Physics 212
Lecture 6

Today's Concept:

Electric Potential

(Defined in terms of Path Integral of Electric Field)

Big Idea

Last time we defined the electric potential energy of charge q in an electric field:

$$\Delta U_{a\to b} = -\int_a^b \vec{F}\cdot d\vec{l} = -\int_a^b q\vec{E}\cdot d\vec{l}$$

The only mention of the particle was through its charge q.

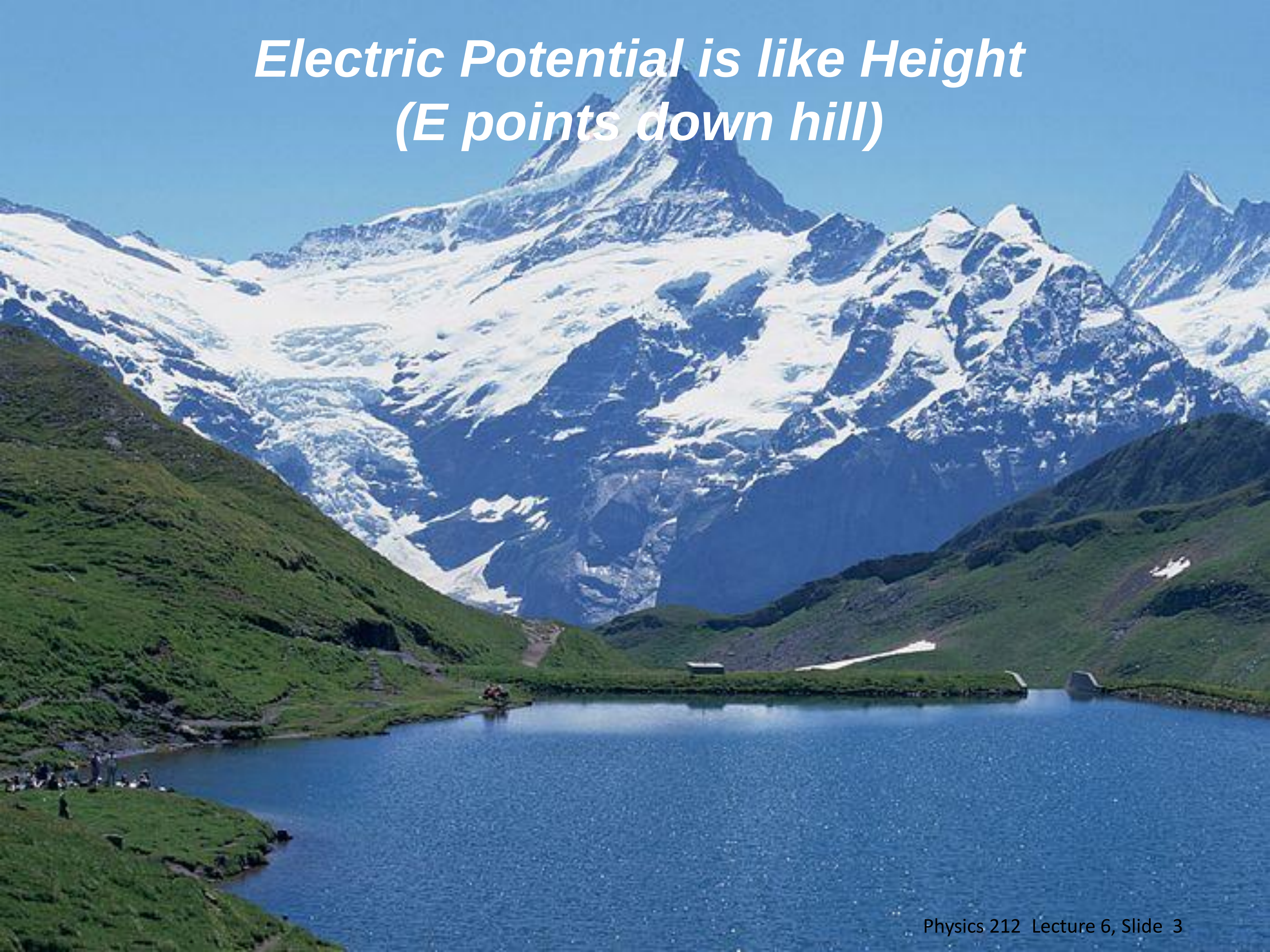
We can obtain a new quantity, the electric potential, which is a PROPERTY OF THE SPACE, as the potential energy per unit charge.

$$\Delta V_{a\to b} \equiv \frac{\Delta U_{a\to b}}{q} = -\int_a^b \vec{E}\cdot d\vec{l}$$

Note the similarity to the definition of another quantity which is also a PROPERTY OF THE SPACE, the electric field.

