ZJU-UIUC Institute

ECE 313– Midterm Exam #1
Probability with Engineering Applications

Instructor: Prof. Piao Chen

25 March 2024

Name: ____

ID Number: ____

Instructions

- You may not use any books, calculators, electronic devices, or notes other than one two-sided sheet of handwritten A4 paper.
- Show all your work to receive full credit for your answers.
- Neatness counts. If we are unable to read your work, we cannot grade it.
- Turn in your entire booklet once you are finished. Scratch papers and normal table are provided in the end.
- This exam contains 12 pages and 4 questions. Total of points is 50.

Distribution of Marks

Question	Points	Score
1	15	
2	15	
3	10	
4	10	
Total:	50	

1. The following questions are unrelated.

   (a) (3 points) The probability that we will have both beans and wine at diner is 0.4. The probability of both wine and dessert is 0.3. The probability of not having beans nor having dessert, but having wine is 0.2. Finally the probability of having dessert given that we have beans and wine is 0.5. What is the probability that we will have wine at dinner?

   (b) (3 points) At a certain bakery the weight X in grams of a loaf of bread may be modeled as a random variable with an $N(503, 25)$ distribution. What is the probability that a random loaf of bread has a weight between 497.9 and 508.1 grams?

   (c) (3 points) For a certain gadget you want to know the expected time until it breaks. This depends on two factors, namely the production quality of the particular gadget and the frequency of use of the person using it. For a randomly selected gadget the frequency of use X in uses per month follows a pareto distribution with parameter $\alpha = 4$ and the production quality Y, expressed in months before it breaks, follows a normal distribution with parameters $\mu = 18$ and $\sigma^2 = 36$. Suppose you know that the time in months until the gadget breaks T depends on these two factors as $T = 15/X + Y$, then what is the expected time until the gadget breaks (expressed in months)?

   (d) (3 points) At an amusement park there are two great roller coasters. For the first roller coaster the time (in minutes) you have to wait in line follows a $U(5, 10)$ distribution and for the second roller coaster the time (in minutes) you have to wait in line follows a $U(8, 11)$ distribution. You decide to roll a die to determine which roller coaster to visit. If the result is 1, 2, 3 or 4 you go to the first roller coaster and if the result is 5 or 6 you go to the second roller coaster. What is the probability that you have to wait in line more than 9 minutes?

   (e) (3 points) At home you perform a test for a certain disease. The test can be either positive or negative (but not anything else). The packaging indicates that the test is 95% sensitive meaning that the probability of a positive result given that you have the disease is 95%. It also indicates that the test is 80% specific meaning that the probability of a negative result given that you do not have the disease is 80%. Suppose that the prevalence of the disease in the population is 70% meaning that 70% of people have the disease. What is the probability that you have the disease given that you tested negative?

2. You are going to visit a friend and should decide if you will go there by train or by car. You decide to flip a (fair) coin and if it is heads you will take the train, while if it is tails you will go by car. The arrival time if you go by train follows a $U(20, 25)$ distribution and the arrival time assuming you go by car follows a $U(15, 30)$ distribution. Let X denote the arrival time.

   (a) (5 points) Derive the distribution function of X. Argue whether X follows a continuous or a discrete distribution or neither.

   (b) (5 points) Determine your expected arrival time (before you know whether you go by train or by car).

   (c) (5 points) Suppose you would use a fair six sided die instead of a coin to determine whether you go by car or by train. You would take the car if the result of the die roll is 1 or 2 and the train if the result is 3, 4, 5 or 6. What would be the distribution function of X in this case?

3. You and I play a tennis match. It is deuce, which means if you win the next two rallies, you win the game; if I win both rallies, I win the game; if we each win one rally, it is deuce again. Suppose the outcome of a rally is independent of other rallies, and you win a rally with probability p. Let W be the event "you win the game," G "the game ends after the next two rallies," and D "it becomes deuce again."

   (a) (5 points) Determine $P(W|G)$.

   (b) (5 points) Determine $P(W)$.

4. (10 points) Consider the one-dimensional Poisson process with intensity λ. Show that the number of points in $[0, t]$, given that the number of points in $[0, 2t]$ is equal to n, has a $Bin(n, 1/2)$ distribution.

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Standard Normal Distribution Table (Right-Tail Probabilities)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002