ECE 313: Problem Set 9: Problems and Solutions

Due: Saturday, Nov 23 at 11:59:00 p.m.
Reading: *ECE 313 Course Notes,* Sections 3.8, 3.10

Note on reading: For most sections of the course notes there are short answer questions at the end of the chapter. We recommend that after reading each section you try answering the short answer questions. Do not hand in; answers to the short answer questions are provided in the appendix of the notes.

Note on turning in homework: You must upload handwritten homework to BB. No typeset homework will be accepted. No late homework will be accepted. Please write on the top right corner of the first page:
NAME AS IT APPEARS ON BB
NETID
SECTION
PROBLEM SET #
Page numbers are encouraged but not required. Five points will be deducted for improper headings.

1. **[Function of a RV]**
   Let X be a uniform random variable where $\mathbb{E}[X] = 1$ and $\mathrm{Var}(X) = 3$. Let $Y = |X|$. Find the pdf $f_Y(u)$ of Y.

2. **[Cauchy Distribution]**
   A random variable X is said to have the Cauchy distribution if its pdf is given by $f_X(u) = \frac{1}{\pi(1+u^2)}$ for $-\infty < u < \infty$ (see Example 3.8.6 of the notes). Let $Y = 1/X$. Find the pdf of Y.

3. **[Generating a Weibull distribution]**
Let X be uniformly distributed on $(0,1)$, and let $Y = g(X)$, where $g(\cdot)$ is a function of X. It is desired that the CDF of Y be the Weibull distribution with shape parameter $\beta > 0$ and scale parameter $\alpha > 0$; that is, $F_Y(v) = 1 - e^{-\left(\frac{v}{\alpha}\right)^{\beta}}$ for $v \geq 0$, and zero else. Find a function $g(\cdot)$ to accomplish this, and check that this indeed gives the desired distribution.

4. **[Binary hypothesis testing]**
Consider the following binary hypothesis testing problem. Under H_0, the random variable X has the pdf f_0, while under H_1, the random variable X has the pdf f_1, where

$$f_0(u) = \begin{cases} \frac{1}{4} & u \in \left[\frac{-1}{2}, \frac{3}{2}\right] \cup \left[\frac{5}{2}, \frac{9}{2}\right], \\ 0 & \text{else} \end{cases}$$

and

$$f_1(u) = \begin{cases} \frac{1}{4}u & u \in [0,2], \\ \frac{-1}{4}u + 1 & u \in (2,4], \\ 0 & \text{else} \end{cases}$$

Assume that $4\pi_0 = \pi_1$.

(a) Find the ML rule.

(b) Find $p_{false\,alarm}$, p_{miss}, and p_e for the ML rule.

(c) Find the MAP rule.

(d) Find $p_{false\,alarm}$, p_{miss}, and p_e for the MAP rule.

5. **[Function of a RV]**
Consider a sphere whose radius is a random variable R with the following PDF:

$$f_R(u) = \begin{cases} 2u & u \in (0,1), \\ 0 & \text{else} \end{cases}$$

(a) What is the average radius of the sphere?

(b) What is the average volume of the sphere?

(c) What is the average surface area of the sphere?

(d) If a sphere of average radius is called an *average sphere*, then does an average sphere have more than, less than, or equal the average volume? Does it have more than, less than, or equal the average surface area?