

Name: _____

Student ID: _____

Group A

For each of the following problems, find the correct answer (tick as appropriate!). No justifications are required. Each problem has exactly one correct solution, which is worth 1 mark. Incorrect solutions (including no answer, multiple answers, or unreadable answers) will be assigned 0 marks; there are no penalties.

- The tangent to $f(t) = (t^5, t^3, t)$ in the point $(1, 1, 1)$ meets the plane $x - y + z = 0$ in
☐ $(0, \frac{2}{3}, \frac{2}{3})$ ☒ $(-\frac{2}{3}, 0, \frac{2}{3})$ ☐ $(\frac{2}{3}, \frac{2}{3}, 0)$ ☐ $(\frac{2}{3}, 0, -\frac{2}{3})$ ☐ no point.
- The arc length of the curve $g(t) = (t \sin(18t), t \cos(18t), 4t^{3/2})$, $t \in [0, 3]$ is
☐ 10 ☐ 36 ☐ 38 ☒ 84 ☐ 148
- For a C^1 -curve $\mathbf{u}: I \rightarrow \mathbb{R}^3 \setminus \{\mathbf{0}\}$ and $t \in I$, the derivative $(d/dt)|\mathbf{u}(t)|^3$ is equal to
☐ $3|\mathbf{u}(t)|^2 \mathbf{u}'(t)$ ☐ $3|\mathbf{u}(t)|^2$ ☒ $3|\mathbf{u}(t)| \mathbf{u}(t) \cdot \mathbf{u}'(t)$ ☐ $\frac{3}{2}|\mathbf{u}(t)|$ ☐ $3\mathbf{u}(t) \cdot \mathbf{u}'(t)$
- The range of a parametric space curve $f: \mathbb{R} \rightarrow \mathbb{R}^3$ with nonzero curvature and $f''(t) = \mathbf{w} \in \mathbb{R}^3 \setminus \{\mathbf{0}\}$ (i.e., a nonzero constant vector) is
☐ a line ☐ an ellipse ☒ a parabola ☐ a hyperbola ☐ non-planar
- The volume of the pyramid ("tetrahedron") with vertices $(2, 0, -1), (3, -1, 0), (0, a, -1), (0, 3, 1)$ is equal to 1 for
☐ $a = -3$ ☐ $a = -2$ ☐ $a = -1$ ☐ $a = 0$ ☒ $a = 1$
- The inverse matrix of $\begin{pmatrix} 5 & 6 \\ 6 & 7 \end{pmatrix}$ is
☐ $\begin{pmatrix} -5 & 6 \\ 6 & -7 \end{pmatrix}$ ☐ $\begin{pmatrix} 5 & -6 \\ -6 & 7 \end{pmatrix}$ ☐ $\begin{pmatrix} -7 & -6 \\ -6 & -5 \end{pmatrix}$ ☒ $\begin{pmatrix} -7 & 6 \\ 6 & -5 \end{pmatrix}$ ☐ $\begin{pmatrix} 7 & -6 \\ -6 & 5 \end{pmatrix}$
- The matrix $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ describes a reflection whose axis in polar coordinates is given by
☐ $\theta = 0^\circ$ ☒ $\theta = 22.5^\circ$ ☐ $\theta = 45^\circ$ ☐ $\theta = 72^\circ$ ☐ $\theta = 90^\circ$
- The maximum rank of $\mathbf{A} \in \mathbb{R}^{4 \times 5}$ with entries ± 1 and exactly two entries equal to -1 is
☐ 1 ☐ 2 ☒ 3 ☐ 4 ☐ 5
- The linear system

$$\begin{array}{rrcr} x_1 & + & x_2 & + & x_3 & = & b \\ x_1 & & & - & x_3 & = & -1 \\ 3x_1 & - & 2x_2 & - & 7x_3 & = & 1 \end{array}$$
 has a solution if
☐ $b = 0$ ☒ $b = -3$ ☐ $b = 3$ ☐ $b = -1$ ☐ $b = 1$
- The distance from the point $(1, -1, 1)$ to the line $x_1 - x_2 = 2x_2 - x_3 = 1$ is
☒ $\frac{1}{2}\sqrt{14}$ ☐ $\frac{11}{4}$ ☐ $\frac{1}{2}\sqrt{11}$ ☐ $\sqrt{5}$ ☐ $\frac{7}{2}$

Time allowed: 40 min

CLOSED BOOK

Good luck!