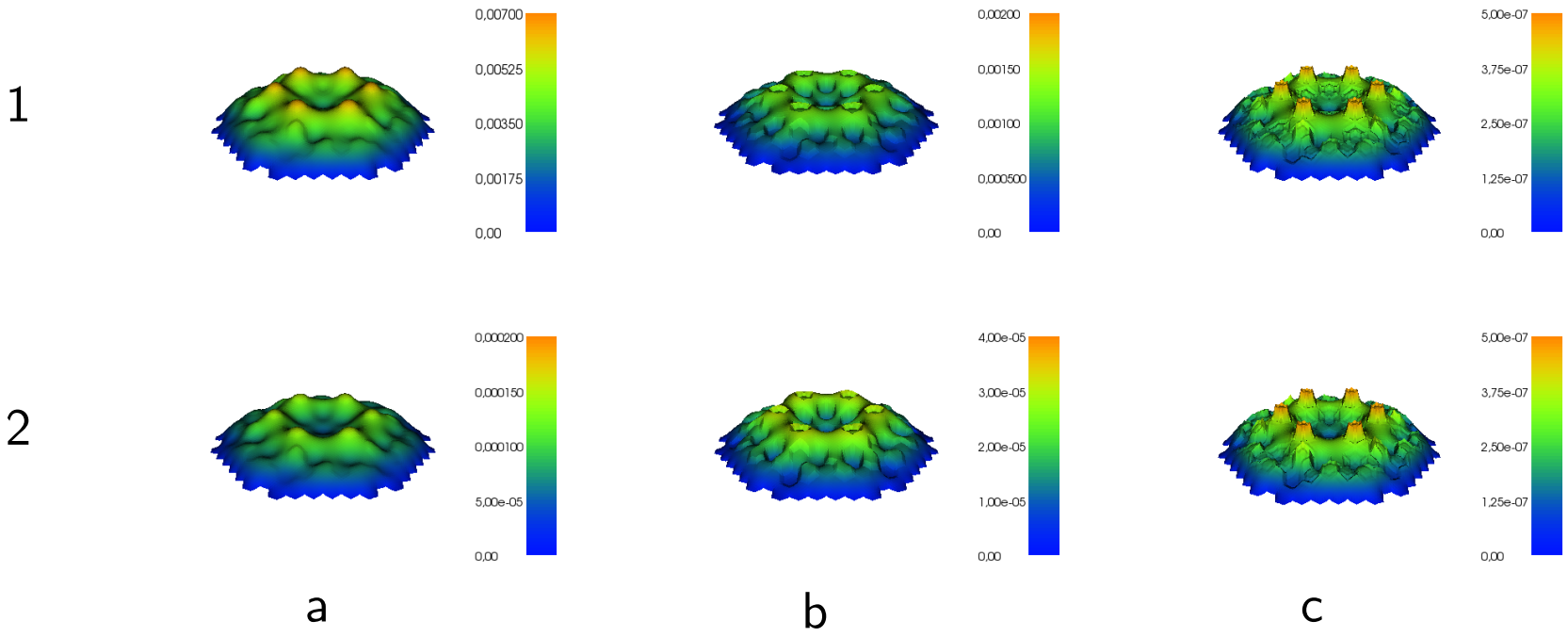
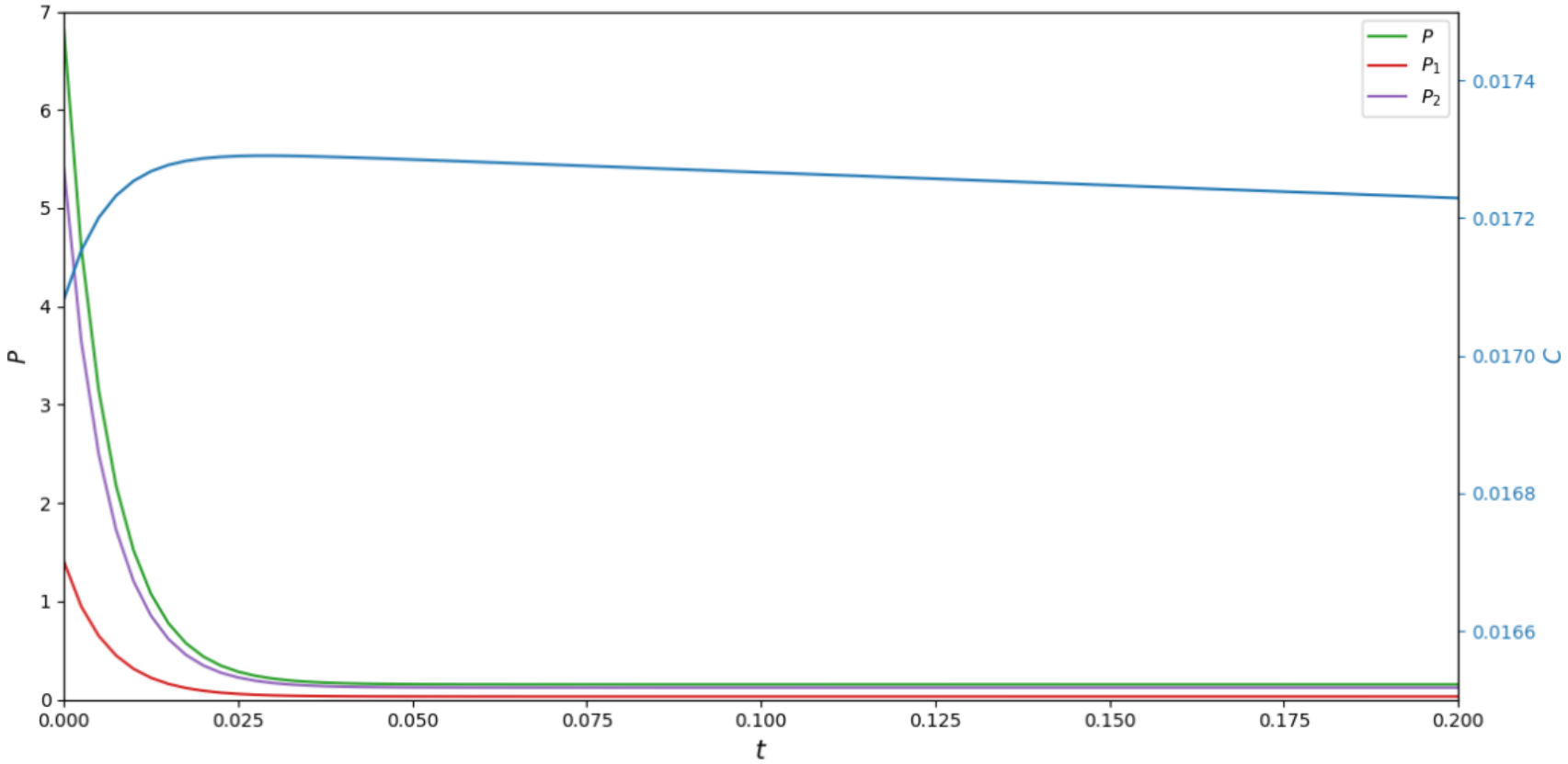
