Math 231 Test #3

Your Name:_____

Please circle your discussion group (2 pt)

1 Gan Yinliang B416	4 Zhang Jinghao B425	7 Loigen Sodian B416
2 Zhang Junwei B424	5 Xu Hang B419	
3 Ke Wentao B419	6 Huang Nuoer B410	

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- You will have one hour for the exam.
- No notes, books or electronics during the exam.
- Do not open this test booklet until a proctor says start.
- For all free response questions, show work that justifies your answer.
- Raise your hand if you have a clarification question.
- Scratch paper is provided. You can ask for more if needed.
- Do not leave early: this disturbs others. If you finish your test early, check your work or just relax.
- Quit working when the test ends and hand your test booklet to proctors.

Question	1	2	3	4	5	6	7	8	Total
Points	9	10	10	18	9	11	15	16	98
Score									

$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$ $R = \infty$

$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$, $R = \infty$

$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$, $R = \infty$

1. (9 points, 3 points each) Determine whether the statement is true or false. Circle the right answer.

   (a) $0.9999999 \ldots = 1$(True or False)

   (b) $\lim_{n\to\infty} \frac{x^n}{n!} = 0$, for every real number x.(True or False)

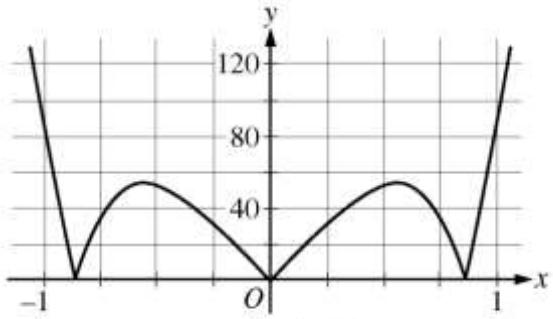
   (c) If $x = f(t)$ and $y = g(t)$ are twice differentiable, then $\frac{d^2y}{dx^2} = \frac{\frac{d^2y}{d^2t}}{\frac{d^2x}{d^2t}}$. (True or False)

2. **(10 pts)** Power series representation of the function $f(x) = \frac{x}{(1+x)^2}$ has the form $f(x) = a_o + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + O(x^6)$. Determine a_4 and a_5

3. (10 pts) Find Taylor polynomial $T_2(x)$ for the function $f(x)$ centered at $a = 1$

$$f(x) = \cot(x).$$

4.(6+7+5 pts) Let $f(x) = \sin(x^2) + \cos(x)$. The graph of $y = |f^{(5)}(x)|$ is show below.



Graph of $y = \left|f^{(5)}(x)\right|$

(a) Write the first four nonzero terms of the Taylor series for this function about $x = 0$.

(b) Find the value of $f^{(6)}(0)$.

(c) Let $P_4(x)$ be the fourth-degree Taylor polynomial for f about $x = 0$. Using information from the graph of $y = |f^{(5)}(x)|$ show above, estimate $|P_4\left(\frac{1}{4}\right) - f\left(\frac{1}{4}\right)|$

5. (8 pts) Identify the polar curve by finding the Cartesian equation.

(a) $\theta = \frac{\pi}{3}$

(b) $r = 2sin\theta$

6. (5+6 pts) (a) A limacon is defined in polar coordinates by the equation $r = 2sin\theta - 1$, sketch the curve.

(b) Find a general formula for the slope of the tangent line of the limacon in the previous problem in terms of θ. Find the slope of the tangent line at the points where $\theta = 0$ and $\theta = \pi/2$.

7. (6+4+5 pts) Consider the parametric curve $x = tcos(t),\ y = tsin(t),\ t > 0$

(a) Write an integral which describes the length of this curve from $t = 0$ to $t = \pi$.

(b) What substitutions could be used to evaluate the integral in part (a)? Do not evaluate the integral.

(c) Set up, but do not evaluate, an integral which describes the surface area obtained by rotating this curve about the x-axis.

8. (16 pts) Find the sum of the series

(a) $\sum_{n=0}^{\infty} [\tan^{-1}(n+1) - \tan^{-1}(n)]$

(b) $\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{2^{3n}}$

(c) $\frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots$

(d) $\frac{1}{0!} - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \cdots$