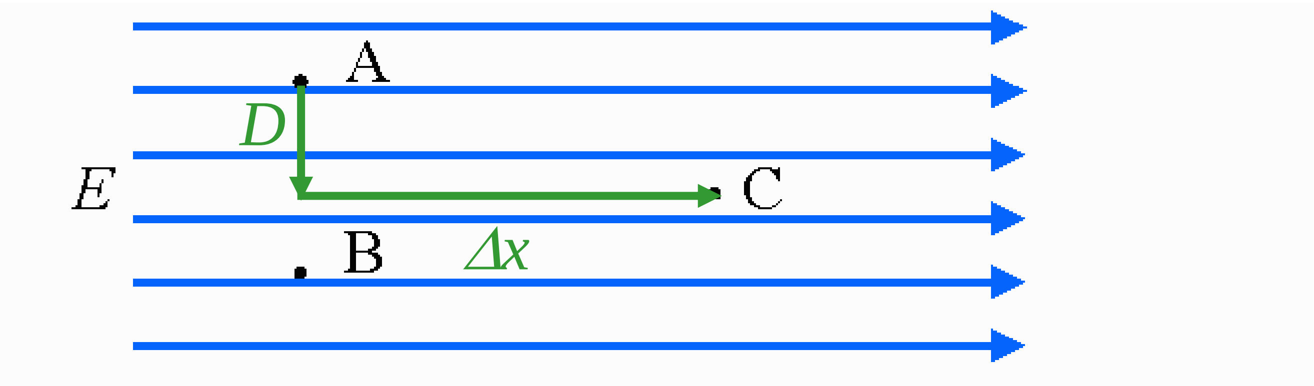
$$\vec{E} \equiv \frac{\vec{F}}{q}$$

Electric Potential is like Height
(E points down hill)



Electric Potential from E field

Consider the three points A, B, and C located in a region of constant electric field as shown.



What is the sign of $\Delta V_{AC} = V_C - V_A$?

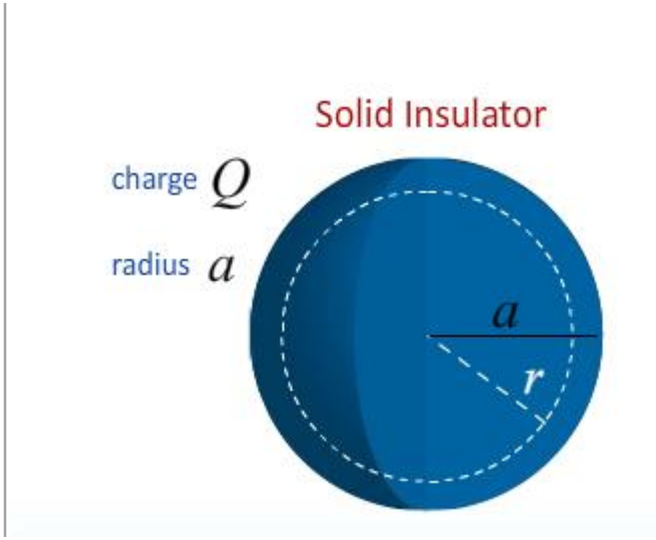
A) $\Delta V_{AC} < 0$ B) $\Delta V_{AC} = 0$ C) $\Delta V_{AC} > 0$

E points down hill

Remember the definition: $\Delta V_{A \to C} = -\int_A^C \vec{E} \cdot d\vec{l}$

Charged Spherical Insulator

I didn't understand the voltage calculation within an insulating sphere.



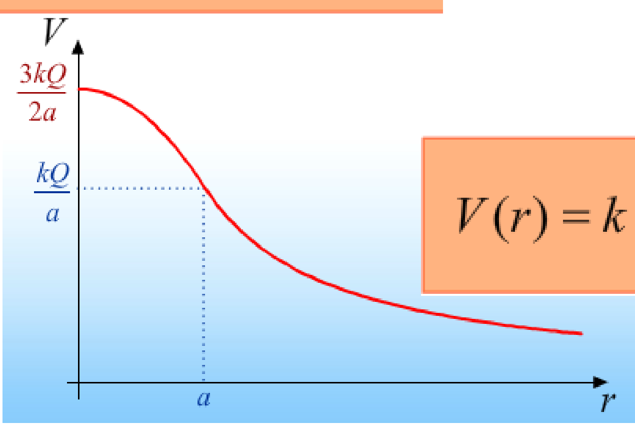
$$V(r) = -\int_{\infty}^{r} \vec{E} \cdot d\vec{l}$$ For $r < a$

$$V(r) = -\int_{\infty}^{a} E\, dr \; - \int_{a}^{r} E\, dr$$

$$V(r) = -\int_{\infty}^{a} k\frac{Q}{r^2}\, dr \; - \int_{a}^{r} k\frac{Q}{a^3}\, r\, dr$$

$$V(r) = k\frac{Q}{a} + k\frac{Q}{2a^3}(a^2 - r^2)$$

$$\boxed{V(r) = k\frac{Q}{2a^3}(3a^2 - r^2)}$$ For $r < a$



$$\boxed{V(r) = k\frac{Q}{r}}$$ For $r > a$

CheckPoint 2

If the electric field is zero in a region of space, what does that tell you about the electric potential in that region, (which statement is always true)?