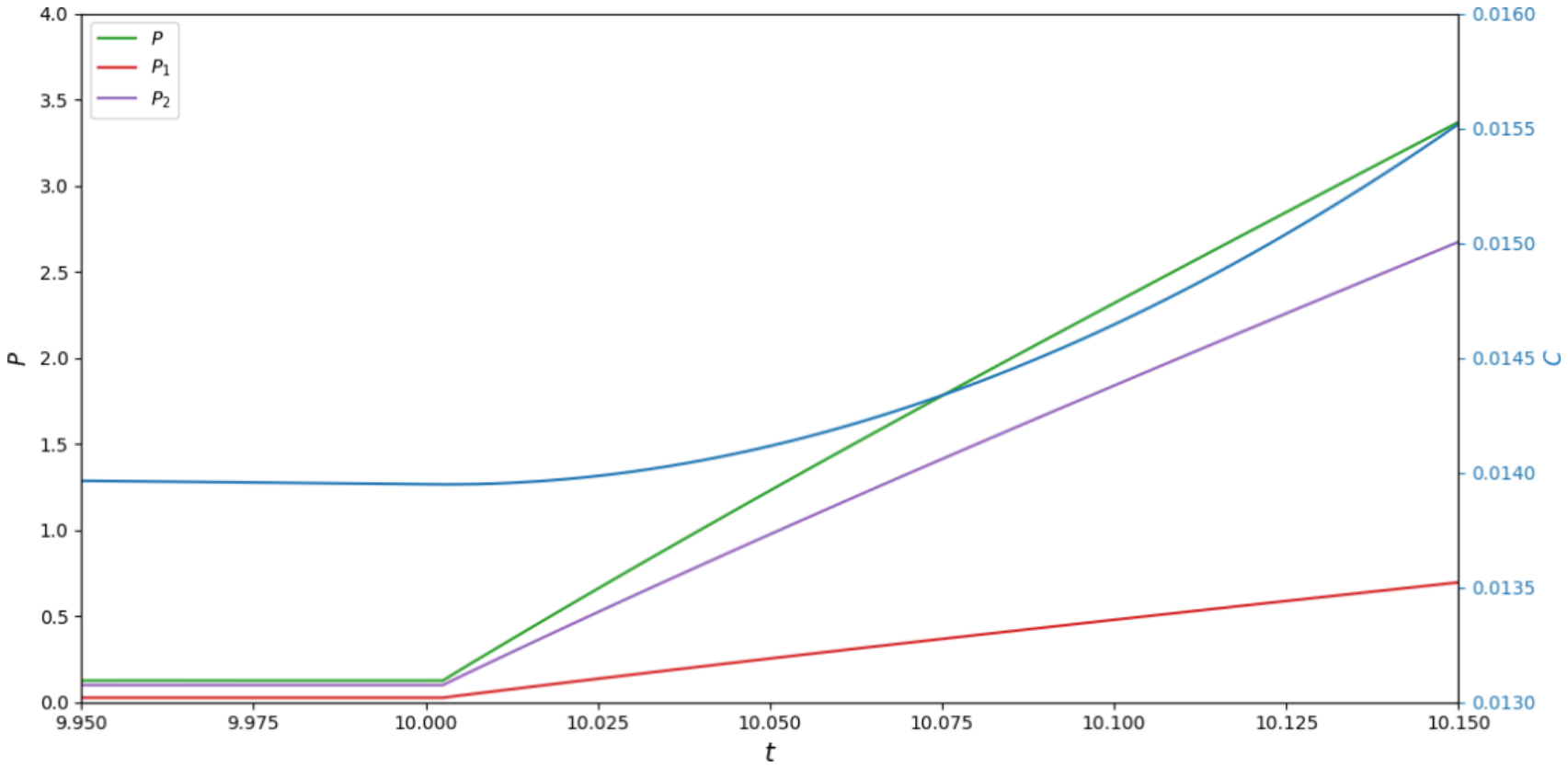


