Menoufia University

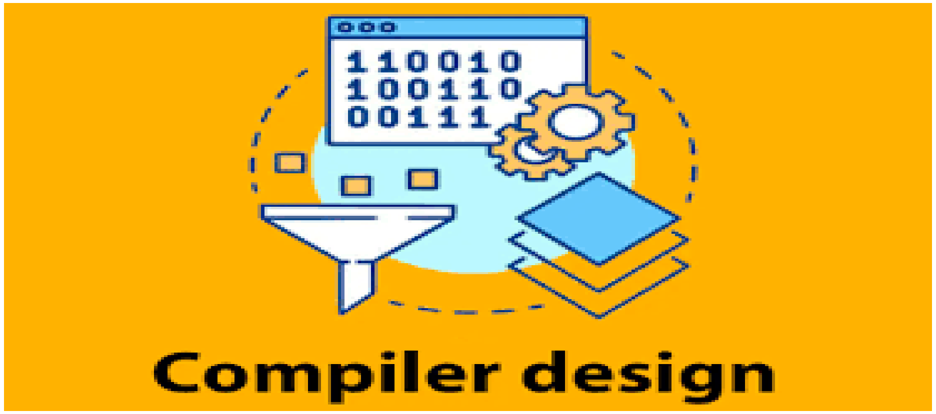
Faculty of computers & Information

Computer Science Department.

Compiler Design

4 Year – first Semester

Lecture 3



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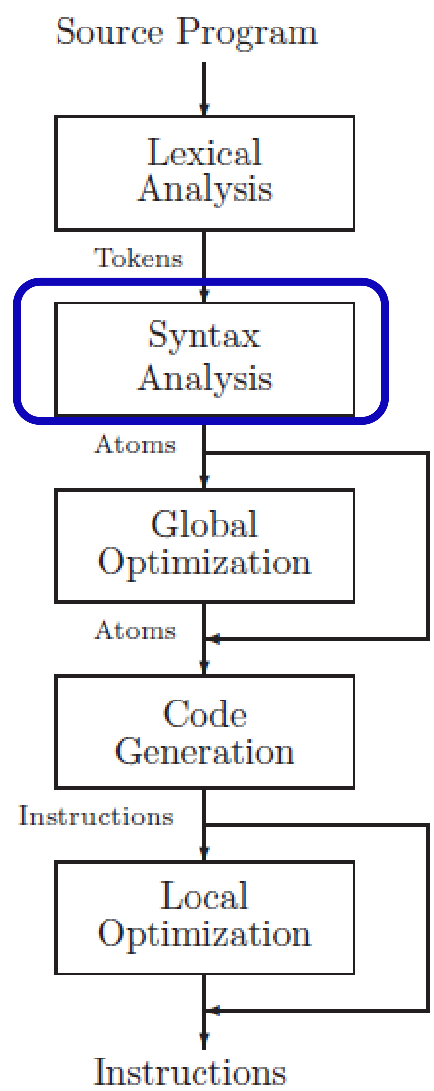
2023-2024

Phases of Compilers

Phases of Compilers

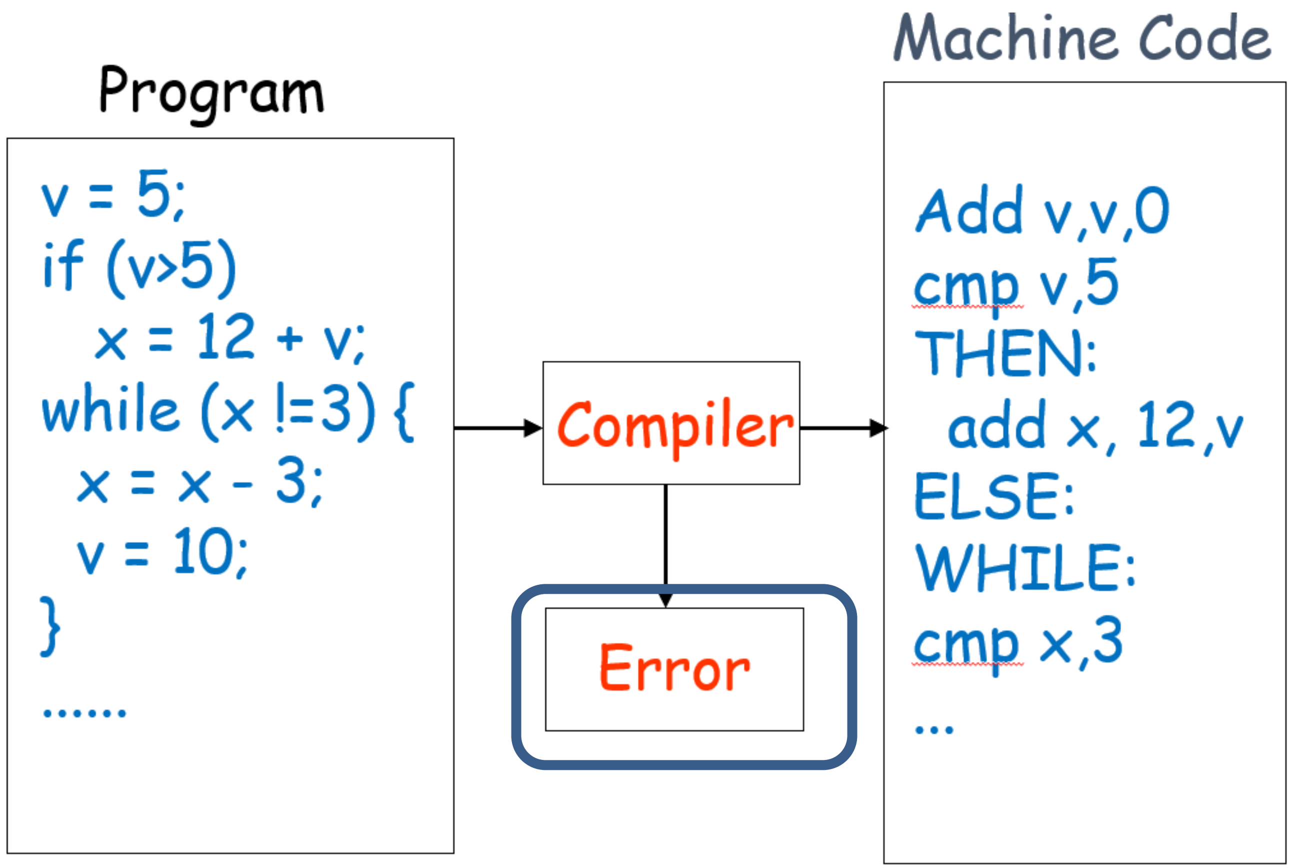
- Lexical Analysis (Scanner)
- Syntax Analysis Phase
- Global Optimization
- Code Generation
- Local Optimization



Outline

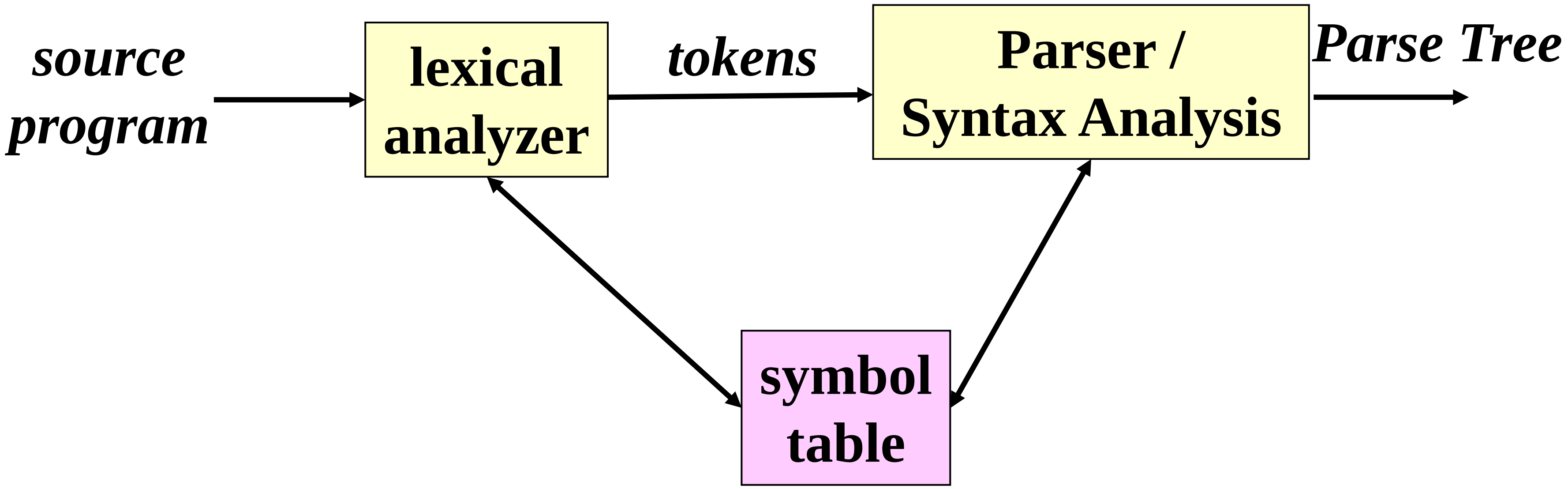
- **Syntax Analysis.**
- Grammar.
- Pushdown Machine.
- Parser.

The Role of a Syntax Analyzer



Syntax Analyzer is used to check for the **prober syntax** and generate the **syntax tree**.

