Lab 9: Solution of Differential Equations

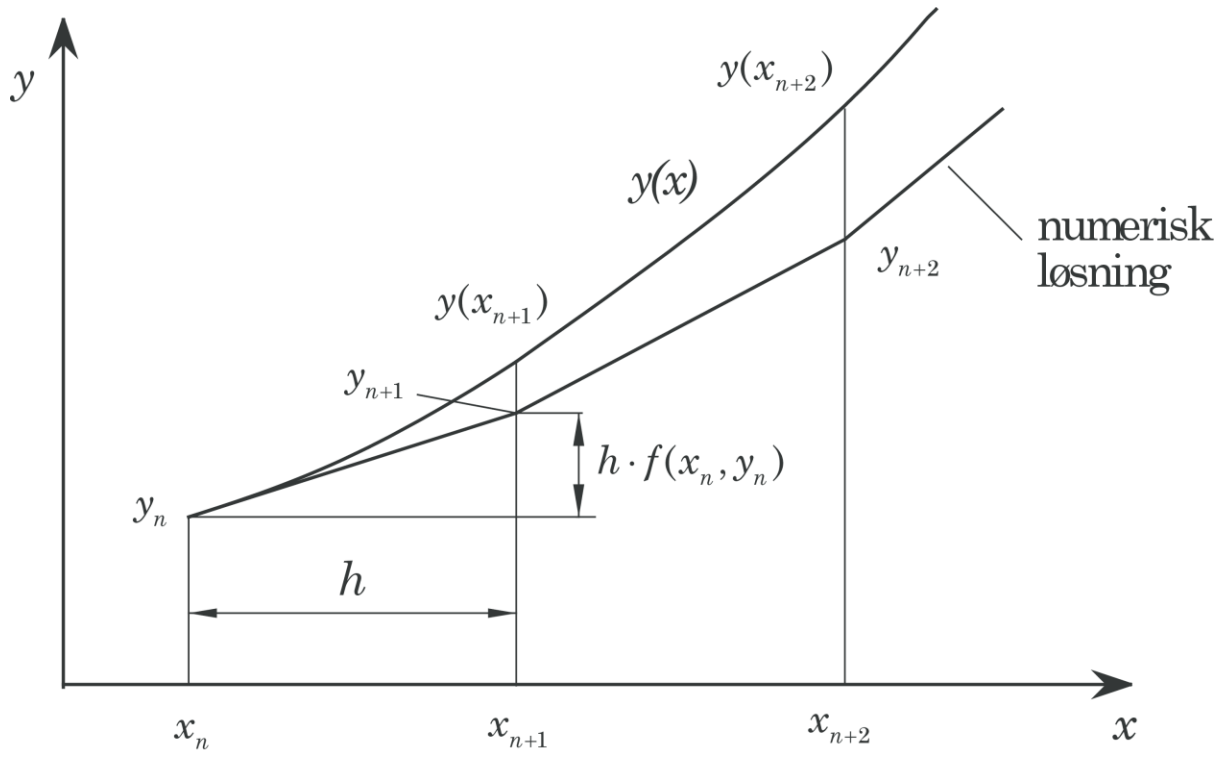
Euler's Method.

Euler's method is the most elementary approximation technique for solving initial-value problems

$$\frac{dy}{dx} = f(x, y), \qquad a \le x \le b, \qquad y(a) = \alpha$$

$$y_{i+1} = y_i + f(x_i, y_i)h$$

This formula is referred to as Euler's (or the Euler-Cauchy or the point-slope) method. A new value of y is predicted using the slope (equal to the first derivative at the original value of x) to extrapolate linearly over the step size h



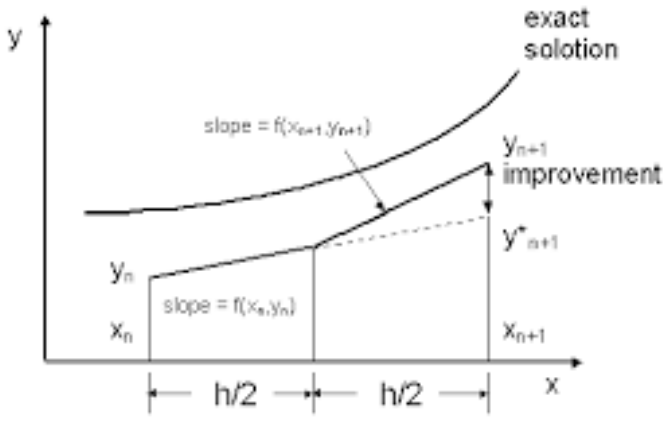
Modified Euler's Method.

The trouble with this most simple method is its lack of accuracy.

In the simple Euler method, we use the slope at the beginning of the interval, y'_i to determine the increment to the function. This technique would be correct only if the function were linear. What we need instead is the correct average slope within the interval. This can be approximated by the mean of the slopes at both ends of the interval.

In Modified Euler's method we use the arithmetic average of the slopes at the beginning and end of the interval to compute y_{i+1}

$$y_{i+1} = y_i + \frac{f(x_i, y_i) + f(x_{i+1}, y_{i+1})}{2} \; h$$



LAB TASKS

1. **Implement the following equation by using Euler's method to approximate soultion with initial value problem with h=0.1**

$$\frac{dy}{dt} = -5y + 5t^2 + 2t, \qquad 0 \le t \le 1, \qquad y(0) = \frac{1}{3}$$

2. **Implement modified Euler method to find an approximate value of y when x =1 subject to y(0) = 1**

$$\frac{dy}{dx} = x + 3y$$