MMR lab seminar

# Numerical Modelling of Neutron Transport in $SP_3$ Approximation by FEM

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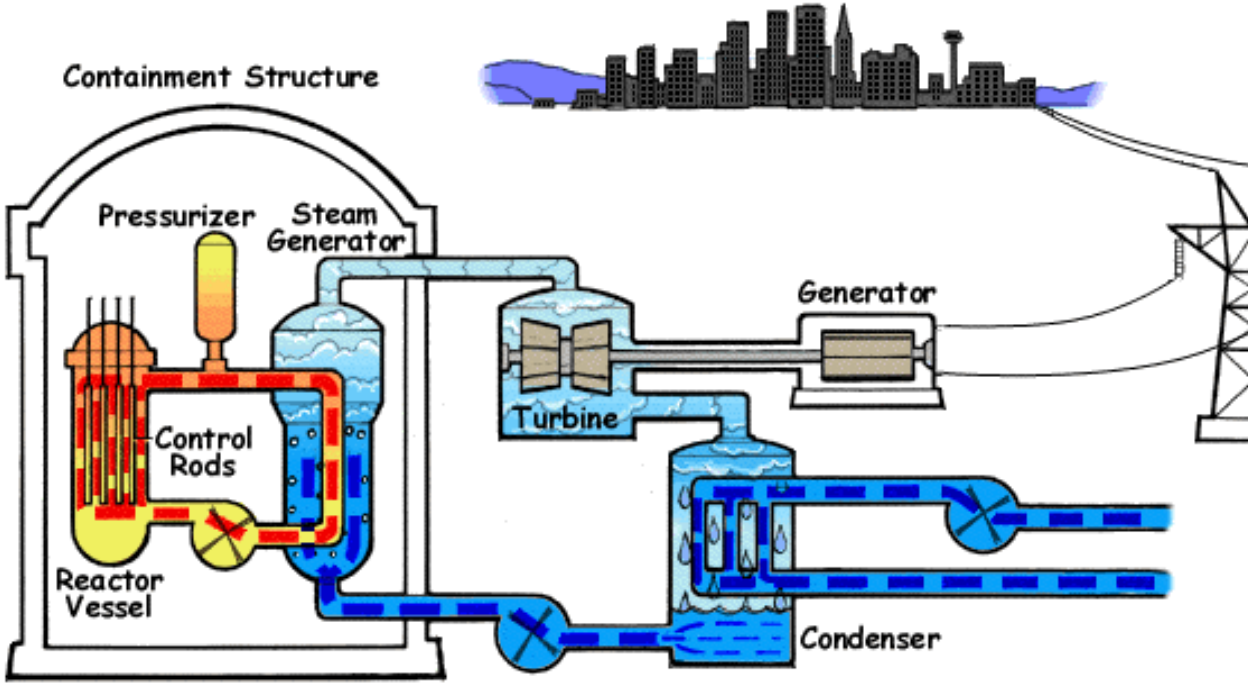
# Motivation


Nuclear
Reactor


Containment Structure
Pressurizer
Steam
Generator
Generator
Turbine
Control
Rods
Reactor
Vessel
Condenser

- Neutron transport equation
    - time, energy, spatial and angular variables (7 unknowns)

- Diffusion approximation
- $SP_3$ approximation

# Equations

Diffusion approximation

$$\frac{1}{v_g}\frac{\partial \phi_g}{\partial t} - \nabla \cdot D_g \nabla \phi_g + \Sigma_{rg}\phi_g = (1-\beta)\chi_g S_n + S_{s,g} + \widetilde{\chi}_g S_d,$$

$$S_n = \sum_{g'=1}^{G} \nu\Sigma_{f,g'}\phi_{g'}, \quad S_{s,g} = \sum_{g \neq g'=1}^{G} \Sigma_{s,g'\to g}\phi_{g'}, \quad S_d = \sum_{m=1}^{M} \lambda_m c_m .$$

SP$_3$ approximation

$$\phi_{0,g} = \phi_g + 2\phi_{2,g}$$

$$\frac{1}{v_g}\frac{\partial \phi_{0,g}}{\partial t} - \frac{2}{v_g}\frac{\partial \phi_{2,g}}{\partial t} - \nabla \cdot D_{0,g} \nabla \phi_{0,g} + \Sigma_{r,g}\phi_{0,g} - 2\Sigma_{r,g}\phi_{2,g} = (1-\beta)\chi_{n,g} S_n + S_{s,g} + \chi_{d,g} S_d,$$

$$-\frac{2}{v_g}\frac{\partial \phi_{0,g}}{\partial t} + \frac{9}{v_g}\frac{\partial \phi_{2,g}}{\partial t} - \nabla \cdot D_{2,g} \nabla \phi_{2,g} + (5\Sigma_{t,g} + 4\Sigma_{r,g})\phi_{2,g} - 2\Sigma_{r,g}\phi_{0,g} = -2(1-\beta)\chi_{n,g} S_n - 2S_{s,g} - 2\chi_{d,g} S_d,$$

Delayed neutron source

$$\frac{\partial c_m}{\partial t} + \lambda_m c_m = \beta_m S_n, \quad m = 1,2,...,M, \quad G = 1,2,...,G$$

# Operator notation

Diffusion approximation

$$\mathbf{u} = \{\phi_1, \phi_2, \ldots, \phi_G\}, \quad \mathbf{c} = \{c_1, c_2, \ldots, c_M\}$$

$$V\frac{\partial \mathbf{u}}{dt} + A\mathbf{u} = (1-\beta)F\mathbf{u} + E\mathbf{c}, \qquad \frac{\partial \mathbf{c}}{dt} + \Lambda\mathbf{c} = Q\mathbf{u}\,.$$

SP$_3$ approximation

$$\mathbf{u_1} = \{\phi_{0,1}, \phi_{0,2}, \cdots, \phi_{0,G}\}, \quad \mathbf{u_2} = \{\phi_{2,1}, \phi_{2,2}, \cdots, \phi_{2,G}\}\,.$$

$$V(\frac{\partial \mathbf{u_1}}{\partial t} - 2\frac{\partial \mathbf{u_2}}{\partial t}) + A_1\mathbf{u_1} + B\mathbf{u_2} = (1-\beta)F(\mathbf{u_1} - 2\mathbf{u_2}) + E\mathbf{c},$$

$$\frac{\partial \mathbf{c}}{dt} + \Lambda\mathbf{c} = Q(\mathbf{u_1} - 2\mathbf{u_2})\,.$$

$$V(-2\frac{\partial \mathbf{u_1}}{\partial t} + 9\frac{\partial \mathbf{u_2}}{\partial t}) + A_2\mathbf{u_2} + B\mathbf{u_1} = -2(1-\beta)F(\mathbf{u_1} - 2\mathbf{u_2}) - 2E\mathbf{c},$$

