# Microstrip Branch-Line Couplers for Crossover Application

Jijun Yao, Cedric Lee, and Swee Ping Yeo

Abstract—The branch-line coupler may be redesigned for crossover application. The bandwidth of such a coupler can be extended by suitably incorporating additional sections into the composite design. Laboratory tests on microstrip prototypes have shown the return loss and isolation of the three- and four-section couplers to be better than 20 dB over bandwidths of 22% and 33%, respectively. The insertion losses and group delays vary by less than  $\pm 0.05$  dB and  $\pm 1$  ns, respectively, for both prototypes.

Index Terms-Microstrip components.

## I. INTRODUCTION

HEN designers of microwave integrated circuits find it difficult to avoid transmission lines cutting across each other's path, they usually opt for air-bridges or under-passes [1], but such structures are not the only means available to them. Designers of Butler matrices [2]–[8], for example, have recently been exploring the possibility of employing other structures that are capable of yielding the following scattering matrix for crossover application:

$$\underline{S} = e^{j\phi} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}. \tag{1}$$

Recognizing that the cross junction [9] formed by a pair of intersecting lines does not in general yield the crossover characteristics specified in (1), researchers have experimented with modifications of various planar coupler structures. The annular-ring coupler reported in [10] has a simple structure, but tests have shown its crossover bandwidth to be limited. Studies of the composite ring-based structures proposed in [11]–[13] have indicated that optimized designs are able to meet 20-dB targets for the return-loss and isolation parameters in (1) over bandwidths of 20%, but these novel couplers may not gain ready acceptance because of their unconventional shapes. Lange couplers have

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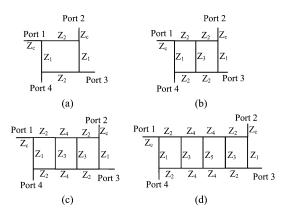


Fig. 1. Multisection branch-line structures (with nominal electrical angles of  $90^{\circ}$  for all line lengths): (a) basic unit, (b) two-section, (c) three-section, and (d) four-section.

also been re-configured to function as crossovers [6], but these inter-digitated structures require wire bonding and fabrication precision.

We revert instead to the multisection branch-line structure, as outlined by Wight *et al.* [14], in our effort to design microstrip couplers for crossover application. The two-section prototype fabricated by Wight *et al.* [14] yields a measured bandwidth of 10% with its isolation meeting the 20-dB specification and its return loss better than 18 dB at the center frequency. Kolodniak *et al.* [15] have experimented with other multisection structures and their window-shaped prototypes yield measured bandwidths of 10% with the isolation meeting the 20-dB target, but the return loss varying between 10–15 dB. For our design optimization in the present paper, we have chosen to impose the 20-dB targets on both isolation and return loss over a bandwidth exceeding the 20% limit faced by the ring-based prototypes tested in [12] and [13].

### II. PRELIMINARY CONSIDERATIONS

Depicted in Fig. 1 are the structures that we considered in detail. Following the formulation furnished by Kolodniak  $et\ al$ . [15], we have employed the analytical procedure based on the odd and even eigenmode approach with ABCD matrices to represent the various sections, as explained in [16]. We have additionally found the symbolic functional capabilities of MATLAB helpful in facilitating our analysis.

Our analytical findings have indicated that it is not possible for the basic branch-line structure sketched in Fig. 1(a) to yield the ideal crossover characteristics listed in (1). If, however, we do not insist on  $|s_{kk}| = 0$  in (1) and allow for  $|s_{kk}| \leq 0.1$  instead, we are then able to derive the following crossover design

