Physics 212
Lecture 5

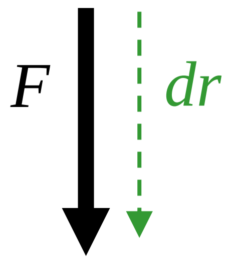
Today's Concept:

Electric Potential Energy

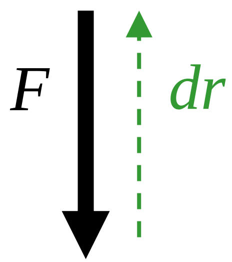
Work (Mechanics Review)

Recall from physics 211:

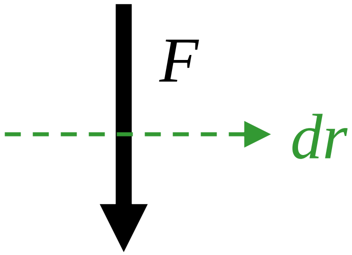
$$W = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r} \qquad W_{TOT} = \Delta K$$



$W > 0$ (e.g. $W_{gravity}$ on object dropped)



$W < 0$ (e.g. $W_{gravity}$ on ball going up)



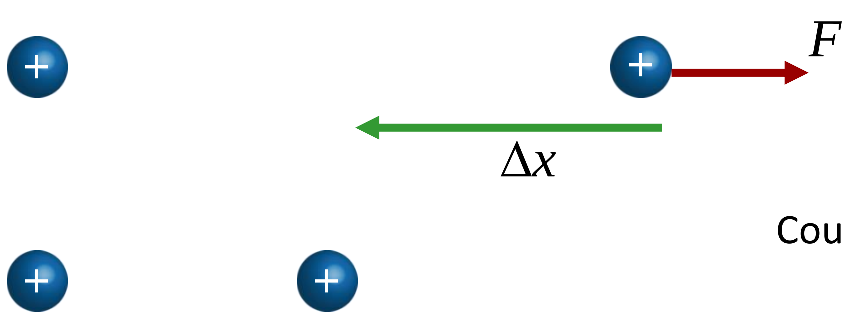
$W = 0$ (e.g. $W_{gravity}$ on moving horizontally)

Potential Energy

$$\Delta U \equiv -W_{conservative}$$

If gravity does negative work, potential energy increases!

Same idea for Coulomb force… if Coulomb force does negative work, potential energy increases.



Coulomb force does negative work
Potential energy increases

Check Point 1

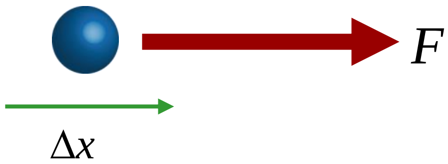
A charge is released from rest in a region of electric field. The charge will start to move

A) In a direction that makes its potential energy increase.

B) In a direction that makes its potential energy decrease.

C) Along a path of constant potential energy.

"It will move in the same direction as F, Work done by force is positive, so the potential energy decreases.."



It will move in the same direction as F

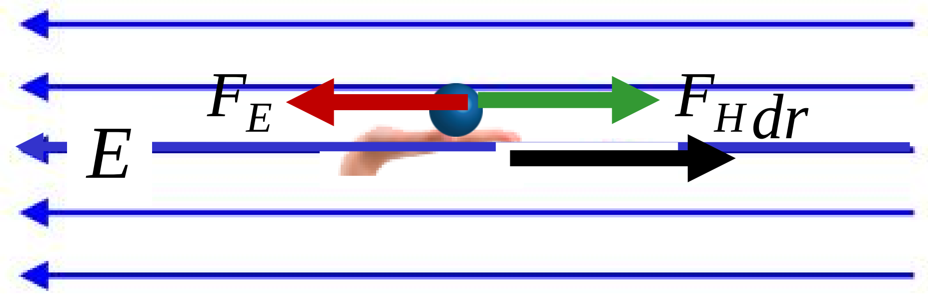
Work done by force is positive

$\Delta U = -$Work, so change in pot. Energy is negative

Nature wants things to move in such a way that PE decreases

Question

You hold a positively charged ball and walk due east in a region that contains an electric field directed due west.



