Physics 212
Lecture 10

Today's Concept:

Kirchhoff's Rules

Last Time

Resistors in series:

Current through is same.

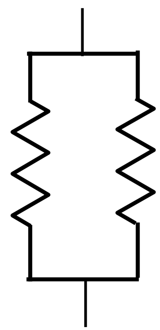
Voltage drop across each resistor i is IR_i

$$R_{effective} = R_1 + R_2$$

Resistors in parallel:

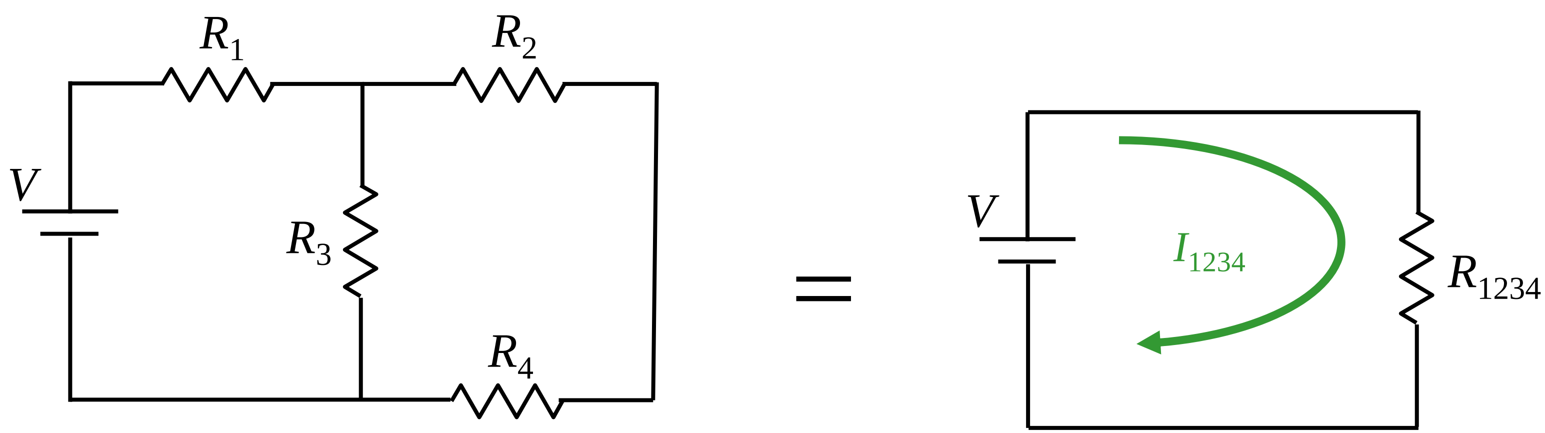
Voltage drop across is same.

Current through is V/R_i

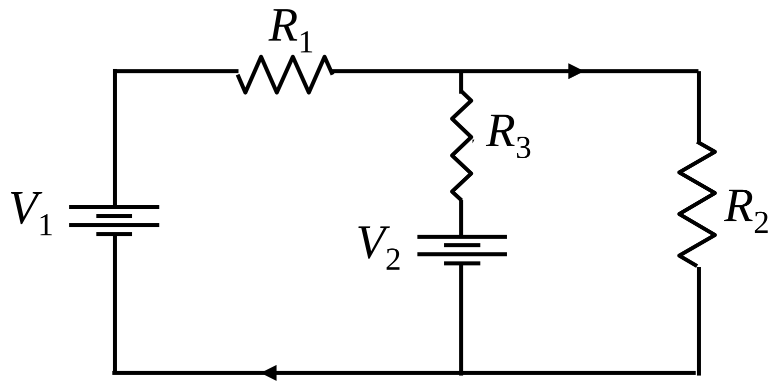


$$\frac{1}{R_{effective}} = \frac{1}{R_1} + \frac{1}{R_2}$$

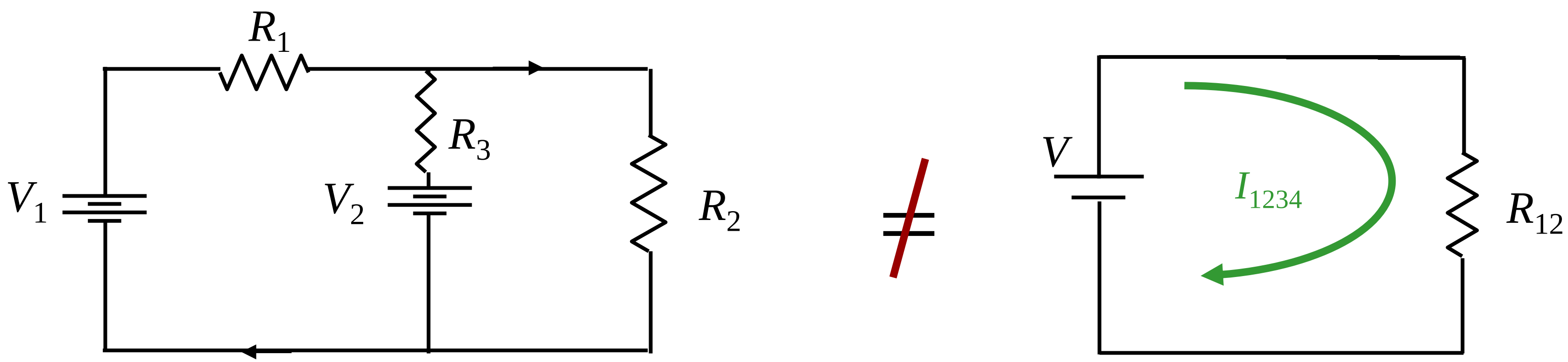
Solved Circuits



New Circuit



How Can We Solve This One?



