Numerical simulation of dynamic processes in a nuclear reactor by state change modal method

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Multiscale Methods and Large-scale Scientific Computing
July 31- August 3, 2017, Yakutsk

Introduction

- Quasistatic method. In this case, an approximate solution is represented as a product of two functions: one of which depends on the time and is related to the amplitude, the second (shape function) describes the spatial distribution. The shape function is often associated with the fundamental eigenfunction of certain eigenvalue problems for neutron diffusion equations.
- When using the quasistatic method, the problem is significantly simplified, thus, it is doubtful to obtain good accuracy for an approximate solution, in particular, in dynamic regimes with a complex redistribution of the neutron flux. For this reason, the more general approach of the modal method has been successfully developed. In this case, the solution is represented in the form of a sum of several dominant eigenvalues with time-dependent coefficients.
- Nonstationary processes can naturally be described on the basis of the approximate solution expansion in time-eigenvalue of α-eigenvalue problem. We deal with an unbound system of equations for the coefficients. It should also be noted that the eigenvalues are complex for both λ- and α-eigenvalue problem. To set the initial state, this leads to the need to solve the appropriate adjoint spectral problems.
- In this paper, we formulate a general strategy for the approximate solution of nonstationary problems of neutron transport in nuclear reactors, which is oriented to fast real-time calculations using the State Change Modal (SCM) method.

Problem statement

Multigroup diffusion approximation

$$\frac{1}{v_g}\frac{\partial \phi_g}{\partial t} - \nabla \cdot D_g \nabla \phi_g + \Sigma_{rg}\phi_g - \sum_{g \neq g'=1}^{G} \Sigma_{s,g' \to g}\phi_{g'} =$$

$$= (1-\beta)\chi_g \sum_{g'=1}^{G} \nu \Sigma_{fg'}\phi_{g'} + \widetilde{\chi}_g \sum_{m=1}^{M} \lambda_m c_m, \quad g = 1, 2, ..., G.$$

The density of sources of delayed neutrons is described as follows:

$$\frac{\partial c_m}{\partial t} + \lambda_m c_m = \beta_m \sum_{g=1}^{G} \nu \Sigma_{fg}\phi_g, \quad m = 1, 2, ..., M,$$

where β_m is is a fraction of delayed neutrons of m type, and

$$\beta = \sum_{m=1}^{M} \beta_m.$$

Initial and boundary conditions

The albedo-type conditions are set at the boundary $\partial\Omega$ of the area Ω:

$$D_g \frac{\partial \phi_g}{\partial n} + \gamma_g \phi_g = 0, \qquad g = 1, 2, ..., G,$$

where n is outer normal to the boundary $\partial\Omega$.

We consider boundary problem with the initial condition:

$$\phi_g(\boldsymbol{x}, 0) = \phi_g^0(\boldsymbol{x}), \quad g = 1, 2, ..., G, \quad c_m(\boldsymbol{x}, 0) = c_m^0(\boldsymbol{x}), \quad m = 1, 2, ..., M.$$

Operator formulation

Let's write the boundary problem in operator form. Define vectors $\phi = \{\phi_1, \phi_2, ..., \phi_G\}$, $\boldsymbol{c} = \{c_1, c_2, ..., c_M\}$ and matrix:

$$V = (v_{gg'}), \quad v_{gg'} = \delta_{gg'} v_g^{-1}, \qquad D = (d_{gg'}), \quad d_{gg'} = -\delta_{gg'} \nabla \cdot D_g \nabla,$$
$$S = (s_{gg'}), \quad s_{gg'} = \delta_{gg'} \Sigma_g - \Sigma_{s,g' \to g}, \qquad R = (r_{gg'}), \quad r_{gg'} = (1-\beta)\chi_g \nu \Sigma_{fg'},$$
$$B = (b_{gm}), \quad b_{gm} = \widetilde{\chi}_g \lambda_m, \qquad \Lambda = (\lambda_{mm'}), \quad \lambda_{mm'} = \lambda_m \delta_{mm'},$$
$$Q = (q_{mg}), \quad q_{mg} = \beta_m \nu \Sigma_{fg}, \qquad g, g' = 1, 2, ..., G, \quad m, m' = 1, 2, ..., M.$$

Using the set definitions, we get boundary problem in operator formulation:

$$V \frac{d\phi}{dt} + (D + S)\phi = R\phi + B\boldsymbol{c},$$
$$\frac{d\boldsymbol{c}}{dt} + \Lambda \boldsymbol{c} = Q\phi.$$

The Cauchy problem is solved under the following initial conditions:

$$\phi(0) = \phi^0, \quad \boldsymbol{c}(0) = \boldsymbol{c}^0,$$

where $\phi^0 = \{\phi_1^0, \phi_2^0, ..., \phi_G^0\}$, $\boldsymbol{c}^0 = \{c_1^0, c_2^0, ..., c_M^0\}$.

Finite element method

Let $H^1(\Omega)$ – sobolev space, $v \in H^1$: v^2 and $|\nabla v|^2$ have a finite integral in Ω. For $\boldsymbol{v} = \{v_1, v_2, ..., v_d\}$ define $V^d = [H^1(\Omega)]^d$. For test functions use notations $\boldsymbol{\xi} = \{\xi_1, \xi_2, ..., \xi_G\}$, $\boldsymbol{\zeta} = \{\zeta_1, \zeta_2, ..., \zeta_M\}$.
In variation formulation we are looking for $\boldsymbol{\phi} \in V^D$, $\boldsymbol{c} \in V^M$ such that:

$$\int_\Omega \left(V \frac{\boldsymbol{\phi}^{n+1} - \boldsymbol{\phi}^n}{\tau} + S\boldsymbol{\phi}^{n+1} \right) \boldsymbol{\xi} d\boldsymbol{x} + \int_\Omega \sum_{g=1}^{G} D_G \nabla \phi_g^{n+1} \nabla \xi_g d\boldsymbol{x} +$$

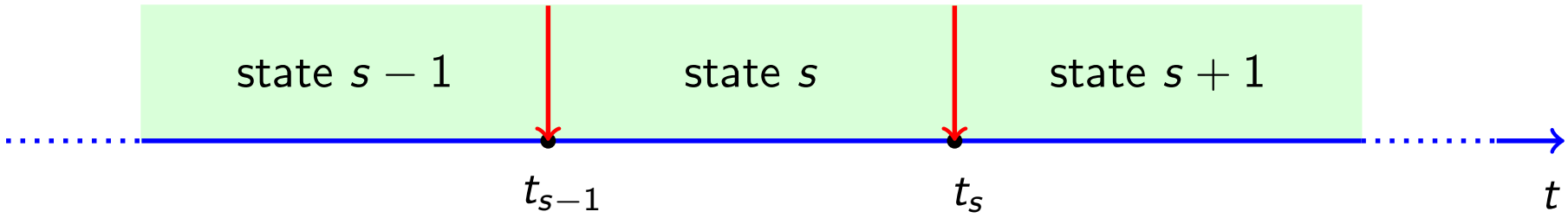
$$+ \int_{\partial\Omega} \sum_{g=1}^{G} \gamma_g \phi_g^{n+1} \xi_g d\boldsymbol{x} = \int_\Omega R\boldsymbol{\phi}^{n+1} \boldsymbol{\xi} d\boldsymbol{x} + \int_\Omega B\boldsymbol{c}^{n+1} \boldsymbol{\xi} d\boldsymbol{x},$$

$$\int_\Omega \boldsymbol{c}^{n+1} \boldsymbol{\zeta} d\boldsymbol{x} = \int_\Omega \widetilde{\Lambda} \boldsymbol{c}^n \boldsymbol{\zeta} d\boldsymbol{x} + \int_\Omega \tau Q \boldsymbol{\phi}^{n+1} \boldsymbol{\zeta} d\boldsymbol{x}$$

for all $\boldsymbol{\xi} \in V^D$, $\boldsymbol{\zeta} \in V^M$.
Let introduce a discrete function spaces $V_h^D \subset V^D$, $V_h^M \subset V^M$ and define discrete variational problem.
The standard Lagrangian finite elements of degree p = 1, 2 and 3 are used.

State change scheme

The state of the reactor is characterized by the constant coefficients of the system of multigroup diffusion equations.



Dynamic processes in a nuclear reactor can be considered as a change of states. At a certain time $t = t_s,\ s = 1, 2, ...$ an instantaneous change of state occurs. The state s is defined by the parameters in equations:

$$V(t) = V(t_s), \quad D(t) = D(t_s), \quad S(t) = S(t_s), \quad R(t) = R(t_s), \quad B(t) = B(t_s)$$

$$\Lambda(t) = \Lambda(t_s), \quad Q(t) = Q(t_s), \quad t_{s-1} < t \leq t_s, \quad s = 1, 2, ...$$

Modal approximation

An approximate description of the non-stationary process at a separate stage is based on modal approximation. An approximate solution is sought in the form of decomposition in eigenfunctions of time and α-eigenvalue problem.
Let's $\boldsymbol{u} = \{\phi, \boldsymbol{c}\}, \quad \phi(t_{s-1}) = \phi^s, \quad \boldsymbol{c}(t_{s-1}) = \boldsymbol{c}^s$. In a separate stage s the following system is considered

$$\boldsymbol{B}\frac{d\boldsymbol{u}}{dt} + \boldsymbol{A}\boldsymbol{u} = 0, \quad t_{s-1} < t \leq t_s,$$

with constants

$$\boldsymbol{A} = \begin{pmatrix} D(t_s) + S(t_s) - R(t_s) & -B(t_s) \\ -Q(t_s) & \Lambda(t_s) \end{pmatrix}, \quad \boldsymbol{B} = \begin{pmatrix} V(t_s) & 0 \\ 0 & I \end{pmatrix}.$$

Supplemented by the corresponding initial condition

$$\boldsymbol{u}(t_{s-1}) = \boldsymbol{u}^s.$$

The main feature of the problems we are considering is that the matrices $\boldsymbol{A}_h$ and $\boldsymbol{B}_h$ are real and asymmetric.