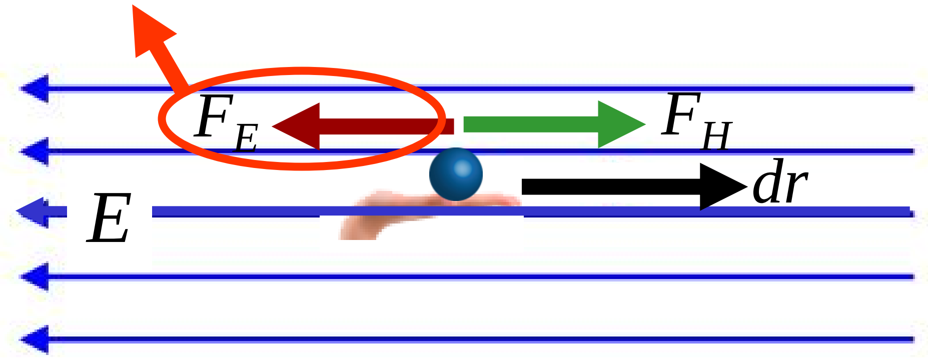
W_H is the work done by the hand on the ball
W_E is the work done by the electric field on the ball

Which of the following statements is true:

A) $W_H > 0$ and $W_E > 0$
B) $W_H > 0$ and $W_E < 0$
C) $W_H < 0$ and $W_E < 0$
D) $W_H < 0$ and $W_E > 0$

Question

Conservative force: $\Delta U = -W_E$



Is ΔU positive, negative or zero?

A) Positive

B) Negative

C) Zero

Example: Two Point Charges

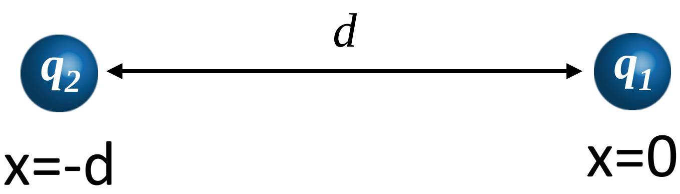
Calculate the change in potential energy for two point charges originally very far apart moved to a separation of "d"

$$\Delta U \equiv -\int_i^f \vec{F} \cdot d\vec{r}$$

$$= -\int_{-\infty}^{-d} F dx$$

$$= \int_{-\infty}^{-d} k \frac{q_1 q_2}{x^2} dx$$

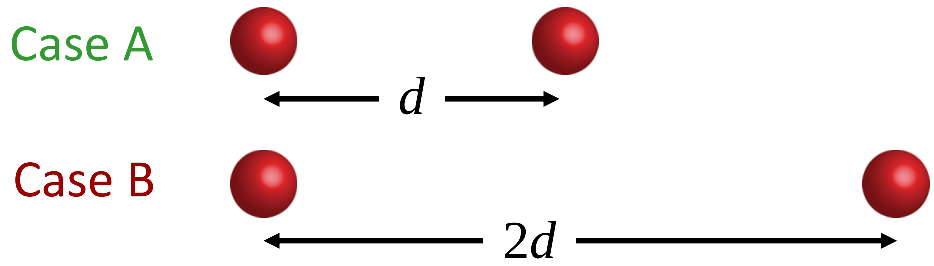
$$= -kq_1 q_2 \left[-\frac{1}{d} - \left(-\frac{1}{\infty} \right) \right] = k \frac{q_1 q_2}{d} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{d}$$



Charged particles with the same sign have an increase in potential energy when brought closer together.

For point charges often choose $r =$ infinity as "zero" potential energy.

Question



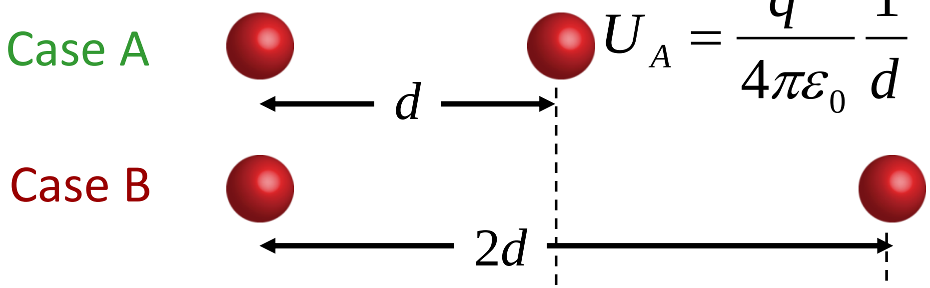
In case A two negative charges which are equal in magnitude are separated by a distance d. In case B the same charges are separated by a distance $2d$. Which configuration has the highest potential energy?

