Chapter 4 – Probability

Experiment – any activity in which there are at least two possible outcomes and the result of the activity cannot be predicted with absolute certainty

Sample space – S – the set of all the possible outcomes from an experiment

Example – Give the sample space for each of the following experiments.

(a) Roll a regular six-sided die once and record the number of spots on the top face.

(b) Flip a coin three times and record the sequence of tosses.

(c) Pick a student at random and record their gender and grade level.

(d) Select parts from an assembly line until you find a defective part. Record the sequence of G's (for good part) and F's (for defective part).

Event – any collection or subset of outcomes from a sample space

Example – Refer to the example above.

(a) For each of the following give the event described.

(i) For Part (a) – Let A = event of an even die roll

(ii) For Part (b) – Let B = event of at least two heads

(iii) For Part (b) – Let C = event that the first and last flips are the same

(iv) For Part (c) – Let D = event that a sophomore or junior is selected

(b) For Part (c) – Describe the following events in words.

(i) $A = \{(Male, Fresh), (Male, Soph), (Male, Junior), (Male, Senior)\}$

(ii) $B = \{(Male, Fresh), (Female, Fresh)\}$

(iii) $C = \{(Male, Junior), (Male, Senior), (Female, Junior), (Female, Senior)\}$

Operations for creating new events

- **Union** $A \cup B$ A or B all outcomes from A or from B or from both
- **Intersection** $A \cap B$ A and B all outcomes shared by both A and B
- **Complement** A' not A all outcomes from S that are not in A

Example – Suppose that a sample space $S = \{a, b, c, d, e, f, g, h\}$. Use the events $A = \{a, b, c\}$, $B = \{b, c, e, g\}$, and $C = \{f, g, h\}$ to find each of the following.

(a) $A \cup B$

(b) $A \cap B$

(c) $A \cap C$

(d) A'

(e) $(A \cap B)'$

(f) $A \cup B \cup C$

(g) $A \cap B \cap C$

Section 4.2 – An Introduction to Probability

Question – We often say "The probability of flipping a coin and getting a head is $\frac{1}{2}$ or 50%." What precisely is meant by this? Use the table below to help give an **interpretation of the probability**.

#Tosses	#Heads	%Heads
10	4	40%
100	44	44%
500	265	53%
1000	485	48.5%
5000	2533	50.66%
10000	5025	50.25%

Example – An experiment consists of rolling a 6-sided die once. Suppose that the die has been rigged or tampered with so that the faces are not equally likely (i.e. it's an unfair die). Suppose that the sample space and corresponding probabilities are given in the following table.

Roll	1	2	3	4	5	6
Probability	0.30	0.25	0.10	0.15	0.05	0.15

(a) What two **conditions** must be true (or checked) **for** this to be a **legitimate distribution**?

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(b) Find the probabilities of the following events.

(i) A = the event that the roll is an even number

(ii) B = the event that the roll is at most 3

(iii) C = the event that the roll is at least 5

Important Rules

- **Complement Rule**: $P(A') = 1 - P(A)$
- **Addition Rule**: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Example – Let A and B be events with $P(A) = 0.3$, $P(B) = 0.4$, $P(A \cap B) = 0.1$. Find the probability that …

(a) … A or B occur.

(b) … A and B occur.

(c) … neither A nor B occur.

(d) … just A (and not B) occurs.

Example (Example 4.17, p. 142) – Marketing research by The Coffee Beanery in Detroit, Michigan, indicates that 70% of all customers put sugar in their coffee, 35% add milk, and 25% use both. Suppose a Coffee Beanery customer is selected at random.

(a) Draw a **Venn diagram** to illustrate the events in this problem.

(b) What is the probability that the customer uses at least one of these two items?

(c) What is the probability that the customer uses neither?

(d) What is the probability that the customer uses just sugar?

(e) What is the probability that the customer uses just one of these two items?

Section 4.4 – Conditional Probability

The probability of an event A may be affected by, or depend on, the occurrence of another event B.