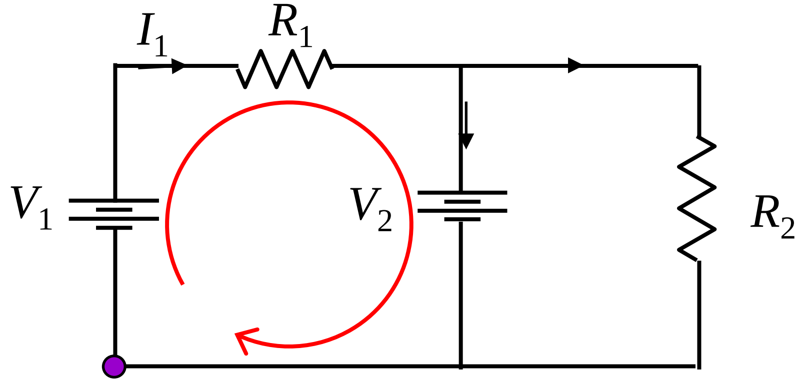
THE ANSWER: Kirchhoff's Rules

Kirchhoff's Voltage Rule

$$\sum \Delta V_i = 0$$

Kirchhoff's Voltage Rule states that the sum of the voltage changes caused by any elements (like wires, batteries, and resistors) around a circuit must be zero.

$$\sum \Delta V_i = -V_1 + IR_1 + V_2 = 0$$



WHY?

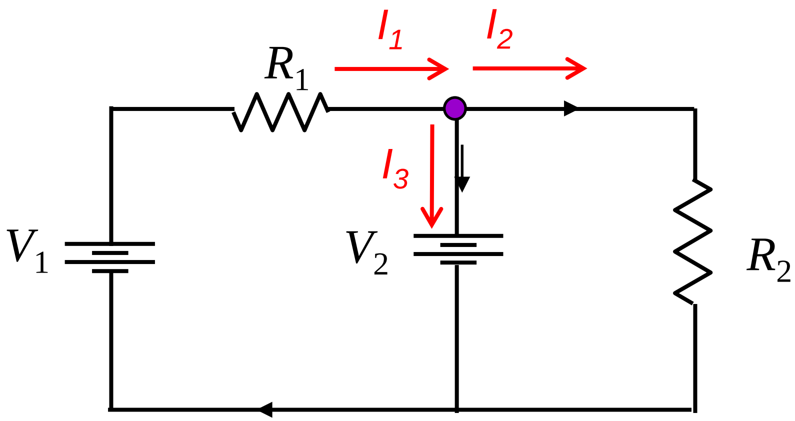
The potential difference between a point and itself is zero!

Kirchhoff's Current Rule

$$\sum I_{in} = \sum I_{out}$$

Kirchhoff's Current Rule states that the sum of all currents entering any given point in a circuit must equal the sum of all currents leaving the same point.

$I_1 = I_2 + I_3$



WHY?

Electric Charge is Conserved

Applying Kirchhoff's Laws in 5 easy steps

1) Label all currents

Choose any direction

2) Label +/– for all elements

Current goes $+ \Rightarrow -$ (for resistors)
Long side is + for battery

3) Choose loop and direction

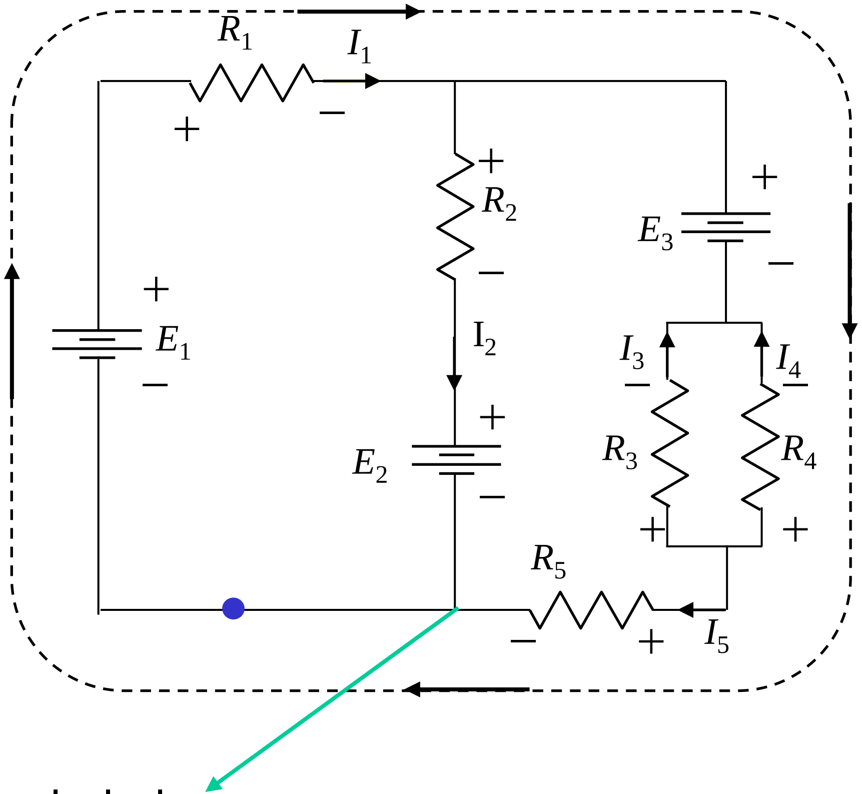
Must start on wire, not element.

4) Write down voltage drops

First sign you hit is sign to use.

