Math 241 Calculus III (Prof. Honold) **Midterm Exam 1** 2022/03/16

Name: ____________________ Student No.: ____________________ Group A

For each of the following problems, find the correct answer (tick as appropriate!). No justifications are required. Each problem has exactly one correct solution, which is worth 1 mark. Incorrect solutions (including no answer, multiple answers, or unreadable answers) will be assigned 0 marks; there are no penalties.

1. The volume of the pyramid ("tetrahedron") with vertices $(b,1,1)$, $(1,-1,-1)$, $(-1,1,-1)$, $(-1,-1,1)$ is equal to 1 for

   ☐ $b=\frac{3}{2}$ ☐ $b=\frac{5}{2}$ ☐ $b=-\frac{3}{2}$ ☐ $b=-\frac{1}{2}$ ☐ $b=\frac{1}{2}$

2. The distance from the point $(1,0,0)$ to the line connecting the points $(0,0,1)$ and $(1,2,3)$ is

   ☐ $\frac{1}{3}\sqrt{21}$ ☐ 3 ☐ $\frac{1}{3}\sqrt{17}$ ☐ $\frac{17}{9}$ ☐ $\frac{21}{9}$

3. The inverse matrix of $\begin{pmatrix} 2 & 3 & 5 \\ 1 & 2 & 3 \\ 0 & 1 & 2 \end{pmatrix}$ has the form $\begin{pmatrix} * & * & * \\ * & c & * \\ * & * & * \end{pmatrix}$ with

   ☐ $c=0$ ☐ $c=1$ ☐ $c=2$ ☐ $c=3$ ☐ $c=4$

4. The reflection of $\mathbb{R}^2$ at the line $\sqrt{3}x+y=0$ is afforded by the matrix

   ☐ $\begin{pmatrix} -1/2 & \sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{pmatrix}$ ☐ $\begin{pmatrix} -1/2 & -\sqrt{3}/2 \\ -\sqrt{3}/2 & 1/2 \end{pmatrix}$ ☐ $\begin{pmatrix} 1/2 & \sqrt{3}/2 \\ \sqrt{3}/2 & -1/2 \end{pmatrix}$

   ☐ $\begin{pmatrix} 1/2 & \sqrt{3}/2 \\ -\sqrt{3}/2 & 1/2 \end{pmatrix}$ ☐ $\begin{pmatrix} 1/2 & -\sqrt{3}/2 \\ -\sqrt{3}/2 & -1/2 \end{pmatrix}$

5. The maximum rank of $\mathbf{A}\in\mathbb{R}^{3\times 4}$ with all row sums and all columns sums equal to zero is

   ☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4

6. The linear system $2x_1-x_2=x_1+ax_2+x_3=x_1-x_2+2x_3=0$ has a nonzero solution for

   ☐ $a=-\frac{1}{4}$ ☐ $a=-\frac{3}{4}$ ☐ $a=\frac{1}{4}$ ☐ $a=\frac{3}{4}$ ☐ $a=-\frac{5}{4}$

7. If $f\colon [0,\pi]\to\mathbb{R}^3$ satisfies $f(0)=(0,0,1)$ and $f'(t)=(2t,\sin t,\cos t)$ then the point $f(\pi)$ is equal to

   ☐ $(\pi,0,0)$ ☐ $(\pi^2,-2,1)$ ☐ $(\pi^2,2,1)$ ☐ $(\pi^2,2,0)$ ☐ $(\pi^2,-2,0)$

8. The twisted cubic $f(t)=(t,t^2,t^3)$, $t\in\mathbb{R}$ intersects the plane $3x-y+2z=4$ in an angle of

   ☐ 0° ☐ 30° ☐ 45° ☐ 60° ☐ 90°

9. The arc length of the curve $g(t)=\left(t^3+3t+1,\sqrt{3}t^2,4t-2\right)$, $t\in[0,2]$ is

   ☐ 36 ☐ 84 ☐ 17 ☐ 12 ☐ 18

10. For a differentiable curve $\gamma=\gamma(t)$ in $\mathbb{R}^3$ and a (constant) vector $\mathbf{u}\in\mathbb{R}^3$ with $|\mathbf{u}|=1$ the derivative $\frac{\mathrm{d}}{\mathrm{d}t}|\gamma-(\gamma\cdot\mathbf{u})\mathbf{u}|^2$ is equal to

    ☐ $2\,|\gamma-(\gamma\cdot\mathbf{u})\mathbf{u}|$ ☐ 2 ☐ $2\,|\gamma-(\gamma\cdot\mathbf{u})\mathbf{u}|\,\big(\gamma'-(\gamma'\cdot\mathbf{u})\mathbf{u}\big)$

    ☐ $2\gamma\cdot\gamma'-2(\gamma\cdot\mathbf{u})(\gamma'\cdot\mathbf{u})$ ☐ $2\,|\gamma'-(\gamma'\cdot\mathbf{u})\mathbf{u}|$

Time allowed: 40 min CLOSED BOOK ***Good luck!***