# Software

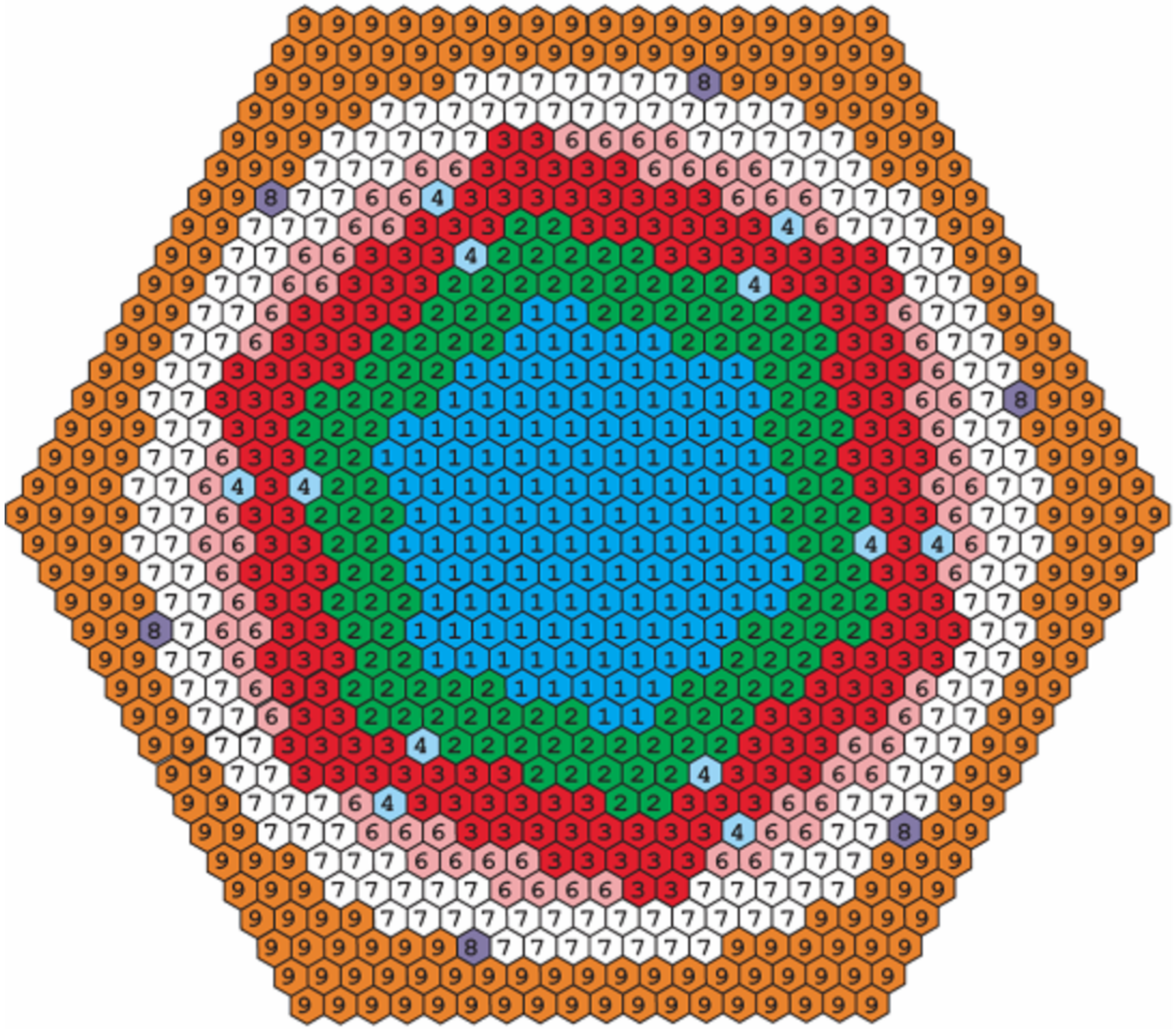
gmsh

SLEPc

FEniCS project

python™

# HWR test problem



2D geometry and stationary

2 groups of instantaneous (G=2) neutrons

Lambda- and alpha- spectral problems

# Spectral problems

Diffusion approximation

SP$_3$ approximation

Lambda-

$$A\mathbf{y} = \lambda^{(k)} F\mathbf{y}$$

$$L\varphi = \lambda^{(k)} M\varphi$$

Alpha-

$$A\mathbf{y} - (1-\beta)F\mathbf{y} - E\mathbf{s} = \lambda^{(\alpha)} V\mathbf{y}$$

$$\Lambda\mathbf{s} - Q\mathbf{y} = \lambda^{(\alpha)}\mathbf{y}$$

$$L\varphi - (1-\beta)M\varphi - I\mathbf{s} = \lambda^{(\alpha)} W\varphi,$$

$$\Lambda\mathbf{s} - R\varphi = \lambda^{(\alpha)}\mathbf{s}\,.$$

$$\varphi = \{\mathbf{y_1}, \mathbf{y_2}\}, \quad L = \begin{pmatrix} A_1 & B \\ B & A_2 \end{pmatrix}, \quad M = \begin{pmatrix} F & -2F \\ -2F & 4F \end{pmatrix}, \quad I = \begin{pmatrix} E \\ -2E \end{pmatrix}, \quad R = \begin{pmatrix} Q & -2Q \end{pmatrix}, \quad W = \begin{pmatrix} V & -2V \\ -2V & 9V \end{pmatrix}$$

# Solution of lambda modes spectral problem

Table: The effective multiplication factor

| $n$ | $p$ | $k_{dif}$ | $\Delta_{dif}, pcm$ | $\delta_{dif}$ | $k_{sp_3}$ | $\Delta_{sp_3}, pcm$ | $\delta_{dif}$ |
|---|---|---|---|---|---|---|---|
| 6 | 1 | 0.991985 | 2.0 | 1.16 | 0.992178 | 5.0 | 0.80 |
| | 2 | 0.991989 | 2.4 | 0.31 | 0.992166 | 3.8 | 0.24 |
| | 3 | 0.991964 | 0.1 | 0.08 | 0.992132 | 0.4 | 0.07 |
| 24 | 1 | 0.991983 | 1.8 | 0.05 | 0.992165 | 3.7 | 0.08 |
| | 2 | 0.991965 | 0.0 | 0.01 | 0.992133 | 0.5 | 0.01 |
| | 3 | 0.991963 | 0.2 | 0.01 | 0.992128 | 0.0 | 0.00 |
| 96 | 1 | 0.991969 | 0.4 | 0.08 | 0.992140 | 1.2 | 0.01 |
| | 2 | 0.991963 | 0.2 | 0.02 | 0.992129 | 0.1 | 0.00 |
| | 3 | 0.991963 | 0.2 | 0.01 | 0.992128 | – | – |
| Ref. | | 0.991965 | | | 0.992128 | | |

# Solution of lambda modes spectral problem

Table: The first 10 eigenvalues for p=3, n=96

| $i$ | diffusion | $SP_3$ |
|---|---|---|
| 1 | $0.991963 + 0.0i$ | $0.992128 + 0.0i$ |
| 2 | $0.983594 + 1.1645\text{e-}05i$ | $0.983793 + 1.2072\text{e-}05i$ |
| 3 | $0.983594 - 1.1645\text{e-}05i$ | $0.983793 - 1.2072\text{e-}05i$ |
| 4 | $0.964240 + 2.1564\text{e-}05i$ | $0.964523 + 2.2337\text{e-}05i$ |
| 5 | $0.964240 - 2.1564\text{e-}05i$ | $0.964523 - 2.2337\text{e-}05i$ |
| 6 | $0.943290 + 0.0i$ | $0.943733 + 0.0i$ |
| 7 | $0.923872 + 0.0i$ | $0.924257 + 0.0i$ |
| 8 | $0.918657 + 0.0i$ | $0.918798 + 0.0i$ |
| 9 | $0.895682 + 3.5570\text{e-}05i$ | $0.896317 + 3.6750\text{e-}05i$ |
| 10 | $0.895682 - 3.5570\text{e-}05i$ | $0.896317 + 3.6750\text{e-}05i$ |

# Solution of alpha modes spectral problem

Table: The first 10 eigenvalues for p=3, n=96

| $i$ | Diffusion | $SP_3$ |
|---|---|---|
| 1 | $42.263 + 0.0i$ | $41.380 + 0.0i$ |
| 2 | $84.867 - 0.06130i$ | $83.821 - 0.06358i$ |
| 3 | $84.867 + 0.06130i$ | $83.821 + 0.06358i$ |
| 4 | $182.914 - 0.11367i$ | $181.471 - 0.11805i$ |
| 5 | $182.914 + 0.11367i$ | $181.471 + 0.11805i$ |
| 6 | $293.017 + 0.0i$ | $290.940 + 0.0i$ |
| 7 | $371.528 + 0.0i$ | $369.374 + 0.0i$ |
| 8 | $515.465 - 0.16397i$ | $512.337 - 0.17197i$ |
| 9 | $515.465 + 0.16397i$ | $512.337 + 0.17197i$ |
| 10 | $518.670 + 0.0i$ | $517.975 + 0.0i$ |