A) Case A
B) Case B

Question Discussion

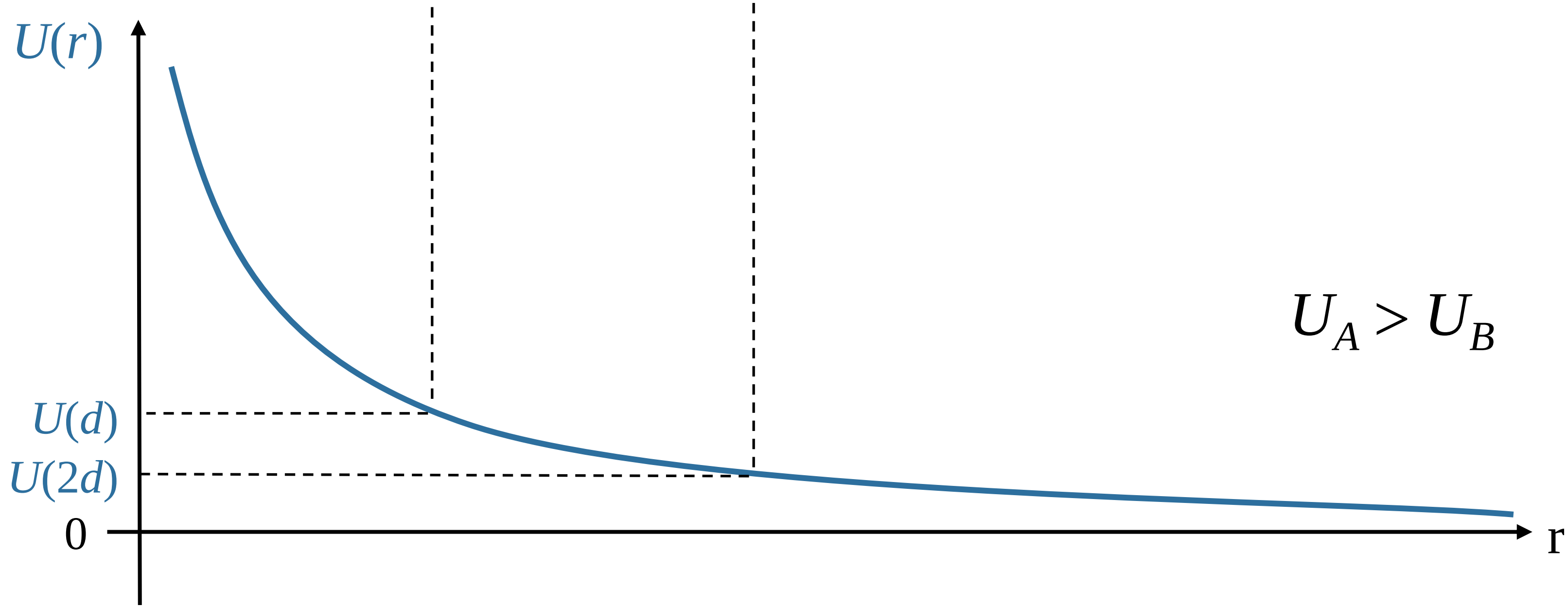
As usual, choose $U = 0$ to be at infinity:

$$U(r) = \frac{q_1 q_2}{4\pi\varepsilon_0}\frac{1}{r}$$



$$U_A = \frac{q^2}{4\pi\varepsilon_0}\frac{1}{d}$$

$$U_B = \frac{q^2}{4\pi\varepsilon_0}\frac{1}{2d}$$

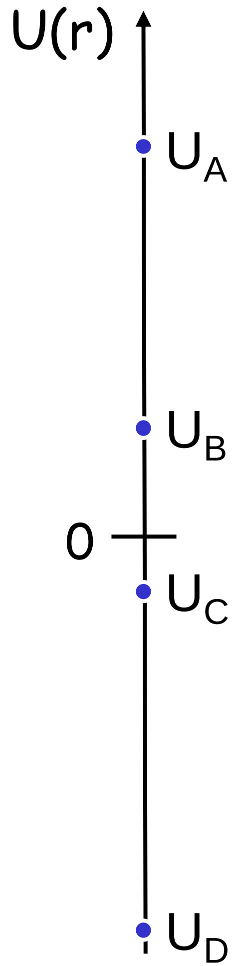


$$U_A > U_B$$

And Remember

U is just a number (not a vector)

