ECE 313: Probability with Engineering Applications

2025 Fall Instructors: Piao Chen & Xu Chen

Homework 1

Name: _____
NetID: _____

Due Date: September 26 23:59, 2025

Problem 1. Describe one event from the sample space of the following experiment, and determine the cardinality of the sample space:

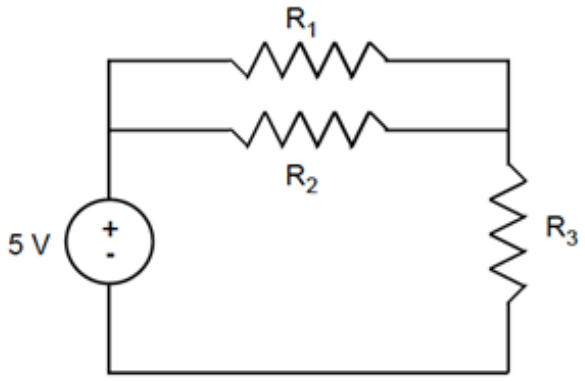
(a) Suppose you have a fair die and a fair coin. First you roll the die and let X be the number shown. If X is even, you toss the coin three times. If X is odd, you toss the coin five times. The outcome of the experiment is the result of the die roll and the coin flips.

(b) You pick a random positive integer less than 2,025 that has exactly one digit 6.

(c) An entire deck of 52 cards is dealt evenly to four people (each receives a hand of 13 cards, excluding the kings and queens).

(d) 5-digit numbers are formed from the integers $1, 2, \ldots, 9$ and no digit can appear more than once.

Problem 2. In this problem, you will construct a probability space $(\Omega, \mathcal{F}, P)$ for the following circuit. Each resistor in this circuit can fail independently, and we want to construct an experiment in which we examine the state of the circuit. When a resistor fails, it behaves as if it were an open circuit. The voltage source never fails.



(a) What is a suitable sample space Ω for this experiment? List all elements in Ω.

(b) Identify the event A corresponding to the statement "current flows through the circuit."

(c) If each resistor fails with a probability p, what is the probability of event A (i.e., $P(A)$)?

Problem 3. Consider sampling r items from a group of n objects, e.g., your pencil case contains n pens and pencils and you select r items from the pencil case. How many possible ways are there to sample r items from a group of n when the sampling is done in the following ways:

(a) Ordered and with replacement.

(b) Ordered and without replacement.

(c) Unordered and with replacement.

(d) Unordered and without replacement.

Problem 4. A closet contains 10 pairs of shoes. If 8 shoes are randomly selected, what is the probability that there will be:

(a) no complete pair.

(b) exactly one complete pair.

Problem 5. Here we consider a simple game called the Prisoner's Dilemma. Two members of a criminal gang, A and B, are arrested and are under investigation. Police don't have enough evidence, which makes them long for the prisoner's confession. Criminals have a choice to confess (C) or to deny (D). To encourage the criminals to confess, police offers a bargain:

- If A and B both confess, each of them serves 2 years in prison.
- If A betrays B (A confesses, B denies), then A will be set free while B will serve 3 years in prison (and vice versa).
- If A and B both remain silent, both of them serve 1 year in prison.

(a) What is the sample space in the Prisoner's Dilemma?

A decides to confess and betray B. B is reluctant to betray A. Therefore, B is thinking of confessing with a probability of 0.3. (B does not know what A will choose and vice-versa).

(b) What is the probability of A being set free, and B serving 3 years in prison?

(c) What is the probability of both A and B serving 2 years in prison?

Problem 6. Bear in mind that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. Then use this to show:

(a) If $A \subset B$ then $P(B-A) = P(B) - P(A)$. (Recall the definition of $B-A$: $B \cap A^c$.)

(b) $P(A) = P(A \cap B) + P(A - B)$. (Hint: Are $A \cap B$ and $A - B$ disjoint?)