Electricity & Magnetism
Lecture 4

Today's Concepts:

A) Conductors

B) Using Gauss' Law

Gauss (not just a good idea, it's the law!)

"What exactly is Gauss's law used to find? It's confusing what exactly it's used to find or how it can be applied?"

$$\int_{closed-surface} \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$

ALWAYS TRUE!

Two uses

1) If know E everywhere on surface can calculate Q_{enc}

(e.g. in metal E = 0)

2) In cases of high symmetry can pull E outside the integral and solve

$$E = \frac{Q_{enclosed}}{A\varepsilon_0}$$

Conductors and Insulators

Conductors = charges free to move

e.g. metals

Insulators = charges fixed

e.g. glass (air is insulator for this class)

Define: Conductors = Charges Free to Move

I didn't understand, why the electric field inside a conductor is zero and why the charge lies at the surface.

Claim: $E = 0$ inside any conductor at equilibrium

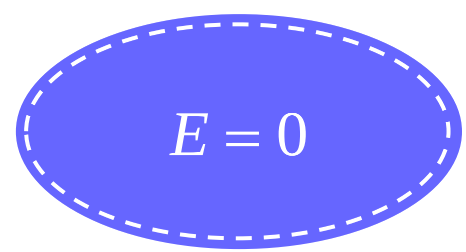
Charges in conductor move to make E field zero inside. (Induced charge distribution).

If $E \neq 0$, then charge feels force and moves!

Claim: Excess charge on conductor only on surface at equilibrium

Why?

- Apply Gauss' Law
  - Take Gaussian surface to be just inside conductor surface
  - $E = 0$ everywhere inside conductor $\longrightarrow$ $\oint \vec{E} \cdot d\vec{A} = 0$
  - Gauss' Law: $\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$ $\longrightarrow$ $\boxed{Q_{enc} = 0}$



Gauss' Law + Conductors + Induced Charges

Could we go over how when there is a placed charge within a hollow conducting sphere the electric field is still zero with in that sphere. Wouldn't Gauss' Law say that because we are containing a charge there would have to be an electric field?

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

ALWAYS TRUE!

If choose a Gaussian surface that is entirely in metal, then $E = 0$ so $Q_{enclosed}$ must also be zero!

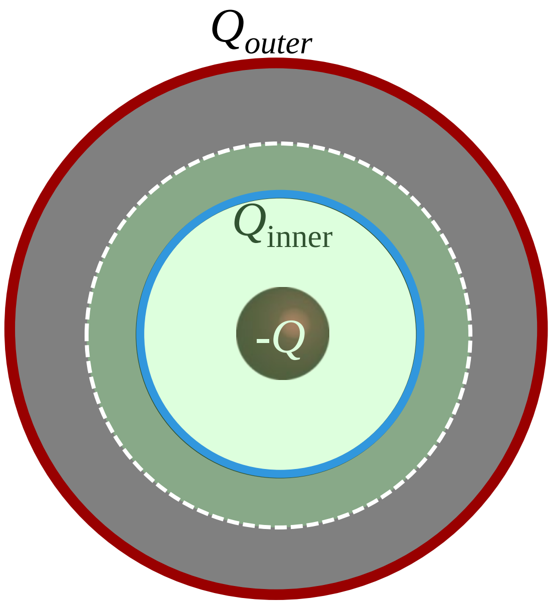
How Does This Work?

Charges in conductor move to surfaces to make $Q_{enclosed} = 0$.

We say charge is induced on the surfaces of conductors

Charge in Cavity of Conductor

A particle with charge $-Q$ is placed in the center of an uncharged conducting hollow sphere. How much charge will be induced on the inner and outer surfaces of the sphere?