- U DOES have a sign



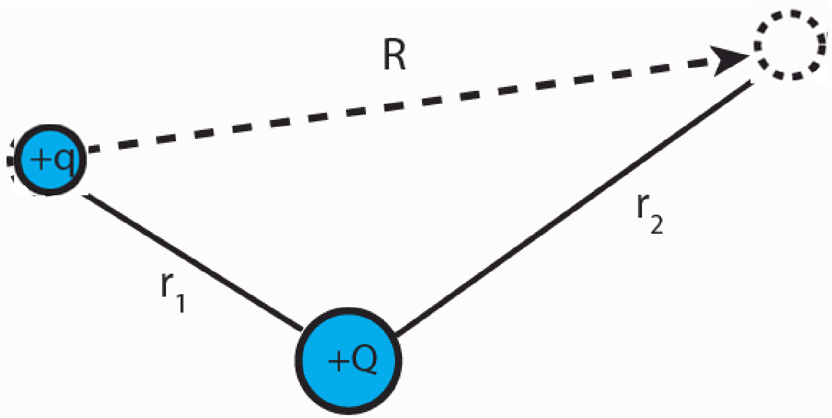
$U_A > U_B$

$U_B > U_C$

$U_C > U_D$

Check Point 2

A charge +q is moved from position 1 to position 2,
What is the change in potential energy?



A $\dfrac{kQq}{R}$

B $\dfrac{kQqR}{r_1^2}$

C $\dfrac{kQqR}{r_2^2}$

D $kQq\left(\dfrac{1}{r_2}-\dfrac{1}{r_1}\right)$

E $kQq\left(\dfrac{1}{r_1}-\dfrac{1}{r_2}\right)$

$$U_1 = \frac{kQq}{r_1} \qquad U_2 = \frac{kQq}{r_2}$$

"The electric potential at 2 is kQq/r2^2 and the electric potential energy at 1 is kQq/r1^2. The change in the potential energy should be U2-U1 no matter what path q takes."

Note: $+q$ moves AWAY from $+Q$.
Its Potential energy MUST DECREASE

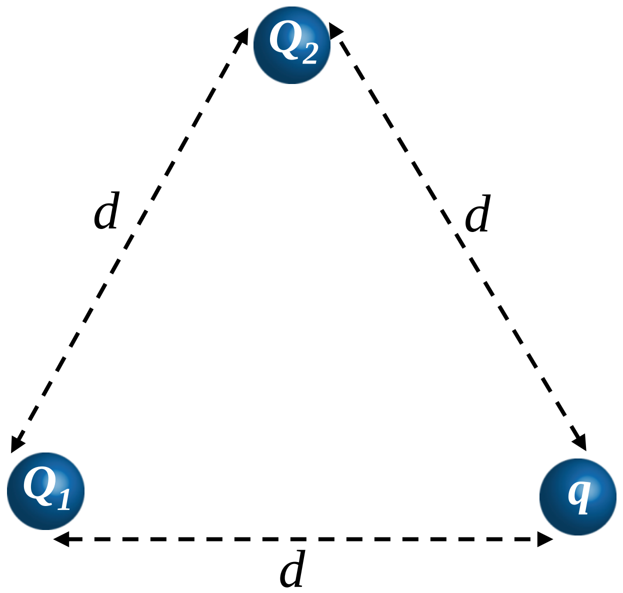
$$\Delta U < 0$$

Potential Energy of Many Charges

Two charges are separated by a distance d. What is the change in potential energy when a third charge q is brought from far away to a distance d from the original two charges?

$$\Delta U = \frac{qQ_1}{4\pi\varepsilon_0}\frac{1}{d} + \frac{qQ_2}{4\pi\varepsilon_0}\frac{1}{d}$$

(superposition)



"Can you go over in further depth, what electric potential energy actually means for a system of charges?"

