

# MIST\_Untitled

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Last modified: December 18, 2025

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# 1 C++

## 1.1 template

```
/*
c++:
ios_base::sync_with_stdio(false);
cin.tie(nullptr), cout.tie(nullptr);
```

python:

```
import sys
input = sys.stdin.readline
sys.stdout.write("-----")
*/
```

## 1.2 random

```
#define accuracy chrono::steady_clock::
    ↪ now().time_since_epoch().count()
mt19937 rng(accuracy);
ll rand(ll l, ll r) {
    uniform_int_distribution<ll> ludo(l, r
    ↪ );
    return ludo(rng);
}
```

## 1.3 gp\_hash

```
#include<ext/pb_ds/assoc_container.hpp>
#include<ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
template <typename p, typename q> using
    ↪ ht = gp_hash_table<p, q>;
```

## 1.4 pbds

```
#include<ext/pb_ds/assoc_container.hpp>
#include<ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
template <typename T>
using o_set = tree<T, null_type, less<T
    ↪ >, rb_tree_tag,
    ↪ tree_order_statistics_node_update
    ↪ >;
/*
find_by_order(k) - returns an iterator
    ↪ to the k-th largest element (0
    ↪ indexed);
order_of_key(k) - the number of elements
    ↪ in the set that are strictly
    ↪ smaller than k;
*/
```

## 1.5 debug

```
string to_string(const string &s) {
    ↪ return '' + s + '';
string to_string(const char *s) { return
    ↪ to_string(string(s));
string to_string(const char c) { return
    ↪ '' + string(1, c) + '';
string to_string(bool b) { return b ? "
    ↪ true" : "false";
template <typename A, typename B> string
    ↪ to_string(pair<A, B> p) {
        return "(" + to_string(p.first) + ", "
    ↪ + to_string(p.second) + ")";
}
template <typename A> string to_string(A
    ↪ v) {
    string res = "{}";
    for (const auto &x : v) {
        res += to_string(x) + ", ";
    }
    res += "}";
    return res;
}
void debug_out() { cerr << endl; }
template <typename Head, typename...
    ↪ Tail> void debug_out(Head H, Tail
    ↪ ... T) {
    cerr << " " << to_string(H);
    debug_out(T...);
}
```

```
}
#define dbg(...) \
    ↪ \ \
    cerr << LINE \
    ↪ __VA_ARGS__ << ":" ["] = ", debug_out \
    ↪ (__VA_ARGS__)
```

## 1.6 stress

```
#!/usr/bin/env bash
wrong="solution"
correct="brute"
gen="gen"
g++ -g solution.cpp -DONPC -o "$wrong"
g++ -g brute.cpp -DONPC -o "$correct"
g++ -g gen.cpp -DONPC -o "$gen"
for ((testNum=0; testNum<$1; testNum++))
do
    ./$gen 2>/dev/null > stdininput
    ./$correct < stdininput 2>/dev/null >
    ↪ outSlow
    ./$wrong < stdininput 2>/dev/null >
    ↪ outWrong
H1=`md5sum outWrong`'
H2=`md5sum outSlow`'
if !(cmp -s "outWrong" "outSlow")
then
    echo "Error found!"
    echo "Input:"
    cat stdininput
    echo "Wrong Output:"
    cat outWrong
    echo "Slow Output:"
    cat outSlow
    exit
fi
done
echo Passed $1 tests
# Usage: ./contest.sh times
```

## 1.7 vscode

```
{
    "key" : "f5",
    "command" : "workbench.action.terminal
    ↪ .sendSequence",
    "args" : {
        "text" : "g++ ${fileBasenameNoExtension}.cpp -
    ↪ ${fileBasenameNoExtension}.h
    ↪ && ./ ${fileBasenameNoExtension} <in.
    ↪ > out.txt\n"
    }
}
```

# 2 Dsa

## 2.1 KMP

```
vector<ll> createLPS(string pattern) {
    ll n = pattern.length(), idx = 0;
    vector<ll> lps(n);
    for (ll i = 1; i < n;) {
        if (pattern[i] == pattern[idx]) {
            lps[i] = idx + 1;
            idx++, i++;
        } else {
            if (idx != 0)
                idx = lps[idx - 1];
            else
                lps[i] = idx, i++;
        }
    }
    return lps;
}
ll kmp(string text, string pattern) {
    ll cnt_of_match = 0, i = 0, j = 0;
    vector<ll> lps = createLPS(pattern);
    while (i < text.length()) {
        if (text[i] == pattern[j])
            if (text[i] == pattern[j])
```

```

    i++, j++; // i = text, j = pattern
  else {
    if (j != 0)
      j = lps[j - 1];
    else
      i++;
  }
  if (j == pattern.length()) {
    cnt_of_match++;
    // the index where match found ->
    // (i - pattern.length());
    j = lps[j - 1];
  }
}
return cnt_of_match;
}

```

## 2.2 Hashing

```

const ll N = 2e5 + 5;
const ll MOD1 = 127657753, MOD2 =
  ↪ 987654319;
const ll p1 = 137, p2 = 277;
ll ip1, ip2;
pair<ll, ll> pw[N], ipw[N];
void prec() {
  pw[0] = {1, 1};
  for (ll i = 1; i < N; i++) {
    pw[i].first = 1LL * pw[i - 1].first
      ↪ * p1 % MOD1;
    pw[i].second = 1LL * pw[i - 1].
      ↪ second * p2 % MOD2;
  }
  ip1 = binaryExp(p1, MOD1 - 2, MOD1);
  ip2 = binaryExp(p2, MOD2 - 2, MOD2);
  ipw[0] = {1, 1};
  for (ll i = 1; i < N; i++) {
    ipw[i].first = 1LL * ipw[i - 1].
      ↪ first * ip1 % MOD1;
    ipw[i].second = 1LL * ipw[i - 1].
      ↪ second * ip2 % MOD2;
  }
}
struct Hashing {
  ll n;
  string s; // 0 -
  ↪ indexed
  vector<pair<ll, ll>> hs; // 1 -
  ↪ indexed
  Hashing() {}
  Hashing(string _s) {
    n = _s.size();
    s = _s;
    hs.emplace_back(0, 0);
    for (ll i = 0; i < n; i++) {
      pair<ll, ll> p;
      p.first = (hs[i].first + 1LL * pw[
        ↪ i].first * s[i] % MOD1) %
        ↪ MOD1;
      p.second = (hs[i].second + 1LL *
        ↪ pw[i].second * s[i] % MOD2)
        ↪ % MOD2;
      hs.push_back(p);
    }
    pair<ll, ll> get_hash(ll l, ll r) {
      // 1 - indexed
      assert(l <= r && r <= n);
      pair<ll, ll> ans;
      ans.first =
        (hs[r].first - hs[l - 1].first +
          ↪ MOD1) * 1LL * ipw[l - 1].
          ↪ first % MOD1;
      ans.second = (hs[r].second - hs[l -
        ↪ 1].second + MOD2) * 1LL *
          ↪ ipw[l - 1].second %
            ↪ MOD2;
      return ans;
    }
    pair<ll, ll> get_hash() { return
      ↪ get_hash(1, n); }
  }
}

```

## 2.3 BigInteger

```

struct BigInteger {
  string str;
  // Constructor to initialize
  // BigInteger with a string
  BigInteger(string s) { str = s; }
  // Overload + operator to add
  // two BigInteger objects
  BigInteger operator+(const BigInteger
    ↪ &b) {
    string a = str, c = b.str;
    ll alen = a.length(), clen = c.
      ↪ length();
    ll n = max(alen, clen);
    if (alen > clen)
      c.insert(0, alen - clen, '0');
    else if (alen < clen)
      a.insert(0, clen - alen, '0');
    string res(n + 1, '0');
    ll carry = 0;
    for (ll i = n - 1; i >= 0; i--) {
      ll digit=(a[i] -'0')+(c[i]-'0'
        ↪ ')
      +carry;
      carry = digit / 10;
      res[i + 1] = digit % 10 + '0
        ↪ ';
    }
    if (carry == 1) {
      res[0] = '1';
      return BigInteger(res);
    } else
      return BigInteger(res.substr
        ↪ (1));
  }
  // Overload - operator to subtract
  // first check which number is greater
  // and then subtract
  BigInteger operator-(const BigInteger
    ↪ &b) {
    string a = str;
    string c = b.str;
    ll alen = a.length(), clen = c.
      ↪ length();
    ll n = max(alen, clen);
    if (alen > clen)
      c.insert(0, alen - clen, '0'
        ↪ );
    else if (alen < clen)
      a.insert(0, clen - alen, '0'
        ↪ );
    if (a < c) {
      swap(a, c);
      swap(alen, clen);
    }
    string res(n, '0');
    ll carry = 0;
    for (ll i = n - 1; i >= 0; i--) {
      ll digit = (a[i] -'0') - (c
        ↪ [i] -'0') - carry;
      if (digit < 0)
        digit += 10, carry = 1;
      else
        carry = 0;
      res[i] = digit + '0';
    }
    // remove leading zeros
    ll i = 0;
    while (i < n && res[i] == '0')
      i++;
    if (i == n)
      return BigInteger("0");
    return BigInteger(res.substr(i));
  }
  // Overload * operator to multiply
  // two BigInteger objects
  BigInteger operator*(const BigInteger
    ↪ &b) {
    string a = str, c = b.str;
    ll alen = a.length(), clen = c.
      ↪ length();
    ll n = alen + clen;
  }
}

```

```

string res(n, '0');
for (ll i = alen - 1; i >= 0; i--) {
    ll carry = 0;
    for (ll j = clen - 1; j >=
        ↪ 0; j--) {
        ll digit =
            (a[i] - '0') * (c[j] -
                ↪ '0') + (res[i +
                    ↪ j + 1] - '0') +
                ↪ carry;
        carry = digit / 10;
        res[i + j + 1] = digit %
            ↪ 10 + '0';
    }
    res[i] += carry;
}
ll i = 0;
while (i < n && res[i] == '0')
    i++;
if (i == n)
    return BigInteger("0");
return BigInteger(res.substr(i));
}

// Overload << operator to output
// BigInteger object
friend ostream &operator<<(ostream &
    ↪ out, const BigInteger &b) {
    out << b.str;
    return out;
}

```

## 2.4 Kadane

```

// return maximum subarray sum.
ll kadense(ll arr[], ll n) {
    ll mxsm = arr[0], curr_s = arr[0];
    for (ll i = 1; i < n; i++) {
        curr_s = max(arr[i], curr_s + arr[i
            ↪ ]);
        mxsm = max(mxsm, curr_s);
    }
    return mxsm;
}

```

## 2.5 Segment\_tree

```

class SEGMENT_TREE {
public:
    vector<ll> v;
    vector<ll> seg;
    SEGMENT_TREE(ll n) {
        v.resize(n + 5);
        seg.resize(4 * n + 5);
    }
    //! initially: ti = 1, low = 1, high =
    //! n (number of elements in the
    //! array);
    void build(ll ti, ll low, ll high) {
        if (low == high) {
            seg[ti] = v[low];
            return;
        }
        ll mid = (low + high) / 2;
        build(2 * ti, low, mid);
        build(2 * ti + 1, mid + 1, high);
        seg[ti] = (seg[2 * ti] + seg[2 * ti
            ↪ + 1]);
    }
    //! initially: ti = 1, low = 1, high =
    //! n (number of elements in the
    //! array), (ql & qr)=user input in
    //! 1 based index;
    ll find(ll ti, ll tl, ll tr, ll ql, ll
        ↪ qr) {
        if (tl > qr || tr < ql) {
            return 0;
        }
        if (tl >= ql and tr <= qr)
            return seg[ti];
        ll mid = (tl + tr) / 2;
        ll l = find(2 * ti, tl, mid, ql, qr)
            ↪ ;
    }
}

```

```

ll r = find(2 * ti + 1, mid + 1, tr,
    ↪ ql, qr);
return (l + r);

//! initially: ti = 1, tl = 1, tr = n
//! (number of elements in the array
//! ), id = user input in 1 based
//! indexing, val = updated value;
void update(ll ti, ll tl, ll tr, ll id
    ↪ , ll val) {
    if (id > tr or id < tl)
        return;
    if (id == tr and id == tl) {
        seg[ti] = val;
        return;
    }
    ll mid = (tl + tr) / 2;
    update(2 * ti, tl, mid, id, val);
    update(2 * ti + 1, mid + 1, tr, id,
        ↪ val);
    seg[ti] = (seg[2 * ti] + seg[2 * ti
        ↪ + 1]);
}
// use 1 based indexing;

```

## 2.6 Fenwick\_tree

```

struct FenwickTree {
    vector<ll> bit; // binary indexed tree
    ll n;
    FenwickTree(ll n) {
        this->n = n;
        bit.assign(n, 0);
    }
    FenwickTree(vector<ll> a) :
        ↪ FenwickTree(a.size()) {
        for (size_t i = 0; i < a.size(); i
            ↪ ++)
            add(i, a[i]);
    }
    ll sum(ll r) {
        ll ret = 0;
        for (; r >= 0; r = (r & (r + 1)) -
            ↪ 1)
            ret += bit[r];
        return ret;
    }
    ll sum(ll l, ll r) { return sum(r) -
        ↪ sum(l - 1); }
    void add(ll idx, ll delta) {
        for (; idx < n; idx = idx | (idx +
            ↪ 1))
            bit[idx] += delta;
    }
    // minimum
    struct FenwickTreeMin {
        vector<ll> bit;
        ll n;
        const ll INF = (ll)1e9;
        FenwickTreeMin(ll n) {
            this->n = n;
            bit.assign(n, INF);
        }
        FenwickTreeMin(vector<ll> a) :
            ↪ FenwickTreeMin(a.size()) {
            for (size_t i = 0; i < a.size(); i
                ↪ ++)
                update(i, a[i]);
        }
        ll getmin(ll r) {
            ll ret = INF;
            for (; r >= 0; r = (r & (r + 1)) -
                ↪ 1)
                ret = min(ret, bit[r]);
            return ret;
        }
        void update(ll idx, ll val) {
            for (; idx < n; idx = idx | (idx +
                ↪ 1))
                bit[idx] = min(bit[idx], val);
        }
    }
}

