Physics PreLab 212-8

Quality is job 1

Name ______________________________

Section ____________ **Date** ________

Impedance

By now you are familiar with the behavior of circuit elements under the influence of a DC voltage source. Rather than using a DC power supply you might want to use an AC power supply. When you studied a DC circuit, you found its resistance, and from it determined how much current would flow. With an AC power supply, the circuit's behavior depends on the frequency of the applied voltage as well as the resistance, the inductance, and the capacitance of the circuit.

In AC circuits, it is no longer sufficient to speak only of resistance because capacitors and inductors also affect current flow. Instead, the term *impedance*, symbolized by Z, is used to describe the current flow. For an AC circuit, the analog of Ohm's Law is

$$V_{rms} = I_{rms} Z \qquad \text{(Eq. 1)}$$

where V_{rms} is the RMS (*root mean square)* voltage, I_{rms} is the RMS current, and Z is the impedance (measured in ohms.) We can not write $V(t) = I(t)Z$ here because of the phase shifts you found in the previous investigation – the current does not necessarily reach a maximum at the same time the voltage does. Note that when used in AC mode, a DMM gives the RMS values of the measured quantities.

The general expression for the impedance of a one loop series *LRC* circuit is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \qquad \text{(Eq. 2)}$$

To calculate the impedance of an AC circuit, one must know not only the resistance R, but also the *capacitive reactance*, X_C, of the capacitor and the *inductive reactance*, X_L, of the inductor. These are both defined below. Note that capacitive reactance and inductive reactance are both frequency dependent, unlike resistance which is independent of the frequency of the AC power source.

Given an AC power source frequency f, the *angular frequency* is defined as $\omega = 2\pi f$. The capacitive reactance and the inductive reactance are defined by

$$X_C = \frac{1}{\omega C} \qquad \text{(Eq. 3)}$$

$$X_L = \omega L \qquad \text{(Eq. 4)}$$

where C is the capacitance and L is the inductance of the circuit. Because the capacitive reactance is inversely proportional to frequency, higher frequency corresponds to lower capacitive reactance. The inductive reactance is directly proportional to frequency and so it is smallest at low frequencies.

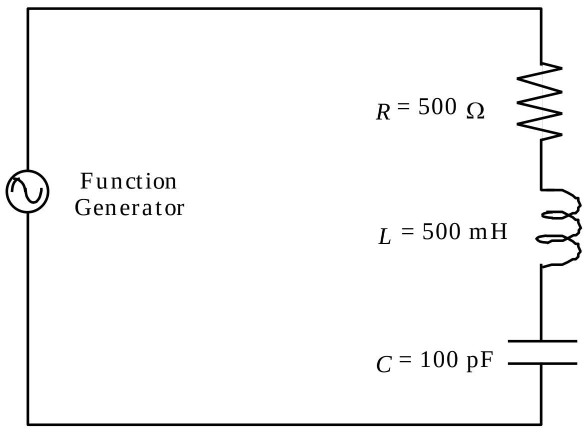

Figure 1. The series *LRC* circuit

Q1[9'] For the circuit of Figure 1 shown above, with $R = 500\Omega$, $C = 100$ pF, and $L = 500$ mH, what are ω, X_L, and X_C at a frequency $f = 15000$ Hz? (pF=picoFarad=10^{-12}F) Explicitly write down your calculations below.

$\omega =$ _______ [radians/sec]

$X_L =$ _______ [Ω]

$X_C =$ _______ [Ω]

Q2[9'] For the circuit of Figure 1, with R = 500Ω, C = 100 pF, and L = 500 mH, what is the impedance Z at a frequency f = 15000 Hz?

Z = ________ [Ω]

Resonance in an *LRC* Circuit

An *LRC* circuit behaves in a manner analogous to a mechanical oscillator with charge on the capacitor and current being the analog of position and the velocity of the mass, respectively.

If you drive an oscillator (mechanical/electrical) such that the frequency of the applied force/voltage matches the natural frequency of the system, a large response results. This special frequency is known as the *resonant frequency*. If we drive the circuit at this frequency, we get the largest current amplitude for a given peak voltage. The phenomenon of resonance in AC circuits is what makes tuning a radio or television possible; when you "tune in" a station, you are actually tuning an *LRC* circuit inside your radio or television for that station's transmitting frequency.

In an *LRC* circuit, the (angular) resonant frequency ω_0 is given by

$$\omega_o = \frac{1}{\sqrt{LC}} \qquad \text{(Eq. 5)}$$

where L is the inductance and C is the capacitance. Notice that the resonant frequency is independent of the resistance in the series circuit.