A) inner = $-Q$, outer = $+Q$

B) inner = $-Q/2$, outer = $+Q/2$

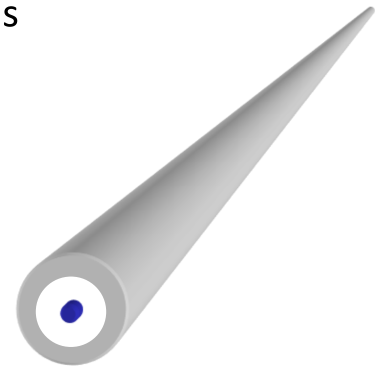
C) inner = 0, outer = 0

D) inner = $+Q/2$, outer = $-Q/2$

E) inner = $+Q$, outer = $-Q$

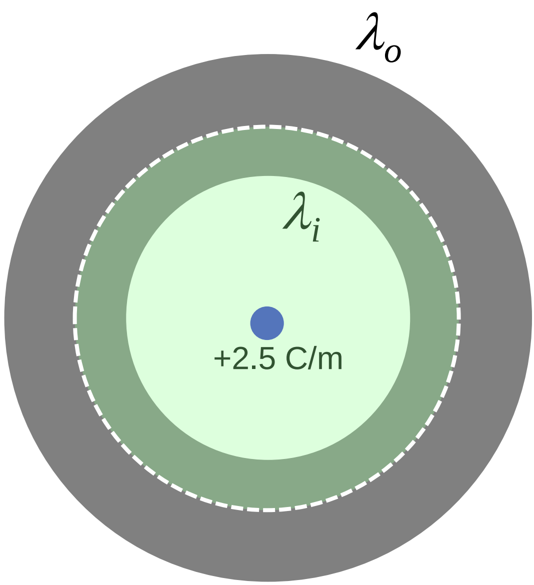
Infinite Cylinders

A long thin wire has a uniform positive charge density of 2.5 C/m. Concentric with the wire is a long thick conducting cylinder, with inner radius 3 cm, and outer radius 5 cm. The conducting cylinder has a net linear charge density of -4C/m.



What is the linear charge density of the induced charge on the inner surface of the conducting cylinder (λ_i) and on the outer surface (λ_o)?

λ_i:	+2.5 C/m	–4 C/m	0	–2.5 C/m	–2.5 C/m
λ_o:	–6.5 C/m	0	-4 C/m	+2.5 C/m	–1.5 C/m
	A	B	C	D	E



Using Gauss' Law to determine E

How do you choose the Gaussian surface???

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

ALWAYS TRUE!

In cases with symmetry can pull E outside and get $E = \frac{Q_{enclosed}}{A\varepsilon_0}$

In General, integral to calculate flux is difficult…. and not useful!

To use Gauss' Law to calculate E, need to choose surface carefully!

1) Want E to be constant and equal to value at location of interest

OR

2) Want E dot $A = 0$ so doesn't add to integral

Gauss' Law Symmetries

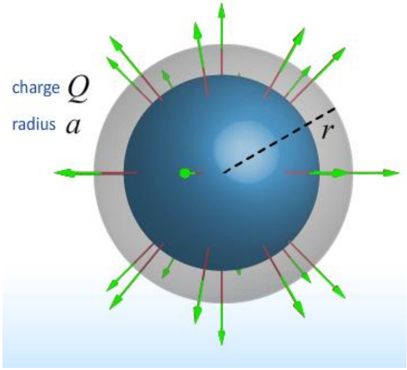
$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

ALWAYS TRUE!