A) The electric potential is zero everywhere in this region.
B) The electric potential is zero at at least one point in this region.
C) The electric potential is constant everywhere in this region.

Remember the definition

$$\Delta V_{A\to B} = -\int_A^B \vec{E}\cdot d\vec{l}$$

E from V

If we can get the potential by integrating the electric field:

$$\Delta V_{a\to b} = -\int_a^b \vec{E}\cdot d\vec{l}$$

We should be able to get the electric field by differentiating the potential

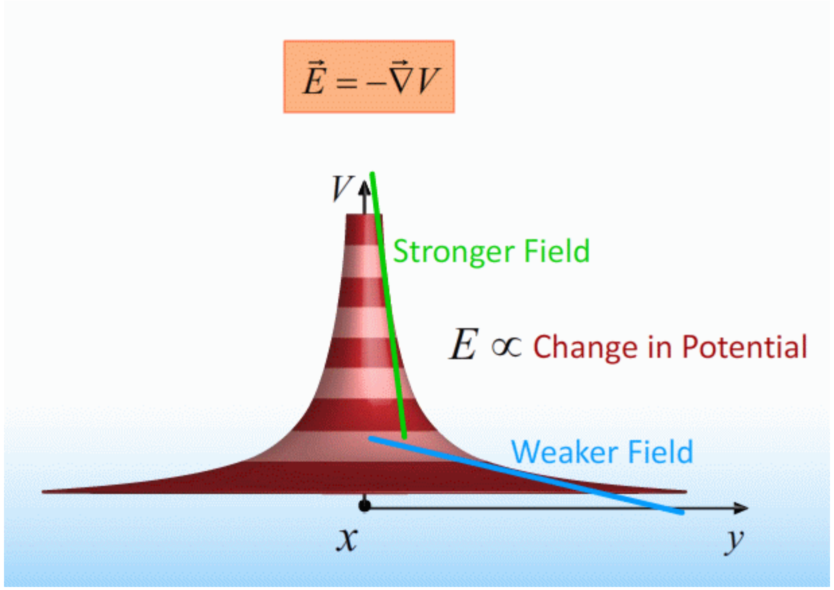
$$\vec{E} = -\vec{\nabla} V$$

In Cartesian coordinates:

$$E_x = -\frac{\partial V}{dx}$$

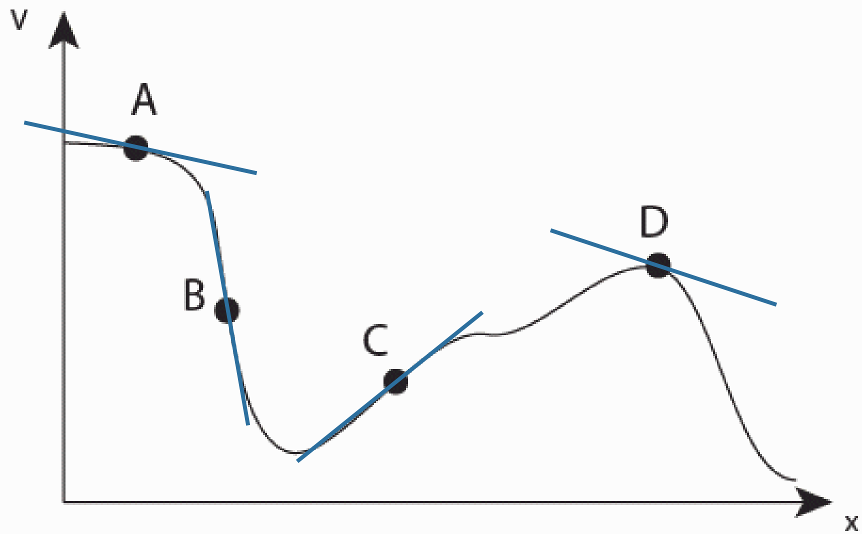
$$E_y = -\frac{\partial V}{dy}$$

$$E_z = -\frac{\partial V}{dz}$$



CheckPoint 1a

2) The electric potential in a certain region is plotted in the following graph



At which point is the magnitude of the electric field greatest?

"A) This is where the greatest potential is."

"B) Steepest slope here.."

