Hello everyone!

1. Title of my talk is “Numerical calculation of spectral problems in SP3 approximation by FEM”. This work was done in collaboration with Alexander Avvakumov from Kurchatov institute, Valery Strizhov and Petr Vabishchevich from Nuclear Safety Institute.

2. Let’s start from introduction! Here you see a nuclear power plant. It is very complex and consists of many systems. But the most important part of course is the nuclear reactor.

Directly, where the chain reaction occurs in a nuclear reactor is called the active zone. Here you can see a schematic version of the active zone. It consists of the assemblies set with fuel, coolant, neutron moderator, control and protection systems.

3. The physical processes in an active zone depend on distribution of neutron flux, whose mathematical description is based on the neutron-transport equation. This equation is integrally-differential one, and depends on time, energy, spatial and angular variables. For practical calculation are used the simplified forms of the neutron transport equation. The most popular and saving sufficient accuracy is the equation system called multi-group diffusion approximation.

For improving the accuracy for both static and transient simulations for reactor core analysis compared with the neutron diffusion theory we can use the SP3 approximation. This is one of simplified form of transport equation. In this regard, it will be very useful to compare the spectral parameters, calculated by both the diffusion and SP3 methods.

4. Diffusion approximation consist of system of equation. Here, Phi is neutron flux, G is number of prompt neutrons equations and M is number of delayed neutrons equations.

The SP3 equations consist of two diffusion type equations with two unknown fluxes: the pseudo

0th moment of angular flux and the second moment of angular flux.

5. The equations are complemented by the following boundary and initial conditions. For diffusion model - Albedo type boundary condition and for SP3 - Marshak type condition. Then we get a boundary problems.

6. For convenience rewrite the boundary problem in operator form. The operators on the right side are responsible for the generation of neutrons. And the operators on the left side are responsible for the loss of neutrons.

7. To characterise the reactor dynamic processes described by Cauchy problem lets consider some spectral problems. The spectral problem, which is known as the lambda-spectral problem, is usually considered. The lambda-spectral problem cannot directly be connected with the dynamic processes in a nuclear reactor. At the best, we can get only the limiting case|the stationary critical state. The more acceptable spectral characteristics for the non-stationary equations are related the alpha-spectral problem.

8. The following software was used for the numerical solution. All these programs are free (open source). The software has been developed using the engineering and scientific calculation library FEniCS. SLEPc has been used for numerical solution of the spectral problems. Python has been used as programming language.

9. To study the properties of the eigenvalues and eigenfunctions of different types, several benchmarks are considered. The first benchmark is the IAEA-2D with reflector. The geometrical model of the IAEA-2D reactor core consists of a set of hexagonal assemblies and is presented in Fig. on the left. 2 group of prompt neutrons and 1 group of delayed neutrons are considered.

The following parameters were varied in the calculation: n - the number of triangles per assembly, p - the order of finite element.

10. As a reference solution we took the solution obtained using the MCNP4C code. These data demonstrate the convergence of the computed eigenvalues with refinement of the mesh and increase the degree of finite element.

11. The results of the solution for test IAEA-2D with a reflector are shown in Table. Here:

k\_dif - effective multiplication factor for the diffusion model;

k\_sp3 - effective multiplication factor for the SP3 model;

Capital delta - absolute deviation from the reference value in pcm (ten to minus 5 degrees);

Delta - the standard deviation of the relative power in percent.

12. In this Fig. you see the power and error distributions using the diffusion (left) and SP3 (right) models. For each assembly the following data are given: the reference solution, the solution for fine grid and the relative error from the reference solution.

13. The results of the first 10 eigenvalues for fine grid are shown in this Table. You can notice that the eigenvalues are very close to each other.

14. The results of the first 10 eigenvalues for fine grid are shown in this Table. On the other hand, the eigenvalues for alpha-spectral problem are well separated.

15. The second benchmark is a model of large heavy-water reactor HWR. The geometry of the HWR test is presented in Fig on the left side. 2 group of prompt neutrons and without taking account delayed neutrons are considered.

16. In this figure you see the solution of lambda-spectral problem. These data demonstrate the convergence of the computed eigenvalues with refinement of the mesh and increase the degree of finite element.

17. The results of the first 10 eigenvalues for fine grid are presented in this Table. The eigenvalues k2; k3; k4; k5; k9; k10 of the lambda-spectral problem are the complex values with small imaginary parts, and the eigenvalues k1; k6; k7; k8 are the real values.

18. The results of the first 10 eigenvalues for fine grid are shown in this Table. The eigenvalues are well separated. The eigenvalues 2,3,4,5,9,10 of the alpha-spectral problem, like for the lambda-spectral problem, are the complex values with small imaginary parts, and the eigenvalues 1,6,7,8 are the real values.

19-21 The eigenfunctions for fundamental eigenvalue (n = 1) of the alpha-spectral problem are shown in Fig. The real part of the eigenfunctions n =2; 3; 4; 5 is shown in Fig. In this Fig. shows the imaginary part of these eigenfunctions. The eigenfunctions of the lambda-spectral and alpha-spectral problems are close to each other in topology.

22. Conclusion.

23. 10x