

# PROBLEM #1

$$(a) P(\text{fruit}) = \frac{2}{3}$$

$$P(\text{1 fruit}) = \frac{3}{5}$$

$$P(\text{orange} | \text{fruit}) = \frac{1+1}{6+9} = \frac{2}{15}$$

$$P(\text{orange} | \text{not fruit}) = \frac{4+1}{11+9} = \frac{5}{20}$$

$$P(\text{Mango} | \text{fruit}) = \frac{1+1}{6+9} = \frac{2}{15}$$

$$P(\text{Mango} | \text{not fruit}) = \frac{0+1}{11+9} = \frac{1}{20}$$

$$P(\text{Peach} | \text{fruit}) = \frac{0+1}{6+9} = \frac{1}{15}$$

$$P(\text{Mango} | \text{not fruit}) = \frac{7+10}{11+9} = \frac{17}{20}$$

$$\frac{0+1}{12+10} = \frac{1}{22}$$

$$P(\text{Red} \mid \text{fruit}) = \frac{0+1}{6+9} = \frac{1}{15}$$

$$P(\text{Red} \mid \text{7 fruit}) = \frac{2+1}{19+9} = \frac{3}{28} = \frac{3}{28}$$

$$P(\text{lemon} \mid \text{fruit}) = \frac{0+1}{6+9} = \frac{1}{15}$$

$$P(\text{lemon} \mid \text{7 fruit}) = \frac{1+1}{19+9} = \frac{2}{28} = \frac{1}{14}$$

$$P(\text{Yellow} \mid \text{fruit}) = \frac{0+1}{6+9} = \frac{1}{15}$$

$$P(\text{Yellow} \mid \text{7 fruit}) = \frac{2+1}{19+9} = \frac{3}{28} = \frac{3}{28}$$

$$P(\text{Blue fruit}) = \frac{0+1}{6+9} = \frac{1}{15}$$

$$P(\text{Blue } \cap \text{ fruit}) = \frac{1+1}{19+9} = \frac{2}{20} = \frac{1}{10}$$

$$P(\text{Apricot } | \text{ fruit}) = \frac{1+1}{6+9} = \frac{2}{15}$$

$$P(\text{Apricot } \cap \text{ fruit}) = \frac{0+1}{19+9} = \frac{1}{20}$$

$$P(\text{Apple } | \text{ fruit}) = \frac{2+1}{6+9} = \frac{3}{15} = \frac{1}{5}$$

$$P(\text{Apple } \cap \text{ fruit}) = \frac{0+1}{19+9} = \frac{1}{20}$$

$$P(\text{Banana } | \text{ fruit}) = \frac{1+1}{6+9} = \frac{2}{15}$$

$$P(\text{Banana } \cap \text{ fruit}) = \frac{0+1}{19+9} = \frac{1}{20}$$

$$P(\text{fruit} | d6) = P(\text{fruit}) \times P(\text{orange fruit}) \\ \times P(\text{Mango fruit}) \\ \times P(\text{Melon fruit})$$

$$= \frac{3}{5} \times \frac{2}{15} \times \frac{2}{15} \times \frac{1}{15} \\ = 0.0004418$$

$$P(\text{1 fruit} | d6) = P(\text{1 fruit}) \times P(\text{orange 1 fruit}) \\ \times P(\text{Mango 1 fruit}) \\ \times P(\text{Melon 1 fruit})$$

$$= \frac{3}{5} \times \frac{5}{20} \times \frac{1}{20} \times \frac{1}{22} \\ = 0.000481 \\ = 0.000340$$

$d6 \rightarrow$  class fruit "No" "Yes"

$$P(\text{fruit} | d7) = P(\text{fruit}) \times P(\text{Orange} | \text{fruit}) \\ \times P(\text{Red} | \text{fruit}) \\ \times P(\text{Lemon} | \text{fruit}) \\ \times P(\text{Yellow} | \text{fruit})$$

$$= \frac{2}{5} \times \frac{2}{15} \times \frac{1}{15} \times \frac{1}{15} \times \frac{1}{15} \\ \approx 0.0158 \times 10^{-3}$$

$$P(7 \text{ fruit} | d7) = P(7 \text{ fruit}) \times P(\text{Orange} | 7\text{fruit}) \\ \times P(\text{Red} | 7\text{fruit}) \\ \times P(\text{Lemon} | 7\text{fruit}) \\ \times P(\text{Yellow} | 7\text{fruit}) \\ = \frac{3}{5} \times \frac{3}{20} \times \frac{3}{20} \times \frac{1}{10} \times \frac{3}{20} \\ \approx 0.3644 \times 10^{-3} \\ \approx 0.3375 \times 10^{-3}$$

$d7 \rightarrow$  class fruit "No"

## PROBLEM #2

R)

Concept drift refers to the gradual change over time of the concept underlying a class.  
such as US President from Barack Obama to Trump.