In cases with symmetry can pull E outside and get

$$E = \frac{Q_{enclosed}}{A\varepsilon_0}$$

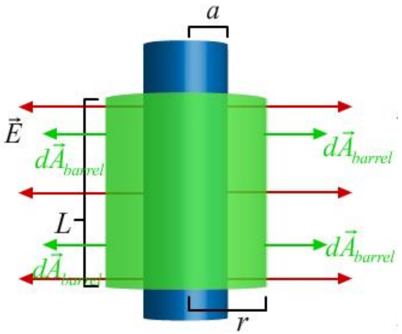
Spherical



$$A = 4\pi r^2$$

$$E = \frac{Q_{enc}}{4\pi\varepsilon_0 r^2}$$

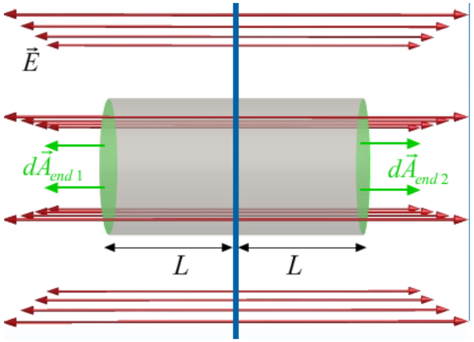
Cylindrical



$$A = 2\pi r L$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

Planar

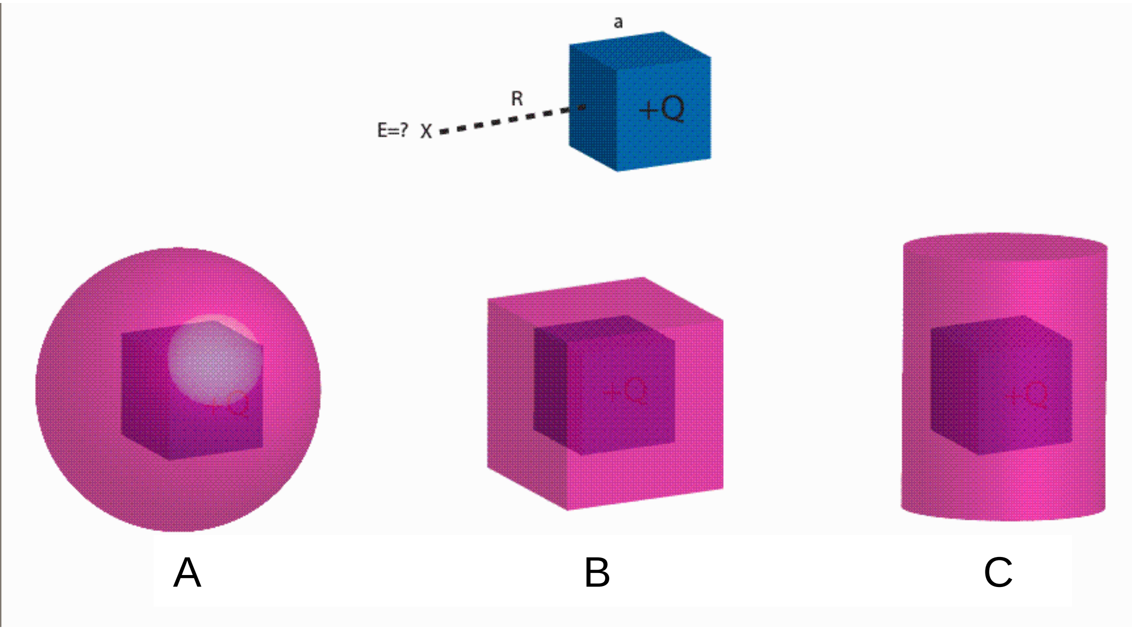


$$A = 2\pi r^2$$

$$E = \frac{\sigma}{2\varepsilon_0}$$

Check Point 1

Which Gaussian Surface would you use to calculate E due to cube of charge?



D) The field cannot be calculated using Gauss' law for the drawn surfaces

E) None of the above

Cube is NOT one of 3 symmetries that works because

THE FIELD AT THE FACE OF THE CUBE

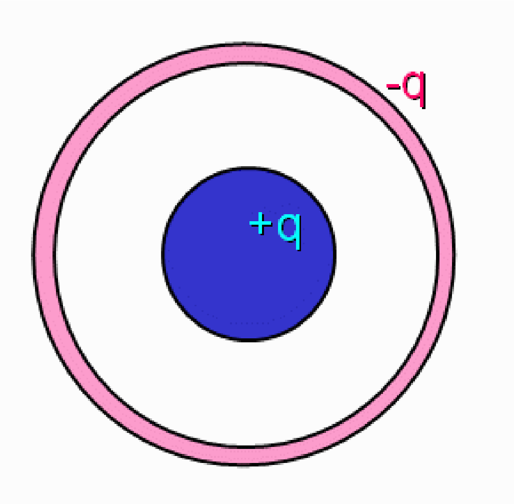
IS NOT

PERPENDICULAR OR PARALLEL

3D	POINT	-	SPHERICAL
2D	LINE	-	CYLINDRICAL
1D	PLANE	-	PLANAR

Check Point 2

A positively charged solid conducting sphere (blue) is inside a negatively charged conducting shell (red).



What is direction of field between blue and red spheres?