Figure: Function $\boldsymbol{u}(\boldsymbol{x}, 0)$ (string 1) and function $\boldsymbol{u}_N(\boldsymbol{x}, 0)$ (ctring 2): a — neutron flux of group 1, b — neutron flux of group 2, c — delayed neutrons source.

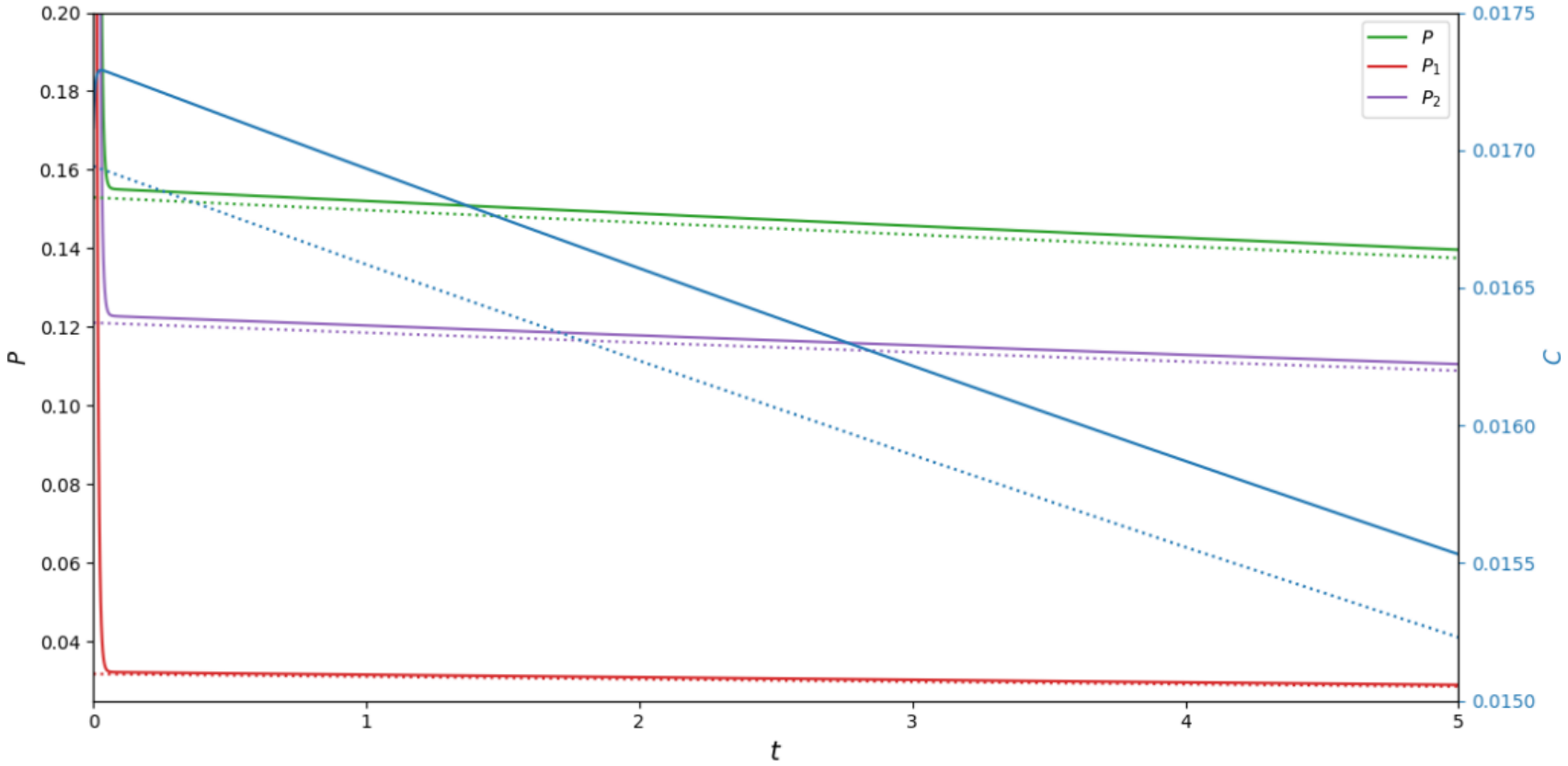
Fast stage t=0



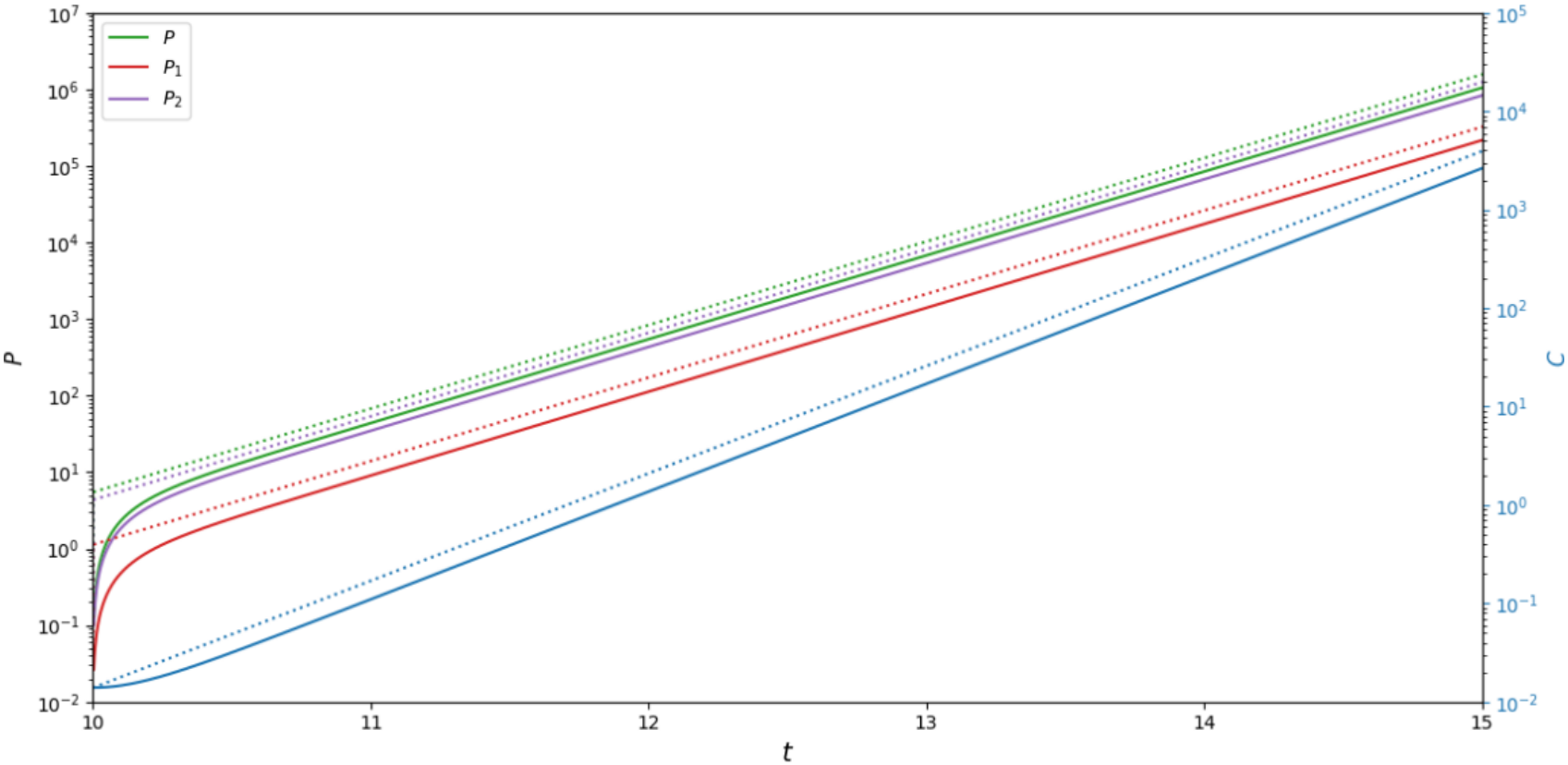
Fast stage t=10



Slow stage t=0



Slow stage t=10



