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Simpson's 3/8 rule:

In numerical integration, Simpson's rules are several approximations for definite integrals, named after Thomas Simpson (1710–1761).

Simpson 3/8 rule for integration can be derived by approximating the given function f (x) with the 3rd order (cubic) polynomial f3(x) .

Simpson's 3/8 rule approximates the integral of f(x) from a to b. This rule is also known as Newton's 3/8 rule .Where a and b are the limits of integration ,in the form $\int$ f(x) dx.

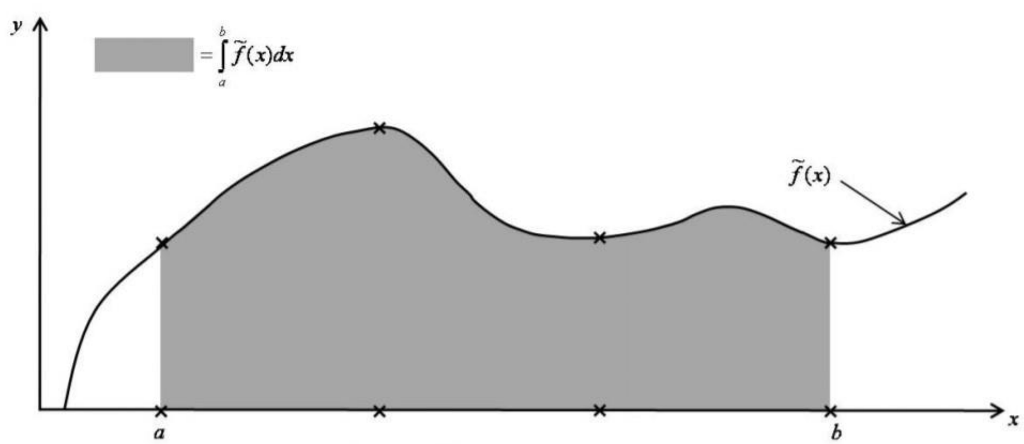

Figure 1 $\tilde{f}(x)$ Cubic function.

$F3(x)=a_0+a_1x^1+a_2x^2+a_3x^3$

$$=\{1,x1,x2,x3\}\begin{bmatrix} a0 \\ a1 \\ a2 \\ a3 \end{bmatrix} \text{ Equation 1}$$

The unknown coefficients a0,a1,a2 and a3 in Equation (3) can be obtained by substituting 4 known coordinate data points { x0, f(x0)},{ x1, f(x1)},{x3,f(x3)} and {x3, f(x3)} into Equation (3).

And if we Write the 4 subtitutions in matrix form :$x0^2$

$$\begin{bmatrix} 1 & x0^1 & x0^2 & x0^3 \\ 1 & x1^1 & x1^2 & x1^3 \\ 1 & x2^1 & x2^2 & x2^3 \\ 1 & x3^1 & x3^2 & x3^3 \end{bmatrix} \begin{bmatrix} a0 \\ a1 \\ a2 \\ a3 \end{bmatrix} = \begin{bmatrix} f(x0) \\ f(x1) \\ f(x2) \\ f(x3) \end{bmatrix}$$

$[A]_{4*4}\,\vec{a}_{4*1} = \vec{f}_{4*1}$

$a=[A]^{-1}\times f$

from Eq.1

$F3(X)= \{1,x1,x2,x3\}\times [A]^{-1}\times f$

From figure 1

X0=a

X1=a+h

$=a+\frac{b-a}{3}$

$=\frac{2a+b}{3}$

X2=a+2h

$=\frac{a+2b}{3}$

X3=a+3h

=b

I=$\int_a^b f(x)$

$\approx \int_a^b f3(x)$ =(b-a)× $\frac{\{f(x0)+3f(x1)+3(f(x2)+f(x3)\}}{8}$

Since

h=$\frac{b-a}{3}$

b-a=3h so equation becomes I≈ $\frac{3h}{8} \times \{f(x0)+f(x1)+f(x2)+f(x3)\}$

$\int ydx = \frac{3h}{8}(y_0+2(y_3+y_6+\ldots+y_{n-3})+3(y_1+y_2+y_4+y_5+\ldots+y_{n-2}+y_{n-1})+y_n)$

Example:

I=$\int_0^4 xe^{2x}dx \approx \frac{3h}{8}\left[f(0)+3f\left(\frac{4}{3}\right)+3f\left(\frac{8}{3}\right)+f(4)\right]$

=$\frac{3(4/3)}{8}[0+3(19.18922)+3(552.33933)+11923.832=6819.209$

Matlab code is given with the other file

Example :

Script21

Enter lower bound:0

Enter Upper bound:3

Enter n must be multiple of 3(must be multiple of 3):9

Value of approximating integration xe^2x from 0 to 3 is 506.5168