"C) At C, the change in Potential is the greatest, so the E-field is the greatest"

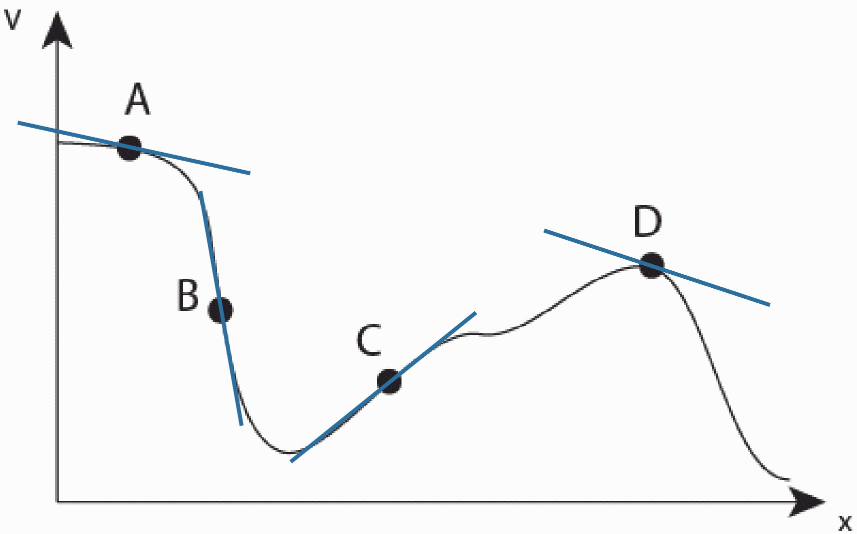
How do we get E from V?

$$\vec{E} = -\vec{\nabla} V \quad \longrightarrow \quad E_x = -\frac{\partial V}{dx} \quad \longrightarrow$$

Look at slopes!

CheckPoint 1b

2) The electric potential in a certain region is plotted in the following graph



At which point is the electric field pointing in the negative x direction?

"B) The slope is negative thus, the E-field should be pointing along the negative x-axis."

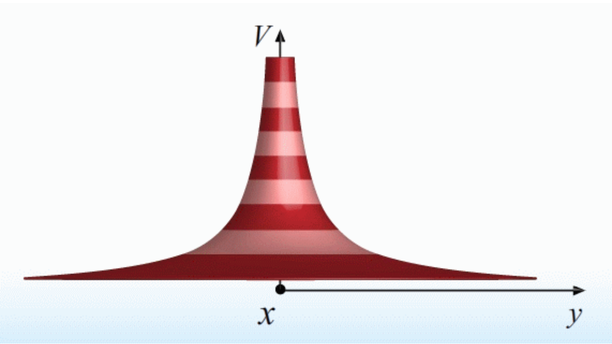
"C) E=-gradient(V). This means where the slope is positive, the E-field is negative.."

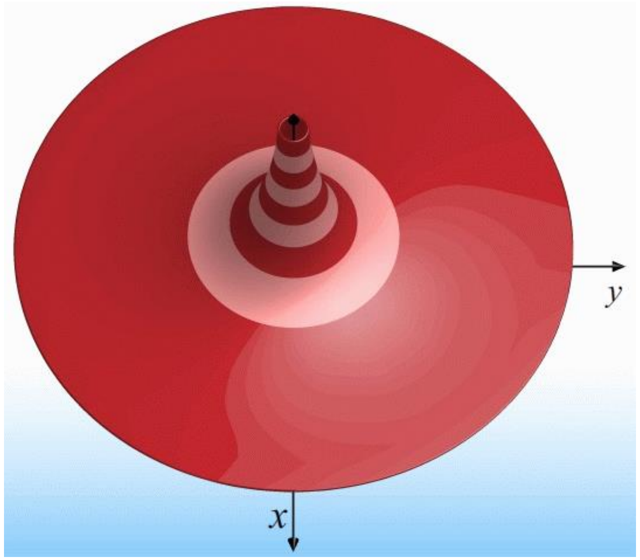
How do we get E from V?

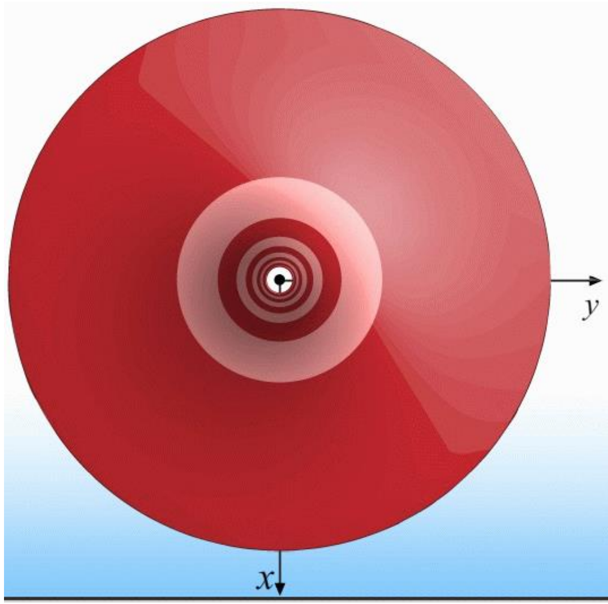
$$\vec{E} = -\vec{\nabla} V \;\rightarrow\; E_x = -\frac{\partial V}{dx} \;\rightarrow\; \text{Look at slopes!}$$

