Electric Current

Physics 212
Lecture 9

Today's Concept:

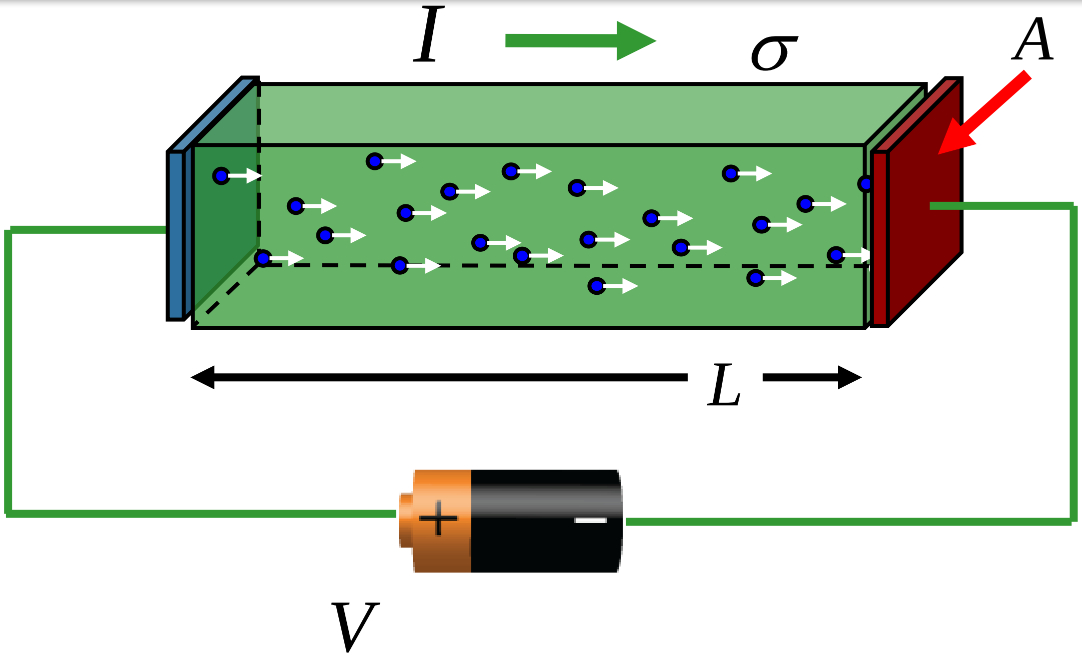
Ohm's Law, Resistors in circuits

Key Concepts:

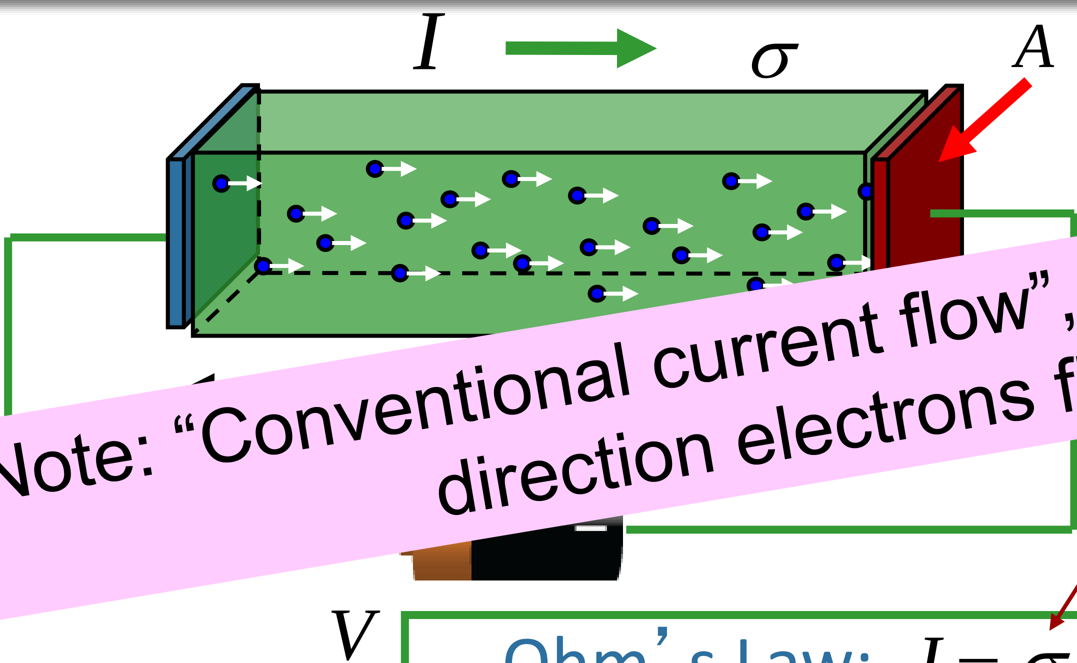
1) How resistance depends on A, L, σ, r
2) How to combine resistors in series and parallel
3) Understanding resistors in circuits

Today's Plan:

1) Review of resistance & prelectures
2) Work out a circuit problem in detail



I
σ
A
L
V



Note: "Conventional current flow", I, is opposite to direction electrons flow

Conductivity – high for good conductors.