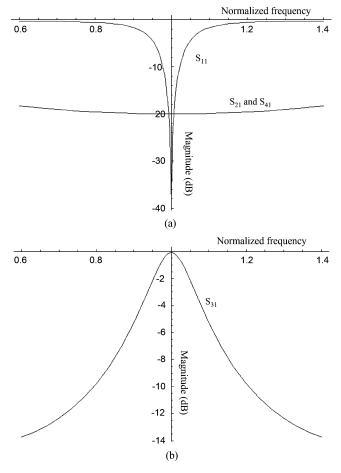


Fig. 2. Simulation plots of scattering-coefficient magnitudes for one-section coupler depicted in Fig. 1(a) where  $Z_c=50~\Omega$  and  $Z_1=Z_2=10~\Omega$ .

condition for the single-section structure based on a return loss of 20 dB:

$$Z_1 = Z_2 = (0.064 Z_c)^2$$
. (2)

The isolation of the single-section coupler will naturally be adversely affected when  $|s_{kk}| = 0$  is not strictly imposed. Since it is evident from the simulation results plotted in Fig. 2 that the bandwidth of the single-section design is limited, our next step is to consider the two-section coupler sketched in Fig. 1(b). There are no design equations provided by Wight *et al.* [14] for the two-section structure, and we thus have to derive them based on the ideal-case specifications listed in (1)

$$Z_1=rac{Z_c^2}{50}$$
 and  $Z_3=rac{50Z_2^2}{Z_c^2}$  for two-section structure. (3)

Fig. 3 presents the simulation plots we generated for a two-section coupler, which has been designed in accordance with (3) for crossover application. Although the return loss is better than 20 dB over a bandwidth of nearly 20%, the  $|s_{21}|$  plot in Fig. 3(a) shows the isolation between Port 1 and Port 2 deteriorating rapidly away from the center frequency. As a result, the insertion-loss plot is not sufficiently flat as can be seen from the variation of  $|s_{31}|$  with frequency in Fig. 3(b).

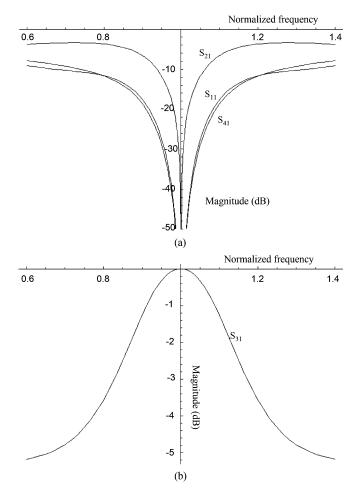


Fig. 3. Simulation plots of scattering-coefficient magnitudes for two-section coupler depicted in Fig. 1(b) where  $Z_c=Z_1=Z_2=Z_3=50~\Omega.$ 

In order to meet our design targets for both isolation and return loss, we have found it necessary to opt for the threeand four-section structures sketched in Fig. 1(c) and (d), respectively. By extending our analysis to include additional sections, we have been able to derive the following design equations based on the crossover specifications listed in (1):

$$Z_3 = Z_4$$
 and  $Z_3 = \frac{2Z_2^2}{Z^1}$  for three-section structure (4)

and

$$Z_1 = \frac{Z_{c1}^2}{100} \quad Z_3 = \frac{50Z_2^2}{Z_{c1}^2} \text{ and } Z_5 = \frac{Z_4^2 Z_{c1}^2}{150Z_2^2}$$
for four-section structure. (5)

We infer from the simulation results plotted in Fig. 4 for the three-section structure that the crossover bandwidth may be increased by incorporating a third section into the composite design in accordance with (4); in addition, we observe from Fig. 4(b) that the insertion-loss plot has become flatter in the process. As demonstrated in Section III, further improvement can be achieved when we progress to the four-section structure based on (5).