# Solution of alpha modes spectral problem

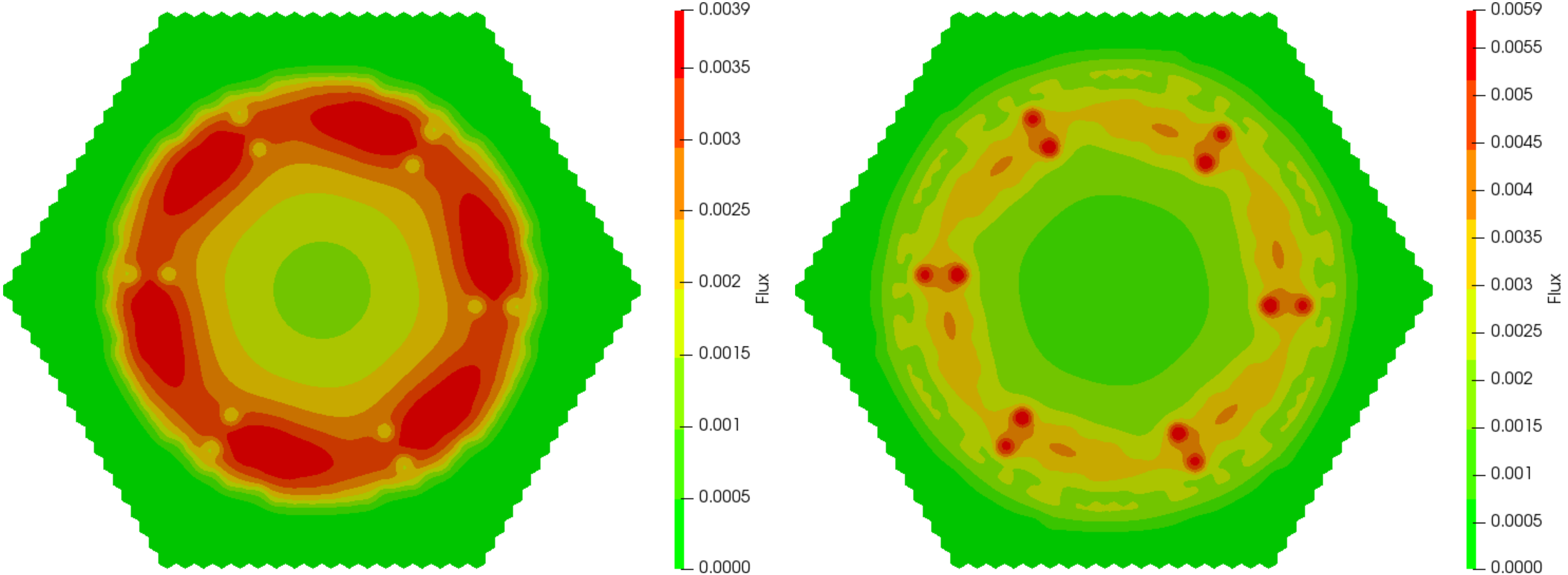



Fig: The eigenfunctions $\phi_1^{(1)}, \phi_2^{(1)}$

# Solution of alpha modes spectral problem

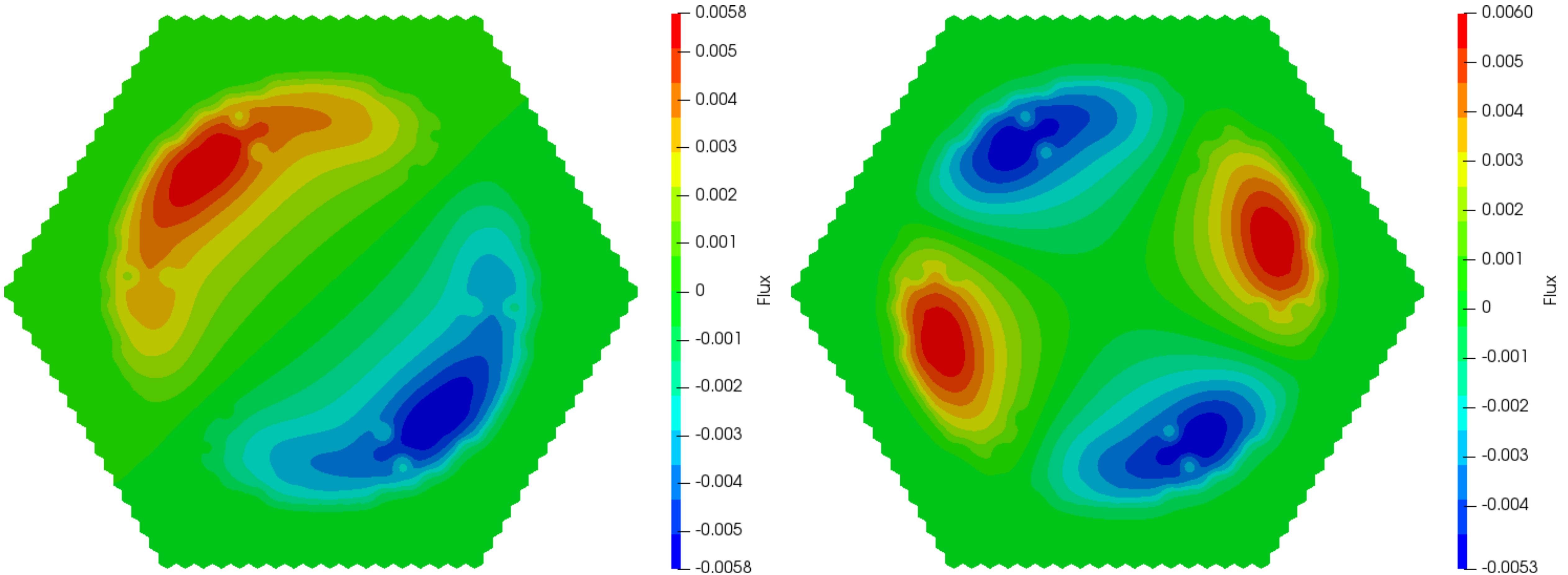