5) Write down node equation $I_{in} = I_{out}$

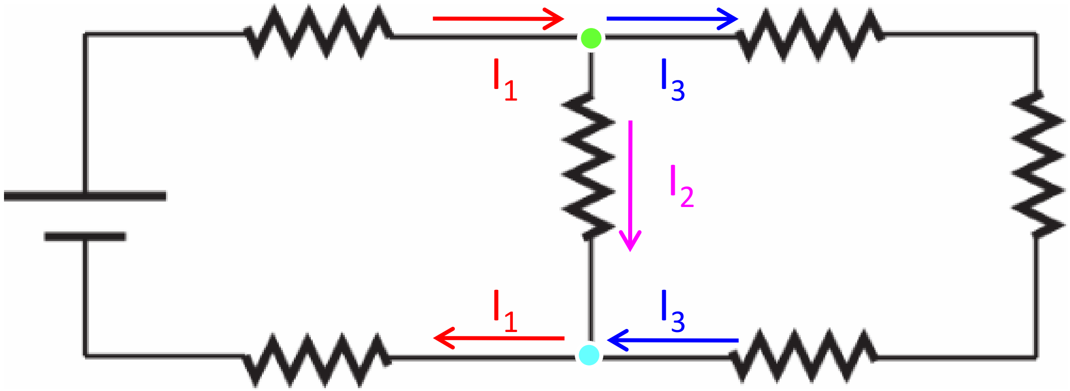
$-E_1 + I_1R_1 + E_3 - I_4R_4 + I_5R_5 = 0$



$I_2 + I_5 = I_1$

Check Point 1

How many potentially different currents are there in the circuit shown?



A. 3 ***B.*** 4 ***C.*** 5 ***D.*** 6 ***E.*** 7

Look at the nodes!

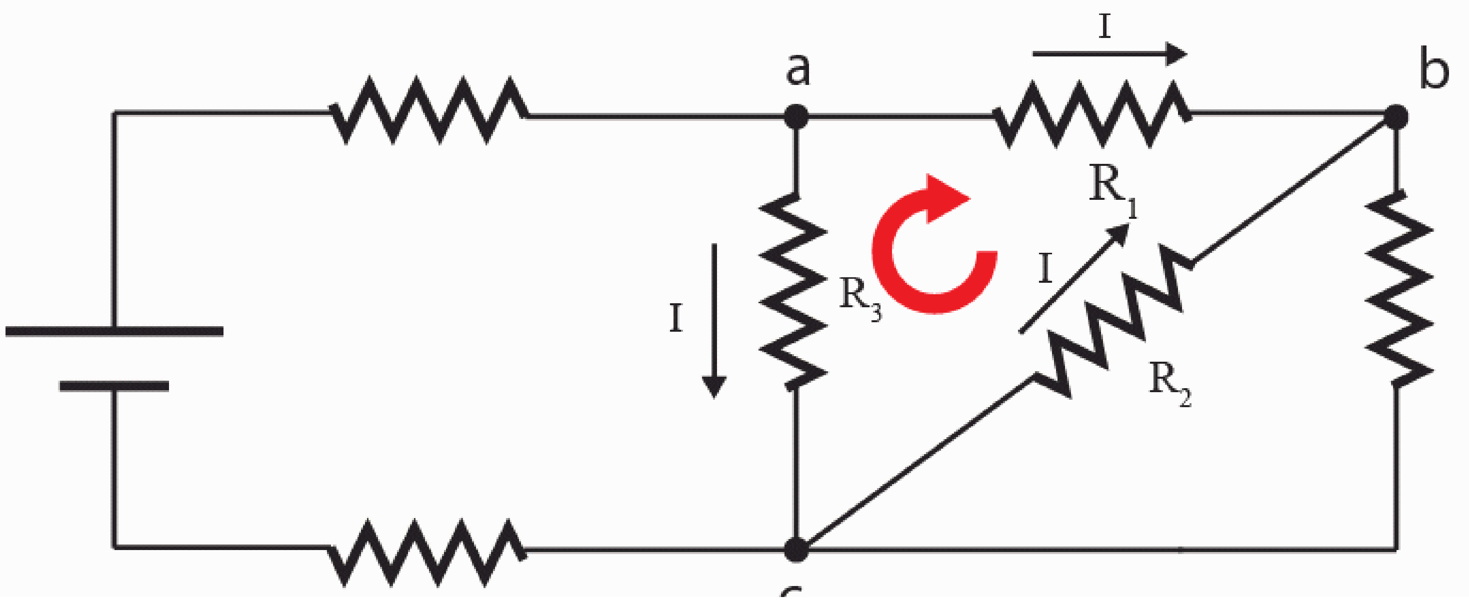
Top node: I_1 flows in, I_2 and I_3 flow out

Bottom node: I_2 and I_3 flow in, I_1 flows out

That’s all of them!

Check Point 2

In the following circuit, consider the loop abc. The direction of the current through each resistor is indicated by black arrows.



