Math 231   Test #2

Your Name: _____

Please circle your discussion group (2 pt)

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2 Zhang Junwei B424	5 Xu Hang B419	
3 Ke Wentao B419	6 Huang Nuoer B410	

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- You will have one hour for the exam.
- No notes, books or electronics during the exam.
- Do not open this test booklet until a proctor says start.
- For all free response questions, show work that justifies your answer.
- Raise your hand if you have a clarification question.
- Scratch paper is provided. You can ask for more if needed.
- Do not leave early: this disturbs others. If you finish your test early, check your work or just relax.
- Quit working when the test ends and hand your test booklet to proctors.

Question	1	2	3	4	5	6	7	8	9	Total
Points	12	14	25	7	8	10	6	10	6	98
Score										

1. (12 points, 3 points each) Determine whether the statement is true or false. Circle the right answer.

(a) If $\sum a_n$ is divergent, then $\sum |a_n|$ is divergent. (True or False)

(b) If $\{a_n\}$ and $\{b_n\}$ are divergent, then $\{a_n + b_n\}$ is divergent. (True or False)

(c) If $\{a_n\}$ is decreasing and $a_n > 0$ for all n, then $\{a_n\}$ is convergent. (True or False)

(d) If $f(n) = a_n$, $\lim_{n \to \infty} a_n = L$ then $\lim_{n \to \infty} f(x) = L$. (True or False)

2. (14 pts) Use integral test to test the following series.

(a) $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

(b) $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

3. (25 pts) Please choose proper series test to decide whether the series is convergent or divergent. Justify your answer.

(a) $\sum_{n=1}^{\infty} \frac{1}{n^{1+1/n}}$

(b) $\sum_{n=1}^{\infty} \frac{n!}{(n+2)!}$

(c) $\sum_{n=1}^{\infty} \frac{\sqrt{4^n+n^2}}{3^n}$

(d) $\sum_{n=1}^{\infty} \frac{3+(\ln n+5)^8}{n^2+6n+\sin n}$

(e) $\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{n^2}$

4. (7 pts) Find a function $f(x)$ whose arc length $L(x)$ from $\left(1, \frac{1}{2}\right)$ to $(x, f(x)), x >$
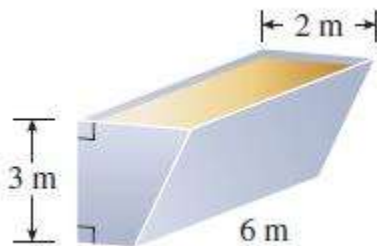
1, is $\frac{1}{2}x^2 + \frac{1}{4}\ln x$

5. (8 pts) Determine whether the following series are absolutely convergent, conditionally convergent or divergent.

(a) $\sum_{n=1}^{\infty} \frac{n^3(2^{-3n})}{n^4+1}$

(b) $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln 3n}$

6. (10 pts) A trough with a trapezoidal cross-section, as shown in the figure, contains vegetable oil with density $1000\ kg/m^3$. Find the hydrostatic force on one end of the trough if it is completely full of oil. ($g \approx 10\ m/s^2$)

7. (6 pts) Let $\{b_n\}$ be a sequence of positive umbers that converges to $\frac{1}{2}$. Determine whether the given series $\sum_{n=1}^{\infty} \frac{b_n^n \cos n\pi}{n}$ is absolutely convergent.

8. (10 pts) $y = \sqrt[3]{x}, 1 \le x \le 8$ is rotated about the x-axis. Set up, but do not evaluate, an integral for the area of the resulting surface.
   (a) by integrating with respect to x

   (b) by integrating with respect to y

9. (6 pts) (a) Approximate the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n^3}$ by using the first 2 terms.

(b) Estimate the maximum error in this approximation.