Physics 212
Lecture 11

Today's Concept:

RC Circuits

The 212 Differential Equations

We describe the world (electrical circuits, problems in heat transfer, control systems, financial markets, etc.) using differential equations

You only need to know the solutions of two basic differential equations

$$\frac{dq(t)}{dt} + \frac{1}{t} q(t) = 0 \quad \rightarrow \quad q(t) = q_0 e^{-t/t}$$

$$\frac{d^2 q(t)}{dt^2} + w^2 q(t) = 0 \quad \rightarrow \quad q(t) = q_0 \sin(wt + f)$$

Capacitors in RC Circuits

Solve by applying Kirchhoff's Rules to circuit. Need to understand some key phrases.

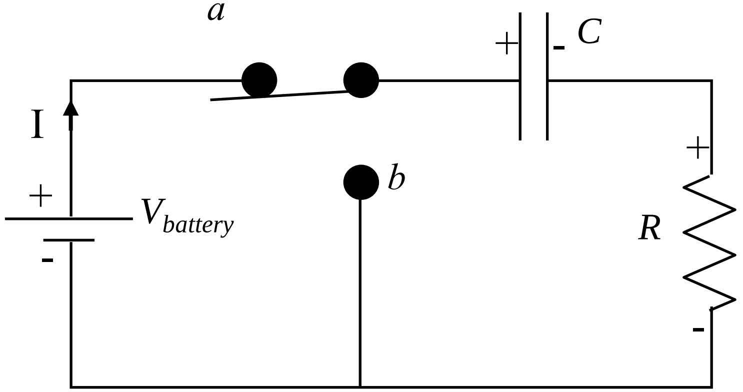
IMMEDIATELY After === Charge on capacitor is same as immediately before

After a LONG TIME === Current through capacitor = 0

After xx seconds === *Exponential solutions*

RC Circuit (Charging)

Capacitor uncharged, Switch is moved to position "a"



Kirchoff's Voltage Rule

$$-V_{battery} + \frac{q}{C} + IR = 0$$

Short Term $(q = q_0 = 0)$

$$-V_{battery} + 0 + I_0 R = 0$$

$$I_0 = \frac{V_{battery}}{R}$$

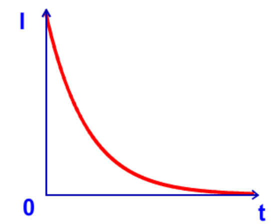
Long Term $(I_c = 0)$

$$-V_{battery} + \frac{q_\infty}{C} + 0 \cdot R = 0$$

$$q_\infty = CV_{battery}$$

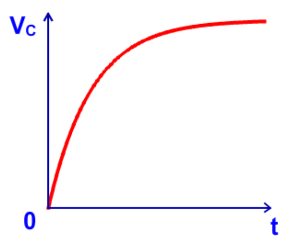
Intermediate

$$-V_{battery} + \frac{q}{C} + \frac{dq}{dt}R = 0$$



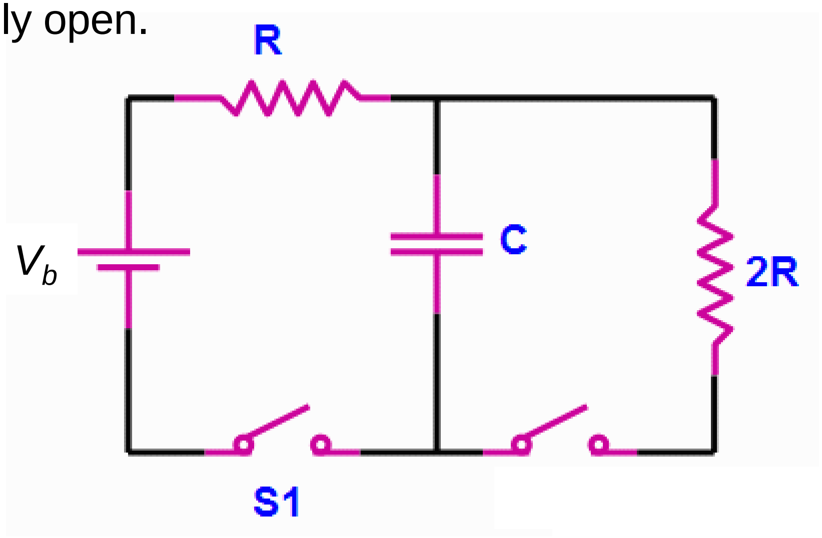
$$q(t) = q_\infty\left(1 - e^{-t/RC}\right)$$

$$I(t) = \frac{dq}{dt} = I_0 e^{-t/RC}$$



CheckPoint 1

A circuit is wired up as shown below. The capacitor is initially uncharged and switches S1 and S2 are initially open.



Close $S1$,
$V_1 =$ voltage across C immediately after
$V_2 =$ voltage across C a long time after