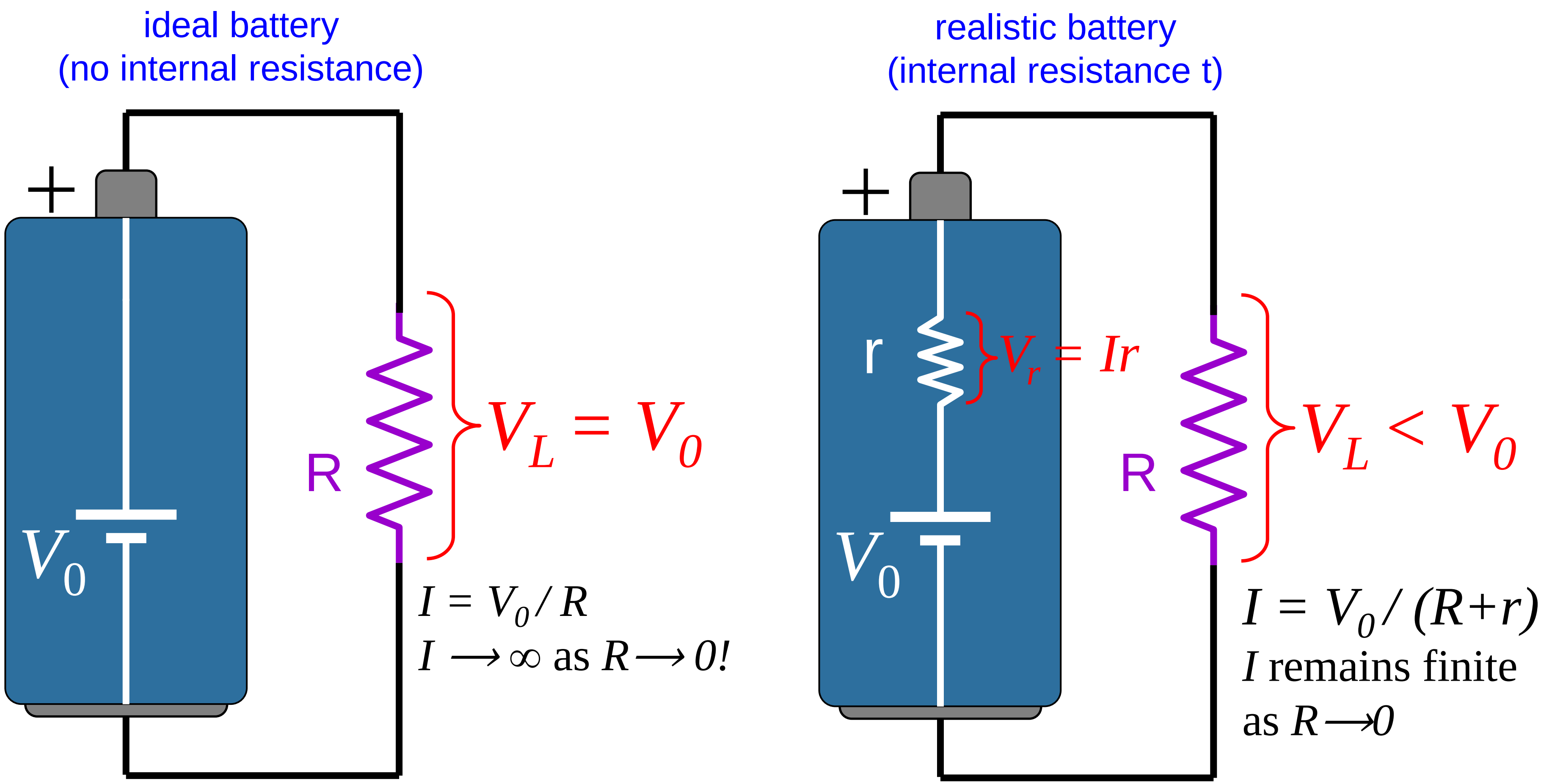
If we are to write Kirchoff's voltage equation for this loop in the clockwise direction starting from point a, what is the correct order of voltage gains/drops that we will encounter for resistors R1, R2 and R3?