Coefficients for assymetric perturbation

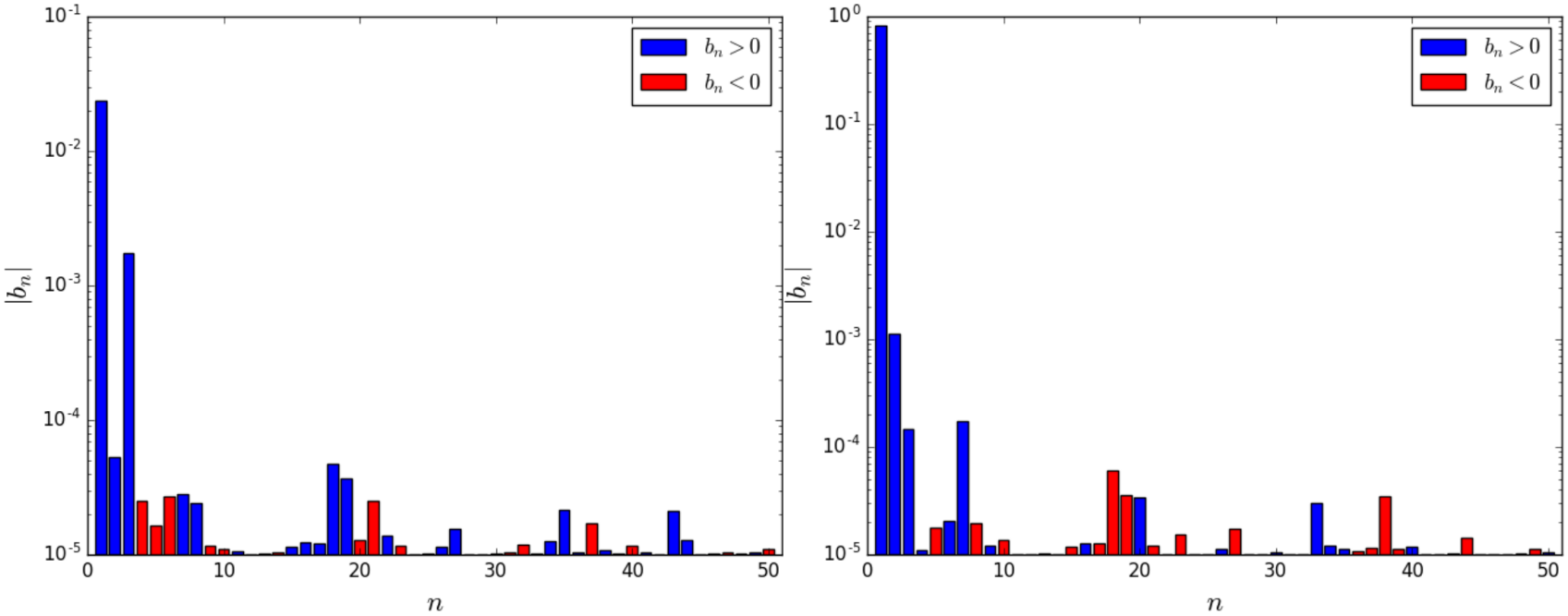


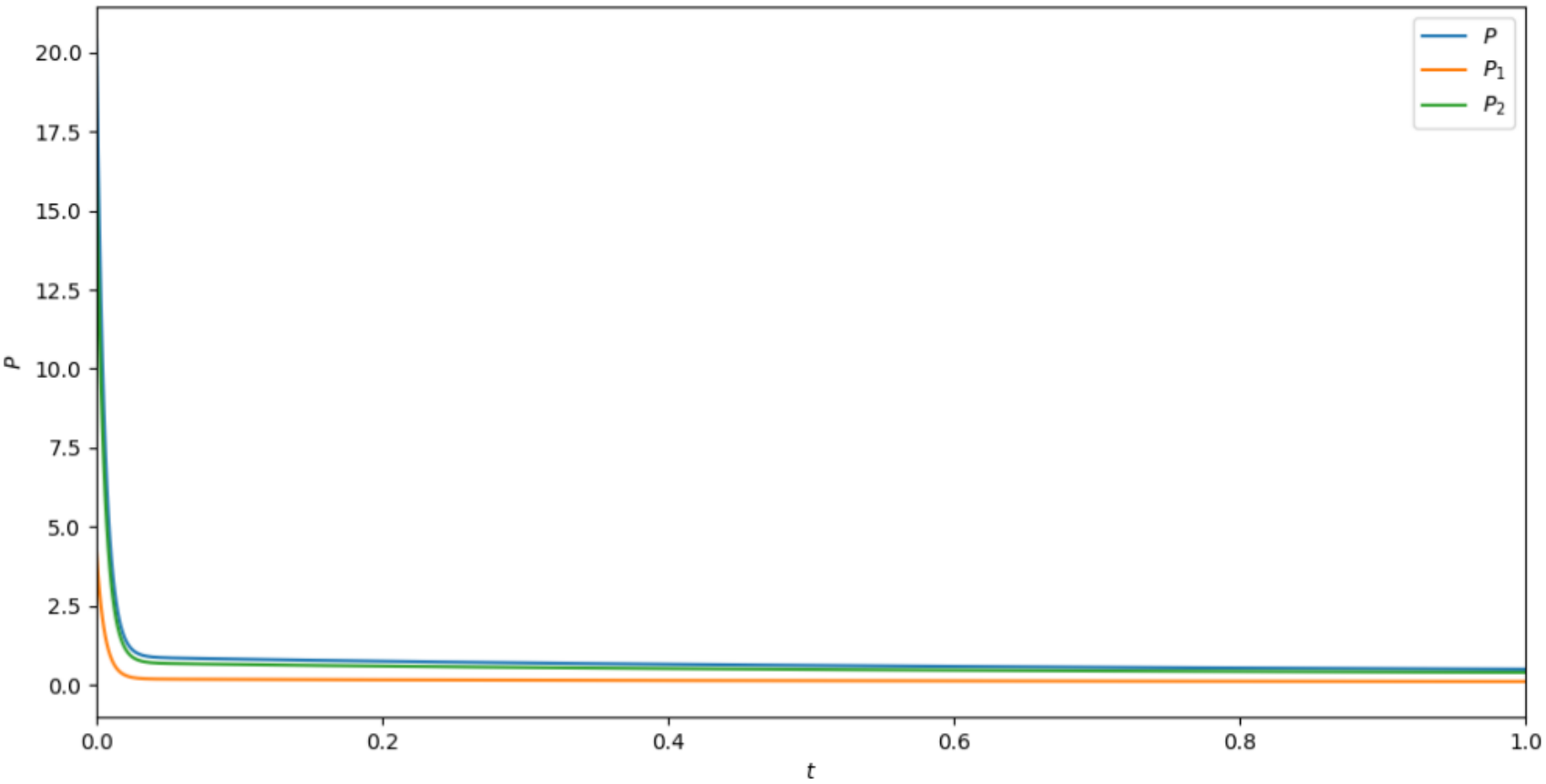
Figure: Approximate solution coefficients t=0 and t=10