A) $V_1 = V_b \quad V_2 = V_b$

B) $V_1 = 0 \quad V_2 = V_b$

C) $V_1 = 0 \quad V_2 = 0$

D) $V_1 = V_b \quad V_2 = 0$

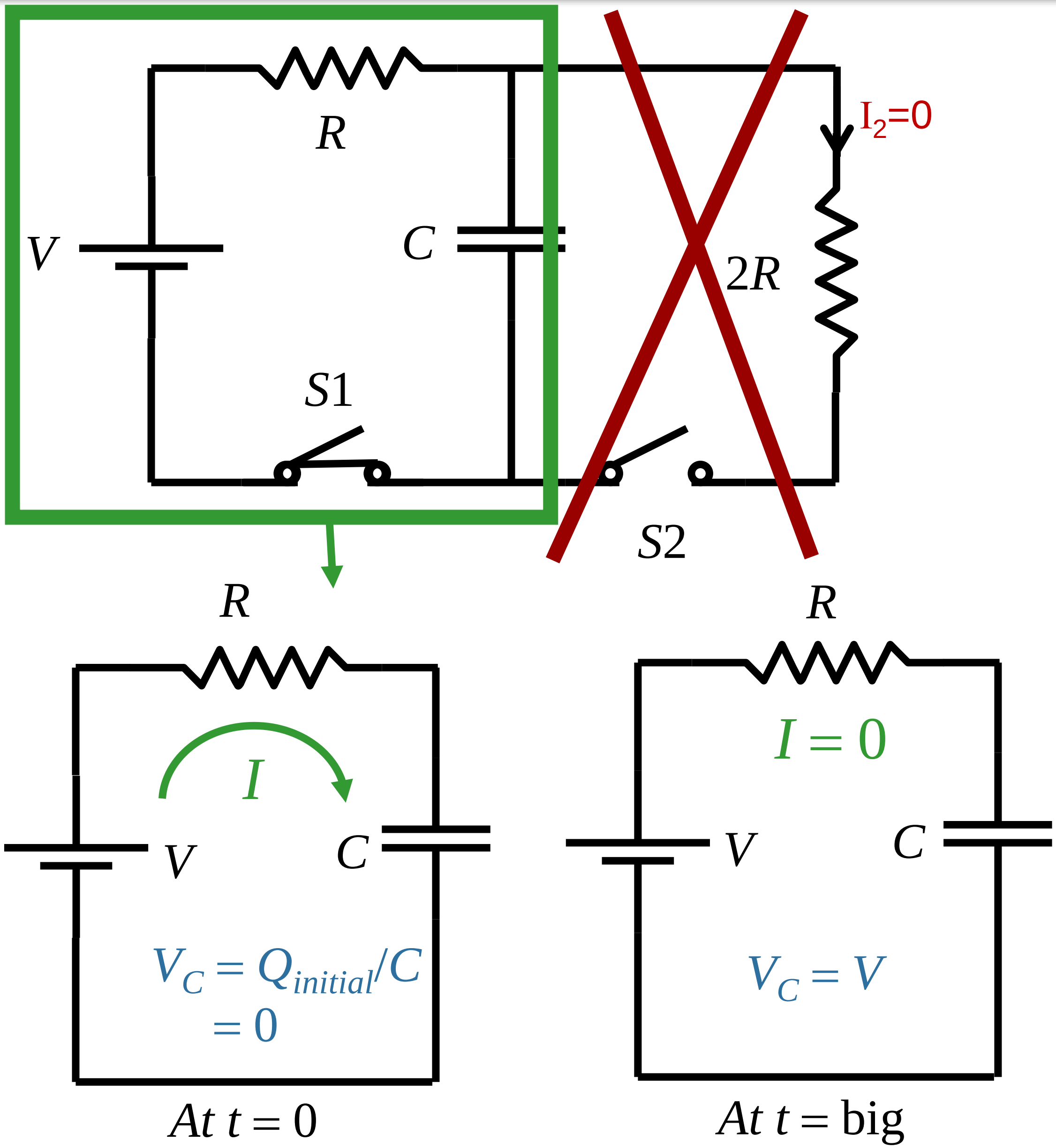
Immediately after the switch S_1 is closed:

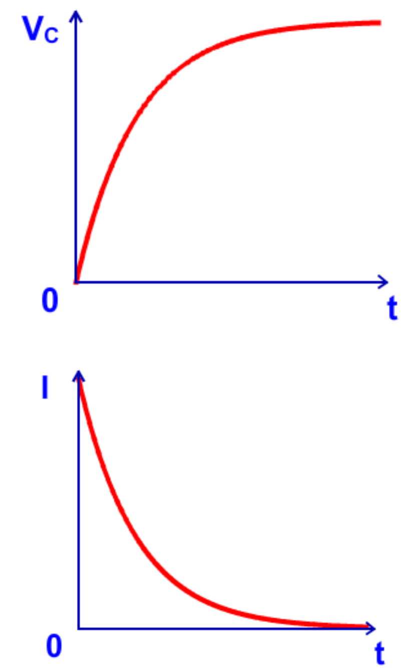
$Q \text{ is same as immediately before}$

After the switch S_1 has been closed for a long time

$I_C = 0$

Close S_1 at $t = 0$
(leave S_2 open)





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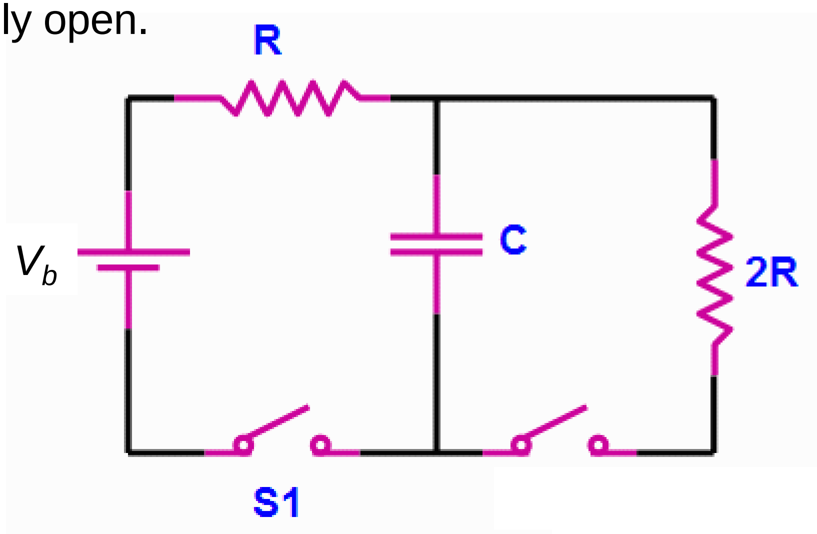


Close $S1$,

$V_1 =$ voltage across C immediately after

$V_2 =$ voltage across C a long time after

A) $V_1 = V_b \quad V_2 = V_b$

B) $V_1 = 0 \quad V_2 = V_b$

C) $V_1 = 0 \quad V_2 = 0$

D) $V_1 = V_b \quad V_2 = 0$

Immediately after the switch S_1 is closed:

Q *is same as immediately before*

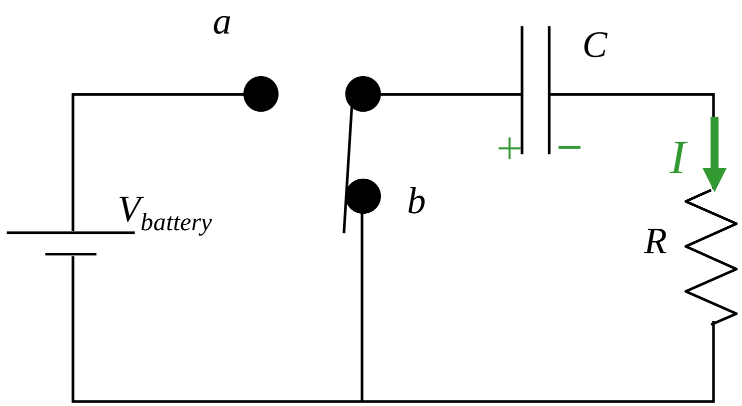
After the switch S_1 has been closed for a long time

$I_C = 0$

RC Circuit (Discharging)

Capacitor has $q_0 = CV_{battery}$, Switch is moved to position "b"

