Lab 5: Numerical Solutions of Non- Linear Equations (Root finding methods)

1. **Open Methods**
   - Newton Raphson Method
   - Secant Method

Open Methods

Newton Raphson

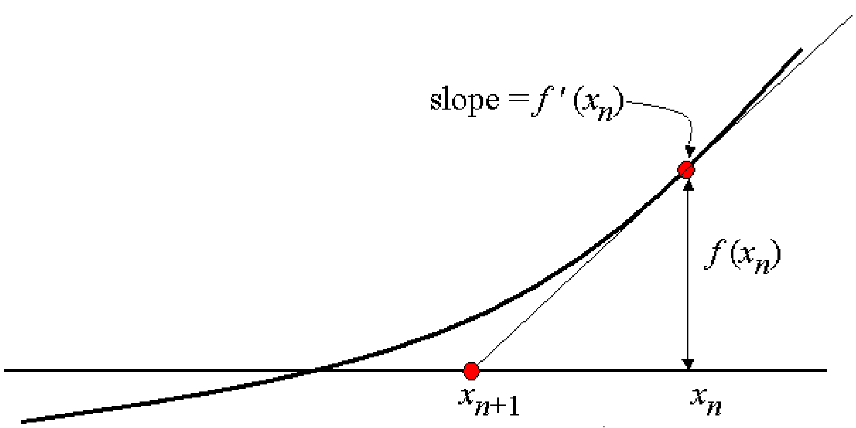
To determine the value of $f(x) = 0$, Start from a single initial estimate, x_o, that is not too far from a root.

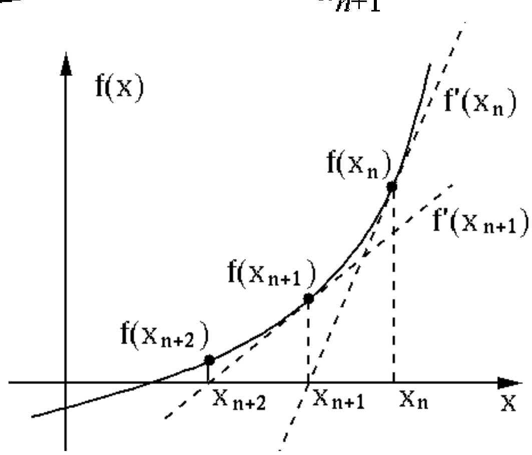
Repeat

$$x_{\mathrm{n}+1} = x_{\mathrm{n}} - \frac{f(x_{\mathrm{n}})}{f'(x_{\mathrm{n}})}$$

Set $x_n = x_{\mathrm{n}+1}$

Until $|f(x_{\mathrm{n}+1})| <$ tolerance level





Secant Method

Algorithm

To determine the value of $f(x) = 0$, given two values x_o and x_1 that are near the root.

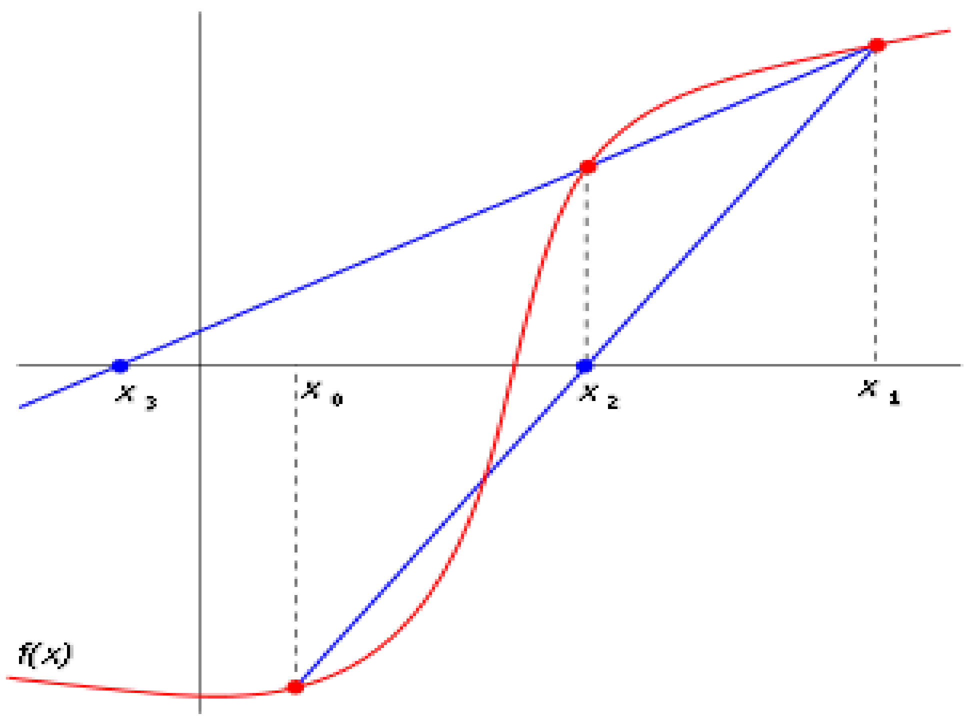
Repeat

$$x_2 = x_1 - f(x_1) * \frac{x_o - x_1}{f(x_o) - f(x_1)}$$

Set $x_o = x_1$

Else $x_1 = x_2$

Until $|f(x_2)| <$ tolerance level



LAB TASKS

1. **The equation $x^2 - 10cosx = 0$ has two solutions, ±1.3793646. Implement Newton's method in R to approximate the solutions to within error = 0 with the following values of x_0.**
   a. x_0 = -100
   b. x_0 = -50
   c. x_0 = -25
   d. x_0 = 25
   e. x_0 = 50
   f. x_0 = 100

   Verify the results with the values given below:

$$f(x) = x^2 - 10cosx \qquad x_{n+1} = x_n - \frac{f(x_n)}{f\prime(x_n)}$$

$$f'(x) = 2x + 10sinx \qquad \varepsilon'_a = \left|x_{n+1}^{new} - x_{n+1}^{old}\right|$$

Iterations	x_n	$f(x_n)$	$f'(x_n)$	x_{n+1}	ε'_a
0	-100	9991.3768	-194.93634	-48.7454385	
1	-48.7454385	2375.6105	-87.503753	-21.5967691	27.1486694
2	-21.5967691	475.65279	-47.035892	-11.4842196	10.11254952
3	-11.4842196	127.193	-14.138743	-2.48815834	8.996061228
4	-2.48815834	14.130939	-11.055485	-1.2099748	1.278183545
5	-1.2099748	-2.0663908	-11.776021	-1.38544925	0.175474457
6	-1.38544925	0.0765929	-12.599622	-1.37937027	0.006078983
7	-1.37937027	7.137E-05	-12.57608	-1.37936459	5.67528E-06
8	-1.379365	6.284833e-11	-1.257606e+01	-1.379365	3.623087e-10

9	-1.379365	-1.242965e-15	-1.257606e+01	-1.379365	0.000000e+00

A real root of given equation is **-1.379365** with error = 0

2. **Implement secant Method to determine a real root of $f(x) = 3x + sinx - e^x$. Using the tolerance value of 10^{-6}.**