α-eigenvalue problem

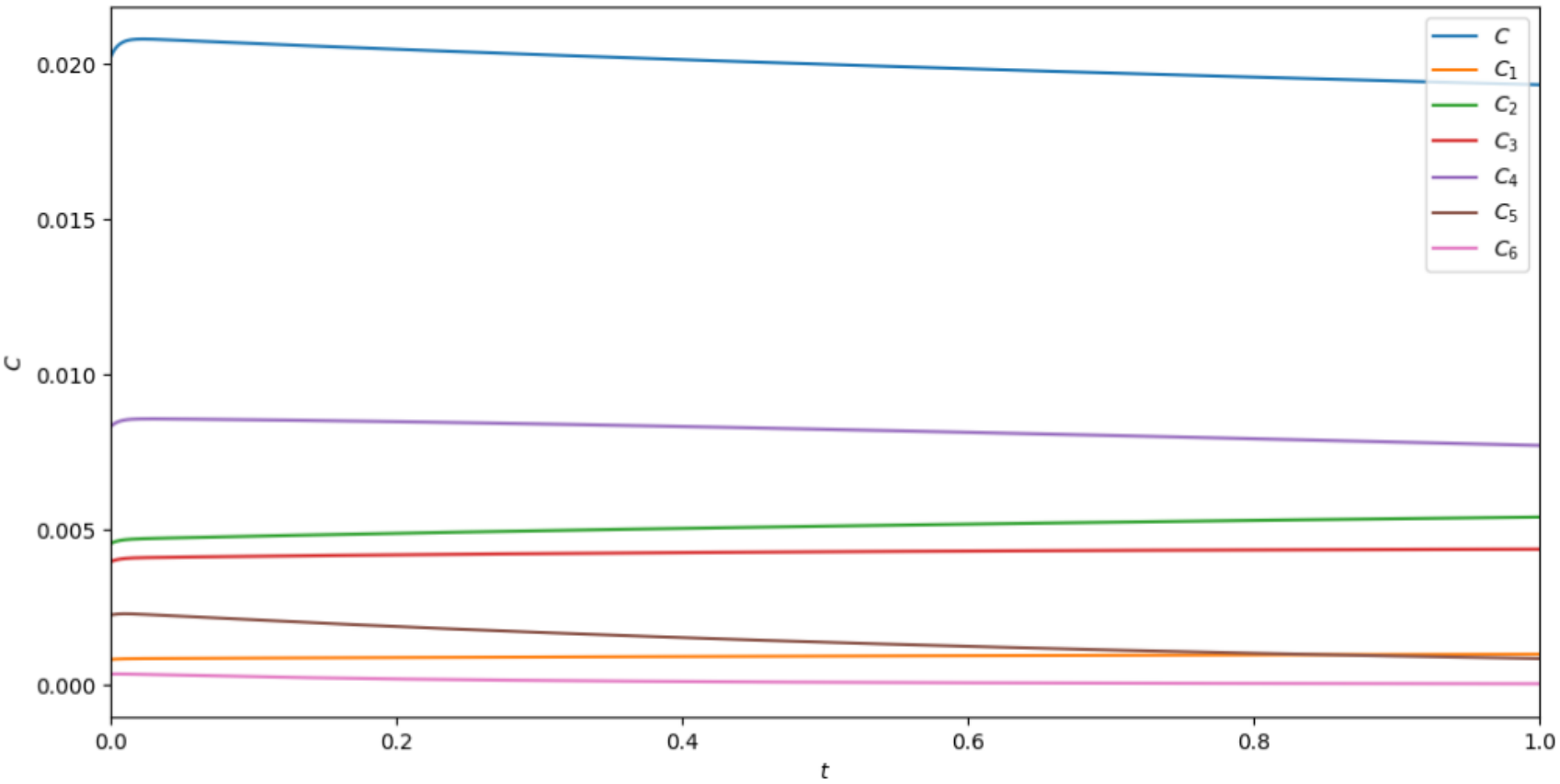
Table: Eigenvalues $\alpha_n = \lambda_n^{(\alpha)}$, $n = 1, 2, ..., 10$ for a six-group delayed neutron problems