```

```
};
```

## 2.7 Segment\_tree\_lazy

```
class SEGMENT_TREE {
public:
    vector<ll> v;
    vector<ll> seg;
    vector<ll> lazy;
    SEGMENT_TREE(ll n) {
        v.resize(n + 5, 0);
        seg.resize(4 * n + 5, 0);
        lazy.resize(4 * n + 5, 0);
    }
    void pull(ll ti) { seg[ti] = (seg[2 * 
        ↪ ti] & seg[2 * ti + 1]); }
    void push(ll ti, ll tl, ll tr) {
        if (lazy[ti] == 0)
            return;
        seg[ti] |= lazy[ti];
        if (tl != tr) {
            lazy[2 * ti] |= lazy[ti];
            lazy[2 * ti + 1] |= lazy[ti];
        }
        lazy[ti] = 0;
    }
    //! 11ially: ti = 1, low = 1, high = n
    //! (number of elements in the array
    //! );
    void build(ll ti, ll low, ll high) {
        lazy[ti] = 0;
        if (low == high) {
            seg[ti] = v[low];
            return;
        }
        ll mid = (low + high) / 2;
        build(2 * ti, low, mid);
        build(2 * ti + 1, mid + 1, high);
        pull(ti);
    }
    //! 11ially: ti = 1, low = 1, high = n
    //! (number of elements in the array
    //! );
    //! & qr) = user input in 1 based
    //! indexing;
    ll query(ll ti, ll tl, ll tr, ll ql,
        ↪ ll qr) {
        push(ti, tl, tr);
        if (tl > qr || tr < ql) {
            return (1LL << 32) - 1;
        }
        if (tl >= ql and tr <= qr)
            return seg[ti];
        ll mid = (tl + tr) / 2;
        ll l = query(2 * ti, tl, mid, ql, qr
            ↪ );
        ll r = query(2 * ti + 1, mid + 1, tr
            ↪ , ql, qr);
        return (l & r);
    }
    //! 11ially: ti = 1, tl = 1, tr = n(
    //! number of elements in the array)
    //!, id =
    //! user input in 1 based indexing,
    //! val = updated value;
    void update(ll ti, ll tl, ll tr, ll
        ↪ idL, ll idR, ll val) {
        push(ti, tl, tr);
        if (idR < tl or tr < idL)
            return;
        if (idL <= tl and tr <= idR) {
            lazy[ti] |= val;
            push(ti, tl, tr);
            return;
        }
        ll mid = (tl + tr) / 2;
        update(2 * ti, tl, mid, idL, idR,
            ↪ val);
        update(2 * ti + 1, mid + 1, tr, idL,
            ↪ idR, val);
        pull(ti);
    }
    // use 1 based indexing for input and
}
```

```
    ↪ queries and update;
};
```

## 2.8 Trie

```
const ll N = 26;
class Node {
public:
    ll EoW;
    Node *child[N];
    Node() {
        EoW = 0;
        for (ll i = 0; i < N; i++)
            child[i] = NULL;
    }
    void insert(Node *node, string s) {
        for (size_t i = 0; i < s.size(); i++)
            ↪ {
                ll r = s[i] - 'A';
                if (node->child[r] == NULL)
                    node->child[r] = new Node();
                node = node->child[r];
            }
        node->EoW += 1;
    }
    ll search(Node *node, string s) {
        for (size_t i = 0; i < s.size(); i++)
            ↪ {
                ll r = s[i] - 'A';
                if (node->child[r] == NULL)
                    return 0;
            }
        return node->EoW;
    }
    void prll(Node *node, string s = "") {
        if (node->EoW)
            cout << s << "\n";
        for (ll i = 0; i < N; i++) {
            if (node->child[i] != NULL) {
                char c = i + 'A';
                prll(node->child[i], s + c);
            }
        }
    }
    bool isChild(Node *node) {
        for (ll i = 0; i < N; i++)
            if (node->child[i] != NULL)
                return true;
        return false;
    }
    bool isJunc(Node *node) {
        ll cnt = 0;
        for (ll i = 0; i < N; i++) {
            if (node->child[i] != NULL)
                cnt++;
        }
        if (cnt > 1)
            return true;
        return false;
    }
    ll trie_delete(Node *node, string s, ll
        ↪ k = 0) {
        if (node == NULL)
            return 0;
        if (k == (ll)s.size()) {
            if (node->EoW == 0)
                return 0;
            if (isChild(node)) {
                node->EoW = 0;
                return 0;
            }
            return 1;
        }
        ll r = s[k] - 'A';
        ll d = trie_delete(node->child[r], s,
            ↪ k + 1);
        ll j = isJunc(node);
        if (d)
            delete node->child[r];
        if (j)
            return 0;
    }
}
```

```

    return d;
}
void delete_trie(Node *node) {
    for (ll i = 0; i < 15; i++) {
        if (node->child[i] != NULL)
            delete_trie(node->child[i]);
    }
    delete node;
}

```

## 2.9 DSU

```

class DisjollSet {
    vector<ll> par, sz, minElmt, maxElmt,
    ↪ cntElmt;

public:
    DisjollSet(ll n) {
        par.resize(n + 1);
        sz.resize(n + 1, 1);
        minElmt.resize(n + 1);
        maxElmt.resize(n + 1);
        cntElmt.resize(n + 1, 1);
        for (ll i = 1; i <= n; i++)
            par[i] = minElmt[i] = maxElmt[i] =
            ↪ i;
    }
    ll findUPar(ll u) {
        if (u == par[u])
            return u;
        return par[u] = findUPar(par[u]);
    }
    void unionBySize(ll u, ll v) {
        ll pU = findUPar(u);
        ll pV = findUPar(v);
        if (pU == pV)
            return;
        if (sz[pU] < sz[pV])
            swap(pU, pV);
        par[pV] = pU;
        sz[pU] += sz[pV];
        cntElmt[pU] += cntElmt[pV];
        minElmt[pU] = min(minElmt[pU],
        ↪ minElmt[pV]);
        maxElmt[pU] = max(maxElmt[pU],
        ↪ maxElmt[pV]);
    }
    ll getMinElementIntheSet(ll u) {
        ↪ return minElmt[findUPar(u)];
    }
    ll getMaxElementIntheSet(ll u) {
        ↪ return maxElmt[findUPar(u)];
    }
    ll getNumofElementIntheSet(ll u) {
        ↪ return cntElmt[findUPar(u)];
    }
};

```

## 2.10 HLD

```

ll par[N], sub_tree_sz[N], heavy[N],
    ↪ wt_from_parent[N], depth[N], head[
    ↪ N],
    position[N];
vector<pair<ll, ll>> gd[N];

// HLD part start
ll dfs(ll node, ll p) {
    par[node] = p;
    sub_tree_sz[node] = 1;
    heavy[node] = -1;

    for (auto [v, w] : gd[node]) {
        if (v == p)
            continue;
        depth[v] = depth[node] + 1;
        wt_from_parent[v] = w;
        sub_tree_sz[node] += dfs(v, node);
        if (heavy[node] == -1 || sub_tree_sz
            ↪ [v] > sub_tree_sz[heavy[node]
            ↪ ]]) {
            heavy[node] = v;
        }
    }
    return sub_tree_sz[node];
}
ll pos;
void decompose(ll node, ll hd) {

```

```

    head[node] = hd;
    position[node] = ++pos;
    if (heavy[node] != -1) {
        decompose(heavy[node], hd);
    }
    for (auto [v, w] : gd[node]) {
        if (v != par[node] && v != heavy[
            ↪ node]) {
            decompose(v, v);
        }
    }
}

// HLD part end
// in main function
ll n, m;
cin >> n;
SEGMENT_TREE seg(n); // Lazy if needed
vector<ll> edge_u(n), edge_v(n),
    ↪ edge_node(n);

for (int i = 1; i < n; i++) {
    ll u, v, wt = 1;
    cin >> u >> v >> wt;
    gd[u].push_back({v, wt});
    gd[v].push_back({u, wt});
    edge_u[i] = u;
    edge_v[i] = v;
}
dfs(1, -1);
pos = 0;
decompose(1, 1);

for (int i = 1; i <= n; i++) {
    // seg.v[position[i]] = val[i]; //
    ↪ for node value
    seg.v[position[i]] = wt_from_parent[i
        ↪ ]; // for edge value
}

// work on a specific edge
for (int i = 1; i < n; i++) {
    ll u = edge_u[i], v = edge_v[i];
    edge_node[i] = (depth[u] > depth[v]) ?
        ↪ u : v;
}

seg.build(1, 1, n);

auto updatePath = [&](ll u, ll v, ll x)
    ↪ {
        while (head[u] != head[v]) {
            if (depth[head[u]] < depth[head[v]])
                swap(u, v);
            seg.update(1, 1, n, position[head[u]
                ↪ ], position[u], x);
            u = par[head[u]];
        }
        if (depth[u] > depth[v])
            swap(u, v);
        // edge value
        if (u != v) {
            seg.update(1, 1, n, position[u] + 1,
            ↪ position[v], x);
        }
        // node value
        // seg.update(1, 1, n, position[u],
        ↪ position[v], x);
    };
}

auto querypath = [&](ll u, ll v) {
    ll ans = -inf;
    while (head[u] != head[v]) {
        if (depth[head[u]] < depth[head[v]])
            swap(u, v);
        ans = max(ans, seg.query(1, 1, n,
            ↪ position[head[u]], position[u
            ↪ ]));
        u = par[head[u]];
    }
    if (depth[u] > depth[v])
        swap(u, v);
    // upward + downward
    if (u != v) {
        ans = max(ans, seg.query(1, 1, n,
            ↪ position[u] + 1, position[v]));
    }
};

```

```

    ↵ ;
}
// only upward
ans = max(ans, seg.query(1, 1, n,
    ↵ position[u], position[v])), // for
    ↵ node value
return ans;
};

seg.update(1, 1, n, position[edge_node[s
    ↵ ]], position[edge_node[s]], x); //  

    ↵ single point update. if path
    ↵ update need call update_path
cout << querypath(x, s) << '\n';

```

## 2.11 Manacher

```

struct Manacher {
    vector<ll> p[2];
    string s;
    // p[1][i] = (max odd length
    //    ↪ palindrome centered at i) / 2 [
    //        ↪ floor division]
    // p[0][i] = same for even, it
    //    ↪ considers the right center
    // e.g. for s = "abba", p[1][3] =
    //    ↪ 3, p[0][2] = 2
    Manacher(string s) {
        this->s = s;
        ll n = s.size();
        p[0].resize(n + 1);
        p[1].resize(n);
        for (ll z = 0; z < 2; z++) {
            for (ll i = 0, l = 0, r = 0; i < n
                ↪ ; i++) {
                ll t = r - i + !z;
                if (i < r)
                    p[z][i] = min(t, p[z][l + t]);
                ll L = i - p[z][i], R = i + p[z
                    ↪ ][i] - !z;
                while (L >= 1 && R + 1 < n && s[
                    ↪ L - 1] == s[R + 1])
                    p[z][i]++;
                L--, R++;
                if (R > r)
                    l = L, r = R;
            }
        }
        bool is_palindrome(ll l, ll r) {
            ll mid = (l + r + 1) / 2, len = r -
            ↪ l + 1;
            return 2 * p[len % 2][mid] + len % 2
            ↪ >= len;
        }
        string get_palin(ll i, bool odd = true
            ↪ ) {
            ll len = p[odd][i];
            return s.substr(i - len, 2 * len + 1
            ↪ - !odd);
        }
    }
}

```

## 2.12 2D prefix Sum

```

pref[i][j] = a[i][j] + pref[i - 1][j] +
    ↪ pref[i][j - 1] - pref[i - 1][j -
    ↪ 1];
Sum of region = pref[row2 + 1][col2 + 1]
    ↪ - pref[row2 + 1][col1] - pref[
    ↪ row1][col2 + 1] + pref[row1][col1
    ↪ ];

```

## 2.13 CRT

```

class CRT {
    typedef long long vlong;
    typedef pair<vlong, vlong> pll;
    vector<pll> equations;

public:
    void clear() { equations.clear(); }
    vlong extended_euclid(vlong a, vlong b
        ↪ , vlong &x, vlong &y) {
        if (b == 0) {

```

```

            x = 1;
            y = 0;
            return a;
        }
        vlong x1, y1;
        vlong d = extended_euclid(b, a % b,
            ↪ x1, y1);
        x = y1;
        y = x1 - y1 * (a / b);
        return d;
    }
    vlong inverse(vlong a, vlong m) {
        vlong x, y;
        vlong g = extended_euclid(a, m, x, y
            ↪ );
        if (g != 1)
            return -1;
        return (x % m + m) % m;
    }
    /** Add equation of the form x = r (
        ↪ mod m) */
    void addEquation(vlong r, vlong m) {
        equations.push_back({r, m});
    }
    pll solve() {
        if (equations.size() == 0)
            return {-1, -1};
        vlong a1 = equations[0].first;
        vlong m1 = equations[0].second;
        a1 %= m1;
        for (int i = 1; i < equations.size()
            ↪ ; i++) {
            vlong a2 = equations[i].first;
            vlong m2 = equations[i].second;
            vlong g = __gcd(m1, m2);
            if (a1 % g != a2 % g)
                return {-1, -1};
            vlong p, q;
            extended_euclid(m1 / g, m2 / g, p,
                ↪ q);
            vlong mod = m1 / g * m2;
            vlong x = ((__int128)a1 * (m2 / g)
                ↪ % mod * q % mod +
                (__int128)a2 * (m1 / g)
                ↪ % mod * p % mod)
                ↪ % mod;
            a1 = x;
            if (a1 < 0)
                a1 += mod;
            m1 = mod;
        }
        return {a1, m1};
    }
}

```

## 2.14 Intersect two arithmetic progression

```

using T = __int128;
// ax + by = __gcd(a, b)
// returns __gcd(a, b)
T extended_euclid(T a, T b, T &x, T &y)
    ↪ {
    T xx = y = 0;
    T yy = x = 1;
    while (b) {
        T q = a / b;
        T t = b;
        b = a % b;
        a = t;
        t = xx;
        xx = x - q * xx;
        x = t;
        t = yy;
        yy = y - q * yy;
        y = t;
    }
    return a;
}
pair<T, T> CRT(T a1, T m1, T a2, T m2) {
    T p, q;
    T g = extended_euclid(m1, m2, p, q);
    if (a1 % g != a2 % g)

```

```

    return make_pair(0, -1);
T m = m1 / g * m2;
p = (p % m + m) % m;
q = (q % m + m) % m;
return make_pair((p * a2 % m * (m1 / g
    ↪ ) % m + q * a1 % m * (m2 / g) %
    ↪ m) % m, m);
}
// intersecting AP of two APs: (a1 + d1x
    ↪ ) and (a2 + d2x)
pair<ll, ll> intersect(ll a1, ll d1, ll
    ↪ a2, ll d2) {
    auto x = CRT(a1 % d1, d1, a2 % d2, d2)
    ↪ ;
    ll a = x.first, d = x.second;
    if (d == -1)
        return {0, 0}; // empty
    ll st = max(a1, a2);
    a = a < st ? a + ((st - a + d - 1) / d
        ↪ ) : a; // while (a < st) a += d;
    return {a, d};
}