Kirchoff's Voltage Rule



$$+\frac{q}{C} + IR = 0$$

Short Term $(q = q_0)$

$$V_{battery} + IR = 0$$

$$I_0 = \frac{-V_{battery}}{R}$$

Long Term $(I_c = 0)$

$$\frac{q_\infty}{C} + 0 \cdot R = 0$$

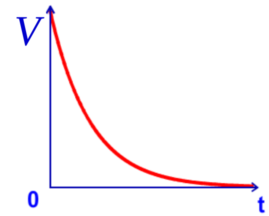
$$q_\infty = 0$$

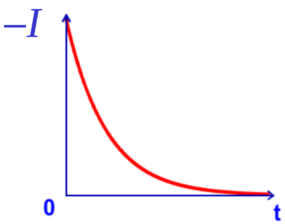
Intermediate

$$+\frac{q}{C} + \frac{dq}{dt}R = 0$$

$$q(t) = q_0 e^{-t/RC}$$

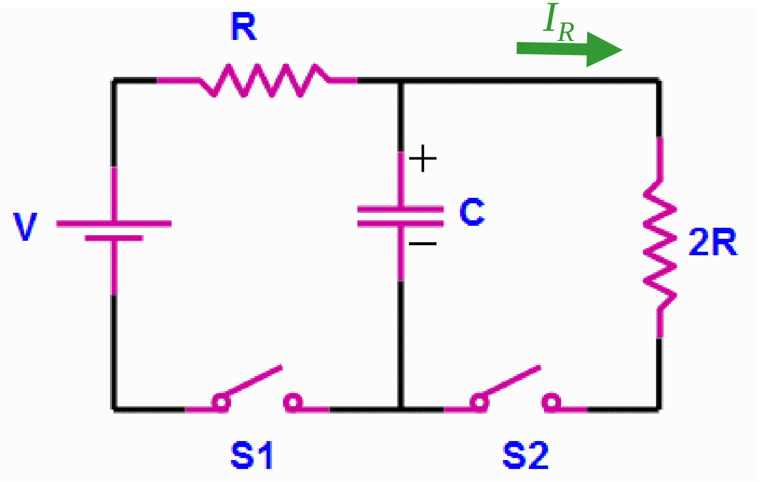
$$I(t) = I_0 e^{-t/RC}$$





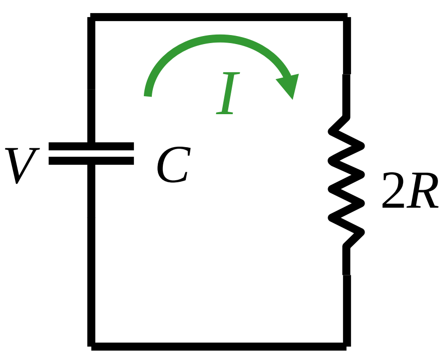
CheckPoint 1c

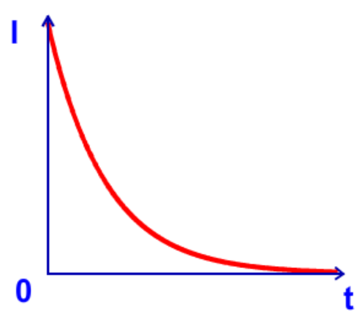
A circuit is wired up as shown below. The capacitor is initially uncharged and switches S1 and S2 are initially open.

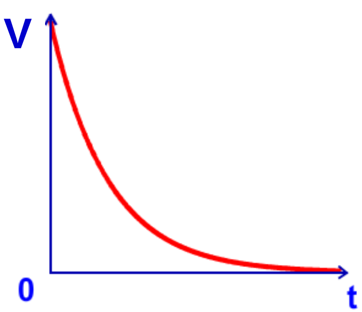


Switch 1 is closed for a long time. Then, switch 1 is opened and switch 2 is closed. What is the current through the right resistor immediately after switch 2 is closed?

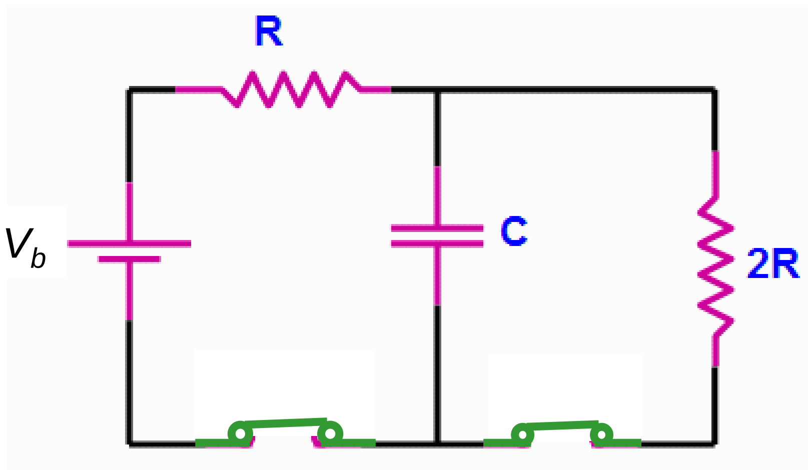
A. $I_R = 0$ ***B.*** $I_R = V/3R$ ***C.*** $I_R = V/2R$ ***D.*** $I_R = V/R$