Modal approximation

The modal approximation corresponds to the representation of the approximate solution ($\boldsymbol{u}_h \approx \boldsymbol{u}_N$) of problem in the following form

$$\boldsymbol{u}_N(\boldsymbol{x}, t) = \sum_{n=1}^{N} a_n(t)\boldsymbol{w}_n(\boldsymbol{x}),$$

where N is the number of dominant eigenvalues of the spectral problem, $\boldsymbol{w}_n(\boldsymbol{x})$ — corresponding eigenfunctions.
Let us define eigenfunctions and eigenvalues as the solution of the α-eigenvalue problem:

$$\boldsymbol{A}_h\boldsymbol{v} = \lambda\boldsymbol{B}_h\boldsymbol{v}.$$

Then we obtain

$$a_n(t)\boldsymbol{w}_n(\boldsymbol{x}) = b_n\mathrm{Re}\big(\exp(-\lambda_n(t - t_{s-1}))\boldsymbol{v}_n(\boldsymbol{x})\big),$$
$$a_{n+1}(t)\boldsymbol{w}_{n+1}(\boldsymbol{x}) = b_{n+1}\mathrm{Im}\big(\exp(-\lambda_n(t - t_{s-1}))\boldsymbol{v}_n(\boldsymbol{x})\big).$$

A special attention should be paid to define the coefficients $a_n(t_{s-1}) = b_n,\ n = 1, 2, ..., N$. For this, the initial condition is involved. For example, in the case of real eigenvalues, we have

$$\boldsymbol{u}_h^s(\boldsymbol{x}) = \sum_{n=1}^{N_h} b_n\boldsymbol{v}_n(\boldsymbol{x}).$$

Time scale processes

The initial condition includes two components

$$\boldsymbol{u}_h^s(\boldsymbol{x}) = (\phi_h^s(\boldsymbol{x}), \boldsymbol{c}_h^s(\boldsymbol{x})).$$

Dynamic behaviour of these components is due to different time-scale processes.

Delayed neutrons source determines **slow processes**, when $\boldsymbol{c}(\boldsymbol{x}, t)$ changes slightly with the reactor state change. In contrast, neutron flux $\phi(\boldsymbol{x}, t)$ determines **fast processes** when the reactor state changes.

By virtue of this separation of dynamic processes, we model the slow phase of the dynamics of the reactor with modal approximation and orientate ourselves on the approximate prediction of the initial state for delayed neutrons, only the function $\boldsymbol{c}_h^s(\boldsymbol{x})$ is approximated. The approximation $\phi_h^s(\boldsymbol{x})$ is not of interest to us, we do not model a fast phase of the state change.

Adjoint spectral problem

Consider the adjoint spectral problem

$$\boldsymbol{A}_h^T \widetilde{\boldsymbol{v}} = \lambda \boldsymbol{B}_h^T \widetilde{\boldsymbol{v}}.$$

The eigenfunctions of problems are orthogonal in the sense of the equality

$$(\boldsymbol{B}_h \boldsymbol{v}_n, \widetilde{\boldsymbol{v}}_m) = 0, \quad m \neq n, \quad m, n = 1, 2, ..., N_h.$$

In view of this, one can obtain

$$b_n = \frac{1}{(\boldsymbol{B}_h \boldsymbol{v}_n, \widetilde{\boldsymbol{v}}_n)} (\boldsymbol{u}_h^s, \boldsymbol{B}_h \widetilde{\boldsymbol{v}}_n), \quad n = 1, 2, ..., N_h.$$

In the approximate solution of problem only the first N coefficients b_n are used:

$$\boldsymbol{c}_h^s(\boldsymbol{x}) \approx \sum_{n=1}^{N} b_n \boldsymbol{c}_n(\boldsymbol{x}),$$

where $\boldsymbol{v}_n(\boldsymbol{x}) = (\phi_n(\boldsymbol{x}), \boldsymbol{c}_n(\boldsymbol{x}))$. In this case, the spectral problems are solved for N dominant eigenvalues.

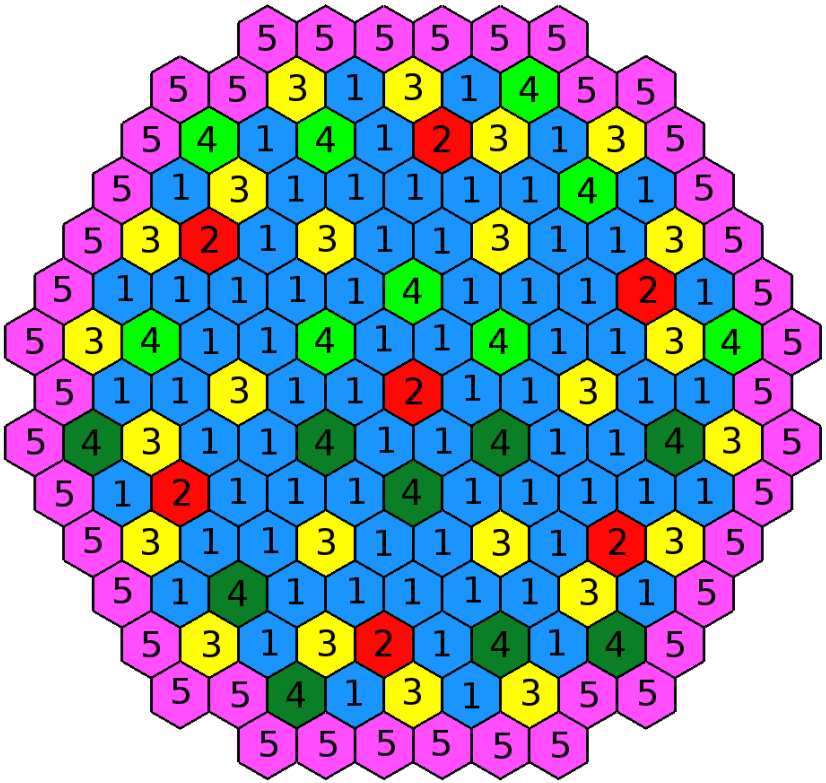
Computation scheme

Off-line calculation. Calculation of the coefficients of the mathematical model of the multigroup diffusion approximation for the isolated reactor states, which is performed in advance. The status passport also includes calculated dominant eigenvalues and eigenfunctions of the α-eigenvalue problem. These data can be supplemented by dominant eigenvalues and eigenvalues of the conjugate eigenvalue problem.

On-line calculation. Real-time modeling is carried out on the basis of the modal solution of the problem. The coefficients in the representation are calculated from the initial condition. The solution for other time intervals is determined according to modal approximation.

VVER-1000 benchmark

The dynamics of the VVER-1000 reactor during the transition from the supercritical mode to the subcritical mode



- two-dimensional
- two group instantaneous and one group of delayed neutrons
- triangles per cassette κ varies from 6 to 96
- order of finite elements p varies from 1 to 3
- two types of perturbation

Supercritical state: α-eigenvalue problem

Table: Eigenvalues $\alpha_n = \lambda_n^{(\alpha)}$, $n = 1, 2, ..., 5$

p	κ	α_1	α_2, α_3	α_4, α_5
	6	-0.22557	0.04241 $\mp$ 3.08808e-06i	0.06588 $\mp$ 4.80449e-07i
1	24	-0.82690	0.03777 $\mp$ 5.37884e-06i	0.06489 $\mp$ 1.37315e-06i
	96	-1.74998	0.03619 $\mp$ 5.69002e-06i	0.06456 $\mp$ 1.40299e-06i
	6	-2.10154	0.03592 $\mp$ 4.96474e-06i	0.06452 $\mp$ 1.21320e-06i
2	24	-2.46601	0.03562 $\mp$ 5.78277e-06i	0.06445 $\mp$ 1.40897e-06i
	96	-2.50375	0.03559 $\mp$ 5.80693e-06i	0.06444 $\mp$ 1.41324e-06i
	6	-2.47975	0.03561 $\mp$ 5.83718e-06i	0.06445 $\mp$ 1.41869e-06i
3	24	-2.50294	0.03559 $\mp$ 5.80783e-06i	0.06444 $\mp$ 1.41341e-06i
	96	-2.51280	0.03558 $\mp$ 5.80954e-06i	0.06444 $\mp$ 1.41362e-06i

Table: Eigenvalues $\alpha_n = \lambda_n^{(\alpha)}$, $n = 6, 7, ..., 10$

p	κ	α_6	α_7	α_8	α_9, α_{10}
1	6	0.07107	0.07214	0.07323	0.07397 $\mp$ 2.04990e-08i
	24	0.07050	0.07167	0.07283	0.07362 $\mp$ 3.65907e-08i
	96	0.07033	0.07152	0.07269	0.07351 $\mp$ 3.91936e-08i
2	6	0.07030	0.07151	0.07268	0.07349 $\mp$ 3.69824e-08i
	24	0.07027	0.07147	0.07265	0.07347 $\mp$ 4.03121e-08i
	96	0.07026	0.07147	0.07265	0.07347 $\mp$ 4.02324e-08i
3	6	0.07027	0.07147	0.07265	0.07347 $\mp$ 4.02573e-08i
	24	0.07026	0.07147	0.07265	0.07347 $\mp$ 4.02248e-08i
	96	0.07026	0.07147	0.07265	0.07347 $\mp$ 4.02332e-08i