```

## 2.15 Find nth value in a recurrence relation in O(logn)

```

[ 1, 1, 1, 0 ] ^ (n - 1) =
[F(n), F(n - 1), F(n - 1), F(n - 2)]
// Function to multiply two 2x2
    ↪ matrices
void multiply(vector<vector<int>> &
    ↪ mat1, vector<vector<int>> &
    ↪ mat2) {
    // Perform matrix multiplication
    int x = mat1[0][0] * mat2[0][0] +
        ↪ mat1[0][1] * mat2[1][0];
    int y = mat1[0][0] * mat2[0][1] +
        ↪ mat1[0][1] * mat2[1][1];
    int z = mat1[1][0] * mat2[0][0] +
        ↪ mat1[1][1] * mat2[1][0];
    int w = mat1[1][0] * mat2[0][1] +
        ↪ mat1[1][1] * mat2[1][1];
    // Update matrix mat1 with the
        ↪ result
    mat1[0][0] = x;
    mat1[0][1] = y;
    mat1[1][0] = z;
    mat1[1][1] = w;
}

// Function to perform matrix
    ↪ exponentiation
void matrixPower(vector<vector<int>> &
    ↪ mat1, int n) {
    // Base case for recursion
    if (n == 0 || n == 1)
        return;
    // Initialize a helper matrix
    vector<vector<int>> mat2 = {{1, 1},
        ↪ {1, 0}};
    // Recursively calculate mat1^(n/2)
    matrixPower(mat1, n / 2);
    // Square the matrix mat1
    multiply(mat1, mat1);
    // If n is odd, multiply by the helper
        ↪ matrix mat2
    if (n % 2 != 0) {
        multiply(mat1, mat2);
    }
}

// Function to calculate the nth
    ↪ Fibonacci number
// using matrix exponentiation
int nthFibonacci(int n) {
    if (n <= 1)
        return n;
    // Initialize the transformation
        ↪ matrix
    vector<vector<int>> mat1 = {{1, 1},
        ↪ {1, 0}};
}

```

```

// Raise the matrix mat1 to the power
    ↪ of (n - 1)
matrixPower(mat1, n - 1);
// The result is in the top-left cell
    ↪ of the matrix
return mat1[0][0];
}

```

## 2.16 All\_solution\_of\_ax+by\_equal\_c

```

// a*x+b*y=c. returns valid x and y if
    ↪ possible.
// all solutions are of the form (x0 + k
    ↪ * b / g, y0 - k * b / g)
bool find_any_solution(ll a, ll b, ll c,
    ↪ ll &x0, ll &y0, ll &g) {
    if (a == 0 and b == 0) {
        if (c)
            return false;
        x0 = y0 = g = 0;
        return true;
    }
    g = extended_euclid(abs(a), abs(b), x0
        ↪ , y0);
    if (c % g != 0)
        return false;
    x0 *= c / g;
    y0 *= c / g;
    if (a < 0)
        x0 *= -1;
    if (b < 0)
        y0 *= -1;
    return true;
}
void shift_solution(ll &x, ll &y, ll a,
    ↪ ll b, ll cnt) {
    x += cnt * b;
    y -= cnt * a;
}
// returns the number of solutions where
    ↪ x is in the range[minx, maxx] and
    ↪ y is
// in the range[miny, maxy]
ll find_all_solutions(ll a, ll b, ll c,
    ↪ ll minx, ll maxx, ll miny, ll maxy
    ↪ ) {
    ll x, y, g;
    if (find_any_solution(a, b, c, x, y, g
        ↪ ) == 0)
        return 0;
    if (a == 0 and b == 0) {
        assert(c == 0);
        return 1LL * (maxx - minx + 1) * (
            ↪ maxx - miny + 1);
    }
    if (a == 0) {
        return (maxx - minx + 1) * (miny <=
            ↪ c / b and c / b <= maxy);
    }
    if (b == 0) {
        return (maxy - miny + 1) * (minx <=
            ↪ c / a and c / a <= maxx);
    }
    a /= g, b /= g;
    ll sign_a = a > 0 ? +1 : -1;
    ll sign_b = b > 0 ? +1 : -1;
    shift_solution(x, y, a, b, (minx - x)
        ↪ / b);
    if (x < minx)
        shift_solution(x, y, a, b, sign_b);
    if (x > maxx)
        return 0;
    ll lx1 = x;
    shift_solution(x, y, a, b, (maxx - x)
        ↪ / b);
    if (x > maxx)
        shift_solution(x, y, a, b, -sign_b);
    ll rx1 = x;
    shift_solution(x, y, a, b, -(miny - y)
        ↪ / a);
    if (y < miny)
        shift_solution(x, y, a, b, -sign_a);
    if (y > maxy)
        shift_solution(x, y, a, b, sign_a);
}

```

```

    return 0;
11 lx2 = x;
shift_solution(x, y, a, b, -(maxy - y)
    ↪ / a);
if (y > maxy)
    shift_solution(x, y, a, b, sign_a);
11 rx2 = x;
if (lx2 > rx2)
    swap(lx2, rx2);
11 lx = max(lx1, lx2);
11 rx = min(rx1, rx2);
if (lx > rx)
    return 0;
return (rx - lx) / abs(b) + 1;
}

int32_t main() {
    ios_base::sync_with_stdio(0);
    cin.tie(0);
    int t, cs = 0;
    cin >> t;
    while (t--) {
        11 a, b, c, x1, x2, y1, y2;
        cin >> a >> b >> c >> x1 >> x2 >> y1
            ↪ >> y2;
        cout << "Case " << ++cs << ":" 
            << find_all_solutions(a, b, -c,
                ↪ x1, x2, y1, y2) << '\n';
    }
    return 0;
}

```

## 2.17 all soln of linear eq

```

struct Combi {
    int n;
    vector<ll> facts, finvs, invs;
    Combi(int _n) : n(_n), facts(_n),
        ↪ finvs(_n), invs(_n) {
        facts[0] = finvs[0] = 1;
        invs[1] = 1;
        for (int i = 2; i < n; i++)
            invs[i] = invs[mod % i] * (-mod /
                ↪ i);
        for (int i = 1; i < n; i++) {
            facts[i] = facts[i - 1] * i;
            finvs[i] = finvs[i - 1] * invs[i];
        }
    }
    inline ll fact(int n) { return facts[n
        ↪ ]; }
    inline ll finv(int n) { return finvs[n
        ↪ ]; }
    inline ll inv(int n) { return invs[n];
        ↪ }
    inline ll ncr(int n, int k) {
        return n < k ? 0 : facts[n] * finvs[
            ↪ k] * finvs[n - k];
    }
};

Combi C(N);

// returns the number of solutions to
// the equation
// x_1 + x_2 + ... + x_n = s and 0 <= l
// <= x_i <= r
11 yo(int n, int s, int l, int r) {
    if (s < l * n)
        return 0;
    s -= l * n;
    r -= l;
    11 ans = 0;
    for (int k = 0; k <= n; k++) {
        11 cur = C.ncr(s - k - k * r + n - 1
            ↪ + 1, n - 1 + 1) * C.ncr(n, k)
            ↪ ;
        if (k & 1)
            ans -= cur;
        else
            ans += cur;
    }
    return ans;
}

int32_t main() {
    ios_base::sync_with_stdio(0);
    cin.tie(0);
}

```

```

    cout << yo(3, 3, 0, 1) << '\n';
    return 0;
}

```

## 2.18 Subset sum sqrt(n)

```

// Sum of elements <= N implies that
// every element is <= N
vector<int> freq(N + 1, 0);
for (int i = 0; i < N; i++) {
    int x;
    cin >> x;
    freq[x]++;
}

vector<pair<int, int>> compressed;
for (int i = 1; i <= N; i++) {
    if (freq[i] > 0)
        compressed.emplace_back(i, freq[i]);
}

vector<int> dp(N + 1, 0);
dp[0] = 1;

for (const auto &[w, k] : compressed) {
    vector<int> ndp = dp;
    for (int p = 0; p < w; p++) {
        int sum = 0;
        for (int multiple = p, count = 0;
            ↪ multiple <= N; multiple += w,
            ↪ count++) {
            if (count > k) {
                sum -= dp[multiple - w * count];
                count--;
            }
            if (sum > 0)
                ndp[multiple] = 1;
            sum += dp[multiple];
        }
        swap(dp, ndp);
    }

    cout << "Possible subset sums are:\n";
    for (int i = 0; i <= N; i++) {
        if (dp[i] > 0)
            cout << i << " ";
    }
}

```

## 2.19 small giant ( $a^x \equiv b \pmod{m}$ , find $x$ , given other)

```

// Returns minimum x for which  $a^x \equiv m$ 
//  $\equiv b \pmod{m}$ , a and m are coprime.
int solve(int a, int b, int m) {
    a %= m, b %= m;
    int n = sqrt(m) + 1;

    int an = 1;
    for (int i = 0; i < n; ++i)
        an = (an * 1ll * a) % m;

    unordered_map<int, int> vals;
    for (int q = 0, cur = b; q <= n; ++q)
        ↪ {
            vals[cur] = q;
            cur = (cur * 1ll * a) % m;
        }

    for (int p = 1, cur = 1; p <= n; ++p)
        ↪ {
            cur = (cur * 1ll * an) % m;
            if (vals.count(cur)) {
                int ans = n * p - vals[cur];
                return ans;
            }
        }
    return -1;
}

// Returns minimum x for which  $a^x \equiv m$ 
//  $\equiv b \pmod{m}$ .
int solve(int a, int b, int m) {

```

```

a %= m, b %= m;
int k = 1, add = 0, g;
while ((g = gcd(a, m)) > 1) {
    if (b == k)
        return add;
    if (b % g)
        return -1;
    b /= g, m /= g, ++add;
    k = (k * 111 * a / g) % m;
}

int n = sqrt(m) + 1;
int an = 1;
for (int i = 0; i < n; ++i)
    an = (an * 111 * a) % m;

unordered_map<int, int> vals;
for (int q = 0, cur = b; q <= n; ++q)
    vals[cur] = q;
    cur = (cur * 111 * a) % m;

for (int p = 1, cur = k; p <= n; ++p)
    cur = (cur * 111 * an) % m;
if (vals.count(cur)) {
    int ans = n * p - vals[cur] + add;
    return ans;
}
return -1;
}

```

## 2.20 Gaussian Elimination

```

class GaussianElimination {
public:
    GaussianElimination(vector<vector<
        double>> matrix, vector<double>
        results)
        : matrix(matrix), results(results)
        , n(matrix.size()) {}

    void solve() {
        fElim();
        bSub();
    }

    vector<vector<double>> matrix;
    vector<double> results, solution;
    ll n;

    void fElim() {
        for (ll i = 0; i < n; ++i) {
            ll maxRow = i;
            for (ll k = i + 1; k < n; ++k)
                if (abs(matrix[k][i]) > abs(
                    matrix[maxRow][i]))
                    maxRow = k;
            swap(matrix[i], matrix[maxRow]);
            swap(results[i], results[maxRow]);
            for (ll k = i + 1; k < n; ++k) {
                double factor = matrix[k][i] /
                    matrix[i][i];
                for (ll j = i; j < n; ++j)
                    matrix[k][j] -= factor *
                        matrix[i][j];
                results[k] -= factor * results[i]
                    ];
            }
        }
    }

    void bSub() {
        solution.resize(n);
        for (ll i = n - 1; i >= 0; --i) {
            solution[i] = results[i];
            for (ll j = i + 1; j < n; ++j)
                solution[i] -= matrix[i][j] *
                    solution[j];
            solution[i] /= matrix[i][i];
        }
    }
};

```

## 2.21 Grundy

```
ll calculateGrundy(ll n, vector<ll> &
```

```

    ↗ grundy, const vector<ll> &moves) {
    if (grundy[n] != -1)
        return grundy[n];
    unordered_set<ll> s;
    for (ll move : moves) {
        if (n >= move) {
            s.insert(calculateGrundy(n - move,
                ↗ grundy, moves));
        }
    }
    ll g = 0;
    while (s.count(g))
        g++;
    return grundy[n] = g;
}

vector<ll> computeGrundy(ll maxN, const
    ↗ vector<ll> &moves) {
    vector<ll> grundy(maxN + 1, -1);
    grundy[0] = 0;
    for (ll i = 1; i <= maxN; ++i) {
        calculateGrundy(i, grundy, moves);
    }
    return grundy;
}

/*
1. Nim = all xor
2. Misere Nim = Nim + corner case: if
    ↗ all piles are 1, reverse(nim)
3. Bogus Nim = Nim
4. Staircase Nim = Odd indexed pile Nim
    ↗ (Even indexed pile doesn't matter,
    ↗ as one player can give bogus moves
    ↗ to drop all even piles to ground)
5. Sprague Grundy = Every impartial game
    ↗ under the normal play convention
    ↗ is equivalent to a one-heap game
    ↗ of nim
*/

```

## 3 Dynamic Programming

### 3.1 LCS

```

/*
Fact about LCS:
1. Longest Increasing Substring
To solve this, we just care about when
    ↗ two char equals. Rest of the
    ↗ things should be neglected.
2. Longest Palindromic Subsequence (LPS)
To solve this, we just take a new string
    ↗ which is the reverse of the
    ↗ original string. Then just call
    ↗ the LCS function to find LPS.
3. Minimum insertions to make a string
    ↗ palindrome To solve this, we just
    ↗ basically do string length - LPS.
    ↗ Why this?
        Let's take an example: string s =
            ↗ aabca; Let's say aca is our
            ↗ LPS. Now we find how many char
            ↗ we need to insert to make the
            ↗ string palindrome while our
            ↗ LPS is fixed.
        a ab c a now to make the string
            ↗ palindrome we just need to
            ↗ insert the reverse of ab after
            ↗ c. So the new string looks
            ↗ like a ab c ba a
4. Minimum Number of Deletions and
    ↗ Insertions to make the string
    ↗ equals To solve this we just find
    ↗ the LCS of those strings then just
    ↗ do: n + m - 2 * LCS.length() where
    ↗ n, m = strings length
*/

