Lab 7: Iterative Methods

Jacobi Method.

Each equation is now solved for the variables in succession

$$x_1 = \frac{b_1 - a_{12}x_2 - a_{13}x_3}{a_{11}}$$

$$x_2 = \frac{b_2 - a_{21}x_1 - a_{23}x_3}{a_{22}}$$

$$x_3 = \frac{b_3 - a_{31}x_1 - a_{32}x_2}{a_{33}}$$

We begin with some initial approximation to the value of the variables. (Each component might be taken equal to zero if no better initial estimates are at hand.) Substituting. These approximations into the right-hand sides of the set of equations generates new approximations that, we hope, are closer to the true value. The new values are substituted in the right-hand sides to generate a second approximation, and the process is repeated until successive values of each of the variables are sufficiently alike

$$x_1^{(n+1)} = \frac{b_1 - a_{12}x_2^{(n)} - a_{13}x_3^{(n)}}{a_{11}}$$

$$x_2^{(n+1)} = \frac{b_2 - a_{21}x_1^{(n)} - a_{23}x_3^{(n)}}{a_{22}}$$

$$x_3^{(n+1)} = \frac{b_3 - a_{31}x_1^{(n)} - a_{32}x_2^{(n)}}{a_{33}}$$

Gauss-Seidel Method.

We begin exactly as with the Jacobi method by rearranging the equations, solving each equation for the variable whose coefficient is dominant in terms of the others. We proceed to improve each x-value in turn, using always the most recent approximations of the other variables. The rate of convergence is more rapid than for the Jacobi method

$$x_1^{(n+1)} = \frac{b_1 - a_{12}x_2^{(n)} - a_{13}x_3^{(n)}}{a_{11}}$$

$$x_2^{(n+1)} = \frac{b_2 - a_{21}x_1^{(n+1)} - a_{23}x_3^{(n)}}{a_{22}}$$

$$x_3^{(n+1)} = \frac{b_3 - a_{31}x_1^{(n+1)} - a_{32}x_2^{(n+1)}}{a_{33}}$$

LAB TASKS

Consider the following system of Linear equations.

$$6x_1 - 2x_2 + x_3 = 11$$

$$x_1 + 2x_2 - 5x_3 = -1$$

$$-2x_1 + 7x_2 + 2x_3 = 5$$

1. **Implement the system of linear equations by using Gauss Jacobi iterative method.**
2. **Implement the system of linear equations by using Gauss Seidel iterative method.**