Hello everyone!

Title of my talk is “Automatic time step selection for numerical solution of neutron diffusion problems”. This work I collaborated with Aleksandr Avvakumov from Kurchatov Institute, Valery Strizhov and Petr Vabishchevich from Nuclear Safety Institute.

Let’s start from introduction!

In computational practice two-layer schemes are mostly used then three-layered or multilayered schemes. The problem of controlling the time step is relatively well worked out.

The main approach is that the error of the approximate solution is estimated at a new time step on the basis of additional calculations. The step is estimated from the theoretical asymptotic dependence of the accuracy on time step and after that correction of step is applied, if necessary, the calculations are repeated.

The algorithm takes into account the features of neutron diffusion problems, for instance, fast changes in the solution or instability with respect to the initial data.

Consider second-order parabolic equation with following boundary and initial conditions.

For convenience, rewrite boundary problem to operator formulation. Then we have Cauchy problem with next initial condition. The problem is considered in a finite-dimensional Hilbert space. We assume that A greater than or equal to zero in H, then for Cauchy problem we have a stability estimate with respect initial condition and right-hand side.

Introduce irregular time grid. For approximate solution the implicit scheme are used.

For approximate solution with these constraints we have a layerwise estimate. Then we obtain a difference estimate.

For the error of the approximate solution zn we have following problem. Here psi n+1 is approximation error. Similarly, we have the difference estimate. The error accumulates and

increases linearly.

Consider the main algorithm for choosing the time step.

1. We select the predicted time step based on the analysis of the solution at the previous steps in time. The predicted time step is determined as following, where gamma is numeric parameter. The default parameter of gamma is one point twenty-five or one and a half.
2. Using the explicit scheme, we find a predicted solution at predicted time. The calculation is done only on one step. Therefore, the possible computational instability does not appear.
3. We estimate the error of approximation by the found predicted solution using the implicit scheme.
4. The time step is estimated by the closeness of the error norm to delta.
5. The solution at a new time step is calculated using the implicit scheme.

Let’s consider modeling neutron flux in a one group diffusion approximation with one group delayed neutron source. With boundary and initial condition. Here phi neutron flux density and c is density of delayed neutron source.

We present the calculated formulas for the choice of the time step for the neutron diffusion equation. Denote vectors and matrix. In our case, the approximation error is.

The needed time step cannot exceed the predicted time step, therefore. We limit the allowable time step by the minimum step t0. The approximation error has the first order in time. In view of this, we set. Then we obtain calculated formula for time step. In this formula (the denominator of the expression), the corrective actions for selecting the time step are clearly reflected, which are associated with the change of the problem operator (the first part), with the dynamics of the solution (second part).

For test problem, we consider benchmark IAEA-2D without a reflector, one group instantaneous and delayed neutrons. Modeling effect of immersion and extraction of control rods.

Let’s define a scenario of dynamic process.

1. We solve spectral problem, which solution we take as initial condition.
2. Then calculation for non-stationary model from 0 sec to 0.5 sec.
3. At the moment of 0.1 sec the value Sigma a for the zone 3 change.

For numerical solution, we use following software.

In this slide presented nuclear power for reference solution. The reference solution is calculated by the implicit scheme on a fine time grid with a fixed time step. Here you can see that at the moment 0.1 sec the nuclear power rapidly changes.