20 Spring Math Exam

1. $$\int x^2(\ln x)^2dx$$

2. $$\int_0^1 x^3\sqrt{1-x^2}dx$$

3. $$\int_0^{\frac{\pi}{2}} 16x\cos^2 x dx$$

4. $$\int_1^{\infty} \frac{\ln x}{x^4}dx$$

5. Power series representation of the function $f(x) = \frac{x}{(1+x)^2}$ has the form:

   $$f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + O(x^6).$$

   Determine a_5

6. Find the interval where the following power series is convergent:

   $$\sum_{n=2}^{\infty} \frac{2^{-n}}{\ln n}(x-1)^n$$

7. Find the area enclosed by the curve $x = a\cos t,\ y = b\sin t$ using parameteric equations.

8. Prove that the following sequence converges and find its limit

   $$a_{n+1} = \sqrt{1+a_n}\ ,\ a_1 = 1$$

9. Determine if the series convergent (give a complete proof indicating which convergence criteria you are using)

   $$\sum_{n=2}^{\infty} \frac{\sin n}{n(\ln n)^2}$$

10. Using Macloaurin series to find the sum of the series

    $$\sum_{n=0}^{\infty} (-1)^n \frac{x^{4n}}{n!}$$

    $$\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{4^{2n}(2n)!}$$

11. Find Taylor polynomial $T_2(x)$ for the function $f(x)$ centered at $a = 1$

$$f(x) = \cot(x)$$

12. Consider the curve in polar coordinates $r = 1 + \cos\theta$
    - Sketch the curve. Find the points on the curve where the tangent line is horizontal or vertical.
    - Find the area enclosed by the curve.
    - Find the length of this curve.
13. Find $f^{(1000)}(0)$ of following function: (Hint: find $\ln(1 + x)$ Taylor series first.)

$$f(x) = \ln(1 + x + x^2 + x^3)$$

14. Consider the following curve:

$$y = \int_1^x \sqrt{\sqrt{t} - 1}\, dt,\ 1 \leq x \leq 16$$

- Find the length of the curve.
- Find the area of the surface obtained by rotating the curve about the y-axis

15. Find all the solutions of the equation: (Hint: consider the cases $x \geq 0$ and $x < 0$ separately)

$$1 + \frac{x}{2!} + \frac{x^2}{4!} + \frac{x^3}{6!} + \frac{x^4}{8!} + \cdots = 0$$