The Bernoulli model is particularly robust with respect to concept drift. It can have good performance when the feature size is small. The most important indicator for a class are less likely to change. Thus, a model that only relies on these features is more likely to maintain a certain level of accuracy in concept drift.

Answer: 2B:

The NB Classifier makes the conditional independence assumption to reduce the number of parameters. i.e. the presence of one term does not affect others. This assumption is very naive for a language model. That's why it is called naive.

Just because the prediction is correct, it does not mean that the estimate and true probability are also same, as the probabilities are directly proportional not equal, but correct estimation does imply accurate prediction.

## Answer 2c

If a document terms do not provide clear evidence for one class versus another, we choose the one that has a higher prior probability.

If one term is not present in the training set, we don't consider its probability 0, as that will reduce all terms to 0. Instead we use Laplace smoothing to handle unseen term

# PROBLEM #3

$$tf \times idf$$

~~Apple, Orange, Lemon, Red~~

$$\text{Taking } Tf = 1 + \log_2 (tf_{t,1} + 1)$$

$$idf = \log_2 \left( \frac{N}{df_{t,1}} \right)$$

d) Orange  $\rightarrow$   $1 + \log_2 (tf_{t,2} + 1)$   
 $= 1 + \log_2 (2 + 1)$   
 $= 1 + 1.386$   
 $= 2.386$

$$idf = \log_2 \left( \frac{5}{1+1} \right) = \cancel{\log_2} 0 \cancel{0}$$

< d) Lemon  $\rightarrow$   $1 + \log_2 (tf_{t,2} + 1)$   
 $= 1 + 1$   
 $= 2$

$$idf = \log_2 \left( \frac{5}{1+1} \right) = \cancel{\log_2} 1.32$$

d) Red  $\rightarrow$   $1 + \log_2 (tf_{t,2} + 1)$   
 $= 1 + \log_2 (1 + 1)$   
 $= 2$

$$idf = \log_2 \left( \frac{5}{1} \right) = \cancel{\log_2} 0.736$$

$$idf:$$

Blue	Yellow	Abricot	Apple	Mango
$\log_2 \left( \frac{5}{3} \right)$	$\log_2 \left( \frac{5}{3} \right)$	$\log_2 \left( \frac{5}{2} \right)$	$\log_2 \left( \frac{5}{3} \right)$	$\log_2 \left( \frac{5}{2} \right)$
$= 1.386$	$= 0.736$	$= 1.32$	$= 0.736$	$= 1.32$
banana				

	Orange	Lemon	Red	Blue	Yellow	Apricot	Apple	Mango	Kawar	Melon
$d_1 <$	0.264	0.286	1.2464	10.736	0.736	1.32	0.736	1.32	1.32	2.32
$d_2 <$	0.02	0.02	1.464	1.2464	1.32	0.736	1.32	1.32	1.32	2.32
$d_3 <$	0.22	0.22	0.736	0.736	0.736	0.736	0.736	0.736	0.736	2.32
$d_4 <$	0.22	0.22	0.736	0.736	0.736	0.736	0.736	0.736	0.736	2.32
$d_5 <$	0.02	0.02	1.32	1.32	1.32	1.32	1.32	1.32	1.32	2.32
$d_6 <$	0.264	0.286	0.736	0.736	0.736	0.736	0.736	0.736	0.736	4.64
$d_7 <$	0.264	0.286	1.2464	1.2464	1.2464	1.2464	1.2464	1.2464	1.2464	2.32

$$\text{Molou} \rightarrow \log_2 \left( \frac{5}{0+1} \right) = 2.32$$

b)

$$El_{\text{fruit}=\text{Yes}} = \frac{1}{2} \begin{pmatrix} 0.264, 1.464, 1.464 \\ 1.464, 3.96, 2.2, 3.96, 3.96 \end{pmatrix}$$

$$= \begin{pmatrix} 0, 1.32, 0.736, 0.736, 0.736 \\ 1.98, 1.1, 1.98, 1.98 \end{pmatrix}$$

$$El_{\text{fruit}=\text{No}} = \frac{1}{3} \begin{pmatrix} 0, 5.28, 3.664, 2.928, 3.664 \\ 1.32 \times 3, 0.736 \times 3, 1.32 \times 3, 1.32 \times 3 \end{pmatrix}$$

$$= \begin{pmatrix} 0, 1.76, 1.22, 0.976, 1.22 \\ 1.32, 0.736, 1.32, 1.32 \end{pmatrix}$$

c)

$$|\vec{El}_{\text{Yes}} - \vec{d}_6| = \sqrt{0.4356 + 0.132 + 0.4356 + 0.4356 + 5.3824} = 2.611$$

$$|\vec{El}_{\text{No}} - \vec{d}_6| = \sqrt{0.1936 + 0.234 + 0.0576 + 0.2346 + 0.0576 + 5.3824} = 2.28$$

