Lab 04: Numerical Solutions of Non- Linear Equations (Root finding methods)

1. **Closed Methods (Bracketing Methods)**
   - Bisection Method
   - False Position Method

Closed Method

Bisection Method

Formula $x_3 = \frac{x_1 + x_2}{2}$

Errors

1. **True Errors**

Percentage Relative Error $\varepsilon_t = \left|\frac{true\ value - Appr\ Value}{True\ value}\right| * 100$

Absolute Error $|true\ value - approximate\ value|$

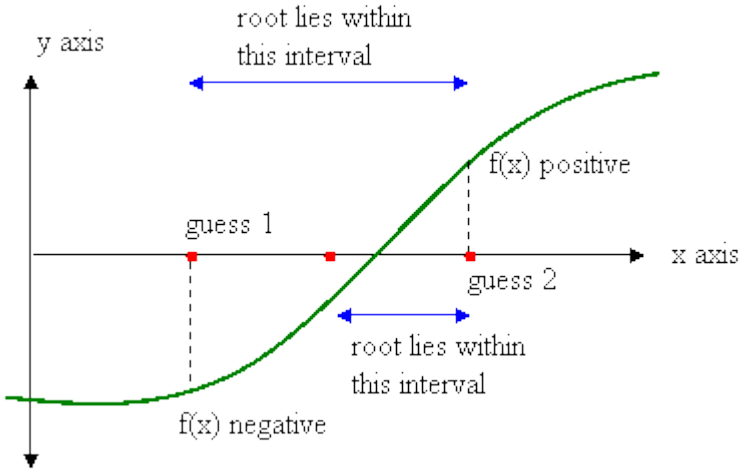
2. **Estimated Error**

Percentage Relative Error $\varepsilon_a = \left|\frac{x_3^{new} - x_3^{old}}{x_3^{new}}\right| * 100$

Absolute Error $|x_3^{new} - x_3^{old}|$

If $f(x_1) * f(x_3) < 0$ set $x_2 = x_3$

$f(x_1) * f(x_3) > 0$ set $x_1 = x_3$



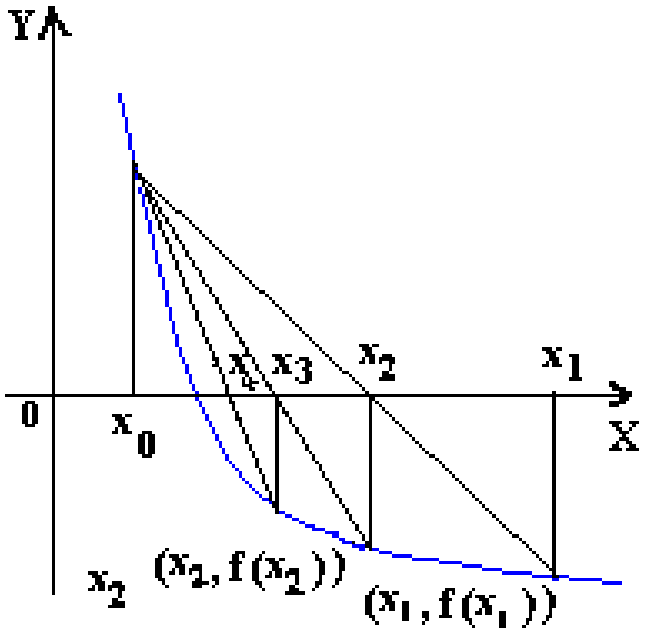
False Position Method

Algorithm

To determine the value of $f(x)$=0, given two values xo and $x1$ that bracket the root: that is $f(xo)$ and $f(x1)$ are of opposite sign.

Formula: $$x_2 = x_1 - f(x_1) * \frac{x_0 - x_1}{f(x_0) - f(x_1)}$$

If $f(x2)$ is of opposite sign to $f(xo)$ Then
Set $x1$= $x2$
Else xo=$x2$



LAB TASKS

A. Implement Bisection Method for the given polynomial in R and find the real root with extimated error.

$$f(x) = 5x^3 - 5x^2 + 6x - 2$$

Using bisection method to locate the root. Employ the initial guesses of $x_l = 0$, and $x_u = 1$ and iteration until the estimated error ε_a fall below the level of 10%.

```
%initialize x1,x2 and x3

 x1=0;
 x2=1;
 x3=[];
```

$x_1 = 0$, $f(x_1 = 0) =$ -2

$x_2 = 1$, $f(x_2) = 4$

$x_3 = \frac{x_1 + x_2}{2}$

```
%iterate by using for or while loop

 for i=2:10
  %your code goes here
  %calculate mid point by the formula
  % x3= x1+x2/2

  % check if f(x1)*f(x3) < 0
  %swap x2 and x3

  %check if f(x1)*f(x3) > 0
  %swap x1 and x3

  %calculate the estimated error
  % ea = abs((x3new - x3old)/x3new) * 100

  %check if ea < 10
  %break the loop

  %print the iterations, x1,x2,x3 f(x3),f(x1)*f(x3) and ea
```

Verify the results with the values in the table given below:

Iterations	x_1	x_2	x_3	**f(x_3)**	**f(x_1)f(x_3)**	ε_a $\varepsilon_a = \left\lvert\frac{x_3^{new} - x_3^{old}}{x_3^{new}}\right\rvert$*100
0	0	1	0.5		-0.75<0 $x_2 = x_3$	--
1	0	0.5	0.25		1.46876>0 $x_1 = x_3$	100%
2	0.25	0.5	0.375		0.13913>0 $x_1 = x_3$	33.3%
3	0.375	0.5	0.4375		-0.01642<0 $x_2 = x_3$	14.285%
4	0.375	0.4375	0.40625		0.00994>0 $x_1 = x_3$	7.692%

The real root of given equation is 0.40625, with 7.692% error.

B. Implement False Position Method for the same polynomial in R and find the real root with estimated error.

$$f(x) = 5x^3 - 5x^2 + 6x - 2$$