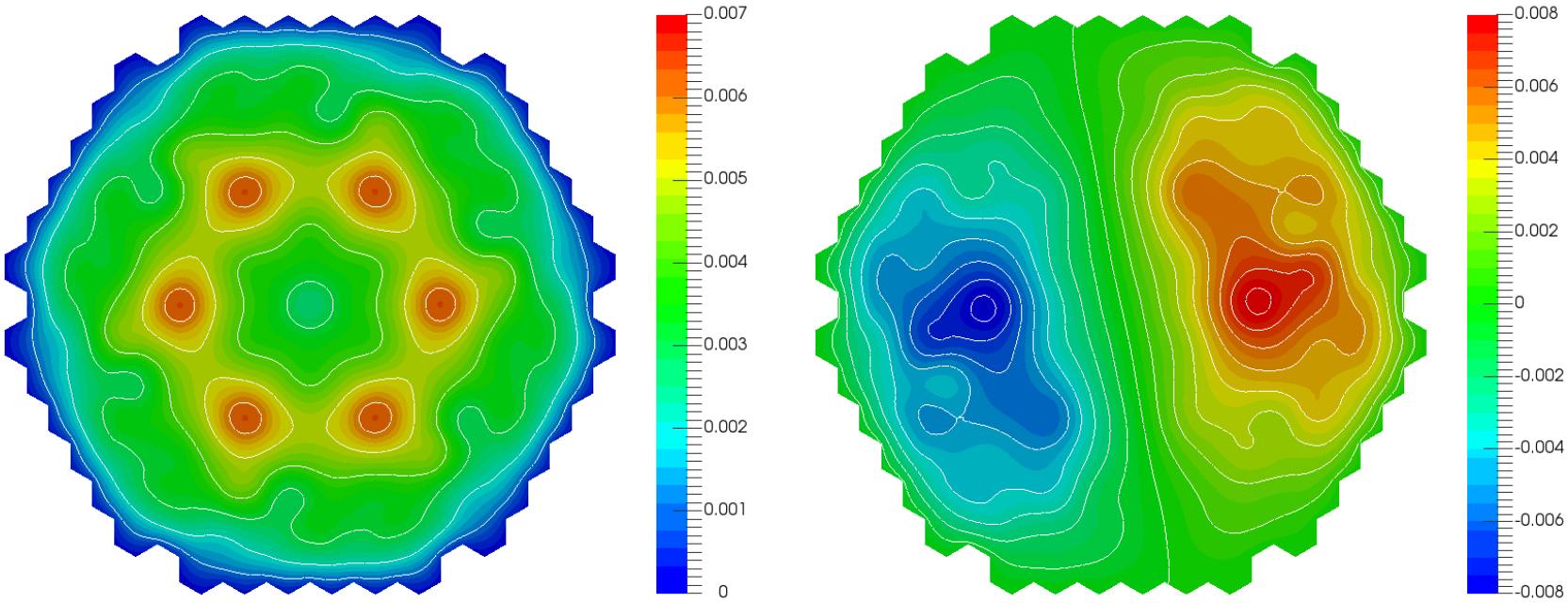


Figure: The eigenfunction $\varphi_1^{(1)}$ (left) and real part of eigenfunctions $\varphi_1^{(2)}$, $\varphi_1^{(3)}$ (right).

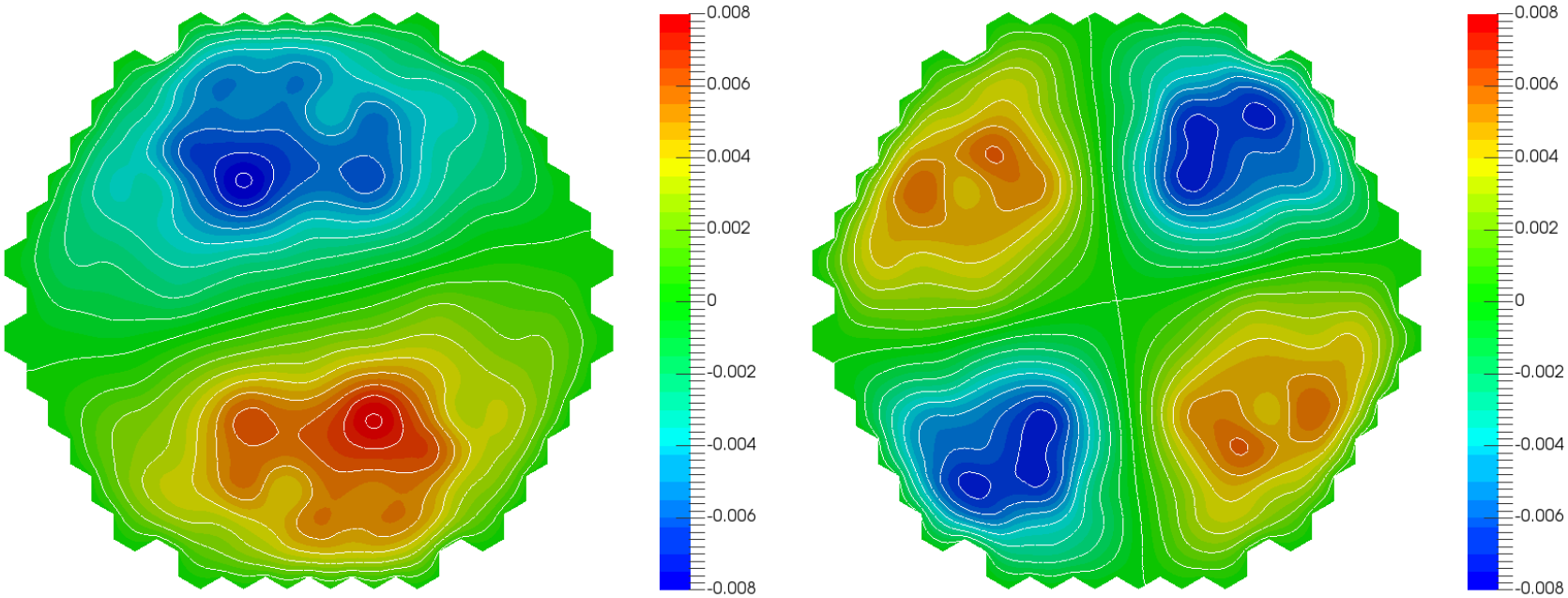


Figure: Imaginary part of eigenfunctions $\varphi_1^{(2)}$, $-\varphi_1^{(3)}$ (left) and real part of eigenfunctions $\varphi_1^{(4)}$, $\varphi_1^{(5)}$ (right).

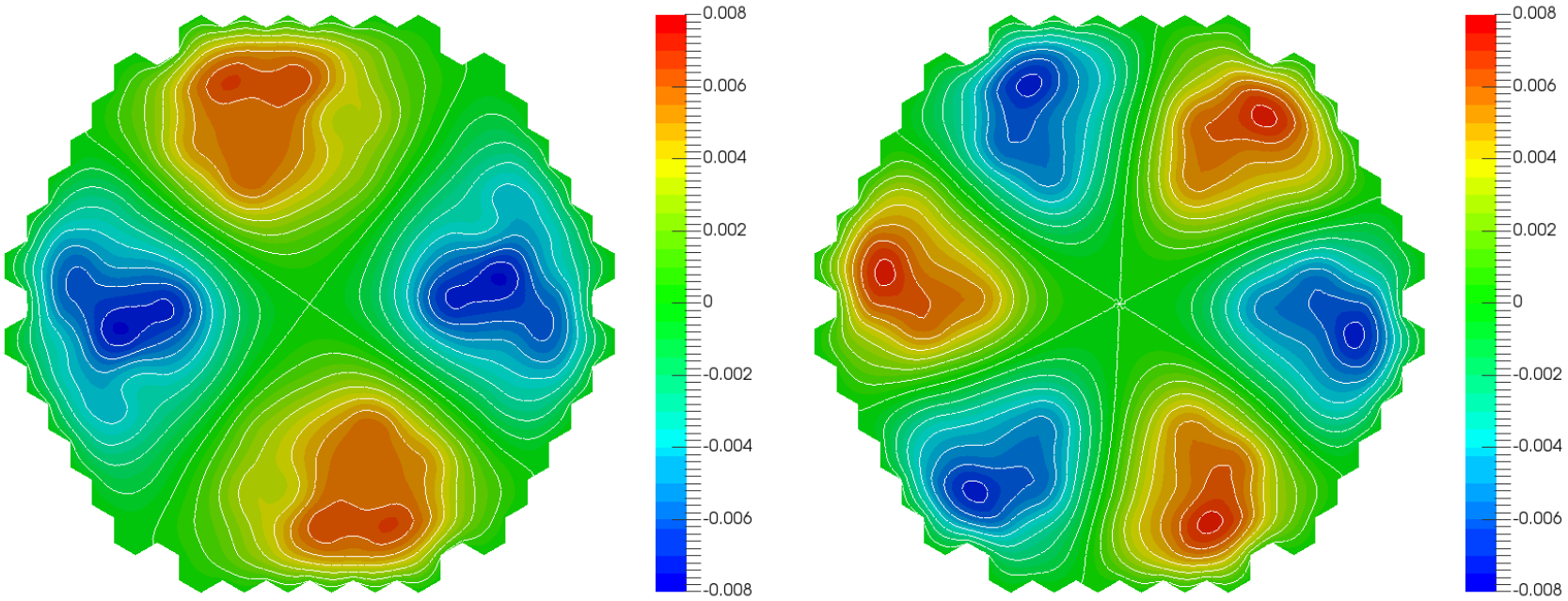


Figure: Imaginary part of eigenfunctions $\varphi_1^{(4)}$, $-\varphi_1^{(5)}$ (left) and eigenfunction $\varphi_1^{(6)}$ (right).

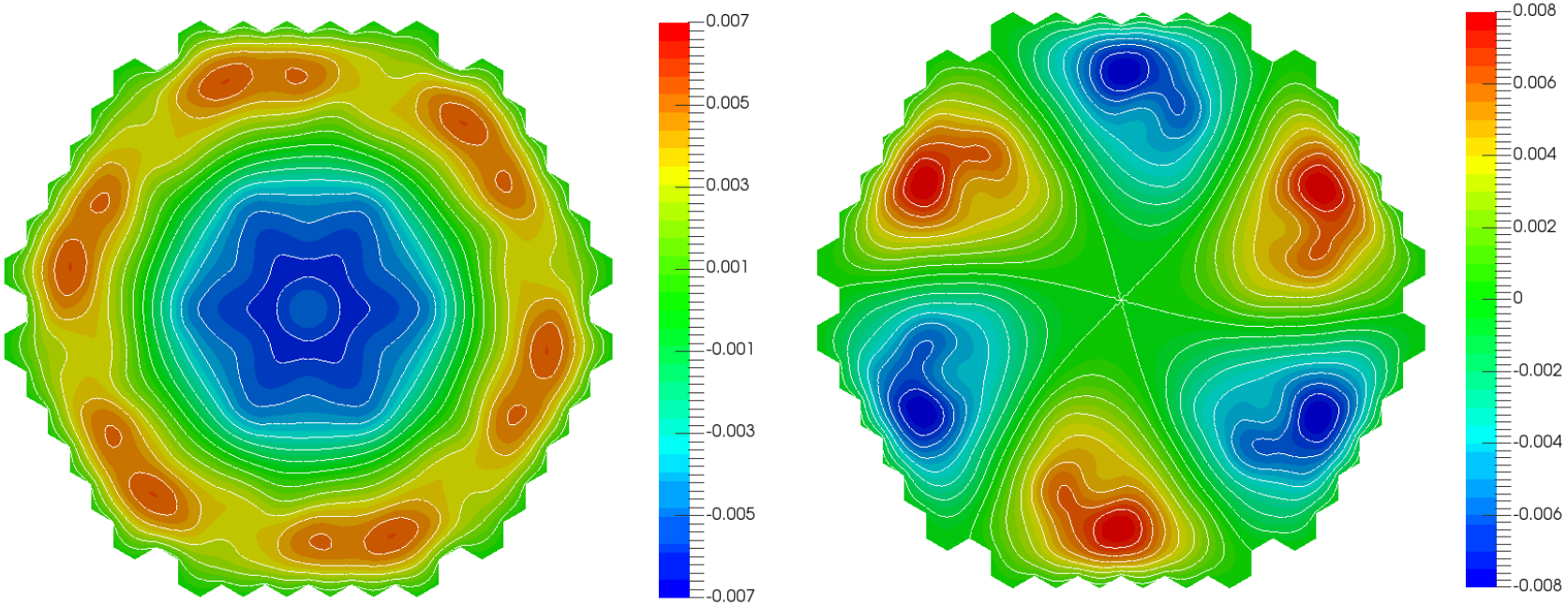


Figure: The eigenfunction $\varphi_1^{(7)}$ (left) and eigenfunction $\varphi_1^{(8)}$ (right).