Fig: Real part of eigenfunctions $\phi_1^{(2)}, \phi_2^{(3)}$ and $\phi_1^{(4)}, \phi_2^{(5)}$

# Solution of alpha modes spectral problem

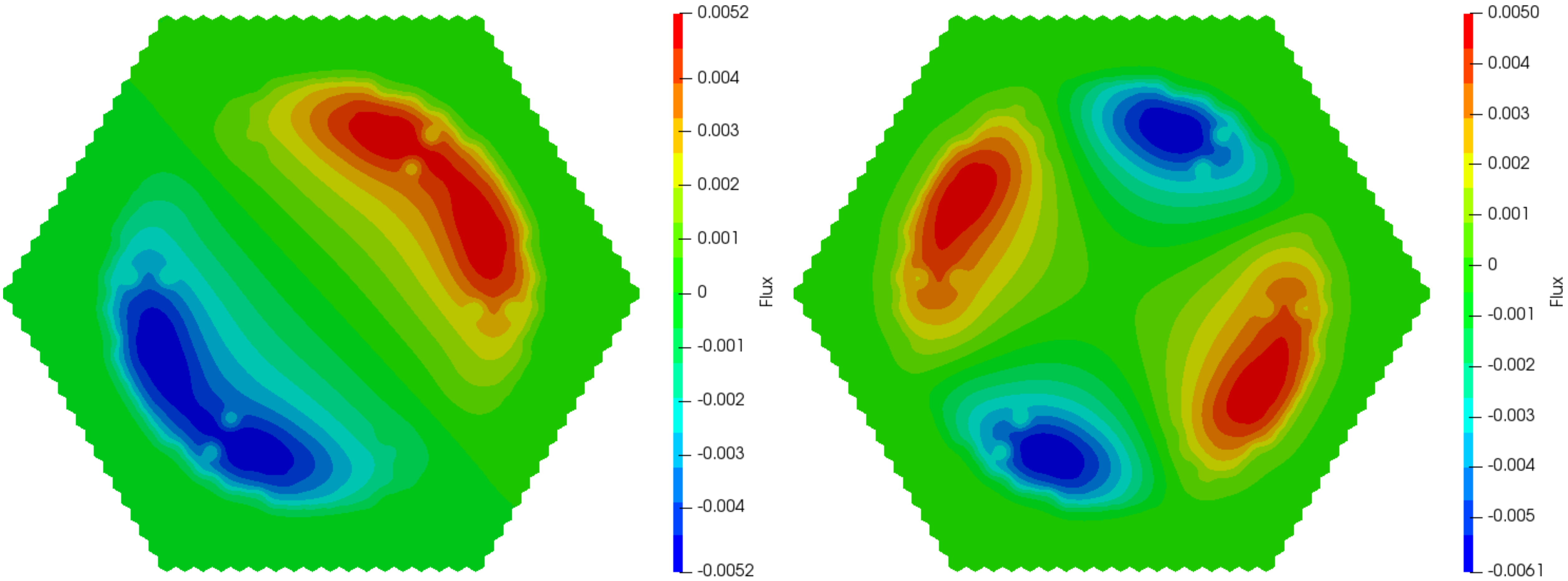
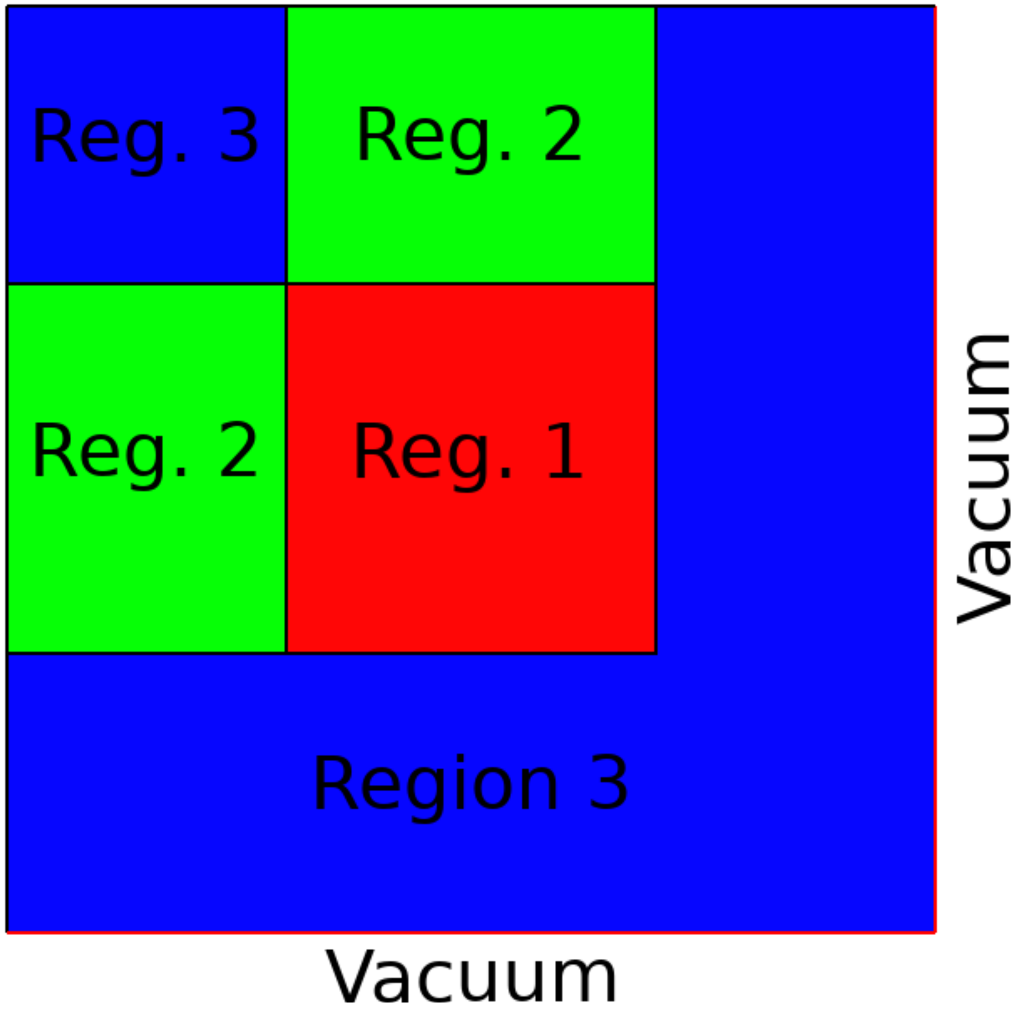