A. drop, drop, drop ***B.*** gain, gain, gain ***C.*** drop, gain, gain
D. gain, drop, drop ***E.*** drop, drop, gain

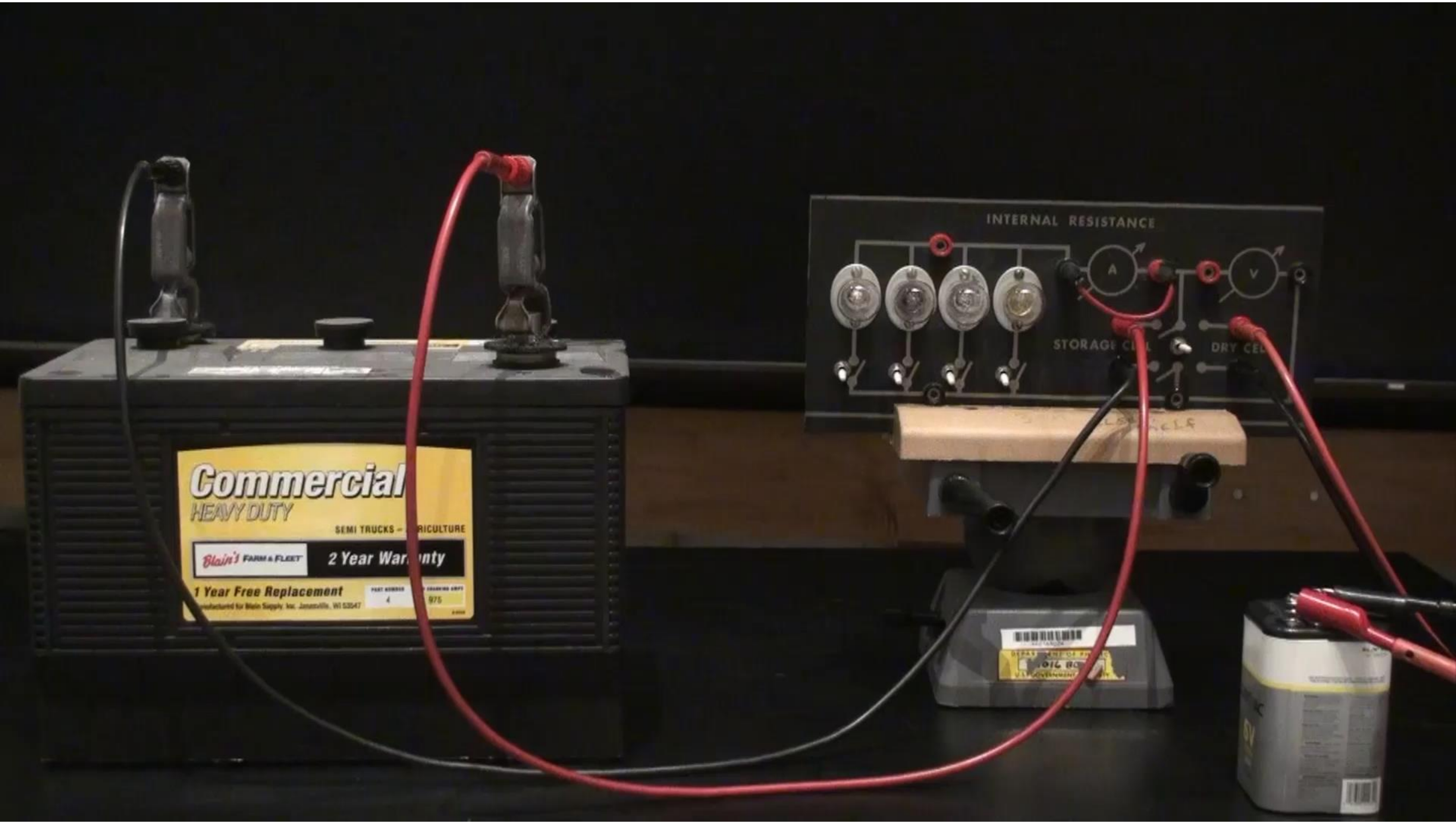
With the current → VOLTAGE DROP

Against the current → VOLTAGE GAIN

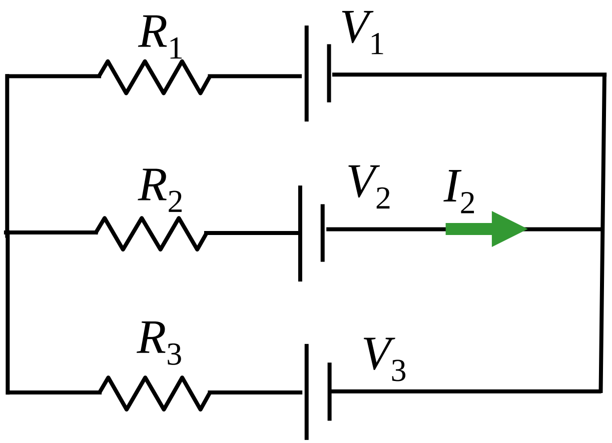
Model for Real Battery: Internal Resistance



Usually, can't supply too much current to the load without voltage "sagging"


Commercial
HEAVY DUTY
2 Year Warranty
1 Year Free Replacement
INTERNAL RESISTANCE
STORAGE CELL
DRY CELL

Calculation



In this circuit, we are given the resistances and battery voltages and are asked to calculate the current through resistor 2.

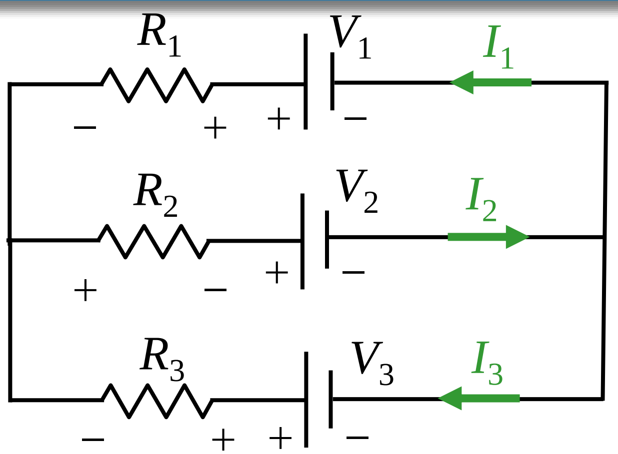
Conceptual Analysis:

- Circuit behavior described by Kirchhoff's Rules:
  - KVR: $\Sigma V_{drops} = 0$
  - KCR: $\Sigma I_{in} = \Sigma I_{out}$

Strategic Analysis

- Write down Loop Equations (KVR)
- Write down Node Equations (KCR)
- Solve

Calculation



In this circuit, assume V_i and R_i are known.

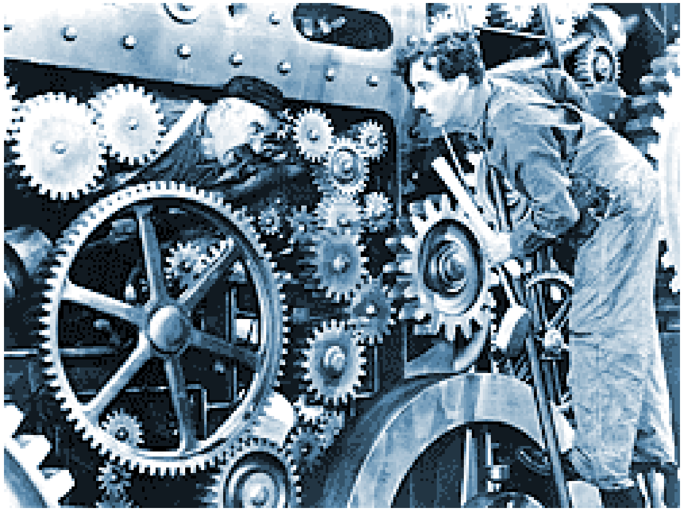
What is I_2 ?