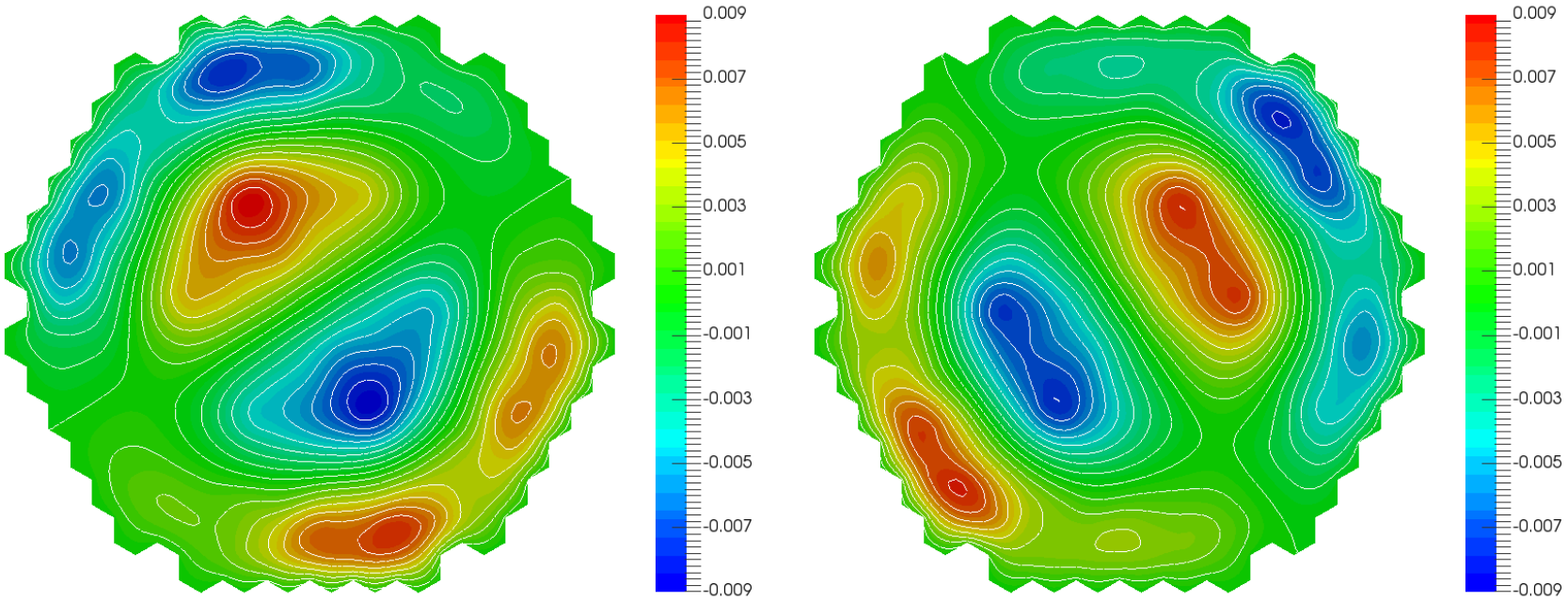


Figure: Real part of eigenfunctions $\varphi_1^{(9)}$, $\varphi_1^{(10)}$ (left) and imaginary part of eigenfunctions $\varphi_1^{(9)}$, $-\varphi_1^{(10)}$ (right).

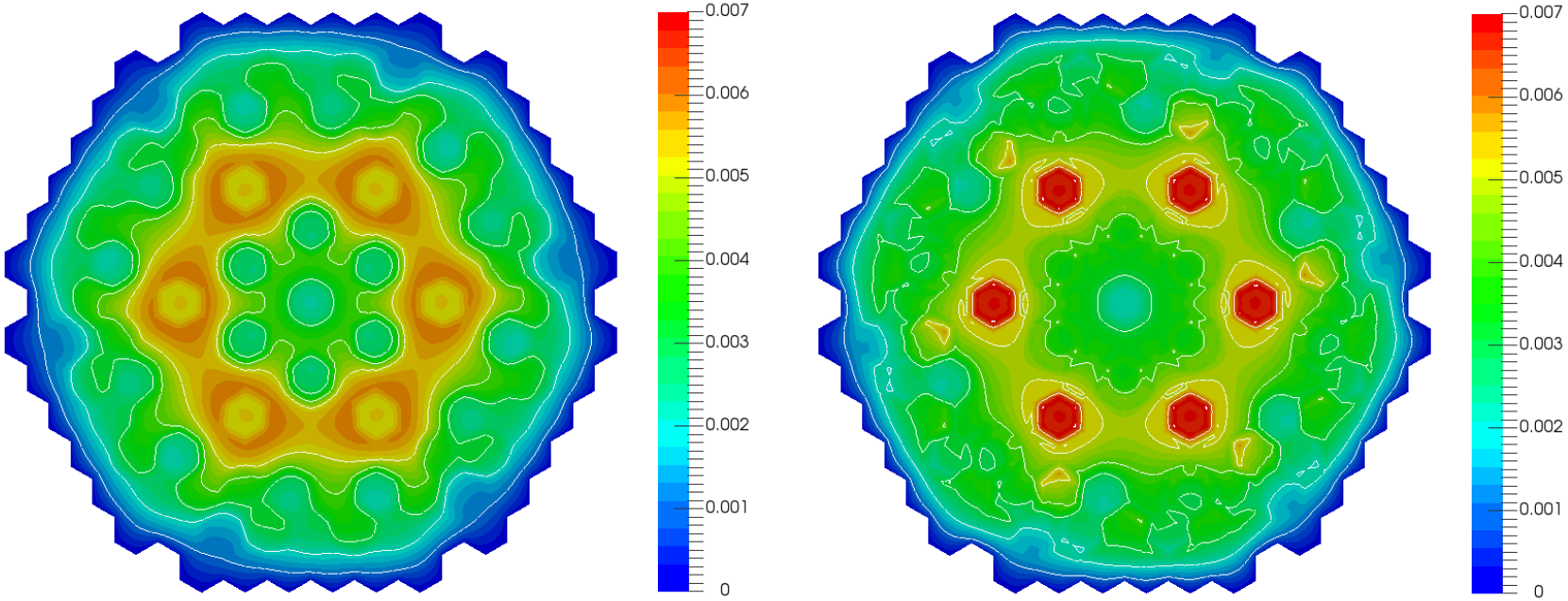


Figure: The eigenfunction $\varphi_2^{(1)}$ (left) and the eigenfunctions $s^{(1)}$ (right).

Adjoint spectral problem

Table: Eigenvalues $\alpha_n = \lambda_n^{(\alpha)}$, $n = 1, 2, ..., 10$ for direct and adjoint problems

n	α_n for direct	α_n for adjoint
1	-2.51280117966	-2.51280117972
2,3	0.0355815000364 $\mp$ 5.80954455861e-06	0.0355815000365 $\mp$ 5.80954421646e-06
4,5	0.0644427013767 $\mp$ 1.41362187449e-06	0.0644427013767 $\mp$ 1.41362190730e-06
6	0.0702618501639	0.0702618501639
7	0.0714652882224	0.0714652882164
8	0.0726456060606	0.0726456060606
9,10	0.0734708921578 $\mp$ 4.02332269037e-08	0.0734708921578 $\mp$ 4.02332146248e-08

Table: Scalar product $(\phi_1^{(n)}, \phi_1^{(m)})$, $n, m = 1, 2, ..., 5$

$n \backslash m$	1	2	3	4	5
1	1.0e-00	1.3e-08	2.2e-08	-3.8e-08	9.8e-09
2	1.3e-08	1.0e-00	-1.6e-08	-1.6e-08	1.4e-08
3	2.2e-08	-1.6e-08	1.0e-00	-9.8e-09	-1.1e-08
4	-3.8e-08	-1.6e-08	-9.8e-09	1.0e-00	-3.9e-10
5	9.8e-09	1.4e-08	-1.1e-08	-3.9e-10	1.0e-00
6	-1.8e-09	4.1e-08	-1.8e-08	-1.1e-08	2.9e-09
7	1.0e-02	1.2e-09	1.1e-08	1.4e-08	-1.6e-08
8	-3.2e-09	-2.0e-07	-3.3e-08	4.0e-09	-1.9e-08
9	-2.2e-08	-3.1e-03	-7.5e-03	3.0e-09	6.3e-09
10	1.6e-09	7.5e-03	-3.1e-03	-1.1e-08	6.3e-09

Table: Scalar product $(\phi_1^{(n)}, \phi_1^{(m)})$, $n, m = 6, 7, ..., 10$

$n \backslash m$	6	7	8	9	10
1	-1.8e-09	1.0e-02	-3.2e-09	-2.2e-08	1.6e-09
2	4.1e-08	1.2e-09	-2.0e-07	-3.1e-03	7.5e-03
3	-1.8e-08	1.1e-08	-3.3e-08	-7.5e-03	-3.1e-03
4	-1.1e-08	1.4e-08	4.0e-09	3.0e-09	-1.1e-08
5	2.9e-09	-1.6e-08	-1.9e-08	6.3e-09	6.3e-09
6	1.0e-00	-4.2e-09	-5.6e-03	4.1e-08	-1.2e-07
7	-4.2e-09	1.0e-00	-2.1e-09	-1.8e-08	8.0e-09
8	-5.6e-03	-2.1e-09	1.0e-00	-5.2e-08	2.3e-07
9	4.1e-08	-1.8e-08	-5.2e-08	1.0e-00	-5.5e-07
10	-1.2e-07	8.0e-09	2.3e-07	-5.5e-07	1.0e-00

Within the modal method, we can not rely on high accuracy when considering a relatively small number of dominant eigenvalues. Therefore, in the example under consideration, we can assume that the eigenvalues are real, and the corresponding eigenfunctions are orthogonal.
Instead of

$$b_n = \frac{1}{(\boldsymbol{B}_h \boldsymbol{v}_n, \widetilde{\boldsymbol{v}}_n)} (\boldsymbol{u}_h^s, \boldsymbol{B}_h \widetilde{\boldsymbol{v}}_n), \quad n = 1, 2, ..., N_h$$

coefficients are used

$$b_n \approx \frac{1}{(\boldsymbol{c}_n, \boldsymbol{c}_n)} (\boldsymbol{c}_h^s, \boldsymbol{c}_n), \quad n = 1, 2, ..., N,$$

to approximate the initial condition.

