ECE 313: Problem Set 7: Problems

Due: Thursday Nov 7th at 11:59:00 p.m.
Reading: *ECE 313 Course Notes,* Sections 3.4 - 3.6.1

Note on reading: For most sections of the course notes there are short answer questions at the end of the chapter. We recommend that after reading each section you try answering the short answer questions. Do not hand in; answers to the short answer questions are provided in the appendix of the notes.

Note on turning in homework: You must upload handwritten homework to BB. No typeset homework will be accepted. No late homework will be accepted. Please write on the top right corner of the first page:
NAME AS IT APPEARS ON BB
NETID
SECTION
PROBLEM SET #
Page numbers are encouraged but not required. Five points will be deducted for improper headings.

1. **[Exponential Distribution]**
   Suppose X has an exponential distribution with parameter $\lambda > 0$. Answer the following in terms of λ.

   (a) Determine $E[X^2]$.

   (b) Determine $P\{\lfloor X^2 \rfloor = 4\}$ where $\lfloor x \rfloor$ is the greatest integer less than or equal to x.

2. **[Poisson process involving fishing]**
   A fisherman catches fish in a river according to a Poisson process with rate equal to 2 fishes per hour. He starts fishing at 6 AM.

   (a) Find the probability that he will catch exactly one fish between 6 AM and 6:30 AM.

   (b) Find the probability that he will catch exactly one fish between 6 AM and 6:30 AM, and a total of exactly 4 fish by 9 AM?

(c) The fisherman needs to catch at least five fish to feed his family. If he fishes from 6 AM until noon, what is the probability that he will be successful in feeding his family.

(d) The fisherman catches 6 fish while fishing from 6 AM until noon. What is the probability that he had caught at least 5 of those between 6 AM and 6:30AM?

3. **[Poisson process involving light bulbs]**
I have a very small house; there are only 4 light bulbs in my house. Each light bulb burns out at an average rate of 0.1 burnouts per light bulb, per day. Burnouts can occur at any time. The number of burnouts in any time interval is independent of the number in any other time interval if and only if the two time intervals are non-overlapping.

(a) If a light bulb burns out at any time during the day, I replace it immediately (thus, for example, there is a nonzero but very small probability of replacing 10,000 light bulbs in any given day). Unfortunately, I have only a three-pack of light bulbs; if more than three light bulbs burn out this week, I will have to go to the store to buy more. What is the probability that I will get through the week (7 days) without going to the store? Your answer should be a number.

(b) I'm tired of changing my own light bulbs, so I'm going to hire a light-bulb-changing service. If one or more light bulbs burn out on any given day, a Light Bulb Technologist (certified by the LBTAA) will visit my house at 8:30pm that evening to change all of the broken bulbs; if no bulbs burn out that day, then the technologist does not visit. I pay a monthly charge that covers up to one visit per week; if the technologist has to visit my house more than once in any given week, I pay an emergency surcharge. What's the probability that I can get through any given week without paying an emergency surcharge? Your answer should be a number.

4. **[Scaling PDFs]**
Suppose that X and Y are the sampled values of two different audio signals. The mean and variance of an audio signal are uninteresting: the mean tells you the bias voltage of the microphone, and the variance tells you the signal loudness. For this reason, the audio signals X and Y are pre-normalized so that $E[X] = E[Y] = 0$ and $\text{Var}(X) = \text{Var}(Y) = 1$.
An audio signal Z is said to be "spiky" if $P\{|Z| > 3\sigma_Z\} > 0.01$, i.e., one-in-hundred samples has a large amplitude.

(a) Suppose that X is a uniformly distributed random variable, scaled so that it has zero mean and unit variance. (1) What is $P\{|X| > \sigma_X\}$? (2) What is $P\{|X| > 3\sigma_X\}$? (3) Is X spiky? Be sure to consider both positive and negative values of X.

(b) Suppose that Y is a Laplacian random variable with a pdf given by:

$$f_Y(u) = \frac{\lambda}{2} e^{-\lambda|u-\mu|}, \qquad -\infty < u < \infty$$

where λ and μ are chosen so that $E[Y] = 0$ and and $\mathrm{Var}(Y) = 1$. (1) What is $P\{|Y| > \sigma_Y\}$? (2) What is $P\{|Y| > 3\sigma_Y\}$? (3) Is Y spiky? Be sure to consider both positive and negative values of Y.

5. **[Scaling a uniform distribution]**
Assume that you have a random number generator that generates a uniformly distributed random variable X over the interval $[-3, 5]$. How would you obtain a random variable Y that is uniformly distributed over $[0, 1]$ from X?