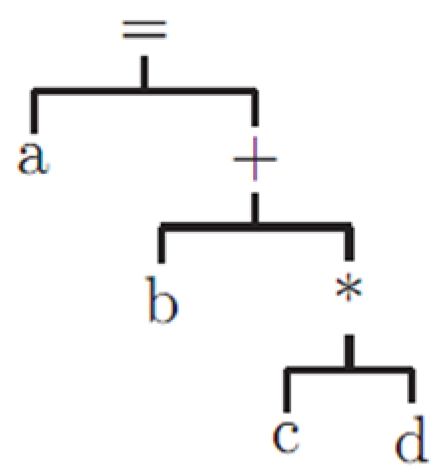
The Role of a Syntax Analyzer



Parse (Syntax) Tree

Syntax Tree is a data structure in which the interior nodes represents operations and leaves represent operands.

$$a = b + c * d$$



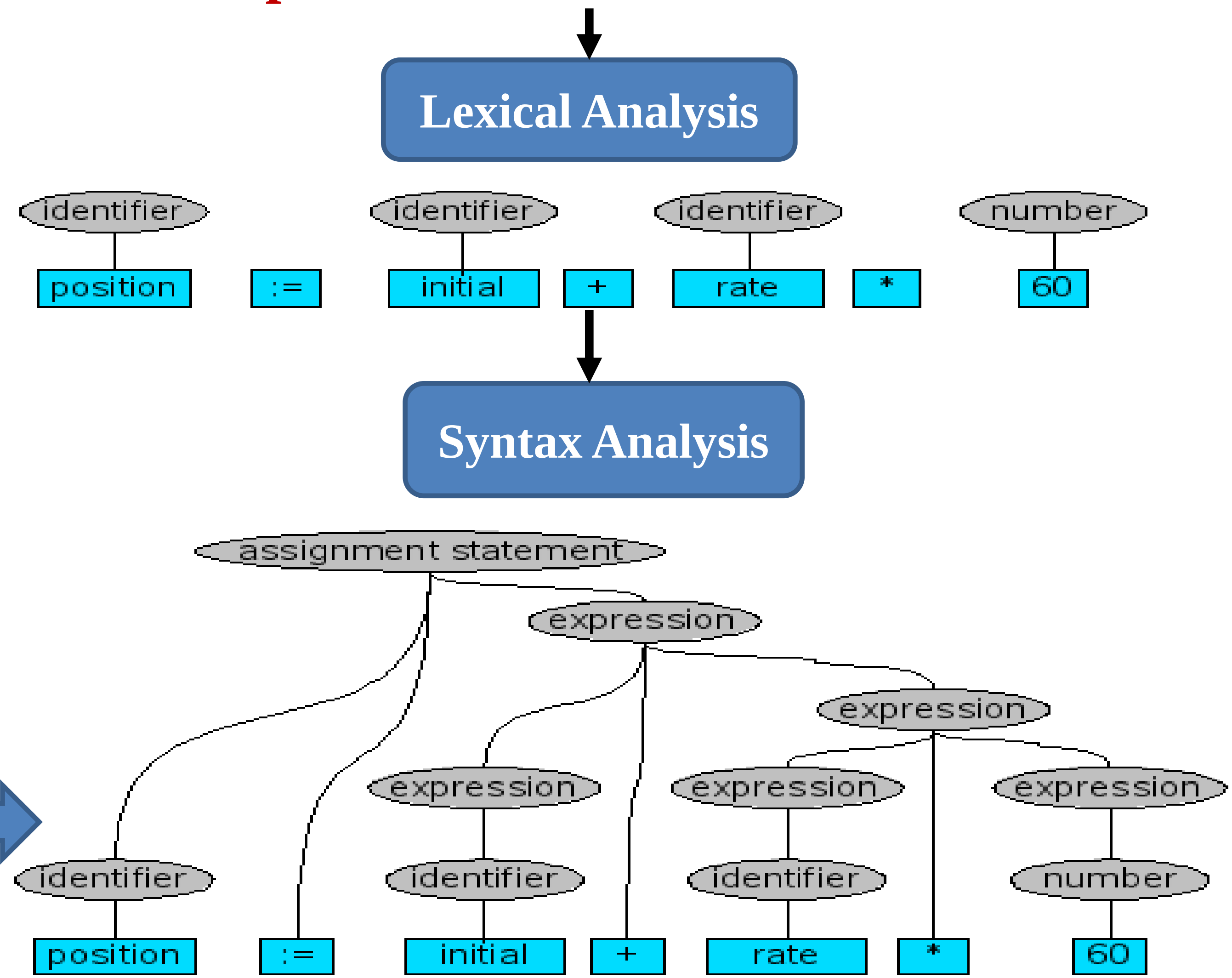
Example

source program

position = initial + rate * 60

Tokens

Parse Tress



Syntax Analysis

Before getting into syntax analysis, we need to cover the concepts of formal **grammar** and **pushdown machine** which are critical to the design of the lexical analyzer.

Outline

- Syntax Analysis.
- **Grammar.**
- Pushdown Machine.
- Parser.

Grammar

Grammar is a list of rules which can be used to describe the structure or syntax of a language. (i.e., The grammar of a language defines the correct form for sentences in that language.)

Example: English language:

$$\langle sentence \rangle \rightarrow \langle noun \rangle \ \langle verb \rangle$$

$$\langle noun \rangle \rightarrow \langle article \rangle \ \langle noun \rangle$$

$$\langle article \rangle \rightarrow a$$

$$\langle article \rangle \rightarrow the$$

$$\langle noun \rangle \rightarrow cat$$

$$\langle noun \rangle \rightarrow dog$$

$$\langle verb \rangle \rightarrow runs$$

$$\langle verb \rangle \rightarrow walks$$

✓ Derivation of "**the dog walks**"

$$\begin{aligned} \langle sentence \rangle &\Rightarrow \langle noun \rangle \ \langle verb \rangle \\ &\Rightarrow \langle article \rangle \ \langle noun \rangle \ \langle verb \rangle \\ &\Rightarrow the \ \ \langle noun \rangle \ \langle verb \rangle \\ &\Rightarrow the \ \ dog \ \ \langle verb \rangle \\ &\Rightarrow the \ \ dog \ \ walks \end{aligned}$$

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$$\langle article \rangle \rightarrow the$$

$$\langle noun \rangle \rightarrow cat$$

$$\langle noun \rangle \rightarrow dog$$

$$\langle verb \rangle \rightarrow runs$$

$$\langle verb \rangle \rightarrow walks$$

✓ Derivation of "**a cat runs**"

$$\begin{aligned} \langle sentence \rangle &\Rightarrow \langle noun \rangle \ \langle verb \rangle \\ &\Rightarrow \langle article \rangle \ \langle noun \rangle \ \langle verb \rangle \\ &\Rightarrow a \ \ \langle noun \rangle \ \langle verb \rangle \\ &\Rightarrow a \ \ cat \ \ \langle verb \rangle \\ &\Rightarrow a \ \ cat \ \ runs \end{aligned}$$

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$$\langle article \rangle \rightarrow a$$

$$\langle article \rangle \rightarrow the$$

$$\langle noun \rangle \rightarrow cat$$

$$\langle noun \rangle \rightarrow dog$$

$$\langle verb \rangle \rightarrow runs$$

$$\langle verb \rangle \rightarrow walks$$

✓ Derivation of "**dog the runs**"

Error

Grammar Components

A **Grammar** is denoted by G and is defined as a **4–tuple** i.e., **G (V, T, S, P)**

Where

V is non empty set of symbols called as **Variables**

T is non empty set of symbols called as **Terminals**

S $\in$**V** is a **Start Variable**

P is set of **productions** or **production rules.**

Grammar

Notation:

- **Variables** are denoted by only UPPER CASE letters and some special Greek letters etc. Variables are also called a Non-Terminals.
- **Terminals** are denoted by lower case letters i.e., a to z and digits 0 to 9 and some special operators like arithmetic operators, relational operators etc.

Production Form

Productions is defined as **mapping function.**

General form of Productions:

