

## PROJECT

1. Consider the Parana dataset from the geoR library (`data(parana)`, `help(parana)`) and **Identify the region of interest, design, response variable and covariates, if any.** Plot what the map would look like

2. Consider elevation `data(elevation)` as a simple linear regression problem, with elevation as the response and north-south coordinate as the explanatory variable. Fit the model and examine the residuals of the model.

Do you consider that a more sophisticated model is necessary for the analysis of spatial variation?

Do you consider that a more sophisticated model is necessary for the analysis of spatial variation, justify your answer.

3. Consider the following models for a data set, whose response  $Y_i = 1, 2, \dots, n$  associated with a sequence of  $x_i$  positions along a spatial axis of one x-dimension.

a)  $Y_i = \alpha + \beta x_i + \epsilon_i$  where  $\alpha$  and  $\beta$  are parameters and the  $\epsilon_i$  are mutually independent with mean zero and variance  $\sigma_\epsilon^2$ .

b)  $Y_i = A + Bx_i + \epsilon_i$  where the  $\epsilon_i$  are as in the previous subsection, but A and B are random variables independent of each other and of the  $\epsilon_i$ , each with zero mean and variances  $\sigma_\alpha^2$  y  $\sigma_\beta^2$ , respectively

- For each of these models, find the expected value and variances of  $Y_i$ , the covariance between  $Y_i$  and

$Y_j$ , for  $i \neq j$ . Plot if possible

4. Look for and reproduce figure 3.2 from the book “Applied Spatial Data Analyses with R”.

5. Consider the following

$$Y \sim \text{Bern}(0.5), X|Y = 0 \sim \mathcal{N}((0, 0), \mathbb{A}) \text{ y } X|Y = 1 \sim \mathcal{N}((1, 2), \mathbb{A})$$

$$\mathbb{A} = \begin{pmatrix} 1 & 0.4 \\ 0.4 & 1 \end{pmatrix}$$

- a) Calculate the optimal Bayesian classifier for a symmetric cost function.
- b) Generate samples  $(X, Y)$  of size  $n = 50, 100, 500$ .
- c) Find and discuss suitable decision trees for these data and compare the best models obtained with cross-validation with the optimal Bayesian classifier, **both visually and quantitatively**.