```

### 3.2 MCM

```
// TC: O(n ^ 3)
const ll N = 1005;
vector<ll> v;
ll dp[N][N], mark[N][N];
```

```

11 MCM(ll i, ll j) {
    if (i == j)
        return dp[i][j] = 0;
    if (dp[i][j] != -1)
        return dp[i][j];
11 mn = INT_MAX;
for (ll k = i; k < j; k++) {
    ll x = mn;
    mn = min(mn, MCM(i, k) + MCM(k + 1,
        ↪ j) + v[i - 1] * v[k] * v[j]);
    if (x != mn)
        mark[i][j] = k;
}
return dp[i][j] = mn;
}

void print_order(ll i, ll j) {
    if (i == j)
        cout << "X" << i;
    else {
        cout << "(";
        print_order(i, mark[i][j]);
        print_order(mark[i][j] + 1, j);
        cout << ")";
    }
}

// memset(dp, -1, sizeof dp);
// print_order(1, n);

```

### 3.3 LIS\_length

```

vector<ll> v = {7, 3, 5, 3, 6, 2, 9, 8};
vector<ll> seq;
/*
here we basically check is the current
    ↪ element from v is greater than the
    ↪ last element of the sequence. if
    ↪ it is then push it to the seq
    ↪ array and if not then replace that
    ↪ index value. let's take an
    ↪ example:
v = 7 3 5 3 6 2 9 8
1st iteration seq = 7;
2nd iteration seq = 3;
3rd iteration seq = 3 5;
4th iteration seq = 3 3;
5th iteration seq = 3 3 6;
6th iteration seq = 2 3 6;
7th iteration seq = 2 3 6 9;
8th iteration seq = 2 3 6 8;
*/
for (auto i : v) {
    auto id = lower_bound(seq.begin(), seq
        ↪ .end(), i);
    if (id == seq.end())
        seq.push_back(i);
    else
        seq[id - seq.begin()] = i;
}
cout << seq.size() << endl;

```

### 3.4 LCIS

```

11 a[100] = {0}, b[100] = {0}, f[100] =
    ↪ {0};
11 n = 0, m = 0;
11 main(void) {
    cin >> n;
    for (ll i = 1; i <= n; i++)
        cin >> a[i];
    cin >> m;
    for (ll i = 1; i <= m; i++)
        cin >> b[i];
    for (ll i = 1; i <= n; i++) {
        ll k = 0;
        for (ll j = 1; j <= m; j++) {
            if (a[i] > b[j] && f[j] > k)
                k = f[j];
            else if (a[i] == b[j] && k + 1 > f
                ↪ [j])
                f[j] = k + 1;
        }
    }
    ll and = 0;
}

```

```

for (ll i = 1; i <= m; i++)
    if (f[i] > ans)
        ans = f[i];
cout << and << endl;
return 0;
}

```

### 3.5 SOS DP

```

// sum over subsets
for (int i = 0; i < B; i++) {
    for (int mask = 0; mask < (1 << B);
        ↪ mask++) {
        if ((mask & (1 << i)) != 0) {
            f[mask] += f[mask ^ (1 << i)];
        }
    }
}

// sum over supersets
for (int i = 0; i < B; i++) {
    for (int mask = (1 << B) - 1; mask >=
        ↪ 0; mask--) {
        if ((mask & (1 << i)) == 0)
            g[mask] += g[mask ^ (1 << i)];
    }
}

// submask
for (int mask = 1; mask < (1 << 5); mask
    ↪ ++ ) {
    for (int submask = mask; submask > 0;
        ↪ submask = ((submask - 1) & mask)
        ↪ ) {
        int subset = mask ^ submask;
    }
}

/***
SOS DP (Sum Over Subsets Dynamic
    ↪ Programming) - 5 Key Points:
1. CORE IDEA: Build DP table dp[i][mask
    ↪ ] = answer considering first i
    ↪ bits of mask Transition: dp[i][
    ↪ mask] = dp[i-1][mask] + dp[i-1][
    ↪ mask ^ (1<<(i-1))]
2. SUBSET SUM: For x|y = x, iterate
    ↪ bits and add contributions from
    ↪ subsets If bit i is set in mask,
    ↪ add dp[i-1][mask without bit i]
3. SUPERSET SUM: For x&y = x, iterate
    ↪ in reverse to handle supersets
    ↪ If bit i is unset in mask, add dp
    ↪ [i-1][mask with bit i set]
**/>

```

### 3.6 BS optimization

```

bitset<100005> bs = 1;
for (auto i : a) {
    bs |= (bs << i);
    // if previous 1 value pos is possible
    ↪ now ith bit or ith sm is also
    ↪ possible
}
cout << bs.count() - 1 << endl;
for (ll i = 1; i <= 100003; i++)
    if (bs[i])
        cout << i << " ";
cout << endl;

```

## 4 Graph

### 4.1 Dijkstra

```

// TC: O(V + ElogV)
typedef pair<ll, ll> pairi;
11 N = 20000 + 5;
vector<vector<pairi>> adj(N);
vector<ll> dis(N, inf), parent(N);

void dijkstra(ll src) {
    priority_queue<pairi, vector<pairi>,
        ↪ greater<pairi>> pq;
    dis[src] = 0;
}

```

```

pq.push({0, src});
while (pq.size()) {
    auto top = pq.top();
    pq.pop();
    for (auto i : adj[top.second]) {
        ll v = i.first;
        ll wt = i.second;
        if (dis[v] > dis[top.second] + wt)
            → {
                dis[v] = dis[top.second] + wt;
                pq.push({dis[v], v});
                parent[v] = top.second
            }
    }
}
ll node = n;
while (parent[node] != node) {
    path.push_back(node);
    node = parent[node];
}
path.push_back(1);

```

## 4.2 BellmanFord

```

// TC : O(V.E)
vector<ll> dist;
vector<ll> parent;
vector<vector<pair<ll, ll>>> adj;
// resize the vectors from main function
void bellmanFord(ll n, ll src) {
    dist[src] = 0;
    for (ll step = 0; step < n; step) {
        for (ll i = 1; i <= n; i++) {
            for (auto it : adj[i]) {
                ll u = i;
                ll v = it.first;
                ll wt = it.second;
                if (dist[u] != inf && ((dist[u]
                    → + wt) < dist[v])) {
                    if (step == n - 1) {
                        cout << "Negative cycle
                            → found\n ";
                        return;
                    }
                    dist[v] = dist[u] + wt;
                    parent[v] = u;
                }
            }
        }
        for (ll i = 1; i <= n; i++)
            cout << dist[i] << " ";
        cout << endl;
    }
}

```

## 4.3 FloydWarshall

```

// TC : O(n ^ 3)
typedef double T;
typedef vector<T> VT;
typedef vector<VT> VVT;
typedef vector<ll> VI;
typedef vector<VI> VVI;
bool FloydWarshall(VVT &w, VVI &prev) {
    ll n = w.size();
    prev = VVI(n, VI(n, -1));
    for (ll k = 0; k < n; k++) {
        for (ll i = 0; i < n; i++) {
            for (ll j = 0; j < n; j++) {
                if (w[i][j] > w[i][k] + w[k][j])
                    → {
                        w[i][j] = w[i][k] + w[k][j];
                        prev[i][j] = k;
                    }
            }
        }
    }
    // check for negative weight cycles
    for (ll i = 0; i < n; i++)
        if (w[i][i] < 0)
            return false;
}

```

```

return true;
}

```

## 4.4 Toposort

```

// TC : O(V + E)
map<ll, vector<ll>> adj;
map<ll, ll> degree;
set<ll> nodes;
vector<ll> ans;
// adj: graph input, degree: cnt
// → indegree,
// node: unique nodes, ans: path
ll c = 0;
void topo_sort() {
    queue<ll> qu;
    // traverse all the nodes and check if
    // → its degree is 0 or not..
    for (ll i : nodes) {
        if (degree[i] == 0)
            qu.push(i);
    }
    while (!qu.empty()) {
        ll top = qu.front();
        qu.pop();
        ans.push_back(top);
        for (ll i : adj[top]) {
            degree[i]--;
            if (degree[i] == 0)
                qu.push(i);
        }
    }
}

```

## 4.5 Kruskal

```

// TC : O(ElogE)
typedef pair<ll, ll> edge;
class Graph {
    vector<pair<ll, edge>> G, T;
    vector<ll> parent;
    ll cost = 0;
public:
    Graph(ll n) {
        for (ll i = 0; i < n; i++)
            parent.push_back(i);
    }
    void add_edges(ll u, ll v, ll wt) { G.
        → push_back({wt, {u, v}}); }
    ll find_set(ll n) {
        if (n == parent[n])
            return n;
        else
            return find_set(parent[n]);
    }
    void union_set(ll u, ll v) { parent[u]
        → = parent[v]; }
    void kruskal() {
        sort(G.begin(), G.end());
        for (auto it : G) {
            ll uRep = find_set(it.second.first
                →);
            ll vRep = find_set(it.second.
                → second);
            if (uRep != vRep) {
                cost += it.first;
                T.push_back(it);
                union_set(uRep, vRep);
            }
        }
        ll get_cost() { return cost; }
        void print() {
            for (auto it : T)
                cout << it.second.first << " " <<
                    → it.second.second << ":" " <<
                        → it.first << endl;
        }
        // g.add_edges(u, v, wt);
    }
}

```

```
// g.kruskal();
```

## 4.6 Prims

```

// TC: O(ElogV)
typedef pair<ll, ll> pll;
class Prims {
    map<ll, vector<pll>> graph;
    map<ll, ll> visited;
public:
    void addEdge(ll u, ll v, ll w) {
        graph[u].push_back({v, w});
        graph[v].push_back({u, w});
    }

    vector<ll> path(pll start) {
        vector<ll> ans;
        priority_queue<pll, vector<pll>,
             $\hookrightarrow$  greater<pll>> pq;
        // cost vs node
        pq.push({start.second, start.first});
         $\hookrightarrow$  ;
        while (!pq.empty()) {
            pair<ll, ll> curr = pq.top();
            pq.pop();
            if (visited[curr.second])
                continue;
            visited[curr.second] = 1;
            ans.push_back(curr.second);
            for (auto i : graph[curr.second])
                 $\hookrightarrow$  {
                    if (visited[i.first])
                        continue;
                    pq.push({i.second, i.first});
                }
            }
        return ans;
    }
}

```

## 4.7 LCA

```

// TC: preprocessing  $O(n\log n)$ , each
//      ↪ query  $O(\log n)$ 
ll n, l;
vector<vector<ll>> adj;
ll timer;
vector<ll> tin, tout;
vector<vector<ll>> up;

void dfs(ll v, ll p) {
    tin[v] = ++timer;
    up[v][0] = p;
    for (ll i = 1; i <= l; ++i)
        up[v][i] = up[up[v][i - 1]][i - 1];

    for (ll u : adj[v]) {
        if (u != p)
            dfs(u, v);
    }
    tout[v] = ++timer;
}

bool is_ancestor(ll u, ll v) { return
    ↪ tin[u] <= tin[v] && tout[u] >=
    ↪ tout[v]; }

ll lca(ll u, ll v) {
    if (is_ancestor(u, v))
        return u;
    if (is_ancestor(v, u))
        return v;
    for (ll i = l; i >= 0; --i) {
        if (!is_ancestor(up[u][i], v))
            u = up[u][i];
    }
    return up[u][0];
}

void preprocess(ll root) {
    tin.resize(n);
    tout.resize(n);
    timer = 0;
    l = ceil(log2(n));
    up.assign(n, vector<ll>(l + 1));
}

```

```
    } dfs(root, root);
```

## 4.8 Rerooting

```

namespace reroot {
const auto exclusive = [](> const auto &a,
    ↪ const auto &base,
    ↪ const auto &
    ↪ merge_into
    ↪ , int
    ↪ vertex)
    ↪ {
int n = (int)a.size();
using Aggregate = decay_t<decltype(
    ↪ base)>;
vector<Aggregate> b(n, base);
for (int bit = (int)lg(n); bit >= 0;
    ↪ --bit) {
    for (int i = n - 1; i >= 0; --i)
        b[i] = b[i >> 1];
    int sz = n - (n & !bit);
    for (int i = 0; i < sz; ++i) {
        int index = (i >> bit) ^ 1;
        b[index] = merge_into(b[index], a[
            ↪ i], vertex, i);
    }
}
return b;
};
// MergeInto : Aggregate * Value *
    ↪ Vertex(int) * EdgeIndex(int) ->
    ↪ Aggregate
// Base : Vertex(int) -> Aggregate
// FinalizeMerge : Aggregate * Vertex(
    ↪ int) * EdgeIndex(int) -> Value
const auto rerooter = [](> const auto &g,
    ↪ const auto &base,
    ↪ const auto &
    ↪ merge_into
    ↪ , const
    ↪ auto &
    ↪ finalize_merge
    ↪ ) {
int n = (int)g.size();
using Aggregate = decay_t<decltype(
    ↪ base(0))>;
using Value = decay_t<decltype(
    ↪ finalize_merge(base(0), 0, 0))>;
vector<Value> root_dp(n), dp(n);
vector<vector<Value>> edge_dp(n),
    ↪ redge_dp(n);

vector<int> bfs, parent(n);
bfs.reserve(n);
bfs.push_back(0);
for (int i = 0; i < n; ++i) {
    int u = bfs[i];
    for (auto v : g[u]) {
        if (parent[u] == v)
            continue;
        parent[v] = u;
        bfs.push_back(v);
    }
}

for (int i = n - 1; i >= 0; --i) {
    int u = bfs[i];
    int p_edge_index = -1;
    Aggregate aggregate = base(u);
    for (int edge_index = 0; edge_index
        ↪ < (int)g[u].size(); ++
        ↪ edge_index) {
        int v = g[u][edge_index];
        if (parent[u] == v) {
            p_edge_index = edge_index;
            continue;
        }
        aggregate = merge_into(aggregate,
            ↪ dp[v], u, edge_index);
    }
    dp[u] = finalize_merge(aggregate, u,
        ↪ p_edge_index);
}
}

