ECE 313: Probability with Engineering Applications

2025 Fall Instructors: Piao Chen & Xu Chen

Homework 3

Name: _____
NetID: _____

Due Date: October 17 23:59, 2025

Problem 1. Consider the random variable X with pmf $P(X = i) = 2^{-i}$ for $i \geq 1$.

(a) Sketch the pmf (given above) and the distribution function for X.

(b) Calculate $P(X \leq 3)$.

(c) Calculate $P(X > 3)$.

(d) Calculate $P(X < 1)$.

(e) Calculate $P(|X - 4| \leq 0.1)$.

(f) Evaluate the following expression:

$$\sum_{k=5}^{\infty} P(X = k)$$

Problem 2. A communication channel receives independent pulses at the rate of 10 pulses per microsecond. The probability of a transmission error is 0.01 for each pulse. Compute the probabilities of:

(a) No errors per microsecond

(b) One error per microsecond

(c) At least one error per microsecond

(d) Exactly two errors per microsecond

Problem 3. Solve the following questions.

(a) Let $X \sim \text{Bin}(n, p)$. Find $P(X \text{ is odd})$ in terms of n and p.

(b) Let $Y \sim \text{Poi}(\lambda)$. Find $P(Y \text{ is odd})$ in terms of λ.

(c) Suppose that $n \to \infty$ and $p \to 0$ such that $np = \lambda$. Verify that your answer in part (a) converges to the answer in part (b).

Hint: when $n \to \infty, p \to 0$, and $np = \lambda$, we have:

$$(1-p)^n \approx e^{-np} = e^{-\lambda}.$$

Problem 4. A graduating ECE student goes to career fair booths in the technology sector. His/her likelihood of receiving an off-campus interview invitation from a certain career fair booth depends on how well he/she did in ECE 313. Specifically, an A in 313 results in a probability $p = 0.9$ of obtaining an invitation, whereas a C in ECE 313 results in a probability $p = 0.2$ of receiving an invitation.

(a) What is the probability that an A student receives his/her first interview when visiting the third booth? How about a C student?

(b) Each student is allowed to visit 5 booths during this career fair. What is the probability that an A student receives exactly two interviews? How about a C student?

(c) Assume that each student visits 5 booths during a typical career fair. Find the probability that an A student in 313 will not get an off-campus interview invitation. Finally, find the probability that a C student in 313 will get at least one invitation.

(d) Assume an A student already visited 10 booths without getting an interview, and a C student already visited 5 booths without getting an interview. If they both go visit 5 additional booths, who has a higher probability of landing an interview?

Problem 5. Assume that the number of jobs arriving to the Blue Waters supercomputer in an interval of t seconds is Poisson distributed with parameter $\lambda = 0.3$. Compute the probabilities of the following events:

(a) Exactly 3 jobs will arrive during a 10s interval.

(b) More than 10 jobs arrive in a period of 20s.(Please keep your answer in analytical form)

(c) The number of job arrivals in an interval of 10s duration is between 2 and 4 (include 2 and 4).

(d) Given that 10 jobs arrive in a period of 30s, what is the conditional probability that 3 jobs arrived in the first 10s?

Hint: Use the Bayes theorem to calculate the conditional probability. Note that the probability of 3 jobs arriving in the first 10s, given that 10 jobs arrived in 30s, equals to the probability of 3 jobs arriving in the first 10s and 7 jobs arriving in the remaining 20s. Also note that the number of arrivals in different time intervals are independent from each other.