1) Label and pick directions for each current

2) Label the + and – side of each element

This is easy for batteries Long side is +

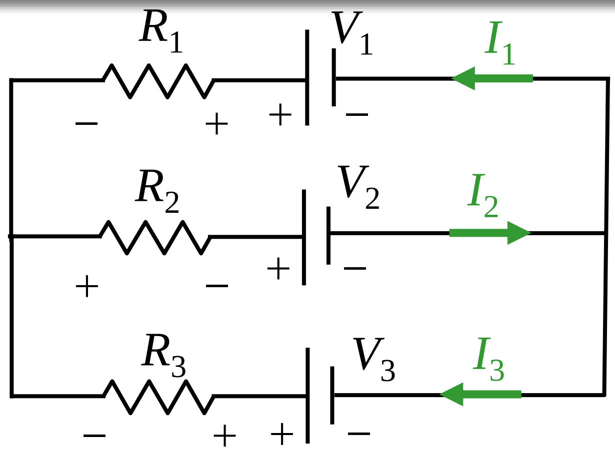
For resistors, the "upstream" side is +



Just turn the crank.

Now write down loop and node equations

Calculation



In this circuit, assume V_i and R_i are known.

What is I_2 ?

How many equations do we need to write down in order to solve for I_2?

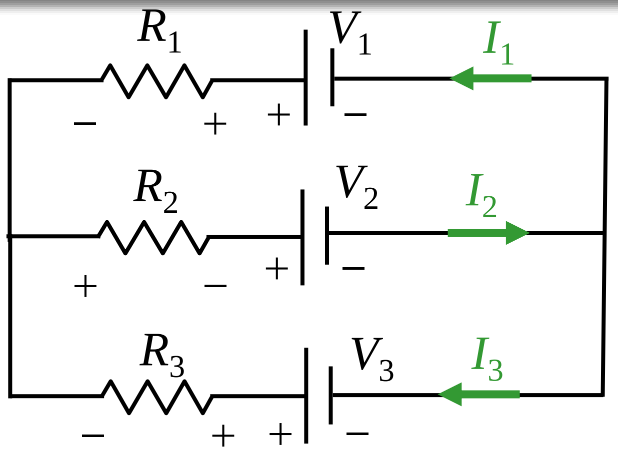
A) 1 B) 2 C) 3 D) 4 E) 5

Why?

- We have 3 unknowns: I_1, I_2, and I_3
- We need 3 independent equations to solve for these unknowns

3) Choose Loops and Directions

Calculation



In this circuit, assume V_i and R_i are known.

What is I_2 ?

Which of the following equations is NOT correct?

A) $I_2 = I_1 + I_3$

B) $-V_1 + I_1R_1 - I_3R_3 + V_3 = 0$

C) $-V_3 + I_3R_3 + I_2R_2 + V_2 = 0$

D) $-V_2 - I_2R_2 + I_1R_1 + V_1 = 0$

4) Write down voltage drops

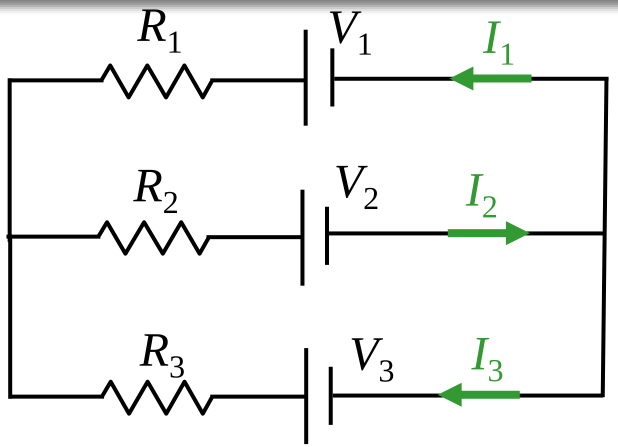
5) Write down node equation

Why?

- (D) is an attempt to write down KVR for the top loop
- Start at negative terminal of V_2 and go clockwise

$V_{gain}\,(-V_2)$ then $V_{gain}\,(-I_2R_2)$ then $V_{gain}\,(-I_1R_1)$ then $V_{drop}\,(+V_1)$

Calculation



In this circuit, assume V_i and R_i are known.

What is I_2 ?

We have the following 4 equations:

1. $I_2 = I_1 + I_3$
2. $-V_1 + I_1R_1 - I_3R_3 + V_3 = 0$
3. $-V_3 + I_3R_3 + I_2R_2 + V_2 = 0$
4. $-V_2 - I_2R_2 - I_1R_1 + V_1 = 0$

We need 3 equations:
Which 3 should we use?

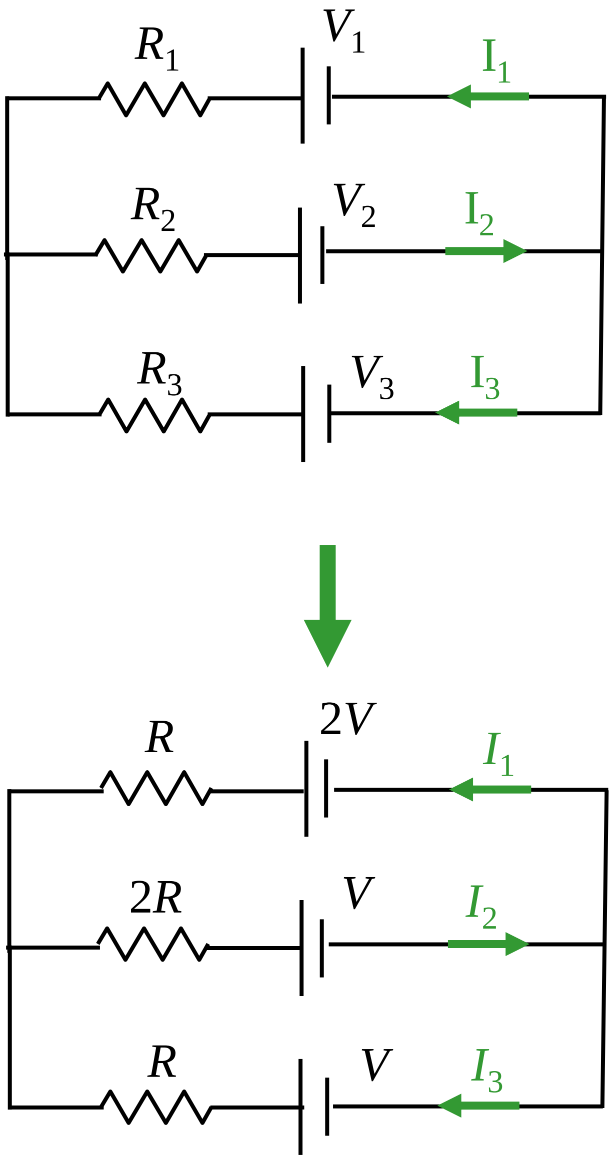
A) Any 3 will do
B) 1, 2, and 4
C) 2, 3, and 4