Fig: imaginary part of eigenfunctions $\phi_1^{(2)}, -\phi_2^{(3)}$ and $\phi_1^{(4)}, -\phi_2^{(5)}$

# TWIGL test problem




2D geometry and non-stationary

2 groups of instantaneous (G=2) and
1 group of delayed (M=2) neutrons

3 scenarios of dynamic process

# Solution of lambda modes spectral problem

Table: The effective multiplication factor

| $n$ | $p$ | $k_{dif}(\gamma = 0.5)$ | $k_{dif}(\gamma = 100)$ | $k_{sp3}$ |
|---|---|---|---|---|
| | 1 | 0.915519 | 0.913286 | 0.916144 |
| 9 | 2 | 0.915519 | 0.913333 | 0.916190 |
| | 3 | 0.915419 | 0.913234 | 0.916094 |
| | 1 | 0.915486 | 0.913288 | 0.916147 |
| 36 | 2 | 0.915423 | 0.913238 | 0.916096 |
| | 3 | 0.915408 | 0.913223 | 0.916076 |
| | 1 | 0.915434 | 0.913245 | 0.916102 |
| 144 | 2 | 0.915409 | 0.913223 | 0.916076 |
| | 3 | 0.915408 | 0.913222 | 0.916073 |

# Scenario №1 (step change)

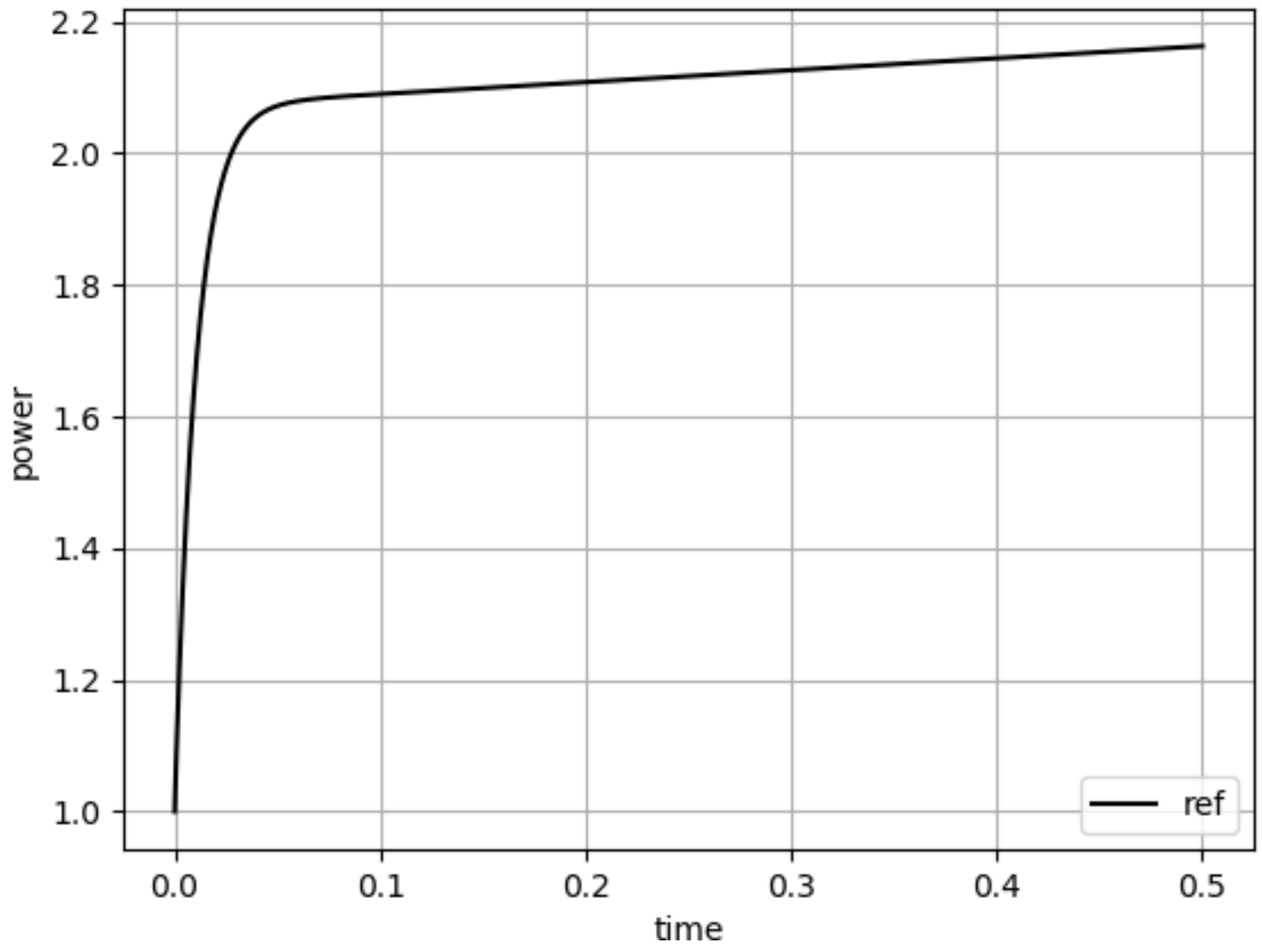



Fig: Reference solution for the $SP_3$ model

$$P(t) = a \sum_{g=1}^{G} \int_{\Omega} \Sigma_{fg} \phi_g \, d\boldsymbol{x}$$

$n = 144$, $p = 3$, $\tau = 0.0001$

# Scenario №1 (step change)

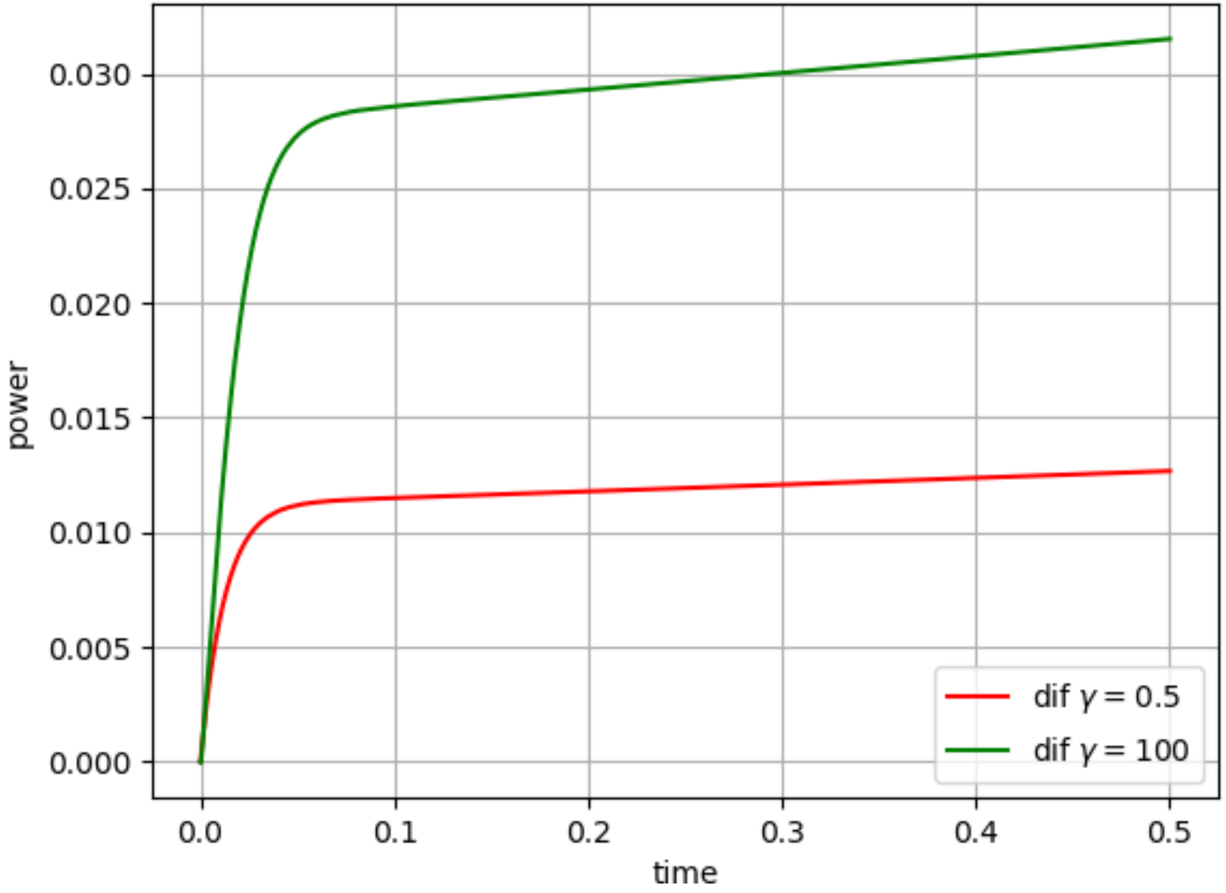



Fig: Differences of the diffusion solution from $SP_3$

| $t$ | Dif ($\gamma = 0.5$) | Dif ($\gamma = 100$) | $SP_3$ |
|---|---|---|---|
| 0.0 | 1.0000 | 1.0000 | 1.0000 |
| 0.1 | 2.0783 | 2.0613 | 2.0898 |
| 0.2 | 2.0961 | 2.0786 | 2.1079 |
| 0.3 | 2.1139 | 2.0959 | 2.1260 |
| 0.4 | 2.1319 | 2.1135 | 2.1442 |
| 0.5 | 2.1500 | 2.1311 | 2.1626 |

