PROBABILITY

• **Probability** implies 'likelihood' or 'chance'. When an event is certain to happen then the probability of occurrence of that event is 1 and when it is certain that the event cannot happen then the probability of that event is 0.
• Hence the value of probability ranges from 0 to 1.

$$P(A) = \frac{\text{Number of favorable outcomes to A}}{\text{Total number of possible outcomes}}$$

Problem Statement:
• A coin is tossed. What is the probability of getting a head?

Solution:
• Number of outcomes favorable to head = 1
• Total number of outcomes = 2 (i.e. head or tail)

PROBABILITY - BASIC CONCEPTS

Random Experiment:
An experiment is said to be a random experiment, if it's out-come can't be predicted with certainty.
Example:
If a coin is tossed, we can't say whether head or tail will appear. So it is a random experiment.

Sample Space
The set of **all possible out-comes** of an experiment is called the sample space. It is denoted by 'S' and its number of elements are n(s).

Example:
In throwing a dice, the number that appears at top is any one of 1,2,3,4,5,6.
So here:
S ={1,2,3,4,5,6} and n(s) = 6

Example:
In the case of a coin, S={Head,Tail} or {H,T} and n(s)=2.

Event
Every subset of a sample space is an event. It is denoted by 'E'.

Example:
In throwing a dice S={1,2,3,4,5,6}, the appearance of an even number will be the

event E={2,4,6}. Clearly E is a subset of S.

Q.A box contains 20 cards, numbered from 1 to 20. A card is drawn from the box at random. Find the probability that the number on the card drawn is (i) even (ii) prime and (iii) multiple of 3.

Ans:- Even number = 2,4,6,8,10,12,14,16,18,20
Prime Number = 2,3,5,7,11,13,17,19
Multiple of 3 = 3,6,9,12,15,18

(i)Probability of getting an Even Number

$$= \frac{\textbf{Number of Even Numbers}}{\textbf{Total Numbers}} = \frac{10}{20}$$

(ii)Probability of getting a prime number

$$= \frac{\textbf{Number of prime Numbers}}{\textbf{Total Numbers}} = \frac{8}{20}$$

(iii)Probability of getting a multiple of 3

$$= \frac{\textbf{Number of mu;ltiples of 3}}{\textbf{Total numbers}} = \frac{6}{20}$$

Mutually Exclusive:
If two or more events can't occur simultaneously, that is no two of them can occur together.

Example:
When a coin is tossed, the event of occurrence of a head and the event of occurrence of a tail are mutually exclusive events.

Mutually independent events
Two or more events are said to be independent if occurrence or non-occurrence of any of them does not affect the probability of occurrence or non-occurrence of the other event.

Example:
When a coin is tossed twice, the event of occurrence of head in the first throw and the event of occurrence of head in the second throw are independent events.

Difference between mutually exclusive and mutually independent events
Mutually exclusiveness is used when the events are taken from the **same experiment**, whereas independence is used when the events are taken from **different experiments.**

ADDITIVE THEOREM OF PROBABILITY - For Mutually Exclusive Events

Statement: If two events A and B are mutually exclusive, the probability of occurrence of either A or B is the sum of the individual probability of A and B.

$$\mathbf{P(A\ or\ B)=P(A)+P(B)}$$

The theorem can be extended to three or more mutually exclusive events.

$$\mathbf{P(A\ or\ B\ or\ C)=P(A)+P(B)+P(C).}$$

Example: A single 6-sided die is rolled. What is the probability of rolling a 2 or a 5?

Event A (Rolling a 2) -> **P(A) =** $\frac{1}{6}$

Event B (Rolling a 5) -> **P(B) =** $\frac{1}{6}$

Probability of rolling a 2 or a 5 -> **P(A or B) = $\frac{1}{6}$ + $\frac{1}{6}$ = 2/6 = $\frac{1}{3}$**

Example: - A bag contains 30 balls numbered 1 to 30. One ball is drawn at random, find the probability that the number of the balls will be the multiple of 5 or 9.

