Principles of Logic Systems

Lecture 1

Introduction:

Principles of Logic Systems is a fundamental course in Computer Engineering that explains how computers represent, process, and control information at the hardware level.

Instead of focusing on software, this course introduces the logical principles used to build computer systems.

Throughout this course, we will study methods of representing information, different numbering systems, and logical operations that are combined to design computer circuits.

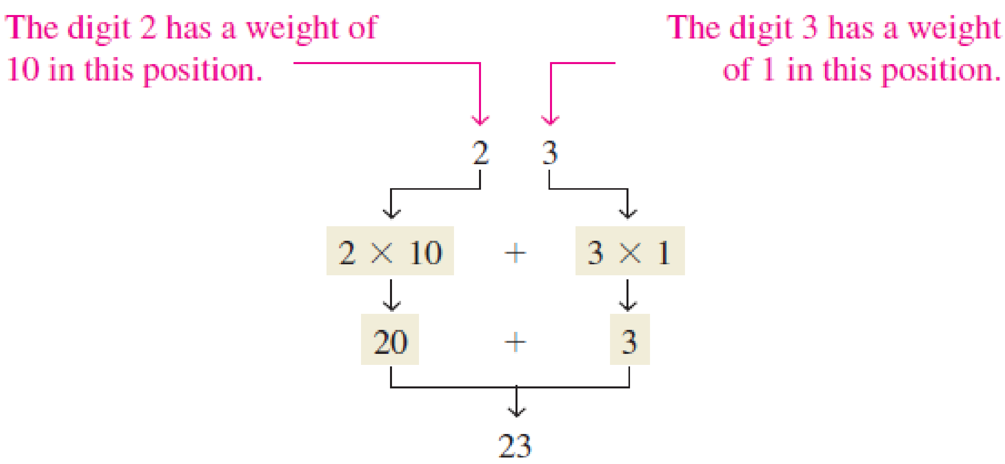
These concepts form the foundation for understanding how computer hardware works and are essential for advanced courses such as computer organization and computer architecture.

By the end of this course, you will understand how simple logical elements are combined to form complex systems.

1. Number systems

1.1 Decimal number system

In the decimal number system, the digits 0 to 9 are used to represent quantities. The value of a digit depends on its position in the number, which determines its magnitude (weight).



By combining digits in different positions, quantities greater than nine can be represented, such as 23 where 2 represents twenty and 3 represents three.

In the decimal number system, the position of each digit determines its weight, which is a positive power of ten.
These weights increase from right to left, starting with $10^0 = 1$

$$\ldots\ 10^5\ 10^4\ 10^3\ 10^2\ 10^1\ 10^0$$

For fractional numbers, the weights are negative powers of ten that decrease from left to right beginning with 10^{-1}.

$$10^2\ 10^1\ 10^0.10^{-1}\ 10^{-2}\ 10^{-3}\ \ldots$$

Decimal point

Example 1:

Express the decimal number 47 as a sum of the values of each digit.

Solution

The digit 4 has a weight of 10, which is 10^1, as indicated by its position. The digit 7 has a weight of 1, which is 10^0, as indicated by its position.

$$\begin{aligned} 47 &= (4 \times 10^1) + (7 \times 10^0) \\ &= (4 \times 10) + (7 \times 1) = \mathbf{40} + \mathbf{7} \end{aligned}$$

Example 2:

Express the decimal number 568.23 as a sum of the values of each digit.

Solution

The whole number digit 5 has a weight of 100, which is 10^2, the digit 6 has a weight of 10, which is 10^1, the digit 8 has a weight of 1, which is 10^0, the fractional digit 2 has a weight of 0.1, which is 10^{-1}, and the fractional digit 3 has a weight of 0.01, which is 10^{-2}.

$$\begin{aligned} 568.23 &= (5 \times 10^2) + (6 \times 10^1) + (8 \times 10^0) + (2 \times 10^{-1}) + (3 \times 10^{-2}) \\ &= (5 \times 100) + (6 \times 10) + (8 \times 1) + (2 \times 0.1) + (3 \times 0.01) \\ &= \mathbf{500} + \mathbf{60} + \mathbf{8} + \mathbf{0.2} + \mathbf{0.03} \end{aligned}$$

Example 3:

Determine the value of each digit in 939

Solution

9 has a value of 900
3 has a value of 30
9 has a value of 9

Example 4:

Determine the value of each digit in 67.924.

Solution:

6 has a value of 60

7 has a value of 7

9 has a value of 9/10 (0.9)

2 has a value of 2/100 (0.02)

4 has a value of 4/1000 (0.004).

