

# MIST\_Untitled

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*Last modified: December 16, 2025*

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# 1 C++

## 1.1 template

```
/*
C++:
ios_base::sync_with_stdio(false);
cin.tie(nullptr), cout.tie(nullptr);

python:
import sys
input = sys.stdin.readline
sys.stdout.write("-----")
*/
```

## 1.2 random

```
#define accuracy chrono::steady_clock::
    now().time_since_epoch().count()
mt19937 rng(accuracy);

ll rand(ll l, ll r) {
    uniform_int_distribution<ll> ludo(l, r
    );
    return ludo(rng);
}
```

## 1.3 gp\_hash

```
#include<ext/pb_ds/assoc_container.hpp>
#include<ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
template <typename p, typename q> using
    ht = gp_hash_table<p, q>;
```

## 1.4 pbds

```
#include<ext/pb_ds/assoc_container.hpp>
#include<ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
template <typename T>
using o_set = tree<T, null_type, less<T
    >, rb_tree_tag,
    tree_order_statistics_node_update
    >;

/*
find_by_order(k) - returns an iterator
    to the k-th largest element (0
    indexed);
order_of_key(k) - the number of elements
    in the set that are strictly
    smaller than k;
*/
```

## 1.5 debug

```
string to_string(const string &s) {
    return '"' + s + '"';
}
string to_string(const char *s) { return
    to_string(string(s)); }
string to_string(const char c) { return
    '"' + string(1, c) + '"'; }
string to_string(bool b) { return b ? "
    true" : "false"; }
template <typename A, typename B> string
    to_string(pair<A, B> p) {
    return "(" + to_string(p.first) + ", "
    + to_string(p.second) + ")";
}
template <typename A> string to_string(A
    v) {
    string res = "{";
    for (const auto &x : v) {
        res += to_string(x) + ", ";
    }
    res += "}";
    return res;
}
void debug_out() { cerr << endl; }
template <typename Head, typename...
    Tail> void debug_out(Head H, Tail
    ... T) {
    cerr << " " << to_string(H);
    debug_out(T...);
}
```

```
}
#define dbg(...)
    \
    cerr << __LINE__ << " : [" << #
    __VA_ARGS__ << "]" = ", debug_out
    (__VA_ARGS__)
```

## 1.6 stress

```
#!/usr/bin/env bash
wrong="solution"
correct="brute"
gen="gen"
g++ -g solution.cpp -DONPC -o "$wrong"
g++ -g brute.cpp -DONPC -o "$correct"
g++ -g gen.cpp -DONPC -o "$gen"

for ((testNum=0;testNum<$1;testNum++))
do
    ./$gen 2>/dev/null > stdinput
    ./$correct < stdinput 2>/dev/
    null > outSlow
    ./$wrong < stdinput 2>/dev/null
    > outWrong
    H1=`md5sum outWrong`
    H2=`md5sum outSlow`
    if !(cmp -s "outWrong" "outSlow"
    )
    then
        echo "Error found!"
        echo "Input:"
        cat stdinput
        echo "Wrong Output:"
        cat outWrong
        echo "Slow Output:"
        cat outSlow
        exit
    fi
done
echo Passed $1 tests
# Usage: ./contest.sh times
```

## 1.7 vscode

```
{
    "key" : "f5",
    "command" : "workbench.action.terminal
    .sendSequence",
    "args" : {
        "text" : "g++ ${
            fileBasenameNoExtension}.cpp -
            o ${fileBasenameNoExtension}
            && ./ ${
            fileBasenameNoExtension} <in.
            <txt> out.txt\n "
        }
    }
}
```

# 2 Dsa

## 2.1 KMP

```
vector<ll> createLPS(string pattern) {
    ll n = pattern.length(), idx = 0;
    vector<ll> lps(n);
    for (ll i = 1; i < n; i++) {
        if (pattern[idx] == pattern[i]) {
            lps[i] = idx + 1;
            idx++, i++;
        } else {
            if (idx != 0)
                idx = lps[idx - 1];
            else
                lps[i] = idx, i++;
        }
    }
    return lps;
}
ll kmp(string text, string pattern) {
    ll cnt_of_match = 0, i = 0, j = 0;
    vector<ll> lps = createLPS(pattern);
    while (i < text.length()) {
```

```

if (text[i] == pattern[j])
    i++, j++; // i = text, j = pattern
else {
    if (j != 0)
        j = lps[j - 1];
    else
        i++;
}
if (j == pattern.length()) {
    cnt_of_match++;
    // the index where match found ->
    // (i - pattern.length());
    j = lps[j - 1];
}
}
return cnt_of_match;
}

```

## 2.2 Hashing

```

const ll N = 2e5 + 5;
const ll MOD1 = 127657753, MOD2 =
    987654319;
const ll p1 = 137, p2 = 277;
ll ip1, ip2;
pair<ll, ll> pw[N], ipw[N];
void prec() {
    pw[0] = {1, 1};
    for (ll i = 1; i < N; i++) {
        pw[i].first = 1LL * pw[i - 1].first
            * p1 % MOD1;
        pw[i].second = 1LL * pw[i - 1].
            second * p2 % MOD2;
    }
    ip1 = binaryExp(p1, MOD1 - 2, MOD1);
    ip2 = binaryExp(p2, MOD2 - 2, MOD2);
    ipw[0] = {1, 1};
    for (ll i = 1; i < N; i++) {
        ipw[i].first = 1LL * ipw[i - 1].
            first * ip1 % MOD1;
        ipw[i].second = 1LL * ipw[i - 1].
            second * ip2 % MOD2;
    }
}
struct Hashing {
    ll n;
    string s; // 0 -
    // indexed
    vector<pair<ll, ll>> hs; // 1 -
    // indexed
    Hashing() {}
    Hashing(string _s) {
        n = _s.size();
        s = _s;
        hs.emplace_back(0, 0);
        for (ll i = 0; i < n; i++) {
            pair<ll, ll> p;
            p.first = (hs[i].first + 1LL * pw[
                i].first * s[i] % MOD1) %
                MOD1;
            p.second = (hs[i].second + 1LL *
                pw[i].second * s[i] % MOD2)
                % MOD2;
            hs.push_back(p);
        }
    }
    pair<ll, ll> get_hash(ll l, ll r) {
        // 1 - indexed
        assert(1 <= l && l <= r && r <= n);
        pair<ll, ll> ans;
        ans.first =
            (hs[r].first - hs[l - 1].first +
                MOD1) * 1LL * ipw[l - 1].
                first % MOD1;
        ans.second = (hs[r].second - hs[l -
            1].second + MOD2) * 1LL *
            ipw[l - 1].second %
            MOD2;
        return ans;
    }
    pair<ll, ll> get_hash() { return
        get_hash(1, n); }
};

```

## 2.3 BigInteger

```

struct BigInteger {
    string str;
    // Constructor to initialize
    // BigInteger with a string
    BigInteger(string s) { str = s; }
    // Overload + operator to add
    // two BigInteger objects
    BigInteger operator+(const BigInteger
        &b) {
        string a = str, c = b.str;
        ll alen = a.length(), clen = c.
            length();
        ll n = max(alen, clen);
        if (alen > clen)
            c.insert(0, alen - clen, '0');
        else if (alen < clen)
            a.insert(0, clen - alen, '0');
        string res(n + 1, '0');
        ll carry = 0;
        for (ll i = n - 1; i >= 0; i--) {
            ll digit = (a[i] - '0') + (c[i] - '0')
                + carry;
            carry = digit / 10;
            res[i + 1] = digit % 10 + '0';
        }
        if (carry == 1) {
            res[0] = '1';
            return BigInteger(res);
        } else
            return BigInteger(res.substr(
                1));
    }
    // Overload - operator to subtract
    // first check which number is greater
    // and then subtract
    BigInteger operator-(const BigInteger
        &b) {
        string a = str;
        string c = b.str;
        ll alen = a.length(), clen = c.
            length();
        ll n = max(alen, clen);
        if (alen > clen)
            c.insert(0, alen - clen, '0');
        else if (alen < clen)
            a.insert(0, clen - alen, '0');
        if (a < c) {
            swap(a, c);
            swap(alen, clen);
        }
        string res(n, '0');
        ll carry = 0;
        for (ll i = n - 1; i >= 0; i--) {
            ll digit = (a[i] - '0') - (c
                [i] - '0') - carry;
            if (digit < 0)
                digit += 10, carry = 1;
            else
                carry = 0;
            res[i] = digit + '0';
        }
        // remove leading zeros
        ll i = 0;
        while (i < n && res[i] == '0')
            i++;
        if (i == n)
            return BigInteger("0");
        return BigInteger(res.substr(i));
    }
    // Overload * operator to multiply
    // two BigInteger objects
    BigInteger operator*(const BigInteger
        &b) {
        string a = str, c = b.str;
        ll alen = a.length(), clen = c.

```

```

    ↪ length();
    ll n = alen + clen;
    string res(n, '0');
    for (ll i = alen - 1; i >= 0; i--) {
        ll carry = 0;
        for (ll j = clen - 1; j >=
            ↪ 0; j--) {
            ll digit =
                (a[i] - '0') * (c[j -
                    ↪ '0']) + (res[i +
                        ↪ j + 1] - '0') +
                    ↪ carry;
            carry = digit / 10;
            res[i + j + 1] = digit %
                ↪ 10 + '0';
        }
        res[i] += carry;
    }
    ll i = 0;
    while (i < n && res[i] == '0')
        i++;
    if (i == n)
        return BigInteger("0");
    return BigInteger(res.substr(i));
}

// Overload << operator to output
// BigInteger object
friend ostream &operator<<(ostream &
    ↪ out, const BigInteger &b) {
    out << b.str();
    return out;
}
};

```

## 2.4 Kadane

```

// return maximum subarray sum.
ll kadense(ll arr[], ll n) {
    ll mxsm = arr[0], curr_s = arr[0];
    for (ll i = 1; i < n; i++) {
        curr_s = max(arr[i], curr_s + arr[i]
            ↪ );
        mxsm = max(mxsm, curr_s);
    }
    return mxsm;
}

```

## 2.5 Segment tree

```

class SEGMENT_TREE {
public:
    vector<ll> v;
    vector<ll> seg;
    SEGMENT_TREE(ll n) {
        v.resize(n + 5);
        seg.resize(4 * n + 5);
    }
    //!! initially: ti = 1, low = 1, high =
        ↪ n (number of elements in the
        ↪ array);
    void build(ll ti, ll low, ll high) {
        if (low == high) {
            seg[ti] = v[low];
            return;
        }
        ll mid = (low + high) / 2;
        build(2 * ti, low, mid);
        build(2 * ti + 1, mid + 1, high);
        seg[ti] = (seg[2 * ti] + seg[2 * ti
            ↪ + 1]);
    }
    //!! initially: ti = 1, low = 1, high =
        ↪ n (number of elements in the
        ↪ array), (ql & qr)=user input in
        ↪ 1 based index;
    ll find(ll ti, ll tl, ll tr, ll ql, ll
        ↪ qr) {
        if (tl > qr || tr < ql) {
            return 0;
        }
        if (tl >= ql and tr <= qr)
            return seg[ti];
        ll mid = (tl + tr) / 2;

```

```

        ll l = find(2 * ti, tl, mid, ql, qr)
            ↪ ;
        ll r = find(2 * ti + 1, mid + 1, tr,
            ↪ ql, qr);
        return (l + r);
    }
    //! initially: ti = 1, tl = 1, tr = n
        ↪ (number of elements in the array
        ↪ ), id = user input in 1 based
        ↪ indexing, val = updated value;
    void update(ll ti, ll tl, ll tr, ll id
        ↪ , ll val) {
        if (id > tr or id < tl)
            return;
        if (id == tr and id == tl) {
            seg[ti] = val;
            return;
        }
        ll mid = (tl + tr) / 2;
        update(2 * ti, tl, mid, id, val);
        update(2 * ti + 1, mid + 1, tr, id,
            ↪ val);
        seg[ti] = (seg[2 * ti] + seg[2 * ti
            ↪ + 1]);
    }
};
// use 1 based indexing;

```

## 2.6 Fenwick tree

```

struct FenwickTree {
    vector<ll> bit; // binary indexed tree
    ll n;
    FenwickTree(ll n) {
        this->n = n;
        bit.assign(n, 0);
    }
    FenwickTree(vector<ll> a) :
        ↪ FenwickTree(a.size()) {
        for (size_t i = 0; i < a.size(); i
            ↪ ++i)
            add(i, a[i]);
    }
    ll sum(ll r) {
        ll ret = 0;
        for (; r >= 0; r = (r & (r + 1)) -
            ↪ 1)
            ret += bit[r];
        return ret;
    }
    ll sum(ll l, ll r) { return sum(r) -
        ↪ sum(l - 1); }
    void add(ll idx, ll delta) {
        for (; idx < n; idx = idx | (idx +
            ↪ 1))
            bit[idx] += delta;
    }
};

// minimum
struct FenwickTreeMin {
    vector<ll> bit;
    ll n;
    const ll INF = (ll)1e9;
    FenwickTreeMin(ll n) {
        this->n = n;
        bit.assign(n, INF);
    }
    FenwickTreeMin(vector<ll> a) :
        ↪ FenwickTreeMin(a.size()) {
        for (size_t i = 0; i < a.size(); i
            ↪ ++i)
            update(i, a[i]);
    }
    ll getmin(ll r) {
        ll ret = INF;
        for (; r >= 0; r = (r & (r + 1)) -
            ↪ 1)
            ret = min(ret, bit[r]);
        return ret;
    }
    void update(ll idx, ll val) {
        for (; idx < n; idx = idx | (idx +
            ↪ 1))
            bit[idx] = min(bit[idx], val);
    }
};

```

```

    }
};

