ECE 313: Problem Set 4: Problems

Due: Friday, Oct 18th at 11:59:00 p.m.
Reading: *ECE 313 Course Notes,* Sections 2.8 - 2.10

Note on reading: For most sections of the course notes there are short answer questions at the end of the chapter. We recommend that after reading each section you try answering the short answer questions. Do not hand in; answers to the short answer questions are provided in the appendix of the notes.

Note on turning in homework: You must upload handwritten homework to BB. No typeset homework will be accepted. No late homework will be accepted. Please write on the top right corner of the first page:
NAME AS IT APPEARS ON BB
NETID
SECTION
PROBLEM SET #
Page numbers are encouraged but not required. Five points will be deducted for improper headings.

1. **[Chip Testing]**
Alice is a graduate student who has designed an integrated circuit (IC) implementing an machine learning accelerator IC in a 45 nm semiconductor process as part of her graduate research. She has just received 50 packaged chips and is getting ready to test them to see if it is working properly. Alice wants to show that her design can classify images with high accuracy p_a. To do that she tests her chip with n images and counts the number E that are incorrectly classified. She obtains an accuracy estimate $\hat{p}_a = 1 - \frac{E}{n}$. Alice hopes to write-up a research paper on her design and submit it to ISSCC, a top circuits conference. All she needs is one working chip that classifies images from the test set with high accuracy in order to report the results (yield is not an issue in papers from academia). However, testing a chip is a slow process and Alice wants to minimize the testing time so she can submit the paper before the deadline.

   (a) Determine the probability distribution of the random variable E representing the misclassification error count.

   (b) Alice tests the first chip using $n = 100$ test images and finds that 95 images are correctly classified. Is it ok for Alice to report that her design gives an accuracy of $p_a = 0.95$? Give reasons.

   (c) How many test vectors should Alice test her chip with so that she can report that the true accuracy of her design p_a lies in the interval $\hat{p}_a \pm 1\%$ with a confidence level greater than 95%?

(d) Since testing is a slow process, Alice decides to do a quick pass through of all her 50 chips by testing with fewer images. She would like to keep the confidence interval same as in Part (c), i.e., $\hat{p}_a \pm 1\%$, but with a relaxed confidence level of 80%. How many test images does she need per chip for this initial pass?

2. [**Maximum Likelihood Parameter Estimation**]
A biased coin when tossed shows a Heads with probability p and Tails with probability $1 - p$.

(a) The biased coin is tossed 10 times and 6 Heads are observed. What is the maximum likelihood estimate $\hat{p}_{ML}$ of p given this observation?

(b) Suppose it is known that $p = 0.05$. The biased coin is now tossed an unknown number n times during which 6 Heads are observed. What is the maximum likelihood estimate $\hat{n}_{ML}$ of n given this observation?

(c) The biased coin is tossed 9 times before the first Head is observed, i.e., the first Head is observed in the 10 flip. What is the maximum likelihood estimate $\hat{p}_{ML}$ of p given this observation?

3. [**Message Sources**]
A binary message source M_2 outputs *bytes* (8 bit words) such as 11010010 with every byte being equally likely. A quaternary message source M_4 produces *words* of length 8 with characters from the set $\{0, 1, 2, 3\}$, such as 32100313, with all such words being equally likely.

(a) What is the probability, p, that a word produced by M_4 is a byte, i.e., every character in the word belongs to the set $\{0, 1\}$?

(b) If a word is equally likely to be drawn from either source, what is the probability it will be a byte?

(c) Given that the word is a byte, what is the probability it came from M_2?

4. [**Debugging a Program**]
While debugging a software program, a student has narrowed it down to one of two bugs. Bug 1 leads to error message 1 with probability $\frac{1}{5}$ and error message 2 with probability $\frac{4}{5}$, while bug 2 leads to error message 1 with probability $\frac{2}{3}$ and error message 2 with probability $\frac{1}{3}$. The student also has the knowledge that the bug 1 is 5 times more likely to appear than bug 2, and that the two bugs cannot exist simultaneously.

(a) What is the probability that error message 1 appears?

(b) Given that error message 1 appears, what is the probability that the system has bug 1?

5. **[Coin Draw and Toss]**
David has 3 coins in his pocket, 2 of which are fair coins while the third is a biased coin with $P(H) = p \neq \frac{1}{2}$. The probability that a coin chosen at random from his pocket will land Tails is $\frac{7}{12}$.

(a) What is the value of p?

(b) David picks two coins at random from his pocket, tosses each coin once, and observes a Head and a Tail. What is the conditional probability that both coins are fair? Assume p is unknown.