To facilitate the connection of our coupler to the external circuitry, we have chosen the default value of 50  $\Omega$  for the  $Z_c$ 

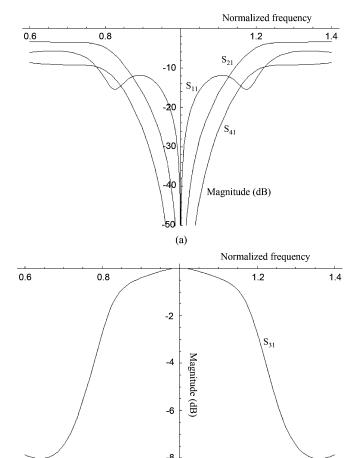


Fig. 4. Simulation plots of scattering-coefficient magnitudes for three-section coupler depicted in Fig. 1(c), where  $Z_c=Z_1=50~\Omega,~Z_2=40~\Omega,$  and  $Z_3=Z_4=64~\Omega.$ 

(b)

lines at the four ports when generating the numerical results presented in Figs. 2–4. For the design optimization performed in Section III, we have found that removing this default constraint allows for further bandwidth improvement and  $Z_c$  will next be permitted to vary over the range of characteristic impedance values typically expected for microstrip lines.

## III. EXPERIMENTAL RESULTS

The number of equations contained in either (4) or (5) is less than the number of coupler dimensions. Hence, each set of equations permits a range of possible designs and we have resorted to optimization in order to search for the maximum-bandwidth design that meets the 20-dB targets imposed on the return loss and isolation parameters in (1). Another optimization target is for the insertion-loss plot to be maximally flat over the operating bandwidth. There is no need for elaborate search algorithms and the commonly used Newton technique is sufficiently efficient in finding the minimum of the following error function:

$$F_e = \sum_{i=1}^{5} w_i g_i \qquad \text{where} \qquad g_1 = \sum_{m=1}^{M} |S_{11}(f_m)|^2$$
$$g_2 = \sum_{m=1}^{M} |S_{22}(f_m)|^2 \qquad g_3 = \sum_{m=1}^{M} |S_{21}(f_m)|^2$$

TABLE I
DIMENSIONS FOR THREE-SECTION PROTOTYPE DEPICTED IN FIG. 5(a)

	Width (mm)	Length (mm)	
Z <sub>c</sub>	3.1	16.3	
$\mathbf{Z}_1$	3.5	15.6	
$Z_2$	6.5	12.9	
$Z_3$	5.5	16.5	
$Z_4$	5.6	12.3	

TABLE II
DIMENSIONS FOR FOUR-SECTION PROTOTYPE DEPICTED IN FIG. 5(b)

	Width (mm)	Length (mm)
Z <sub>c</sub>	3.0	16.1
$Z_1$	2.4	14.4
$Z_2$	7.2	12.9
$Z_3$	6.0	14.4
$Z_4$	4.1	13.9
$Z_5$	0.5	17.5

$$g_4 = \sum_{m=1}^{M} |S_{41}(f_m)|^2 \qquad g_5 = \sum_{m=1}^{M} (|S_{31}(f_m)| - 1)^2$$

$$f_m = f_L + \frac{(m-1)(f_H - f_L)}{(M-1)}, \quad \text{for } m = 1, 2, \dots, M$$

$$f_L = \text{lowest frequency of requisite bandwidth}$$

 $f_H = \text{highest frequency of requisite bandwidth.}$  (6)

Spurious effects due to junction parasitics and other hardware imperfections have been taken into consideration. A survey of the numerical results we amassed during the course of our simulation trials indicates that a tolerance limit of  $\pm 0.1$  mm should be sufficient for the fabrication of our prototypes. Although we have incorporated into (6) the possibility of choosing different weights for  $g_i$  (where  $i=1,2,\ldots,5$ ), the optimization results are found to be satisfactory even when we retain the default setting of  $w_i=1$ .

Listed in Tables I and II are the coupler dimensions returned by the optimization process for the three- and four-section designs, respectively (when implemented in microstrip form on Rogers 4003C substrate with thickness of 0.78 mm, relative permittivity of 3.38, and loss tangent of 0.0027). Depicted in Fig. 5 are our two prototypes (drawn to scale). Since we have allowed for  $Z_c \neq 50~\Omega$  during the iterative process to optimize the coupler designs, we need to insert step junctions between the  $Z_c$  lines and the 50- $\Omega$  lines connecting the coupler to the external circuitry.