$P(A)$ = original or unconditional probability of A happening

$P(A \mid B)$ = conditional probability of A happening given that B has occurred

Example – A class contains 26 students of which 15 are freshmen, 14 are business majors, and 10 are both freshmen and business majors. (Note: Let F = event of picking a freshman; let B = event of picking a business major.)

(a) Draw and fill in a Venn diagram.

(b) Suppose that a person is picked at random from the class. What is the probability that they are a freshman?

(c) After learning that a student is a business major, what is the chance that the student is a freshman?

Definition – Provided $P(B) > 0$, the **conditional probability** of event A given that event B has occurred is

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

Example (Exercise 4.121, p. 165) – According to Trail Count, the annual survey of San Jose's off-street bicycle and pedestrian trail users, approximately 24% use trails daily. Suppose 12% use trails daily and exercise, and 8% use trails daily for commuting. Suppose a trail user is randomly selected.

(a) Given that the person uses trails daily, what is the probability that he uses the trails for exercise?

(b) Given that the person uses trails daily, what is the probability that he uses the trails for commuting?

(c) If the person uses trails daily, what is the probability that he does not use the trail for commuting?

Example – A large statistics course has 80 students enrolled. Each student is cross-classified according to their gender and their grade level. The results are presented in the table below.

	Freshman	Sophomore	Junior	Senior	
Male	2	10	16	15	43
Female	1	8	14	14	37
	3	18	30	29	

Suppose that one student from the course is selected at random. Find the probability of each of the following events.

(a) The student is a male.

(b) The student is a sophomore.

(c) The student is a female junior.

(d) The student is a female or a junior.

(e) If the student is a male, what is the chance they are a freshman?

(f) If the student is not a senior, what is the chance they are a female?

Section 4.5– Independence

Definition – Events A and B are **independent** if and only if $P(A \mid B) = P(A)$. If A and B are not independent, they are said to be **dependent** events.

What does this mean or say? – It means that the probability of event A is the same whether or not event B has occurred.

That is,

$P(A)$ = probability of event A before we learn about event B occurring

is the same as

$P(A \mid B)$ = probability of event A after we learn about event B occurring

This means that the occurrence of event B did not affect the chance of event A, i.e. that they're independent.

Example – Suppose that we roll a balanced or fair die having the probability distribution below. Suppose that we define the events $A = \{1,3,5\}$, $B = \{4,5,6\}$, and $C = \{3,4,5,6\}$.

Roll	1	2	3	4	5	6
Probability	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

(a) Are events A and B independent?

(b) Are events A and C independent?

Important Notes

- If events A and B are independent, then so are the pairs
  (i) A and B' (ii) A' and B (iii) A' and B'

- Instead of checking if events are independent, we sometimes are told (or can assume) that they are. In this case, to use the independence we'll often use the following result.

  Multiplication Rule for Independent Events
  If events A and B are independent, then $P(A \cap B) = P(A) \times P(B)$

Example – Suppose that events A, B, and C are independent with $P(A) = 0.2$, $P(B) = 0.4$, and $P(C) = 0.7$. Find the following probabilities.

(a) $P(A \cap B)$

(b) $P(B \cap C')$

(c) $P(A \cap B' \cap C)$

Example (Example 4.36, p. 171) – Better Bedding in East Hartford, Connecticut, claims that 99.4% of all its mattress deliveries are on time. Suppose two mattress deliveries are selected at random.

(a) What is the probability that both mattresses will be delivered on time?

(b) What is the probability that both mattresses will be delivered late?

(c) What is the probability that exactly one mattress will be delivered on time?

<u>Example</u> – The following describes the proportions of M&Ms in a bowl. Suppose that three M&Ms are selected independently from one another.

Color	Red	Yellow	Orange	Green	Blue	Brown
Proportion	0.20	0.20	0.10	0.10	0.10	0.30

(a) What is the probability that all three are brown?

(b) What is the probability that none are yellow?

(c) What is the probability that exactly one is orange?

(d) What is the probability that at least one is green?