ECE 313: Probability with Engineering Applications

2025 Fall   Instructors: Piao Chen & Xu Chen

Homework 5

Name: ____________________

Student ID: ____________________

Due Date: November 07 23:59, 2025

Problem 1. Let the random variable X have the following cumulative distribution function (CDF):

$$F(x) = \begin{cases} 0, & x < 0, \\ x^2, & 0 \leq x \leq 1, \\ 1, & x > 1. \end{cases}$$

(a) Find the median of X.

(b) Find the first quartile ($q_{0.25}$) and the third quartile ($q_{0.75}$) of X.

Problem 2. Assume that the number of buses arriving at a bus stop in an interval of t seconds, denoted by N_t, follows a Poisson distribution with parameter $\lambda = 0.3t$. Compute the probabilities of the following events:

(a) Exactly 3 buses arrive during a 10-second interval.

(b) At most 10 buses arrive during a 20-second interval.

(c) The number of arrivals during a 10-second interval is between 2 and 4 (inclusive).

Problem 3. Let $\{N_t, t \geq 0\}$ be a Poisson process with rate $\lambda > 0$. Answer the following questions; your answers may include λ.

(a) Compute the conditional probability

$$P(N_6 - N_4 = 4 \mid N_5 - N_4 = 1).$$

(b) Compute the conditional probability

$$P(N_7 - N_2 = 0 \mid N_4 - N_3 = 0).$$

(c) The interval $(1, 4]$ is divided into three equal parts: $(1, 2]$, $(2, 3]$, and $(3, 4]$. Given that $N_4 - N_1 = 6$, find the probability that there are $(2, 1, 3)$ arrivals in these three subintervals, respectively.

Problem 4. The lifetime of the memory chips produced by a factory is exponentially distributed with parameter $\lambda = 0.2$ (years^{-1}). Suppose John bought a computer with a memory chip produced by this factory and after five years it is still working. What is the conditional probability it will still work for at least three more years?

Problem 5. (a) Find the PDF of the minimum of two independent exponential random variables with parameter λ.

Hint: Work with $1 - F_X(x)$, where $F_X(x)$ is the CDF of the minimum. Use the independence property.

(b) You have a digital device that requires two batteries to operate. To be on the safe side, you buy three types of batteries (marked as 1, 2, 3), each of which has a lifetime that is exponentially distributed with parameter λ, and operates/fails independently of all the other batteries. Initially, you install two batteries, say 1 and 2. When one of these two batteries fails, you replace it with battery 3. What is the expected total time until your device stops working?

(c) In the scenario of part (b), what is the probability that battery 1 is the last battery that still works?