```

## 2.7 Segment\_tree\_lazy

```

class SEGMENT_TREE {
public:
    vector<ll> v;
    vector<ll> seg;
    vector<ll> lazy;
    SEGMENT_TREE(ll n) {
        v.resize(n + 5, 0);
        seg.resize(4 * n + 5, 0);
        lazy.resize(4 * n + 5, 0);
    }
    void pull(ll ti) { seg[ti] = (seg[2 *
        ↪ ti] & seg[2 * ti + 1]); }
    void push(ll ti, ll tl, ll tr) {
        if (lazy[ti] == 0)
            return;
        seg[ti] |= lazy[ti];
        if (tl != tr) {
            lazy[2 * ti] |= lazy[ti];
            lazy[2 * ti + 1] |= lazy[ti];
        }
        lazy[ti] = 0;
    }
    ///! llially: ti = 1, low = 1, high = n
    ↪ (number of elements in the array
    ↪ );
    void build(ll ti, ll low, ll high) {
        lazy[ti] = 0;
        if (low == high) {
            seg[ti] = v[low];
            return;
        }
        ll mid = (low + high) / 2;
        build(2 * ti, low, mid);
        build(2 * ti + 1, mid + 1, high);
        pull(ti);
    }
    ///! llially: ti = 1, low = 1, high = n
    ↪ (number of elements in the array
    ↪ ), (ql
    ///! & qr) = user input in 1 based
    ↪ indexing;
    ll query(ll ti, ll tl, ll tr, ll ql,
        ↪ ll qr) {
        push(ti, tl, tr);
        if (tl > qr || tr < ql) {
            return (1LL << 32) - 1;
        }
        if (tl >= ql and tr <= qr)
            return seg[ti];
        ll mid = (tl + tr) / 2;
        ll l = query(2 * ti, tl, mid, ql, qr
            ↪ );
        ll r = query(2 * ti + 1, mid + 1, tr
            ↪ , ql, qr);
        return (l & r);
    }
    ///! llially: ti = 1, tl = 1, tr = n(
    ↪ number of elements in the array)
    ↪ , id =
    ///! user input in 1 based indexing,
    ↪ val = updated value;
    void update(ll ti, ll tl, ll tr, ll
        ↪ idL, ll idR, ll val) {
        push(ti, tl, tr);
        if (idR < tl or tr < idL)
            return;
        if (idL <= tl and tr <= idR) {
            lazy[ti] |= val;
            push(ti, tl, tr);
            return;
        }
        ll mid = (tl + tr) / 2;
        update(2 * ti, tl, mid, idL, idR,
            ↪ val);
        update(2 * ti + 1, mid + 1, tr, idL,
            ↪ idR, val);
        pull(ti);
    }
}

```

```

// use 1 based indexing for input and
↪ queries and update;
};

```

## 2.8 Trie

```

const ll N = 26;
class Node {
public:
    ll EoW;
    Node *child[N];
    Node() {
        EoW = 0;
        for (ll i = 0; i < N; i++)
            child[i] = NULL;
    }
};

void insert(Node *node, string s) {
    for (size_t i = 0; i < s.size(); i++)
        ↪ {
            ll r = s[i] - 'A';
            if (node->child[r] == NULL)
                node->child[r] = new Node();
            node = node->child[r];
        }
    node->EoW += 1;
}

ll search(Node *node, string s) {
    for (size_t i = 0; i < s.size(); i++)
        ↪ {
            ll r = s[i] - 'A';
            if (node->child[r] == NULL)
                return 0;
        }
    return node->EoW;
}

void prll(Node *node, string s = "") {
    if (node->EoW)
        cout << s << "\n";
    for (ll i = 0; i < N; i++) {
        if (node->child[i] != NULL) {
            char c = i + 'A';
            prll(node->child[i], s + c);
        }
    }
}

bool isChild(Node *node) {
    for (ll i = 0; i < N; i++)
        if (node->child[i] != NULL)
            return true;
    return false;
}

bool isJunc(Node *node) {
    ll cnt = 0;
    for (ll i = 0; i < N; i++) {
        if (node->child[i] != NULL)
            cnt++;
    }
    if (cnt > 1)
        return true;
    return false;
}

ll trie_delete(Node *node, string s, ll
    ↪ k = 0) {
    if (node == NULL)
        return 0;
    if (k == (ll)s.size()) {
        if (node->EoW == 0)
            return 0;
        if (isChild(node)) {
            node->EoW = 0;
            return 0;
        }
        return 1;
    }
    ll r = s[k] - 'A';
    ll d = trie_delete(node->child[r], s,
        ↪ k + 1);
    ll j = isJunc(node);
    if (d)
        delete node->child[r];
    if (j)

```

```

    return 0;
    return d;
}
void delete_trie(Node *node) {
    for (ll i = 0; i < 15; i++) {
        if (node->child[i] != NULL)
            delete_trie(node->child[i]);
    }
    delete node;
}

```

## 2.9 DSU

```

class DisjollSet {
    vector<ll> par, sz, minElmt, maxElmt,
        ↪ cntElmt;

public:
    DisjollSet(ll n) {
        par.resize(n + 1);
        sz.resize(n + 1, 1);
        minElmt.resize(n + 1);
        maxElmt.resize(n + 1);
        cntElmt.resize(n + 1, 1);
        for (ll i = 1; i <= n; i++)
            par[i] = minElmt[i] = maxElmt[i] =
                ↪ i;
    }
    ll findUPar(ll u) {
        if (u == par[u])
            return u;
        return par[u] = findUPar(par[u]);
    }
    void unionBySize(ll u, ll v) {
        ll pU = findUPar(u);
        ll pV = findUPar(v);
        if (pU == pV)
            return;
        if (sz[pU] < sz[pV])
            swap(pU, pV);
        par[pV] = pU;
        sz[pU] += sz[pV];
        cntElmt[pU] += cntElmt[pV];
        minElmt[pU] = min(minElmt[pU],
            ↪ minElmt[pV]);
        maxElmt[pU] = max(maxElmt[pU],
            ↪ maxElmt[pV]);
    }
    ll getMinElementIntheSet(ll u) {
        ↪ return minElmt[findUPar(u)];
    }
    ll getMaxElementIntheSet(ll u) {
        ↪ return maxElmt[findUPar(u)];
    }
    ll getNumofElementIntheSet(ll u) {
        ↪ return cntElmt[findUPar(u)];
    }
};

```

## 2.10 HLD

```

ll par[N], sub_tree_sz[N], heavy[N],
    ↪ wt_from_parent[N], depth[N], head[
    ↪ N],
    position[N];
vector<pair<ll, ll>> gd[N];

// HLD part start
ll dfs(ll node, ll p) {
    par[node] = p;
    sub_tree_sz[node] = 1;
    heavy[node] = -1;

    for (auto [v, w] : gd[node]) {
        if (v == p)
            continue;
        depth[v] = depth[node] + 1;
        wt_from_parent[v] = w;
        sub_tree_sz[node] += dfs(v, node);
        if (heavy[node] == -1 || sub_tree_sz
            ↪ [v] > sub_tree_sz[heavy[node]
            ↪ ]) {
            heavy[node] = v;
        }
    }
    return sub_tree_sz[node];
}
ll pos;

```

```

void decompose(ll node, ll hd) {
    head[node] = hd;
    position[node] = ++pos;
    if (heavy[node] != -1) {
        decompose(heavy[node], hd);
    }
    for (auto [v, w] : gd[node]) {
        if (v != par[node] && v != heavy[
            ↪ node]) {
            decompose(v, v);
        }
    }
}

// HLD part end
// in main function
ll n, m;
cin >> n;
SEGMENT_TREE seg(n); // Lazy if needed
vector<ll> edge_u(n), edge_v(n),
    ↪ edge_node(n);

for (int i = 1; i < n; i++) {
    ll u, v, wt = 1;
    cin >> u >> v >> wt;
    gd[u].push_back({v, wt});
    gd[v].push_back({u, wt});
    edge_u[i] = u;
    edge_v[i] = v;
}

dfs(1, -1);
pos = 0;
decompose(1, 1);

for (int i = 1; i <= n; i++) {
    // seg.v[position[i]] = val[i]; //
    ↪ for node value
    seg.v[position[i]] = wt_from_parent[i]
    ↪ ; // for edge value
}

// work on a specific edge
for (int i = 1; i < n; i++) {
    ll u = edge_u[i], v = edge_v[i];
    edge_node[i] = (depth[u] > depth[v]) ?
    ↪ u : v;
}

seg.build(1, 1, n);
auto updatePath = [&](ll u, ll v, ll x)
    ↪ {
    while (head[u] != head[v]) {
        if (depth[head[u]] < depth[head[v]])
            swap(u, v);
        seg.update(1, 1, n, position[head[u]
            ↪ ], position[u], x);
        u = par[head[u]];
    }
    if (depth[u] > depth[v])
        swap(u, v);
    // edge value
    if (u != v) {
        seg.update(1, 1, n, position[u] + 1,
            ↪ position[v], x);
    }
    // node value
    // seg.update(1, 1, n, position[u],
    ↪ position[v], x);
};

auto queryPath = [&](ll u, ll v) {
    ll ans = -inf;
    while (head[u] != head[v]) {
        if (depth[head[u]] < depth[head[v]])
            swap(u, v);
        ans = max(ans, seg.query(1, 1, n,
            ↪ position[head[u]], position[u]
            ↪ ));
        u = par[head[u]];
    }
    if (depth[u] > depth[v])
        swap(u, v);
    // upward + downward
    if (u != v) {
        ans = max(ans, seg.query(1, 1, n,

```

```

    ↪ position[u] + 1, position[v]))
    ↪ ;
}
// only upward
// ans = max(ans, seg.query(1, 1, n,
    ↪ position[u], position[v])); // for
    ↪ node value
return ans;
};
seg.update(1, 1, n, position[edge_node[s]
    ↪ ], position[edge_node[s]], x); //
    ↪ single point update. if path
    ↪ update need call update path
cout << querypath(x, s) << '\n';

```

## 2.11 Manacher

```

struct Manacher {
    vector<ll> p[2];
    string s;
    // p[1][i] = (max odd length
        ↪ palindrome centered at i) / 2 [
        ↪ floor division]
    // p[0][i] = same for even, it
        ↪ considers the right center
    // e.g. for s = "abbabba", p[1][3] =
        ↪ 3, p[0][2] = 2
    Manacher(string s) {
        this->s = s;
        ll n = s.size();
        p[0].resize(n + 1);
        p[1].resize(n);
        for (ll z = 0; z < 2; z++) {
            for (ll i = 0, l = 0, r = 0; i < n
                ↪ ; i++) {
                ll t = r - i + !z;
                if (i < r)
                    p[z][i] = min(t, p[z][l + t]);
                ll L = i - p[z][i], R = i + p[z]
                    ↪ ][i] - !z;
                while (L >= 1 && R + 1 < n && s[
                    ↪ L - 1] == s[R + 1])
                    p[z][i]++, L--, R++;
                if (R > r)
                    l = L, r = R;
            }
        }
        bool is_palindrome(ll l, ll r) {
            ll mid = (l + r + 1) / 2, len = r -
                ↪ l + 1;
            return 2 * p[len % 2][mid] + len % 2
                ↪ >= len;
        }
        string get_palin(ll i, bool odd = true
            ↪ ) {
            ll len = p[odd][i];
            return s.substr(i - len, 2 * len + 1
                ↪ - !odd);
        }
    };
};

```

## 2.12 2D prefix Sum

```

pref[i][j] = a[i][j] + pref[i - 1][j] +
    ↪ pref[i][j - 1] - pref[i - 1][j -
    ↪ 1];
Sum of region = pref[row2 + 1][col2 + 1]
    ↪ - pref[row2 + 1][col1] - pref[
    ↪ row1][col2 + 1] + pref[row1][col1
    ↪ ];

```

## 2.13 CRT

```

class CRT {
    typedef long long vlong;
    typedef pair<vlong, vlong> pll;
    vector<pll> equations;
public:
    void clear() { equations.clear(); }
    vlong extended_euclid(vlong a, vlong b
        ↪ , vlong &x, vlong &y) {

```

```

        if (b == 0) {
            x = 1;
            y = 0;
            return a;
        }
        vlong x1, y1;
        vlong d = extended_euclid(b, a % b,
            ↪ x1, y1);
        x = y1;
        y = x1 - y1 * (a / b);
        return d;
    }
    vlong inverse(vlong a, vlong m) {
        vlong x, y;
        vlong g = extended_euclid(a, m, x, y
            ↪ );
        if (g != 1)
            return -1;
        return (x % m + m) % m;
    }
    /** Add equation of the form x = r (
        ↪ mod m) */
    void addEquation(vlong r, vlong m) {
        ↪ equations.push_back({r, m}); }
    pll solve() {
        if (equations.size() == 0)
            return {-1, -1};
        vlong a1 = equations[0].first;
        vlong m1 = equations[0].second;
        a1 %= m1;
        for (int i = 1; i < equations.size()
            ↪ ; i++) {
            vlong a2 = equations[i].first;
            vlong m2 = equations[i].second;

            vlong g = __gcd(m1, m2);
            if (a1 % g != a2 % g)
                return {-1, -1};
            vlong p, q;
            extended_euclid(m1 / g, m2 / g, p,
                ↪ q);

            vlong mod = m1 / g * m2;
            vlong x = ((__int128)a1 * (m2 / g)
                ↪ % mod * q % mod +
                ↪ ((__int128)a2 * (m1 / g)
                ↪ % mod * p % mod)
                ↪ % mod;
            a1 = x;
            if (a1 < 0)
                a1 += mod;
            m1 = mod;
        }
        return {a1, m1};
    }
};

```

## 2.14 Intersect two arithmetic progression

```

using T = __int128;
// ax + by = __gcd(a, b)
// returns __gcd(a, b)
T extended_euclid(T a, T b, T &x, T &y)
    ↪ {
    T xx = y = 0;
    T yy = x = 1;
    while (b) {
        T q = a / b;
        T t = b;
        b = a % b;
        a = t;
        t = xx;
        xx = x - q * xx;
        x = t;
        t = yy;
        yy = y - q * yy;
        y = t;
    }
    return a;
}
pair<T, T> CRT(T a1, T m1, T a2, T m2) {
    T p, q;
    T g = extended_euclid(m1, m2, p, q);

```



```

if (a1 % g != a2 % g)
    return make_pair(0, -1);
T m = m1 / g * m2;
p = (p % m + m) % m;
q = (q % m + m) % m;
return make_pair((p * a2 % m * (m1 / g
    ↪ ) % m + q * a1 % m * (m2 / g) %
    ↪ m) % m, m);
}
// intersecting AP of two APs: (a1 + d1x
    ↪ ) and (a2 + d2x)
pair<ll, ll> intersect(ll a1, ll d1, ll
    ↪ a2, ll d2) {
    auto x = CRT(a1 % d1, d1, a2 % d2, d2)
        ↪ ;
    ll a = x.first, d = x.second;
    if (d == -1)
        return {0, 0}; // empty
    ll st = max(a1, a2);
    a = a < st ? a + ((st - a + d - 1) / d
        ↪ ) : a; // while (a < st) a += d;
    return {a, d};
}