$$\boxed{\text{Ohm's Law: } J = \sigma E}$$

Observables:

$$V = EL$$
$$I = JA$$

$$\rightarrow \quad I/A = \sigma V/L \quad \rightarrow \quad I = V/(L/\sigma A)$$

$$R = \frac{L}{\sigma A}$$

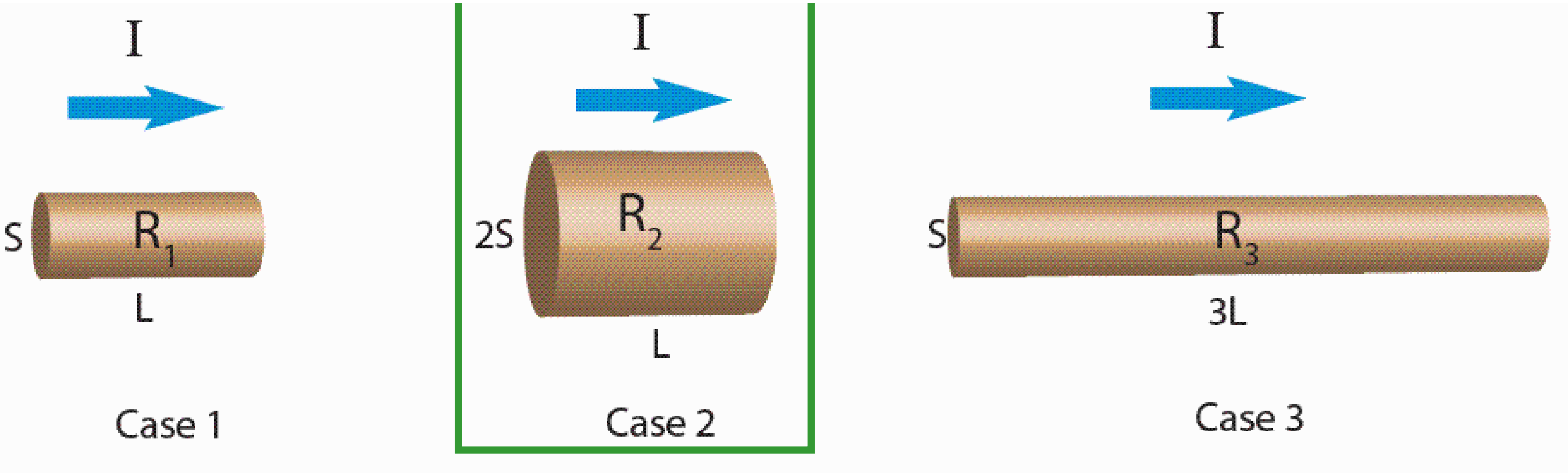
$$\boxed{I = V/R}$$

R = Resistance

$$\rho = 1/\sigma$$

Check Point 1

The SAME amount of current I passes through three different resistors. R_2 has twice the cross-sectional area and the same length as R_1, and R_3 is three times as long as R_1 but has the same cross-sectional area as R_1.



In which case is the CURRENT DENSITY through the resistor the smallest?

A. Case 1 ***B.*** Case 2 ***C.*** Case 3

$$J \equiv \frac{I}{A} \longrightarrow J_1 = J_3 = 2J_2$$

Same Current $\longrightarrow$ $J \propto \dfrac{1}{A}$

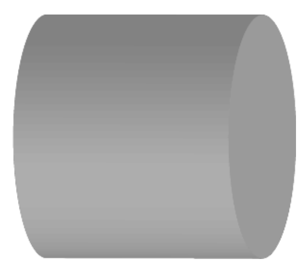
This is just like Plumbing!

I is like flow rate of water (gallons/hour)
V is like pressure
R is how hard it is for water to flow in a pipe

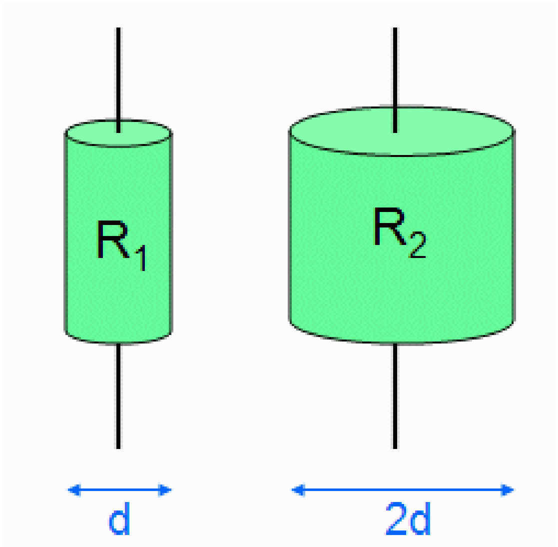
$$R = \frac{L}{\sigma A}$$

To make R big, make L long or A small

To make R small, make L short or A big

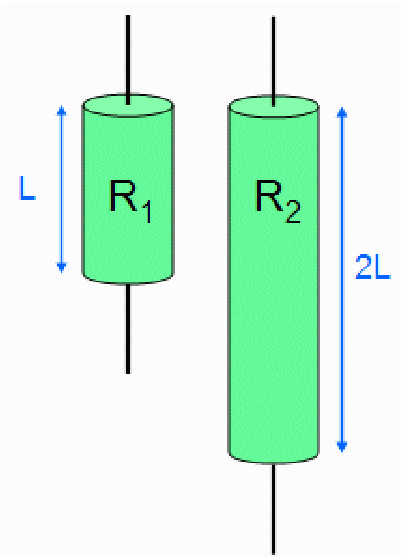


Check Point 2a



- A: $V_1 > V_2$
- B: $V_1 = V_2$
- C: $V_1 < V_2$

Check Point 2b



- A: $V_1 > V_2$
- B: $V_1 = V_2$
- C: $V_1 < V_2$

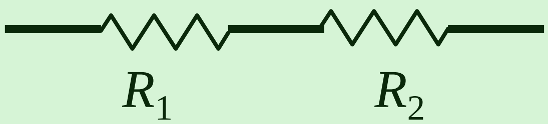
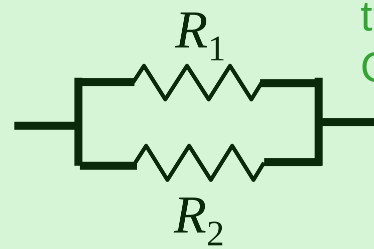
Same current through both resistors