The symmetric perturbation

In the supercritical mode, due to the sufficiently large magnitude of the main eigenvalue, the regular regime of the reactor is rapidly developing, where

$$\boldsymbol{u}(\boldsymbol{x}, t) \approx a_1 \exp(-\alpha_1 t)\boldsymbol{v}_1^0(\boldsymbol{x}).$$

Here $\boldsymbol{v}_1^0(\boldsymbol{x})$ is the first mode of the supercritical state. We consider the problem with the transition from this supercritical state at $t_0 = 0$ to the subcritical state.
The subcritical stage is characterized by a 15% increase in the coefficient Σ_2 for material 4 in the VVER-1000 test diffusion constants. Thus, the dynamics of the reactor is as follows:

$$\Sigma_2 \longrightarrow 1.15\Sigma_2 \quad (\text{material } 4).$$

The initial state is characterized by specifying the initial conditions at $t_0 = 0$ as

$$\boldsymbol{u}(\boldsymbol{x}, 0) = \boldsymbol{v}_1^0(\boldsymbol{x}).$$

Table: Subcritical state: $\alpha_n = \lambda_n^{(\alpha)}$, $n = 1, 2, ..., 5$

p	κ	α_1	α_2, α_3	α_4, α_5
	6	0.03602	0.05760 $\mp$ 1.49652e-06i	0.06890 $\mp$ 4.92606e-07i
1	24	0.02656	0.05502 $\mp$ 2.06007e-06i	0.06804 $\mp$ 1.01253e-06i
	96	0.02276	0.05411 $\mp$ 2.16813e-06i	0.06774 $\mp$ 1.03843e-06i
	6	0.02250	0.05404 $\mp$ 1.81823e-06i	0.06772 $\mp$ 8.73562e-07i
2	24	0.02144	0.05380 $\mp$ 2.19400e-06i	0.06765 $\mp$ 1.04253e-06i
	96	0.02125	0.05376 $\mp$ 2.20812e-06i	0.06763 $\mp$ 1.04715e-06i
	6	0.02139	0.05379 $\mp$ 2.22579e-06i	0.06764 $\mp$ 1.05369e-06i
3	24	0.02124	0.05376 $\mp$ 2.20883e-06i	0.06763 $\mp$ 1.04736e-06i
	96	0.02122	0.05376 $\mp$ 2.20951e-06i	0.06763 $\mp$ 1.04756e-06i

Table: Subcritical state: $\alpha_n = \lambda_n^{(\alpha)}$, $n = 6, 7, ..., 10$

p	κ	α_6	α_7	α_8	α_9, α_{10}
	6	0.07276	0.07363	0.07369	0.07466 $\mp$ 2.47162e-08i
1	24	0.07222	0.07316	0.07329	0.07429 $\mp$ 1.08814e-08i
	96	0.07204	0.07301	0.07316	0.07417 $\mp$ 1.38093e-08i
	6	0.07203	0.07300	0.07315	0.07416 $\mp$ 1.26708e-08i
2	24	0.07199	0.07296	0.07312	0.07413 $\mp$ 1.50527e-08i
	96	0.07198	0.07295	0.07312	0.07413 $\mp$ 1.49196e-08i
	6	0.07198	0.07295	0.07312	0.07413 $\mp$ 1.52256e-08i
3	24	0.07198	0.07295	0.07312	0.07413 $\mp$ 1.49141e-08i
	96	0.07198	0.07295	0.07311	0.07413 $\mp$ 1.49178e-08i

For an approximate solution, we use the following formulation:

$$\boldsymbol{u}_N(\boldsymbol{x}, t) = \sum_{n=1}^{N} b_n \exp(-\mathrm{Re}\,\alpha_n\, t)\boldsymbol{v}_n(\boldsymbol{x}),$$

where the coefficients $b_n,\ n = 1, 2, ..., N$ are calculated according to the given initial condition.
We distinguish two phases of the dynamic process: fast and slow. In the fast phase, the initial condition: from the function $\boldsymbol{u}(\boldsymbol{x}, 0)$ to the function $\boldsymbol{u}_N(\boldsymbol{x}, 0)$. The slow phase is associated with the evolution of the solution according to above equation. Within the state change modal technology, the fast phase is not modeled at all.