```

## 2.15 Find nth value in a recurrence relation in O(logn)

```

[ 1, 1; 1, 0 ] ^ (n - 1) =
[F(n), F(n - 1); F(n - 1), F(n - 2)]
// Function to multiply two 2x2
    ↪ matrices
void multiply(vector<vector<int>> &
    ↪ mat1, vector<vector<int>> &
    ↪ mat2) {
    // Perform matrix multiplication
    int x = mat1[0][0] * mat2[0][0] +
        ↪ mat1[0][1] * mat2[1][0];
    int y = mat1[0][0] * mat2[0][1] +
        ↪ mat1[0][1] * mat2[1][1];
    int z = mat1[1][0] * mat2[0][0] +
        ↪ mat1[1][1] * mat2[1][0];
    int w = mat1[1][0] * mat2[0][1] +
        ↪ mat1[1][1] * mat2[1][1];

    // Update matrix mat1 with the
        ↪ result
    mat1[0][0] = x;
    mat1[0][1] = y;
    mat1[1][0] = z;
    mat1[1][1] = w;
}

// Function to perform matrix
    ↪ exponentiation
void matrixPower(vector<vector<int>> &
    ↪ mat1, int n) {
    // Base case for recursion
    if (n == 0 || n == 1)
        return;
    // Initialize a helper matrix
    vector<vector<int>> mat2 = {{1, 1},
        ↪ {1, 0}};

    // Recursively calculate mat1^(n/2)
    matrixPower(mat1, n / 2);

    // Square the matrix mat1
    multiply(mat1, mat1);

    // If n is odd, multiply by the helper
        ↪ matrix mat2
    if (n % 2 != 0) {
        multiply(mat1, mat2);
    }
}

// Function to calculate the nth
    ↪ Fibonacci number
// using matrix exponentiation
int nthFibonacci(int n) {
    if (n <= 1)
        return n;

    // Initialize the transformation
        ↪ matrix
    vector<vector<int>> mat1 = {{1, 1},

```

```

    ↪ {1, 0}};

    // Raise the matrix mat1 to the power
        ↪ of (n - 1)
    matrixPower(mat1, n - 1);

    // The result is in the top-left cell
        ↪ of the matrix
    return mat1[0][0];
}

```

## 2.16 All solution of ax+by=equal\_c

```

// a*x+b*y=c. returns valid x and y if
    ↪ possible.
// all solutions are of the form (x0 + k
    ↪ * b / g, y0 - k * b / g)
bool find_any_solution(ll a, ll b, ll c,
    ↪ ll &x0, ll &y0, ll &g) {
    if (a == 0 and b == 0) {
        if (c)
            return false;
        x0 = y0 = g = 0;
        return true;
    }
    g = extended_euclid(abs(a), abs(b), x0
        ↪ , y0);
    if (c % g != 0)
        return false;
    x0 *= c / g;
    y0 *= c / g;
    if (a < 0)
        x0 *= -1;
    if (b < 0)
        y0 *= -1;
    return true;
}

void shift_solution(ll &x, ll &y, ll a,
    ↪ ll b, ll cnt) {
    x += cnt * b;
    y -= cnt * a;
}

// returns the number of solutions where
    ↪ x is in the range[minx, maxx] and
    ↪ y is
// in the range[miny, maxy]
ll find_all_solutions(ll a, ll b, ll c,
    ↪ ll minx, ll maxx, ll miny, ll maxy
    ↪ ) {
    ll x, y, g;
    if (find_any_solution(a, b, c, x, y, g
        ↪ ) == 0)
        return 0;
    if (a == 0 and b == 0) {
        assert(c == 0);
        return 1LL * (maxx - minx + 1) * (
            ↪ maxy - miny + 1);
    }
    if (a == 0) {
        return (maxx - minx + 1) * (miny <=
            ↪ c / b and c / b <= maxy);
    }
    if (b == 0) {
        return (maxy - miny + 1) * (minx <=
            ↪ c / a and c / a <= maxx);
    }
    a /= g, b /= g;
    ll sign_a = a > 0 ? +1 : -1;
    ll sign_b = b > 0 ? +1 : -1;
    shift_solution(x, y, a, b, (minx - x)
        ↪ / b);
    if (x < minx)
        shift_solution(x, y, a, b, sign_b);
    if (x > maxx)
        return 0;
    ll lx1 = x;
    shift_solution(x, y, a, b, (maxx - x)
        ↪ / b);
    if (x > maxx)
        shift_solution(x, y, a, b, -sign_b);
    ll rx1 = x;
    shift_solution(x, y, a, b, -(miny - y)
        ↪ / a);
    if (y < miny)
        shift_solution(x, y, a, b, -sign_a);

```



```

if (y > maxy)
    return 0;
ll lx2 = x;
shift_solution(x, y, a, b, -(maxy - y)
    ↪ / a);
if (y > maxy)
    shift_solution(x, y, a, b, sign_a);
ll rx2 = x;
if (lx2 > rx2)
    swap(lx2, rx2);
ll lx = max(lx1, lx2);
ll rx = min(rx1, rx2);
if (lx > rx)
    return 0;
return (rx - lx) / abs(b) + 1;
}

int32_t main() {
    ios_base::sync_with_stdio(0);
    cin.tie(0);
    int t, cs = 0;
    cin >> t;
    while (t--) {
        ll a, b, c, x1, x2, y1, y2;
        cin >> a >> b >> c >> x1 >> x2 >> y1
            ↪ >> y2;
        cout << "Case " << ++cs << ": "
            << find_all_solutions(a, b, -c,
            ↪ x1, x2, y1, y2) << '\n';
    }
    return 0;
}

```

## 2.17 all soln of linear eq

```

struct Combi {
    int n;
    vector<ll> facts, finvs, invs;
    Combi(int n) : n(n), facts(n),
        ↪ finvs(n), invs(n) {
        facts[0] = finvs[0] = 1;
        invs[1] = 1;
        for (int i = 2; i < n; i++)
            invs[i] = invs[mod % i] * (-mod /
            ↪ i);
        for (int i = 1; i < n; i++) {
            facts[i] = facts[i - 1] * i;
            finvs[i] = finvs[i - 1] * invs[i];
        }
        inline ll fact(int n) { return facts[n]
            ↪ ; }
        inline ll finv(int n) { return finvs[n]
            ↪ ; }
        inline ll inv(int n) { return invs[n];
            ↪ }
        inline ll ncr(int n, int k) {
            return n < k ? 0 : facts[n] * finvs[
            ↪ k] * finvs[n - k];
        }
    };
    Combi C(N);
    // returns the number of solutions to
    ↪ the equation
    // x_1 + x_2 + ... + x_n = s and 0 ≤ x_i ≤ r
    ↪ ≤ x_i ≤ r
    ll yo(int n, int s, int l, int r) {
        if (s < l * n)
            return 0;
        s -= l * n;
        r -= l;
        ll ans = 0;
        for (int k = 0; k ≤ n; k++) {
            ll cur = C.ncr(s - k - k * r + n - 1
            ↪ + 1, n - 1 + 1) * C.ncr(n, k)
            ↪ ;
            if (k & 1)
                ans -= cur;
            else
                ans += cur;
        }
        return ans;
    }
}

int32_t main() {
    ios_base::sync_with_stdio(0);

```

```

cin.tie(0);
cout << yo(3, 3, 0, 1) << '\n';
return 0;
}

```

## 2.18 Subset sum sqrt(n)

```

// Sum of elements ≤ N implies that
    ↪ every element is ≤ N
vector<int> freq(N + 1, 0);
for (int i = 0; i < N; i++) {
    int x;
    cin >> x;
    freq[x]++;
}

vector<pair<int, int>> compressed;
for (int i = 1; i ≤ N; i++) {
    if (freq[i] > 0)
        compressed.emplace_back(i, freq[i]);
}

vector<int> dp(N + 1, 0);
dp[0] = 1;
for (const auto &[w, k] : compressed) {
    vector<int> ndp = dp;
    for (int p = 0; p < w; p++) {
        int sum = 0;
        for (int multiple = p, count = 0;
            ↪ multiple ≤ N; multiple += w,
            ↪ count++) {
            if (count > k) {
                sum -= dp[multiple - w * count];
                count--;
            }
            if (sum > 0)
                ndp[multiple] = 1;
            sum += dp[multiple];
        }
        swap(dp, ndp);
    }
    cout << "Possible subset sums are:\n";
    for (int i = 0; i ≤ N; i++) {
        if (dp[i] > 0)
            cout << i << " ";
    }
}

```

## 2.19 small giant ( $a^x = b \pmod m$ , find $x$ , given other)

```

// Returns minimum x for which  $a^x \pmod m$ 
    ↪ =  $b \pmod m$ , a and m are coprime.
int solve(int a, int b, int m) {
    a %= m, b %= m;
    int n = sqrt(m) + 1;

    int an = 1;
    for (int i = 0; i < n; ++i)
        an = (an * 1ll * a) % m;

    unordered_map<int, int> vals;
    for (int q = 0, cur = b; q ≤ n; ++q)
        ↪ {
        vals[cur] = q;
        cur = (cur * 1ll * a) % m;
        }

    for (int p = 1, cur = 1; p ≤ n; ++p)
        ↪ {
        cur = (cur * 1ll * an) % m;
        if (vals.count(cur)) {
            int ans = n * p - vals[cur];
            return ans;
        }
    }
    return -1;
}

// Returns minimum x for which  $a^x \pmod m$ 
    ↪ =  $b \pmod m$ .

```

```

int solve(int a, int b, int m) {
    a %= m, b %= m;
    int k = 1, add = 0, g;
    while ((g = gcd(a, m)) > 1) {
        if (b == k)
            return add;
        if (b % g)
            return -1;
        b /= g, m /= g, ++add;
        k = (k * 1ll * a / g) % m;
    }

    int n = sqrt(m) + 1;
    int an = 1;
    for (int i = 0; i < n; ++i)
        an = (an * 1ll * a) % m;

    unordered_map<int, int> vals;
    for (int q = 0, cur = b; q <= n; ++q)
        ↪ {
            vals[cur] = q;
            cur = (cur * 1ll * a) % m;
        }

    for (int p = 1, cur = k; p <= n; ++p)
        ↪ {
            cur = (cur * 1ll * an) % m;
            if (vals.count(cur)) {
                int ans = n * p - vals[cur] + add;
                return ans;
            }
        }
    return -1;
}

```

## 2.20 Gaussian Elimination

```

class GaussianElimination {
public:
    GaussianElimination(vector<vector<
        ↪ double>> matrix, vector<double>
        ↪ results)
        : matrix(matrix), results(results)
        ↪ , n(matrix.size()) {}

    void solve() {
        fElim();
        bSub();
    }

    vector<vector<double>> matrix;
    vector<double> results, solution;
    ll n;

    void fElim() {
        for (ll i = 0; i < n; ++i) {
            ll maxRow = i;
            for (ll k = i + 1; k < n; ++k)
                if (abs(matrix[k][i]) > abs(
                    ↪ matrix[maxRow][i]))
                    maxRow = k;
            swap(matrix[i], matrix[maxRow]);
            swap(results[i], results[maxRow]);
            for (ll k = i + 1; k < n; ++k) {
                double factor = matrix[k][i] /
                    ↪ matrix[i][i];
                for (ll j = i; j < n; ++j)
                    matrix[k][j] -= factor *
                        ↪ matrix[i][j];
                results[k] -= factor * results[i]
                    ↪ ;
            }
        }
    }

    void bSub() {
        solution.resize(n);
        for (ll i = n - 1; i >= 0; --i) {
            solution[i] = results[i];
            for (ll j = i + 1; j < n; ++j)
                solution[i] -= matrix[i][j] *
                    ↪ solution[j];
            solution[i] /= matrix[i][i];
        }
    }
};

```

## 2.21 Grundy

```

ll calculateGrundy(ll n, vector<ll> &
    ↪ grundy, const vector<ll> &moves) {
    if (grundy[n] != -1)
        return grundy[n];
    unordered_set<ll> s;
    for (ll move : moves) {
        if (n >= move) {
            s.insert(calculateGrundy(n - move,
                ↪ grundy, moves));
        }
    }
    ll g = 0;
    while (s.count(g))
        g++;
    return grundy[n] = g;
}

vector<ll> computeGrundy(ll maxN, const
    ↪ vector<ll> &moves) {
    vector<ll> grundy(maxN + 1, -1);
    grundy[0] = 0;
    for (ll i = 1; i <= maxN; ++i) {
        calculateGrundy(i, grundy, moves);
    }
    return grundy;
}

```

## 3 Dynamic Programming

### 3.1 LCS

```

/*
Fact about LCS:
1. Longest Increasing Substring
To solve this, we just care about when
    ↪ two char equals. Rest of the
    ↪ things should be neglected.
2. Longest Palindromic Subsequence (LPS)
To solve this, we just take a new string
    ↪ which is the reverse of the
    ↪ original string. Then just call
    ↪ the LCS function to find LPS.
3. Minimum insertions to make a string
    ↪ palindrome To solve this, we just
    ↪ basically do string length - LPS.
    ↪ Why this?
    Let's take an example: string s =
        ↪ aabca; Let's say aca is our
        ↪ LPS. Now we find how many char
        ↪ we need to insert to make the
        ↪ string palindrome while our
        ↪ LPS is fixed.
        a ab c a now to make the string
            ↪ palindrome we just need to
            ↪ insert the reverse of ab after
            ↪ c. So the new string looks
            ↪ like a ab c ba a
4. Minimum Number of Deletions and
    ↪ Insertions to make the string
    ↪ equals To solve this we just find
    ↪ the LCS of those string then just
    ↪ do: n + m - 2 * LCS.length() where
    ↪ n, m = strings length
*/

```

### 3.2 MCM

```

// TC: O(n ^ 3)
const ll N = 1005;
vector<ll> v;
ll dp[N][N], mark[N][N];
ll MCM(ll i, ll j) {
    if (i == j)
        return dp[i][j] = 0;
    if (dp[i][j] != -1)
        return dp[i][j];
    ll mn = INT_MAX;
    for (ll k = i; k < j; k++) {
        ll x = mn;
        mn = min(mn, MCM(i, k) + MCM(k + 1,
            ↪ j) + v[i - 1] * v[k] * v[j]);
        if (x != mn)
            mark[i][j] = k;
    }
}