For our three-section prototype, the plots presented in Fig. 6 show generally good agreement between the numerical results and the experimental data (taken by the HP8510C network analyzer). The measured isolation and return loss meet their 20-dB targets over a 22% bandwidth (between 2.3–2.9 GHz), which is wider than that observed from the previous plots in Fig. 4 for the preliminary design based on (4) with the default setting of  $Z_c=50~\Omega$ . Varying between 0.3–0.4 dB over the entire bandwidth, the insertion-loss plot in Fig. 6(c) is also flatter when compared with its counterpart in Fig. 4(b).

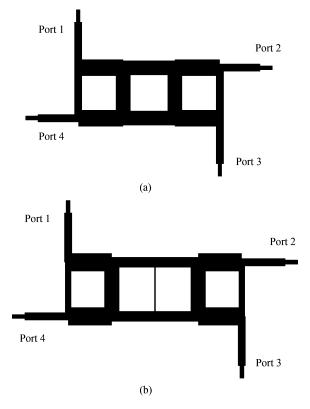


Fig. 5. Printed circuit board (PCB) layout of: (a) three- and (b) four-section prototypes with dimensions listed in Tables I and II, respectively.

It is possible to increase the crossover bandwidth by opting for the four-section design instead, as can be seen from the plots presented in Fig. 7, which similarly show close agreement between the numerical and measured results. Adopting the common 20-dB benchmark for both isolation and return loss, we infer from the measured plots that our four-section prototype is suitable for crossover application over a 33% bandwidth (between 2.2–3.1 GHz) where the insertion loss remains constant at  $0.4 \pm 0.05$  dB and the group delay varies between 8.5-10 ns.

In comparison, we note that the various microstrip couplers reported in [14] and [15] have been designed to function as crossovers over bandwidths of only 10%. Actually, there are no results presented in [15] for the two-section coupler, as the focus of Kolodniak *et al.*is on the window-shaped prototypes, which, however, do not meet the 20-dB target for the return loss over the entire bandwidth. It should be pointed out that other implementations are possible for the cascaded branch-line couplers as well, e.g., the rectangular-coaxial [3] and slot-line [4] versions have recently been designed for crossover application, but their return-loss plots also do not satisfy the 20-dB specification over the entire bandwidth.

# IV. ADDITIONAL EXTENSIONS

The progressive improvements we observed for the results obtained thus far for the one-, two-, three-, and four-section designs indicate the possibility of further enhancement by incorporating additional sections into the branch-line structure. The design data we subsequently derived for the five-, six-, and

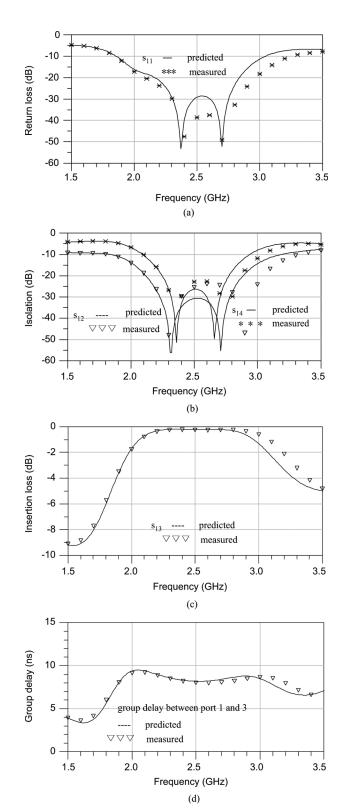


Fig. 6. Predicted and measured results for three-section prototype depicted in Fig. 5(a) with dimensions listed in Table I.

seven-section designs are listed in Table III (where, for the purpose of completeness, we also reproduce the corresponding data for the structures considered in Sections II and III).