Potential Energy of Many Charges

What is the change in potential energy when we bring in three identical charges, from infinitely far away, to the points on an equilateral triangle shown.

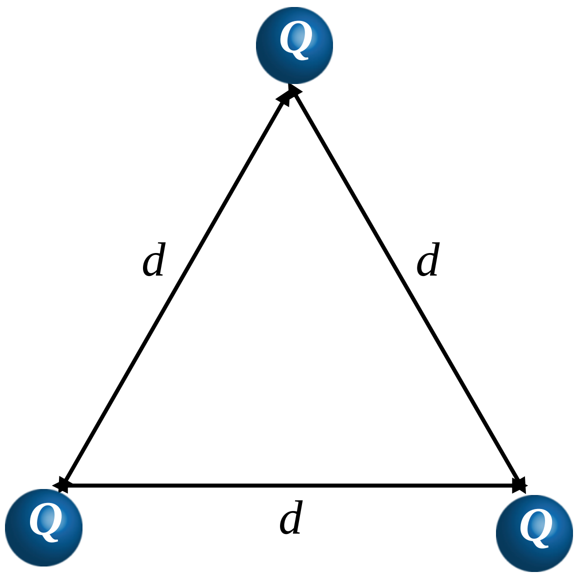
A) 0

B) $\Delta U = \frac{Q^2}{4\pi\varepsilon_0}\frac{1}{d}$

C) $\Delta U = 2\frac{Q^2}{4\pi\varepsilon_0}\frac{1}{d}$

D) $\Delta U = 3\frac{Q^2}{4\pi\varepsilon_0}\frac{1}{d}$

E) $\Delta U = 6\frac{Q^2}{4\pi\varepsilon_0}\frac{1}{d}$



Work by E to bring in first charge: $W_1 = 0$

Work by E to bring in second charge : $W_2 = -\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{d}$

Work by E to bring in third charge : $W_3 = -\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{d} - \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{d} = -\frac{2}{4\pi\varepsilon_0}\frac{Q^2}{d}$

Potential Energy of Many Charges

Suppose one of the charges is negative. Now what is the change in potential energy when we bring the three charges in from infinitely far away?