```

```

    return dp[i][j] = mn;
}

void print_order(ll i, ll j) {
    if (i == j)
        cout << "X" << i;
    else {
        cout << "(";
        print_order(i, mark[i][j]);
        print_order(mark[i][j] + 1, j);
        cout << ")";
    }
}

// memset(dp, -1, sizeof dp);
// print_order(1, n);

```

### 3.3 LIS length

```

vector<ll> v = {7, 3, 5, 3, 6, 2, 9, 8};
vector<ll> seq;
/*
here we basically check is the current
    ↪ element from v is greater than the
    ↪ last element of the sequence. if
    ↪ it is then push it to the seq
    ↪ array and if not then replace that
    ↪ index value. let's take an
    ↪ example:
v = 7 3 5 3 6 2 9 8
1st iteration seq = 7;
2nd iteration seq = 3;
3rd iteration seq = 3 5;
4th iteration seq = 3 3;
5th iteration seq = 3 3 6;
6th iteration seq = 2 3 6;
7th iteration seq = 2 3 6 9;
8th iteration seq = 2 3 6 8;
*/
for (auto i : v) {
    auto id = lower_bound(seq.begin(), seq
        ↪ .end(), i);
    if (id == seq.end())
        seq.push_back(i);
    else
        seq[id - seq.begin()] = i;
}
cout << seq.size() << endl;

```

### 3.4 LCIS

```

ll a[100] = {0}, b[100] = {0}, f[100] =
    ↪ {0};
ll n = 0, m = 0;
ll main(void) {
    cin >> n;
    for (ll i = 1; i <= n; i++)
        cin >> a[i];
    cin >> m;
    for (ll i = 1; i <= m; i++)
        cin >> b[i];
    for (ll i = 1; i <= n; i++) {
        ll k = 0;
        for (ll j = 1; j <= m; j++) {
            if (a[i] > b[j] && f[j] > k)
                k = f[j];
            else if (a[i] == b[j] && k + 1 > f
                ↪ [j])
                f[j] = k + 1;
        }
    }
    ll and = 0;
    for (ll i = 1; i <= m; i++)
        if (f[i] > ans)
            ans = f[i];
    cout << and << endl;
    return 0;
}

```

### 3.5 SOS DP

```

// sum over subsets
for (int i = 0; i < B; i++) {
    for (int mask = 0; mask < (1 << B);
        ↪ mask++) {

```

```

        if ((mask & (1 << i)) != 0) {
            f[mask] += f[mask ^ (1 << i)];
        }
    }
}

// sum over supersets
for (int i = 0; i < B; i++) {
    for (int mask = (1 << B) - 1; mask >=
        ↪ 0; mask--) {
        if ((mask & (1 << i)) == 0)
            g[mask] += g[mask ^ (1 << i)];
    }
}

// submask
for (int mask = 1; mask < (1 << 5); mask
    ↪ ++){
    for (int submask = mask; submask > 0;
        ↪ submask = (submask - 1) & mask)
        ↪ {
        int subset = mask ^ submask;
    }
}

/**
SOS DP (Sum Over Subsets Dynamic
    ↪ Programming) - 5 Key Points:
1. CORE IDEA: Build DP table dp[i][mask
    ↪ ] = answer considering first i
    ↪ bits of mask Transition: dp[i][
    ↪ mask] = dp[i-1][mask] + dp[i-1][
    ↪ mask ^ (1<<(i-1))]
2. SUBSET SUM: For x|y = x, iterate
    ↪ bits and add contributions from
    ↪ subsets If bit i is set in mask,
    ↪ add dp[i-1][mask without bit i]
3. SUPERSET SUM: For x&y = x, iterate
    ↪ in reverse to handle supersets
    ↪ If bit i is unset in mask, add dp
    ↪ [i-1][mask with bit i set]
**/

```

### 3.6 BS optimization

```

bitset<100005> bs = 1;
for (auto i : a) {
    bs |= (bs << i);
    // if previous 1 value pos is possible
    ↪ now ith bit or ith sm is also
    ↪ possible
}
cout << bs.count() - 1 << endl;
for (ll i = 1; i <= 100003; i++)
    if (bs[i])
        cout << i << " ";
cout << endl;

```

## 4 Graph

### 4.1 Dijkstra

```

// TC: O(V + ElogV)
typedef pair<ll, ll> pairi;
ll N = 20000 + 5;
vector<vector<pairi>> adj(N);
vector<ll> dis(N, inf), parent(N);

void dijkstra(ll src) {
    priority_queue<pairi, vector<pairi>,
        ↪ greater<pairi>> pq;
    dis[src] = 0;
    pq.push({0, src});
    while (pq.size()) {
        auto top = pq.top();
        pq.pop();
        for (auto i : adj[top.second]) {
            ll v = i.first;
            ll wt = i.second;
            if (dis[v] > dis[top.second] + wt)
                ↪ {
                dis[v] = dis[top.second] + wt;
                pq.push({dis[v], v});
                parent[v] = top.second;
            }
        }
    }
}

```

```

    }
}
}
ll node = n;
while (parent[node] != node) {
    path.push_back(node);
    node = parent[node];
}
path.push_back(1);

```

## 4.2 BellmanFord

```

// TC : O(V.E)
vector<ll> dist;
vector<ll> parent;
vector<vector<pair<ll, ll>>> adj;
// resize the vectors from main function
void bellmanFord(ll n, ll src) {
    dist[src] = 0;
    for (ll step = 0; step < n; step++) {
        for (ll i = 1; i <= n; i++) {
            for (auto it : adj[i]) {
                ll u = i;
                ll v = it.first;
                ll wt = it.second;
                if (dist[u] != inf && ((dist[u]
                    ↪ + wt) < dist[v])) {
                    if (step == n - 1) {
                        cout << "Negative cycle
                            ↪ found\n";
                        return;
                    }
                    dist[v] = dist[u] + wt;
                    parent[v] = u;
                }
            }
        }
    }
    for (ll i = 1; i <= n; i++)
        cout << dist[i] << " ";
    cout << endl;
}

```

## 4.3 FloydWarshall

```

// TC : O(n ^ 3)
typedef double T;
typedef vector<T> VT;
typedef vector<VT> VVT;
typedef vector<ll> VI;
typedef vector<VI> VVI;
bool FloydWarshall(VVT &w, VVI &prev) {
    ll n = w.size();
    prev = VVI(n, VI(n, -1));
    for (ll k = 0; k < n; k++) {
        for (ll i = 0; i < n; i++) {
            for (ll j = 0; j < n; j++) {
                if (w[i][j] > w[i][k] + w[k][j])
                    ↪ {
                        w[i][j] = w[i][k] + w[k][j];
                        prev[i][j] = k;
                    }
            }
        }
    }
    // check for negative weight cycles
    for (ll i = 0; i < n; i++)
        if (w[i][i] < 0)
            return false;
    return true;
}

```

## 4.4 Toposort

```

// TC : O(V + E)
map<ll, vector<ll>> adj;
map<ll, ll> degree;
set<ll> nodes;
vector<ll> ans;
// adj: graph input, degree: cnt
    ↪ indegree,
// node: unique nodes, ans: path

```

```

ll c = 0;
void topo_sort() {
    queue<ll> qu;
    // traverse all the nodes and check if
    ↪ its degree is 0 or not..
    for (ll i : nodes) {
        if (degree[i] == 0)
            qu.push(i);
    }
    while (!qu.empty()) {
        ll top = qu.front();
        qu.pop();
        ans.push_back(top);
        for (ll i : adj[top]) {
            degree[i]--;
            if (degree[i] == 0) {
                qu.push(i);
            }
        }
    }
}

```

## 4.5 Kruskal

```

// TC : O(ElogE)
typedef pair<ll, ll> edge;
class Graph {
    vector<pair<ll, edge>> G, T;
    vector<ll> parent;
    ll cost = 0;
public:
    Graph(ll n) {
        for (ll i = 0; i < n; i++)
            parent.push_back(i);
    }
    void add_edges(ll u, ll v, ll wt) { G.
        ↪ push_back({wt, {u, v}}); }
    ll find_set(ll n) {
        if (n == parent[n])
            return n;
        else
            return find_set(parent[n]);
    }
    void union_set(ll u, ll v) { parent[u]
        ↪ = parent[v]; }
    void kruskal() {
        sort(G.begin(), G.end());
        for (auto it : G) {
            ll uRep = find_set(it.second.first
                ↪ );
            ll vRep = find_set(it.second.
                ↪ second);
            if (uRep != vRep) {
                cost += it.first;
                T.push_back(it);
                union_set(uRep, vRep);
            }
        }
    }
    ll get_cost() { return cost; }
    void print() {
        for (auto it : T)
            cout << it.second.first << " " <<
                ↪ it.second.second << ": " <<
                ↪ it.first << endl;
    }
};
// g.add_edges(u, v, wt);
// g.kruskal();

```

## 4.6 Prims

```

// TC: O(ElogV)
typedef pair<ll, ll> pll;
class Prims {
    map<ll, vector<pll>> graph;
    map<ll, ll> visited;
public:
    void addEdge(ll u, ll v, ll w) {
        graph[u].push_back({v, w});
        graph[v].push_back({u, w});
    }

```

```

}
vector<ll> path(pll start) {
    vector<ll> ans;
    priority_queue<pll, vector<pll>,
        greater<pll>> pq;
    // cost vs node
    pq.push({start.second, start.first});
    while (!pq.empty()) {
        pair<ll, ll> curr = pq.top();
        pq.pop();
        if (visited[curr.second])
            continue;
        visited[curr.second] = 1;
        ans.push_back(curr.second);
        for (auto i : graph[curr.second])
            if (visited[i.first])
                continue;
        pq.push({i.second, i.first});
    }
    return ans;
}
};

```

## 4.7 LCA

```

// TC: preprocessing O(nlogn), each
//      query O(logn)
ll n, l;
vector<vector<ll>> adj;
ll timer;
vector<ll> tin, tout;
vector<vector<ll>> up;

void dfs(ll v, ll p) {
    tin[v] = ++timer;
    up[v][0] = p;
    for (ll i = 1; i <= l; ++i)
        up[v][i] = up[up[v][i-1]][i-1];
    for (ll u : adj[v]) {
        if (u != p)
            dfs(u, v);
    }
    tout[v] = ++timer;
}

bool is_ancestor(ll u, ll v) { return
    tin[u] <= tin[v] && tout[u] >=
    tout[v]; }

ll lca(ll u, ll v) {
    if (is_ancestor(u, v))
        return u;
    if (is_ancestor(v, u))
        return v;
    for (ll i = l; i >= 0; --i) {
        if (!is_ancestor(up[u][i], v))
            u = up[u][i];
    }
    return up[u][0];
}

void preprocess(ll root) {
    tin.resize(n);
    tout.resize(n);
    timer = 0;
    l = ceil(log2(n));
    up.assign(n, vector<ll>(l + 1));
    dfs(root, root);
}

```

## 4.8 Rerooting

```

namespace reroot {
    const auto exclusive = [] (const auto &a,
        const auto &base,
        const auto &
        merge_into
        , int
        vertex)
        {
    int n = (int)a.size();

```

```

using Aggregate = decay_t<decltype(
    base)>;
vector<Aggregate> b(n, base);
for (int bit = (int)lg(n); bit >= 0;
    --bit) {
    for (int i = n - 1; i >= 0; --i)
        b[i] = b[i >> 1];
    int sz = n - (n & !bit);
    for (int i = 0; i < sz; ++i) {
        int index = (i >> bit) ^ 1;
        b[index] = merge_into(b[index], a[
            i], vertex, i);
    }
}
return b;
};

// MergeInto : Aggregate * Value *
//              Vertex(int) * EdgeIndex(int) ->
//              Aggregate
// Base : Vertex(int) -> Aggregate
// FinalizeMerge : Aggregate * Vertex(
//              int) * EdgeIndex(int) -> Value
const auto rerooter = [] (const auto &g,
    const auto &base,
    const auto &
    merge_into
    , const
    auto &
    finalize_merge
    ) {
    int n = (int)g.size();
    using Aggregate = decay_t<decltype(
        base(0))>;
    using Value = decay_t<decltype(
        finalize_merge(base(0), 0, 0))>;
    vector<Value> root_dp(n), dp(n);
    vector<vector<Value>> edge_dp(n),
        redge_dp(n);

    vector<int> bfs, parent(n);
    bfs.reserve(n);
    bfs.push_back(0);
    for (int i = 0; i < n; ++i) {
        int u = bfs[i];
        for (auto v : g[u]) {
            if (parent[u] == v)
                continue;
            parent[v] = u;
            bfs.push_back(v);
        }
    }

    for (int i = n - 1; i >= 0; --i) {
        int u = bfs[i];
        int p_edge_index = -1;
        Aggregate aggregate = base(u);
        for (int edge_index = 0; edge_index
            < (int)g[u].size(); ++
            edge_index) {
            int v = g[u][edge_index];
            if (parent[u] == v) {
                p_edge_index = edge_index;
                continue;
            }
            aggregate = merge_into(aggregate,
                dp[v], u, edge_index);
        }
        dp[u] = finalize_merge(aggregate, u,
            p_edge_index);
    }

    for (auto u : bfs) {
        dp[parent[u]] = dp[u];
        edge_dp[u].reserve(g[u].size());
        for (auto v : g[u])
            edge_dp[u].push_back(dp[v]);
        auto dp_exclusive = exclusive(
            edge_dp[u], base(u),
            merge_into, u);
        redge_dp[u].reserve(g[u].size());
        for (int i = 0; i < (int)
            dp_exclusive.size(); ++i)
            redge_dp[u].push_back(
                finalize_merge(dp_exclusive[
                    i], u, i));
    }
}