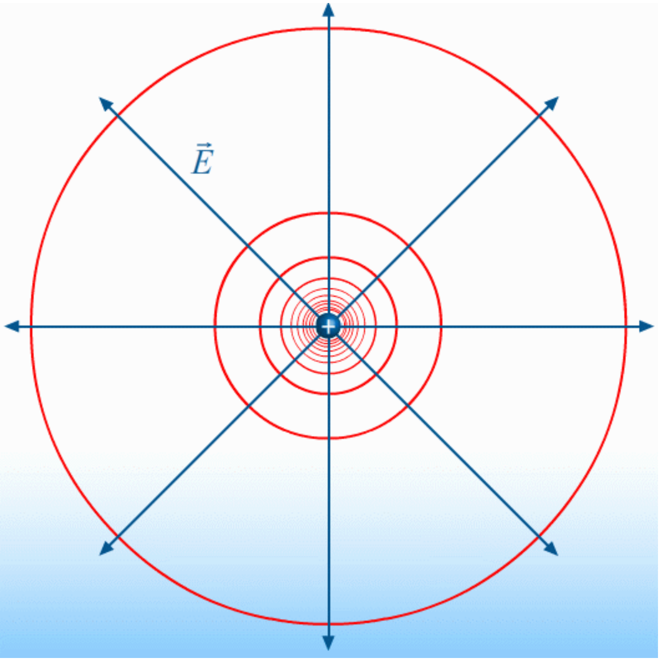
Equipotentials

Equipotentials are the locus of points having the same potential.







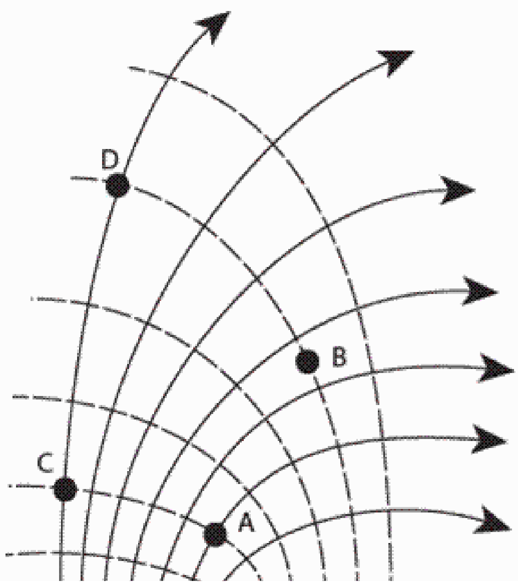


Equipotentials are
ALWAYS
perpendicular to the electric field lines.

The SPACING of the equipotentials indicates
The STRENGTH of the electric field.

CheckPoint 3b

The field-line representation of the E-field in a certain region in space is shown below. The dashed lines represent equipotential lines.