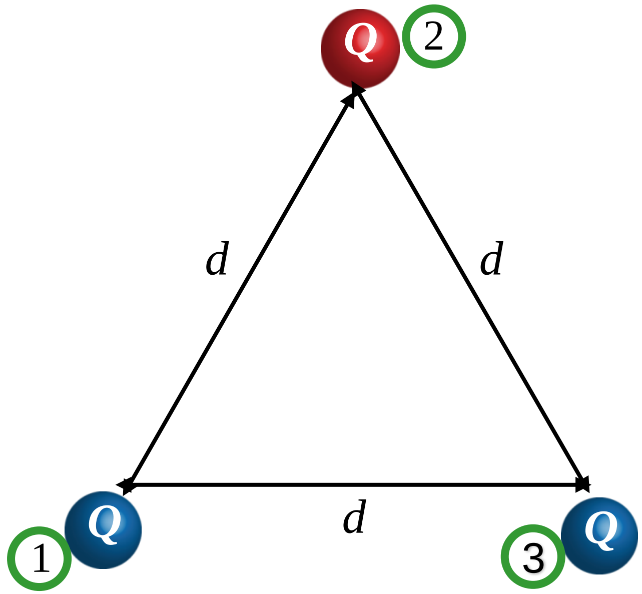
A) 0

B) $\Delta U = +1 \dfrac{Q^2}{4\pi\varepsilon_0} \dfrac{1}{d}$

C) $\Delta U = -1 \dfrac{Q^2}{4\pi\varepsilon_0} \dfrac{1}{d}$

D) $\Delta U = +2 \dfrac{Q^2}{4\pi\varepsilon_0} \dfrac{1}{d}$

E) $\Delta U = -2 \dfrac{Q^2}{4\pi\varepsilon_0} \dfrac{1}{d}$



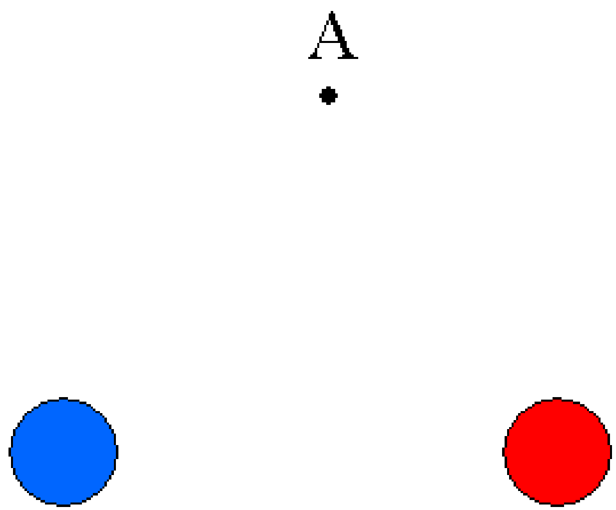
Work by E to bring in first charge: $W_1 = 0$

Work by E to bring in second charge : $W_2 = +\dfrac{1}{4\pi\varepsilon_0} \dfrac{Q^2}{d}$

Work by E to bring in third charge : $W_3 = +\dfrac{1}{4\pi\varepsilon_0} \dfrac{Q^2}{d} - \dfrac{1}{4\pi\varepsilon_0} \dfrac{Q^2}{d} = 0$

Check Point 3

Two charges with equal magnitude but opposite sign are located at equal distances from the point labeled A



If a third charge is brought in from far away to point A, how does the potential energy of the collection of charges change?

Increases	Decreases	Same	Depends on sign of charge
A	B	C	D

"Because the signs are oppositely charged, newly added kqq/d and the -kqq/d will cancel out"

Check Point 4

A positive charge is placed on the left side of a negative charge. The magnitude of the negative charge is twice that of the positive charge.

Is there any (finite) location that a third charge can be placed such that the total potential energy of the system does not change?

- YES, as long as the third charge is positive
- YES, as long as the third charge is negative
- YES, no matter what the third charge is
- NO

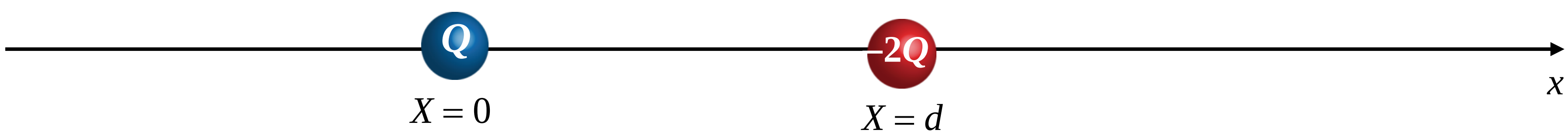
C) “As long as the third charge is twice as far from the larger negative charge as it is the smaller positive charge, the total potential energy of the system will be unaffected.“

D) “A third charge cannot cancel out because the magnitudes of the two initial charges are not equal..”

LET’S DO THE CALCULATION!

Example

A positive charge q is placed at $x = 0$ and a negative charge $-2q$ is placed at $x = d$. At how many different places along the x axis could another positive charge be placed without changing the total potential energy of the system?



