Lab 10: Solution of Differential Equations

R.K. Method.

The simple Euler method comes from using just one term from the Taylor series for $y(x)$ expanded about$x = x_0$. The modified Euler method can be derived from using two terms. What if we use more terms of the Taylor series? Two German mathematicians, Runge and Kutta, developed algorithms from using more than two terms of the series.

The most popular RK methods are fourth order. As with the second-order approaches there are an infinite number of versions. The following is the most commonly used form, and we therefore call it the classical fourth-order RK method:

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

Where

$$k_1 = f(x_i, y_i)h$$

$$k_2 = f\left(x_i + \frac{1}{2}h,\ y_i + \frac{1}{2}k_1\right)h$$

$$k_3 = f\left(x_i + \frac{1}{2}h,\ y_i + \frac{1}{2}k_2\right)h$$

$$k_4 = f(x_i + h,\ y_i + k_3)h$$

LAB TASKS

1. **Implement the following equation by using RK method to approximate soultion with initial value problem with h=0.5**

$$\frac{dy}{dx} = 2x^2 - y, \qquad y(0) = -1 \qquad 0 \leq x \leq 2$$

2. **Determine y at $x = 1$ for the following equation, using fourth-order Runge-Kutta method with $h = 0.2$.**

$$\frac{dy}{dx} = \frac{1}{(x+y)}, \qquad y(0) = 2$$