```

```

    root_dp[u] = finalize_merge(
        n > 1 ? merge_into(dp_exclusive
            ↪ [0], edge_dp[u][0], u, 0)
            ↪ : base(u), u,
        -1);
    for (int i = 0; i < (int)g[u].size()
        ↪ ; ++i) {
        dp[g[u][i]] = redge_dp[u][i];
    }
    return make_tuple(move(root_dp), move(
        ↪ edge_dp), move(redge_dp));
};
} // namespace reroot

int main() {
    ll n;
    cin >> n;
    vector<vector<ll>> g(n);
    // everything should be 0 based.

    using Aggregate = int;
    using Value = int;

    auto base = [] (int vertex) ->
        ↪ Aggregate {
        // task here
    };
    auto merge_into = [] (Aggregate
        ↪ vertex_dp, Value neighbor_dp,
        ↪ int vertex, int edge_index) ->
        ↪ Aggregate {
        // task here
    };
    auto finalize_merge = [] (Aggregate
        ↪ vertex_dp, int vertex, int
        ↪ edge_index) -> Value {
        // task here
    };
    auto [reroot_result, edge_dp, redge_dp
        ↪ ] = reroot::rerooter(g, base,
        ↪ merge_into, finalize_merge);
}

```

## 4.9 Centroid\_Tree

```

const ll n = 1e5;
vector<ll> sz(n + 5), dead(n + 5);
function<void(ll, ll)> calculate_sz =
    ↪ [&] (ll u, ll p) {
    sz[u] = 1;
    for (auto v : adj[u]) {
        if (v != p and !dead[v]) {
            calculate_sz(v, u);
            sz[u] += sz[v];
        }
    }
    return;
};

function<ll(ll, ll, ll)> find_centroid
    ↪ = [&] (ll u, ll p, ll total) ->
    ↪ ll {
    for (auto v : adj[u]) {
        if (v != p and !dead[v] and 2 * sz
            ↪ [v] > total)
            return find_centroid(v, u, total
                ↪ );
    }
    return u;
};

function<void(ll)> decompose = [&] (ll
    ↪ u) -> void {
    // if needed change the parameter
    calculate_sz(u, -1);
    ll center = find_centroid(u, -1, sz[
        ↪ u]);
    // calculate the ans here
    dead[center] = 1;
    for (auto v : adj[center]) {
        if (!dead[v])
            decompose(v);
    }
};

```

```

// call decompose only
decompose(1);

```

## 4.10 Euler\_ckt

```

unordered_map<ll, ll> Start, End, Val;
unordered_map<ll, pair<ll, ll>> Range;
ll start = 0;
void dfs(ll node) {
    visited[node] = true;
    Start[node] = start++;
    for (auto child : adj[node]) {
        if (!visited[child])
            dfs(child);
    }
    End[node] = start - 1;
}
dfs(1);
vector<ll> FlatArray(start + 5);
for (auto i : Start) {
    FlatArray[i.second] = Val[i.first];
    Range[i.first] = {i.second, End[i.
        ↪ first]};
}

```

## 4.11 Min Cost Max Flow

```

#include <bits/stdc++.h>
using namespace std;
const int N = 3e5 + 9;

// Works for both directed, undirected
// ↪ and with negative cost too
// doesn't work for negative cycles
// for undirected edges just make the
// ↪ directed flag false
// Complexity: O(min(E^2 * V log V, E
// ↪ logV * flow))
using T = long long;
const T inf = 1LL << 61;
struct MCMF {
    struct edge {
        int u, v;
        T cap, cost;
        int id;
        edge(int _u, int _v, T _cap, T _cost
            ↪ , int _id) {
            u = _u;
            v = _v;
            cap = _cap;
            cost = _cost;
            id = _id;
        }
    };
    int n, s, t, mxid;
    T flow, cost;
    vector<vector<int>> g;
    vector<edge> e;
    vector<T> d, potential, flow_through;
    vector<int> par;
    bool neg;
    MCMF() {}
    MCMF(int _n) { // 0-based indexing
        n = _n + 10;
        g.assign(n, vector<int>());
        neg = false;
        mxid = 0;
    }
    void add_edge(int u, int v, T cap, T
        ↪ cost, int id = -1,
        ↪ bool directed = true) {
        if (cost < 0)
            neg = true;
        g[u].push_back(e.size());
        e.push_back(edge(u, v, cap, cost, id
            ↪ ));
        g[v].push_back(e.size());
        e.push_back(edge(v, u, 0, -cost, -1)
            ↪ );
        mxid = max(mxid, id);
        if (!directed)
            add_edge(v, u, cap, cost, -1, true
                ↪ );
    }
    bool dijkstra() {

```



```

par.assign(n, -1);
d.assign(n, inf);
priority_queue<pair<T, T>, vector<
    ↪ pair<T, T>>, greater<pair<T, T
    ↪ >>> q;
d[s] = 0;
q.push(pair<T, T>(0, s));
while (!q.empty()) {
    int u = q.top().second;
    T nw = q.top().first;
    q.pop();
    if (nw != d[u])
        continue;
    for (int i = 0; i < (int)g[u].size
        ↪ (); i++) {
        int id = g[u][i];
        int v = e[id].v;
        T cap = e[id].cap;
        T w = e[id].cost + potential[u]
        ↪ - potential[v];
        if (d[u] + w < d[v] && cap > 0)
            {
                d[v] = d[u] + w;
                par[v] = id;
                q.push(pair<T, T>(d[v], v));
            }
    }
}
for (int i = 0; i < n; i++) {
    if (d[i] < inf)
        d[i] += (potential[i] -
        ↪ potential[s]);
}
for (int i = 0; i < n; i++) {
    if (d[i] < inf)
        potential[i] = d[i];
}
return d[t] != inf; // for max flow
    ↪ min cost
// return d[t] <= 0; // for min cost
    ↪ flow
}
T send_flow(int v, T cur) {
    if (par[v] == -1)
        return cur;
    int id = par[v];
    int u = e[id].u;
    T w = e[id].cost;
    T f = send_flow(u, min(cur, e[id].
    ↪ cap));
    cost += f * w;
    e[id].cap -= f;
    e[id ^ 1].cap += f;
    return f;
}
// returns {maxflow, mincost}
pair<T, T> solve(int _s, int _t, T
    ↪ goal = inf) {
    s = _s;
    t = _t;
    flow = 0, cost = 0;
    potential.assign(n, 0);
    if (neg) {
        // Run Bellman-Ford to find
        ↪ starting potential on the
        ↪ starting graph
        // If the starting graph (before
        ↪ pushing flow in the residual
        ↪ graph) is a
        // DAG, then this can be
        ↪ calculated in O(V + E) using
        ↪ DP: potential(v) =
        // min({potential[u] + cost[u][v
        ↪ ]) for each u -> v and
        ↪ potential[s] = 0
        d.assign(n, inf);
        d[s] = 0;
        bool relax = true;
        for (int i = 0; i < n && relax; i
            ↪ ++){
            relax = false;
            for (int u = 0; u < n; u++) {
                for (int k = 0; k < (int)g[u].
                    ↪ size(); k++) {

```

```

        int id = g[u][k];
        int v = e[id].v;
        T cap = e[id].cap, w = e[id]
        ↪ ].cost;
        if (d[v] > d[u] + w && cap >
            ↪ 0) {
            d[v] = d[u] + w;
            relax = true;
        }
    }
}
for (int i = 0; i < n; i++)
    if (d[i] < inf)
        potential[i] = d[i];
while (flow < goal && dijkstra())
    flow += send_flow(t, goal - flow);
flow_through.assign(mxid + 10, 0);
for (int u = 0; u < n; u++) {
    for (auto v : g[u]) {
        if (e[v].id >= 0)
            flow_through[e[v].id] = e[v ^
            ↪ 1].cap;
    }
}
return make_pair(flow, cost);
};
int main() {
    ios_base::sync_with_stdio(0);
    cin.tie(0);
    int n;
    cin >> n;
    assert(n <= 10);
    MCMF F(2 * n);
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < n; j++) {
            int k;
            cin >> k;
            F.add_edge(i, j + n, 1, k, i * 20
            ↪ + j);
        }
    }
    int s = 2 * n + 1, t = s + 1;
    for (int i = 0; i < n; i++) {
        F.add_edge(s, i, 1, 0);
        F.add_edge(i + n, t, 1, 0);
    }
    auto ans = F.solve(s, t).second;
    long long w = 0;
    set<int> se;
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < n; j++) {
            int p = i * 20 + j;
            if (F.flow_through[p] > 0) {
                se.insert(j);
                w += F.flow_through[p];
            }
        }
    }
    assert(se.size() == n && w == n);
    cout << ans << '\n';
    return 0;
}

```

## 4.12 SCC

```

unordered_map<ll, vector<ll>> adj,
    ↪ InvAdj;
stack<ll> order;
unordered_map<ll, bool> visited;
unordered_map<ll, vector<ll>> all_scc;
unordered_map<ll, ll> compId;
void dfs_for_start(ll curr) {
    visited[curr] = 1;
    for (auto i : adj[curr])
        if (!visited[i])
            dfs_for_start(i);
    order.push(curr);
}
vector<ll> curr_comp;
void dfs_for_scc(ll curr) {
    visited[curr] = 1;

```

```

for (auto i : InvAdj[curr])
    if (!visited[i])
        dfs_for_scc(i);
curr_comp.push_back(curr);
}
inline void scc() {
    ll n, e, u, v;
    cin >> n >> e;
    for (ll i = 0; i < e; i++) {
        cin >> u >> v;
        adj[u].push_back(v);
        InvAdj[v].push_back(u);
    }
    for (ll i = 1; i <= n; i++)
        if (!visited[i])
            dfs_for_start(i);
    visited.clear();
    while (!order.empty()) {
        if (!visited[order.top()]) {
            curr_comp.clear();
            dfs_for_scc(order.top());
            ll sz = all_scc.size() + 1;
            all_scc[sz] = curr_comp;
            for (auto i : curr_comp)
                compId[i] = sz;
        }
        order.pop();
    }
}
// no. of ways and min cost of connecting
// the sccs
const ll MOD = 1e9 + 7, N = 1e5 + 2, INF
    = 1e18 + 2;
ll n, m, comp[N];
vector<ll> adj[N], rev[N];
bitset<N> vis;
void DFS1(ll u, stack<ll> &TS) {
    vis[u] = true;
    for (ll v : adj[u])
        if (!vis[v])
            DFS1(v, TS);
    TS.push(u);
}
void DFS2(ll u, const ll scc_no, ll &
    min_cost, ll &ways, vector<ll> &
    cost) {
    vis[u] = true;
    comp[u] = scc_no;
    for (ll v : rev[u])
        if (!vis[v]) {
            if (min_cost == cost[v])
                ++ways;
            else if (min_cost > cost[v]) {
                ways = 1;
                min_cost = cost[v];
            }
            DFS2(v, scc_no, min_cost, ways,
                cost);
        }
}
signed main() {
    FIO cin >> n;
    vector<ll> cost(n + 1);
    for (ll i = 1; i <= n; ++i)
        cin >> cost[i];
    cin >> m;
    while (m--) {
        ll u, v;
        cin >> u >> v;
        adj[u].push_back(v);
        rev[v].push_back(u);
    }
    ll tot = 0, ways = 1;
    stack<ll> TS;
    for (ll i = 1; i <= n; ++i)
        if (!vis[i])
            DFS1(i, TS);
    vis.reset();
    ll scc_no = 0;
    while (!TS.empty()) {
        ll u = TS.top();
        TS.pop();
        if (!vis[u]) {
            ll tmp_cst = cost[u], tmp_ways =

```

```

        1;
        DFS2(u, ++scc_no, tmp_cst,
            tmp_ways, cost);
        tot += tmp_cst;
        ways = (ways * tmp_ways) % MOD;
    }
    cout << tot << ' ' << ways;
} // TC: O(V+E)

```

#### 4.13 0-1 BFS

```

vector<ll> d(n, INF);
d[s] = 0;
deque<ll> q;
q.push_front(s);
while (!q.empty()) {
    ll v = q.front();
    q.pop_front();
    for (auto edge : adj[v]) {
        ll u = edge.first;
        ll w = edge.second;
        if (d[v] + w < d[u]) {
            d[u] = d[v] + w;
            if (w == 1)
                q.push_back(u);
            else
                q.push_front(u);
        }
    }
}

```

#### 4.14 Hull

```

// Convex Hull
#pragma GCC target("avx2")
#pragma GCC optimize("O3")
#pragma GCC optimize("unroll-loops")
#include <bits/stdc++.h>
using namespace std;

typedef long long int ll;
typedef long double ld;
typedef pair<ll, ll> pl;
typedef vector<ll> vl;
typedef complex<ll> pt;

#define G(x)
    x;

    x;
    cin >> x;
#define F(i, l, r) for (ll i = l; i < (r)
    ); ++i)
#define A(a) (a).begin(), (a).end()
#define CRS(a, b) (conj(a) * (b)).Y
#define K first
#define V second
#define X real()
#define Y imag()
#define N 100010

namespace std {
    bool operator<(pt a, pt b) { return a.X
        == b.X ? a.Y < b.Y : a.X < b.X; }
} // namespace std

bool in_hull(pt p, vector<pt> &hu,
    vector<pt> &hd) {
    if (p == *hu.begin() || p == *hd.begin()
        )
        return false; // change to true if
        border counts as inside
    if (p < *hu.begin() || *hd.begin() < p
        )
        return false;
    auto u = upper_bound(A(hu), p);
    auto d = lower_bound(hd.rbegin(), hd.
        rend(), p);
    return CRS(*u - p, *(u - 1) - p) > 0
        && CRS(*(d - 1) - p, *d - p) >
        0;
    // change to >= if border counts as "
    inside"

```

```

}
void do_hull(vector<pt> &pts, vector<pt>
    ↪ &h) {
    for (pt p : pts) {
        while (h.size() > 1 && CRS(h.back()
            ↪ - p, h[h.size() - 2] - p) <=
            ↪ 0)
            // change to < 0 if border points
            ↪ included
            h.pop_back();
        h.push_back(p);
    }
}
pair<vector<pt>, vector<pt>> get_hull(
    ↪ vector<pt> &pts) {
    vector<pt> hu, hd;
    sort(A(pts)), do_hull(pts, hu);
    reverse(A(pts)), do_hull(pts, hd);
    return {hu, hd};
}
vector<pt> full_hull(vector<pt> &pts) {
    auto h = get_hull(pts);
    h.K.pop_back(), h.V.pop_back();
    for (pt p : h.V)
        h.K.push_back(p);
    return h.K;
}
int main() {
    G(n) vector<pt> v;
    F(i, 0, n) { G(x) G(y) v.push_back({x,
        ↪ y}); }
    vector<pt> h = full_hull(v);
}