A) Outward B) Inward C) Zero

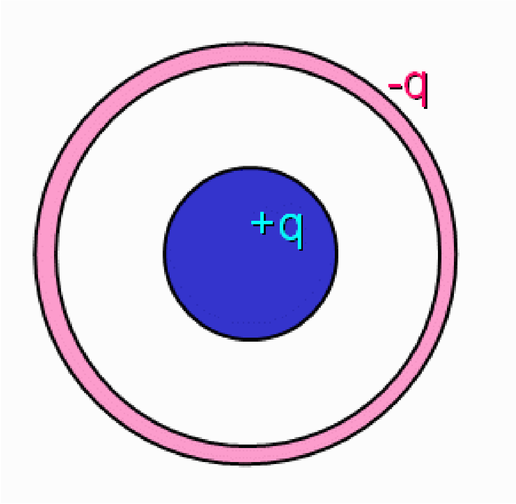
Careful: what does inside mean?
This is always true for a solid conductor (within the material of the conductor)
Here we have a charge "inside"

A) "Gauss's law, the region between the spheres encloses a positive charge, and thus the field must point outward."

C) "Within the boundaries of a conductor, the electric field will be 0."

Check Point 3

A positively charged solid conducting sphere (blue) is inside a negatively charged conducting shell (red).



What is direction of field OUTSIDE the red sphere?

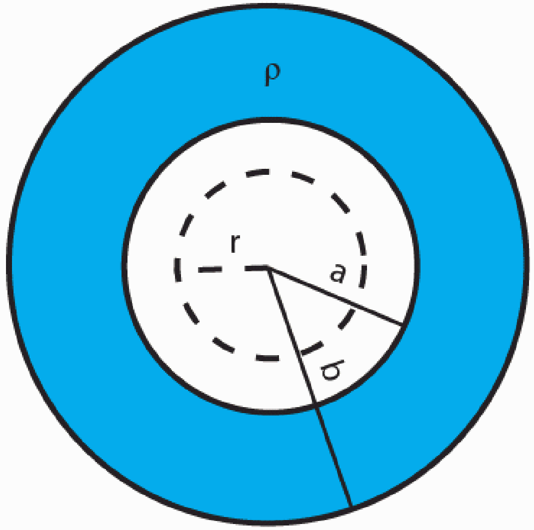
A) Outward

B) Inward

C) Zero

Check Point 4

A spherical insulating shell has inner radius a, and outer radius b, and uniform charge density ρ



What is magnitude of E at dashed line (r)?

A) $\dfrac{\rho}{\varepsilon_0}$

B) Zero

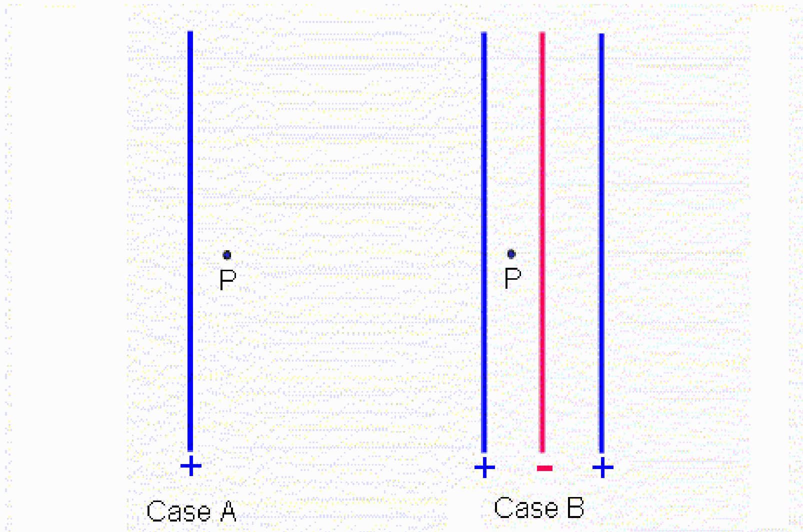
C) $\dfrac{\rho(b^2 - a^2)}{3\varepsilon_0 r^2}$

D) None of above

"Since the charge enclosed by r<a = 0, the electric field must also be 0 by Gauss' Law."

Check Point 5

10) In both cases shown below, the colored lines represent positive (blue) and negative (red) charged planes. The magnitudes of the charge per unit area on each plane is the same.



In which case is E at point P the biggest?

