Hille Series Trajectory Tracing

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December 20, 2018

Contents

- Hille Series Trajectory Tracing
- Introduction
- Version Info

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Alec Hoyland 2018-12-20 10:59

Introduction

The Hille series is equivalent to a discretized Taylor series under the limit

$$\lim_{\Delta t \to 0} \sum_{n=0}^{\infty} \frac{t^n}{n!(\Delta t)^n} \mathrm{D}^n f(a) = f(a+t)$$

for $t > 0$ and D^n is the finite difference operator of order n.

For a discrete time step Δt, the trajectory f can be predicted at future times $a+t$. The number of historical trajectory points needed depends on the order of the approximation. When expanded, this equation yields:

$$\left[1 + \frac{t}{\Delta t}\mathrm{D}^1 + \frac{t^2}{2(\Delta t)^2}\mathrm{D}^2 + ...\right] f(a)$$

The finite difference operator combines past trajectory points in proportions according to order. The first three terms of the Hille series in matrix form are:

$$\left(1 \, \frac{t}{\Delta t} \, \frac{t^2}{2(\Delta t)^2}\right) \begin{pmatrix} 001 \\ -\frac{1}{2}0\frac{1}{2} \\ 1-11 \end{pmatrix} \begin{pmatrix} f(a) \\ f(a-\Delta t) \\ f(a-2\Delta t) \end{pmatrix}$$

Version Info

```
git clone git@github.com:hasselmonians/hasselmo-tracking.git /home/ahoyland/code/hasselmo-tracking
git checkout  d82cda35b6f0d3478ede32f60df691868132c5c3
git clone git@github.com:alec-hoyland/srinivas.gs_mtools.git /home/ahoyland/code/srinivas.gs_mtools
git checkout  c21986bb074dadb0258f494f6e0a024d05f21714
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