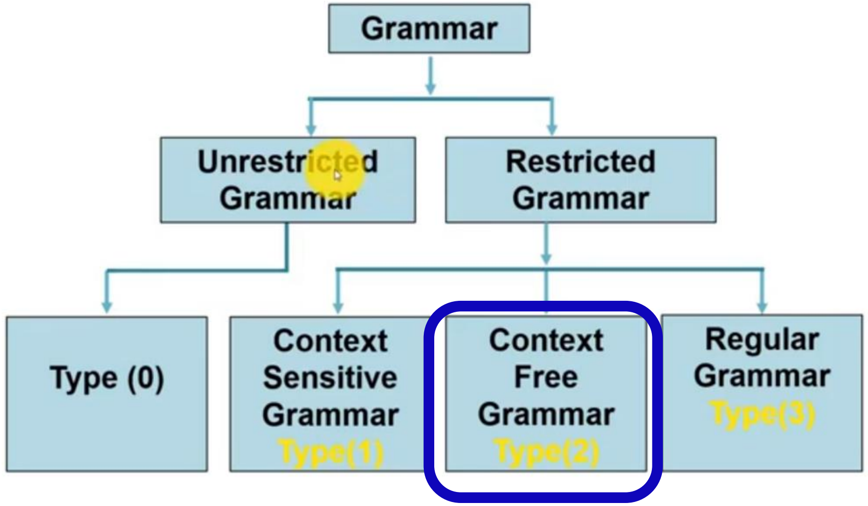
$$P : \alpha \to \beta$$

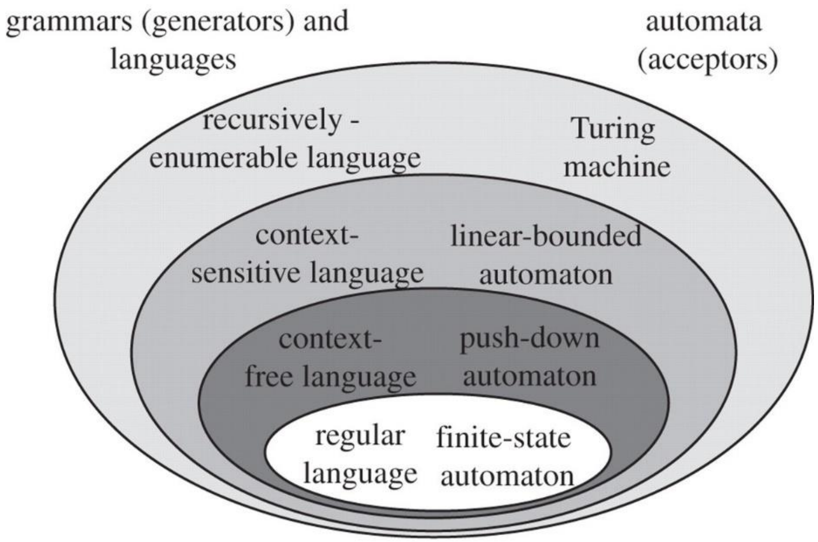
$$\alpha \in (V \cup T)^{+}$$

$$\beta \in (V \cup T)^{*}$$

Types of Grammar

Chomsky Hierarchy





the traditional Chomsky hierarchy

Examples

Example-1:

Let $G = (V, T, S, P)$ is a grammar.

Where

$V = \{S, A, B\}$

$T = \{a, b\}$

S is a Start Variable

And Productions P are given below:

$$
\boxed{\begin{aligned} S &\rightarrow ASB \\ A &\rightarrow aSb \mid \varepsilon \\ B &\rightarrow bSa \mid \varepsilon \end{aligned}}
$$

ε or λ is Null String

Examples

Example-2:

Let G = (V, T, S, P) is a grammar.

Where

V = {S, A, B}

T = {a, b, +}

S is a Start Variable

And Productions P are given below:

$$\boxed{\begin{aligned} &Sa \to ASB \\ &Sb \to aSb \mid \varepsilon \\ &BA \to bSB \mid A + B \end{aligned}}$$

ε or λ is Null String

Derivation of grammar

The grammar specifies a language in the following way:

Beginning with the starting nonterminal, any of the rewriting rules are applied repeatedly to produce a sentential form, which may contain a mix of terminals and nonterminals.

A **derivation** is a sequence of rewriting rules, applied to the starting nonterminal, ending with a string of terminals.

Examples

Example-3:

$$S \rightarrow aSb$$

$$S \rightarrow \lambda$$

- Derivation of string **ab**:

$$S \underset{1}{\Rightarrow} aSb \underset{2}{\Rightarrow} ab$$

$$S \rightarrow aSb \qquad S \rightarrow \lambda$$

Examples

Example-3:

$$S \rightarrow aSb$$

$$S \rightarrow \lambda$$

- Derivation of string **aabb**:

$$S \underset{1}{\Rightarrow} aSb \underset{1}{\Rightarrow} aaSbb \underset{2}{\Rightarrow} aabb$$

$$S \rightarrow aSb \qquad\qquad S \rightarrow \lambda$$

Grammar

Examples: Describe the language of this grammar

Example-3:

$$S \rightarrow aSb$$

$$S \rightarrow \lambda$$

- Derivation of string **aaabbb**:

$$S \underset{1}{\Rightarrow} aSb \underset{1}{\Rightarrow} aaSbb \underset{1}{\Rightarrow} aaaSbbb \underset{2}{\Rightarrow} aaabbb$$

- Derivation of string **aaaabbbb**:

$$S \overset{1}{\Rightarrow} aSb \overset{1}{\Rightarrow} aaSbb \overset{1}{\Rightarrow} aaaSbbb$$

$$\overset{1}{\Rightarrow} aaaaSbbbb \overset{2}{\Rightarrow} aaaabbbb$$

Language of the grammar

$$L(G) = \{a^n b^n : n \geq 0\}$$

Grammar

Examples: Describe the language of this grammar

Example-4:

$$S \to Ab$$

$$A \to aAb$$

$$A \to \lambda$$

Derivations:

$$S \to Ab \to b$$

$$S \to Ab \to aAbb \to abb$$

$$S \to Ab \to aAbb \to aaAbbb \to aabbb$$

Language of the grammar

$$L(G) = \{a^n b^n b : n \geq 0\}$$

Examples: Describe the language of this grammar

Example-5:

$$S \to 0S0$$

$$S \to 1S1$$

$$S \to 0$$

$$S \to 1$$

Derivations:

$$S \to 0S0 \to 000$$

$$S \to 0S0 \to 01S10 \to 01010$$

Language of the grammar

Palindromes of **odd** length over the alphabet $\{0,1\}$

Examples: Describe the language of this grammar

Example-6:

$$S \to (S)$$
$$S \to \lambda$$

$$L(G) = \{(^n)^n : n \geq 0\}$$

Describes parentheses:

((( )))

$$S \to (S)$$
$$S \to SS$$
$$S \to \lambda$$

Describes parentheses:

( ) ((( ))) (( ))

Derivation Order

There are two type of derivations which are:

- **Leftmost derivation** is one in which the **left-most nonterminal** is always the one to which a rule is applied.
- **Rightmost derivation** is one in which the **right-most nonterminal** is always the one to which a rule is applied.

Derivation Order

Example:

$$S \to aAB$$

$$A \to bBb$$

$$B \to A \mid \lambda$$

Leftmost derivation:

$$S \Rightarrow aAB \Rightarrow abBbB \Rightarrow abAbB \Rightarrow abbBbbB$$

$$\Rightarrow abbbbB \Rightarrow abbbb$$

Rightmost derivation:

$$S \Rightarrow aAB \Rightarrow aA \Rightarrow abBb \Rightarrow abAb$$

$$\Rightarrow abbBbb \Rightarrow abbbb$$

Review

Show three different derivations using the grammar shown below:

1. $S \to a\ S\ A$
2. $S \to B\ A$
3. $A \to a\ b$
4. $B \to b\ A$

Solution:

$S \Rightarrow a\ S\ A \Rightarrow a\ B\ A\ A \Rightarrow a\ B\ a\ b\ A \Rightarrow a\ B\ a\ b\ a\ b \Rightarrow a\ b\ A\ a\ b\ a\ b \Rightarrow a\ b\ a\ b\ a\ b\ a\ b$

$S \Rightarrow a\ S\ A \Rightarrow a\ S\ a\ b \Rightarrow a\ B\ A\ a\ b \Rightarrow a\ b\ A\ A\ a\ b \Rightarrow a\ b\ a\ b\ A\ a\ b \Rightarrow a\ b\ a\ b\ a\ b\ a\ b$

$S \Rightarrow B\ A \Rightarrow b\ A\ A \Rightarrow b\ a\ b\ A \Rightarrow b\ a\ b\ a\ b$

Note that in the solution to this problem we have shown that it is possible to have more than one derivation for the same string: **ababab**.

Classes of Grammars

Figure 3.1: Classes of grammars

$A, B, C, ...$	A single nonterminal
$a, b, c, ...$	A single terminal
$..., X, Y, Z$	A single terminal or nonterminal
$..., x, y, z$	A string of terminals
α, β, γ	A string of terminals and nonterminals

Classes of Grammars

- 0. **Unrestricted**: An unrestricted grammar is one in which there are no restrictions on the rewriting rules.
- 1. **Context-Sensitiv**e: A context-sensitive grammar is one in which each rule must be of the form:

$\alpha A\gamma \rightarrow \alpha\beta\gamma$ where each of α, β, and γ is any string of terminals and nonterminals

- 2. **Context-Free**: A context-free grammar is one in which each rule must be of the form:

$A \rightarrow \alpha$ where A represents a single nonterminal and α is any string of terminals and nonterminals.

- 3. **Right Linear**: A right linear grammar is one in which each rule is of the form:

$A \rightarrow aB$ or $A \rightarrow a$

Classify each of the following grammar rules according to Chomsky's classification of grammars (in each case give the largest - i.e. most restricted - classification type that applies):

1. $aSb \rightarrow aAcBb$
2. $B \rightarrow aA$
3. $S \rightarrow aBc$
4. $S \rightarrow aBc$
5. $Ab \rightarrow b$
6. $AB \rightarrow BA$

1. *Type 1, Context-Sensitive*
2. *Type 3, Right Linear*
3. *Type 0, Unrestricted*
4. *Type 2, Context-Free*
5. *Type 1, Context-Sensitive*
6. *Type 0, Unrestricted*

Give a right linear grammar for each of the languages of Sample Problem

- Strings over {0,1} containing an odd number of 0's.
- 1. S → 0
- 2. S → 1S
- 3. S → 0A
- 4. A → 1
- 5. A → 1A
- 6. A → 0S

- Strings over {0,1} which contain exactly three 0's.
- 1. S → 1S
- 2. S → 0A
- 3. A → 1A
- 4. A → 0B
- 5. B → 1B
- 6. B → 0C
- 7. B → 0
- 8. C → 1C
- 9. C → 1

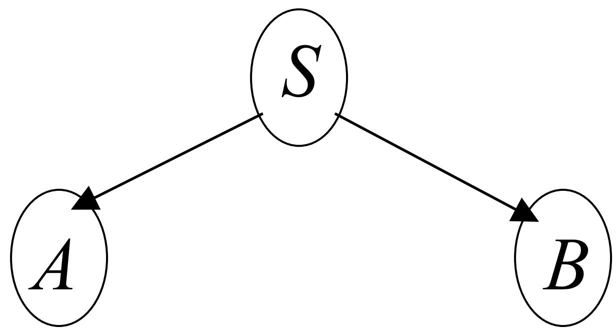
Derivation Tree

Derivation Tree is a tree in which each **node** corresponds to a **nonterminal** in a sentential form and each **leaf node** corresponds to a **terminal** symbol in the derived string.

