Visualizing quadric surfaces

Today, we will be exploring various quadric surfaces. Please go to:

`http://dunfield.info/quadrics`

For each surface, please write the general equation of the surface, as well as any restrictions on the constants. Then, use the graphics to answer the questions which follow.

1. Elliptic paraboloid:
   (a) Each slice $x = c$ is a parabola. If we view all of these slices as living in the same yz-plane, how do these parabolas differ? Use the first picture to figure this out, and then confirm your answer algebraically from the equation.
   (b) In the second picture, what happens if either A or B is 0? What if they both are? Should any of these objects be called "elliptic" paraboloids?
   (c) What would happen if the sliders included negative values for A and B and we made both A and B negative?
2. Hyperbolic paraboloid:
   (a) What does the horizontal cross section given by $z = 0$ look like? Check on the first picture, and also look at the equation when $z = 0$. Is this still a hyperbola?
   (b) How would $z = y^2 - x^2$ look different than $z = x^2 - y^2$?
3. Ellipsoid:
   (a) What needs to happen for an ellipsoid to be a sphere?
   (b) The sliders don't actually go all the way to 0. Make the values as small as you can and zoom in to verify this; you'll find you have a very small sphere. (It's radius is 0.1, as it happens.) Why didn't I make the sliders go all the way to 0?
   (c) If you want to see an interesting effect, maximize all of the sliders and zoom in until you're inside the ellipsoid. You can rotate around and look at the interior of the surface. (This works best with the gridlines on.)
4. Double cone:
   (a) Why aren't any of the vertical or horizontal cross sections parabolas?
   (b) Explain what happens when either $A = 0$ or $B = 0$. Why don't you get a cone?
   (c) Similarly, what are the cross sections given by $x = 0$ or $y = 0$? Are these hyperbolas?
5. Hyperboloid of one sheet:
   (a) Once again, the sliders don't go all the way to 0. Why not? Make all of them as small as possible and zoom in to see the resulting hyperboloid. What other quadric surface does the picture resemble?

(b) Look at the equation. What should happen for the slices $x = A$ or $x = -A$? Check this in the first picture; recall that $A = 1$ there.

(c) Does there always have to be a "hole" through the hyperboloid, or could the sides touch at the origin? In other words, could the cross section given by z=0 ever be a point instead of an ellipse? Experiment with the second picture; be sure to look directly from the top and zoom in before just assuming that the hole is gone.

6. Hyperboloid of two sheets:

   (a) Go back to the equation and figure out why larger values of A and B make the hyperboloid flatter, not steeper.

   (b) Does there always need to be a gap between the two sheets, or could they touch?

   (c) Fix that $A = B = C$ and suppose we make A very small. What other quadric surface is very similar to the resulting picture? Is it reasonable to say that the this family of hyperboloids "limit" to the other surface? Is there a way to justify this algebraically?