n	supercritical state	subcritical state
1	-6.25232556772	0.0107985727746
2,3	0.0119801965951 $\mp$ 2.473859e-07	0.0124470473769 $\mp$ 3.393001e-08
4,5	0.0125793204697 $\mp$ 1.409007e-08	0.0126093618587 $\mp$ 9.323010e-09
6	0.0126317468066	0.0126453184341
7	0.0126413530651	0.0126526726807
8	0.0126504165154	0.0126539227972
9,10	0.0126565535405 $\mp$ 2.957858e-10	0.0126613234777 $\mp$ 1.072561e-10

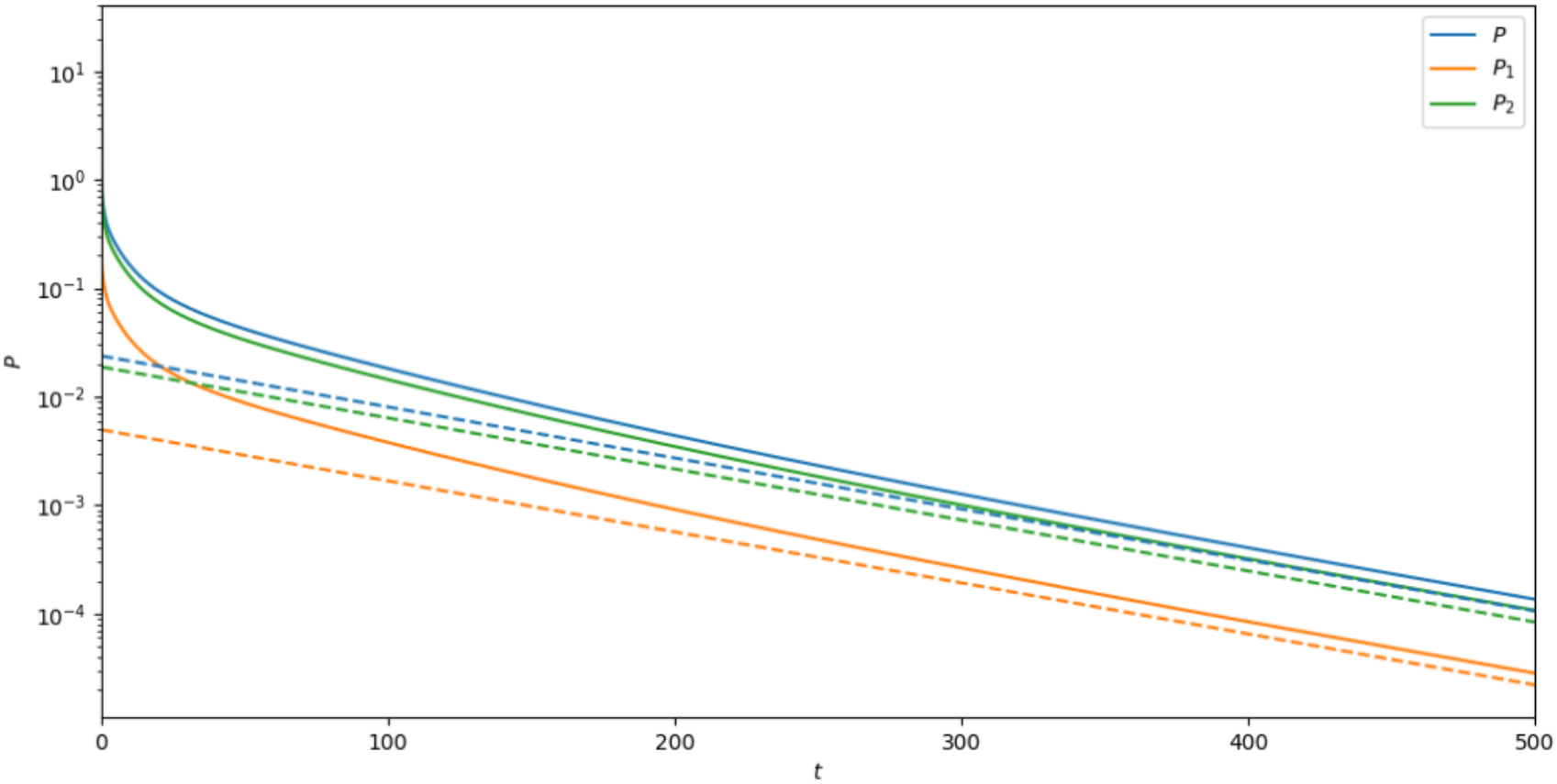
Fast stage: neutronic power



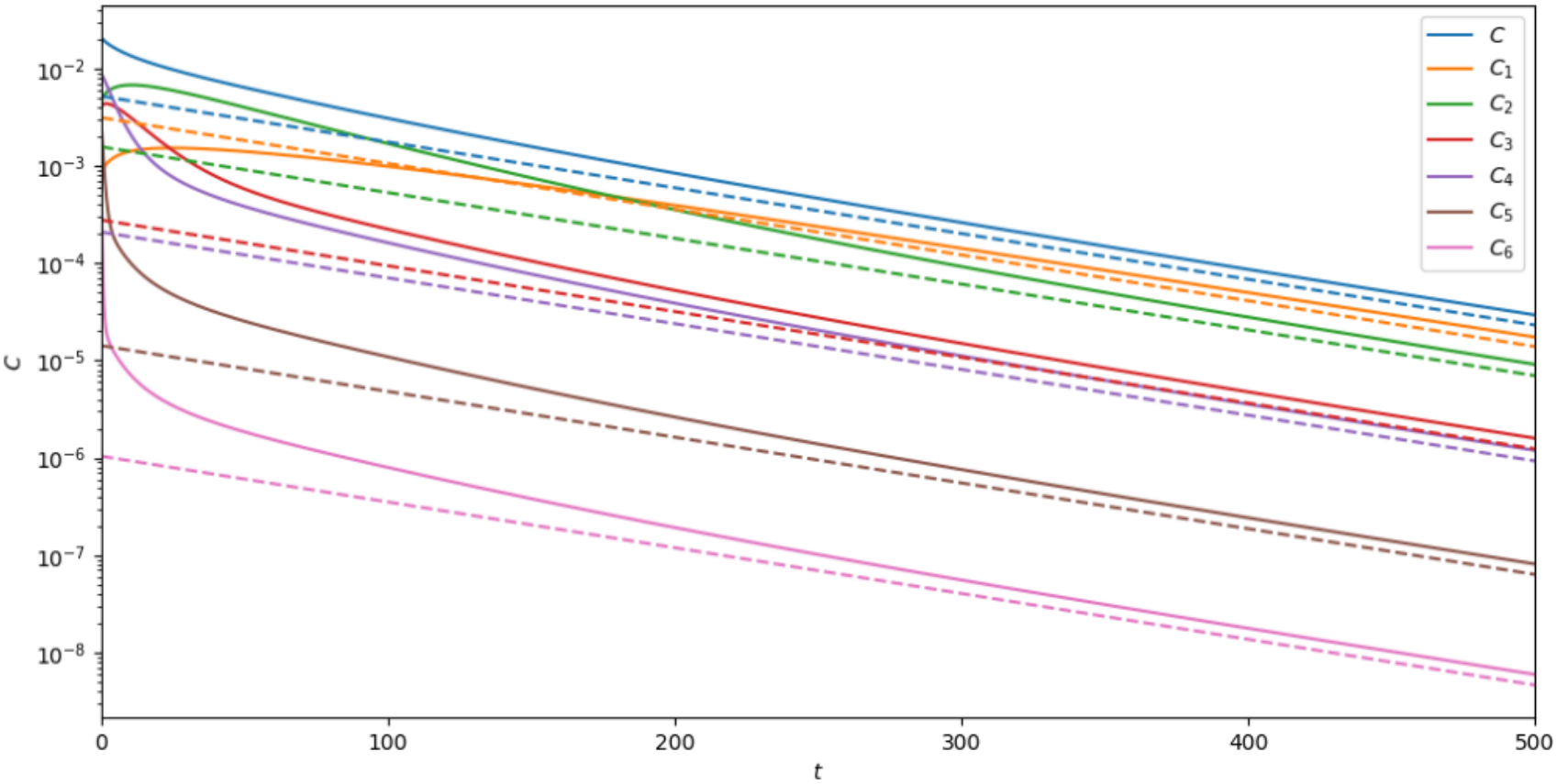
Fast stage: source of delayed neutron



Slow stage: neutronic power



Slow stage: source of delayed neutron



Thank you for your attention!