```

## 4.15 Dynamic Hull

```

// Dynamic Convex Hull
#pragma GCC target("avx2")
#pragma GCC optimize("O3")
#pragma GCC optimize("unroll-loops")
#include <bits/stdc++.h>
using namespace std;

typedef long long int ll;
typedef long double ld;
typedef pair<ll, ll> pl;
typedef vector<ll> vl;
typedef complex<ll> pt;

#define G(x) ll x; cin >> x;
#define F(i, l, r) for (ll i = l; i < (r
    ↪ ); ++i)
#define A(a) (a).begin(), (a).end()
#define CRS(a, b) (conj(a) * (b)).Y
#define X real()
#define Y imag()
#define N 100010

namespace std {
    bool operator<(pt a, pt b) { return a.X
        ↪ == b.X ? a.Y < b.Y : a.X < b.X; }
} // namespace std

// helper function for dyn_in_hull
bool in(pt p, set<pt> &h) {
    if (h.empty() || p < *h.begin() || *h.
        ↪ rbegin() < p)
        return false;
    auto i = h.upper_bound(p), j = i--;
    return CRS(*j - p, *i - p) > 0; //
    ↪ change to >= if border counts as
    ↪ "inside"
}

// returns true if p contained in
    ↪ dynamic hull hu / hd
bool in_hull(pt p, set<pt> &hu, set<pt>
    ↪ &hd) { return in(p, hu) && in(-p,
    ↪ hd); }

// helper function for dyn_add
void fix_bad(set<pt>::iterator i, set<pt>
    ↪ &h, bool l) {
    if (i == --h.begin() || i == h.end())

```

```

        return;
        pt p = *i;
        h.erase(p);
        if (!in(p, h))
            h.insert(p);
        else
            fix_bad(l ? --h.lower_bound(p) : h.
                ↪ upper_bound(p), h, l);
    }

// helper function for dyn_add_to_hull
void add(pt p, set<pt> &h) {
    if (in(p, h))
        return;
    h.insert(p);
    fix_bad(--h.lower_bound(p), h, true);
    fix_bad(h.upper_bound(p), h, false);
}

// adds p to dynamic hull hu / hd
void add_to_hull(pt p, set<pt> &hu, set<
    ↪ pt> &hd) { add(p, hu), add(-p, hd)
    ↪ ; }

int main() {
    G(n) set<pt> hu, hd;
    F(i, 0, n) { G(x) G(y) add_to_hull({x,
        ↪ y}, hu, hd); }
}

```

## 4.16 Count Simple Cycle

```

void findNumberOfSimpleCycles(int N,
    ↪ vector<vector<int>> adj) {
    int ans = 0;
    int dp[(1 << N)][N];
    memset(dp, 0, sizeof dp);
    for (int mask = 0; mask < (1 << N);
        ↪ mask++) {
        int nodeSet = __builtin_popcountll(
            ↪ mask);
        int firstSetBit = __builtin_ffsl(
            ↪ mask);
        if (nodeSet == 1)
            dp[mask][firstSetBit] = 1;
        else {
            for (int j = firstSetBit + 1; j <
                ↪ N; j++) {
                if ((mask & (1 << j))) {
                    int newNodeSet = mask ^ (1 <<
                        ↪ j);
                    for (int k = 0; k < N; k++) {
                        if ((newNodeSet & (1 << k))
                            ↪ && adj[k][j]) {
                            dp[mask][j] += dp[
                                ↪ newNodeSet][k];
                            if (adj[j][firstSetBit] &&
                                ↪ nodeSet > 2)
                                ans += dp[mask][j];
                        }
                    }
                }
            }
        }
    }
    cout << ans << endl;
}

```

## 5 Misc

### 5.1 Max Pos and Next Greater

```

const ll MXX = 1e5 + 5;
ll mxtree[4 * MXX], arr[MXX];
void maxtree(ll idx, ll left, ll right)
    ↪ {
    if (left == right)
        mxtree[idx] = left;
    else {
        ll mid = (left + right) / 2;
        maxtree(idx * 2, left, mid);
        maxtree(idx * 2 + 1, mid + 1, right)
            ↪ ;
        ll left = mxtree[idx * 2];
        ll right = mxtree[idx * 2 + 1];
    }
}

```

```

    if (arr[left] < arr[right])
        mxtree[idx] = right;
    else
        mxtree[idx] = left;
}
}
ll mxPos(ll idx, ll tleft, ll tright, ll
    ↪ qlf, ll qright) {
    if (qlf > qright)
        return -1;
    if (qlf == tleft and qright ==
        ↪ tright)
        return mxtree[idx];
    ll tmid = (tleft + tright) / 2;
    ll left = mxPos(idx * 2, tleft, tmid,
        ↪ qlf, min(qright, tmid));
    ll right = mxPos(idx * 2 + 1, tmid +
        ↪ 1, tright, max(qlf, tmid + 1),
        ↪ qright);
    ll ans;
    if (left == -1)
        ans = right;
    else if (right == -1)
        ans = left;
    else if (arr[left] < arr[right])
        ans = right;
    else
        ans = left;
    return ans;
}
ll main() {
    ll t = 1, n, q, a, b;
    cin >> t;
    while (t--) {
        cin >> n >> q;
        for (ll i = 0; i < n; i++)
            cin >> arr[i];
        stack<ll> stk;
        ll nge[n];
        stk.push(0);
        for (ll i = 1; i < n; i++) {
            while (stk.size() and arr[stk.top()
                ↪ ()] < arr[i]) {
                nge[stk.top()] = i;
                stk.pop();
            }
            stk.push(i);
        }
        while (stk.size()) {
            nge[stk.top()] = n;
            stk.pop();
        }
        ll ans[n];
        ans[n - 1] = 0;
        for (ll i = n - 2; i >= 0; i--) {
            ll tmp = nge[i];
            if (tmp == n)
                ans[i] = 0;
            else
                ans[i] = ans[tmp] + 1;
        }
        mxtree(1, 0, n - 1);
        for (ll i = 0; i < q; i++) {
            cin >> a >> b;
            if (a > b)
                swap(a, b);
            cout << ans[mxPos(1, 0, n - 1, a -
                ↪ 1, b - 1)] << "\n";
        }
    }
}

```

## 5.2 Knight Move

```

ll X[8]={2,1,-1,-2,-2,-1,1,2};
ll Y[8]={1,2,2,1,-1,-2,-2,-1};

```

## 5.3 Matrix Exponentiation

```

typedef long long LL;
LL arr[60][60], res[60][60], tmp
    ↪ [60][60], m;
void matMul(LL a[][60], LL b[][60], LL
    ↪ mod) {

```

```

    for (ll i = 0; i < m; i++)
        for (ll j = 0; j < m; j++) {
            tmp[i][j] = 0;
            for (ll k = 0; k < m; k++) {
                tmp[i][j] += (a[i][k] * b[k][j])
                    ↪ % mod;
                tmp[i][j] %= mod;
            }
        }
}

void power(LL n, LL mod) {
    for (ll i = 0; i < m; i++)
        for (ll j = 0; j < m; j++)
            if (i == j)
                res[i][j] = 1;
            else
                res[i][j] = 0;
    while (n) {
        if (n & 1) {
            matMul(res, arr, mod);
            for (ll i = 0; i < m; i++)
                for (ll j = 0; j < m; j++)
                    res[i][j] = tmp[i][j];
            n--;
        } else {
            matMul(arr, arr, mod);
            for (ll i = 0; i < m; i++)
                for (ll j = 0; j < m; j++)
                    arr[i][j] = tmp[i][j];
            n /= 2;
        }
    }
}

```

## 5.4 Ternary Search

```

double ternary_search(double l, double r
    ↪ ) {
    double eps = 1e-9; // error limit
    while (r - l > eps) {
        double m1 = l + (r - l) / 3, m2 = r
            ↪ - (r - l) / 3;
        double f1 = f(m1), f2 = f(m2); //
            ↪ evaluates the function at m1,
            ↪ m2
        if (f1 < f2)
            l = m1;
        else
            r = m2;
    }
    return f(l); // return the maximum of
        ↪ f(x) in [l, r]
}

```

## 6 Number Theory

### 6.1 Leap\_year

```

bool isLeap(ll n) {
    if (n % 100 == 0)
        return (n % 400 == 0);
    else
        return (n % 4 == 0);
}
// leap year between l and r
ll calNum(ll y) { return (y / 4) - (y /
    ↪ 100) + (y / 400); }
ll leapNum(ll l, ll r) { return calNum(r
    ↪ ) - calNum(--l); }

```

### 6.2 Two Line Intersection

```

ll cross(ll x1, ll y1, ll x2, ll y2, ll
    ↪ x3, ll y3) {
    return (x2 - x1) * (y3 - y1) - (y2 -
        ↪ y1) * (x3 - x1);
}

bool intersect(ll x1, ll y1, ll x2, ll
    ↪ y2, ll x3, ll y3, ll x4, ll y4) {
    ll c1 = cross(x1, y1, x2, y2, x3, y3),
        ↪ c2 = cross(x1, y1, x2, y2, x4,
        ↪ y4),

```

```

c3 = cross(x3, y3, x4, y4, x1, y1),
    ↪ c4 = cross(x3, y3, x4, y4,
    ↪ x2, y2);
if ((!c1 && min(x1, x2) <= x3 && x3 <=
    ↪ max(x1, x2) && min(y1, y2) <=
    ↪ y3 &&
    y3 <= max(y1, y2)) |
    (!c2 && min(x1, x2) <= x4 && x4 <=
    ↪ max(x1, x2) && min(y1, y2)
    ↪ <= y4 &&
    y4 <= max(y1, y2)) |
    (!c3 && min(x3, x4) <= x1 && x1 <=
    ↪ max(x3, x4) && min(y3, y4)
    ↪ <= y1 &&
    y1 <= max(y3, y4)) |
    (!c4 && min(x3, x4) <= x2 && x2 <=
    ↪ max(x3, x4) && min(y3, y4)
    ↪ <= y2 &&
    y2 <= max(y3, y4)))
    return true;
return (c1 > 0) != (c2 > 0) && (c3 >
    ↪ 0) != (c4 > 0);
}

```

### 6.3 Binary\_exponentiation

```

ll binaryExp(ll base, ll power, ll MOD =
    ↪ mod) {
    ll res = 1;
    while (power) {
        if (power & 1)
            res = (res * base) % MOD;
        base = ((base % MOD) * (base % MOD))
            ↪ % MOD;
        power /= 2;
    }
    return res;
}
/*
task: a ^ b ^ c
binaryExp(a, binaryExp(b, c, mod - 1),
    ↪ mod)
*/

```

### 6.4 Count\_divisor

```

ll maxVal = 1e6 + 1;
vector<ll> countDivisor(maxVal, 0);
void countingDivisor() {
    for (ll i = 1; i < maxVal; i++)
        for (ll j = i; j < maxVal; j += i)
            countDivisor[j]++;
}
// TC: nlog(n)
// count the number of divisors of all
    ↪ numbers in a range.

```

### 6.5 Check\_prime

```

bool prime(ll n) {
    if (n < 2)
        return false;
    if (n <= 3)
        return true;
    if (!(n % 2) || !(n % 3))
        return false;
    for (ll i = 5; i * i <= n; i += 6) {
        if (!(n % i) || !(n % (i + 2)))
            return false;
    }
    return true;
}
// TC: sqrt(n) / 6;

```

### 6.6 SPF

```

// smallest prime factor using sieve
const ll N = 1e7 + 5;
ll spf[N];
void smallestPrimeFactorUsingSieve() {
    for (ll i = 2; i < N; i++) {
        if (spf[i] == 0) {

```

```

            for (ll j = i; j < N; j += i) {
                if (spf[j] == 0)
                    spf[j] = i;
            }
        }
    }
}
// smallest factor of a number
ll factor(ll n) {
    ll a;
    if (n % 2 == 0)
        return 2;
    for (a = 3; a * a <= n; a += 2) {
        if (n % a == 0)
            return a;
    }
    return n;
}
// complete factorization
ll r;
while (n > 1) {
    r = factor(n);
    cout << r << '\n';
    n /= r;
}

```

### 6.7 Sieve

```

const ll N = 1e7 + 5;
ll prime[N];
void sieveOfEratosthenes() {
    for (ll i = 2; i < N; i++)
        prime[i] = 1;
    for (ll i = 4; i < N; i += 2)
        prime[i] = 0;
    for (ll i = 3; i * i < N; i++) {
        if (prime[i]) {
            for (ll j = i * i; j < N; j += i *
                ↪ 2)
                prime[j] = 0;
        }
    }
}

```

### 6.8 Optimize\_sieve

```

vector<ll> sieve(const ll N, const ll Q
    ↪ = 17, const ll L = 1 << 15) {
    static const ll rs[] = {1, 7, 11, 13,
        ↪ 17, 19, 23, 29};
    struct P {
        P(ll p) : p(p) {}
        ll p;
        ll pos[8];
    };
    auto approx_prime_count = [](const ll
        ↪ N) -> ll {
        return N > 60184 ? N / (log(N) -
            ↪ 1.1) : max(1., N / (log(N) -
            ↪ 1.11)) + 1;
    };
    const ll v = sqrt(N), vv = sqrt(v);
    vector<bool> isp(v + 1, true);
    for (ll i = 2; i <= vv; ++i)
        if (isp[i]) {
            for (ll j = i * i; j <= v; j += i)
                isp[j] = false;
        }
    const ll rsize = approx_prime_count(N
        ↪ + 30);
    vector<ll> primes = {2, 3, 5};
    ll psize = 3;
    primes.resize(rsize);

    vector<P> sprimes;
    size_t pbeg = 0;
    ll prod = 1;
    for (ll p = 7; p <= v; ++p) {
        if (!isp[p])
            continue;
        if (p <= Q)
            prod *= p, ++pbeg, primes[psize++]
            ↪ = p;
    }
}