Derivation Tree for aab

$$S \to AB \qquad A \to aaA \mid \lambda \qquad B \to Bb \mid \lambda$$

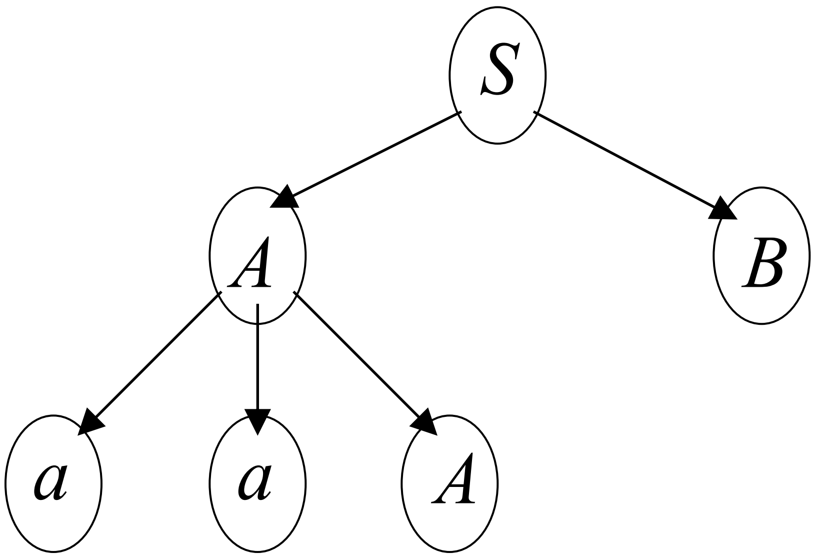
$$S \Rightarrow AB$$



Derivation Tree for aab

$$S \rightarrow AB \qquad A \rightarrow aaA \mid \lambda \qquad B \rightarrow Bb \mid \lambda$$

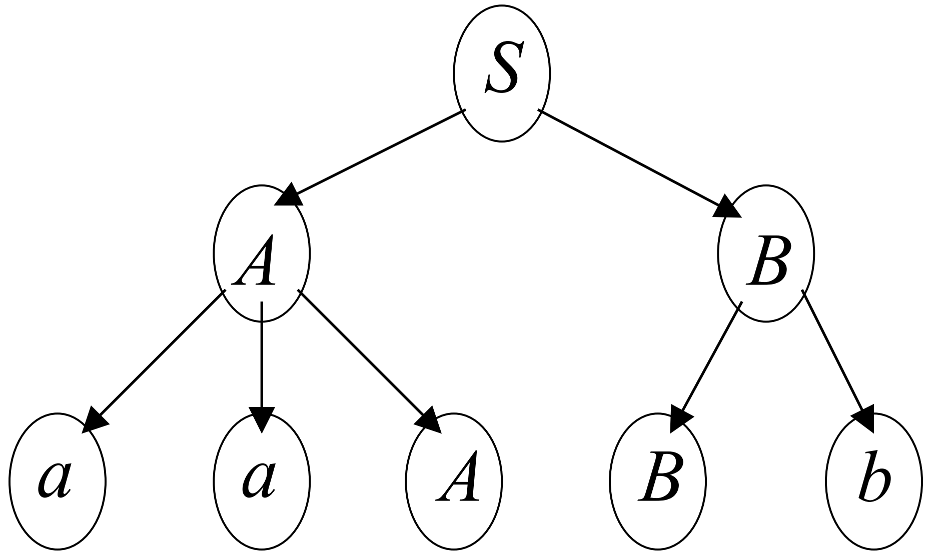
$$S \Rightarrow AB \Rightarrow aaAB$$



Derivation Tree for aab

$$S \to AB \qquad A \to aaA \mid \lambda \qquad B \to Bb \mid \lambda$$

$$S \Rightarrow AB \Rightarrow aaAB \Rightarrow aaABb$$

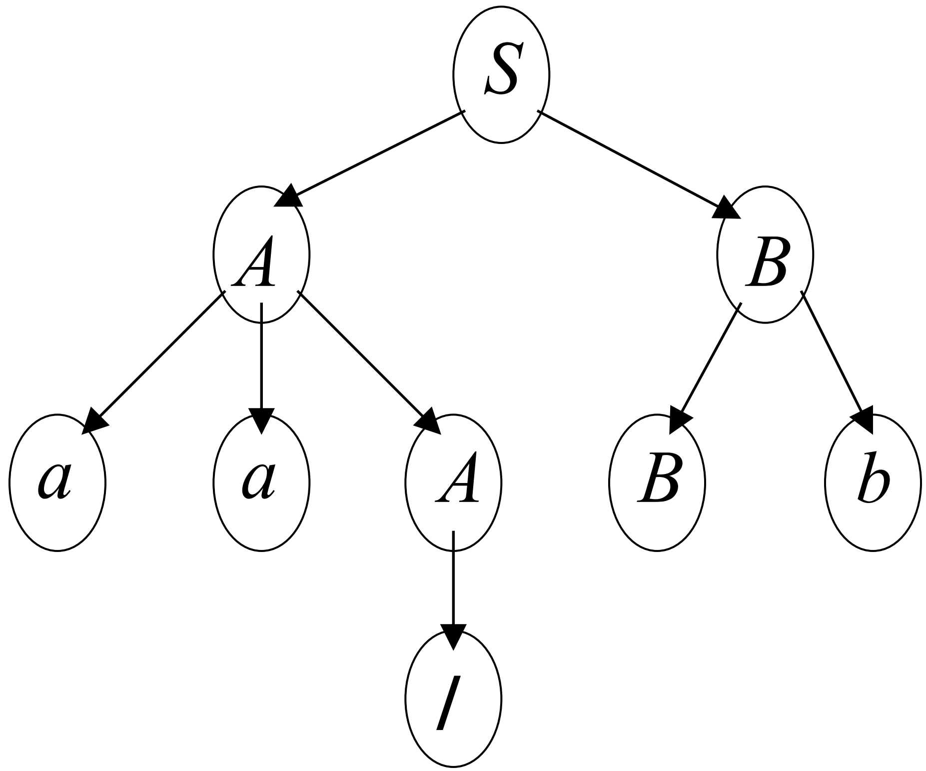


Grammar

Derivation Tree for aab

$$S \rightarrow AB \qquad A \rightarrow aaA \mid \lambda \qquad B \rightarrow Bb \mid \lambda$$

$$S \Rightarrow AB \Rightarrow aaAB \Rightarrow aaABb \Rightarrow aaBb$$

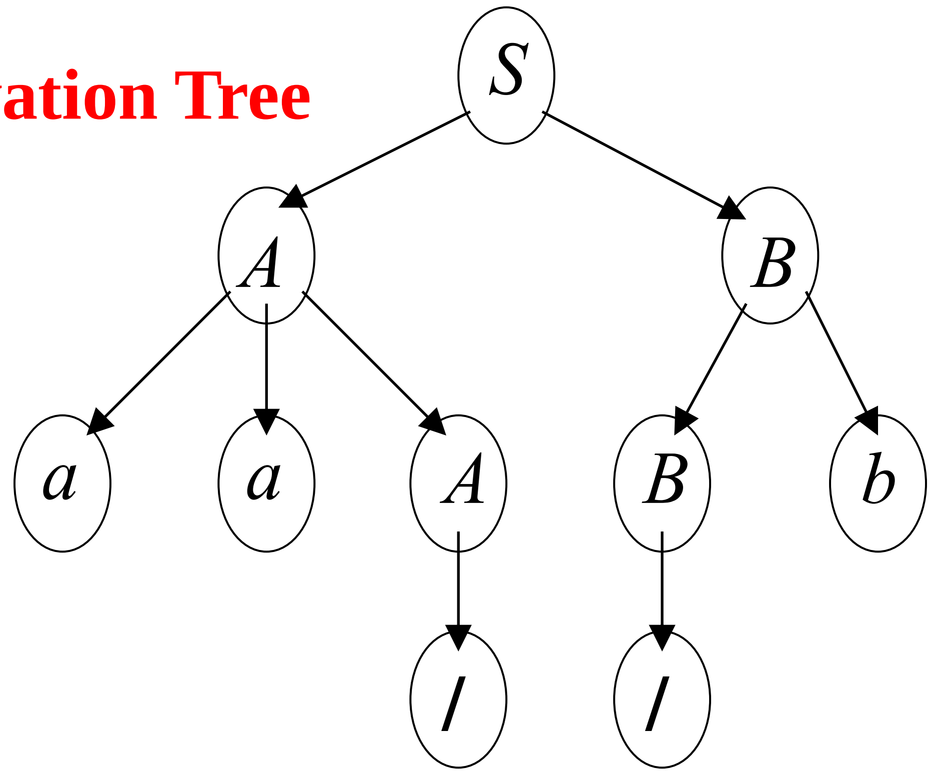


Derivation Tree for aab

$$S \rightarrow AB \qquad A \rightarrow aaA \mid \lambda \qquad B \rightarrow Bb \mid \lambda$$

$$S \Rightarrow AB \Rightarrow aaAB \Rightarrow aaABb \Rightarrow aaBb \Rightarrow aab$$