Compare voltages across resistors

$$R \propto \frac{L}{A}$$

$$V = IR \propto \frac{L}{A}$$

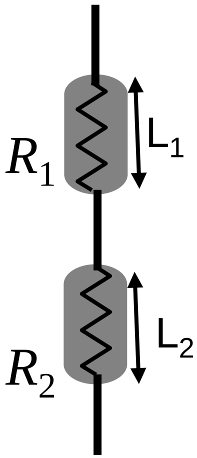
Resistor Summary

	Series	Parallel
	Every loop with R_1 also has R_2 	 There is a loop that contains ONLY R_1 and R_2
Wiring	Each resistor on the <u>same</u> wire.	Each resistor on a <u>different</u> wire.
Voltage	<u>Different</u> for each resistor. $V_{total} = V_1 + V_2$	<u>Same</u> for each resistor. $V_{total} = V_1 = V_2$
Current	<u>Same</u> for each resistor $I_{total} = I_1 = I_2$	<u>Different</u> for each resistor $I_{total} = I_1 + I_2$
Resistance	<u>Increases</u> $R_{eq} = R_1 + R_2$	<u>Decreases</u> $1/R_{eq} = 1/R_1 + 1/R_2$

Resistors and Capacitors

Can we go over why Capacitors and Resistors are inverses in series and parallel? Like more of a physical reason not just "the math works that way"

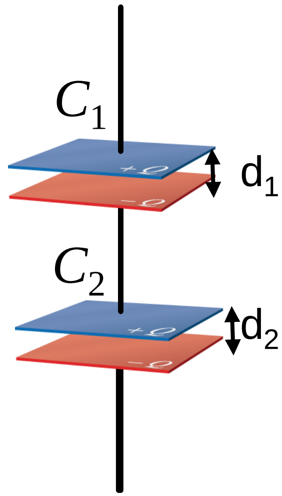
Series, you are adding lengths



$$R = \rho \frac{L}{A}$$

$$R_{tot} = \rho \frac{L_1 + L_2}{A} = \rho \frac{L_1}{A} + \rho \frac{L_2}{A} = R_1 + R_2$$

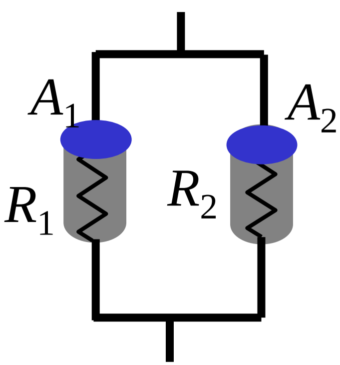
$$= R_1 + R_2$$



$$C = \varepsilon \frac{A}{d} \qquad \frac{1}{C} = \frac{d}{\varepsilon A}$$

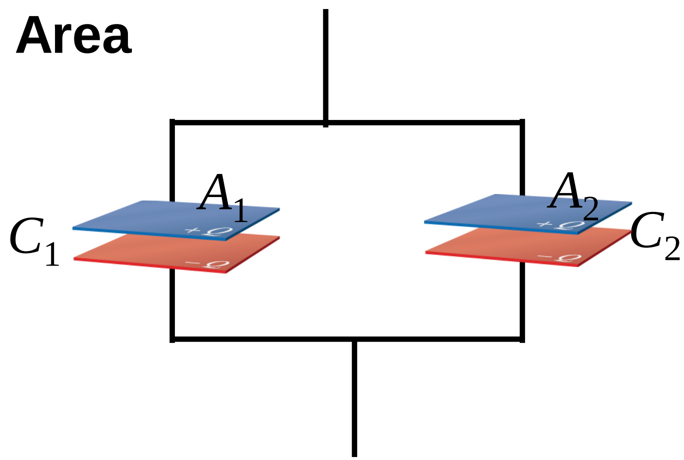
$$\frac{1}{C_{tot}} = \frac{d_1 + d_2}{\varepsilon A} = \frac{d_1}{\varepsilon A} + \frac{d_2}{\varepsilon A} = \frac{1}{C_1} + \frac{1}{C_2}$$

Parallel, you are adding Area



$$\frac{1}{R} = \frac{A}{\rho L}$$

$$\frac{1}{R_{tot}} = \frac{1}{R_1} + \frac{1}{R_2}$$

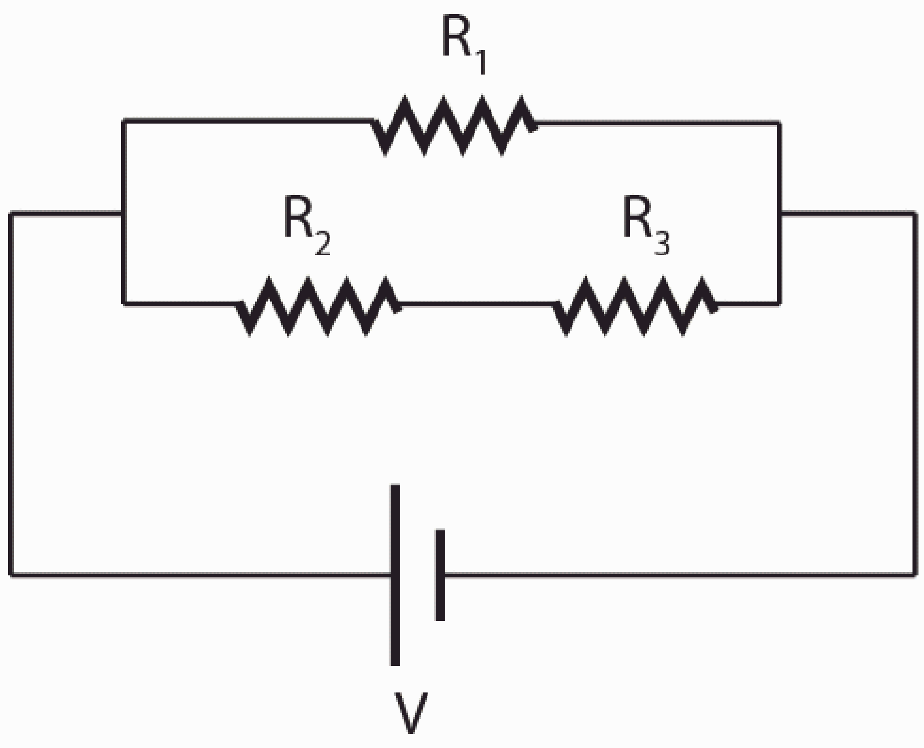


$$C = \varepsilon \frac{A}{d}$$

$$C_{tot} = C_1 + C_2$$

Check Point 3a

Three resistors are connected to a battery with emf V as shown. The resistances of the resistors are all the same, i.e. $R_1 = R_2 = R_3 = R$.