Exercises:

1. What weight does the digit 7 have in each of the following numbers?
   (a) 1370 (b) 6725 (c) 7051 (d) 58.72
2. Express each of the following decimal numbers as a sum of the products obtained by multiplying each digit by its appropriate weight:
   (a) 51 (b) 137 (c) 1492 (d) 106.58

Homework:

1. What is the weight of 7 in each of the following decimal numbers?
   (a) 1947 (b) 1799 (c) 1979
2. Express each of the following decimal numbers as a power of ten:
   (a) 1000 (b) 10000000 (c) 1000000000
3. Give the value of each digit in the following decimal numbers:
   (a) 263 (b) 5436 (c) 234543

1.2 Binary number system

The binary number system represents quantities using only two digits, 0 and 1, and is therefore a base-two system. The value of each binary digit depends on its position, and the weights are powers of two.

Binary numbers are weighted numbers. For whole binary numbers, the right-most bit (Least Significant Bit (LSB)) has a weight of 2^0=1 and the weights increase from right to left as powers of two.
The left-most bit is the (Most Significant Bit (MSB)), and its weight depends on the number of bits.

Binary fractional numbers are represented using bits to the right of the binary point. The left-most fractional (MSB) bit has a weight of 2^{-1} = 0.5, and the weights decrease from left to right as negative powers of two.

The weight structure of a binary number is:

$$2^{n-1} \ldots 2^3\, 2^2\, 2^1\, 2^0\, .\, 2^{-1}\, 2^{-2} \ldots 2^{-n}$$

Binary point

Table 1: Binary Weights

Positive Powers of Two (Whole Numbers)								
2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
256	128	64	32	16	8	4	2	1

Negative Powers of Two (Fractional Number)					
2^{-1}	2^{-2}	2^{-3}	2^{-4}	2^{-5}	2^{-6}
1/2 0.5	1/4 0.25	1/8 0.125	1/16 0.625	1/32 0.03125	1/64 0.015625

Q/ how to count in binary?

A/ Counting in the binary system is similar to counting in the decimal system, but only two digits (0 and 1) are used.

When all possible combinations of a given number of bits are used, an additional bit position is added to continue counting.

Decimal Number	Binary Number			
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
8	1	0	0	0
9	1	0	0	1
10	1	0	1	0
11	1	0	1	1
12	1	1	0	0
13	1	1	0	1
14	1	1	1	0
15	1	1	1	1

In a binary system, the maximum number that can be represented using *n* bits is $\mathbf{2^n-1}$. This means that with *n* bits, values from 0 up to 2^n-1 can be represented.

For example: 5 bits represent values from 0 to 31, and 6 bits represent values from 0 to 63

1.3 Binary-to-Decimal Conversion

The decimal value of any binary number can be found by adding the weights of all bits that are 1 and discarding the weights of all bits that are 0.

Example:

Convert the binary whole number 1101101 to decimal.

Solution

Determine the weight of each bit that is a 1, and then find the sum of the weights to get the decimal number.

$$\begin{array}{rl} \text{Weight:} & 2^6\ 2^5\ 2^4\ 2^3\ 2^2\ 2^1\ 2^0 \\ \text{Binary number:} & 1\ \ 1\ \ 0\ \ 1\ \ 1\ \ 0\ \ 1 \end{array}$$

$$\begin{aligned} 1101101 &= 2^6 + 2^5 + 2^3 + 2^2 + 2^0 \\ &= 64 + 32 + 8 + 4 + 1 = \mathbf{109} \end{aligned}$$

Example:

Convert the fractional binary number 0.1011 to decimal.

Solution

Determine the weight of each bit that is a 1, and then sum the weights to get the decimal fraction.

$$\begin{array}{rllll} \text{Weight:} & 2^{-1} & 2^{-2} & 2^{-3} & 2^{-4} \\ \text{Binary number:} & 0\,.\,1 & 0 & 1 & 1 \end{array}$$

$$\begin{aligned} 0.1011 &= 2^{-1} + 2^{-3} + 2^{-4} \\ &= 0.5 + 0.125 + 0.0625 = \mathbf{0.6875} \end{aligned}$$

Exercises:

1. Convert the binary number 10010001 to decimal.
2. Convert the binary number 10.111 to decimal.

Homework:

1. What is the largest decimal number that can be represented in binary with eight bits?
2. Determine the weight of the 1 in the binary number 10000.
3. Convert the binary number 10111101.011 to decimal.