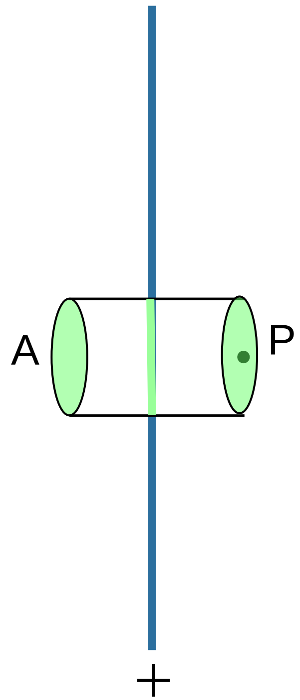
A) A B) B C) the same

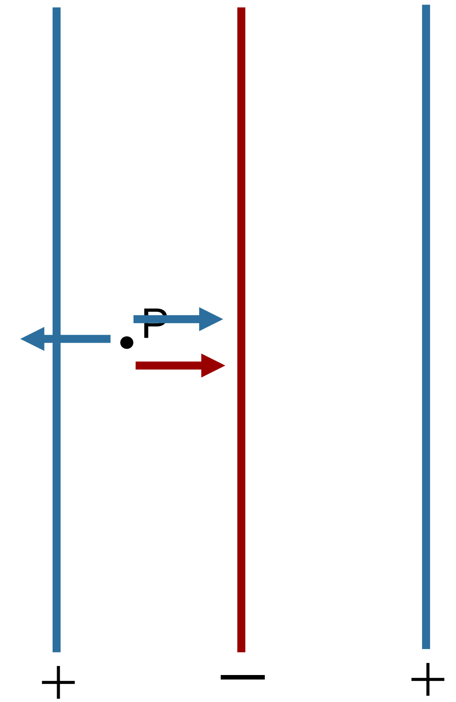
B) In case B, the surrounding planes both emit a field in the same direction, so they "add together," and as such, the field experienced at P is stronger in case B.

C) “The two positive planes in case B have a net 0 effect because they are on opposite sides of point P, so they can be ignored. Both cases can be thought of as having only one plane.”

Gauss's Law and Superposition

Lets do calculation!