Why?

- We need 3 INDEPENDENT equations
- Equations 2, 3, and 4 are NOT INDEPENDENT

  $\text{Eqn } 2 + \text{Eqn } 3 = -\text{Eqn } 4$
- We must choose Equation 1 and any two of the remaining (2, 3, and 4)

Calculation



In this circuit, assume V_i and R_i are known.

What is I_2 ?

We have 3 equations and 3 unknowns.

$$I_2 = I_1 + I_3$$
$$V_1 + I_1 R_1 - I_3 R_3 + V_3 = 0$$
$$V_2 - I_2 R_2 - I_1 R_1 + V_1 = 0$$

Now just need to solve ☺

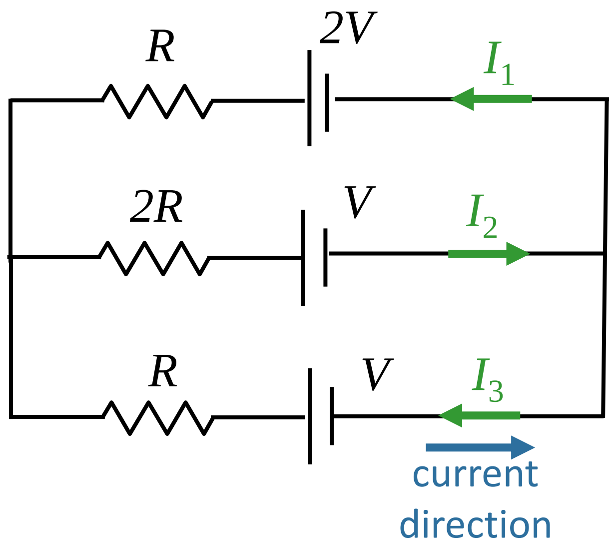
The solution will get very messy!

Simplify: assume $V_2 = V_3 = V$

$$V_1 = 2V$$
$$R_1 = R_3 = R$$
$$R_2 = 2R$$

Calculation: Simplify

In this circuit, assume V and R are known. What is I_2 ?



We have 3 equations and 3 unknowns.

$$I_2 = I_1 + I_3$$

$$-2V + I_1R - I_3R + V = 0 \quad \text{(outside)}$$

$$-V - I_2(2R) - I_1R + 2V = 0 \quad \text{(top)}$$

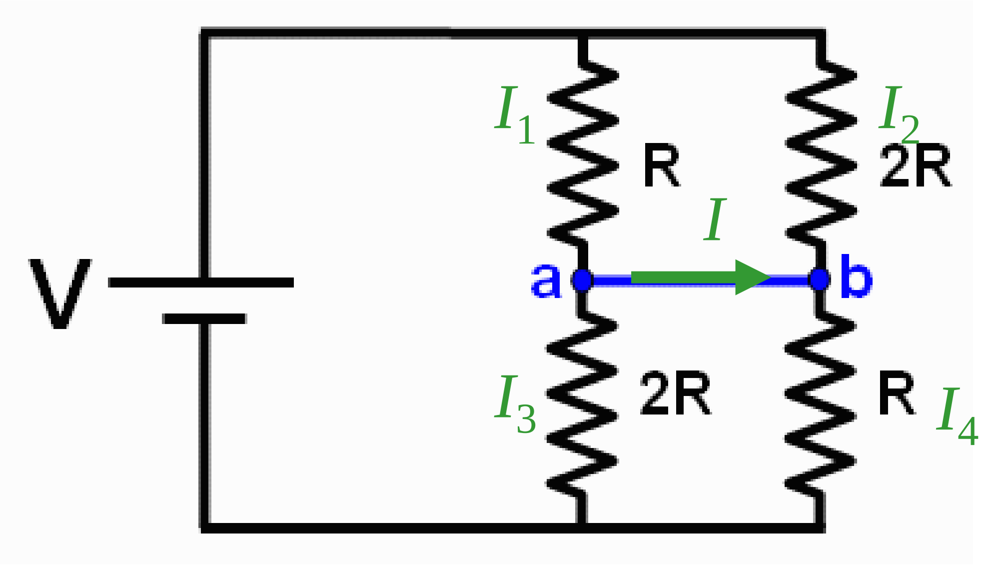
With this simplification, you can verify:

$$I_2 = (\ 1/5)\ V/R$$

$$I_1 = (\ 3/5)\ V/R$$

$$I_3 = (-2/5)\ V/R$$

Check Point 3a



Which of the following best describes the current flowing in the blue wire connecting points **a** and **b**?