Number of multiple of 5 (Event A) = (5,10,15,20,25 ,30) = 6
Number of multiple of 9(Event B) = (9,18,27) = 3

Sample space = Total no. of Event = 30

P(A) = 6/30
P(B) = 3/30
P(A U B) = P(A) + P(B)
6/30 + 3/30 =9/30 =3/10.

MULTIPLICATIVE THEOREM - For Independent Events

Statement - If two events A and B are independent the probability that both will occur is equal to the product of their individual probabilities.

P(A and B)= P(A) x P(B)

The theorem can be extended to three or more independent events.

P(A and B and C)=P(A)x P(B)x P(C).

Example: A bag contains 5 white and 3 black balls. Two Balls are drawn at random one after another without replacement. Find the probability that Both balls are black.

P(A) = Probability of drawing a black ball in the first attempt=3/(3+5)=3/8.
P(B) = Probability of drawing again a black ball in the second attempt=2/(2+5)=2/7.
P(A and B) = P(A) x P(B)= 3/8 x 2/7 =3/28.

Example: You have a cowboy hat, a top hat, and an Indonesian hat called a songkok. You also have four shirts: white, black, green, and pink. If you choose one hat and one shirt at random, what is the probability that you choose the songkok and the black shirt?

P(A) = Probability of choosing a songkok = ⅓

P(B) = Probability of choosing a black shirt = ¼

P(A ∩ B) = P(A) x P(B)= 1/12

Conditional Probability - For Dependent Events

The probability of event A given event B equals the probability of event A and event B divided by the probability of event A.

$$P(A / B) = \frac{P(A \cap B)}{P(B)}$$

Q. A family has two children , what is the probability that both the children are boys. Given that at least one of them is a boy.

Event A - Both the children are boys.
Event B - At least one of them is a boy.
Sample space={bb,bg,gb,gg}
P(A) = ¼ = {bb}
P(B) = ¾ = {bg,gb,bb}
P(A∩B) = {bb} -> Probability that they are two boys given that one of them is a boy
P(A∩B) = 1/4
P(A|B) = Probability that both children are boy , given that one child is a boy.
P(A|B) = P(A∩B)/P(B)=(1/4)/3/4)=1/3.

Q. In a school there are 1000 students, out of which 430 are girls, it is known that out of 430 girls 10% girls are studying in class 12 . A student chosen randomly from the school, what is the probability that the chosen one is a student of class 12?. It is given that the chosen student is a girl.

Event A - The chosen one is a student of class 12
Event B - The chosen student is a girl -> 430/1000
$P(A \cap B)$ = 43/1000 -> Probability that she is of class 12 given that she's a girl
$P(A|B) = P(A \cap B)/P(B)$
= 43/1000/ 430/10000 = 1/10

Q. A New movie released, for a married couple the probability that the husband will watch the movie is 70%. The probability that the wife watches the movie is 65 %. The probability that both will watch the movie is 60%. If the husband is watching the movie, what is the probability that the wife is also watching the movie?

Event A - The husband watches the movie - 70/100
Event B - The wife watches the movie - 65/100

$P(A \cap B)$ - Probability that both will watch the movie 60/100

$P(A|B) = P(A \cap B)/P(B)$
= 60/70 = 6/7

owner	Probability Have pet animal	Probability Don't have Pet animal	total
male	0.41	0.08	0.49
female	0.45	0.06	0.51
total	0.86	0.14	1

Q. What is the probability that the randomly selected person is a Male? Given that the selected person has a Pet animal?

Event A - Selected person is male - P(A) - 0.49
Event B - Selected person has a pet - P(B) - 0.86
P(A∩B) - Person who has a pet is a male - 0.41

$P(A|B) = P(A \cap B)/P(B)$
$= 0.41/0.86$
$= 0.4777$