We infer from the bandwidth data presented in the bottom row of Table III that a steady improvement can be expected,

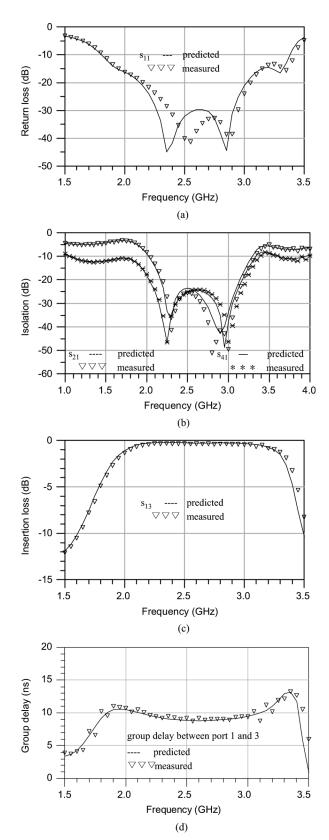


Fig. 7. Predicted and measured results for four-section prototype depicted in Fig. 5(b) with dimensions listed in Table II.

e.g., the simulation results show that the seven-section design is capable of yielding a 55% bandwidth (based on 20-dB targets for both isolation and return loss), but the overall coupler

TABLE III
KEY COUPLER PARAMETERS OF MULTISECTION BRANCH-LINE
DESIGNS FOR CROSSOVER APPLICATION

	Number of Sections N for Branch-line Structure*							
	1	2	3	4	5	6	7	
Z <sub>c</sub>	79.0	71.3	50.0	43.0	40.1	36.9	35.3	
$Z_1$	20.0	100	80.0	66.2	73.0	82.5	73.5	
$Z_2$	20.0	48.0	36.5	25.2	21.2	23.3	21.1	
$\mathbb{Z}_3$		25.1	35.0	28.6	25.2	38.6	44.3	
$Z_4$			32.1	25.5	25.4	25.3	25.0	
$Z_5$				49.0	94.3	79.0	93.8	
$Z_6$					42.9	37.3	34.8	
$Z_7$						100	99.5	
$Z_8$							36.4	
band- width	1%	9%	23%	33%	42%	49%	55%	

<sup>\*</sup> with all line lengths based on nominal electrical angles of 90° and all bandwidths based on 20-dB targets for both isolation and return loss

size has become unacceptably large for use in microwave integrated circuits with space limitations. If there is a need to employ the N-section design with  $N \geq 5$ , it is suggested that some appropriate size-reduction technique be attempted (such as the approaches proposed in [5] or [17]).

# V. CONCLUSION

Although commonly employed as a quadrature coupler in microwave integrated circuits, the microstrip branch-line coupler may also be redesigned for crossover application, but the bandwidth of the two-section prototype reported in [14] is only 10%. We have shown that the crossover bandwidth of such a coupler can be increased by opting for the three- or four-section design. The results we obtained from simulations and measurements have confirmed that the return-loss and isolation parameters of our three- and four-section prototypes meet their 20-dB targets over bandwidths of 22% and 33%, respectively, with the insertion loss and group delay varying by less than  $\pm 0.05~{\rm dB}$  and  $\pm 1~{\rm ns}$ , respectively.

Our simulation results have also indicated that additional bandwidth improvements can be gained by extending to five-, six-, and seven-section designs. However, such couplers may not be readily adopted by circuit designers because of size constraints (unless size-reduction techniques such as those proposed in [5] or [17] are attempted as well).