```

```

for (auto u : bfs) {
    dp[parent[u]] = dp[u];
    edge_dp[u].reserve(g[u].size());
    for (auto v : g[u])
        edge_dp[u].push_back(dp[v]);
    auto dp_exclusive = exclusive(
        ↪ edge_dp[u], base(u),
        ↪ merge_into, u);
    redge_dp[u].reserve(g[u].size());
    for (int i = 0; i < (int)
        ↪ dp_exclusive.size(); ++i)
        redge_dp[u].push_back(
            ↪ finalize_merge(dp_exclusive[
                ↪ i], u, i));
    root_dp[u] = finalize_merge(
        n > 1 ? merge_into(dp_exclusive
            ↪ [0], edge_dp[u][0], u, 0)
            ↪ : base(u), u,
        -1);
    for (int i = 0; i < (int)g[u].size()
        ↪ ; ++i) {
        dp[g[u][i]] = redge_dp[u][i];
    }
}
return make_tuple(move(root_dp), move(
    ↪ edge_dp), move(redge_dp));
} // namespace reroot
int main() {
    ll n;
    cin >> n;
    vector<vector<ll>> g(n);
    // everything should be 0 based.

    using Aggregate = int;
    using Value = int;
    auto base = [](int vertex) ->
        ↪ Aggregate {
        // task here
    };
    auto merge_into = [](Aggregate
        ↪ vertex_dp, Value neighbor_dp,
        ↪ int vertex, int edge_index) ->
        ↪ Aggregate {
        // task here
    };
    auto finalize_merge = [](Aggregate
        ↪ vertex_dp, int vertex, int
        ↪ edge_index) -> Value {
        // task here
    };
    auto [reroot_result, edge_dp, redge_dp
        ↪ ] = reroot::rerooter(g, base,
        ↪ merge_into, finalize_merge);
}

```

## 4.9 Centroid\_Tree

```

const ll n = 1e5;
vector<ll> sz(n + 5), dead(n + 5);
function<void(ll, ll)> calculate_sz =
    ↪ [&](ll u, ll p) {
    sz[u] = 1;
    for (auto v : adj[u]) {
        if (v != p and !dead[v]) {
            calculate_sz(v, u);
            sz[u] += sz[v];
        }
    }
    return;
};

function<ll(ll, ll, ll)> find_centroid
    ↪ = [&](ll u, ll p, ll total) ->
    ↪ ll {
    for (auto v : adj[u]) {
        if (v != p and !dead[v] and 2 * sz
            ↪ [v] > total)
            return find_centroid(v, u, total
                ↪ );
    }
    return u;
};

```

```

function<void(ll)> decompose = [&] (ll
    ↪ u) -> void {
    // if needed change the parameter
    calculate_sz(u, -1);
    ll center = find_centroid(u, -1, sz[
        ↪ u]);
    // calculate the ans here
    dead[center] = 1;
    for (auto v : adj[center]) {
        if (!dead[v])
            decompose(v);
    }
    // call decompose only
    decompose(1);
}

```

## 4.10 Euler\_ckt

```

unordered_map<ll, ll> Start, End, Val;
unordered_map<ll, pair<ll, ll>> Range;
ll start = 0;
void dfs(ll node) {
    visited[node] = true;
    Start[node] = start++;
    for (auto child : adj[node]) {
        if (!visited[child])
            dfs(child);
    }
    End[node] = start - 1;
}
dfs(1);
vector<ll> FlatArray(start + 5);
for (auto i : Start) {
    FlatArray[i.second] = Val[i.first];
    Range[i.first] = {i.second, End[i].
        ↪ first}};
}

```

## 4.11 Min Cost Max Flow

```

#include <bits/stdc++.h>
using namespace std;
const int N = 3e5 + 9;

// Works for both directed, undirected
// and with negative cost too
// doesn't work for negative cycles
// for undirected edges just make the
// directed flag false
// Complexity: O(min(E^2 * V log V, E
// logV * flow))
using T = long long;
const T inf = 1LL << 61;
struct MCMF {
    struct edge {
        int u, v;
        T cap, cost;
        int id;
        edge(int _u, int _v, T _cap, T _cost
            ↪ , int _id) {
            u = _u;
            v = _v;
            cap = _cap;
            cost = _cost;
            id = _id;
        }
        ;
        int n, s, t, mxid;
        T flow, cost;
        vector<vector<int>> g;
        vector<edge> e;
        vector<T> d, potential, flow_through;
        vector<int> par;
        bool neg;
        MCMF() {}
        MCMF(int _n) { // 0-based indexing
            n = _n + 10;
            g.assign(n, vector<int>());
            neg = false;
            mxid = 0;
        }
        void add_edge(int u, int v, T cap, T
            ↪ cost, int id = -1,

```

```

        bool directed = true) {
    if (cost < 0)
        neg = true;
    g[u].push_back(e.size());
    e.push_back(edge(u, v, cap, cost, id
                    ));
    g[v].push_back(e.size());
    e.push_back(edge(v, u, 0, -cost, -1)
                    );
    mxid = max(mxid, id);
    if (!directed)
        add_edge(v, u, cap, cost, -1, true
                    );
}
bool dijkstra() {
    par.assign(n, -1);
    d.assign(n, inf);
    priority_queue<pair<T, T>, vector<
                    pair<T, T>>, greater<pair<T, T
                    >>> q;
    d[s] = 0;
    q.push(pair<T, T>(0, s));
    while (!q.empty()) {
        int u = q.top().second;
        T nw = q.top().first;
        q.pop();
        if (nw != d[u])
            continue;
        for (int i = 0; i < (int)g[u].size
              () ; i++) {
            int id = g[u][i];
            int v = e[id].v;
            T cap = e[id].cap;
            T w = e[id].cost + potential[u]
                  - potential[v];
            if (d[u] + w < d[v] && cap > 0)
                {
                    d[v] = d[u] + w;
                    par[v] = id;
                    q.push(pair<T, T>(d[v], v));
                }
        }
        for (int i = 0; i < n; i++) {
            if (d[i] < inf)
                d[i] += (potential[i] -
                          potential[s]);
        }
        for (int i = 0; i < n; i++) {
            if (d[i] < inf)
                potential[i] = d[i];
        }
        return d[t] != inf; // for max flow
        // min cost
        // return d[t] <= 0; // for min cost
        // flow
    }
    T send_flow(int v, T cur) {
        if (par[v] == -1)
            return cur;
        int id = par[v];
        int u = e[id].u;
        T w = e[id].cost;
        T f = send_flow(u, min(cur, e[id].
                               cap));
        cost += f * w;
        e[id].cap -= f;
        e[id ^ 1].cap += f;
        return f;
    }
    // returns {maxflow, mincost}
    pair<T, T> solve(int _s, int _t, T
                     goal = inf) {
        s = _s;
        t = _t;
        flow = 0, cost = 0;
        potential.assign(n, 0);
        if (neg) {
            // Run Bellman-Ford to find
            // starting potential on the
            // starting graph
            // If the starting graph (before
            // pushing flow in the residual
            // graph) is a
    }
}

```

```

// DAG, then this can be
// calculated in O(V + E) using
// DP: potential(v) =
// min({potential[u] + cost[u][v
// ]}) for each u -> v and
// potential[s] = 0
d.assign(n, inf);
d[s] = 0;
bool relax = true;
for (int i = 0; i < n && relax; i
      ++ ) {
    relax = false;
    for (int u = 0; u < n; u++) {
        for (int k = 0; k < (int)g[u].
                    size(); k++) {
            int id = g[u][k];
            int v = e[id].v;
            T cap = e[id].cap, w = e[id].
                cost;
            if (d[v] > d[u] + w && cap >
                 0) {
                d[v] = d[u] + w;
                relax = true;
            }
        }
    }
    for (int i = 0; i < n; i++)
        if (d[i] < inf)
            potential[i] = d[i];
}
while (flow < goal && dijkstra())
    flow += send_flow(t, goal - flow);
flow_through.assign(mxid + 10, 0);
for (int u = 0; u < n; u++) {
    for (auto v : g[u]) {
        if (e[v].id >= 0)
            flow_through[e[v].id] = e[v ^
                                         1].cap;
    }
}
return make_pair(flow, cost);
};

int main() {
    ios_base::sync_with_stdio(0);
    cin.tie(0);
    int n;
    cin >> n;
    assert(n <= 10);
    MCMF F(2 * n);
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < n; j++) {
            int k;
            cin >> k;
            F.add_edge(i, j + n, 1, k, i * 20
                       + j);
        }
    }
    int s = 2 * n + 1, t = s + 1;
    for (int i = 0; i < n; i++) {
        F.add_edge(s, i, 1, 0);
        F.add_edge(i + n, t, 1, 0);
    }
    auto ans = F.solve(s, t).second;
    long long w = 0;
    set<int> se;
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < n; j++) {
            int p = i * 20 + j;
            if (F.flow_through[p] > 0) {
                se.insert(j);
                w += F.flow_through[p];
            }
        }
    }
    assert(se.size() == n && w == n);
    cout << ans << '\n';
    return 0;
}

```

## 4.12 SCC

unordered\_map<ll, vector<ll>> adj,

```

    ↪ InvAdj;
stack<ll> order;
unordered_map<ll, bool> visited;
unordered_map<ll, vector<ll>> all_scc;
unordered_map<ll, ll> compId;
void dfs_for_start(ll curr) {
    visited[curr] = 1;
    for (auto i : adj[curr])
        if (!visited[i])
            dfs_for_start(i);
    order.push(curr);
}
vector<ll> curr_comp;
void dfs_for_scc(ll curr) {
    visited[curr] = 1;
    for (auto i : InvAdj[curr])
        if (!visited[i])
            dfs_for_scc(i);
    curr_comp.push_back(curr);
}
inline void scc() {
    ll n, e, u, v;
    cin >> n >> e;
    for (ll i = 0; i < e; i++) {
        cin >> u >> v;
        adj[u].push_back(v);
        InvAdj[v].push_back(u);
    }
    for (ll i = 1; i <= n; i++)
        if (!visited[i])
            dfs_for_start(i);
    visited.clear();
    while (!order.empty()) {
        if (!visited[order.top()])
            curr_comp.clear();
        dfs_for_scc(order.top());
        ll sz = all_scc.size() + 1;
        all_scc[sz] = curr_comp;
        for (auto i : curr_comp)
            compId[i] = sz;
    }
    order.pop();
}
// no.of ways and min cost of connecting
    ↪ the sccs
const ll MOD = 1e9 + 7, N = 1e5 + 2, INF
    ↪ = 1e18 + 2;
ll n, m, comp[N];
vector<ll> adj[N], rev[N];
bitset<N> vis;
void DFS1(ll u, stack<ll> &TS) {
    vis[u] = true;
    for (ll v : adj[u])
        if (!vis[v])
            DFS1(v, TS);
    TS.push(u);
}
void DFS2(ll u, const ll scc_no, ll &
    ↪ min_cost, ll &ways, vector<ll> &
    ↪ cost) {
    vis[u] = true;
    comp[u] = scc_no;
    for (ll v : rev[u])
        if (!vis[v]) {
            if (min_cost == cost[v])
                ++ways;
            else if (min_cost > cost[v]) {
                ways = 1;
                min_cost = cost[v];
            }
            DFS2(v, scc_no, min_cost, ways,
                ↪ cost);
        }
}
signed main() {
    FIO cin >> n;
    vector<ll> cost(n + 1);
    for (ll i = 1; i <= n; ++i)
        cin >> cost[i];
    cin >> m;
    while (m--) {
        ll u, v;
        cin >> u >> v;

```

```

        adj[u].push_back(v);
        rev[v].push_back(u);
    }
    ll tot = 0, ways = 1;
    stack<ll> TS;
    for (ll i = 1; i <= n; ++i)
        if (!vis[i])
            DFS1(i, TS);
    vis.reset();
    ll scc_no = 0;
    while (!TS.empty()) {
        ll u = TS.top();
        TS.pop();
        if (!vis[u]) {
            ll tmp_cst = cost[u], tmp_ways =
                ↪ 1;
            DFS2(u, ++scc_no, tmp_cst,
                ↪ tmp_ways, cost);
            tot += tmp_cst;
            ways = (ways * tmp_ways) % MOD;
        }
    }
    cout << tot << ' ' << ways;
} // TC: O(V+E)

```

## 4.13 0-1 BFS

```

vector<ll> d(n, INF);
d[s] = 0;
deque<ll> q;
q.push_front(s);
while (!q.empty()) {
    ll v = q.front();
    q.pop_front();
    for (auto edge : adj[v]) {
        ll u = edge.first;
        ll w = edge.second;
        if (d[v] + w < d[u]) {
            d[u] = d[v] + w;
            if (w == 1)
                q.push_back(u);
            else
                q.push_front(u);
        }
    }
}

```

## 4.14 Hull

```

// Convex Hull
#pragma GCC target("avx2")
#pragma GCC optimize("O3")
#pragma GCC optimize("unroll-loops")
#include <bits/stdc++.h>
using namespace std;
typedef long long int ll;
typedef long double ld;
typedef pair<ll, ll> pl;
typedef vector<ll> vl;
typedef complex<ll> pt;
#define G(x)
    ↪ \
    ↪ \
    ll x;
    ↪ \
#define F(i, l, r) for (ll i = l; i < (r
    ↪ ); ++i)
#define A(a) (a).begin(), (a).end()
#define CRS(a, b) (conj(a) * (b)).Y
#define K first
#define V second
#define X real()
#define Y imag()
#define N 100010
namespace std {
bool operator<(pt a, pt b) { return a.X
    ↪ == b.X ? a.Y < b.Y : a.X < b.X; }
} // namespace std
bool in_hull(pt p, vector<pt> &hu,
    ↪ vector<pt> &hd,

```

```

if (p == *hu.begin() || p == *hd.begin
    ↪ ())
return false; // change to true if
    ↪ border counts as inside
if (p < *hu.begin() || *hd.begin() < p
    ↪ )
return false;
auto u = upper_bound(A(hu), p);
auto d = lower_bound(hd.rbegin(), hd.
    ↪ rend(), p);
return CRS(*u - p, *(u - 1) - p) > 0
    && CRS(*(d - 1) - p, *d - p) >
    ↪ 0;
// change to >= if border counts as "
    ↪ inside"
}

void do_hull(vector<pt> &pts, vector<pt>
    ↪ &h) {
    for (pt p : pts) {
        while (h.size() > 1 && CRS(h.back()
            ↪ - p, h[h.size() - 2] - p) <=
            ↪ 0)
            // change to < 0 if border points
            ↪ included
        h.pop_back();
        h.push_back(p);
    }
}

pair<vector<pt>, vector<pt>> get_hull(
    ↪ vector<pt> &pts) {
    vector<pt> hu, hd;
    sort(A(pts)), do_hull(pts, hu);
    reverse(A(pts)), do_hull(pts, hd);
    return {hu, hd};
}

vector<pt> full_hull(vector<pt> &pts) {
    auto h = get_hull(pts);
    h.K.pop_back(), h.V.pop_back();
    for (pt p : h.V)
        h.K.push_back(p);
    return h.K;
}

int main() {
    G(n) vector<pt> v;
    F(i, 0, n) { G(x) G(y) v.push_back({x,
        ↪ y}); }
    vector<pt> h = full_hull(v);
}

