Hello everyone!

1. Title of my talk is “Automatic time step selection for numerical solution of neutron diffusion problems”. This work I collaborated with Petr Vabishchevich from Nuclear Safety Institute.

2. Let’s start from introduction! Here you see a nuclear power station. It is very complex and consists of many systems. But the most important part of course is the nuclear reactor.

3. Here you see a schematic version of a nuclear power station. Directly, where the chain reaction occurs is called the active zone. It consists of the assemblies set with fuel, coolant, neutron moderator, control and protection systems.

4. The physical processes in an active zone depend on distribution of neutron flux, whose mathematical description is based on the neutron-transport equation. This equation is integrally-differential one, and depends on time, energy, spatial and angular variables. For practical calculation are used the simplified forms of the neutron transport equation. The most popular and saving sufficient accuracy is the equation system called multogroup diffusion approximation.

5. The problem of the time step control is relatively well developed for the Cauchy problem solution of differential equations systems. The basic approach is to use additional calculations at a new time step to estimate the approximate solution.

Additional calculations for estimating the error of the approximate solution can be carried out in different ways. The best-known strategy is connected with the solution of the problem on a separate time interval using the given step (the first solution) and with a step, two times smaller (the second solution).

This way of selecting the time step is related to the class of a posteriori accuracy estimation methods. The decision as to suitable the time step or the re-calculation is accepted only after the calculation is completed.

6. In this figure you can see the nuclear power versus time. At the moment of time t=0 we have a feature associated with instability due to the initial data. Second feature is associated with rapidly down of nuclear power.

7. If we use a uniform grid in time, then in these special places we will be a loss of accuracy. So, we need an algorithm of time step selection that takes into account these features without losing accuracy.

8. Consider the basic algorithm of selection the time step. We select the time step based on the analysis of the previous step solutions. The predicted time step is determined as following, where gamma is numerical parameter. The default value of gamma is 1.5. Using explicit scheme, we can obtain a predicted solution at predicted time. The calculation is performed only at single time step therefore the possible computational instability does not appear.

9. We estimate the approximation error using the calculated predicted solution by the implicit scheme.

10. The time step is estimated by the closeness of the error norm to error parameter delta.

11. The solution at a new time step is calculated with a time step by the implicit scheme.

12. Let's consider neutron diffusion equation in one-group approximation with one-group delayed neutron sources. Neutron flux dynamics is considered within a bounded 2D domain Omega with a convex boundary deltaOmega. Boundary problem considered with boundary and initial condions.

13. After mathematical manipulations from Layerwise estimate, form difference estimate, and others we get calculated formulas for time step. The needed time step cannot exceed the predicted time step. We limit the time step by the minimum time step. We obtain next calculated formula. Here you can see, the corrective actions which are associated with the change of problem operator (the first part) and with the dynamic of solution (second part).

17. The accuracy of the solution was evaluated by a reference solution, which uses a numerical solution on a sufficiently detailed grid in time by the implicit scheme with a fixed time step. Figure show the integral powers for insersion or withdrawal of the control rods respectively.

18. The error is estimated as this, where P\_ref is the reference solution, P is the solution obtained by using the time stepping algorithm. We took a minimum time step tau\_0 = tau\_ref. Figure show the error varepsilon\_P, when the control rods are taken for immersion and extraction, respectively, for different values of the parameter delta. Here we see that the error converges as the parameter delta decreases.

19. Figure shows the time steps for insersion or withdrawal of rods, respectively. It is seen that first there is a rapid growth of the time step with a specified accuracy of delta.

Then the *catch* of a sudden change in power occurs with a strong decrease in the time step.

Further, the time step grows to a certain point and remains at the same level, which is controlled by the error delta.

20. Table shows the various output data for different values of the parameter delta, where max(epsilon\_P) is the maximum power error, n is the number of steps in time, and t is the calculation time in seconds. The reference solution: the number of steps in time is 50000, the calculation time is 2130 seconds.