Hence  $d_6 \rightarrow^u \text{Yes}$

$$|\vec{d}_{no} - \vec{v}(d_7)| = \sqrt{0.7144 + 0.05936}$$

$$= \sqrt{(2.64 - 1.76)^2 + (1.464 - 1.22)^2 + (0.976 - 0.736)^2 + (1.22 - 0.781)^2}$$

$$= 1.06$$

Then  $d_7 \rightarrow "No"$

3(d)

$$\cos(\vec{d}_1, \vec{d}_6) = \frac{\vec{d}_1 \cdot \vec{d}_6}{|\vec{d}_1| |\vec{d}_6|} \quad \cancel{\phi}$$
$$= \frac{0.882 \cancel{1.00} 2349}{(3.00) (5.99)}$$
$$= \cancel{0.882} 0.864$$

$$\cos(\vec{d}_2, \vec{d}_6) = 0.882 \quad \cancel{0.88}$$

$$\cos(\vec{d}_3, \vec{d}_6) = 0.887 \quad \cancel{0.88}$$

$$\cos(\vec{d}_4, \vec{d}_6) = 0.878 \quad \cancel{0.88}$$

$$\cos(\vec{d}_5, \vec{d}_6) = 0.922 \quad \cancel{0.92}$$

---

	3
$d_5$	0.922
$d_3$	0.887
$d_2$	0.882

$d_6 \rightarrow$  "No" No

$$\cos(\delta_1, \delta_7) = 0.999$$

$$\cos(\delta_2, \delta_7) = 0.908$$

$$\cos(\delta_3, \delta_7) = 0.839$$

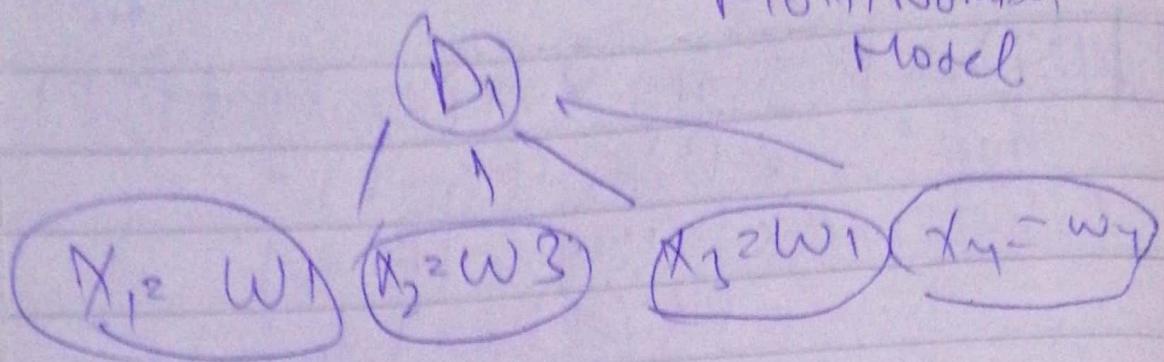
$$\cos(\delta_4, \delta_7) = 0.8688$$

$$\cos(\delta_5, \delta_7) = 0.912$$

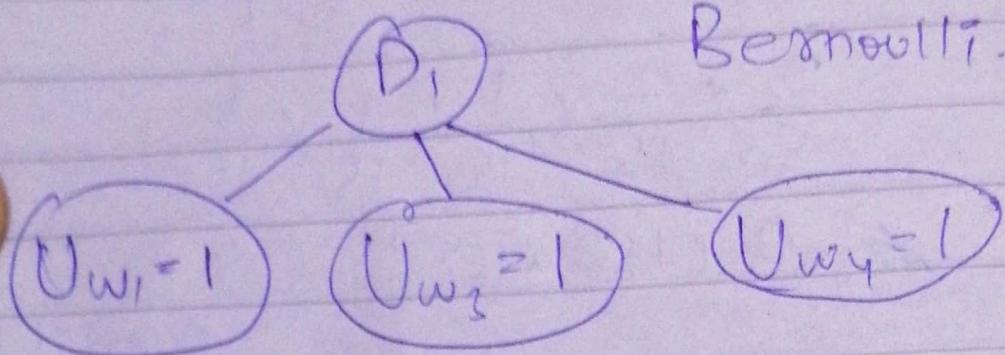
( ~~$\delta_6, \delta_7$~~ ,  $\delta_7$ )  $\rightarrow {}^u N_6$