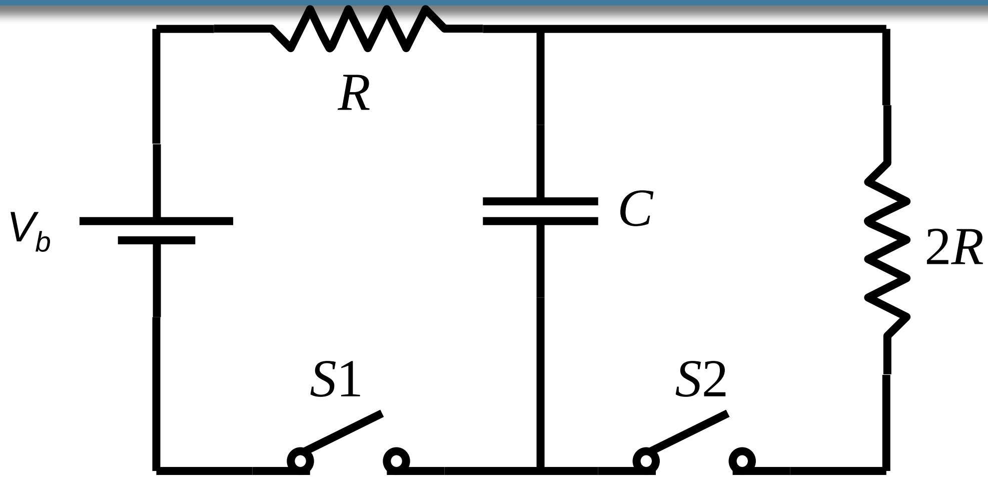


CheckPoint 1 d

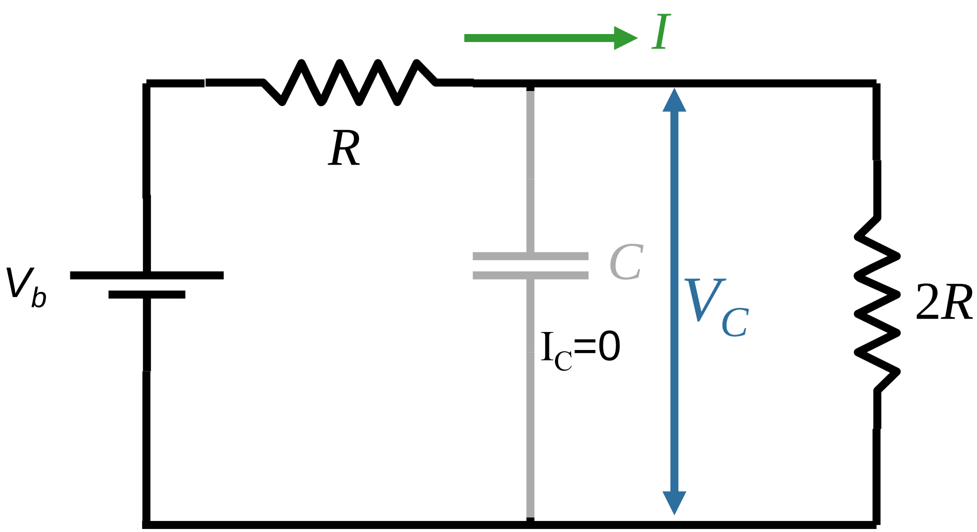


Now suppose both switches are closed. What is the voltage across the capacitor after a very long time?

A. $V_C = 0$ ***B.*** $V_C = V_b$ ***C.*** $V_C = 2V_b/3$



Close both $S1$ and $S2$ and wait a long time…



No current flows through the capacitor after a long time. This will always be the case in any static circuit!!

Outer Loop

$$IR + 2IR - V_b = 0$$

$$I = V_b/(3R)$$

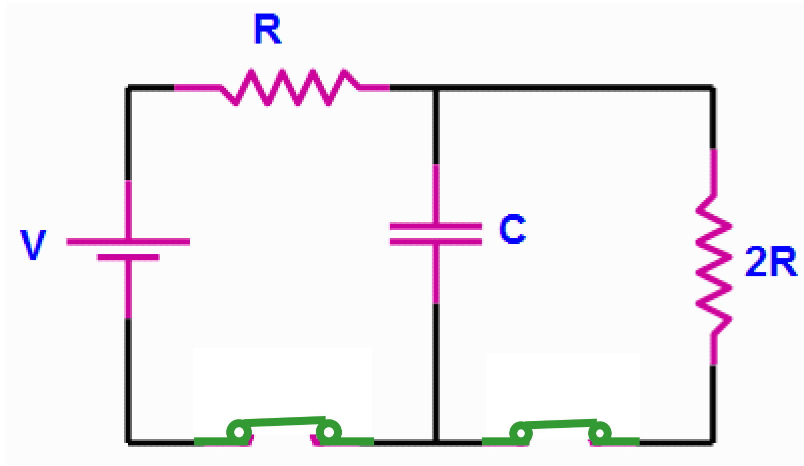
Right Loop

$$+V_C - 2IR = 0$$

$$V_C = 2IR$$

$$\rightarrow \quad V_C = (2/3)V_b$$

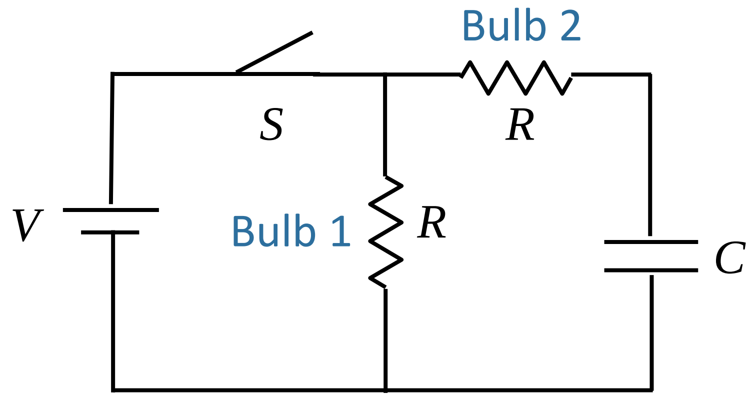
CheckPoint 1 d



Now suppose both switches are closed. What is the voltage across the capacitor after a very long time?