A. Positive current flows from *a* to *b*

B. Positive current flows from *b* to *a*

C. No current flows between *a* and *b*

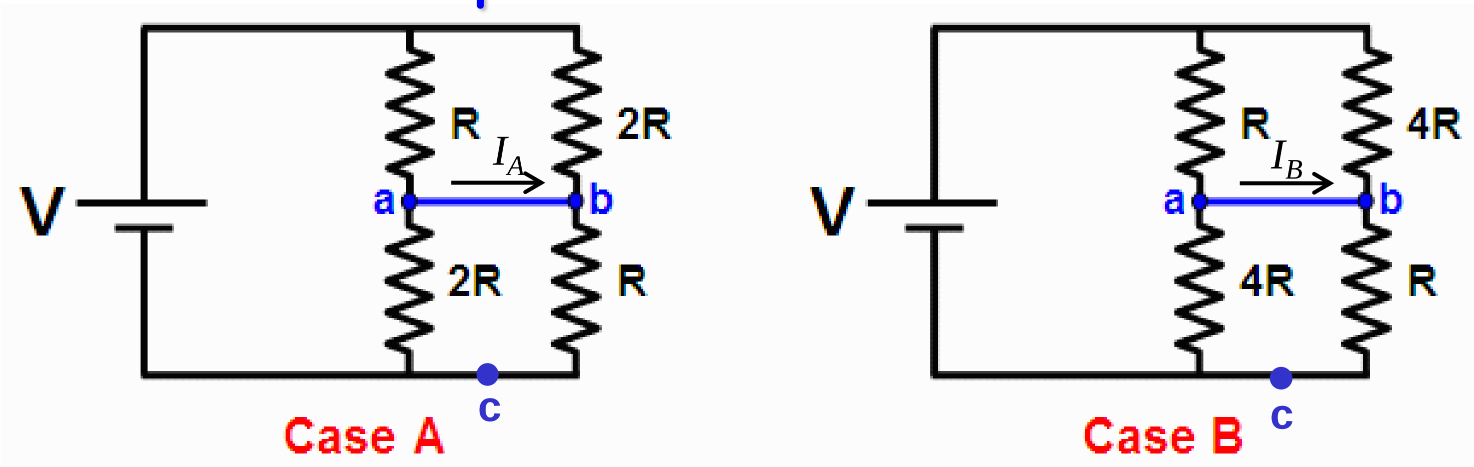
$I_1 R - I_2\,(2R) = 0$ ➡ $I_2 = ½\ I_1$

$I_4 R - I_3\,(2R) = 0$ ➡ $I_4 = 2\ I_3$

$I = I_1 - I_3$

$I + I_2 = I_4$ ➡ $I_1 - I_3 + ½\,I_1 = 2I_3$ ➡ $I_1 = 2I_3$ ➡ $I = +I_3$

Check point 3b



which case is the current flowing in the blue wire connecting points **a** and **b** the largest?

A. Case A ***B.*** Case B ***C.*** They are both the same

Current will flow from left to right in both cases.

In both cases, $V_{ac} = V/2$

$$I_{2R} = 2I_{4R}$$

$$I_A = I_R - I_{2R}$$
$$= I_R - 2I_{4R}$$

$$I_B = I_R - I_{4R}$$

Summary

1) Label all currents

Choose any direction

2) Label +/– for all elements

Current goes + ⇒ – (for resistors)

Long side is + for battery

3) Choose loop and direction

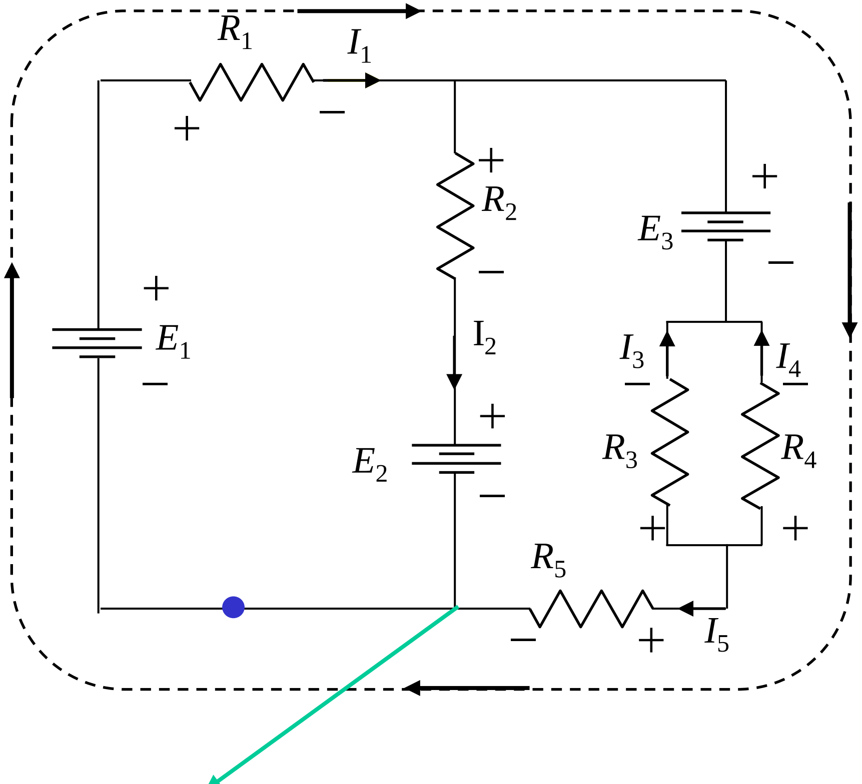
Must start on wire, not element.

4) Write down voltage drops

First sign you hit is sign to use.

5) Write down node equation $I_{in} = I_{out}$

$-E_1 + I_1R_1 + E_3 - I_4R_4 + I_5R_5 = 0$



$I_2 + I_5 = I_1$