Derivation Tree

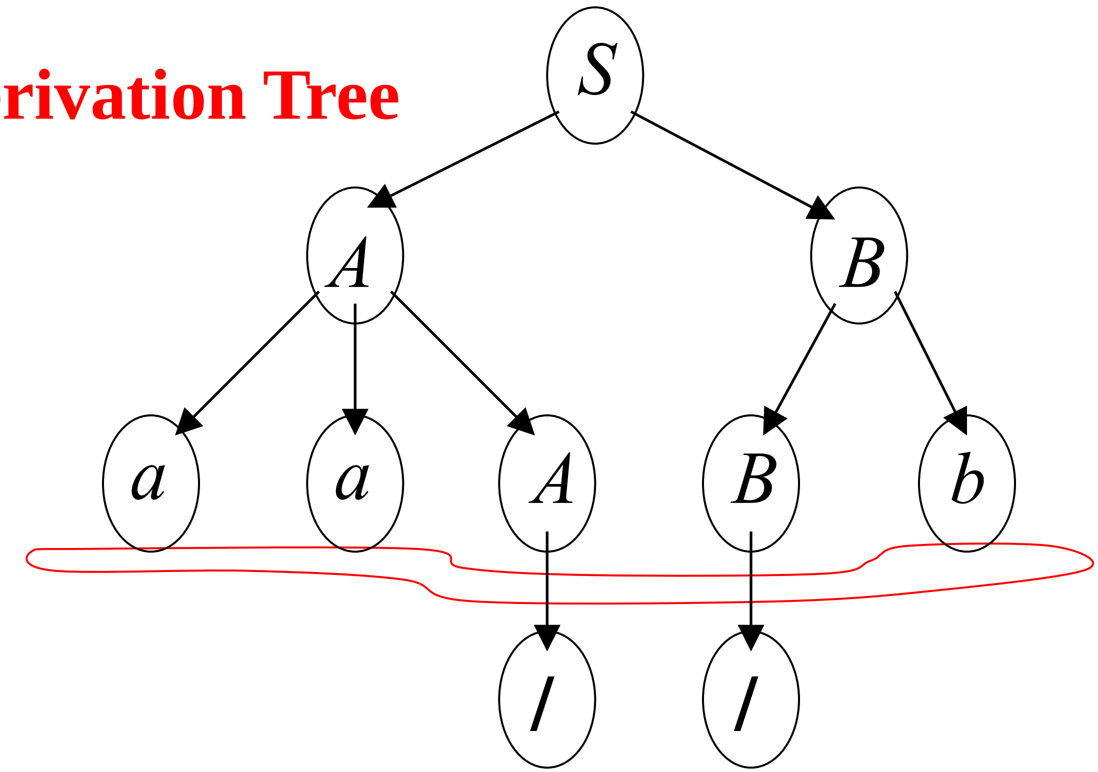


Derivation Tree for aab

$$S \rightarrow AB \qquad A \rightarrow aaA \mid \lambda \qquad B \rightarrow Bb \mid \lambda$$

$$S \Rightarrow AB \Rightarrow aaAB \Rightarrow aaABb \Rightarrow aaBb \Rightarrow aab$$

Derivation Tree



yield

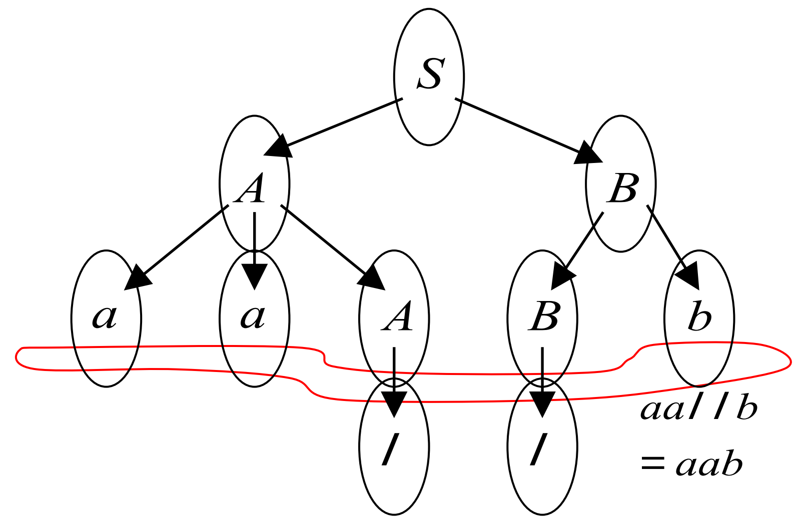
$$aa\lambda\lambda b$$

$$= aab$$

Derivation Tree for aab

$$S \rightarrow AB \qquad A \rightarrow aaA \mid \lambda \qquad B \rightarrow Bb \mid \lambda$$

Derivation Tree



Leftmost:

$$S \Rightarrow AB \Rightarrow aaAB \Rightarrow aaB \Rightarrow aaBb \Rightarrow aab$$

Rightmost:

$$S \Rightarrow AB \Rightarrow ABb \Rightarrow Ab \Rightarrow aaAb \Rightarrow aab$$

Ambiguous Grammar

A context-free grammar G is ambiguous
if there is a string $w \in L(G)$ which has:

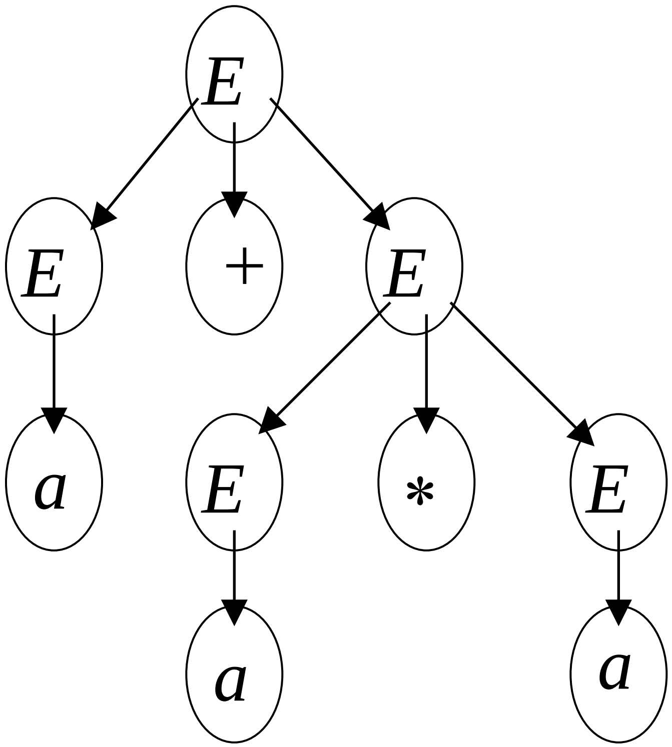
two different derivation trees
or
two leftmost derivations

(Two different derivation trees give two
different leftmost derivations and vice-versa)

Ambiguity

A context-free grammar is said to be **ambiguous** if there is **more than one derivation tree for a particular string**.

$$\boxed{E \to E + E \ \mid E * E \mid (E) \mid a}$$



A leftmost derivation for

$$\boxed{a + a * a}$$

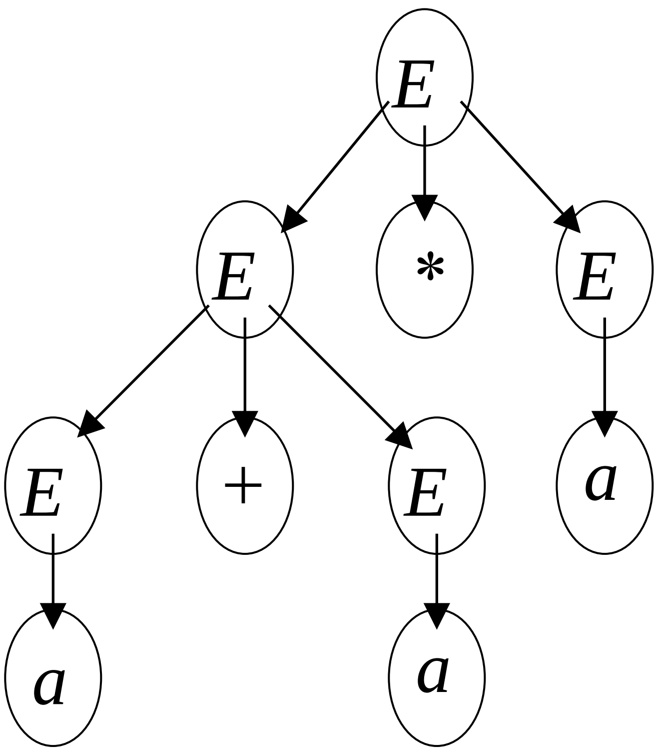
$$E \Rightarrow E + E \Rightarrow a + E \Rightarrow a + E * E$$

$$\Rightarrow a + a * E \Rightarrow a + a * a$$

Ambiguity

A context-free grammar is said to be **ambiguous** if there is **more than one derivation tree for a particular string**.