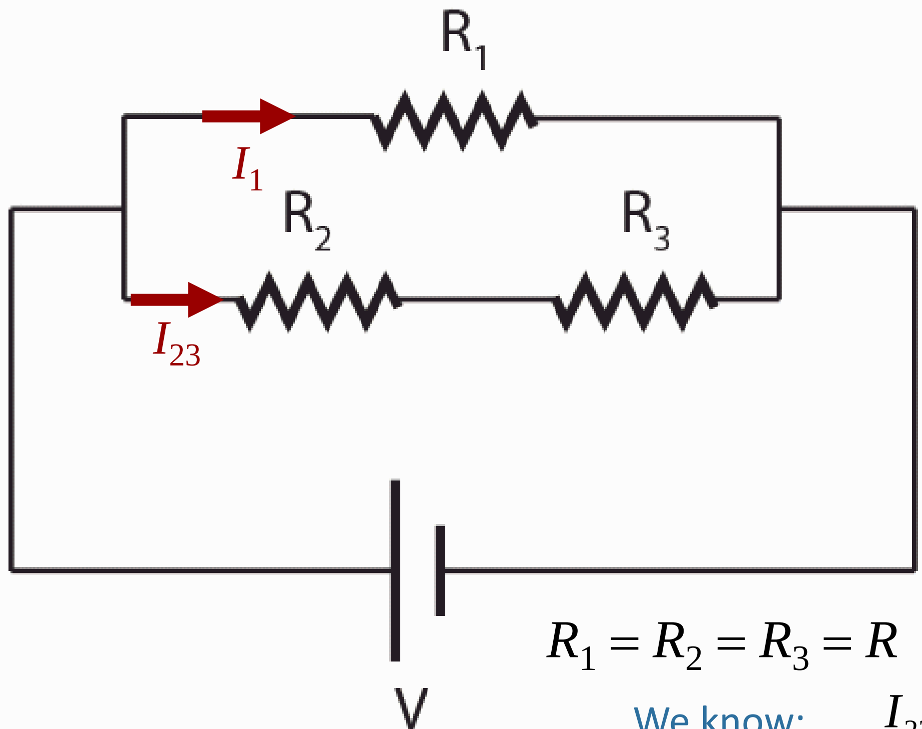
Compare the current through R_2 with the current through R_3:

A. $I_2 > I_3$ ***B.*** $I_2 = I_3$ ***C.*** $I_2 < I_3$

R_2 in series with R_3 → Current through R_2 and R_3 is the same

$$I_{23} = \frac{V}{R_2 + R_3}$$

Check Point 3b



$$R_1 = R_2 = R_3 = R$$

Compare the current through R_1 with the current through R_2

A $I_1/I_2 = 1/2$

B $I_1/I_2 = 1$

C $I_1/I_2 = 2$

D $I_1/I_2 = 3$

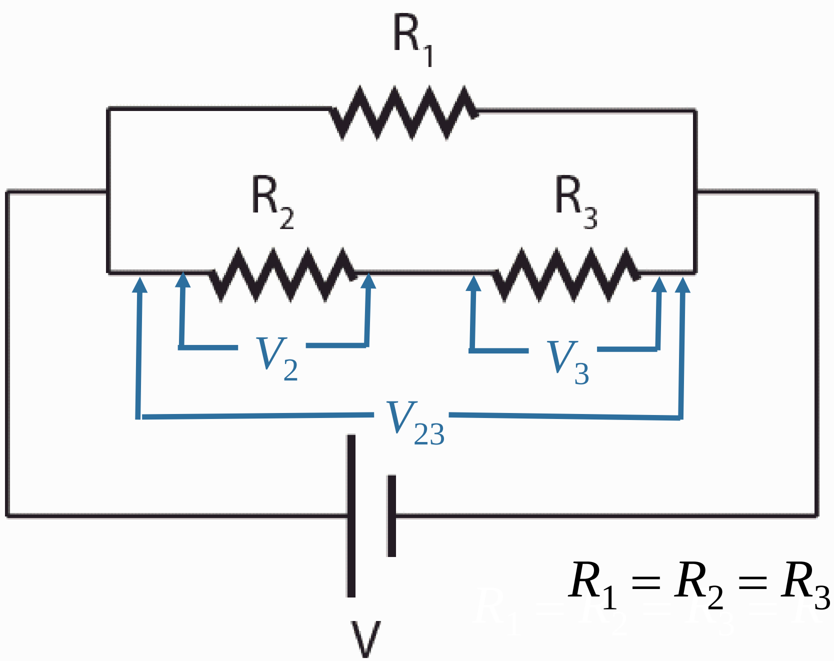
E $I_1/I_2 = 4$

We know: $I_{23} = \dfrac{V}{R_2 + R_3}$

Similarly: $I_1 = \dfrac{V}{R_1}$

$$\longrightarrow \quad I_1 = I_{23}\frac{R_2 + R_3}{R_1}$$

Check Point 3C



$$R_1 = R_2 = R_3 = R$$

Compare the voltage across R_2 with the voltage across R_3

A $V_2 > V_3$

B $V_2 = V_3 = V$

C $V_2 = V_3 < V$

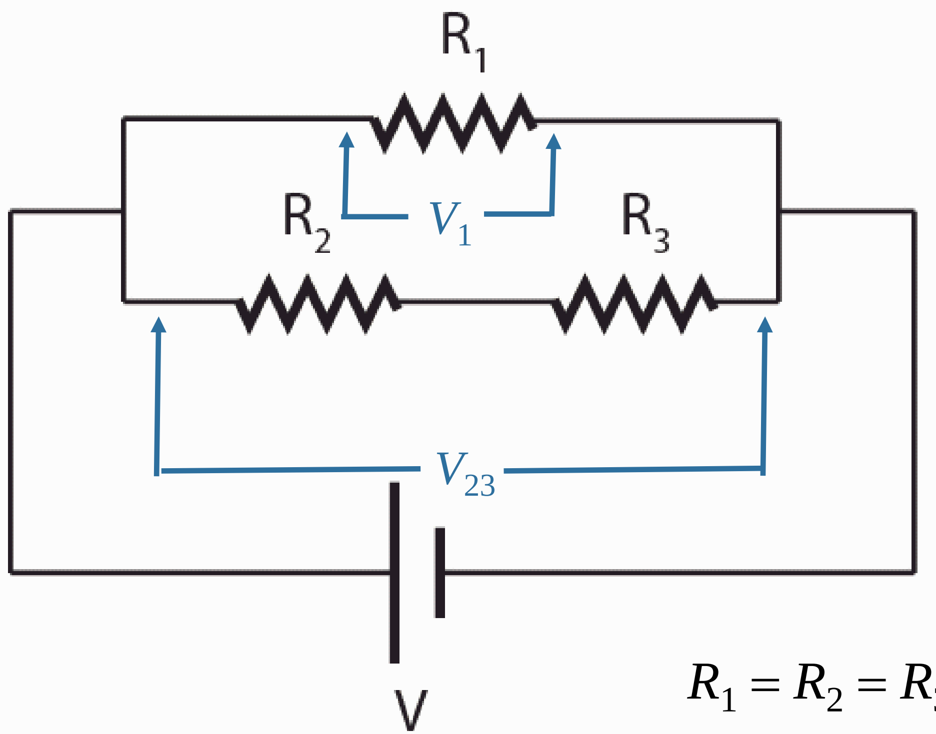
D $V_2 < V_3$

$$V_2 = I_2 R_2$$

$$V_3 = I_3 R_3$$

$I_2 = I_3$ (Series)
$R_2 = R_3$ (Problem statement)

Check Point 3D



R

$$R_1 = R_2 = R_3 = R$$

Compare the voltage across R_1 with the voltage across R_2

R_1 in parallel with series combination of R_2 and R_3

A $V_1 = V_2 = V$

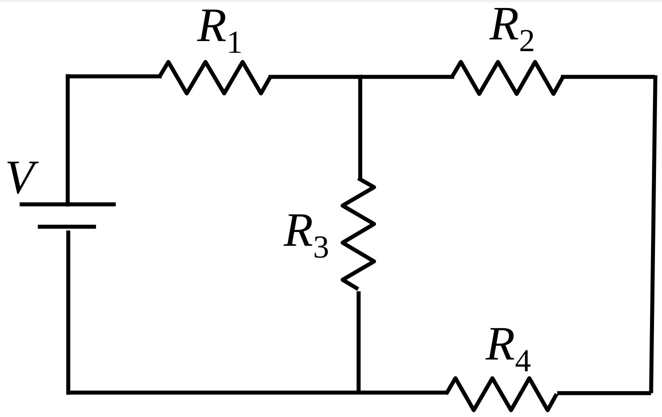
B $V_1 = ½\, V_2 = V$

C $V_1 = ½\, V_2 = 1/5\, V$

D $V_1 = 2V_2 = V$

E $V_1 = ½\, V_2 = ½\, V$

Calculation



In the circuit shown: $V = 18V$,

$R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 3\Omega$, and $R_4 = 4\Omega$.

What is V_2, the voltage across R_2?

Conceptual Analysis:

Ohm's Law: when current *I* flows through resistance R, the potential drop V is given by: $V = IR$.

Resistances are combined in series and parallel combinations

$$R_{series} = R_a + R_b$$
$$(1/R_{parallel}) = (1/R_a) + (1/R_b)$$

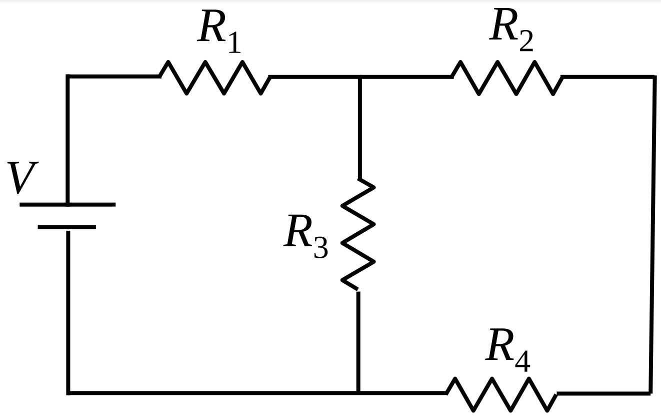
Strategic Analysis:

Combine resistances to form equivalent resistances

Evaluate voltages or currents from Ohm's Law

Expand circuit back using knowledge of voltages and currents

Calculation



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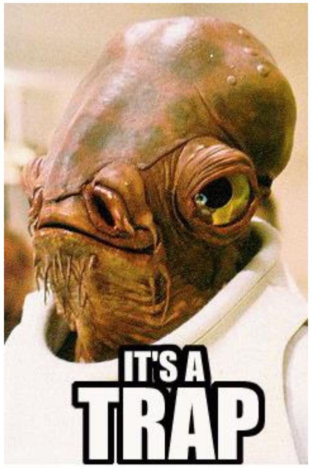


Combine Resistances:

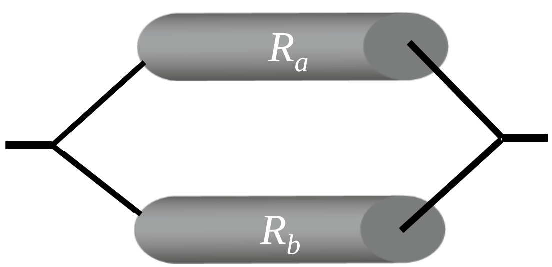
R_1 and R_2 are connected:

A) in series

B) in parallel

C) neither in series nor in parallel

Parallel Combination

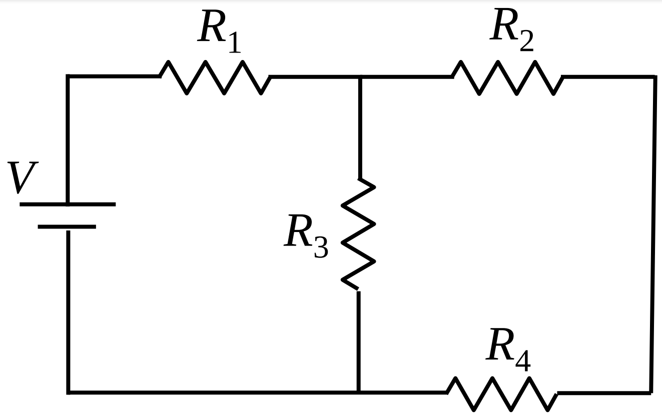


Parallel: Can make a loop that contains only those two resistors

Series Combination

Series : Every loop with resistor 1 also has resistor 2.

Calculation



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$R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 3\Omega$, and $R_4 = 4\Omega$.

What is V_2, the voltage across R_2?

We first will combine resistances R_2, R_4:

Which of the following is true?

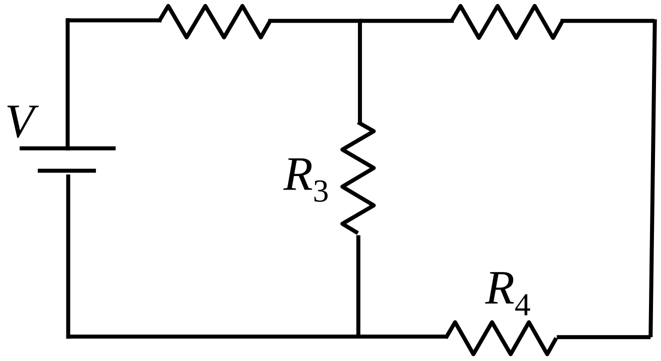
A) R_2 and R_4 are connected in series

B) R_2 and R_4 are connected in parallel

C) R_2 and R_4 are neither in series nor in parallel

D) R_2 and R_4 are both in series and in parallel

Calculation



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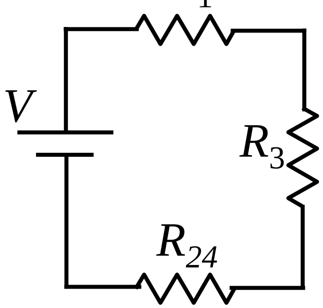
$R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 3\Omega$, and $R_4 = 4\Omega$.

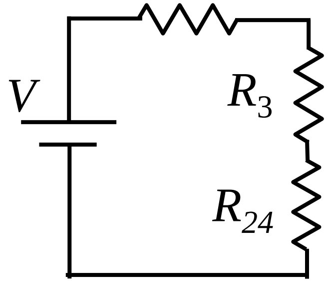
What is V_2, the voltage across R_2?

R_2 and R_4 are connected in series (R_{24})

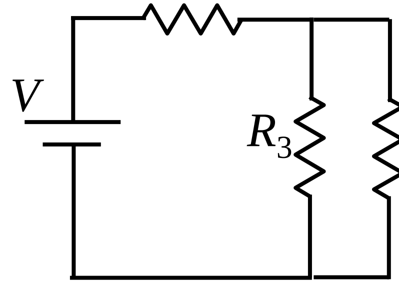
Redraw the circuit using the equivalent resistor R_{24} = series combination of R_2 and R_4.



(A)

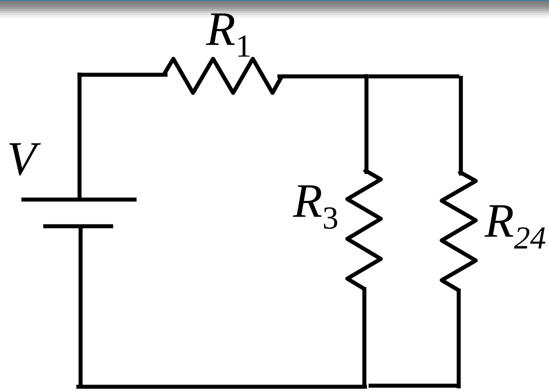


(B)



(C)

Calculation



In the circuit shown: $V = 18V$,

$R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 3\Omega$, and $R_4 = 4\Omega$.

What is V_2, the voltage across R_2?

Combine Resistances:

R_2 and R_4 are connected in series $= R_{24}$
R_3 and R_{24} are connected in parallel $= R_{234}$

What is the value of R_{234}?