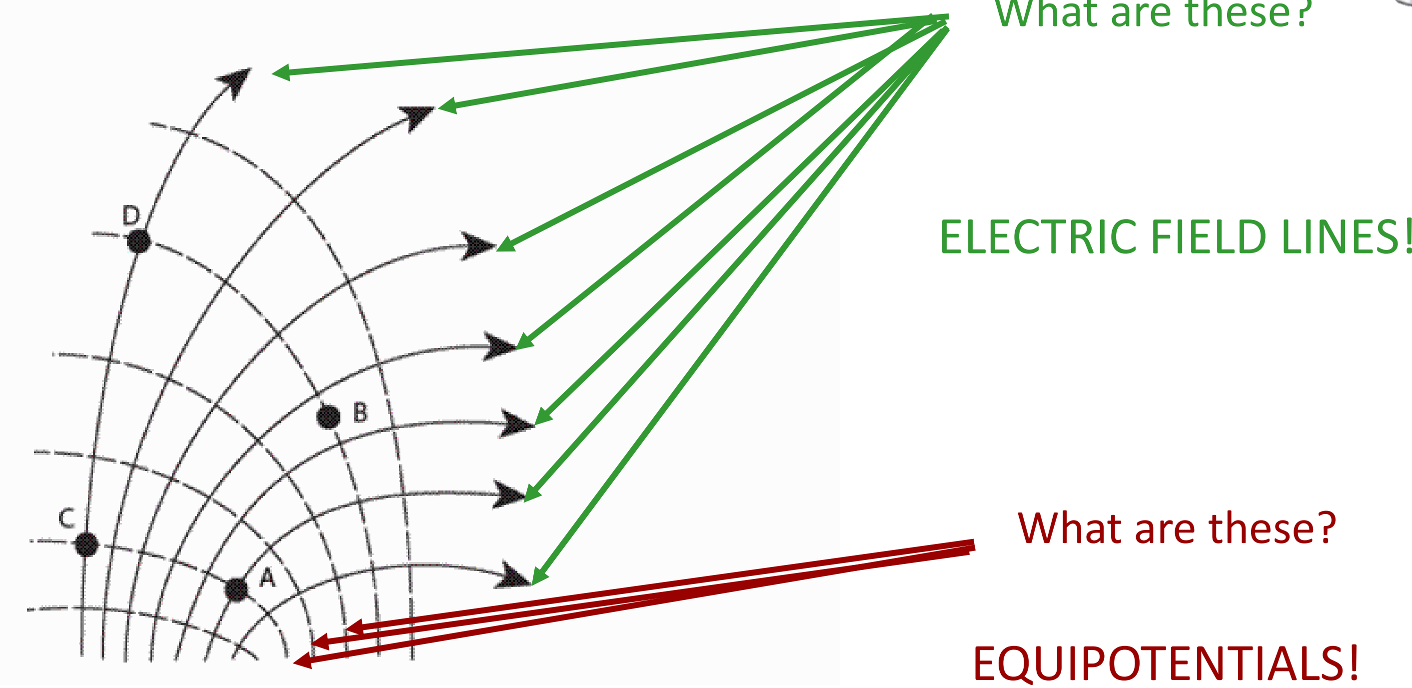


Compare the work needed to move a NEGATIVE charge from A to B, with that required to move it from C to D

A) More work from A to B
B) More work from C to D
C) Same
D) Can not determine w/o performing calculation

Hint

The field-line representation of the E-field in a certain region in space is shown below. The dashed lines represent equipotential lines.



What is the sign of W_{AC} = work done by E field to move negative charge from A to C ?

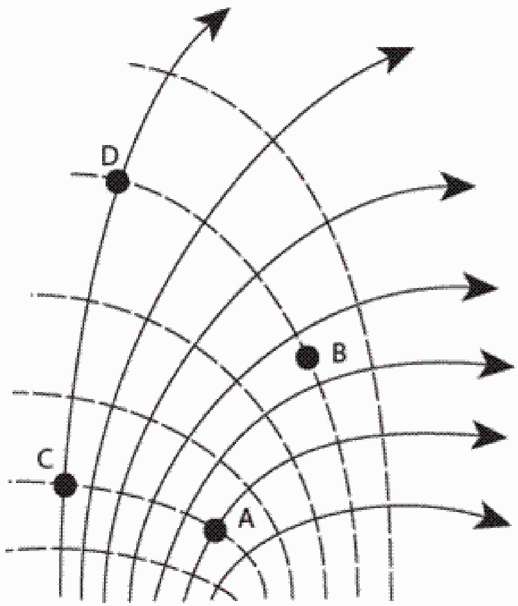
A) $W_{AC} < 0$

B) $W_{AC} = 0$

C) $W_{AC} > 0$

CheckPoint 3b Again?

The field-line representation of the E-field in a certain region in space is shown below. The dashed lines represent equipotential lines.



Compare the work needed to move a NEGATIVE charge from A to B, with that required to move it from C to D

A) More work from A to B — "The field is stronger in the region from A to B than from C to D."

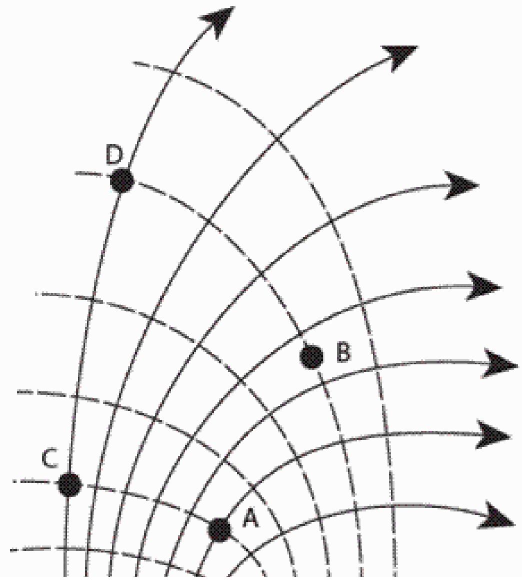
B) More work from C to D — "The distance between the two points is greater."

C) Same — "same number of equipotential jumps."

D) Can not determine w/o performing calculation

CheckPoint 3c

The field-line representation of the E-field in a certain region in space is shown below. The dashed lines represent equipotential lines.