```

## 4.15 Dynamic Hull

```

// Dynamic Convex Hull
#pragma GCC target("avx2")
#pragma GCC optimize("O3")
#pragma GCC optimize("unroll-loops")
#include <bits/stdc++.h>
using namespace std;

typedef long long int ll;
typedef long double ld;
typedef pair<ll, ll> pl;
typedef vector<ll> vl;
typedef complex<ll> pt;

#define G(x) ll x; cin >> x;
#define F(i, l, r) for (ll i = l; i < (r
    ↪ ); ++i)
#define A(a) (a).begin(), (a).end()
#define CRS(a, b) (conj(a) * (b)).Y
#define X real()
#define Y imag()
#define N 100010

namespace std {
    bool operator<(pt a, pt b) { return a.X
        ↪ == b.X ? a.Y < b.Y : a.X < b.X; }
} // namespace std

// helper function for dyn_in_hull
bool in(pt p, set<pt> &h) {
    if (h.empty() || p < *h.begin() || *h.
        ↪ rbegin() < p)
        return false;
}

```

```

auto i = h.upper_bound(p), j = i--;
return CRS(*j - p, *i - p) > 0; //
    ↪ change to >= if border counts as
    ↪ "inside"
}

// returns true if p contained in
    ↪ dynamic hull hu / hd
bool in_hull(pt p, set<pt> &hu, set<pt>
    ↪ &hd) { return in(p, hu) && in(-p,
    ↪ hd); }

// helper function for dyn_add
void fix_bad(set<pt>::iterator i, set<pt
    ↪ > &h, bool l) {
    if (i == --h.begin() || i == h.end())
        return;
    pt p = *i;
    h.erase(p);
    if (!in(p, h))
        h.insert(p);
    else
        fix_bad(l ? --h.lower_bound(p) : h.
            ↪ upper_bound(p), h, l);
}

// helper function for dyn_add_to_hull
void add(pt p, set<pt> &h) {
    if (in(p, h))
        return;
    h.insert(p);
    fix_bad(--h.lower_bound(p), h, true);
    fix_bad(h.upper_bound(p), h, false);
}

// adds p to dynamic hull hu / hd
void add_to_hull(pt p, set<pt> &hu, set<
    ↪ pt> &hd) { add(p, hu), add(-p, hd)
    ↪ ; }

int main() {
    G(n) set<pt> hu, hd;
    F(i, 0, n) { G(x) G(y) add_to_hull({x,
        ↪ y}, hu, hd); }
}

```

## 4.16 Count SImple Cycle

```

void findNumberOfSimpleCycles(int N,
    ↪ vector<vector<int>> adj) {
    int ans = 0;
    int dp[(1 << N)][N];
    memset(dp, 0, sizeof dp);
    for (int mask = 0; mask < (1 << N);
        ↪ mask++) {
        int nodeSet = __builtin_popcountll(
            ↪ mask);
        int firstSetBit = __builtin_ffsl(
            ↪ mask);
        if (nodeSet == 1)
            dp[mask][firstSetBit] = 1;
        else {
            for (int j = firstSetBit + 1; j <
                ↪ N; j++) {
                if ((mask & (1 << j))) {
                    int newNodeSet = mask ^ (1 <<
                        ↪ j);
                    for (int k = 0; k < N; k++) {
                        if ((newNodeSet & (1 << k)) &
                            ↪ & adj[k][j]) {
                            dp[mask][j] += dp[
                                ↪ newNodeSet][k];
                            if (adj[j][firstSetBit] &&
                                ↪ nodeSet > 2)
                                ans += dp[mask][j];
                        }
                    }
                }
            }
        }
    }
    cout << ans << endl;
}

```

## 5 Misc

### 5.1 Max Pos and Next Greater

```

const ll MXX = 1e5 + 5;
ll mxtree[4 * MXX], arr[MXX];
void maxtree(ll idx, ll left, ll right)
→ {
    if (left == right)
        mxtree[idx] = left;
    else {
        ll mid = (left + right) / 2;
        maxtree(idx * 2, left, mid);
        maxtree(idx * 2 + 1, mid + 1, right)
    → ;
        ll left = mxtree[idx * 2];
        ll right = mxtree[idx * 2 + 1];
        if (arr[left] < arr[right])
            mxtree[idx] = right;
        else
            mxtree[idx] = left;
    }
}
ll mxPos(ll idx, ll tleft, ll tright, ll
→ qleft, ll qright) {
    if (qleft > qright)
        return -1;
    if (qleft == tleft and qright ==
→ tright)
        return mxtree[idx];
    ll tmid = (tleft + tright) / 2;
    ll left = mxPos(idx * 2, tleft, tmid,
→ qleft, min(qright, tmid));
    ll right = mxPos(idx * 2 + 1, tmid +
→ 1, tright, max(qleft, tmid + 1),
→ qright);
    ll ans;
    if (left == -1)
        ans = right;
    else if (right == -1)
        ans = left;
    else if (arr[left] < arr[right])
        ans = right;
    else
        ans = left;
    return ans;
}
ll main() {
    ll t = 1, n, q, a, b;
    cin >> t;
    while (t--) {
        cin >> n >> q;
        for (ll i = 0; i < n; i++)
            cin >> arr[i];
        stack<ll> stk;
        ll nge[n];
        stk.push(0);
        for (ll i = 1; i < n; i++) {
            while (stk.size() and arr[stk.top
→ ()] < arr[i]) {
                nge[stk.top()] = i;
                stk.pop();
            }
            stk.push(i);
        }
        while (stk.size()) {
            nge[stk.top()] = n;
            stk.pop();
        }
        ll ans[n];
        ans[n - 1] = 0;
        for (ll i = n - 2; i >= 0; i--) {
            ll tmp = nge[i];
            if (tmp == n)
                ans[i] = 0;
            else
                ans[i] = ans[tmp] + 1;
        }
        maxtree(1, 0, n - 1);
        for (ll i = 0; i < q; i++) {
            cin >> a >> b;
            if (a > b)
                swap(a, b);
            cout << ans[mxPos(1, 0, n - 1, a -
→ 1, b - 1)] << "\n";
        }
    }
}

```

```

    }
}

```

### 5.2 Knight Move

```

ll X[8]={2,1,-1,-2,-2,-1,1,2};
ll Y[8]={1,2,2,1,-1,-2,-2,-1};

```

### 5.3 Matrix Exponentiation

```

using vvi = vector<vector<ll>>;
vvi martixMul(vvi &a, vvi &b) {
    ll r = a.size(), c = b[0].size();
    vvi ans(r, vector<ll>(c, 0));
    for (ll i = 0; i < r; i++) {
        for (ll j = 0; j < c; j++) {
            ans[i][j] = 0;
            for (ll k = 0; k < r; k++) {
                ans[i][j] += (a[i][k] * b[k][j])
            → % mod;
                ans[i][j] %= mod;
            }
        }
    }
    return ans;
}
vvi martixExp(vvi base, ll power, ll MOD
→ = mod) {
    vvi ans = base; // change the ans
    while (power) {
        if (power & 1)
            ans = martixMul(ans, base);
        base = martixMul(base, base);
        power /= 2;
    }
    return ans;
}

```

### 5.4 Ternary Search

```

double ternary_search(double l, double r
→ ) {
    double eps = 1e-9; // error limit
    while (r - l > eps) {
        double m1 = l + (r - l) / 3, m2 = r
→ - (r - l) / 3;
        double f1 = f(m1), f2 = f(m2); //
→ evaluates the function at m1,
→ m2
        if (f1 < f2)
            l = m1;
        else
            r = m2;
    }
    return f(l); // return the maximum of
    return f(x) in [l, r]
}

```

## 6 Number Theory

### 6.1 Leap\_year

```

bool isLeap(ll n) {
    if (n % 100 == 0)
        return (n % 400 == 0);
    else
        return (n % 4 == 0);
}
// leap year between l and r
ll calNum(ll y) { return (y / 4) - (y /
→ 100) + (y / 400); }
ll leapNum(ll l, ll r) { return calNum(r
→ ) - calNum(--l); }

```

### 6.2 Two Line Intersection

```

ll cross(ll x1, ll y1, ll x2, ll y2, ll
→ x3, ll y3) {
    return (x2 - x1) * (y3 - y1) - (y2 -
→ y1) * (x3 - x1);
}

```

```

}

bool intersect(ll x1, ll y1, ll x2, ll
    ↪ y2, ll x3, ll y3, ll x4, ll y4) {
    ll c1 = cross(x1, y1, x2, y2, x3, y3),
    ↪ c2 = cross(x1, y1, x2, y2, x4, y4),
    ↪ c3 = cross(x3, y3, x4, y4, x1, y1),
    ↪ c4 = cross(x3, y3, x4, y4, x2, y2);
    if ((!c1 && min(x1, x2) <= x3 && x3 <=
        ↪ max(x1, x2) && min(y1, y2) <=
        ↪ y3 &&
        y3 <= max(y1, y2)) ||
        (!c2 && min(x1, x2) <= x4 && x4 <=
            ↪ max(x1, x2) && min(y1, y2)
            ↪ <= y4 &&
            y4 <= max(y1, y2)) ||
        (!c3 && min(x3, x4) <= x1 && x1 <=
            ↪ max(x3, x4) && min(y3, y4)
            ↪ <= y1 &&
            y1 <= max(y3, y4)) ||
        (!c4 && min(x3, x4) <= x2 && x2 <=
            ↪ max(x3, x4) && min(y3, y4)
            ↪ <= y2 &&
            y2 <= max(y3, y4)))
    return true;
    return (c1 > 0) != (c2 > 0) && (c3 >
        ↪ 0) != (c4 > 0);
}

```

### 6.3 Binary\_exponentiation

```

ll binaryExp(ll base, ll power, ll MOD =
    ↪ mod) {
    ll res = 1;
    while (power) {
        if (power & 1)
            res = (res * base) % MOD;
        base = ((base % MOD) * (base % MOD))
            ↪ % MOD;
        power /= 2;
    }
    return res;
}

/*
task: a ^ b ^ c
binaryExp(a, binaryExp(b, c, mod - 1),
    ↪ mod)
*/

```

### 6.4 Count\_divisor

```

ll maxVal = 1e6 + 1;
vector<ll> countDivisor(maxVal, 0);
void countingDivisor() {
    for (ll i = 1; i < maxVal; i++)
        for (ll j = i; j < maxVal; j += i)
            countDivisor[j]++;
}

// TC: nlog(n)
// count the number of divisors of all
    ↪ numbers in a range.

```

### 6.5 Check\_prime

```

bool prime(ll n) {
    if (n < 2)
        return false;
    if (n <= 3)
        return true;
    if (!(n % 2) || !(n % 3))
        return false;
    for (ll i = 5; i * i <= n; i += 6) {
        if (!(n % i) || !(n % (i + 2)))
            return false;
    }
    return true;
}

// TC: sqrt(n) / 6;

```

### 6.6 SPF

```

// smallest prime factor using seive
const ll N = 1e7 + 5;
ll spf[N];
void smallestPrimeFactorUsingSeive() {
    for (ll i = 2; i < N; i++) {
        if (spf[i] == 0) {
            for (ll j = i; j < N; j += i) {
                if (spf[j] == 0)
                    spf[j] = i;
            }
        }
    }
}

// smallest factor of a number
ll factor(ll n) {
    ll a;
    if (n % 2 == 0)
        return 2;
    for (ll a = 3; a * a <= n; a += 2) {
        if (n % a == 0)
            return a;
    }
    return n;
}

// complete factorization
ll r;
while (n > 1) {
    r = factor(n);
    cout << r << "\n";
    n /= r;
}

```

### 6.7 Seive

```

const ll N = 1e7 + 5;
ll prime[N];
void sieveOfEratosthenes() {
    for (ll i = 2; i < N; i++)
        prime[i] = 1;
    for (ll i = 4; i < N; i += 2)
        prime[i] = 0;
    for (ll i = 3; i * i < N; i++) {
        if (prime[i]) {
            for (ll j = i * i; j < N; j += i *
                ↪ 2)
                prime[j] = 0;
        }
    }
}

```

### 6.8 Optimize\_seive

```

vector<ll> sieve(const ll N, const ll Q
    ↪ = 17, const ll L = 1 << 15) {
    static const ll rs[] = {1, 7, 11, 13,
        ↪ 17, 19, 23, 29};
    struct P {
        P(ll p) : p(p) {}
        ll p;
        ll pos[8];
    };
    auto approx_prime_count = [](const ll
        ↪ N) -> ll {
        return N > 60184 ? N / (log(N) -
            ↪ 1.1) : max(1., N / (log(N) -
            ↪ 1.11)) + 1;
    };
    const ll v = sqrt(N), vv = sqrt(v);
    vector<bool> isp(v + 1, true);
    for (ll i = 2; i <= vv; ++i)
        if (isp[i]) {
            for (ll j = i * i; j <= v; j += i)
                isp[j] = false;
        }
    const ll rsize = approx_prime_count(N
        ↪ + 30);
    vector<ll> primes = {2, 3, 5};
    ll psize = 3;
    primes.resize(rsize);
    vector<P> sprimes;
    size_t pbeg = 0;
    ll prod = 1;
}

```

```

for (ll p = 7; p <= v; ++p) {
    if (!isp[p])
        continue;
    if (p <= Q)
        prod *= p, ++pbeg, primes[psize]++;
    auto pp = P(p);
    for (ll t = 0; t < 8; ++t) {
        ll j = (p <= Q) ? p : p * p;
        while (j % 30 != rs[t])
            j += p << 1;
        pp.pos[t] = j / 30;
    }
    sprimes.push_back(pp);
}

vector<unsigned char> pre(prod, 0xFF);
for (size_t pi = 0; pi < pbeg; ++pi) {
    auto pp = sprimes[pi];
    const ll p = pp.p;
    for (ll t = 0; t < 8; ++t) {
        const unsigned char m = ~(1 << t);
        for (ll i = pp.pos[t]; i < prod; i
             += p)
            pre[i] &= m;
    }
}
const ll block_size = (L + prod - 1) /
    prod * prod;
vector<unsigned char> block(block_size
    );
unsigned char *pblock = block.data();
const ll M = (N + 29) / 30;
for (ll beg = 0; beg < M; beg +=
     block_size, pblock -= block_size
     ) {
    ll end = min(M, beg + block_size);
    for (ll i = beg; i < end; i += prod)
        {
            copy(pre.begin(), pre.end(),
                  pblock + i);
        }
    if (beg == 0)
        pblock[0] &= 0xFE;
    for (size_t pi = pbeg; pi < sprimes.
         size(); ++pi) {
        auto &pp = sprimes[pi];
        const ll p = pp.p;
        for (ll t = 0; t < 8; ++t) {
            ll i = pp.pos[t];
            const unsigned char m = ~(1 << t
                );
            for (; i < end; i += p)
                pblock[i] &= m;
            pp.pos[t] = i;
        }
        for (ll i = beg; i < end; ++i) {
            for (ll m = pblock[i]; m > 0; m &=
                 m - 1) {
                primes[psize++] = i * 30 + rs[
                    __builtin_ctz(m)];
            }
        }
    }
    assert(psize <= rsize);
    while (psize > 0 && primes[psize - 1]
          > N)
        --psize;
    primes.resize(psize);
    return primes;
}
// it takes 500ms for generating prime
// upto 1e9

