ENME403- Linear Systems Control and System Identification

Parameter ID Project

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Introduction
The goal of this project was to identify unknown parameters in the Bouc-Wen model. This can be done through least squares, gradient descent, integral-based and overall least squares. Accurate modelling leads better decision of structural systems.

Methods

Part 1
The project data was given describes the characteristic of the Bouc-Wen model considering the nonlinear stiffness k. The Bouc-Wen model is given by the equation:

$$m\ddot{v} + c\dot{v} + \ \alpha k_0 v \ + (1-\alpha)k_0 z = -m\ddot{v}_g \tag{1}$$

where m is the mass, c is the damping and $\ddot{v}_g$ is the input acceleration. $v, \dot{v}, \ddot{v}$ are the displacement, velocity and acceleration of the system, respectively. k represents the stiffness of the system which is changing over time. Since the Bouc-Wen model assumes the hysteretic restoring force $f(t)$, which comprises a linear spring element of stiffness, k_0, and a hysteretic component in parallel in the equation:

$$f(t) = \alpha k_0 v + (1-\alpha)k_0 z \tag{2}$$

Equation (2) can be used back in Equation (1) which would give the following:

$$f(t) = -m\ddot{v}_g - m\ddot{v} - c\dot{v} \tag{3}$$

where $\ddot{v}_g, \ddot{v}, \dot{v}, m,$ and c are all know values given in the project data. The data does not contain the initial stiffness k_0 or the post-yielding ratio α. Thus, these unknowns can be solved by simplifying Equation (2) to the following:

$$f(t) = \theta_1 v + \theta_2 z \tag{4}$$

where $\theta_1 = \ \alpha k_0$ and $\theta_2 = (1-\alpha)k_0$. To identify the parameters k_0 and α the least squares and gradient descent can be considered.

Least Squares
The matrix form of Equation (5) can be written as

$$F = A\theta + E \rightarrow \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_n \end{bmatrix} = \begin{bmatrix} v_1 & z_1 \\ v_2 & z_2 \\ \vdots & \vdots \\ v_n & z_n \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_n \end{bmatrix} \tag{5}$$

and to obtain the square of the errors $J(\theta)$ can be rewritten as

$$J(\theta) = E^T E = (F - A\theta)^T (F - A\theta) = e_1^2 + e_2^2 + \cdots + e_n^2 \tag{6}$$

$$\frac{dJ(\theta)}{d\theta} = -2A^T F + 2A^T A\theta = 0 \tag{7}$$

$$\theta = (A^T A)^{-1} A^T F \tag{8}$$

Thus, once the value of $\theta_1$and θ_2 are calculated, α and k_0 can be obtained from

$$\alpha = {}^{1}\!\Big/\!\left(\frac{\theta_2}{\theta_1} + 1\right); \ k_0 = {}^{\theta_1}\!/_{\alpha} \tag{9}$$

This process can be summarised by the block diagram in Figure 1.

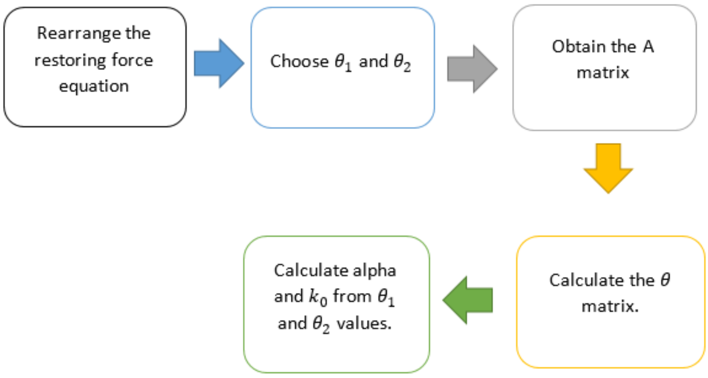

Figure 1: Block Diagram of the method used for this task.

Gradient Descent

The gradient descent can be also used as an alternative method to linear least squares method in obtaining the values of k_0 and α. The procedure, however, differs to that of the least squares. Gradient descent algorithm can be summarised in Figure 2.