A. $V_C = 0$ ***B.*** $V_C = V_b$ ***C.*** $V_C = 2V_b/3$

DEMO – Clicker Question 1

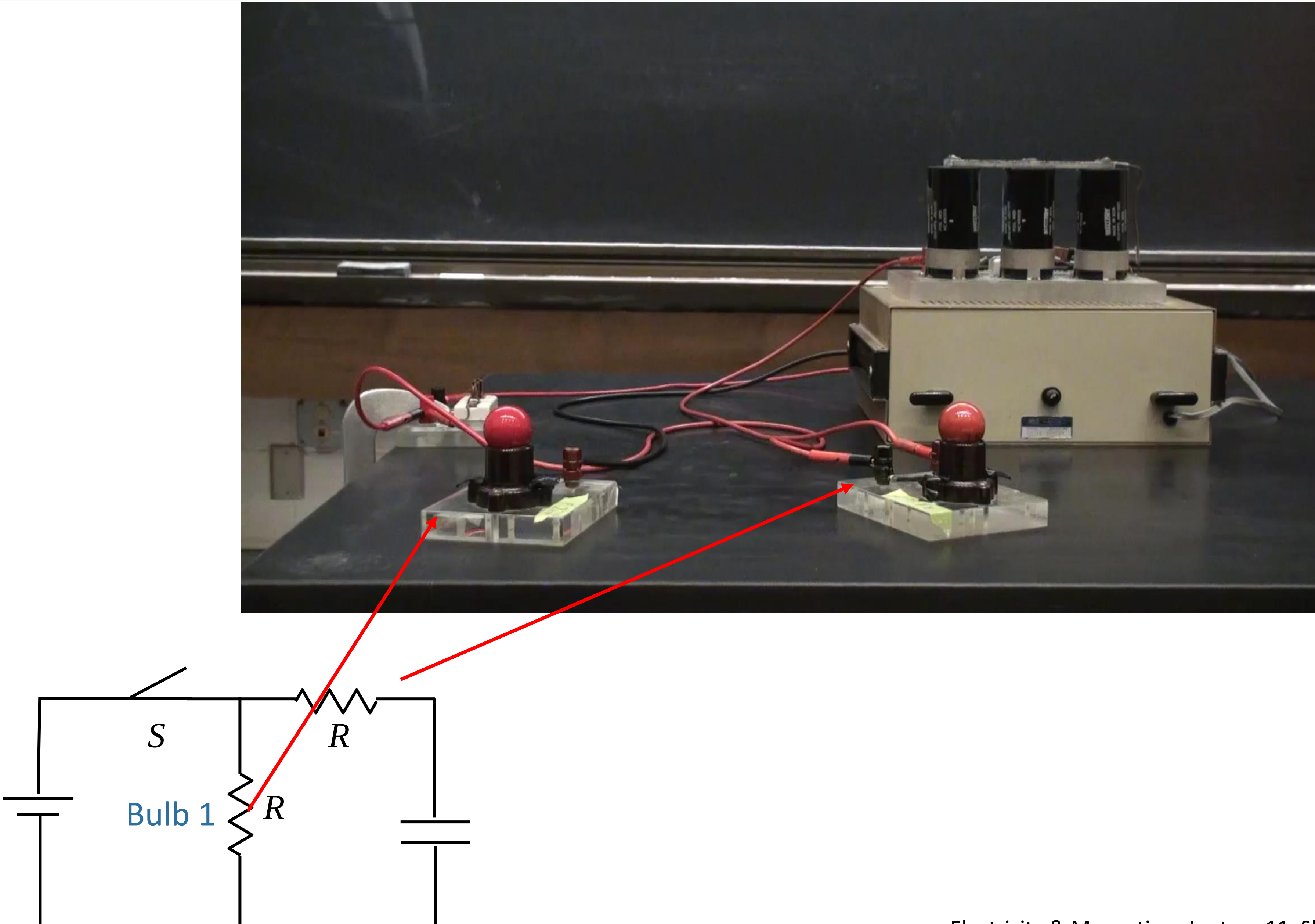


What will happen after I close the switch?

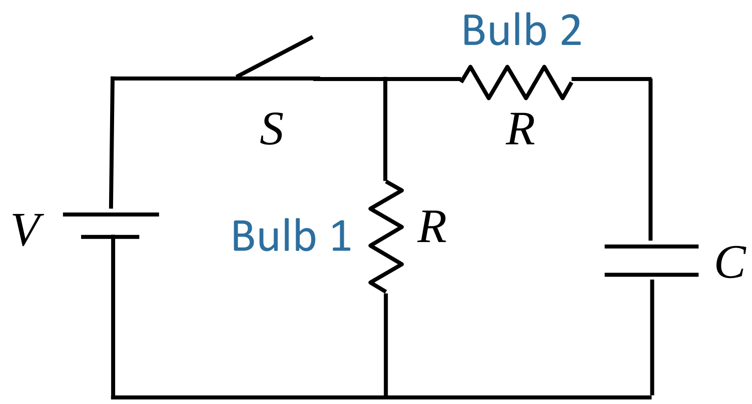
A) Both bulbs come on and stay on.
B) Both bulbs come on but then bulb 2 fades out.
C) Both bulbs come on but then bulb 1 fades out.
D) Both bulbs come on and then both fade out.

No initial charge on capacitor → $V(\text{bulb 1}) = V(\text{bulb 2}) = V$ → Both bulbs light

