# Hille Series Trajectory Tracing

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### Introduction

The Hille series is equivalent to a discretized Taylor series under the limit

$$\lim_{\Delta t \to 0} \sum_{n=0}^{\infty} \frac{t^n}{n!(\Delta t)^n} \mathrm{D}^n f(a) = f(a+t)$$

for $t > 0$ and $\mathrm{D}^n$ is the finite difference operator of order $n$.

For a discrete time step $\Delta t$, the trajectory $f$ can be predicted at future times $a+t$. The number of historical trajectory points needed depends on the order of the approximation. When expanded, this equation yields:

$$\left[1 + \frac{t}{\Delta t}\mathrm{D}^1 + \frac{t^2}{2(\Delta t)^2}\mathrm{D}^2 + ...\right] f(a)$$

The finite difference operator combines past trajectory points in proportions according to order. The first three terms of the Hille series in matrix form are:

$$\begin{pmatrix} 1 & \frac{t}{\Delta t} & \frac{t^2}{2(\Delta t)^2} \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \\ -\frac{1}{2} & 0 & \frac{1}{2} \\ & 1-11 & \end{pmatrix} \begin{pmatrix} f(a) \\ f(a-\Delta t) \\ f(a-2\Delta t) \end{pmatrix}$$

### Version Info

```
git clone git@github.com:hasselmonians/hasselmo-tracking.git /home/ahoyland/code/hasselmo-tracking
git checkout  aa538356dbd80452d0e9a200e3c49f8c5c41aea7
git clone git@github.com:alec-hoyland/srinivas.gs_mtools.git /home/ahoyland/code/srinivas.gs_mtools
git checkout  c21986bb074dadb0258f494f6e0a024d05f21714
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