Fig: Reference solutions

# Scenario №1 (step change)

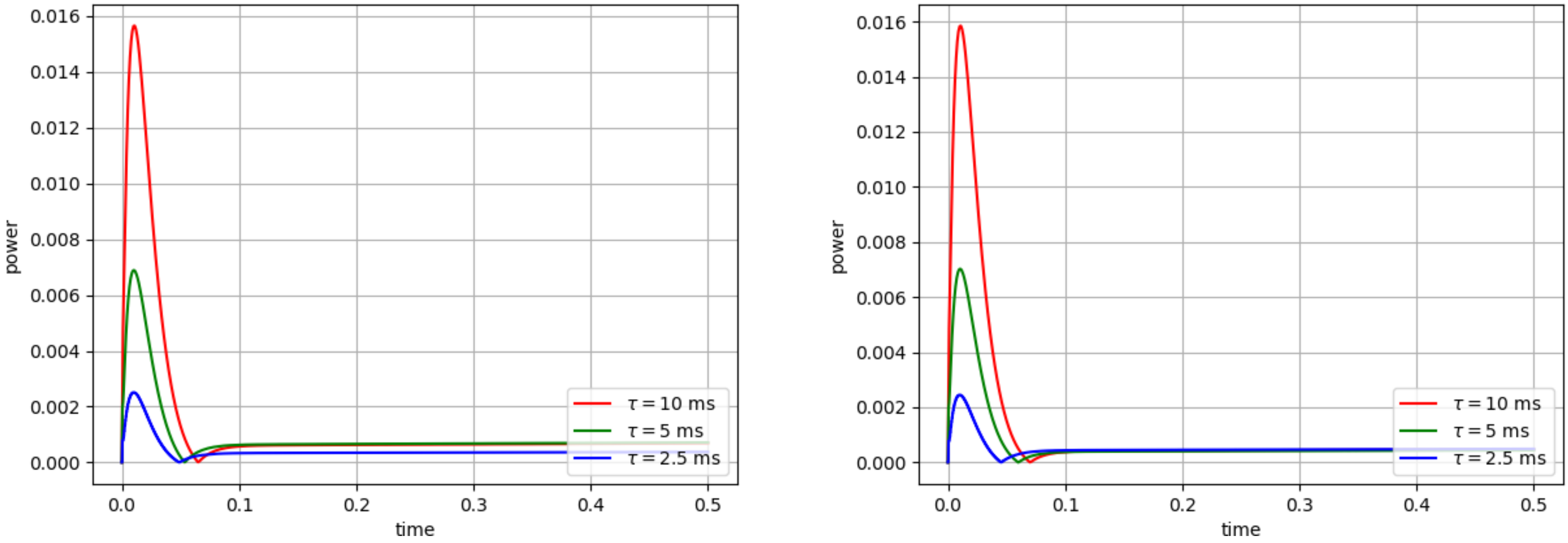



Fig: Differences at different time steps for diffusion and $SP_3$ models

# Scenario №2 (linear change)

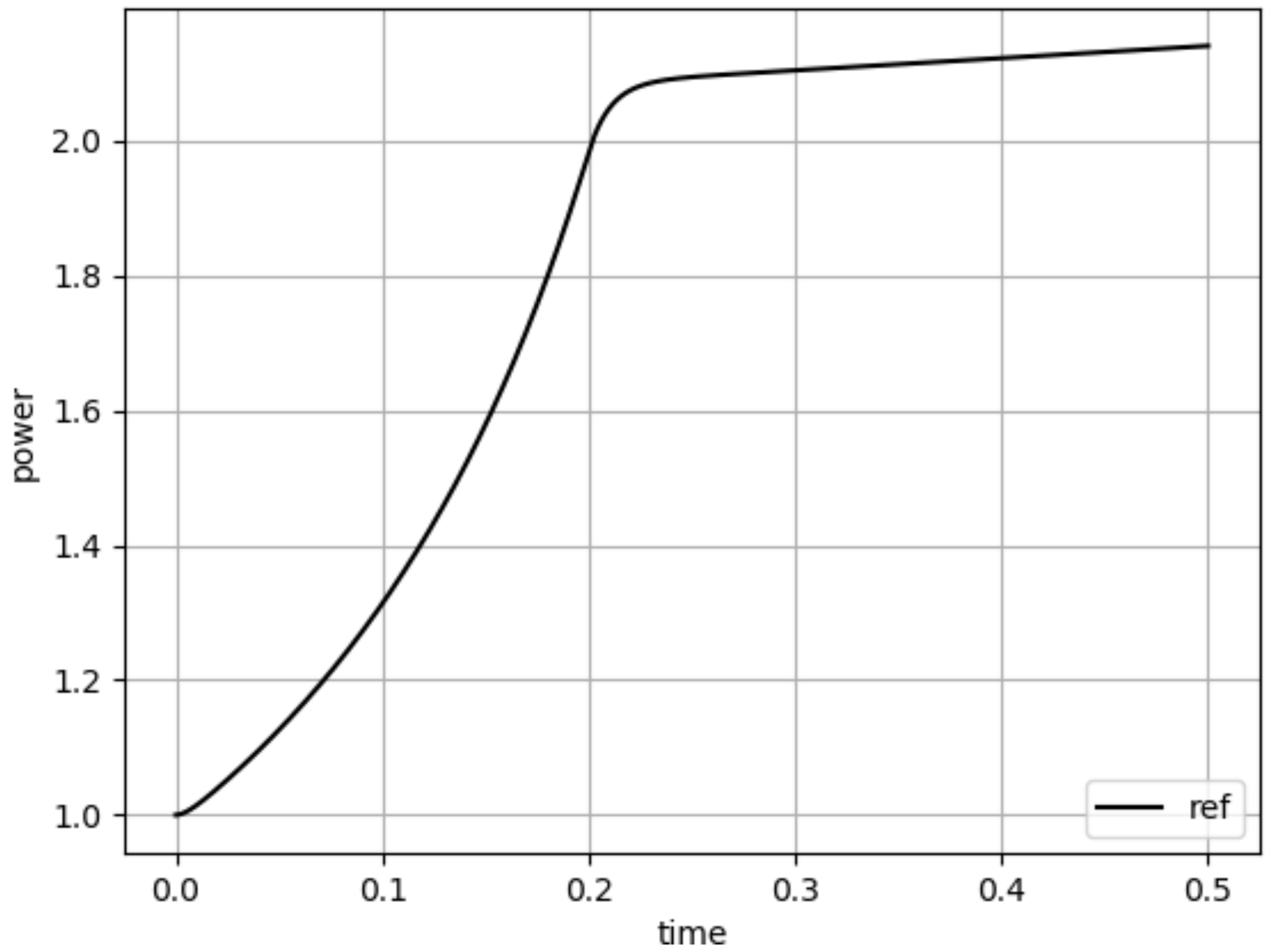



Fig: Reference solution for the $SP_3$ model

$n = 144,\ p = 3,\ \tau = 0.0001$

# Scenario №2 (linear change)

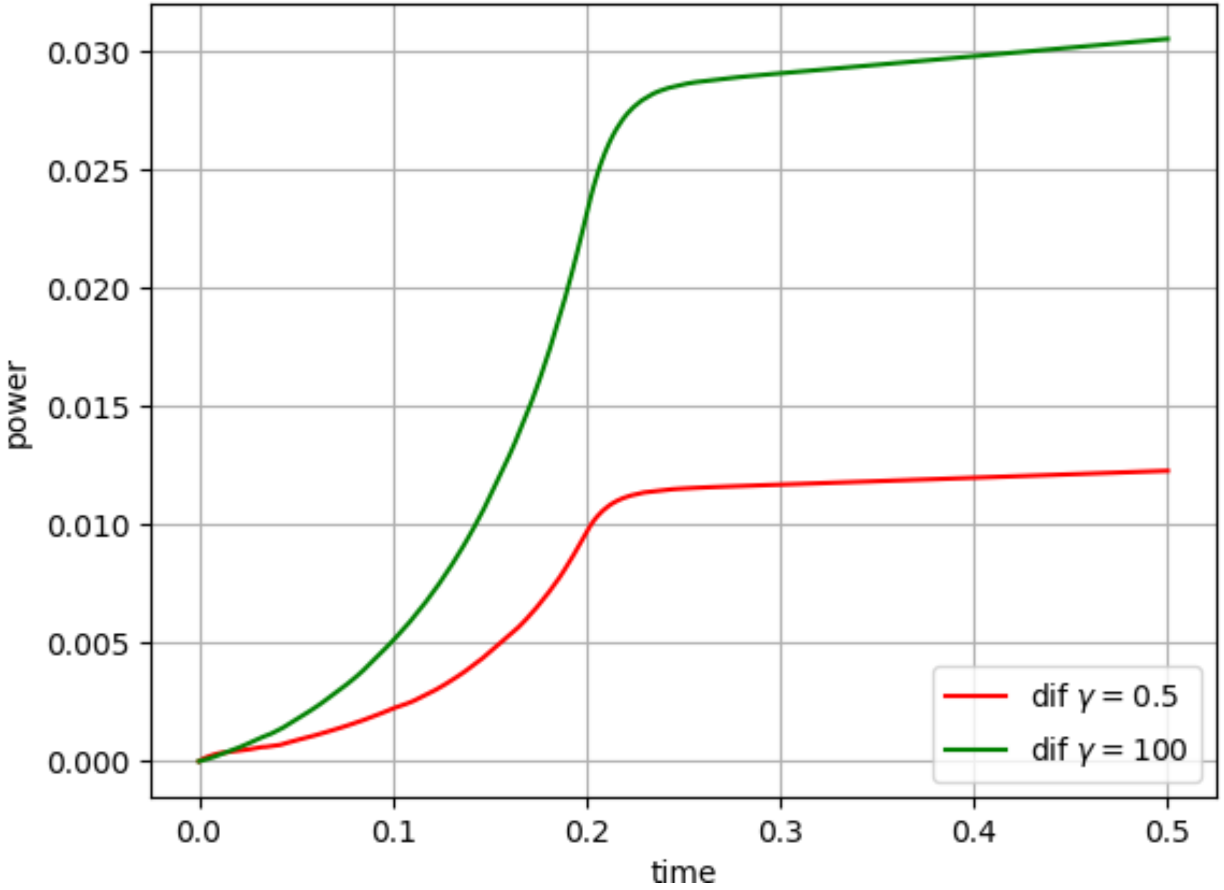



Fig: Differences of the diffusion solution from $SP_3$

| $t$ | Dif ($\gamma = 0.5$) | Dif ($\gamma = 100$) | $SP_3$ |
|---|---|---|---|
| 0.0 | 1.0000 | 1.0000 | 1.0000 |
| 0.1 | 1.3112 | 1.3083 | 1.3134 |
| 0.2 | 1.9729 | 1.9595 | 1.9826 |
| 0.3 | 2.0921 | 2.0747 | 2.1038 |
| 0.4 | 2.1099 | 2.0921 | 2.1219 |
| 0.5 | 2.1278 | 2.1096 | 2.1401 |