No final current through capacitor → $V(\text{bulb 2}) = 0$


S
R
Bulb 1
R

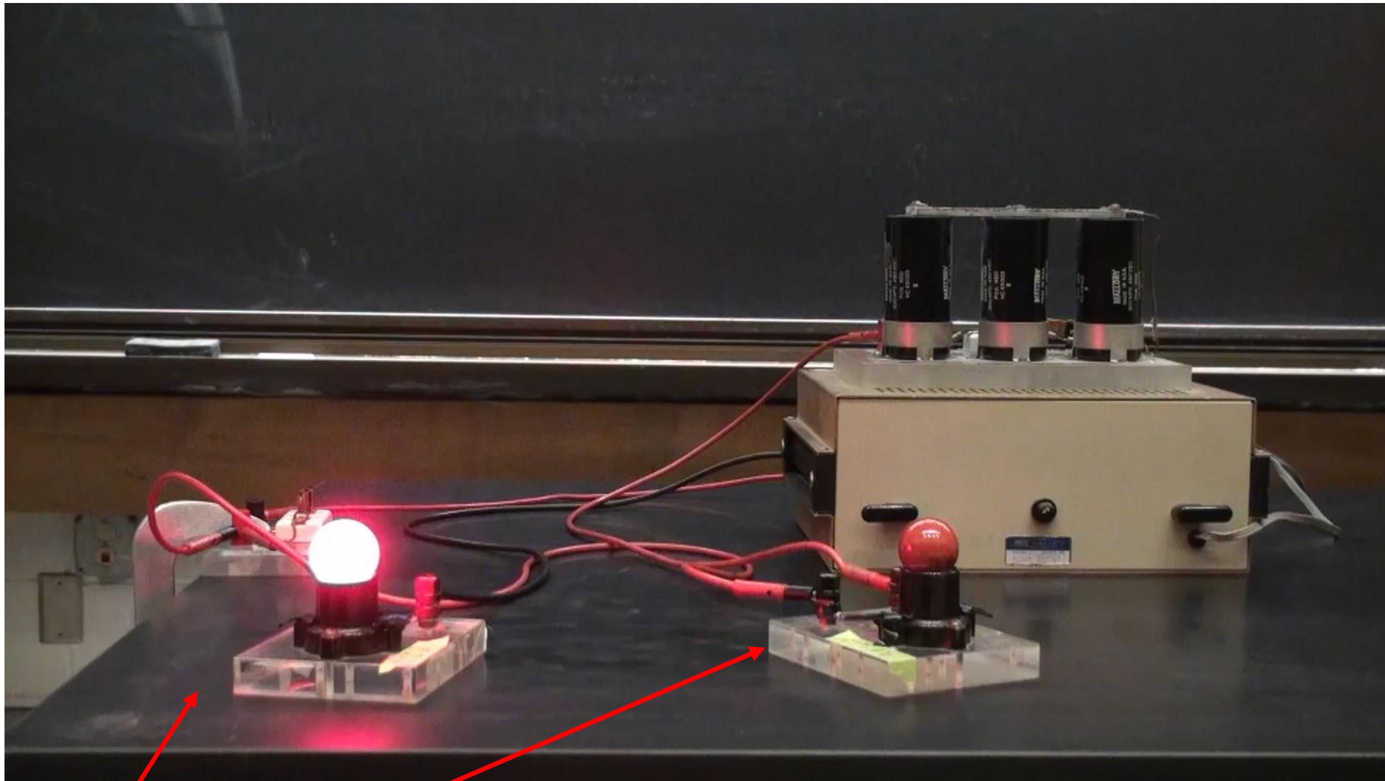
DEMO Clicker Question 2

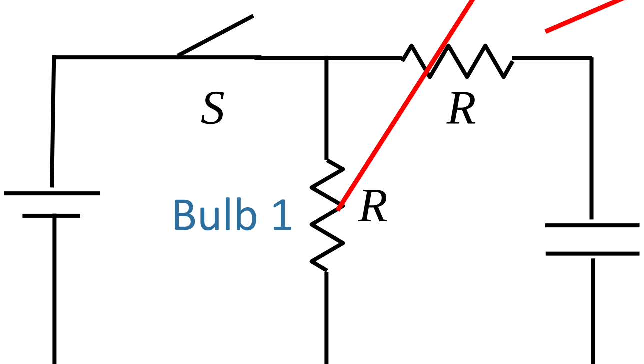


Suppose the switch has been closed a long time.
Now what will happen after opening the switch?

A) Both bulbs come on and stay on.
B) Both bulbs come on but then bulb 2 fades out.
C) Both bulbs come on but then bulb 1 fades out.
D) Both bulbs come on and then both fade out.

Capacitor has charge (=CV) → Capacitor discharges through both resistors

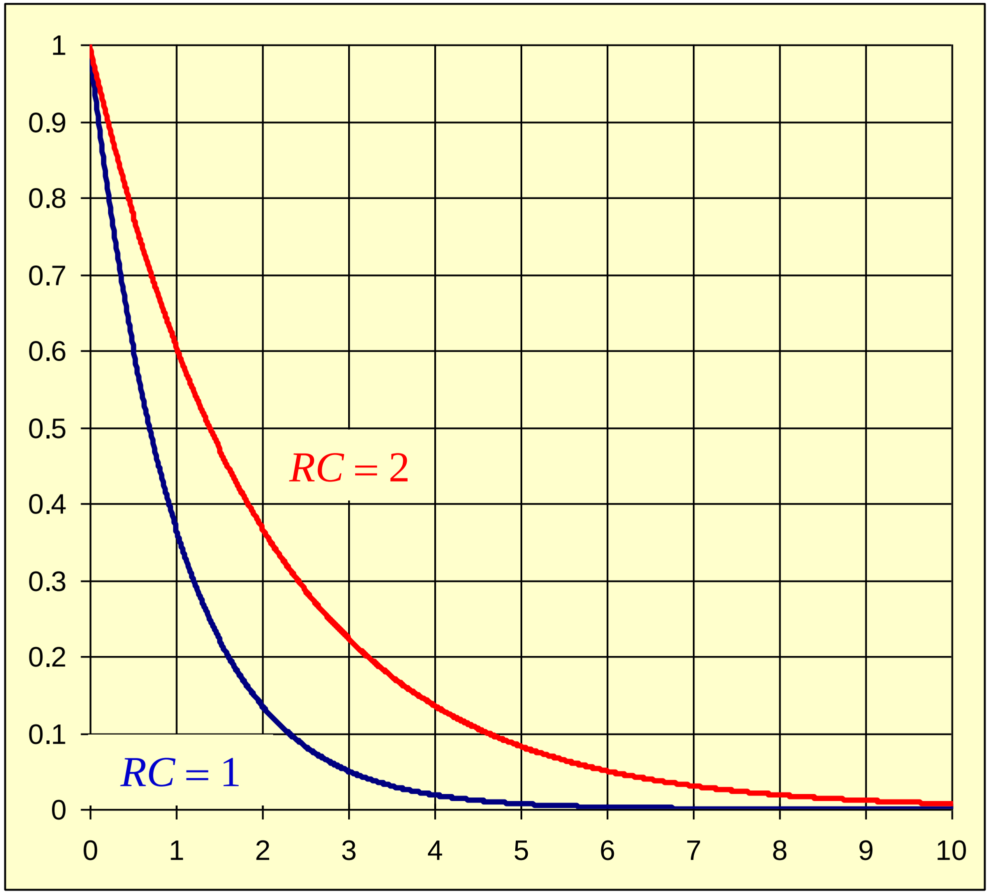



S
R
Bulb 1
R

How do Exponentials Work?

$$Q(t) = Q_0 e^{-t/RC}$$

$$\frac{Q(t)}{Q_0}$$



t

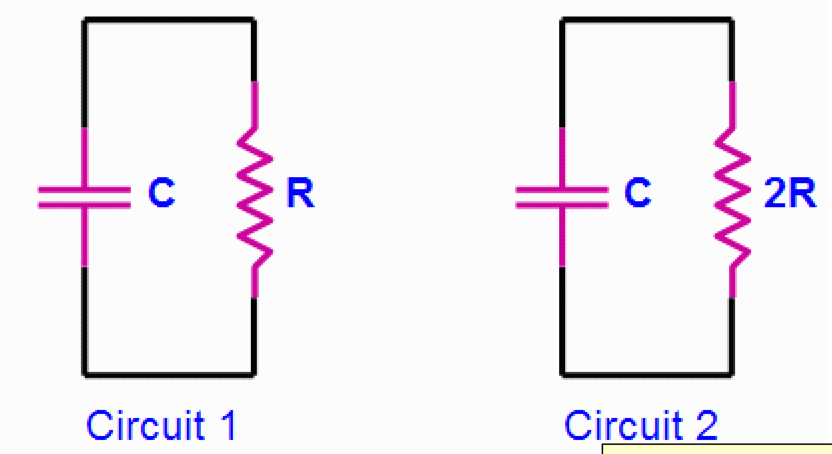
Time constant:

$\tau = RC$

The bigger τ is,
the longer it takes to get the same change…

CheckPoint 2

The two circuits shown below contain identical capacitors that hold the same charge at t = 0. Circuit 2 has twice as much resistance as circuit 1.



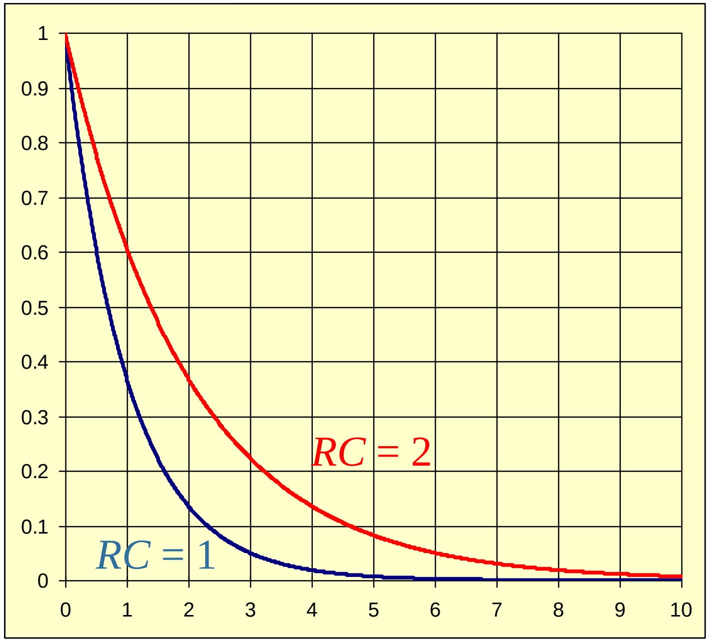
Which circuit has the largest time constant?

