1. Recall Properties of the Cross Product:

   11 Properties of the Cross Product If **a**, **b**, and **c** are vectors and c is a scalar, then

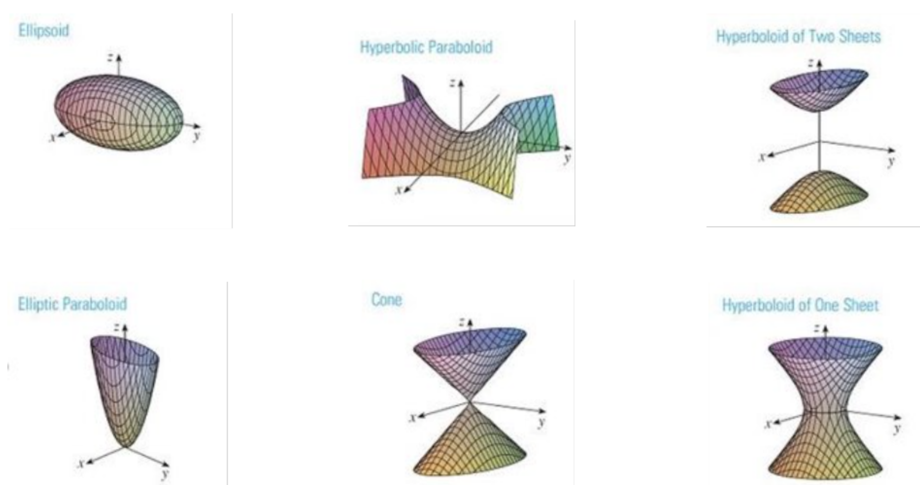
   1. $\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$
   2. $(c\mathbf{a}) \times \mathbf{b} = c(\mathbf{a} \times \mathbf{b}) = \mathbf{a} \times (c\mathbf{b})$
   3. $\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = \mathbf{a} \times \mathbf{b} + \mathbf{a} \times \mathbf{c}$
   4. $(\mathbf{a} + \mathbf{b}) \times \mathbf{c} = \mathbf{a} \times \mathbf{c} + \mathbf{b} \times \mathbf{c}$
   5. $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}$
   6. $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$

   (a) Prove that
   $$(\mathbf{a} - \mathbf{b}) \times (\mathbf{a} + \mathbf{b}) = 2(\mathbf{a} \times \mathbf{b})$$

   (b) Prove that
   $$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) + \mathbf{b} \times (\mathbf{c} \times \mathbf{a}) + \mathbf{c} \times (\mathbf{a} \times \mathbf{b}) = \mathbf{0}$$

2. Find the distance between the lines $(1, -1, 0) + \mathbb{R}(0, 1, 1)$ and $(2, 0, 1) + \mathbb{R}(2, -1, 0)$

3. Match the graphs and the standard forms of six basic types of quadric surfaces



a) $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ b) $\frac{z^2}{c^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ c) $\frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ d) $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ e) $\frac{z}{c} = \frac{x^2}{a^2} - \frac{y^2}{b^2}$ f) $-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

1. (a)

$$\begin{aligned}(\mathbf{a}-\mathbf{b})\times(\mathbf{a}+\mathbf{b}) &= (\mathbf{a}-\mathbf{b})\times\mathbf{a}+(\mathbf{a}-\mathbf{b})\times\mathbf{b}\\ &= \mathbf{a}\times\mathbf{a}+(-\mathbf{b})\times\mathbf{a}+\mathbf{a}\times\mathbf{b}+(-\mathbf{b})\times\mathbf{b} \quad \textbf{(1 point)}\\ &= (\mathbf{a}\times\mathbf{a})-(\mathbf{b}\times\mathbf{a})+(\mathbf{a}\times\mathbf{b})-(\mathbf{b}\times\mathbf{b})\\ &= \mathbf{0}-(\mathbf{b}\times\mathbf{a})+(\mathbf{a}\times\mathbf{b})-\mathbf{0}\\ &= (\mathbf{a}\times\mathbf{b})+(\mathbf{a}\times\mathbf{b})\\ &= 2(\mathbf{a}\times\mathbf{b}) \quad \textbf{(1 point)}\end{aligned}$$

(b)

$$\begin{aligned}&\mathbf{a}\times(\mathbf{b}\times\mathbf{c})+\mathbf{b}\times(\mathbf{c}\times\mathbf{a})+\mathbf{c}\times(\mathbf{a}\times\mathbf{b})\\ &=[(\mathbf{a}\cdot\mathbf{c})\mathbf{b}-(\mathbf{a}\cdot\mathbf{b})\mathbf{c}]+[(\mathbf{b}\cdot\mathbf{a})\mathbf{c}-(\mathbf{b}\cdot\mathbf{c})\mathbf{a}]+[(\mathbf{c}\cdot\mathbf{b})\mathbf{a}-(\mathbf{c}\cdot\mathbf{a})\mathbf{b}] \quad \textbf{(1 point)}\\ &=(\mathbf{a}\cdot\mathbf{c})\mathbf{b}-(\mathbf{a}\cdot\mathbf{b})\mathbf{c}+(\mathbf{a}\cdot\mathbf{b})\mathbf{c}-(\mathbf{b}\cdot\mathbf{c})\mathbf{a}+(\mathbf{b}\cdot\mathbf{c})\mathbf{a}-(\mathbf{c}\cdot\mathbf{a})\mathbf{b}\\ &=\mathbf{0} \quad \textbf{(1 point)}\end{aligned}$$

2. span a plane C at the line $L_1 = (2,0,1)+\mathbb{R}(2,-1,0)$ which is parallel to the line $L_2 = (1,-1,0)+\mathbb{R}(0,1,1)$, the normal vector of the plane can be $n = \langle 1,2,-2\rangle$ **(1 point)**; choose a point $(1,-1,0)$ on the line L_2 and a point $(2,0,1)$ on the plane C to make a vector $v = \langle 1,1,1\rangle$ **(1 point)**, the distance is the projection of v on n, $d = |\frac{v\cdot n}{|n|}| = |\frac{1\times1+1\times2+1\times(-2)}{\sqrt{1^2+2^2+(-2)^2}}| = \frac{1}{3}$ **(1 point)**.

3. **(0.5 point each)**
   (a) Ellipsoid
   (b) Cone
   (c) Elliptic Paraboloid
   (d) Hyperboloid of One Sheet
   (e) Hyperbolic Paraboloid
   (f) Hyperboloid of Two Sheets