Fig: Reference solutions

# Scenario №2 (linear change)

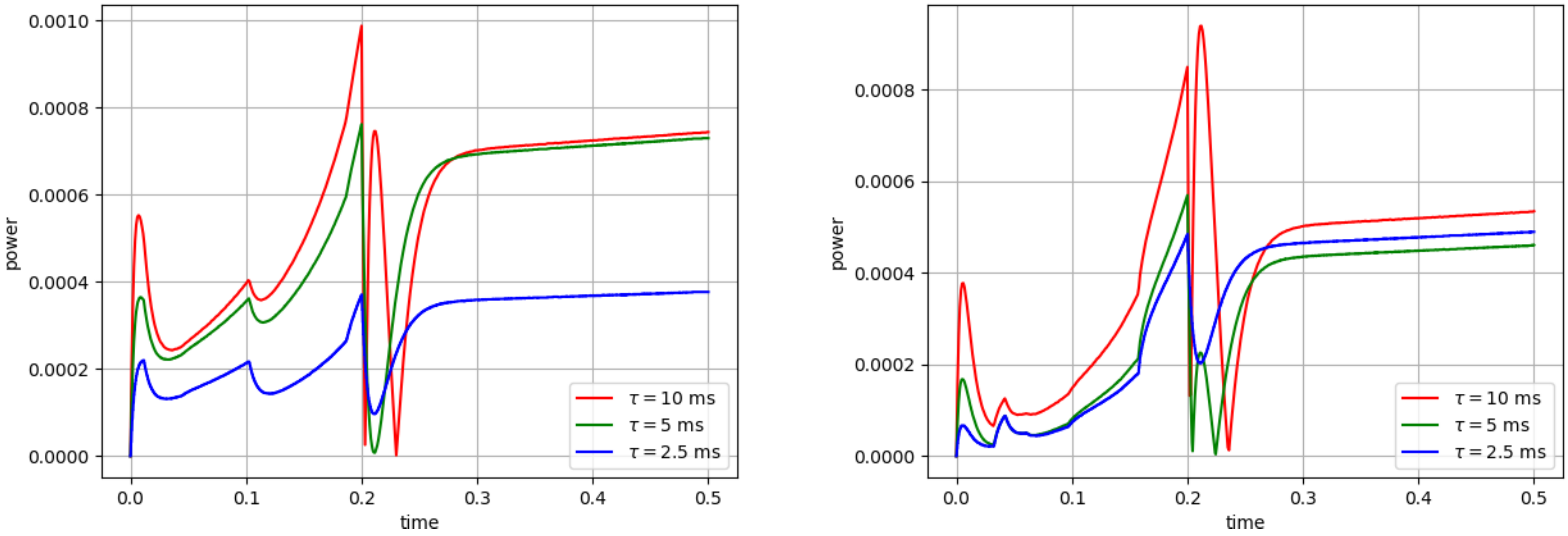



Fig: Differences at different time steps for diffusion and $SP_3$ models

# Scenario №3 (combined)

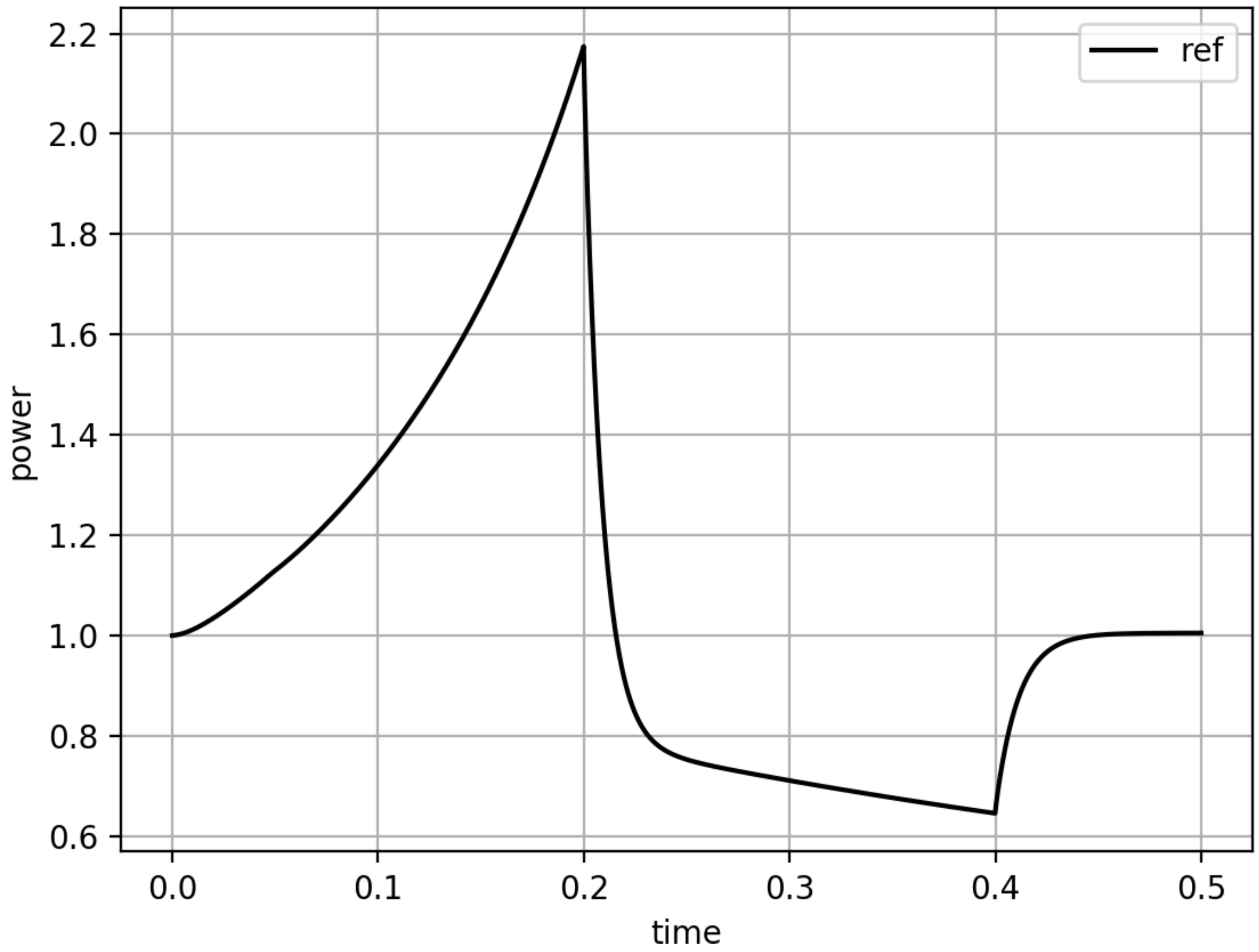



Fig: Reference solution for the $SP_3$ model

| | | |
|---|---|---|
| 0.0 | 0.2 | Linear change |
| 0.2 | 0.2 | Step change |
| 0.2 | 0.4 | Linear change |
| 0.4 | 0.4 | Step change |