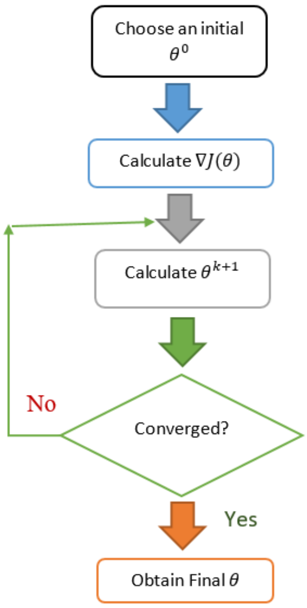

Figure 2: shows the block diagram of the algorithm of gradient descent method.

Part 2

Integral-Based Method

The change in stiffness $k(t)$ is determined by the initial stiffness k_0, and the differential function $\frac{\dot{z}}{\dot{v}}$. One definition of the differential function for a non-degrading yielding system is defined as:

$$\frac{\dot{z}}{\dot{v}} = 1 - 0.5 * (sign(\dot{v}z) + 1)\left|\frac{z}{dy}\right|^2 \tag{10}$$

where dy is a positive value and is the yielding deformation. The sign indicates the signum function. The method used in this part to find the dy parameter is the integral-based method. The method can be simplified by the steps in Figure 2.

The first step this method is to rearrange the $\frac{\dot{z}}{\dot{v}}$ function which leads to:

$$\dot{z} = \dot{v} - 0.5\dot{v}(sign(\dot{v}z) + 1)z^2 \left|\frac{1}{dy}\right|^2 \tag{11}$$

where $K = \left|\frac{1}{dy}\right|^2$ and $sign(\dot{v}z) + 1$ is defined as

$$sign(\dot{v}z) + 1 = \begin{cases} 2 & \dot{v}z > 0 \\ 1 & \dot{v}z = 0 \\ 0 & \dot{v}z < 0 \end{cases} = \begin{bmatrix} 1 \\ 2 \\ \vdots \\ 0 \end{bmatrix} \tag{12}$$

The singun in Equation (7) would represent a matrix since $\dot{v}$ and z are both matrices. For simplification, the parameter $S = -0.5\dot{v}(sign(\dot{v}z) + 1)z^2$ is used. Thus, Equation (6) becomes:

$$\dot{z} = \dot{v} + SK \tag{12}$$

Integrating both sides

$$\int_{t_1}^{t_2} \dot{z} = \int_{t_1}^{t_2} \dot{v} + \int_{t_1}^{t_2} SK \tag{14}$$

$$z(t_2) - z(t_1) = v(t_2) - v(t_1) - \left(\frac{\Delta t}{2}\left(S_{t_1} + S_{t_2}\right) + 2\left(S_{t_1+1} + S_{t_1+2} + \cdots + S_{t_2-1}\right)\right)K \tag{15}$$

$$z(t_2) - z(t_1) - v(t_2) + v(t_1) = -\left(\frac{\Delta t}{2}\left(S_{t_1} + S_b\right) + 2\left(S_{t_1+1} + S_{t_1+2} + \cdots + S_{t_2-1}\right)\right)K \tag{16}$$

The next step is to equate the Equation (11) with the least squares matrix vector form $Y = XK + E$. Therefore,

$$y_{ba} = z(t_2) - z(t_1) - v(t_2) + v(t_1) \tag{17}$$

$$X_{ba} = -\left(\frac{\Delta t}{2}\left(S_{t_1} + S_b\right) + 2\left(S_{t_1+1} + S_{t_1+2} + \cdots + S_{t_2-1}\right)\right) \tag{18}$$

Thus, the Y and X matrices

$$Y = \begin{bmatrix} y_{t_1} \\ y_{t_2} \\ \vdots \\ y_{t_n} \end{bmatrix}, X = \begin{bmatrix} x_{t_1} \\ x_{t_2} \\ \vdots \\ x_{t_n} \end{bmatrix} \tag{19}$$

Thus, K can be solved by the least square to obtain dy.

$$K = (X^T X)^{-1} X^T Y \tag{20}$$

$$dy = \sqrt{1/K} \tag{21}$$

This procedure of implementing the integral-based method to find the value of dy can be summarised by the block diagram in Figure 2.