```

```

    auto pp = P(p);
    for (ll t = 0; t < 8; ++t) {
        ll j = (p <= Q) ? p : p * p;
        while (j % 30 != rs[t])
            j += p << 1;
        pp.pos[t] = j / 30;
    }
    sprimes.push_back(pp);
}

vector<unsigned char> pre(prod, 0xFF);
for (size_t pi = 0; pi < pbeg; ++pi) {
    auto pp = sprimes[pi];
    const ll p = pp.p;
    for (ll t = 0; t < 8; ++t) {
        const unsigned char m = ~(1 << t);
        for (ll i = pp.pos[t]; i < prod; i
            += p)
            pre[i] &= m;
    }
}
const ll block_size = (L + prod - 1) /
    prod * prod;
vector<unsigned char> block(block_size
    );
unsigned char *pblock = block.data();
const ll M = (N + 29) / 30;
for (ll beg = 0; beg < M; beg +=
    block_size, pblock -= block_size
    ) {
    ll end = min(M, beg + block_size);
    for (ll i = beg; i < end; i += prod)
        copy(pre.begin(), pre.end(),
            pblock + i);
    if (beg == 0)
        pblock[0] &= 0xFE;
    for (size_t pi = pbeg; pi < sprimes.
        size(); ++pi) {
        auto &pp = sprimes[pi];
        const ll p = pp.p;
        for (ll t = 0; t < 8; ++t) {
            ll i = pp.pos[t];
            const unsigned char m = ~(1 << t
                );
            for (; i < end; i += p)
                pblock[i] &= m;
            pp.pos[t] = i;
        }
    }
    for (ll i = beg; i < end; ++i) {
        for (ll m = pblock[i]; m > 0; m &=
            m - 1) {
            primes[psize++] = i * 30 + rs[
                __builtin_ctz(m)];
        }
    }
}
assert(psize <= rsize);
while (psize > 0 && primes[psize - 1]
    > N)
    --psize;
primes.resize(psize);
return primes;
}
// it takes 500ms for generating prime
    upto 1e9

```

## 6.9 nth\_prime\_number

```

vector<ll> nth_prime;
const ll MX = 86200005;
bitset<MX> visited;
void optimized_prime() {
    nth_prime.push_back(2);
    for (ll i = 3; i < MX; i += 2) {
        if (visited[i])
            continue;
        nth_prime.push_back(i);
        if (1ll * i * i > MX)
            continue;
        for (ll j = i * i; j < MX; j += i +

```

```

        i)
            visited[j] = true;
    }
}

```

## 6.10 nCr

```

// 1:
// more space, less time
const ll MAX = 1e7 + 5;
vector<ll> fact(MAX), ifact(MAX), inv(
    MAX);
void factorial() {
    inv[1] = fact[0] = ifact[0] = 1;
    for (ll i = 2; i < MAX; i++)
        inv[i] = inv[mod % i] * (mod - mod /
            i) % mod;
    for (ll i = 1; i < MAX; i++)
        fact[i] = (fact[i - 1] * i) % mod;
    for (ll i = 1; i < MAX; i++)
        ifact[i] = ifact[i - 1] * inv[i] %
            mod;
}
ll nCr(ll n, ll r) {
    if (r < 0 || r > n)
        return 0;
    return (ll)fact[n] * ifact[r] % mod *
        ifact[n - r] % mod;
}
// 2:
// less space, more time
const ll MAX = 1e7 + 10;
vector<ll> fact(MAX), inv(MAX);
void factorial() {
    fact[0] = 1;
    for (ll i = 1; i < MAX; i++)
        fact[i] = (i * fact[i - 1]) % mod;
}
ll binaryExp(ll a, ll n, ll M = mod){};
// needs to implement
void inverse() {
    for (ll i = 0; i < MAX; ++i)
        inv[i] = binaryExp(fact[i], mod - 2)
            ;
}
ll nCr(ll a, ll b) {
    if (a < b or a < 0 or b < 0)
        return 0;
    ll de = (inv[b] * inv[a - b]) % mod;
    return (fact[a] * de) % mod;
}
// 3:
// nCr mod m where m is not prime
ll C_mod_p(ll n, ll k, ll p) {
    if (k > n)
        return 0;
    vector<ll> fac(p);
    fac[0] = 1;
    for (int i = 1; i < p; i++)
        fac[i] = fac[i - 1] * i % p;
    ll res = 1;
    while (n || k) {
        ll ni = n % p, ki = k % p;
        if (ki > ni)
            return 0;
        res = res * fac[ni] % p * modInv(fac
            [ki], p) % p * modInv(fac[ni -
            ki], p) %
            p;
        n /= p;
        k /= p;
    }
    return res;
}
// compute nCr mod composite m (non-
    prime)
ll nCr_mod_m(ll n, ll k, ll m) {
    // Step 1: factorize m
    vector<int> primes;
    int tmp = m;
    for (int i = 2; i * i <= tmp; i++) {
        if (tmp % i == 0) {
            primes.push_back(i);

```



```

    while (tmp % i == 0)
        tmp /= i;
    }
    if (tmp > 1)
        primes.push_back(tmp);
    // Step 2: compute result mod each
    ↪ prime
    vector<ll> rem, mod;
    for (int p : primes) {
        rem.push_back(C_mod_p(n, k, p));
        mod.push_back(p);
    }
    // Step 3: Chinese Remainder Theorem (
    ↪ combine)
    ll res = 0;
    for (int i = 0; i < (int)mod.size(); i
        ↪ ++){
        ll Mi = m / mod[i];
        ll invMi = binaryExp(Mi, mod[i] - 2,
            ↪ mod[i]); // modular inverse
        res = (res + rem[i] * Mi % m * invMi
            ↪ % m) % m;
    }
    return res;
}

```

### 6.11 Factorial\_mod

```

// n! mod p : Here P is mod value
// For binaryExp we call 1.6 function
ll factmod(ll n, ll p) {
    ll res = 1;
    while (n > 1) {
        res = (res * binaryExp(p - 1, n / p,
            ↪ p)) % p;
        for (ll i = 2; i <= n % p; ++i)
            res = (res * i) % p;
        n /= p;
    }
    return (res % p);
}

```

### 6.12 PHI

// the positive integers less than or  
 ↪ equal to n that are relatively  
 ↪ prime to n.

```

ll phi(ll n) {
    ll result = n;
    for (ll i = 2; i * i <= n; i++) {
        if (n % i == 0) {
            while (n % i == 0)
                n /= i;
            result -= result / i;
        }
    }
    if (n > 1)
        result -= result / n;
    return result;
}
// PHI of 1 to N
const int N = 1e6 + 9;
int phi[N];
int phiS[N];
void totient() {
    for (int i = 1; i < N; i++)
        phi[i] = i;
    for (int i = 2; i < N; i++) {
        if (phi[i] == i) {
            for (int j = i; j < N; j += i)
                phi[j] -= phi[j] / i;
        }
    }
    phiS[0] = phi[0];
    for (int i = 1; i < N; i++)
        phiS[i] = phiS[i - 1] + phi[i];
}

```

### 6.13 Catalan

```

void catalan(ll n) {

```

```

    ll res = 1;
    cout << res << " ";
    for (ll i = 1; i < n; i++) {
        res = (res * (4 * i - 2)) / (i + 1);
        cout << res << " ";
    }
}

```

### 6.14 Extended\_GCD

```

// return {x,y} such that ax + by = gcd(
    ↪ a,b)
ll extended_euclid(ll a, ll b, ll &x, ll
    ↪ &y) {
    if (b == 0) {
        x = 1;
        y = 0;
        return a;
    }
    ll x1, y1;
    ll d = extended_euclid(b, a % b, x1,
        ↪ y1);
    x = y1;
    y = x1 - y1 * (a / b);
    return d;
}
ll inverse(ll a, ll m) {
    ll x, y;
    ll g = extended_euclid(a, m, x, y);
    if (g != 1)
        return -1;
    return (x % m + m) % m;
}

```

### 6.15 Large Mod

```

ll mod(string &num, ll a) {
    ll res = 0;
    for (ll i = 0; i < num.length(); i++)
        res = (res * 10 + num[i] - '0') % a;
    return res;
}

```

### 6.16 Factorial\_Divisor

```

ll factorialDivisors(ll n) {
    ll result = 1;
    for (ll i = 0; i < allPrimes.size(); i
        ↪ ++){
        ll p = allPrimes[i];
        ll exp = 0;
        while (p <= n) {
            exp = exp + (n / p);
            p = p * allPrimes[i];
        }
        result = result * (exp + 1);
    }
    return result;
}

```

### 6.17 Number\_conversion

```

// 10 - ary to m - ary
char a[16] = {'0', '1', '2', '3', '4', '5',
    ↪ '6', '7', '8', '9', 'A', 'B',
    ↪ 'C', 'D', 'E', 'F'};
string tenToM(ll n, ll m) {
    ll temp = n;
    string result = "";
    while (temp != 0) {
        result = a[temp % m] + result;
        temp /= m;
    }
    return result;
}
// m - ary to 10 - ary
string num = "0123456789ABCDE";
ll mToTen(string n, ll m) {
    ll multi = 1;
    ll result = 0;
    for (ll i = n.size() - 1; i >= 0; i--)
        ↪ {
            result += num.find(n[i]) * multi;
            multi *= m;
        }
}

```

```

    }
    return result;
}

```

## 6.18 Number\_of\_1\_in\_bit\_till\_N

```

ll cntOnes(ll n) {
    ll cnt = 0;
    for (ll i = 1; i <= n; i <= 1) {
        ll x = (n + 1) / (i <= 1);
        cnt += x * i;
        if ((n + 1) % i && n & i)
            cnt += (n + 1) % i;
    }
    return cnt;
}

```

## 6.19 Disarrangement

```

ll disarrange(ll n) {
    if (n == 1)
        return 0;
    if (n == 2)
        return 1;
    return (n - 1) * (disarrange(n - 1) +
        ↪ disarrange(n - 2));
}
// D(n) = (n!)/e

```

## 6.20 Millar\_Rabin

```

bool check_composite(ll n, ll a, ll d,
    ↪ ll s) {
    ll x = binaryExp(a, d, n);
    if (x == 1 || x == n - 1)
        return false;
    for (ll r = 1; r < s; r++) {
        x = (u128)x * x % n;
        if (x == n - 1)
            return false;
    }
    return true;
};
bool MillerRabin(ll n, ll iter = 5) {
    // returns true if n is probably prime
    ↪ , else returns false.
    if (n < 4)
        return n == 2 || n == 3;
    ll s = 0;
    ll d = n - 1;
    while ((d & 1) == 0) {
        d >>= 1;
        s++;
    }
    for (ll i = 0; i < iter; i++) {
        ll a = 2 + rand() % (n - 3);
        if (check_composite(n, a, d, s))
            return false;
    }
    return true;
}

```

## 6.21 Modular\_operation

```

// Addition :
ll mod_add(ll a, ll b, ll MOD = mod) {
    a = a % MOD, b = b % MOD;
    return ((a + b) % MOD) + MOD) % MOD;
}
// Subtraction :
ll mod_sub(ll a, ll b, ll MOD = mod) {
    a = a % MOD, b = b % MOD;
    return ((a - b) % MOD) + MOD) % MOD;
}
// Multiplication :
ll mod_mul(ll a, ll b, ll MOD = mod) {
    a = a % MOD, b = b % MOD;
    return ((a * b) % MOD) + MOD) % MOD;
}
// Division :
ll mminvprime(ll a, ll b) { return
    ↪ binaryExp(a, b - 2, b); }
ll mod_div(ll a, ll b, ll MOD = mod) {
    a = a % MOD, b = b % MOD;

```

```

    return (mod_mul(a, mminvprime(b, MOD),
        ↪ MOD) + MOD) % MOD;
}

```

## 6.22 MSLCM

```

// For a given number N, maximum sum LCM
    ↪ indicates the set of numbers
    ↪ whose LCM
// is N and summation is maximum. Let,
    ↪ MSLCM(N) denote this maximum sum
    ↪ of
// numbers. Given the value of N you
    ↪ will have to find the value:
    ↪ summation of
// MSLCM(i) from i to 2n
ll MSLCM(ll n) {
    ll l = 1, r, val, ret = 0;
    while (l <= n) {
        val = n / l, r = n / val;
        ret += val * ((l + r) * (r - l + 1)
            ↪ / 2);
        l = r + 1;
    }
    return ret - 1;
}

```

## 6.23 Find numbers in between [L, R] which are divisible by all Array elements

```

void solve(ll *arr, ll N, ll L, ll R) {
    ll LCM = arr[0];
    for (ll i = 1; i < N; i++) {
        LCM = (LCM * arr[i]) / (__gcd(LCM,
            ↪ arr[i]));
    }
    if ((LCM < L && LCM * 2 > R) || LCM >
        ↪ R) {
        return;
    }
    ll k = (L / LCM) * LCM;
    //
    if (k < L)
        k = k + LCM;
    for (ll i = k; i <= R; i = i + LCM)
        cout << i << ' ';
}

```

## 6.24 PollarRho

```

namespace PollardRho {
    mt19937 rnd(chrono::steady_clock::now
        ↪ ().time_since_epoch().count());
    const int P = 1e6 + 9;
    ll seq[P];
    int primes[P], spf[P];
    inline ll add_mod(ll x, ll y, ll m) {
        return (x + y) < m ? x : x - m;
    }
    inline ll mul_mod(ll x, ll y, ll m) {
        ll res = __int128(x) * y % m;
        return res;
        // ll res = x * y - (ll)((long
            ↪ double)x * y / m + 0.5) * m;
        // return res < 0 ? res + m : res;
    }
    inline ll pow_mod(ll x, ll n, ll m) {
        ll res = 1 % m;
        for (; n; n >>= 1) {
            if (n & 1) res = mul_mod(res, x, m
                ↪ );
            x = mul_mod(x, x, m);
        }
        return res;
    }
    // O(it * (logn)^3), it = number of
        ↪ rounds performed
    inline bool miller_rabin(ll n) {
        if (n <= 2 || (n & 1 ^ 1)) return (n
            ↪ == 2);
        if (n < P) return spf[n] == n;
        ll c, d, s = 0, r = n - 1;