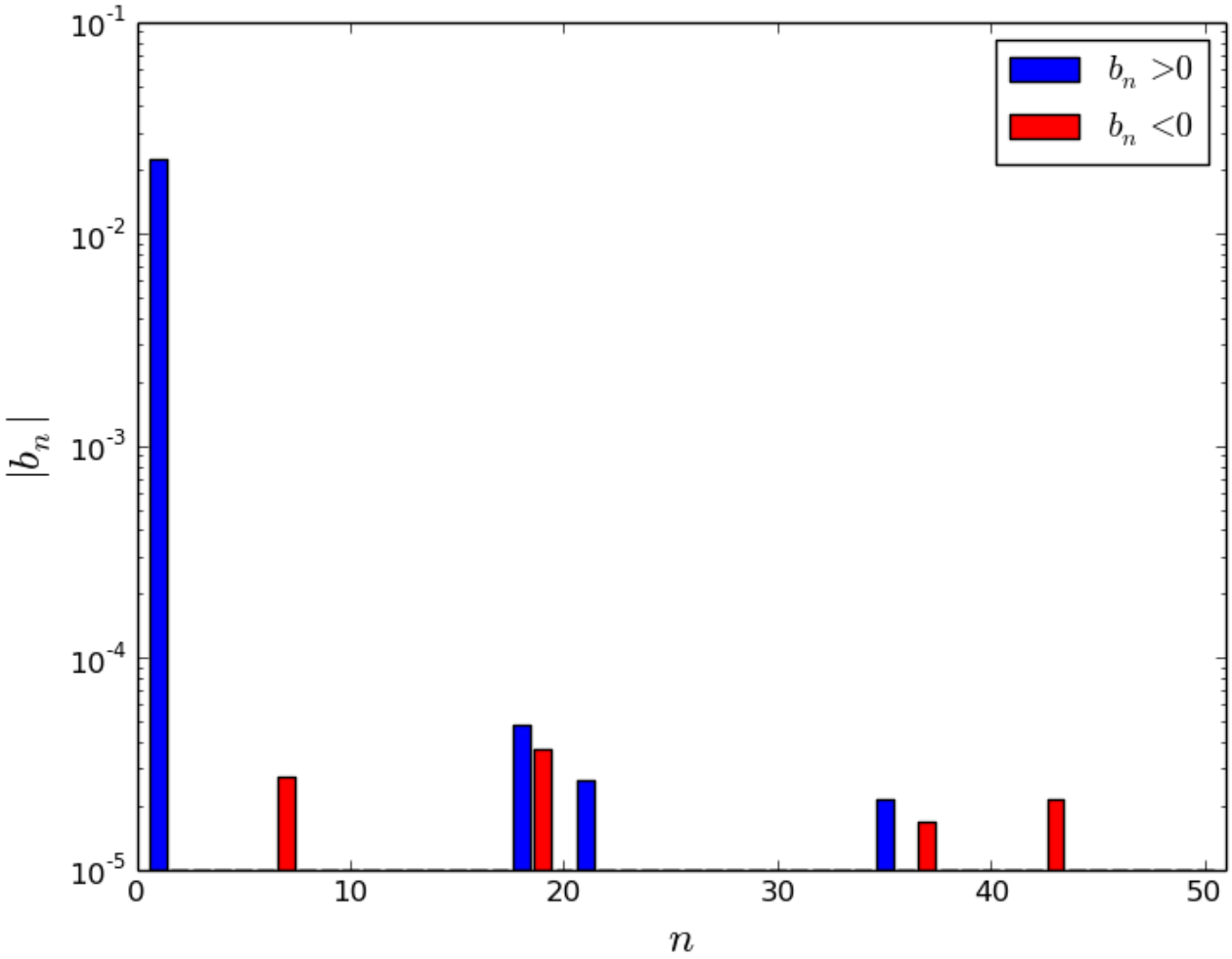


Figure: Approximate solution coefficients

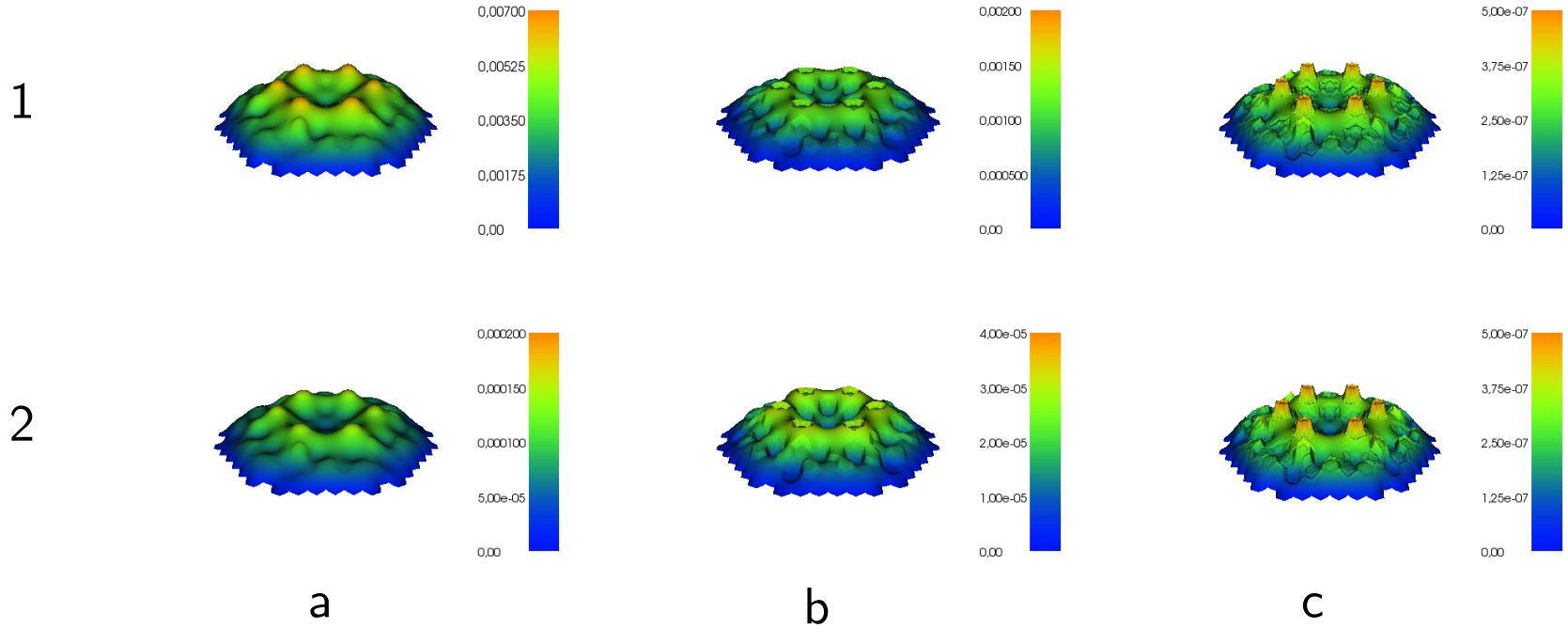


Figure: Function $\boldsymbol{u}(\boldsymbol{x}, 0)$ (string 1) and function $\boldsymbol{u}_N(\boldsymbol{x}, 0)$ (ctring 2): a — neutron flux of group 1, b — neutron flux of group 2, c — delayed neutrons source.

Let us pay attention to the substantial restructuring of the solution, which is illustrated by large changes in the neutron flux amplitudes for the first and second groups.

The asymmetric perturbation

Consider a more complex transition to a subcritical state. The subcritical stage will be characterized by a different increase in the coefficient Σ_2 for material 4 in the diffusion constants in the upper and lower half of the reactor cross-section. Now let the reactor dynamics corresponds to the following transformation

$$\Sigma_2 \longrightarrow \begin{cases} 1.1\Sigma_2, & \text{material 4 (top part)}, \\ 1.2\Sigma_2, & \text{material 4 (bottom part)}. \end{cases}$$

All eigenvalues for this reactor state are real.

Table: Subcritical asymmetric state: $\alpha_n = \lambda_n^{(\alpha)}$, $n = 1, 2, ..., 5$

p	κ	α_1	α_2	α_3	α_4	α_5
	6	0.03347	0.05728	0.05788	0.06884	0.06889
1	24	0.02333	0.05467	0.05528	0.06797	0.06802
	96	0.01925	0.05374	0.05436	0.06768	0.06772
	6	0.01894	0.05367	0.05429	0.06765	0.06770
2	24	0.01782	0.05343	0.05405	0.06758	0.06762
	96	0.01763	0.05339	0.05401	0.06757	0.06761
	6	0.01777	0.05342	0.05404	0.06758	0.06762
3	24	0.01762	0.05339	0.05400	0.06757	0.06761
	96	0.01760	0.05338	0.05400	0.06757	0.06761

Table: Subcritical asymmetric state: $\alpha_n = \lambda_n^{(\alpha)}$, $n = 6, 7, ..., 10$

p	κ	α_6	α_7	α_8	α_9	α_{10}
	6	0.07274	0.07355	0.07369	0.07464	0.07468
1	24	0.07220	0.07309	0.07329	0.07427	0.07430
	96	0.07202	0.07294	0.07316	0.07415	0.07419
	6	0.07201	0.07293	0.07314	0.07414	0.07417
2	24	0.07197	0.07289	0.07311	0.07412	0.07415
	96	0.07196	0.07288	0.07311	0.07411	0.07414
	6	0.07196	0.07289	0.07311	0.07411	0.07415
3	24	0.07196	0.07288	0.07311	0.07411	0.07414
	96	0.07196	0.07288	0.07311	0.07411	0.07414