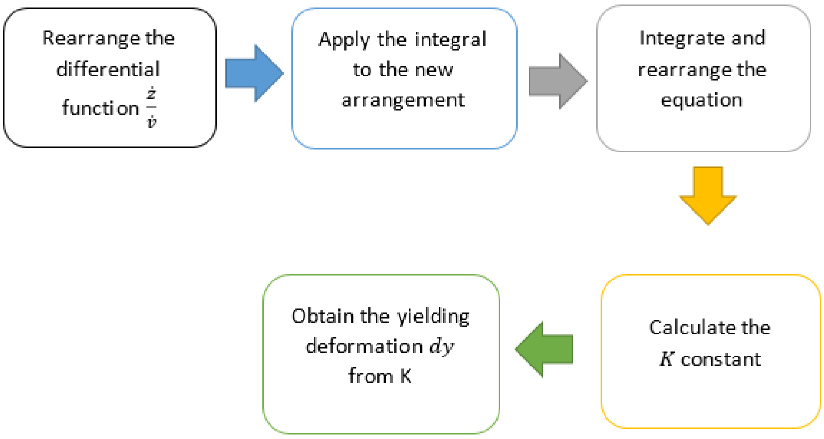

Figure 3: Integral-based method block diagram.

Overall Least Squares

Overall least squares is another method that can implemented to identify the wielding deformation dy, which is the breaking point that separates the elastic segment from the plastic segment. The idea in this method is to estimate the value of unknown $\dot{z}$. Equation (12) can be rearranged to

$$\frac{z_{t_2} - z_{t_1}}{\Delta t} - \dot{v} = SK \tag{22}$$

where $\dot{z} = \frac{z_{t_2} - z_{t_1}}{\Delta t}$ and $S = -0.5\dot{v}(sign(\dot{v}z) + 1)z^2$.

Results

Part 1

The MATLAB code results for α and k_0 are 0.2771 and 1.9312, respectfully.

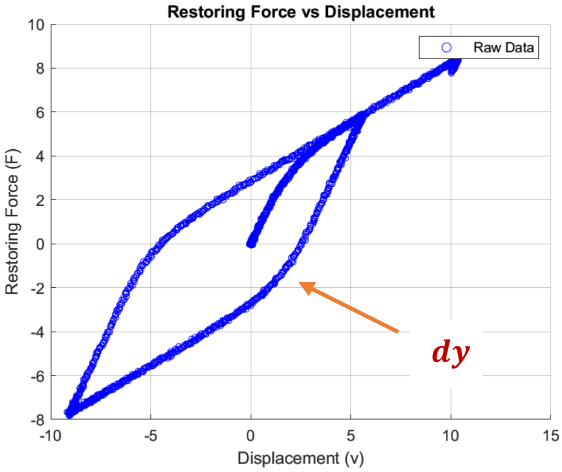

Figure 4: shows the shape of hysteretic restoring force vs displacement. dy is the yielding point of the plot.

Figure 4 shows the cost function $J(\theta)$ or the error plot against different values of θ. The red dot in Figure 4 and 5 shows the minimum value of $J(\theta)$. This value represent the least cost point in the $J(\theta)$ plot.

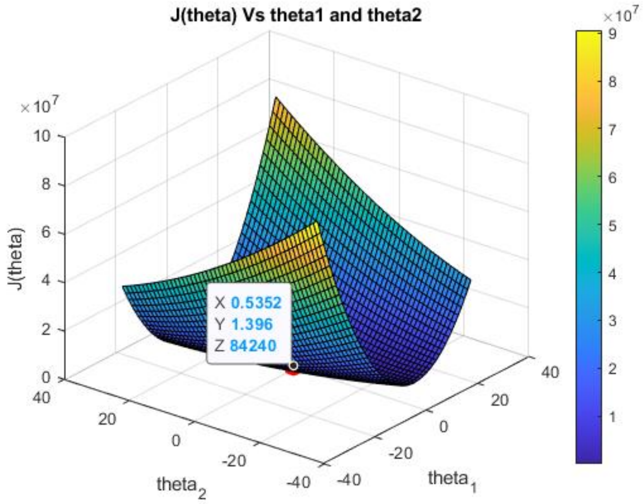

Figure 5: The error surface or $J(\theta)$ of the system.

Figure 6 shows contour plot of the $J(\theta)$, θ_1, and θ_2 matrices.

