ECE 313: Problem Set 10: Problems

Due: Saturday, Nov 30 at 11:59:00 p.m.
Reading: *ECE 313 Course Notes,* Sections 4.1, 4.2, 4.3

Note on reading: For most sections of the course notes there are short answer questions at the end of the chapter. We recommend that after reading each section you try answering the short answer questions. Do not hand in; answers to the short answer questions are provided in the appendix of the notes.

Note on turning in homework: You must upload handwritten homework to BB. No typeset homework will be accepted. No late homework will be accepted. Please write on the top right corner of the first page:
NAME AS IT APPEARS ON BB
NETID
SECTION
PROBLEM SET #
Page numbers are encouraged but not required. Five points will be deducted for improper headings.

1. **[Joint pmf]**
Two fair six-sided dice are rolled. One of the dice shows Z_1 pips (pips are small dots on each face of a six-sided die), the other shows Z_2 pips. The random variables X and Y are defined as follows:

$$X = \min(Z_1, Z_2)$$
$$Y = \max(Z_1, Z_2)$$

(a) Sketch the support, in the (u, v) plane, of the joint pmf $p_{X,Y}(u, v)$.

(b) Find the joint pmf $p_{X,Y}(u, v)$.

(c) Find the marginal pmf of Y.

(d) Find $E[Y - X]$.

2. **[Joint PDF]**
X and Y are two random variables with the following joint pdf:

$$f_{X,Y}(u,v) = \begin{cases} A(1-|u-v|), & 0<u<1, 0<v<1; \\ 0, & \text{else.} \end{cases}$$

(a) Find A.

(b) Find marginal pdfs for X and Y.

(c) Find $P\{X > Y\}$.

(d) Find $P\{X + Y < 1 | X > 1/2\}$.

3. **[Joint pdf]**
Consider a pair of continuous-valued random variables X and Y whose joint pdf is given by the following pyramid, for some constant height A:

$$f_{X,Y}(u,v) = \begin{cases} Au, & 0 \le u \le v \le 1; \\ Av, & 0 \le v \le u \le 1; \\ 0, & \text{otherwise.} \end{cases}$$

(a) What is the value of the constant A?

(b) Find the marginal pdf of Y.

(c) Find $f_{X|Y}(u|v)$.

(d) Find $E[X + Y]$.