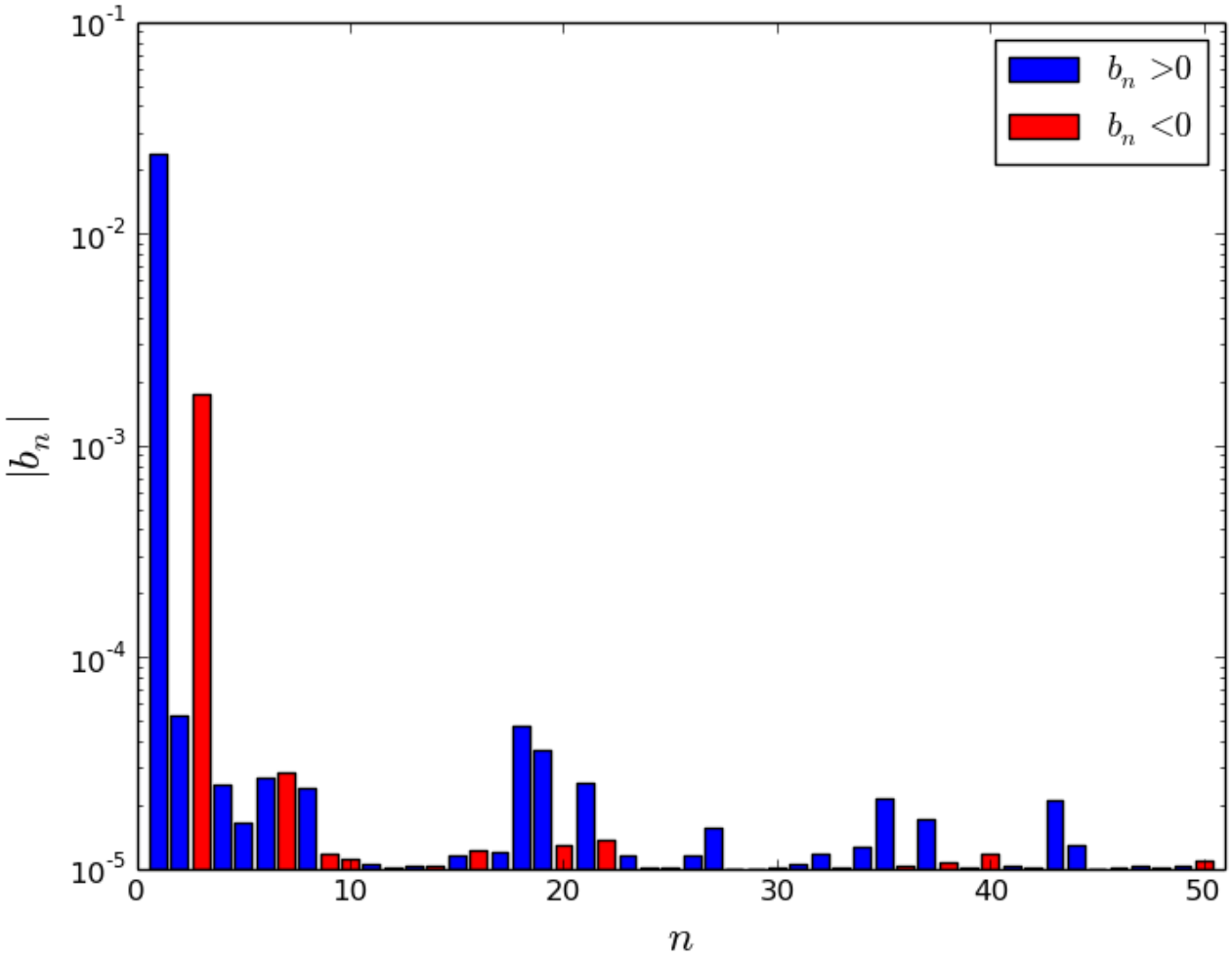


Figure: Approximate solution coefficients

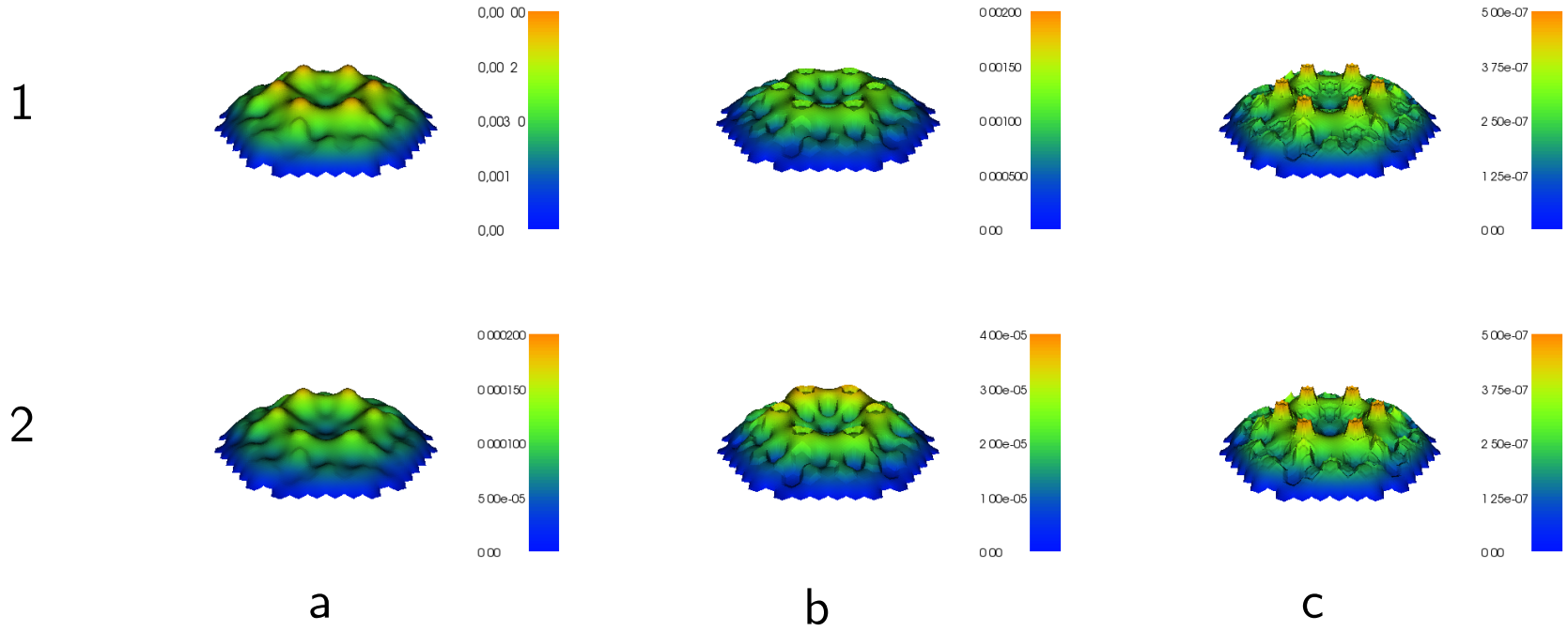
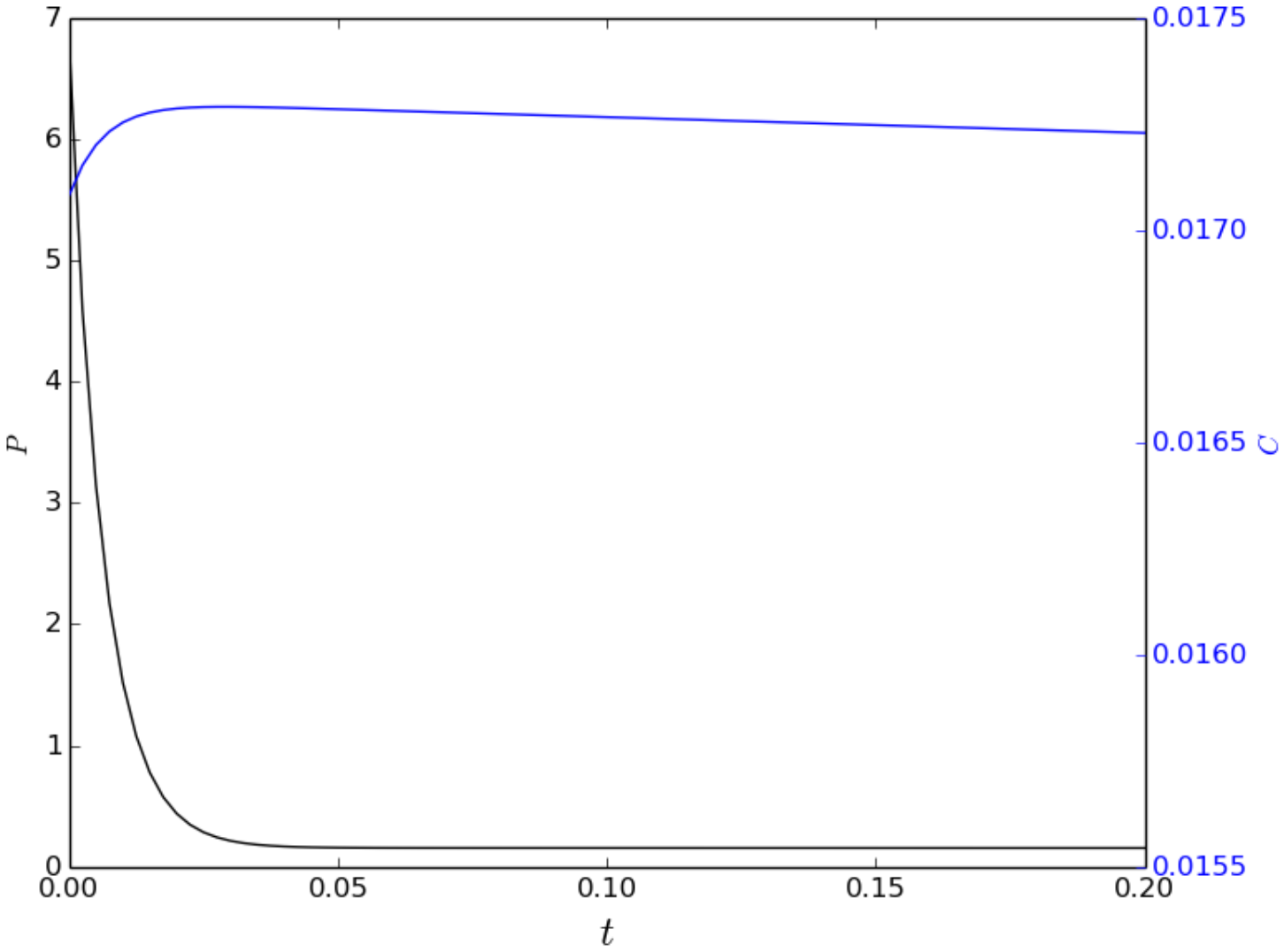


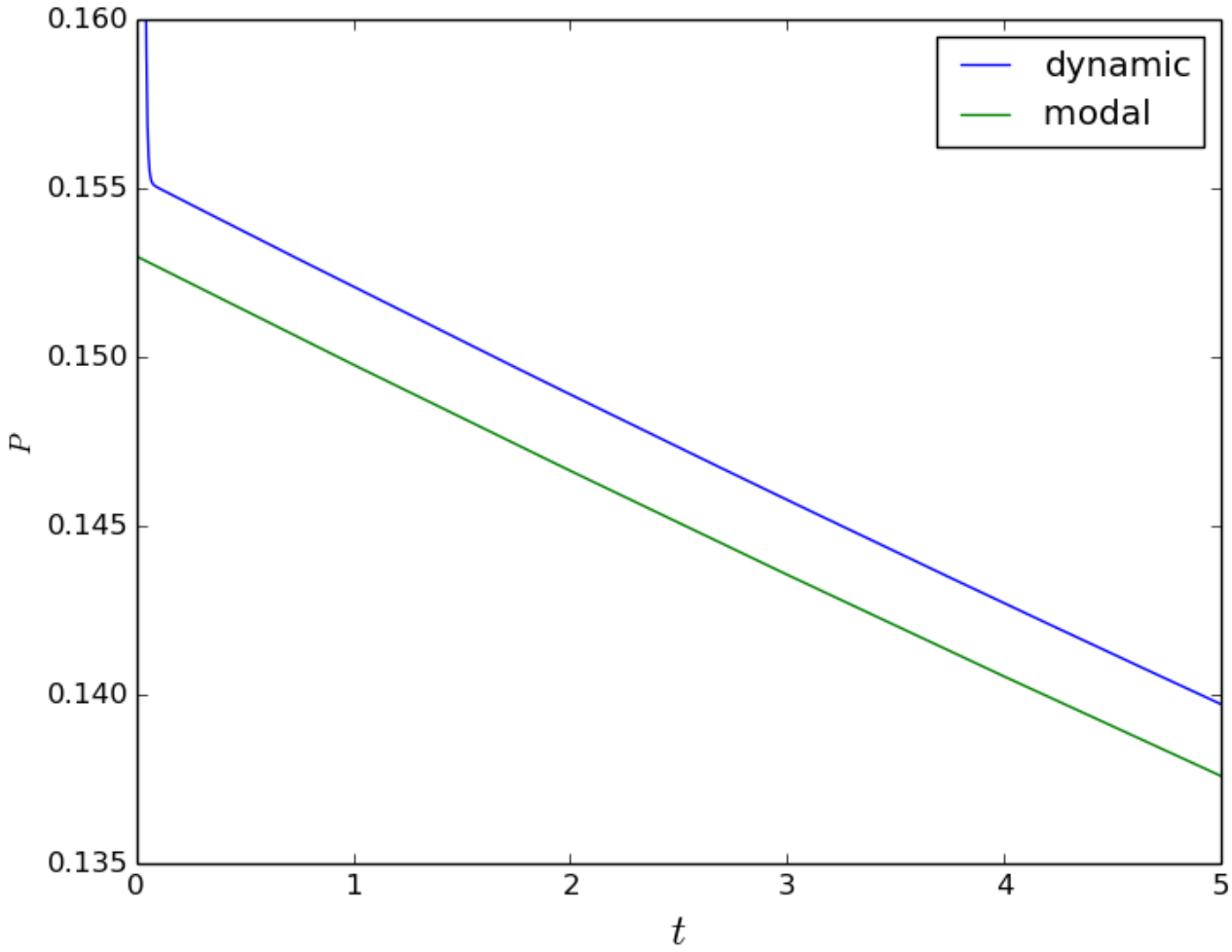
Figure: Function $\boldsymbol{u}(\boldsymbol{x}, 0)$ (string 1) and function $\boldsymbol{u}_N(\boldsymbol{x}, 0)$ (string 2) for asymmetric perturbation: a — neutron flux of group 1, b — neutron flux of group 2, c — delayed neutrons source.

In the case under consideration, the approximate solution contains several modes and can not be described only by the first mode. Calculations of the end of the fast phase are performed at $N = 10$.