Compare the work needed to move a NEGATIVE charge from A to B, with that required to move it from A to D

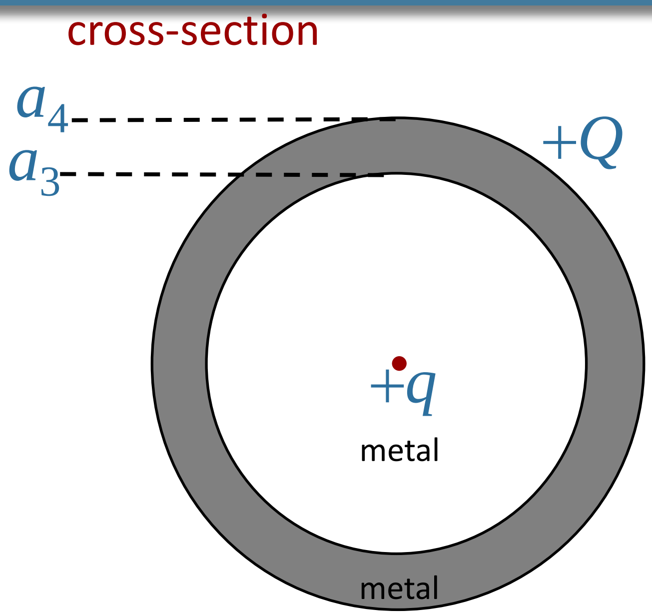
A) More work from A to B — "C and D is on the same electric field so the work is zero.."

B) More work from A to D — "The distance between the two points is greater."

C) Same — "same number of equipotential jumps."

D) Can not determine w/o performing calculation

Calculation for Potential



Point charge q at center of spherical shell of inner and outer radii a_3, and a_4. The shell carries charge Q.

What is V as a function of r?

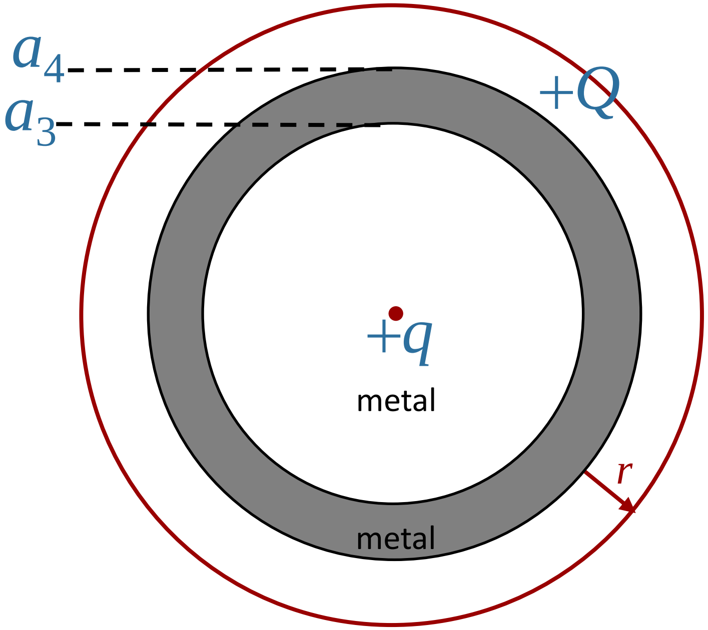
Conceptual Analysis:

- Charges q and Q will create an E field throughout space
- $V(r) = -\int_{r_0}^{r} \vec{E} \cdot d\vec{\ell}$

Strategic Analysis:

- Spherical symmetry: Use Gauss' Law to calculate E everywhere
- Integrate E to get V

Calculation: Quantitative Analysis



$r > a_4$: What is $E(r)$ outside shell?

A) 0

B) $\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$

C) $\frac{1}{2\pi\varepsilon_0}\frac{Q+q}{r}$

D) $\frac{1}{4\pi\varepsilon_0}\frac{Q+q}{r^2}$

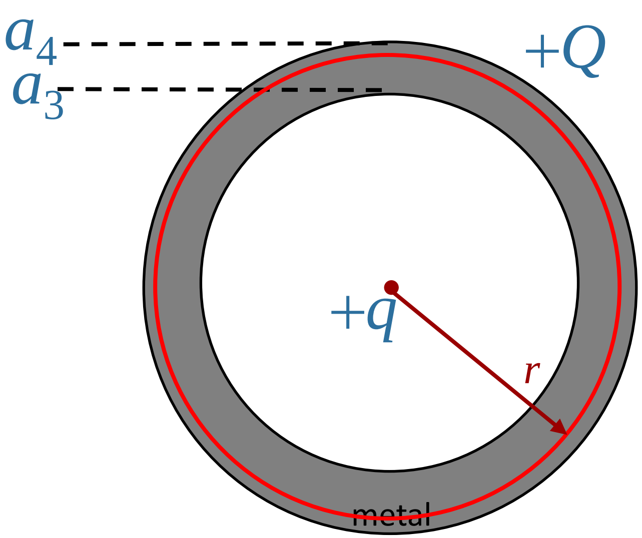
E) $\frac{1}{4\pi\varepsilon_0}\frac{Q-q}{r^2}$

Why?

Gauss' law: $\int \vec{E}\cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$

Calculation: Quantitative Analysis

cross-section



$a_3 < r < a_4$: What is $E(r)$ Inside metal sphere?

A) 0 B) $\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$ C) $\frac{1}{2\pi\varepsilon_0}\frac{q}{r}$

D) $\frac{1}{4\pi\varepsilon_0}\frac{-q}{r^2}$ E) $\frac{1}{4\pi\varepsilon_0}\frac{Q-q}{r^2}$

Applying Gauss' law, what is $Q_{enclosed}$ for red sphere shown?