```

```

for (; !(r & 1); r >>= 1, s++) {}
// each iteration is a round
for (int i = 0; primes[i] < n &&
    ↪ primes[i] < 32; i++) {
    c = pow_mod(primes[i], r, n);
    for (int j = 0; j < s; j++) {
        d = mul_mod(c, c, n);
        if (d == 1 && c != 1 && c != (n
            ↪ - 1)) return false;
        c = d;
    }
    if (c != 1) return false;
}
return true;
}
void init() {
    int cnt = 0;
    for (int i = 2; i < P; i++) {
        if (!spf[i]) primes[cnt++] = spf[i]
            ↪ i;
        for (int j = 0, k; (k = i * primes
            ↪ [j]) < P; j++) {
            spf[k] = primes[j];
            if (spf[i] == spf[k]) break;
        }
    }
}
// returns O(n^(1/4))
ll pollard_rho(ll n) {
    while (1) {
        ll x = rnd() % n, y = x, c = rnd()
            ↪ % n, u = 1, v, t = 0;
        ll *px = seq, *py = seq;
        while (1) {
            *py++ = y = add_mod(mul_mod(y, y
                ↪ , n), c, n);
            *py++ = y = add_mod(mul_mod(y, y
                ↪ , n), c, n);
            if ((x = *px++) == y) break;
            v = u;
            u = mul_mod(u, abs(y - x), n);
            if (!u) return __gcd(v, n);
            if (++t == 32) {
                t = 0;
                if ((u = __gcd(u, n)) > 1 && u
                    ↪ < n) return u;
            }
        }
        if (t && (u = __gcd(u, n)) > 1 && u
            ↪ u < n) return u;
    }
}
vector<ll> factorize(ll n) {
    if (n == 1) return vector<ll>();
    if (miller_rabin(n)) return vector<
        ↪ ll> {n};
    vector<ll> v, w;
    while (n > 1 && n < P) {
        v.push_back(spf[n]);
        n /= spf[n];
    }
    if (n >= P) {
        ll x = pollard_rho(n);
        v = factorize(x);
        w = factorize(n / x);
        v.insert(v.end(), w.begin(), w.end()
            ↪ ());
    }
    return v;
}
}
// auto f = PollardRho::factorize(n);
// sort(f.begin(), f.end());
// cout << f.size() << '\n';
// for (auto x: f) cout << x << ' ';
// ↪ cout << '\n';

```

## 6.25 2D Seg Tree

```

template <typename T> class BIT2D {
private:
    const int n, m;
    vector<vector<T>> bit;
public:

```

```

BIT2D(int n, int m) : n(n), m(m), bit(
    ↪ n + 1, vector<T>(m + 1)) {}
// adds val to the point (r, c) */
void add(int r, int c, T val) {
    r++, c++;
    for (; r <= n; r += r & -r) {
        for (int i = c; i <= m; i += i & -
            ↪ i) {
            bit[r][i] += val;
        }
    }
}
// sum of points with row in [0, r]
// ↪ and column in [0, c] */
T rect_sum(int r, int c) {
    r++, c++;
    T sum = 0;
    for (; r > 0; r -= r & -r) {
        for (int i = c; i > 0; i -= i & -i
            ↪ ) {
            sum += bit[r][i];
        }
    }
    return sum;
}
// sum of points with row in [r1, r2]
// ↪ and column in [c1, c2] */
T rect_sum(int r1, int c1, int r2, int
    ↪ c2) {
    return rect_sum(r2, c2) - rect_sum(
        ↪ r2, c1 - 1) - rect_sum(r1 - 1,
        ↪ c2) +
        rect_sum(r1 - 1, c1 - 1);
}
};

```

## 6.26 Next Prev Smaller

```

vector<ll> next_smallest(vector<ll> v) {
    ll n = v.size();
    vector<ll> nextS(n, n);
    stack<ll> st;
    for (int i = 0; i < n; i++) {
        while (!st.empty() and v[i] < v[st.
            ↪ top()]) {
            nextS[st.top()] = i;
            st.pop();
        }
        st.push(i);
    }
    return nextS;
}
vector<ll> prev_smallest(vector<ll> v) {
    ll n = v.size();
    vector<ll> prevS(n, -1);
    stack<ll> st;
    for (int i = 0; i < n; ++i) {
        while (!st.empty() && v[i] < v[st.
            ↪ top()]) {
            st.pop();
            if (!st.empty())
                prevS[i] = st.top();
            st.push(i);
        }
    }
    return prevS;
}

```

7 Information

7.1 Numbers with Most Divisors

| Max Value ( $N$ ) | Number with Most Divisors ( $n$ ) | Number of Divisors ( $\tau(n)$ ) |
|-------------------|-----------------------------------|----------------------------------|
| $10^3$            | 83,160                            | 128                              |
| $10^6$            | 720,720                           | 240                              |
| $10^7$            | 9,609,600                         | 640                              |
| $10^8$            | 98,280,000                        | 672                              |
| $10^9$            | 735,134,400                       | 1,344                            |
| $10^{10}$         | 7,242,460,800                     | 2,688                            |
| $10^{11}$         | 73,346,256,000                    | 5,376                            |
| $10^{12}$         | 936,966,912,400                   | 10,752                           |

7.2 Combinatorics Information

**Distinct-element counts over all subarrays:** To count the number of distinct elements across all subarrays, consider every ending index  $i$ . By tracking the last occurrence of each value, we can determine how far left we can extend while keeping all elements distinct. This gives the maximum valid window  $[L_i, i]$ . The number of subarrays ending at  $i$  that have all distinct elements is simply the window size  $(i - L_i + 1)$ . Summing these values for all  $i$  yields the total distinct-element count over all subarrays.

**Bitwise OR over all subarrays:** For subarrays ending at index  $i$ , the bitwise OR value can only increase as the subarray grows, since once a bit becomes 1, it never returns to 0. Let  $S_{i-1}$  be the set of OR values of all subarrays ending at  $i - 1$ . To compute the OR values for subarrays ending at  $i$ , take each value in  $S_{i-1}$ , OR it with  $a_i$ , and also include the single element OR value  $a_i$ . After merging duplicates, the resulting set contains all OR values achieved by subarrays ending at  $i$ . Summing all these values over every  $i$  yields the total bitwise OR over all subarrays.

**Bitwise XOR over all subarrays:** Let  $\text{pref}[i]$  be the prefix XOR up to index  $i$ . The XOR of a subarray  $[l, r]$  is  $\text{pref}[r] \oplus \text{pref}[l - 1]$ . Consider a single bit position. This bit is 1 in the subarray XOR exactly when the two prefixes differ at that bit. Count how many prefixes have bit 0 and how many have bit 1. The number of contributing pairs is their product. Summing this contribution over all bits gives the total XOR value of all subarrays.

**XOR of all pairs:** For any bit position, the XOR of two values is 1 at that bit if and only if one value has the bit set and the other does not. If  $c_1$  numbers have the bit 1 and  $c_0$  have the bit 0, then the number of pairs contributing a 1 at that bit is  $c_0 \cdot c_1$ . Summing these contributions over all bit positions yields the XOR of all unordered pairs.

**XOR of OR of every subarray:** A bit in the OR of a subarray is 1 if the subarray contains at least one element with that bit set. If a bit appears in  $k$  positions, count how many subarrays include at least one such position. If this number is odd, the bit remains in the final XOR; otherwise it cancels out.

**Tree: total length of all simple paths:** Removing an edge splits the tree into parts of sizes  $s$  and  $n - s$ . Any path connecting one node from each part must use

this edge, giving  $s(n - s)$  such paths. Multiplying by the edge weight and summing over all edges yields the total path length.

**Tree: distinct elements over all simple paths:** Using DSU-on-tree, maintain the largest child’s data structure for each subtree and merge smaller children into it. This efficiently tracks all distinct values appearing along all paths.

**Minimum Hamming distance over all cyclic shifts:** The Hamming distance for each shift equals the number of mismatched positions. Map equal characters to +1 and unequal characters to  $-1$ , then use convolution to compute match counts for all shifts simultaneously. More matches imply a smaller Hamming distance.

**Total Hamming distance over all pairs:** For each bit, if  $c$  values have that bit set and  $n - c$  do not, then  $c(n - c)$  pairs differ at that bit. Summing over all bits yields the total Hamming distance.

**Sum of products over all subsequences:** Expanding the product  $(1 + a_1)(1 + a_2) \cdots (1 + a_n)$  selects either 1 or  $a_i$  from each term, corresponding exactly to skipping or including  $a_i$ . Every subsequence product appears once; subtracting 1 removes the empty subsequence.

**Widths over all subsequences:** After sorting, element  $a_i$  is maximum in  $2^i$  subsequences and minimum in  $2^{n-i-1}$  subsequences. Summing the contributions of all elements gives the total width.

**Optimal weighted sum with range increments:** A difference array counts how many range operations affect each position. Sorting both the array values and their frequencies and pairing largest with largest maximizes the total sum.

**Sum of divisors for 1 to  $N$ :** Each integer  $i$  is a divisor of all multiples  $i, 2i, 3i, \dots$ . Adding  $i$  to the divisor sum of each of these values for all  $i$  from 1 to  $N$  computes all divisor sums.

**Sum of absolute differences over all pairs:** After sorting, each  $a_i$  contributes to the absolute difference with all smaller elements on its left and all larger elements on its right. Prefix sums compute these contributions efficiently.

**Sum of inversion counts over all permutations:** Each pair  $(i, j)$  with  $i < j$  has a  $1/2$  probability of appearing as an inversion in a random permutation. With  $\frac{n(n-1)}{2}$  such pairs, the expected number of inversions is  $\frac{n(n-1)}{4}$ . Multiplying by  $n!$  gives the total inversion sum.

**Inversion sum over binary strings with  $x$  zeros and  $y$  ones:** There are  $xy$  zero-one pairs, and each such pair contributes an inversion in exactly half the binary strings of length  $x + y$  containing  $x$  zeros and  $y$  ones. Multiplying by the total count  $\binom{x+y}{x}$  yields the inversion sum.

8 Mathematics

8.1 Area Formulas

|                   |  |
|-------------------|--|
| Rectangle         | length $\times$ width  |
| Square            | side <sup>2</sup>  |
| Triangle          | $\frac{1}{2} \times$ base $\times$ height  |
| Parallelogram     | base $\times$ height   |
| Pyramid (no base) | $\frac{1}{2} \times$ (perimeter of base) $\times$ (slant height)   |
| Polygon           | $\frac{1}{2}  \sum_{i=1}^n (x_i y_{i+1} - x_{i+1} y_i) $<br>$a + \frac{b}{2} - 1$ (for lattice co-ordinates) |

$a$  = interior lattice pts,  $b$  = boundary pts.

8.2 Volume Formulas

|                   |  |
|-------------------|--|
| Cube              | side <sup>3</sup>                                  |
| Rectangular Prism | length $\times$ width $\times$ height              |
| Cylinder          | $\pi \times$ radius <sup>2</sup> $\times$ height   |
| Sphere            | $\frac{4}{3} \pi \times$ radius <sup>3</sup>       |
| Pyramid           | $\frac{1}{3} \times$ (base area) $\times$ (height) |

8.3 Surface Area Formulas

|                   |  |
|-------------------|--|
| Cube              | $6 \times$ side <sup>2</sup>   |
| Rectangular Prism | $2(lw + lh + wh)$ ( $l$ = length, $w$ = width, $h$ = height)         |
| Cylinder          | $2\pi r(r + h)$  |
| Sphere            | $4\pi r^2$   |
| Pyramid           | base area $+ \frac{1}{2} \times$ (perimeter) $\times$ (slant height) |

8.4 Triangles

|                |  |
|----------------|--|
| Semiperimeter  | $s = \frac{a+b+c}{2}$                        |
| Area           | $A = \sqrt{s(s-a)(s-b)(s-c)}$                |
| Circumradius   | $R = \frac{abc}{4A}$                         |
| Inradius       | $r = \frac{A}{s}$                            |
| Median         | $m_a = \frac{1}{2} \sqrt{2b^2 + 2c^2 - a^2}$ |
| Angle bisector | $s_a = \sqrt{\frac{bc}{1-(a/(b+c))^2}}$      |

Side lengths:  $a, b, c$ .

8.5 Sum Equations

$$\sum_{i=k}^n c^i = \frac{c^{n+1} - c^k}{c - 1}$$

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^n i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

$$\sum_{i=1}^n i^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

$$\sum_{i=1}^n (2i-1) = n^2$$

8.6 Logarithmic Basic

|   |  |
|---|--|
| $\log_b 1 = 0$  | $\log_b b = 0$                         |
| $b^{\log_b a} = a$                                      | $x^{\log_b y} = y^{\log_b x}$          |
| $\log_a b = \frac{1}{\log_b a}$                         | $\log_a x = \frac{\log_b x}{\log_b a}$ |
| $\log_b (AB) = \log_b A + \log_b B$                     |  |
| $\log_b \left(\frac{A}{B}\right) = \log_b A - \log_b B$ |  |
| $\log_a c = \log_a b \times \log_b c$                   |  |
| $\log_b (A^x) = x \log_b A$                             |  |

8.7 Series

Catalan:  $C_n = \frac{1}{n+1} \binom{2n}{n}$ ,  $C_n = \sum_{k=0}^n C_k C_{n-k}$   
Arithmetic:  $a_n = a + (n-1)d$ ,  $s_n = \frac{n}{2}(2a + (n-1)d)$   
Geometric:  $a_n = ar^{n-1}$ ,  $s_n = \frac{a(1-r^n)}{1-r}$   
Derangements:  $D_n = n! \sum_{k=0}^n \frac{(-1)^k}{k!}$ ,  $D_n = \left\lfloor \frac{n!}{e} + \frac{1}{2} \right\rfloor$   
Fibonacci:  $f_n = \frac{\phi^n - (1-\phi)^n}{\sqrt{5}}$ ,  $\phi = \frac{1+\sqrt{5}}{2}$

8.8 Pick's Theorem

$A = I - \frac{1}{2}B + 1$  ( $I$  = interior points,  $B$  = boundary points)

8.9 Stars and Bars

Number of solutions of  $x_1 + \dots + x_k = n$ :  
 $\binom{n-1}{k-1}$  when  $x_i > 0$ ;  $\binom{n+k-1}{k-1}$  when  $x_i \geq 0$ .

8.10 Facts

$\lceil \frac{a}{b} \rceil = \lfloor \frac{a-1}{b} \rfloor + 1$   
Sum  $l$  to  $r$ :  $\frac{l+r}{2}(r-l+1)$   
 $\lfloor \frac{\lfloor n/a \rfloor}{b} \rfloor = \lfloor \frac{n}{ab} \rfloor$

8.11 LCM

$\text{lcm}(a, n) + \text{lcm}(n-a, n) = \frac{n^2}{\text{gcd}(a, n)}$   
 $\text{SUM} = \frac{n}{2} \left( \sum_{d|n} \varphi(d) d + 1 \right)$