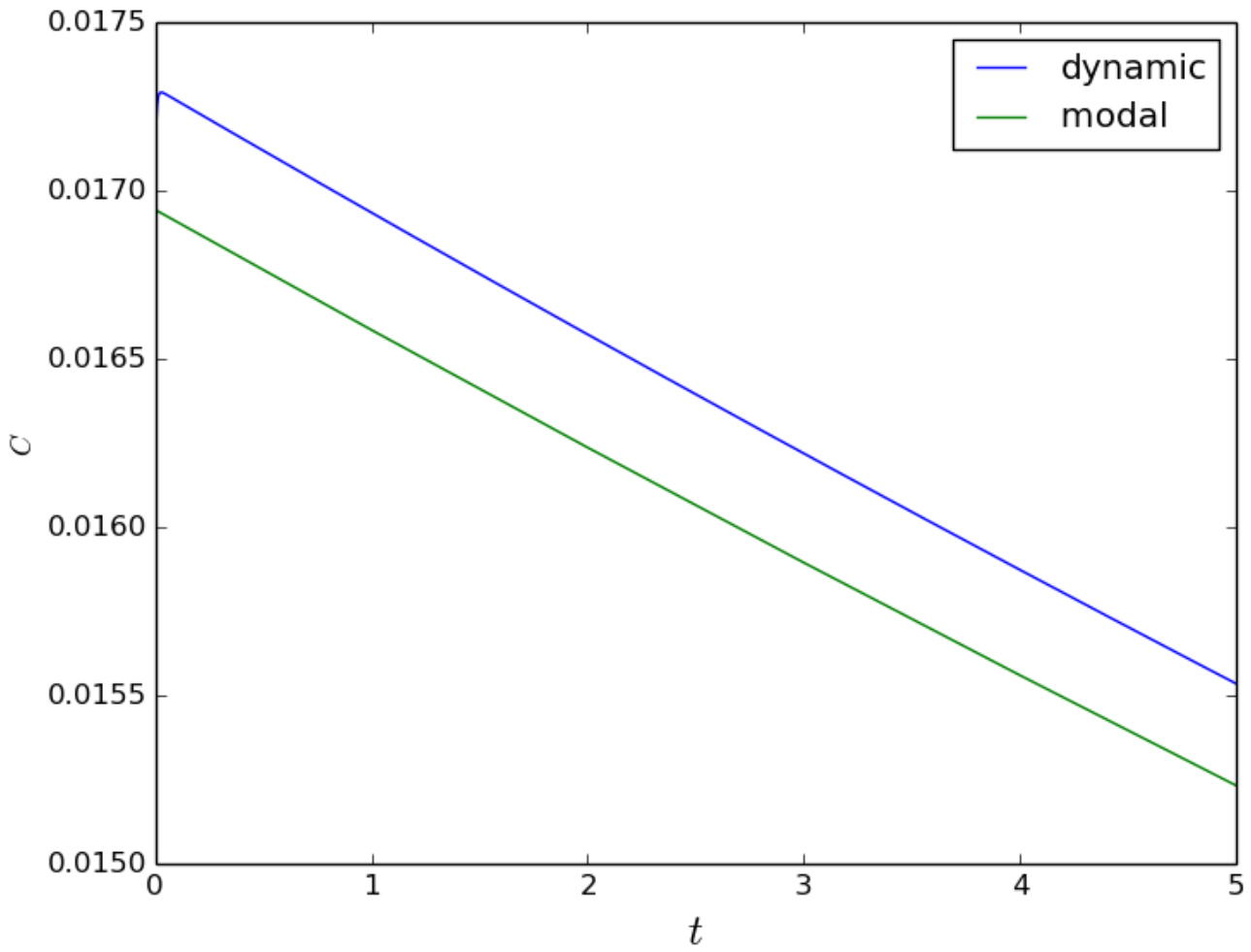
Fast stage of reactor state: neutronic power



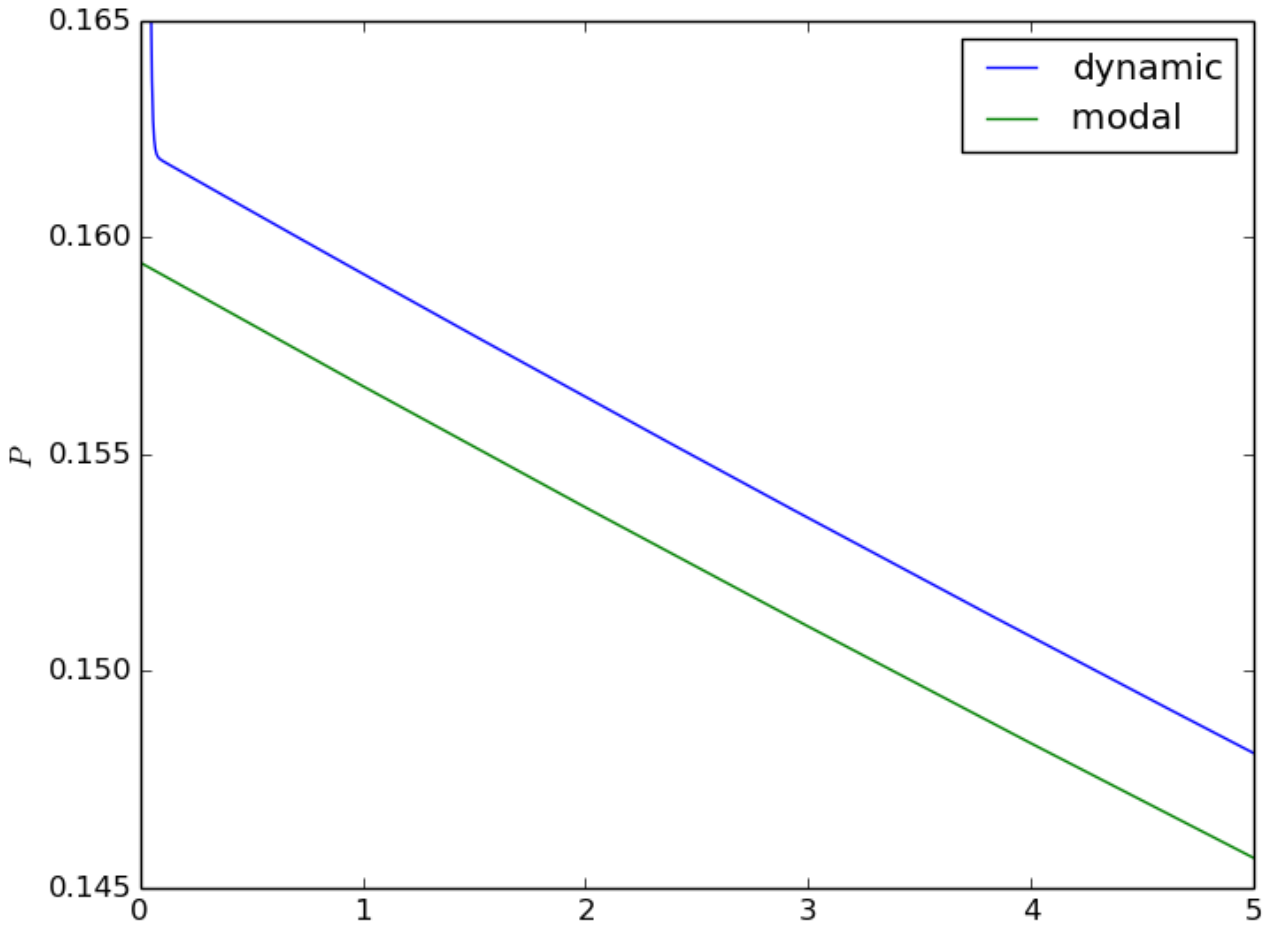
Slow stage of reactor state: neutronic power



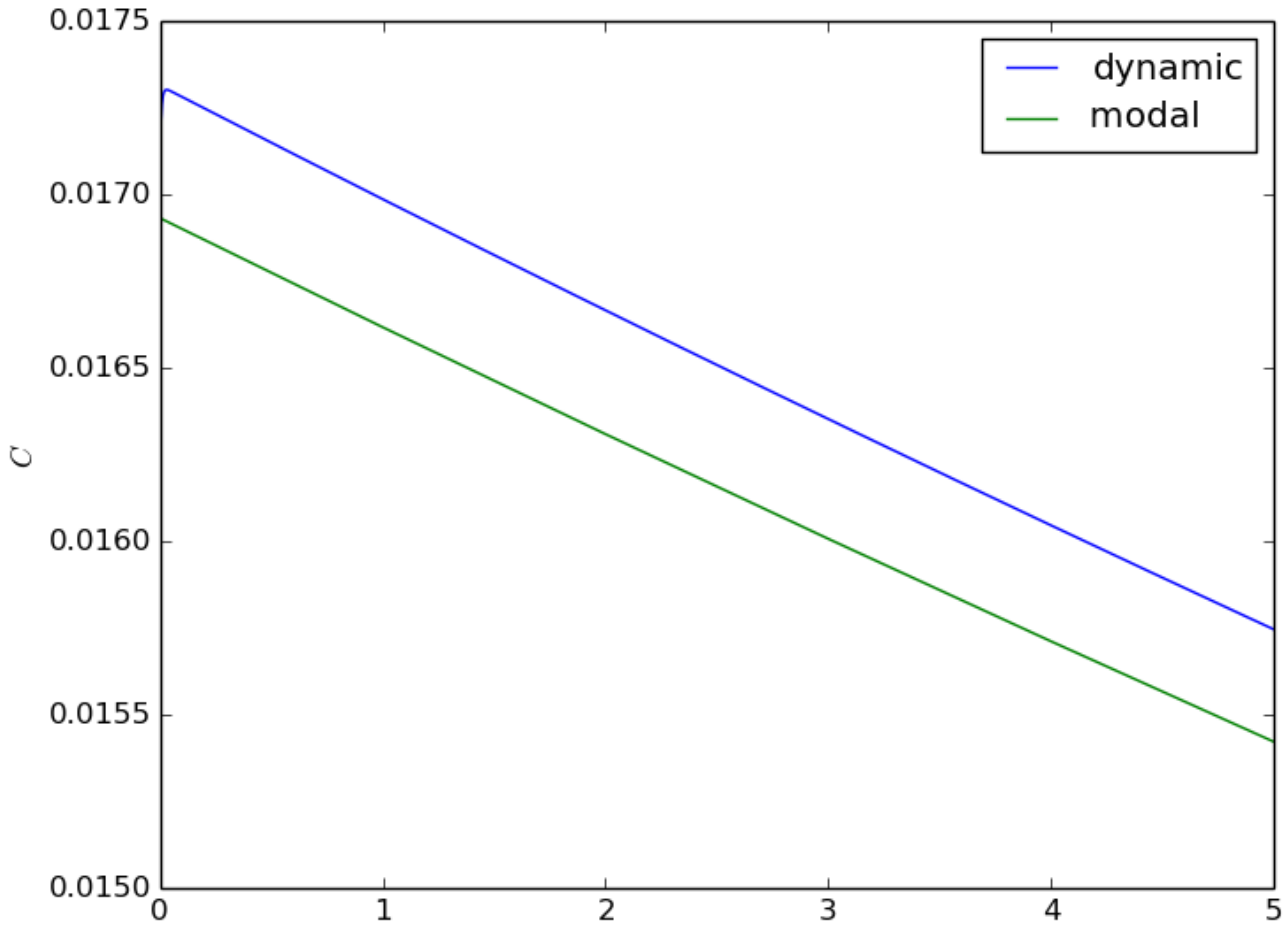
Slow stage of reactor state: delayed neutrons sourse



Slow stage of reactor state for the asymmetric perturbation: neutronic power



Slow stage of reactor state for the asymmetric perturbation: delayed neutrons sourse



Conclusion

- The problem of simulation of reactor dynamic processes is considered on the basis of multigroup neutron diffusion equations accounting for delayed neutrons. The modal approximation is used.
- In the developed SCM method a fast phase and slow phase is allocated
- Offline calculation, online calculation
- Finite Element Method, FEniCS, SLEPc
- The modeling of the reactor state change as a transfer from one state to another state, symmetric and assymetric perturbation
- Comparison of the calculational results obtained by using two methods, one based on modal approximation and another based on the full dynamics calculation

Thank you for your attention!