```

## 6.9 nth\_prime\_number

```

vector<ll> nth_prime;
const ll MX = 86200005;
bitset<MX> visited;
void optimized_prime() {
    nth_prime.push_back(2);
    for (ll i = 3; i < MX; i += 2) {
        if (visited[i])

```

```

        continue;
        nth_prime.push_back(i);
        if (i * i * i > MX)
            continue;
        for (ll j = i * i; j < MX; j += i +
            i)
            visited[j] = true;
    }
}

```

## 6.10 nCr

```

// 1:
// more space, less time
const ll MAX = 1e7 + 5;
vector<ll> fact(MAX), ifact(MAX), inv(
    MAX);
void factorial() {
    inv[1] = fact[0] = ifact[0] = 1;
    for (ll i = 2; i < MAX; i++)
        inv[i] = inv[mod % i] * (mod - mod /
            i) % mod;
    for (ll i = 1; i < MAX; i++)
        fact[i] = (fact[i - 1] * i) % mod;
    for (ll i = 1; i < MAX; i++)
        ifact[i] = ifact[i - 1] * inv[i] %
            mod;
}
ll nCr(ll n, ll r) {
    if (r < 0 || r > n)
        return 0;
    return (ll)fact[n] * ifact[r] % mod *
        ifact[n - r] % mod;
}
// 2:
// less space, more time
const ll MAX = 1e7 + 10;
vector<ll> fact(MAX), inv(MAX);
void factorial() {
    fact[0] = 1;
    for (ll i = 1; i < MAX; i++)
        fact[i] = (i * fact[i - 1]) % mod;
}
ll binaryExp(ll a, ll n, ll M = mod){};
void inverse() {
    for (ll i = 0; i < MAX; ++i)
        inv[i] = binaryExp(fact[i], mod - 2);
}
ll nCr(ll a, ll b) {
    if (a < b or a < 0 or b < 0)
        return 0;
    ll de = (inv[b] * inv[a - b]) % mod;
    return (fact[a] * de) % mod;
}
// 3:
// nCr mod m where m is not prime
ll C_mod_p(ll n, ll k, ll p) {
    if (k > n)
        return 0;
    vector<ll> fac(p);
    fac[0] = 1;
    for (int i = 1; i < p; i++)
        fac[i] = fac[i - 1] * i % p;
    ll res = 1;
    while (n || k) {
        ll ni = n % p, ki = k % p;
        if (ki > ni)
            return 0;
        res = res * fac[ni] % p * modInv(fac
            [ki], p) % p * modInv(fac[ni -
            ki], p) %
            p;
        n /= p;
        k /= p;
    }
    return res;
}
// compute nCr mod composite m (non-
// prime)
ll nCr_mod_m(ll n, ll k, ll m) {
    // Step 1: factorize m
    // Step 2: calculate nCr mod each prime
    // Step 3: calculate result mod m
}
```

```

vector<int> primes;
int tmp = m;
for (int i = 2; i * i <= tmp; i++) {
    if (tmp % i == 0) {
        primes.push_back(i);
        while (tmp % i == 0)
            tmp /= i;
    }
}
if (tmp > 1)
    primes.push_back(tmp);

// Step 2: compute result mod each
// prime
vector<ll> rem, mod;
for (int p : primes) {
    rem.push_back(C_mod_p(n, k, p));
    mod.push_back(p);
}
// Step 3: Chinese Remainder Theorem (
// combine)
ll res = 0;
for (int i = 0; i < (int)mod.size(); i
    //++)
{
    ll Mi = m / mod[i];
    ll invMi = binaryExp(Mi, mod[i] - 2,
        // mod[i]); // modular inverse
    res = (res + rem[i] * Mi % m * invMi
        // % m) % m;
}
return res;
}

```

## 6.11 Factorial\_mod

```

// n! mod p : Here P is mod value
// For binaryExp we call 1.6 function
ll factmod(ll n, ll p) {
    ll res = 1;
    while (n > 1) {
        res = (res * binaryExp(p - 1, n / p,
            // p)) % p;
        for (ll i = 2; i <= n % p; ++i)
            res = (res * i) % p;
        n /= p;
    }
    return (res % p);
}

```

## 6.12 PHI

```

// the positive integers less than or
// equal to n that are relatively
// prime to n.
ll phi(ll n) {
    ll result = n;
    for (ll i = 2; i * i <= n; i++) {
        if (n % i == 0) {
            while (n % i == 0)
                n /= i;
            result -= result / i;
        }
    }
    if (n > 1)
        result -= result / n;
    return result;
}

// PHI of 1 to N
const int N = 1e6 + 9;
int phi[N];
int phiS[N];
void totient() {
    for (int i = 1; i < N; i++)
        phi[i] = i;
    for (int i = 2; i < N; i++) {
        if (phi[i] == i) {
            for (int j = i; j < N; j += i)
                phi[j] -= phi[j] / i;
        }
    }
    phiS[0] = phi[0];
    for (int i = 1; i < N; i++)
        phiS[i] = phiS[i - 1] + phi[i];
}

```

```

}

```

## 6.13 Catalan

```

void catalan(ll n) {
    ll res = 1;
    cout << res << " ";
    for (ll i = 1; i < n; i++) {
        res = (res * (4 * i - 2)) / (i + 1);
        cout << res << " ";
    }
}

```

## 6.14 Extended\_GCD

```

// return {x,y} such that ax + by = gcd(
// a,b)
ll extended_euclid(ll a, ll b, ll &x, ll
    // &y) {
    if (b == 0) {
        x = 1;
        y = 0;
        return a;
    }
    ll x1, y1;
    ll d = extended_euclid(b, a % b, x1,
        // y1);
    x = y1;
    y = x1 - y1 * (a / b);
    return d;
}
ll inverse(ll a, ll m) {
    ll x, y;
    ll g = extended_euclid(a, m, x, y);
    if (g != 1)
        return -1;
    return (x % m + m) % m;
}

```

## 6.15 Large Mod

```

ll mod(string &num, ll a) {
    ll res = 0;
    for (ll i = 0; i < num.length(); i++)
        res = (res * 10 + num[i] - '0') % a;
    return res;
}

```

## 6.16 Factorial\_Divisor

```

ll factorialDivisors(ll n) {
    ll result = 1;
    for (ll i = 0; i < allPrimes.size(); i
        //++)
    {
        ll p = allPrimes[i];
        ll exp = 0;
        while (p <= n) {
            exp = exp + (n / p);
            p = p * allPrimes[i];
        }
        result = result * (exp + 1);
    }
    return result;
}

```

## 6.17 Number\_conversion

```

// 10 - ary to m - ary
char a[16] = {'0', '1', '2', '3', '4',
    // '5', '6', '7', '8', '9', 'A', 'B', 'C',
    // 'D', 'E', 'F'};
string tenToM(ll n, ll m) {
    ll temp = n;
    string result = "";
    while (temp != 0) {
        result = a[temp % m] + result;
        temp /= m;
    }
    return result;
}

// m - ary to 10 - ary
string num = "0123456789ABCDE";
ll mToTen(string n, ll m) {

```

```

ll multi = 1;
ll result = 0;
for (ll i = n.size() - 1; i >= 0; i--)
    ↪ {
        result += num.find(n[i]) * multi;
        multi *= m;
    }
return result;
}

```

## 6.18 Number\_of\_1\_in\_bit\_till\_N

```

ll cntOnes(ll n) {
    ll cnt = 0;
    for (ll i = 1; i <= n; i <<= 1) {
        ll x = (n + 1) / (i << 1);
        cnt += x * i;
        if ((n + 1) % i && n & i)
            cnt += (n + 1) % i;
    }
    return cnt;
}

```

## 6.19 Disarrangement

```

ll disarrange(ll n) {
    if (n == 1)
        return 0;
    if (n == 2)
        return 1;
    return (n - 1) * (disarrange(n - 1) +
        ↪ disarrange(n - 2));
}
// D(n) = (n!)/e

```

## 6.20 Millar\_Rabin

```

bool check_composite(ll n, ll a, ll d,
    ↪ ll s) {
    ll x = binaryExp(a, d, n);
    if (x == 1 || x == n - 1)
        return false;
    for (ll r = 1; r < s; r++) {
        x = (u128)x * x % n;
        if (x == n - 1)
            return false;
    }
    return true;
};
bool MillerRabin(ll n, ll iter = 5) {
    // returns true if n is probably prime
    // , else returns false.
    if (n < 4)
        return n == 2 || n == 3;
    ll s = 0;
    ll d = n - 1;
    while ((d & 1) == 0) {
        d >>= 1;
        s++;
    }
    for (ll i = 0; i < iter; i++) {
        ll a = 2 + rand() % (n - 3);
        if (check_composite(n, a, d, s))
            return false;
    }
    return true;
}

```

## 6.21 Modular\_operation

```

// Addition :
ll mod_add(ll a, ll b, ll MOD = mod) {
    a = a % MOD, b = b % MOD;
    return (((a + b) % MOD) + MOD) % MOD;
}
// Subtraction :
ll mod_sub(ll a, ll b, ll MOD = mod) {
    a = a % MOD, b = b % MOD;
    return (((a - b) % MOD) + MOD) % MOD;
}
// Multiplication :
ll mod_mul(ll a, ll b, ll MOD = mod) {
    a = a % MOD, b = b % MOD;
    return (((a * b) % MOD) + MOD) % MOD;
}

```

```

}
// Division :
ll mmInvprime(ll a, ll b) { return
    ↪ binaryExp(a, b - 2, b); }
ll mod_div(ll a, ll b, ll MOD = mod) {
    a = a % MOD, b = b % MOD;
    return (mod_mul(a, mmInvprime(b, MOD),
        ↪ MOD) + MOD) % MOD;
}

```

## 6.22 MSLCM

```

// For a given number N, maximum sum LCM
// indicates the set of numbers
// whose LCM
// is N and summation is maximum. Let,
// MSLCM(N) denote this maximum sum
// of
// numbers. Given the value of N you
// will have to find the value:
// summation of
// MSLCM(i) from i to 2n
ll MSLCM(ll n) {
    ll l = 1, r, val, ret = 0;
    while (l <= n) {
        val = n / l, r = n / val;
        ret += val * ((l + r) * (r - l + 1)
            ↪ / 2);
        l = r + 1;
    }
    return ret - 1;
}

```

## 6.23 Find numbers in between [L, R] which are divisible by all Array elements

```

void solve(ll *arr, ll N, ll L, ll R) {
    ll LCM = arr[0];
    for (ll i = 1; i < N; i++) {
        LCM = (LCM * arr[i]) / (__gcd(LCM,
            ↪ arr[i]));
    }
    if ((LCM < L && LCM * 2 > R) || LCM >
        ↪ R) {
        return;
    }
    ll k = (L / LCM) * LCM;
    //
    if (k < L)
        k = k + LCM;
    for (ll i = k; i <= R; i = i + LCM)
        cout << i << ' ';
}

```

## 6.24 PollarRho

```

namespace PollardRho {
    mt19937 rnd(chrono::steady_clock::now
        ↪ ().time_since_epoch().count());
    const int P = 1e6 + 9;
    ll seq[P];
    int primes[P], spf[P];
    inline ll add_mod(ll x, ll y, ll m) {
        return (x += y) < m ? x : x - m;
    }
    inline ll mul_mod(ll x, ll y, ll m) {
        ll res = __int128(x) * y % m;
        return res;
        // ll res = x * y - (long
        // ↪ double)x * y / m + 0.5) * m;
        // return res < 0 ? res + m : res;
    }
    inline ll pow_mod(ll x, ll n, ll m) {
        ll res = 1 % m;
        for (; n; n >>= 1) {
            if (n & 1) res = mul_mod(res, x,
                ↪ );
            x = mul_mod(x, x, m);
        }
        return res;
    }
}

```

```

// O(it * (logn)^3), it = number of
// rounds performed
inline bool miller_rabin(ll n) {
    if (n <= 2 || (n & 1 ^ 1)) return (n
        ↪ == 2);
    if (n < P) return spf[n] == n;
    ll c, d, s = 0, r = n - 1;
    for (; !(r & 1); r >>= 1, s++) {}
    // each iteration is a round
    for (int i = 0; primes[i] < n &&
        ↪ primes[i] < 32; i++) {
        c = pow_mod(primes[i], r, n);
        for (int j = 0; j < s; j++) {
            d = mul_mod(c, c, n);
            if (d == 1 && c != 1 && c != (n
                ↪ - 1)) return false;
            c = d;
        }
        if (c != 1) return false;
    }
    return true;
}
void init() {
    int cnt = 0;
    for (int i = 2; i < P; i++) {
        if (!spf[i]) primes[cnt++] = spf[i
            ↪ ] = i;
        for (int j = 0, k; (k = i * primes
            ↪ [j]) < P; j++) {
            spf[k] = primes[j];
            if (spf[i] == spf[k]) break;
        }
    }
    // returns O(n^(1/4))
    ll pollard_rho(ll n) {
        while (1) {
            ll x = rnd() % n, y = x, c = rnd()
                ↪ % n, u = 1, v, t = 0;
            ll *px = seq, *py = seq;
            while (1) {
                *py++ = y = add_mod(mul_mod(y, y
                    ↪ , n), c, n);
                *py++ = y = add_mod(mul_mod(y, y
                    ↪ , n), c, n);
                if ((x = *px++) == y) break;
                v = u;
                u = mul_mod(u, abs(y - x), n);
                if (!u) return __gcd(v, n);
                if (++t == 32) {
                    t = 0;
                    if ((u = __gcd(u, n)) > 1 && u
                        ↪ < n) return u;
                }
                if (t && (u = __gcd(u, n)) > 1 &&
                    ↪ u < n) return u;
            }
            vector<ll> factorize(ll n) {
                if (n == 1) return vector<ll>();
                if (miller_rabin(n)) return vector<
                    ↪ ll>{n};
                vector<ll> v, w;
                while (n > 1 && n < P) {
                    v.push_back(spf[n]);
                    n /= spf[n];
                }
                if (n >= P) {
                    ll x = pollard_rho(n);
                    v = factorize(x);
                    w = factorize(n / x);
                    v.insert(v.end(), w.begin(), w.end
                        ↪ ());
                }
                return v;
            }
            // auto f = PollardRho::factorize(n);
            // sort(f.begin(), f.end());
            // cout << f.size() << '\n';
            // for (auto x: f) cout << x << ' ';
            ↪ cout << '\n';
        }
    }