Q3[7'] For the circuit of Figure 1, with R = 500Ω , C = 100 pF, and L = 500 mH, what is the angular resonant frequency ω_0?

ω_0 =________ [radians/sec]

Q4[2'] What is the resonance frequency f_0?

f_0 =_______ [cycles/sec]

The Quality of an *LRC* Circuit

As you can see from Eq. 5, a wide range of values for L and C can be used to achieve any particular resonance frequency. Notice again that the resistance R does not enter in the determination of ω_0. However, the way in which the circuit reacts to frequencies *near* the resonance is greatly affected by all three parameters R, L, and C.

Recall that the average power consumed by a series *LRC* circuit is

$$P_{avg} = I_{rms}^2 R \qquad \text{(Eq. 6a)}$$

$$P_{avg} = \varepsilon_{rms} I_{rms} \cos \Phi \qquad \text{(Eq. 6b)}$$

with $\cos\Phi = R/Z$ called the *power factor* . When the circuit is driven at the resonance frequency, $Z = R$ and $\cos\Phi = 1$ which yields a maximum in the average power. For frequencies above or below the resonance frequency, the power dissipated in the resistor is less. The shape of the average power versus frequency curve is affected by the values of R and L in the circuit as shown in the figure below.

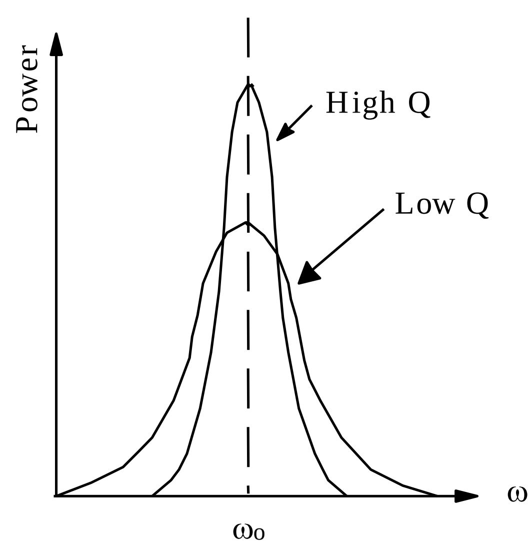

Figure 2. Series *LRC* circuit response

For a sharp resonance, defined as one with a narrow width around the resonance frequency, a circuit is said to have a high Q ("quality factor"). This is desirable if one is tuning a circuit to react only to those frequencies close to resonance, such as tuning in a radio station.

The Q factor may be expressed in many ways:

$$
\begin{aligned}
Q = \frac{2\pi E}{\Delta E} &= \frac{\sqrt{X_{L_O} X_{C_O}}}{R} = \frac{1}{R}\sqrt{\frac{L}{C}} \\
&= \frac{X_{L_O}}{R} = \frac{\omega_O L}{R} \\
&= \frac{X_{C_O}}{R} = \frac{1}{R\omega_O C} \\
&\approx \frac{\omega_O}{\Delta\omega} = \frac{f_O}{\Delta f}
\end{aligned}
\qquad \text{(Eq. 7)}
$$

where $X_{L_0} = \omega_0 L$

$$X_{C_0} = \frac{1}{\omega_0 C}$$

E = total energy of the circuit

and ΔE = energy lost due to resistance in one cycle

with the latter expression of Eq. 7 being a valid approximation when Q is greater than 2 or 3. The quantity Δf is called the full width at half maximum (FWHM) and is defined as follows: at frequencies $\pm\Delta f/2$ from the resonance frequency, the average power consumed by the circuit drops by a factor of 2 compared to the power at resonance. For a given circuit, you can measure Q by making a plot like the one shown in Figure 2.

Q5[7'] Using Equation 7, for the circuit of Figure 1, with R = 500Ω, C = 100 pF, and L = 500 mH, calculate the Q factor of the circuit.

Q = ________

Q6[4'] Using Equation 7, for the circuit of Figure 1, with R = 500 Ω, C = 100 pF, and L = 500 mH, what are the frequencies at the half power points $f_{low} = f_O - \Delta f/2$ and $f_{high} = f_O + \Delta f/2$?

f_{low} = ________ [Hz]

f_{high} = ________ [Hz]

Q7[12'] At resonance ($f=f_O$), the RMS voltage V_C across the capacitor is related to the Q factor and to the RMS voltage V_{fg} across the function generator by a very simple relation. Find this relation using what you know about the values of Q and Z at resonance. (Hint: drawing the phasor diagram can help). Show your work.

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