## PROBLEM #2 D

Multinomial Model



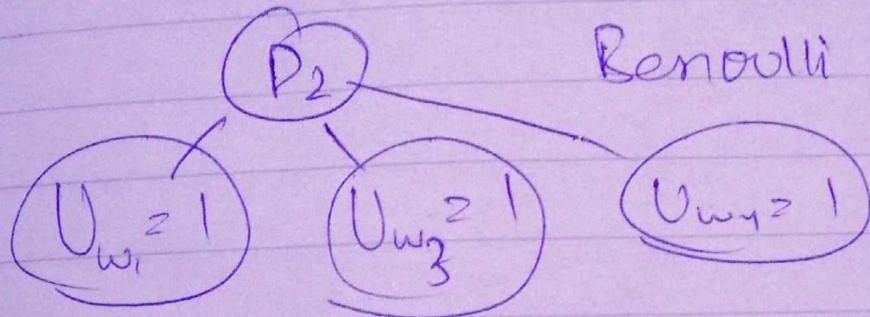
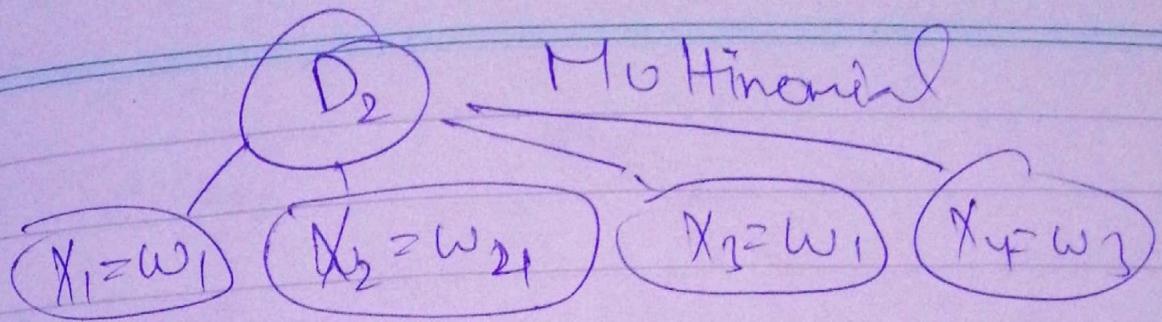
Bernoulli.



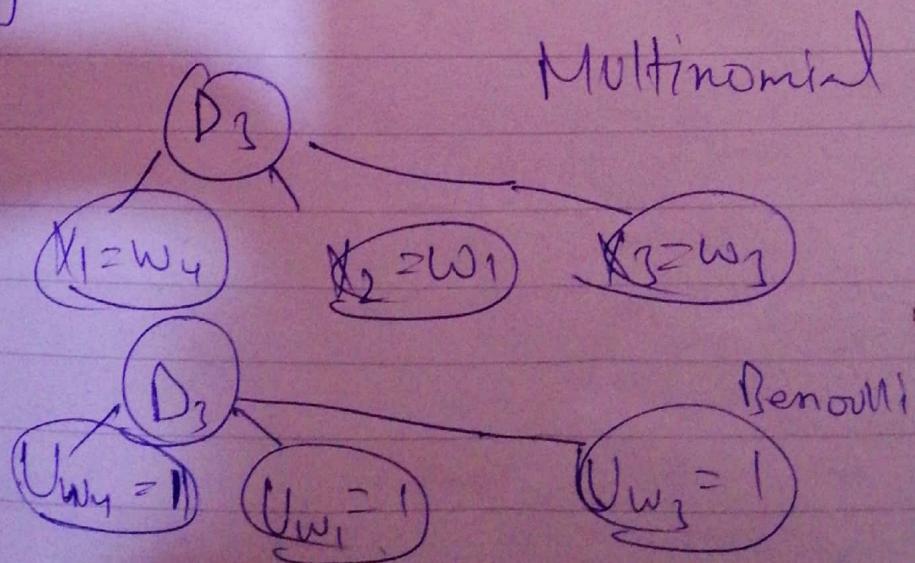
$D_1$  has different representation as bernoulli representation does not count frequency of features (terms) only if they are present or not.

Multinomial Rep is same for  $\emptyset_1, D_1$  and  $D_2$

Binomial Rep is same for all  $\emptyset, D_1, D_2$  and  $D_3$



$D_2$  has different representation as Multinomial representation considers frequency of features (count) while Bernoulli only consider presence regardless of count.



$D_3$  has same representation due to same term count.