A 0

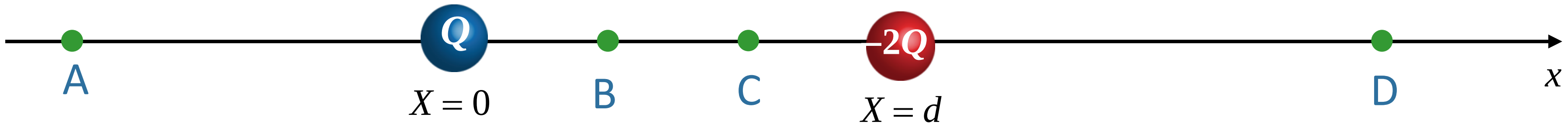
B) 1

C) 2

D) 3

Example

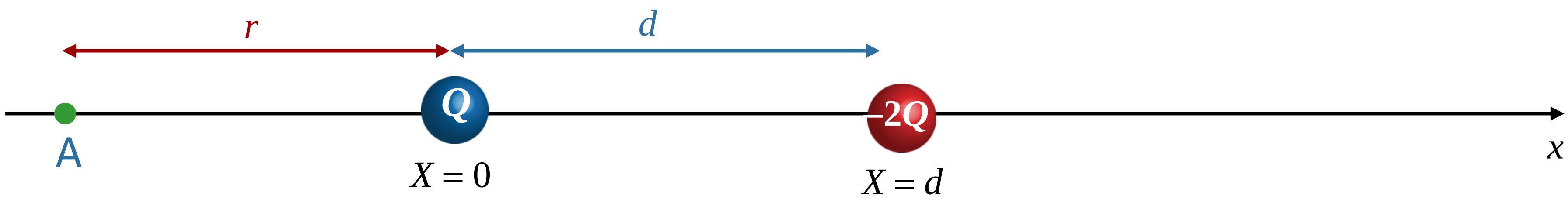
At which two places can a positive charge be placed without changing the total potential energy of the system?



A) A & B
B) A & C
C) B & C
D) B & D
E) A & D

Let's calculate the positions of A and B

Lets work out where A is



$$\Delta U = +\frac{1}{4\pi\varepsilon_0}\frac{Qq}{r} - \frac{1}{4\pi\varepsilon_0}\frac{2Qq}{r+d}$$

Set $\Delta U = 0$

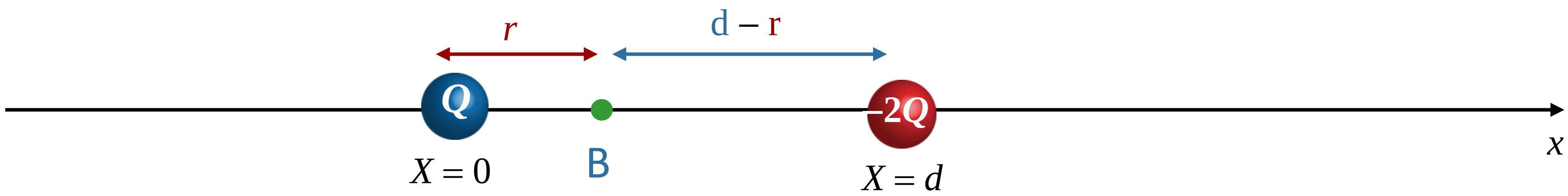
$$\frac{1}{r} = \frac{2}{r+d}$$

$$\boxed{r = d}$$

Makes Sense!

q is twice as far from $-2Q$ as it is from $+Q$

Lets work out where B is



Setting $\Delta U = 0$ → $\dfrac{1}{r} = \dfrac{2}{d-r}$

$$2r = d - r$$

$$\boxed{r = \frac{d}{3}}$$

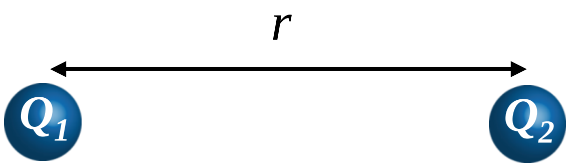
Makes Sense!

q is twice as far from -2Q as it is from + Q

Takeaways

For a pair of charges:

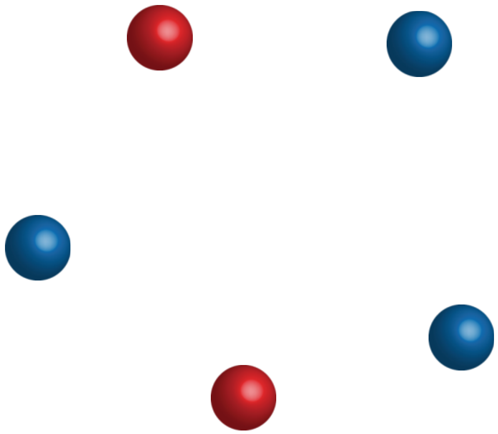
$$U = k\frac{q_1 q_2}{r}$$



(We usually choose $\mathrm{U} = 0$ to be where the charges are far apart)

For a collection of charges:

Sum up $U = k\dfrac{q_1 q_2}{r}$ **for all pairs**



Next: electric potential