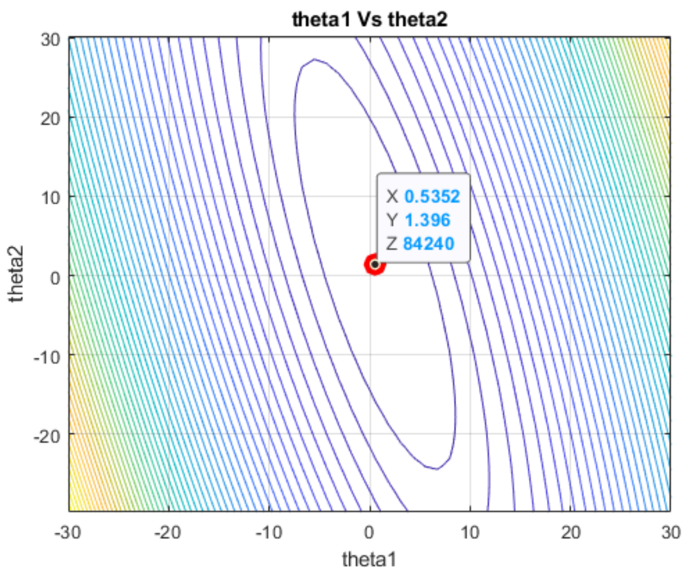

Figure 6: The contour map of the system.

Part 2

Table 1 shows value of K and dy with different number of segmentations S in Equation (12). The table contains the number segmentation, which is a factor of 2001. This is because there is 2001 data points. For instance, dividing the area of S into one segment with initial point being 1 and the endpoint 2001, the value of dy is 2.0529.

Table 1: Number of partitions vs the values of K and dy.

Number of Partitions	K	dy
1	0.2373	2.0529
3	0.2367	2.0554
23	0.2362	2.0578
29	0.2363	2.0572
69	0.2362	2.0577
87	0.2369	2.0597
667	0.2371	2.0538

Discussion

Part 1

Post-yielding ratio is ratio of k_y is post-yielding stiffness over the initial stiffness k_0 in $\gamma = {k_y}/{k_0}$ [1]. A ratio of $\gamma = 0$ indicates the ideal elastic-plastic model and a ratio that is $\gamma > 0$ or $\gamma < 0$ indicates a positive and a negative post-yield stiffness [1]. In the case of this paper, the post ratio $\alpha = {\theta_1}/{k_0}$.

The red sphere in Figure 5 represents the minima or the lowest value of the cost plot. This point might be useful for reducing the costs for a Bouc-Wen model.

One of the advantages of using the least squares is the reliability factor of the method. The method is a simple algorithm of adding/multiplying matrices together to get particular θ matrix. One advantage of gradient descent can be reducing the amount of thetas or iterations by the magnitude of the learning rate α. However, if α is too big, then algorithm might miss the minimum point because the step might be too big and if α is too small, then the algorithm would take a large number of iteration to reach convergence because the step might be too small. Moreover, another disadvantage to this method is that if the cost function had more than a single minimum point, it might possible miss the global minima if it reached the local minima. This would indicate a false minimum cost and lead to the wrong θ values. The method followed to obtain the results was the least squares. This because of the ease of implementation of the method in MATLAB.

Part 2

Table 1 showed the values of dy for different number of segments. The table confirms that the true value of $dy = 2.05636$ by taking the average of the all data of dy. In Figure 4, dy can be seen within the ranges found and it is around 2.056 in Table 1. This dy value would the yielding point in the restoring force vs displacement plot.

Conclusion

Linear least squares and gradient descent can predict k_0 and α. Integral-based and overall least squares methods can predict the value of dy. Least-squares method can be used for quick analysis of small number of data samples, however, overall least squares can be implemented when the sample size can be large as long as the chosen α is within the right range.

References

[1] L. Ye1, X. Lu1, Q. Ma, G. Cheng, S. Song, Z. Miao and P. Pan, “STUDY ON THE INFLUENCE OF POST-YIELDING STIFFNESS TO THE SEISMIC RESPONSE OF BUILDING STRUCTURES,” [Online]. Available: https://www.researchgate.net/publication/255595509_STUDY_ON_THE_INFLUENCE_OF_POST-YIELDING_STIFFNESS_TO_THE_SEISMIC_RESPONSE_OF_BUILDING_STRUCTURES.