$$\boxed{E \to E + E \;\mid\; E * E \;\mid\; (E) \;\mid\; a}$$



Another leftmost derivation for

$$\boxed{a + a * a}$$

$$E \Rightarrow E * E \Rightarrow E + E * E \Rightarrow a + E * E$$

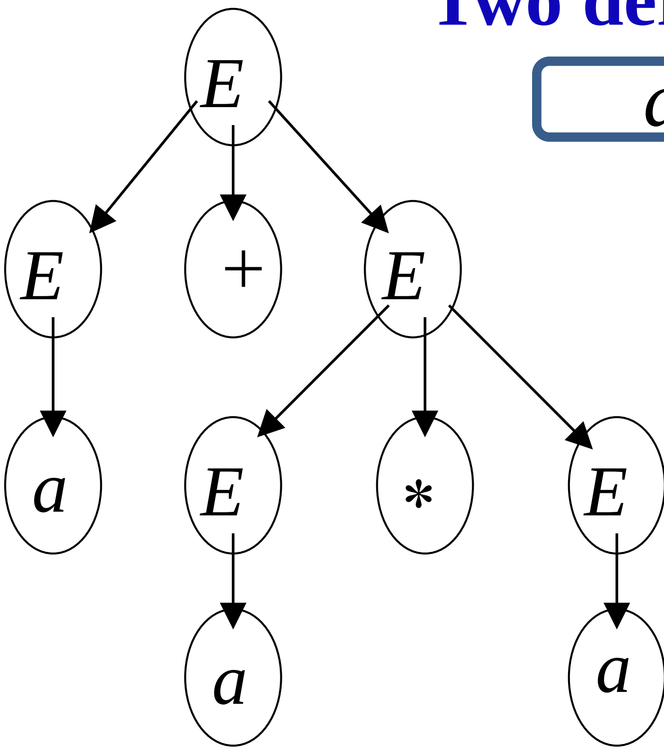
$$\Rightarrow a + a * E \Rightarrow a + a * a$$

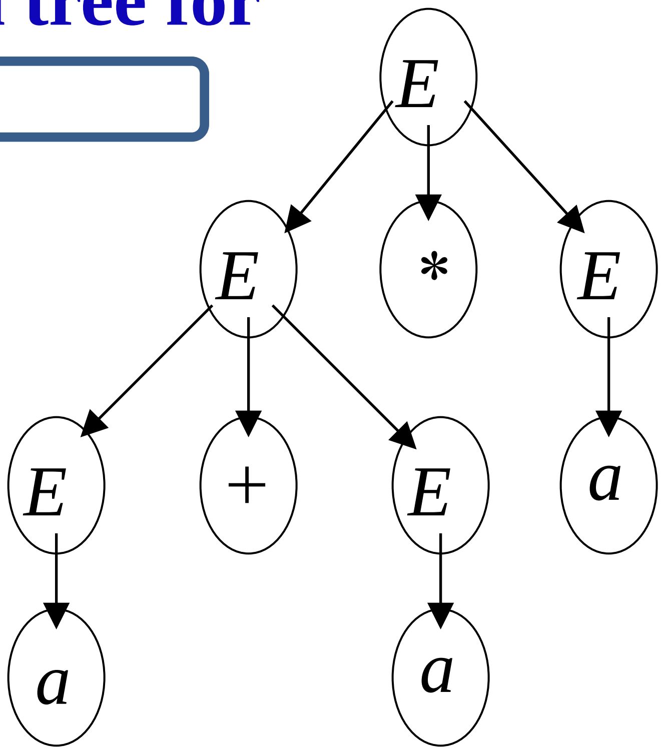
Ambiguity

$$E \rightarrow E+E \;\mid\; E*E \;\mid\; (E) \;\mid\; a$$

Two derivation tree for

$$\boxed{a+a*a}$$

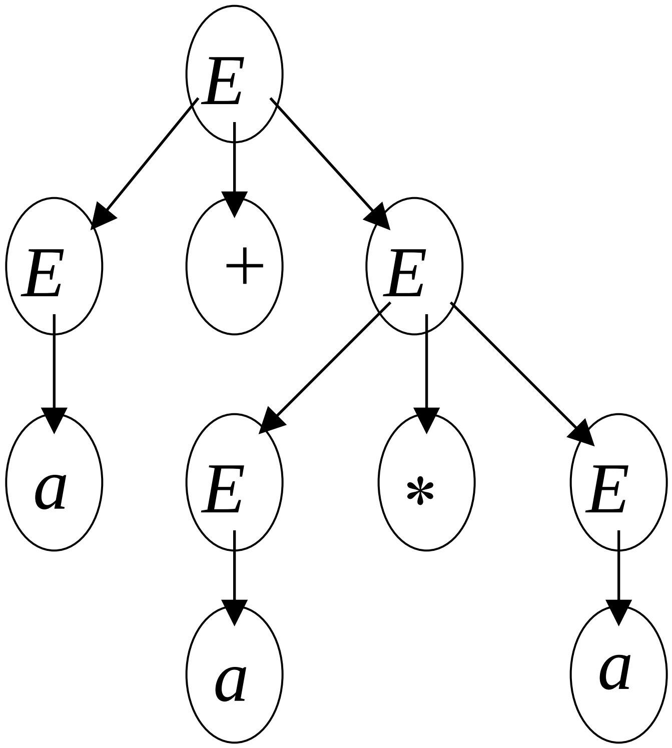


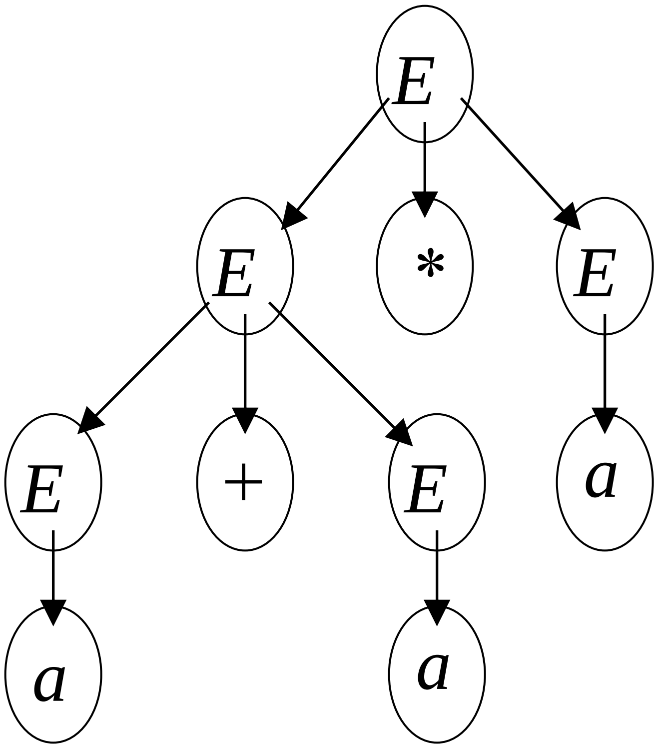


Ambiguity

take $a = 2$

$$a + a * a = 2 + 2 * 2$$





Ambiguity

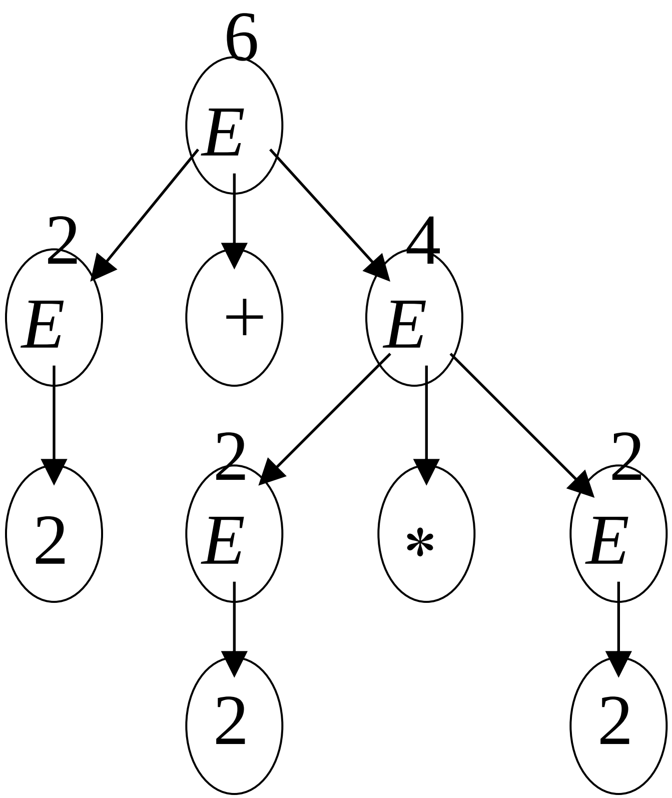
Good Tree

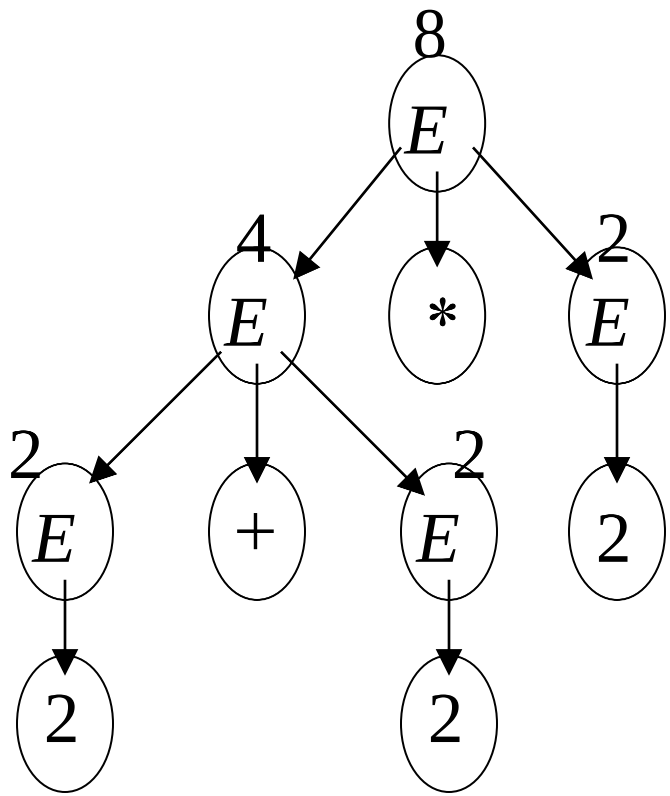
$$2+2*2=6$$

Compute expression result using the tree

Bad Tree

$$2+2*2=8$$





Ambiguity

A successful example:

Ambiguous Grammar

$$E \to E + E$$
$$E \to E * E$$
$$E \to (E)$$
$$E \to a$$

Equivalent

Non-Ambiguous Grammar

$$E \to E + T \mid T$$
$$T \to T * F \mid F$$
$$F \to (E) \mid a$$

Grammar

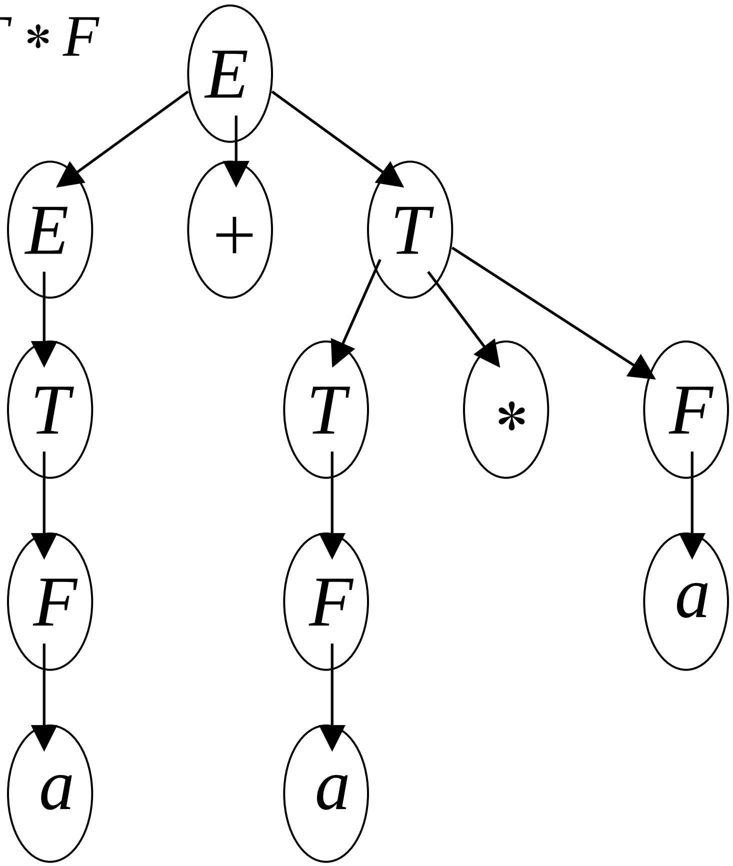
Ambiguity

$$E \Rightarrow E+T \Rightarrow T+T \Rightarrow F+T \Rightarrow a+T \Rightarrow a+T*F$$
$$\Rightarrow a+F*F \Rightarrow a+a*F \Rightarrow a+a*a$$

$$\boxed{\begin{aligned} E &\to E+T \mid T \\ T &\to T*F \mid F \\ F &\to (E) \mid a \end{aligned}}$$

Unique derivation tree for

$$\boxed{a + a * a}$$



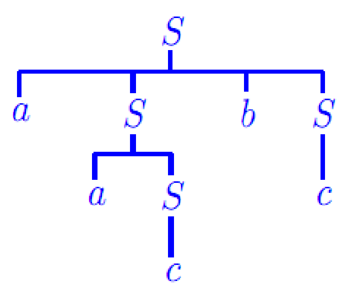
Example

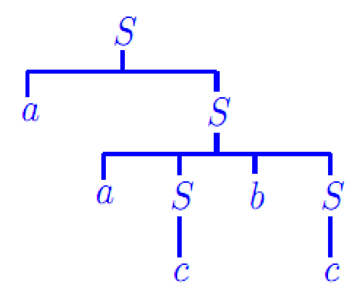
Determine whether the following grammar is ambiguous. If so, show two different derivation trees for the same string of terminals, and show a left-most derivation corresponding to each tree.

$$S \rightarrow aSbS$$

$$S \rightarrow aS$$

$$S \rightarrow c$$





$$S \Rightarrow a\ S\ b\ S \Rightarrow a\ a\ S\ b\ S \Rightarrow a\ a\ c\ b\ S \Rightarrow a\ a\ c\ b\ c$$

$$S \Rightarrow a\ S \Rightarrow a\ a\ S\ b\ S \Rightarrow a\ a\ c\ b\ S \Rightarrow a\ a\ c\ b\ c$$

Outline

- Syntax Analysis
- Grammar
- **Pushdown Machine**
- Parser

Pushdown Machine

Pushdown Machine

A **Pushdown machines** can be used for syntax analysis, just as finite state machines are used for lexical analysis.

A pushdown machine consists of:

- ✓ **A finite set of states**
- ✓ **A finite set of input symbols**
- ✓ **An infinite stack**
- ✓ **A state transition function**

Pushdown Machine

Pushdown Machine

- **Rows** are labeled by **stack symbols** and the **columns** are labeled by **input symbols**.
- ← character is used as an **endmarker**, indicating the end of the input string,
- ∇ symbol is a stack symbol which we are using to mark the bottom of the stack so that we can test for the **empty stack** condition.
- **Each cell** of those tables shows a **stack operation** (**push() or pop**), an input pointer **function** (**advance or retain**), and the **next state**. **Accept and Reject** are exits from the machine.

- A state transition function which takes as arguments the **current state**, the **current input symbol**, and the **symbol currently on top of the stack**; its result is the **new state of the machine**.
- On each state transition the machine may perform one of the stack operations, **push(X)** or **pop**, where **X** is one of the **stack symbols**.
- A state transition may include an exit from the machine labeled either **Accept** or **Reject**

Pushdown Machine

Example

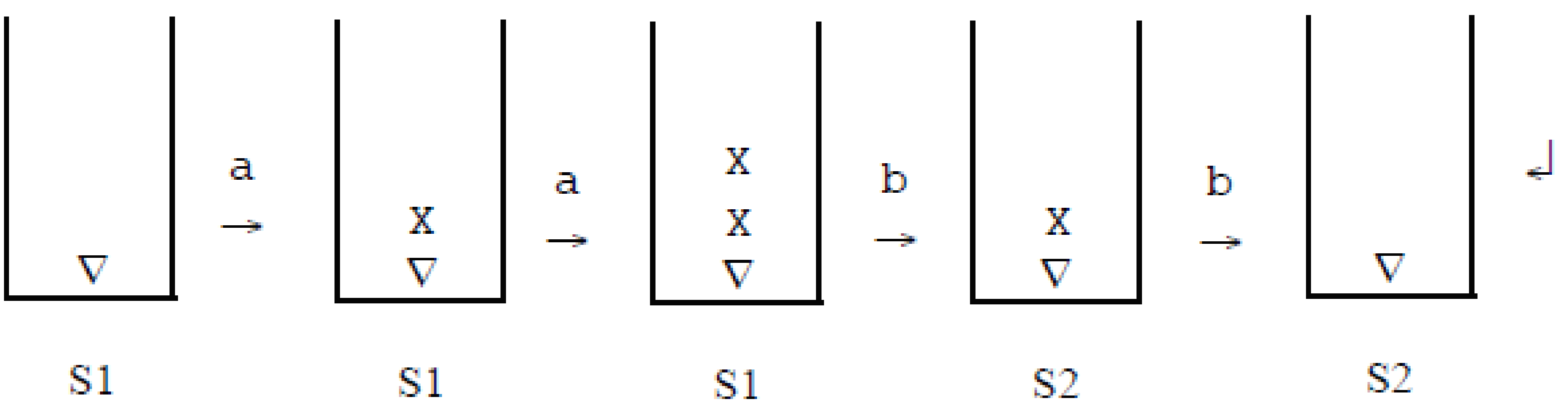
S1	a	b	↵
X	Push (X) Advance S1	Pop Advance S2	Reject
∇	Push (X) Advance S1	Reject	Accept

S2	a	b	↵
X	Reject	Pop Advance S2	Reject
∇	Reject	Reject	Accept

Initial Stack

$$S \rightarrow ASB$$
$$S \rightarrow \epsilon$$
$$A \rightarrow a$$
$$B \rightarrow b$$