```

## 6.25 2D Seg Tree

```

template <typename T> class BIT2D {
private:
    const int n, m;
    vector<vector<T>> bit;
public:
    BIT2D(int n, int m) : n(n), m(m), bit(
        ↪ n + 1, vector<T>(m + 1)) {}
    // adds val to the point (r, c) */
    void add(int r, int c, T val) {
        r++, c++;
        for (; r <= n; r += r & -r) {
            for (int i = c; i <= m; i += i & -
                ↪ i) {
                bit[r][i] += val;
            }
        }
    }
    // sum of points with row in [0, r]
    // and column in [0, c] */
    T rect_sum(int r, int c) {
        r++, c++;
        T sum = 0;
        for (; r > 0; r -= r & -r) {
            for (int i = c; i > 0; i -= i & -i
                ↪ ) {
                sum += bit[r][i];
            }
        }
        return sum;
    }
    // sum of points with row in [r1, r2]
    // and column in [c1, c2] */
    T rect_sum(int r1, int c1, int r2, int
        ↪ c2) {
        return rect_sum(r2, c2) - rect_sum(
            ↪ r2, c1 - 1) - rect_sum(r1 - 1,
            ↪ c2) +
            rect_sum(r1 - 1, c1 - 1);
    }
}

```

## 6.26 Next Prev Smaller

```

vector<ll> next_smallest(vector<ll> v) {
    ll n = v.size();
    vector<ll> nextS(n, n);
    stack<ll> st;
    for (int i = 0; i < n; i++) {
        while (!st.empty() && v[i] < v[st.
            ↪ top()]) {
            nextS[st.top()] = i;
            st.pop();
        }
        st.push(i);
    }
    return nextS;
}
vector<ll> prev_smallest(vector<ll> v) {
    ll n = v.size();
    vector<ll> prevS(n, -1);
    stack<ll> st;
    for (int i = 0; i < n; ++i) {
        while (!st.empty() && v[i] < v[st.
            ↪ top()])
            st.pop();
        if (!st.empty())
            prevS[i] = st.top();
        st.push(i);
    }
    return prevS;
}

```

## 7 Information

## 7.1 Numbers with Most Divisors

Max Value ( $N$ )	Number with Most Divisors ( $n$ )	Number of Divisors ( $\tau(n)$ )
$10^3$	83,160	128
$10^6$	720,720	240
$10^7$	9,609,600	640
$10^8$	98,280,000	672
$10^9$	735,134,400	1,344
$10^{10}$	7,242,460,800	2,688
$10^{11}$	73,346,256,000	5,376
$10^{12}$	936,966,912,400	10,752

## 7.2 Totient Function inside:

- For a prime  $p$ :  $\phi(p) = p - 1$
- For a prime power  $p^e$ :  $\phi(p^e) = (p - 1)p^{e-1}$
- Divisor sum identity:  $\sum_{d|n} \phi(d) = n$
- Sum of integers coprime to  $n$ :  

$$f(n) = \sum_{\substack{1 \leq i \leq n \\ \gcd(i,n)=1}} i = \frac{\phi(n) \cdot n}{2}$$
- If  $\text{GCD}(n, i) = x$  then,  $\phi\left(\frac{n}{x}\right) = 1$
- Sum of GCD over all pairs  $(i, j)$  with  $1 \leq i, j \leq n$ :  

$$\sum_{i=1}^n \sum_{d|i} d \cdot \phi\left(\frac{i}{d}\right)$$
- Sum of LCM over all pairs  $(i, j)$  with  $1 \leq i, j \leq n$ :  

$$\sum_{i=1}^n \sum_{d|i} i \cdot f\left(\frac{i}{d}\right)$$

## 7.3 Bézout's Identity and GCD Properties

$\gcd(a, b) = g$  implies there exist integers  $x, y$  such that  $ax + by = g$ .

- All integers of the form  $ax + by$  are exactly the multiples of  $g$ .
- Adding or subtracting multiples doesn't change the gcd:  $a \equiv b \pmod{g} \iff g | (a - b)$ .
- If  $\gcd(a, b) = 1$  then any integer can be formed; if  $\gcd(a, b) = g$  then any multiple of  $g$  can be formed.
- $\gcd(a, b) = \gcd(a - b, b) = \gcd(a, b - a)$ .
- If  $\gcd(a, b) = g$  then  $\gcd\left(\frac{a}{g}, \frac{b}{g}\right) = 1$ .
- $\gcd(ka, kb) = k \gcd(a, b)$ .
- If  $\gcd(a, m) = 1$ , Bézout gives  $ax + my = 1$ , hence  $ax \equiv 1 \pmod{m}$ , so  $x$  is the modular inverse of  $a$  mod  $m$  (important when modulus is needed and  $m$  is not prime).

## 7.4 Combinatorics Information

**Distinct-element counts over all subarrays:** To count the number of distinct elements across all subarrays, consider every ending index  $i$ . By tracking the last occurrence of each value, we can determine how far left we can extend while keeping all elements distinct. This gives the maximum valid window  $[L_i, i]$ . The number of subarrays ending at  $i$  that have all distinct elements is

simply the window size  $(i - L_i + 1)$ . Summing these values for all  $i$  yields the total distinct-element count over all subarrays.

**Bitwise OR over all subarrays:** For subarrays ending at index  $i$ , the bitwise OR value can only increase as the subarray grows, since once a bit becomes 1, it never returns to 0. Let  $S_{i-1}$  be the set of OR values of all subarrays ending at  $i - 1$ . To compute the OR values for subarrays ending at  $i$ , take each value in  $S_{i-1}$ , OR it with  $a_i$ , and also include the single element OR value  $a_i$ . After merging duplicates, the resulting set contains all OR values achieved by subarrays ending at  $i$ . Summing all these values over every  $i$  yields the total bitwise OR over all subarrays.

**Bitwise XOR over all subarrays:** Let  $\text{pref}[i]$  be the prefix XOR up to index  $i$ . The XOR of a subarray  $[l, r]$  is  $\text{pref}[r] \oplus \text{pref}[l-1]$ . Consider a single bit position. This bit is 1 in the subarray XOR exactly when the two prefixes differ at that bit. Count how many prefixes have bit 0 and how many have bit 1. The number of contributing pairs is their product. Summing this contribution over all bits gives the total XOR value of all subarrays.

**XOR of all pairs:** For any bit position, the XOR of two values is 1 at that bit if and only if one value has the bit set and the other does not. If  $c_1$  numbers have the bit 1 and  $c_0$  have the bit 0, then the number of pairs contributing a 1 at that bit is  $c_0 \cdot c_1$ . Summing these contributions over all bit positions yields the XOR of all unordered pairs.

**XOR of OR of every subarray:** A bit in the OR of a subarray is 1 if the subarray contains at least one element with that bit set. If a bit appears in  $k$  positions, count how many subarrays include at least one such position. If this number is odd, the bit remains in the final XOR; otherwise it cancels out.

**Tree: total length of all simple paths:** Removing an edge splits the tree into parts of sizes  $s$  and  $n - s$ . Any path connecting one node from each part must use this edge, giving  $s(n - s)$  such paths. Multiplying by the edge weight and summing over all edges yields the total path length.

**Tree: distinct elements over all simple paths:** Using DSU-on-tree, maintain the largest child's data structure for each subtree and merge smaller children into it. This efficiently tracks all distinct values appearing along all paths.

**Minimum Hamming distance over all cyclic shifts:** The Hamming distance for each shift equals the number of mismatched positions. Map equal characters to +1 and unequal characters to -1, then use convolution to compute match counts for all shifts simultaneously. More matches imply a smaller Hamming distance.

**Total Hamming distance over all pairs:** For each bit, if  $c$  values have that bit set and  $n - c$  do not, then  $c(n - c)$  pairs differ at that bit. Summing over all bits yields the total Hamming distance.

**Sum of products over all subsequences:** Expanding the product  $(1+a_1)(1+a_2) \cdots (1+a_n)$  selects either 1 or  $a_i$  from each term, corresponding exactly to skipping or

including  $a_i$ . Every subsequence product appears once; subtracting 1 removes the empty subsequence.

**Widths over all subsequences:** After sorting, element  $a_i$  is maximum in  $2^i$  subsequences and minimum in  $2^{n-i-1}$  subsequences. Summing the contributions of all elements gives the total width.

**Optimal weighted sum with range increments:** A difference array counts how many range operations affect each position. Sorting both the array values and their frequencies and pairing largest with largest maximizes the total sum.

**Sum of divisors for 1 to  $N$ :** Each integer  $i$  is a divisor of all multiples  $i, 2i, 3i, \dots$ . Adding  $i$  to the divisor sum of each of these values for all  $i$  from 1 to  $N$  computes all divisor sums.

**Sum of absolute differences over all pairs:** After sorting, each  $a_i$  contributes to the absolute difference with all smaller elements on its left and all larger elements on its right. Prefix sums compute these contributions efficiently.

**Sum of inversion counts over all permutations:** Each pair  $(i, j)$  with  $i < j$  has a  $1/2$  probability of appearing as an inversion in a random permutation. With  $\frac{n(n-1)}{2}$  such pairs, the expected number of inversions is  $\frac{n(n-1)}{4}$ . Multiplying by  $n!$  gives the total inversion sum.

**Inversion sum over binary strings with  $x$  zeros and  $y$  ones:** There are  $xy$  zero-one pairs, and each such pair contributes an inversion in exactly half the binary strings of length  $x + y$  containing  $x$  zeros and  $y$  ones. Multiplying by the total count  $\binom{x+y}{x}$  yields the inversion sum.

## 8 Mathematics

### 8.1 Area Formulas

Rectangle	length $\times$ width
Square	side <sup>2</sup>
Triangle	$\frac{1}{2} \times \text{base} \times \text{height}$
Parallelogram	base $\times$ height
Pyramid (no base)	$\frac{1}{2} \times (\text{perimeter of base}) \times (\text{slant height})$
Polygon	$\frac{1}{2}  \sum_{i=1}^n (x_i y_{i+1} - x_{i+1} y_i) $ $a + \frac{b}{2} - 1$ (for lattice coordinates)

$a$  = interior lattice pts,  $b$  = boundary pts.

### 8.2 Volume Formulas

Cube	side <sup>3</sup>
Rectangular Prism	length $\times$ width $\times$ height
Cylinder	$\pi \times \text{radius}^2 \times \text{height}$
Sphere	$\frac{4}{3}\pi \times \text{radius}^3$
Pyramid	$\frac{1}{3} \times (\text{base area}) \times (\text{height})$

### 8.3 Surface Area Formulas

Cube	$6 \times \text{side}^2$
Rectangular Prism	$2(lw + lh + wh)$ ( $l$ = length, $w$ = width, $h$ = height)
Cylinder	$2\pi r(r + h)$
Sphere	$4\pi r^2$
Pyramid	base area $+ \frac{1}{2} \times (\text{perimeter}) \times (\text{slant height})$

### 8.4 Triangles

Semiperimeter	$s = \frac{a+b+c}{2}$
Area	$A = \sqrt{s(s-a)(s-b)(s-c)}$
Circumradius	$R = \frac{abc}{4A}$
Inradius	$r = \frac{A}{s}$
Median	$m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$
Angle bisector	$s_a = \sqrt{\frac{bc}{1-(a/(b+c))^2}}$

Side lengths:  $a, b, c$ .

### 8.5 Sum Equations

$$\sum_{i=k}^n c^i = \frac{c^{n+1} - c^k}{c - 1} \quad \sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6} \quad \sum_{i=1}^n i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

$$\sum_{i=1}^n i^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30} \quad \sum_{i=1}^n (2i-1) = n^2$$

### 8.6 Logarithmic Basic

$\log_b 1 = 0$	$\log_b b = 0$
$b^{\log_b a} = a$	$x^{\log_b y} = y^{\log_b x}$
$\log_a b = \frac{1}{\log_b a}$	$\log_a x = \frac{\log_b x}{\log_b a}$
$\log_b(AB) = \log_b A + \log_b B$	
$\log_b\left(\frac{A}{B}\right) = \log_b A - \log_b B$	
$\log_a c = \log_a b \times \log_b c$	
$\log_b(A^x) = x \log_b A$	

### 8.7 Series

Catalan:  $C_n = \frac{1}{n+1} \binom{2n}{n}$ ,  $C_n = \sum_{k=0}^n C_k C_{n-k}$   
 Arithmetic:  $a_n = a + (n-1)d$ ,  $s_n = \frac{n}{2}(2a + (n-1)d)$   
 Geometric:  $a_n = ar^{n-1}$ ,  $s_n = \frac{a(1-r^n)}{1-r}$   
 Derangements:  $D_n = n! \sum_{k=0}^n \frac{(-1)^k}{k!}$ ,  $D_n = \left\lfloor \frac{n!}{e} + \frac{1}{2} \right\rfloor$   
 Fibonacci:  $f_n = \frac{\phi^n - (1-\phi)^n}{\sqrt{5}}$ ,  $\phi = \frac{1+\sqrt{5}}{2}$

### 8.8 Pick's Theorem

$A = I - \frac{1}{2}B + 1$  ( $I$  = interior points,  $B$  = boundary points)

### 8.9 Stars and Bars

Number of solutions of  $x_1 + \dots + x_k = n$ :  
 $\binom{n-1}{k-1}$  when  $x_i > 0$ ;  $\binom{n+k-1}{k-1}$  when  $x_i \geq 0$ .

### 8.10 Facts

$\lceil \frac{a}{b} \rceil = \lfloor \frac{a-1}{b} \rfloor + 1$   
 Sum  $l$  to  $r$ :  $\frac{l+r}{2}(r-l+1)$   
 $\left\lfloor \frac{\lfloor n/a \rfloor}{b} \right\rfloor = \left\lfloor \frac{n}{ab} \right\rfloor$

### 8.11 LCM

$$\text{lcm}(a, n) + \text{lcm}(n-a, n) = \frac{n^2}{\text{gcd}(a, n)}$$

$$\text{SUM} = \frac{n}{2} \left( \sum_{d|n} \varphi(d)d + 1 \right)$$