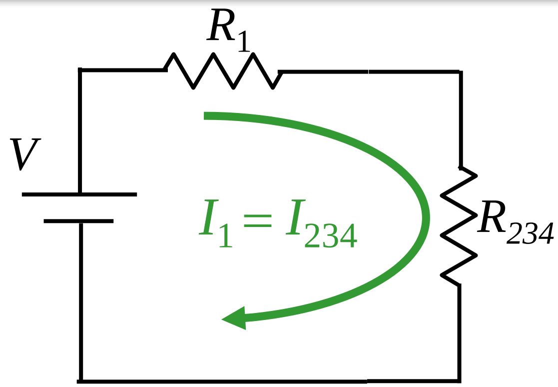
A) $R_{234} = 1\ \Omega$ B) $R_{234} = 2\ \Omega$ C) $R_{234} = 4\ \Omega$ D) $R_{234} = 6\ \Omega$

R_2 and R_4 in series → $R_{24} = R_2 + R_4 = 2\Omega + 4\Omega = 6\Omega$

R_3 and R_{24} are connected in parallel

$(1/R_{parallel}) = (1/R_a) + (1/R_b)$

Calculation



In the circuit shown: $V = 18V$,

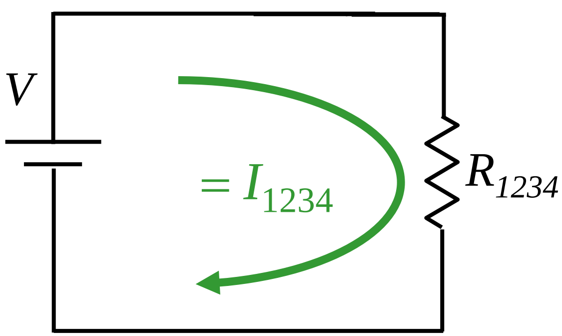
$R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 3\Omega$, and $R_4 = 4\Omega$.

$R_{24} = 6\Omega$ $R_{234} = 2\Omega$

What is V_2, the voltage across R_2?

R_1 and R_{234} are in series. $R_{1234} = 1 + 2 = 3\ \Omega$

Our next task is to calculate the total current in the circuit

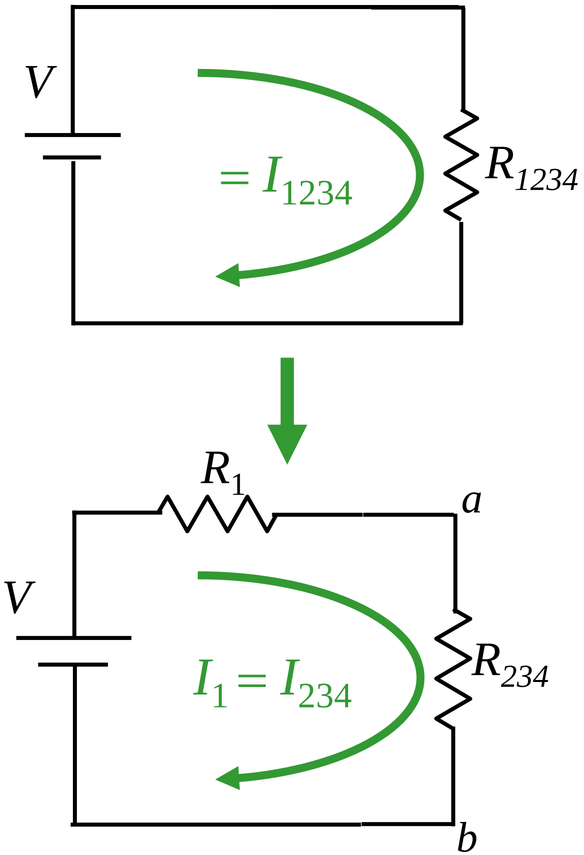


Ohm's Law tells us: $I_{1234} = V/R_{1234}$

$= 18 / 3$

$= 6 \text{ Amps}$

Calculation



In the circuit shown: $V = 18V$,

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$R_{24} = 6\Omega$ $\quad R_{234} = 2\Omega$ $\quad I_{1234} = 6\ A$

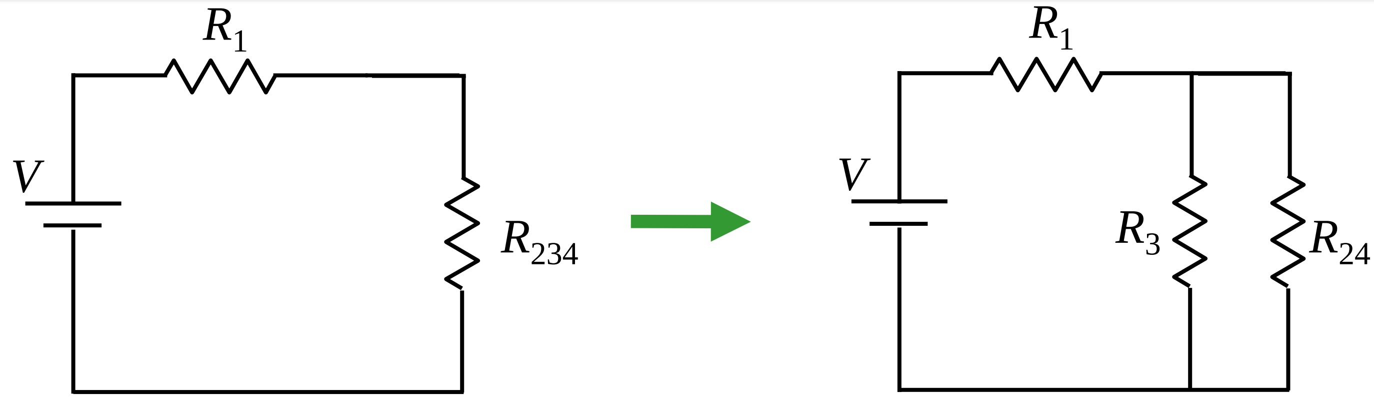
What is V_2, the voltage across R_2?

$I_{234} = I_{1234}$ Since R_1 in series with R_{234}

What is V_{ab}, the voltage across R_{234} ?

A) $V_{ab} = 1\ V$ B) $V_{ab} = 2\ V$ C) $V_{ab} = 9\ V$ D) $V_{ab} = 12\ V$ E) $V_{ab} = 16\ V$

Calculation



$V = 18V$
$R_1 = 1\Omega$
$R_2 = 2\Omega$
$R_3 = 3\Omega$
$R_4 = 4\Omega$
$R_{24} = 6\Omega$
$R_{234} = 2\Omega$
$I_{1234} = 6$ Amps
$I_{234} = 6$ Amps
$V_{234} = 12V$
What is V_2?

Which of the following are true?