## REFERENCES

- [1] T. S. Horng, "A rigorous study of microstrip crossovers and their possible improvements," *IEEE Trans. Microw. Theory Tech.*, vol. 42, no. 9, pp. 1802–1806, Sep. 1994.
- [2] S. Gruszczynski and K. Wincza, "Broadband 4 × 4 Butler matrices as a connection of symmetrical multi-section coupled-line 3-dB directional couplers and phase correction networks," *IEEE Trans. Microw. Theory Tech.*, vol. 57, no. 1, pp. 1–9, Jan. 2009.
- [3] Y. Wang, M. Ke, and M. J. Lancaster, "A Ka-band Butler matrix with antenna array based on micro-machined rectangular coaxial structures," in Eur. Microw. Conf. Dig., 2009, pp. 739–742.
- [4] T. A. Denidni and N. Nedil, "Experimental investigation of a new Butler matrix using slot-line technology for beamforming antenna arrays," in *Proc. IET Microw. Antennas Propag.*, Jul. 2008, vol. 2, no. 7, pp. 641–649.

- [5] W. L. Chen, G. M. Wang, and C. X. Zhang, "Fractal-shaped switchedbeam antenna with reduced size and broadside beam," *Electron. Lett.*, vol. 44, no. 19, pp. 1110–1111, Sep. 2008.
- [6] T. N. Kaifas and J. N. Sahalos, "On the design of a single-layer wideband Butler matrix for switched-beam UMTS system applications," *IEEE. Antennas Propag. Mag.*, vol. 48, no. 6, pp. 193–204, Dec. 2006.
- [7] A. S. Liu, H. S. Wu, C. K. C. Tzuang, and R. B. Wu, "Ka-band 32 GHz planar integrated switched-beam smart antenna," in *IEEE MTT-S Int. Microw. Symp. Dig.*, 2005, pp. 565–568.
- [8] M. Bona, L. Manholm, J. P. Starski, and B. Svensson, "Low-loss compact Butler matrix for a microstrip antenna," *IEEE Trans. Microw. Theory Tech.*, vol. 50, no. 9, pp. 2069–2075, Sep. 2002.
- [9] S. C. Wu, H. Y. Yang, N. G. Alexopoulos, and I. Wolff, "A rigorous dispersive characterization of microstrip cross and T junctions," *IEEE Trans. Microw. Theory Tech.*, vol. 38, no. 12, pp. 1837–1844, Dec. 1000
- [10] F. Tefiku and E. Yamashita, "Improved analysis method for multi-port microstrip annular-ring power dividers," *IEEE Trans. Microw. Theory Tech.*, vol. 42, no. 3, pp. 376–382, Mar. 1994.
- [11] F. C. de Ronde, "Octave-wide matched symmetrical reciprocal fourand five-ports," in *IEEE MTT-S Int. Microw. Symp. Dig.*, 1982, pp. 521–523.
- [12] Y. Chen and S. P. Yeo, "A symmetrical four-port microstrip coupler for crossover application," *IEEE Trans. Microw. Theory Tech.*, vol. 55, no. 11, pp. 2434–2438, Nov. 2007.
- [13] Y. C. Chiou, J. Y. Kuo, and H. R. Lee, "Design of compact symmetric four-port crossover junction," *IEEE Microw. Wireless Compon. Lett.*, vol. 19, no. 9, pp. 545–547, Sep. 2009.
- [14] J. S. Wight, W. J. Chudobiak, and V. Makios, "A microstrip and stripline crossover structure," *IEEE Trans. Microw. Theory Tech.*, vol. MTT-24, no. 5, p. 270, May 1976.
- [15] D. Kholodniak, G. Kalinin, E. Vernoslova, and I. Vendik, "Wide-band 0-dB branch-line directional couplers," in *IEEE MTT-S Int. Microw.* Symp. Dig., 2000, pp. 1307–1310.
- [16] J. Reed and G. J. Wheeler, "A method of analysis of symmetrical four port networks," *IRE Trans. Microw. Theory Tech.*, vol. MTT-4, no. 4, pp. 246–252, Oct. 1956.
- [17] K. W. Eccleston and S. H. M. Ong, "Compact planar microstripline branch-line and rat-race couplers," *IEEE Trans. Microw. Theory Tech.*, vol. 51, no. 10, pp. 2119–2125, Oct. 2003.



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