A) q B) $-q$ C) 0

How is this possible?

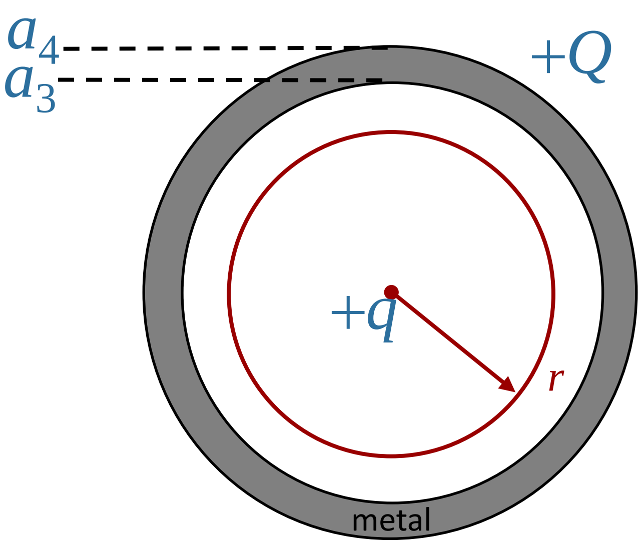
$-q$ must be induced at $r = a_3$ surface → charge at $r = a_4$ surface $= Q + q$

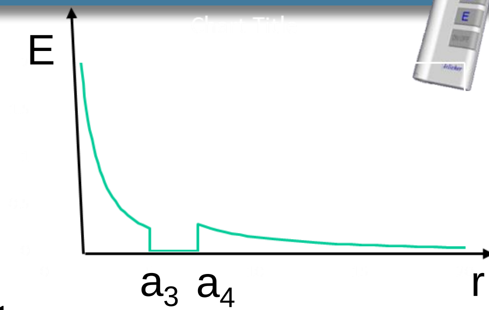
$$\sigma_3 = \frac{-q}{4\pi a_3^2}$$

$$\sigma_4 = \frac{Q+q}{4\pi a_4^2}$$

Calculation: Quantitative Analysis

cross-section





Continue on in…

$a_3 < r < a_4$: $\quad E = 0$

$r < a_3$: $\quad E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r^2}$

To find V:

1) Choose r_0 such that $V(r_0) = 0$ (usual: r_0 = infinity)
2) Integrate!

$$V(r) = -\int_{r_0}^{r} \vec{E} \cdot d\vec{\ell}$$

$r > a_4$: $\quad V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q+q}{r}$

$a_3 < r < a_4$:

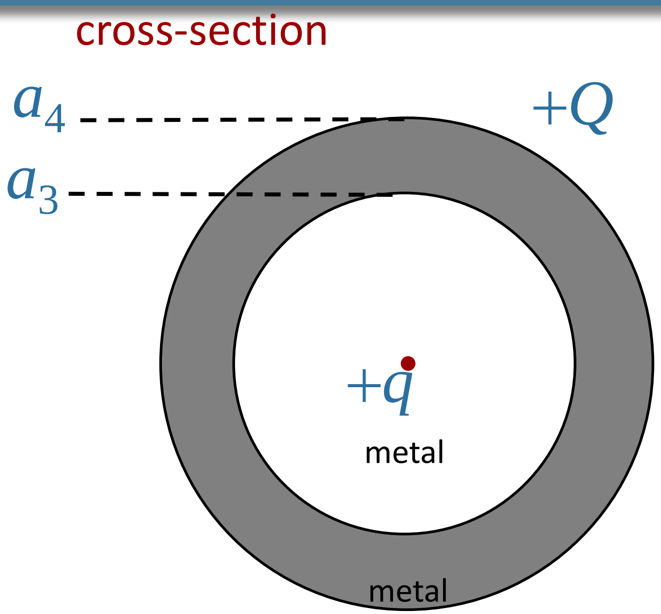
A) $V = 0$

B) $V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q+q}{a_4}$

C) $V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q+q}{a_3}$

D) $V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q+q}{r}$

Calculation: Quantitative Analysis



To find V:

1) Choose r_0 such that $V(r_0) = 0$ (usual: r_0 = infinity)
2) Integrate!

$r > a_4$: $V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q+q}{r}$

$a_3 < r < a_4$: $V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q+q}{a_4}$

$$V(r) = -\int_{r_0}^{r} \vec{E} \cdot d\vec{\ell}$$

$r < a_3$:

A) $V = 0$

B) $V = \dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{Q+q}{a_4} + \dfrac{q}{r}\right)$

C) $V = \dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{Q+q}{a_3} + \dfrac{q}{r}\right)$

D) $V = \dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{Q+q}{a_4} + \dfrac{q}{r} - \dfrac{q}{a_3}\right)$