A) Circuit 1

B) Circuit 2

C) Same

$$\tau = R_{equiv}\, C$$



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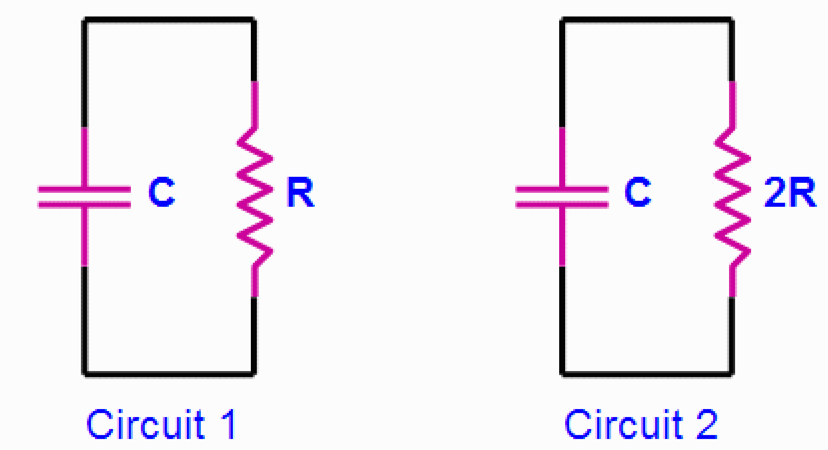
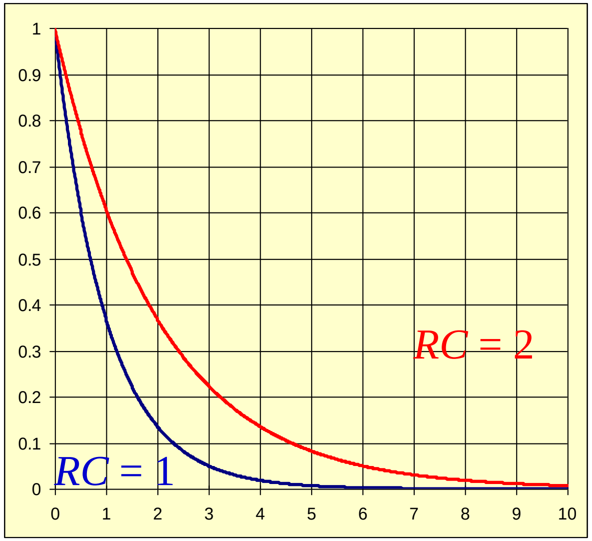


Which of the following statements best describes the charge remaining on each of the the two capacitors for for any time after t = 0?

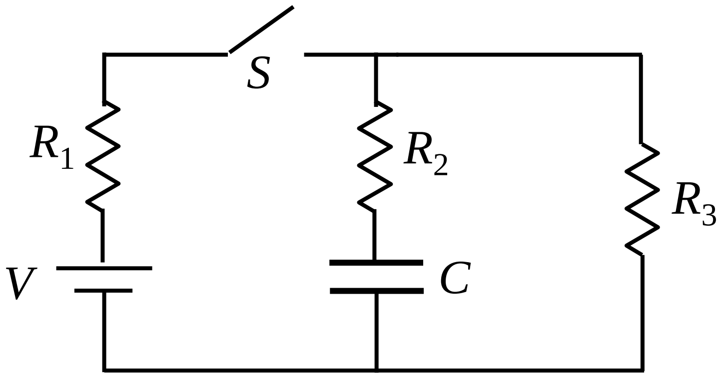
A $Q_1 < Q_2$
B $Q_1 > Q_2$
C $Q_1 = Q_2$
D $Q_1 < Q_2$ at first and then $Q_1 > Q_2$ after a long time
E $Q_1 > Q_2$ at first and then $Q_1 < Q_2$ after a long time

$$\boxed{Q = Q_0 e^{-t/RC}}$$

Look at plot!



Calculation



In this circuit, assume V, C, and R_i are known.
C initially uncharged and then switch S is closed.

What is the voltage across the capacitor after a long time ?

Conceptual Analysis:

Circuit behavior described by Kirchhoff's Rules:

$$\Sigma V_{drops} = 0$$

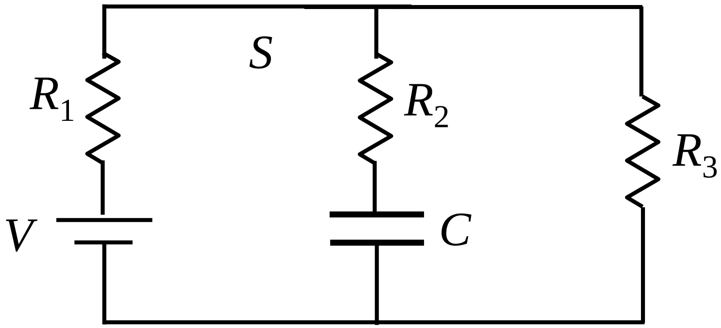
$$\Sigma I_{in} = \Sigma I_{out}$$

S closed and C charges to some voltage with some time constant

Strategic Analysis

Determine currents and voltages in circuit a long time after S closed

Calculation



In this circuit, assume V, C, and R_i are known.
C initially uncharged and then switch S is closed.

What is the voltage across the capacitor after a long time ?

Immediately after S is closed:

what is I_2, the current through C

what is V_C, the voltage across C?

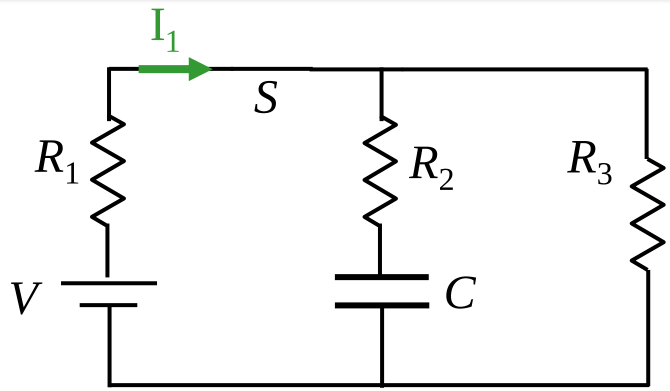
A) Only $I_2 = 0$ B) Only $V_C = 0$ C) Both I_2 and $V_C = 0$ D) Neither I_2 nor $V_C = 0$

Why?