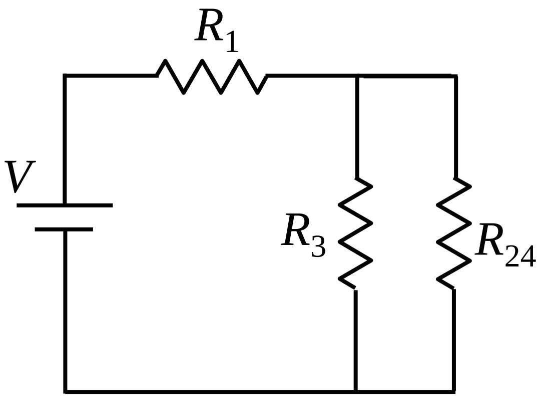
A) $V_{234} = V_{24}$ B) $I_{234} = I_{24}$ C) Both A+B D) None

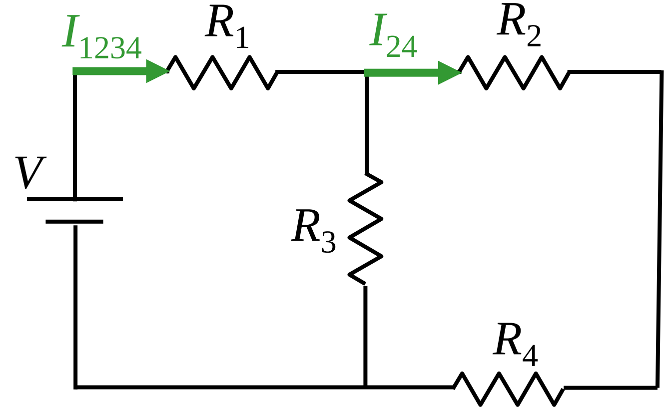
R_3 and R_{24} were combined in parallel to get R_{234} → Voltages are same!

Ohm's Law

$$I_{24} = V_{24} / R_{24}$$
$$= 12 / 6$$
$$= 2 \text{ Amps}$$

Calculation





$V = 18V$
$R_1 = 1\Omega$
$R_2 = 2\Omega$
$R_3 = 3\Omega$
$R_4 = 4\Omega.$
$R_{24} = 6\Omega$
$R_{234} = 2\Omega$
$I_{1234} = 6 \text{ Amps}$
$I_{234} = 6 \text{ Amps}$
$V_{234} = 12V$
$V_{24} = 12V$
$I_{24} = 2 \text{ Amps}$
What is V_2?

Which of the following are true?

A) $V_{24} = V_2$ B) $I_{24} = I_2$ C) Both A+B D) None

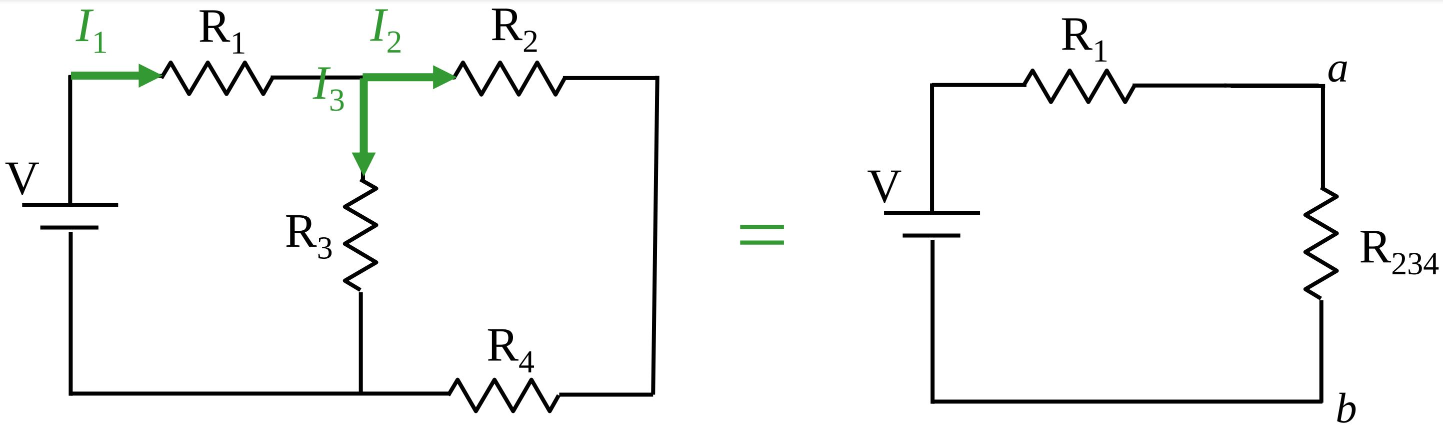
R_2 and R_4 where combined in series to get R_{24} → Currents are same!

Ohm's Law

The Problem Can Now Be Solved!

$$V_2 = I_2 \; R_2$$
$$= 2 \text{ x } 2$$
$$= 4 \text{ Volts!}$$

Quick Follow-Ups



$V = 18V$
$R_1 = 1\Omega$
$R_2 = 2\Omega$
$R_3 = 3\Omega$
$R_4 = 4\Omega$
$R_{24} = 6\Omega$
$R_{234} = 2\Omega$
$V_{234} = 12V$
$V_2 = 4V$
$I_{1234} = 6$ Amps

What is I_3 ?

A) $I_3 = 2\ A$ B) $I_3 = 3\ A$ C) $I_3 = 4\ A$

$\mathrm{V}_3 = \mathrm{V}_{234} = 12\mathrm{V}$

What is I_1 ?

We know $I_1 = I_{1234} = 6\ A$

NOTE: $I_2 = V_2/R_2 = 4/2 = 2\ A$ ➡ $I_1 = I_2 + I_3$ Make Sense?

Current and Resistance

Key Concepts:

1) How resistance depends on A, L, σ, r

2) How to combine resistors in series and parallel

3) Understanding resistors in circuits

4) Solve a network circuit