Superposition

Gauss's Law

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

$$E(2A) = \frac{Q_{enc}}{\varepsilon_0}$$

$$E(2A) = \frac{\sigma A}{\varepsilon_0}$$

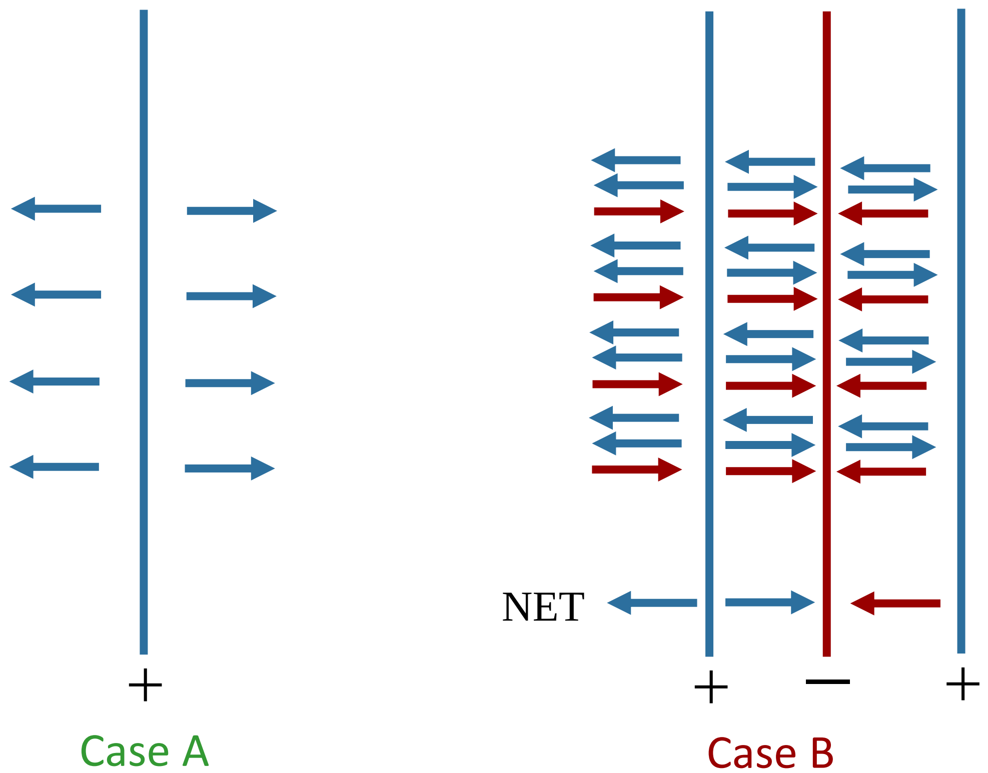
$$E = \frac{\sigma}{2\varepsilon_0}$$

$$E = +\frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0}$$

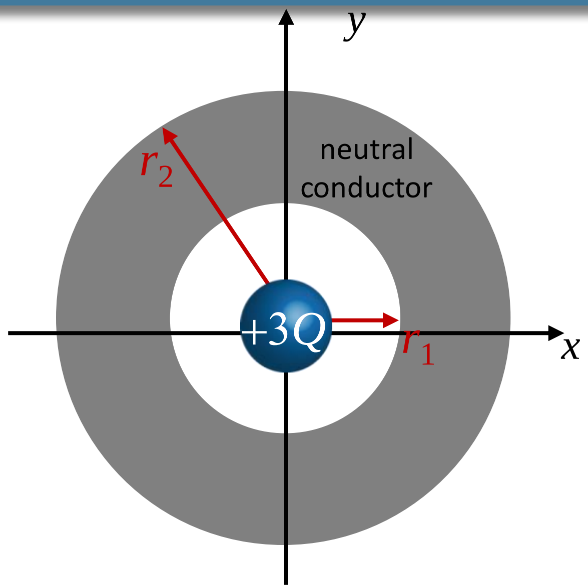
$$E = \frac{\sigma}{2\varepsilon_0}$$

Superposition:

Can you explain about the infinite sheets of charge problem? Like draw out the directions to where they are going during lecture?



Calculation



Point charge $+3Q$ at center of neutral conducting shell of inner radius r_1 and outer radius r_2.

a) What is E everywhere?

First question: Do we have enough symmetry to use Gauss' Law to determine E?

Yes, Spherical Symmetry (what does this mean???)

A) Direction of E is along $\hat{x}$

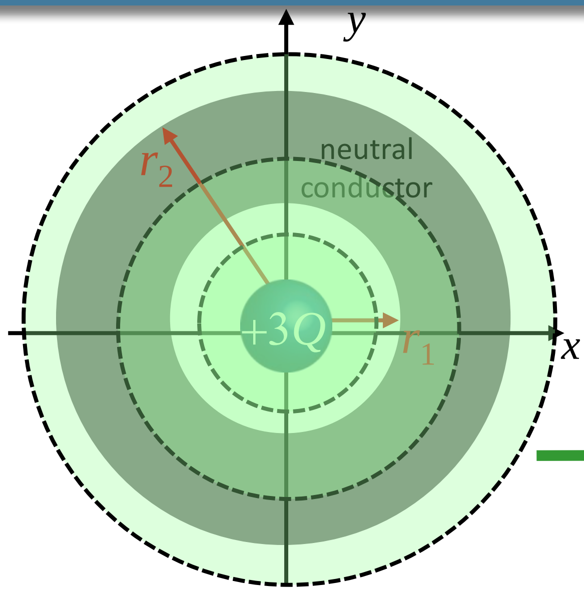
B) Direction of E is along $\hat{y}$

C) Direction of E is along $\hat{r}$

D) None of the above

Magnitude of E depends only on R

Calculation



Point charge $+3Q$ at center of neutral conducting shell of inner radius r_1 and outer radius r_2.

A) What is E everywhere?

We know:

magnitude of E is *fcn* of r

direction of E is along $\hat{r}$

We can use Gauss' Law to determine E

Use Gaussian surface = sphere centered on origin

$r < r_1$

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

$$E 4\pi r^2 = \frac{3Q}{\varepsilon_0}$$

$$E = \frac{3Q}{4\pi r^2 \varepsilon_0}$$

$r_1 < r < r_2$

A) $E = \dfrac{3Q}{4\pi r^2 \varepsilon_0}$

B) $E = \dfrac{3Q}{4\pi r_1^2 \varepsilon_0}$

C) $E = 0$

$r > r_2$

A) $E = \dfrac{3Q}{4\pi r^2 \varepsilon_0}$

B) $E = \dfrac{3Q}{4\pi (r - r_2)^2 \varepsilon_0}$

C) $E = 0$