We are told that C is initially uncharged $(V_C = Q/C)$

I_2 cannot be zero because $V_{R2}+V_C = V_{R3}$ (Right Loop)

Calculation



In this circuit, V, C, and R_i are known.
C initially uncharged and then switch S is closed.

What is the voltage across the capacitor after a long time ?

Immediately after S is closed, what is I_1, the current through R_1 ?

A. $\dfrac{V}{R_1}$

B. $\dfrac{V}{R_1 + R_3}$

C. $\dfrac{V}{R_1 + R_2 + R_3}$

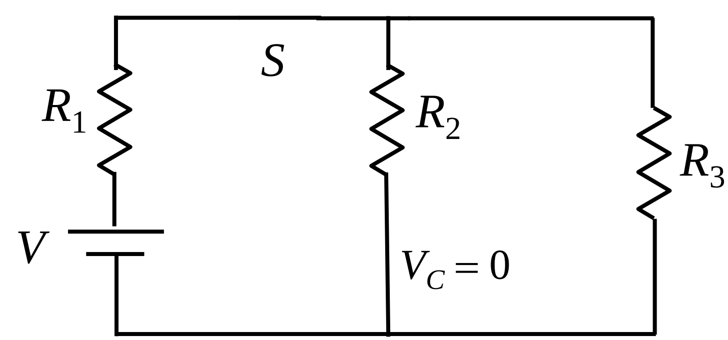
D. $\dfrac{V}{R_1 + \dfrac{R_2 R_3}{R_2 + R_3}}$

E. $V \dfrac{R_1 + R_2 + R_3}{R_1 R_2 + R_2 R_3 + R_1 R_3}$

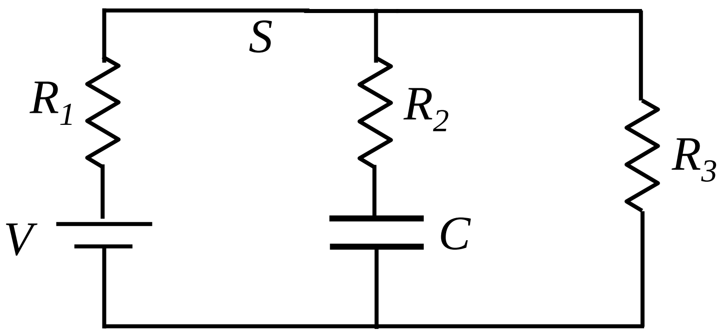
Why?

Draw circuit just after S closed (knowing $V_C = 0$)

R_1 is in series with the parallel combination of R_2 and R_3



Calculation



In this circuit, assume V, C, and R_i are known.
C initially uncharged and then switch S is closed.

What is the voltage across the capacitor after a long time ?

After S has been closed "for a long time", what is I_2, the current through R_2 ?

$\dfrac{V}{R_2}$

A

$\dfrac{V}{R_1}$

B

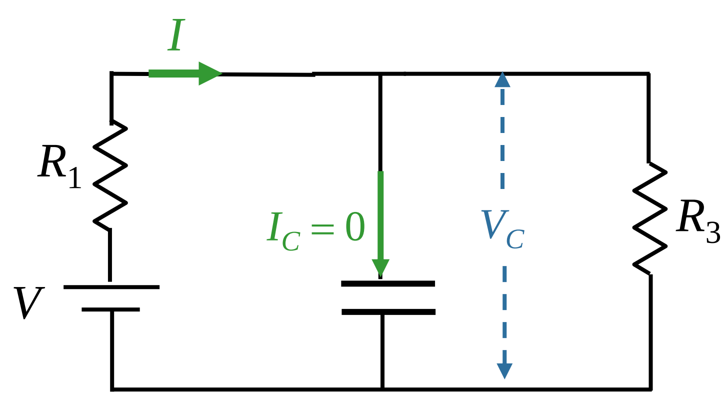
0

C

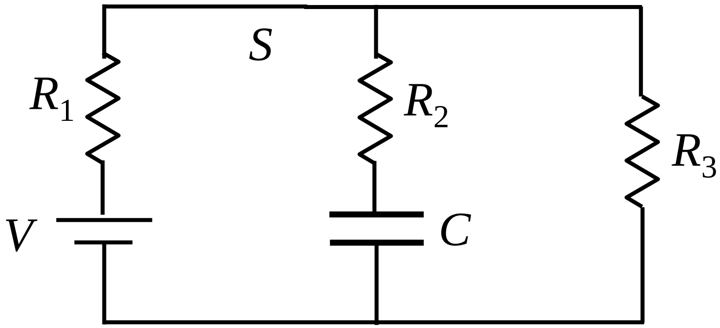
Why?

After a long time in a static circuit, the current through any capacitor approaches 0 !

This means we Redraw circuit with open circuit in middle leg



Calculation



In this circuit, assume V, C, and R_i are known.
C initially uncharged and then switch S is closed.

What is the voltage across the capacitor after a long time ?

After S has been closed "for a long time", what is V_C, the voltage across C ?

A: $V\dfrac{R_3}{R_1+R_3}$

B: $V\dfrac{R_2}{R_1+R_2}$

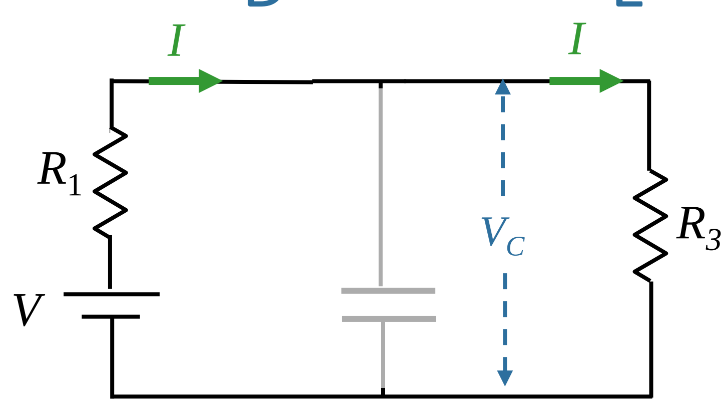
C: V

D: $V\dfrac{R_2}{R_1+\dfrac{R_2R_3}{R_2+R_3}}$

E: 0

Why?

– $V_C = V_3 = IR_3 = (V/(R_1+R_3))R_3$



Next up --> Magnetism!