$n = 144$, $p = 3$, $\tau = 0.0001$

# Scenario №3 (combined)

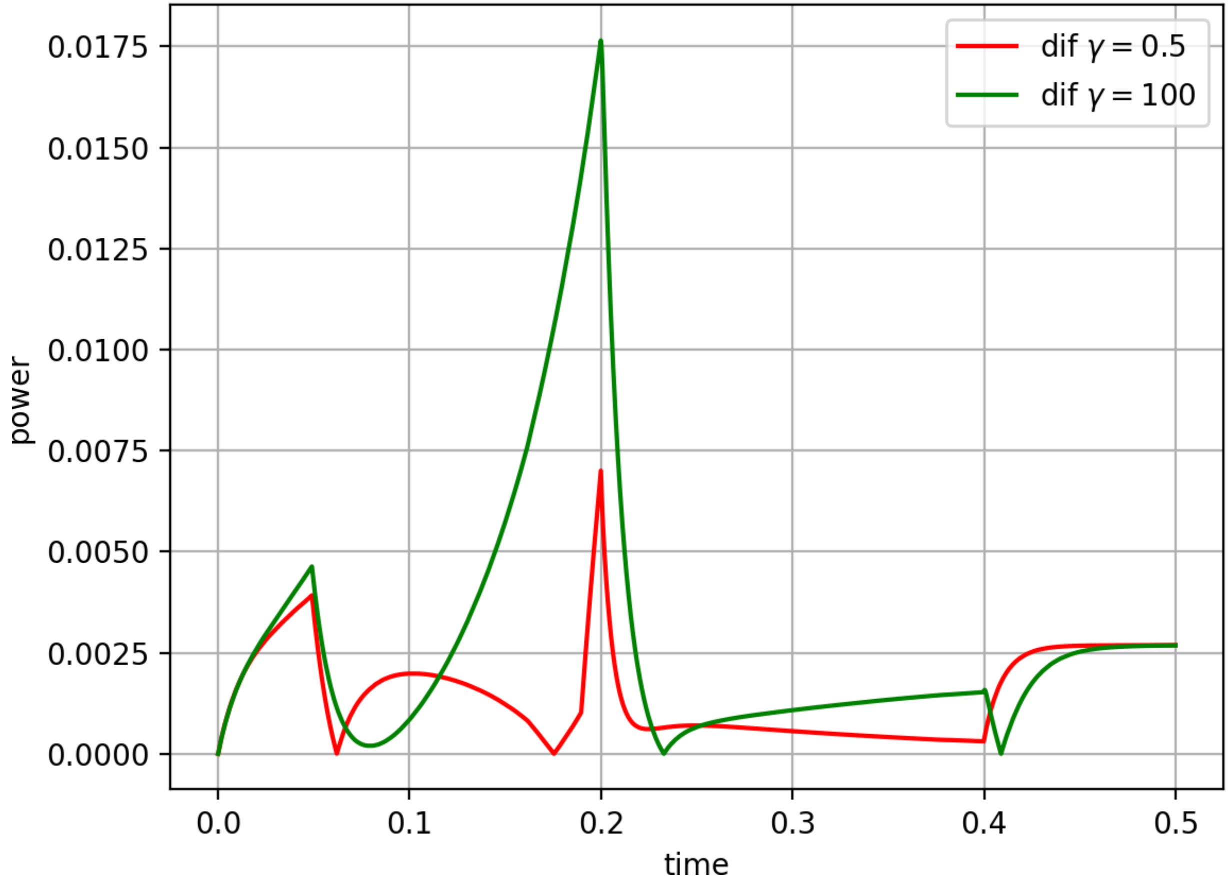
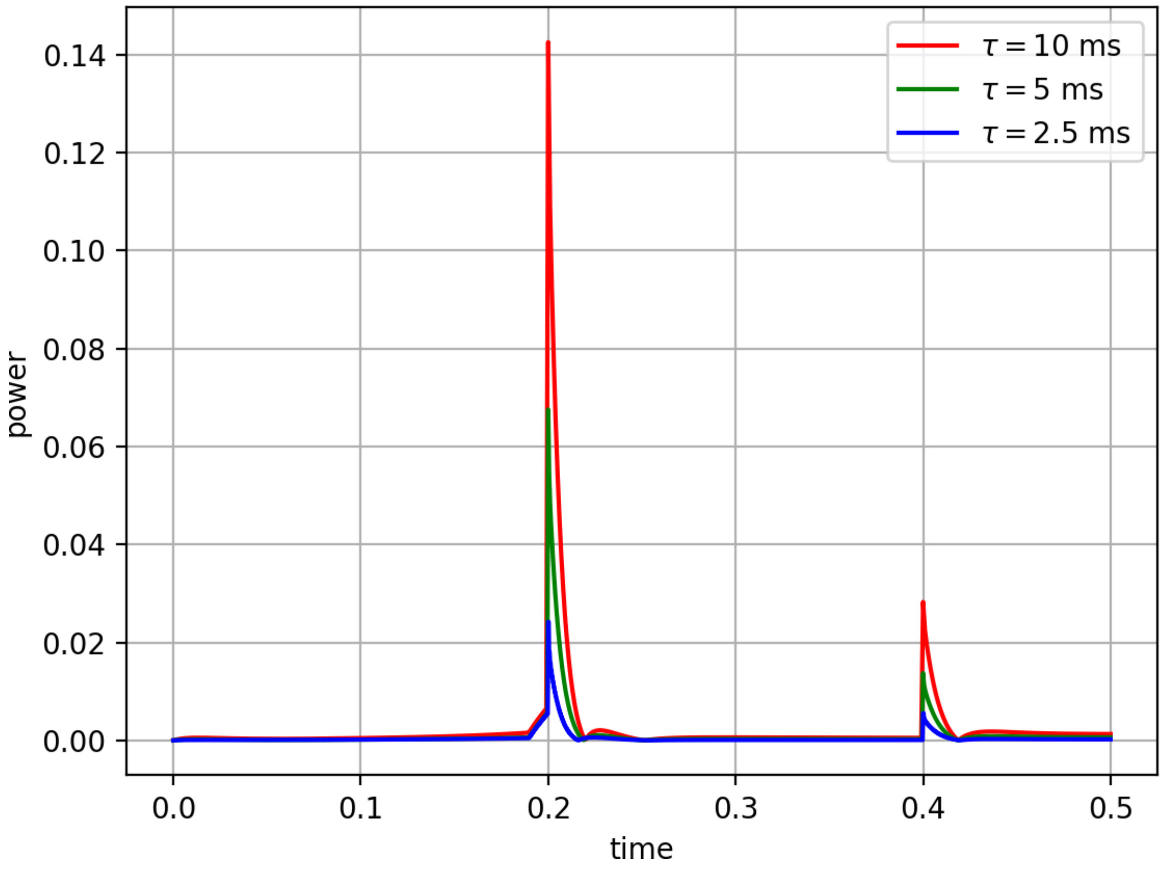



Fig: Differences of the diffusion solution from $SP_3$

| $t$ | Dif ($\gamma = 0.5$) | Dif ($\gamma = 100$) | $SP_3$ |
|---|---|---|---|
| 0.0 | 1.0000 | 1.0000 | 1.0000 |
| 0.1 | 1.3422 | 1.3393 | 1.3402 |
| 0.2 | 2.1686 | 2.1580 | 2.1757 |
| 0.3 | 0.7103 | 0.7119 | 0.7109 |
| 0.4 | 0.6452 | 0.6470 | 0.6455 |
| 0.5 | 1.0024 | 1.0024 | 1.0051 |

Fig: Reference solutions

# Scenario №3 (combined)



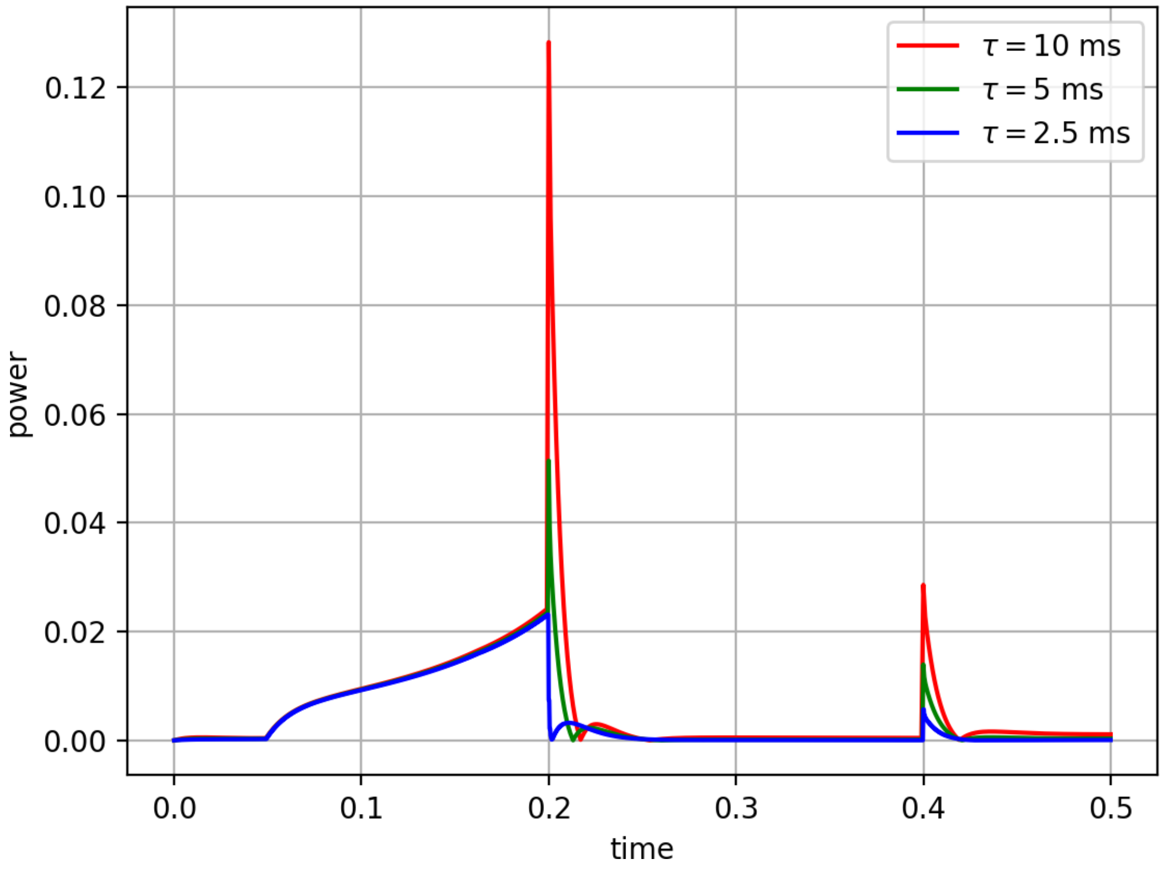




Fig: Differences at different time steps for diffusion and $SP_3$ models

# Conclusion

- Compared the spectral parameters and non-stationary solutions, calculated by both the diffusion and $SP_3$ options using the FEM.

- Solution of the lambda- and alpha- spectral problems has been tested for the HWR reactor benchmark test. Three scenarios for non-stationary TWIGL benchmark were solved.

- Of particular interest is the problem associated with appearance of complex eigenvalues and eigenfunctions. It was found that this tendency occurs for both the diffusion and $SP_3$ solutions of the HWR reactor test.

Thank you for your attention!