Lab 8: Decomposition Methods

LU Decomposition:

Step 1(LU decomposition step):

Coefficient matrix $[\boldsymbol{A}]$ is factored or "decomposed" into lower $[\boldsymbol{L}]$ and upper $[\boldsymbol{U}]$ triangular matrices.

$$[\boldsymbol{A}][\boldsymbol{X}] = \boldsymbol{B}$$

$$[\boldsymbol{L}][\boldsymbol{U}][\boldsymbol{X}] = \boldsymbol{B}$$

Step 2(Substitution step):

$[\boldsymbol{L}]$ and $[\boldsymbol{U}]$ are used to determine a solution $[\boldsymbol{X}]$ for a right-hand side $\boldsymbol{B}$.

This step itself consists of two steps.

1. First an intermediate vector $[\boldsymbol{Y}]$ is generated by forward substitution.

$$[\boldsymbol{L}][\boldsymbol{Y}] = \boldsymbol{B}$$

2. Then, the result is substituted, to get the$[\boldsymbol{X}]$, which can be solved by back substitution.

$$[\boldsymbol{U}][\boldsymbol{X}] = [\boldsymbol{Y}]$$

Methods:

1. **Doolittle** decomposition, provided that: all of $[\boldsymbol{L}]$ diagonal elements is 1 and diagonal elements $[\boldsymbol{U}]$ is not zero.
2. **Crout** decomposition, provided that: diagonal elements $[\boldsymbol{L}]$ is not zero and all of $[\boldsymbol{U}]$ diagonal elements is 1.
3. **Cholesky** decomposition, provided that: $[\boldsymbol{A}]$ is symmetric matrix. And $[\boldsymbol{A}] = [\boldsymbol{L}]\,[\boldsymbol{L}]^T$ with $[\boldsymbol{L}]$and $[\boldsymbol{L}]^T$ diagonal elements is not zero

LAB TASKS

Consider the following matrices

$$\begin{bmatrix} 6 & 15 & 55 \\ 15 & 55 & 225 \\ 55 & 225 & 979 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 152.6 \\ 585.6 \\ 2488.8 \end{bmatrix}$$

1. **Implement and find solutions by using Cholesky Decomposition method.**