Show the sequence of stacks for the input string aabb ←



Accept

Pushdown Machine

Example

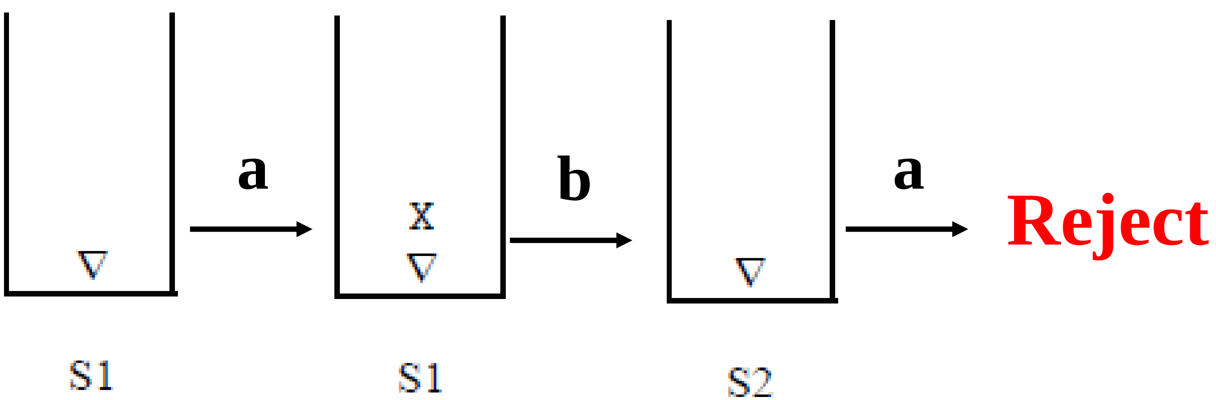
S1	a	b	↵
X	Push (X) Advance S1	Pop Advance S2	Reject
∇	Push (X) Advance S1	Reject	Accept

S2	a	b	↵
X	Reject	Pop Advance S2	Reject
∇	Reject	Reject	Accept

Initial Stack

$$S \rightarrow ASB$$
$$S \rightarrow \epsilon$$
$$A \rightarrow a$$
$$B \rightarrow b$$

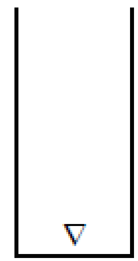
Show the sequence of stacks for the input string aba ←



Exercise

- pushdown machine to accept any string of well-balanced parentheses

S1	(	)	↲
X	Push (X) Advance S1	Pop Advance S1	Reject
∇	Push (X) Advance S1	Reject	Accept



Initial Stack

$$S \rightarrow (S)$$
$$S \rightarrow \lambda$$

- Show the sequence of stacks for the input string (()

Outline

- Syntax Analysis
- Grammar
- Pushdown Machine
- **Parser**

Parser

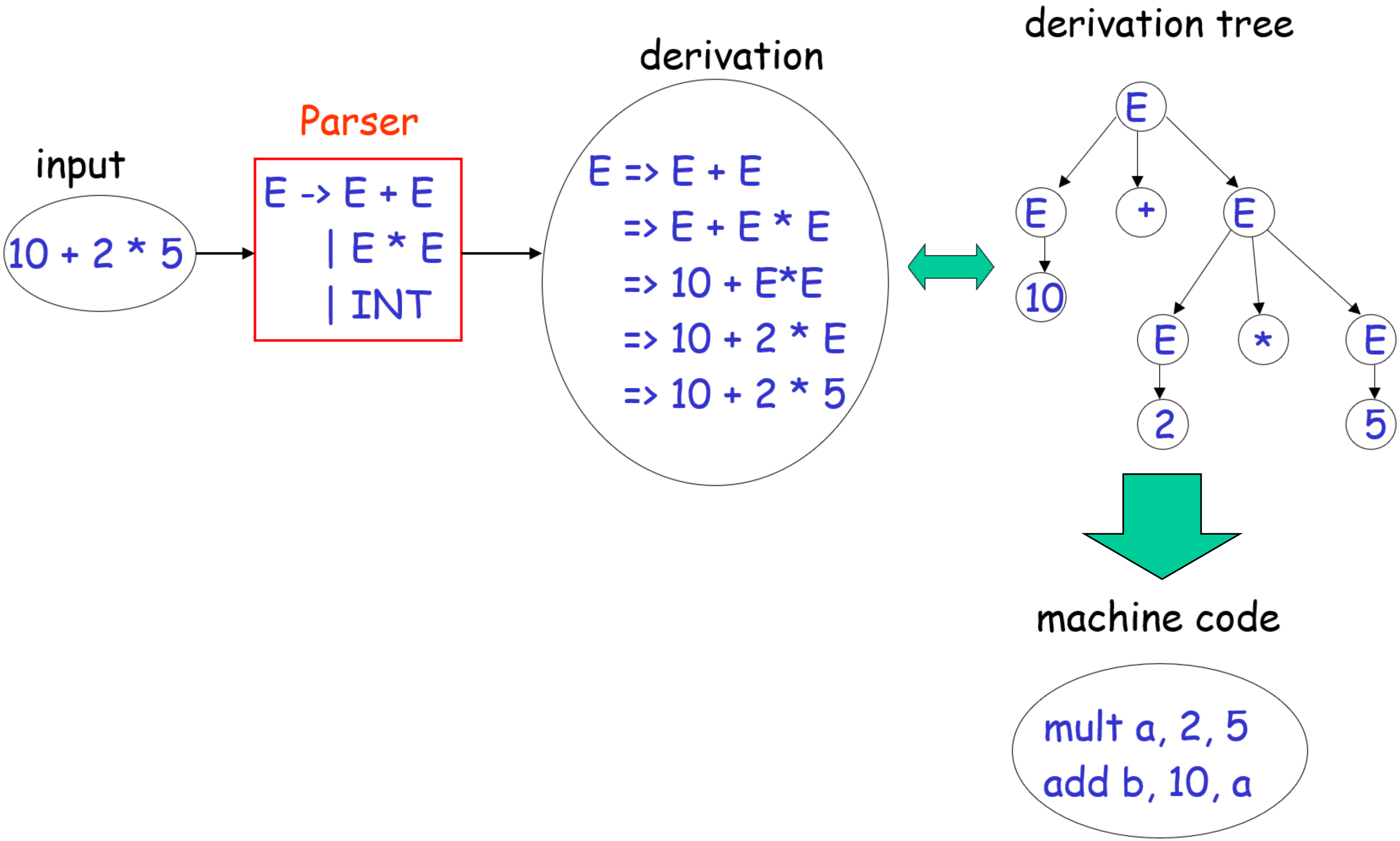
A parser knows the grammar of the programming language

```
PROGRAM → STMT_LIST
STMT_LIST→ STMT; STMT_LIST | STMT;
STMT→ EXPR | IF_STMT | WHILE_STMT
                | { STMT_LIST }

EXPR → EXPR + EXPR |  EXPR - EXPR | ID
IF_STMT→  if (EXPR) then STMT
          | if (EXPR) then STMT else STMT
WHILE_STMT→ while (EXPR) do STMT
```

Parser

Parser



Pushdown Translator

- An infix expression is one in which the operation is placed between the two operands.

- A postfix expression is one in which the two operands precede the operation:

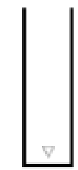
Infix	**Postfix**
2 + 3	**2 3 +**
2 + 3 * 5	**2 3 5 * +**
2 * 3 + 5	**2 3 * 5 +**
(2 + 3) * 5	**2 3 + 5 ***

Pushdown Translator

S1	a	+	*	(	)	↵
E	Reject	push(+)	push(*)	Reject	pop retain S3	pop retain
E_p	Reject	pop out(+)	push(*)	Reject	pop retain S2	pop retain S2
L	push(E) out(a)	Reject	Reject	push(L)	Reject	Reject
L_p	push(E) out(a)	Reject	Reject	push(L)	Reject	Reject
L_s	push(E) out(a)	Reject	Reject	push(L)	Reject	Reject
+	push(E_p) out(a)	Reject	Reject	push(L_p)	Reject	Reject
*	pop out(a*)	Reject	Reject	push(L_s)	Reject	Reject
∇	push(E) out(a)	Reject	Reject	push(L)	Reject	Accept

S2	)	↵
+	pop out(+) retain,S3	pop out(+) retain,S1
*	pop out(*) S1	Reject

S3	)
L	Rep(E) S1
L_p	Rep(E_p) S1
E	pop retain
L_s	pop retain S2
∇	Reject

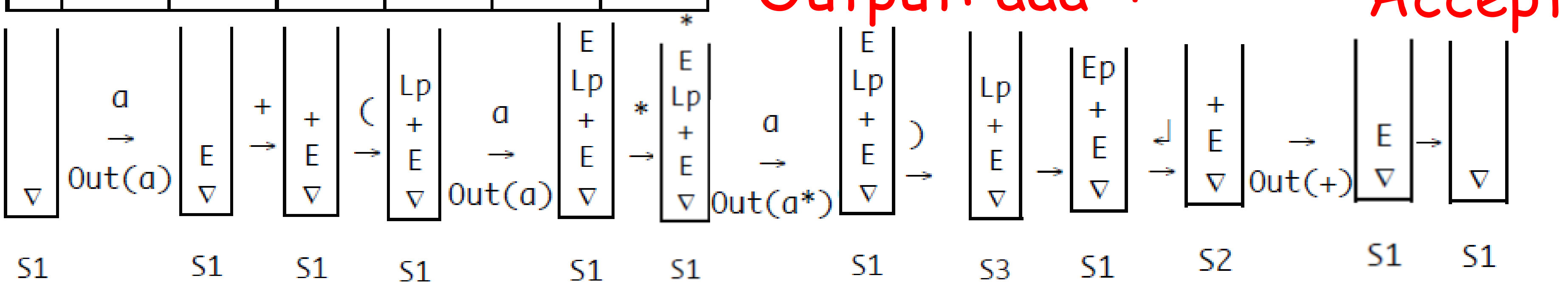


Initial Stack

- Show the sequence of stacks and states which the pushdown machine of Figure would go through if the input were: a+(a*a)

Output: aaa*+

Accept



THANKS

for your attention