Electricity & Magnetism Lecture 2

Today's Concepts:

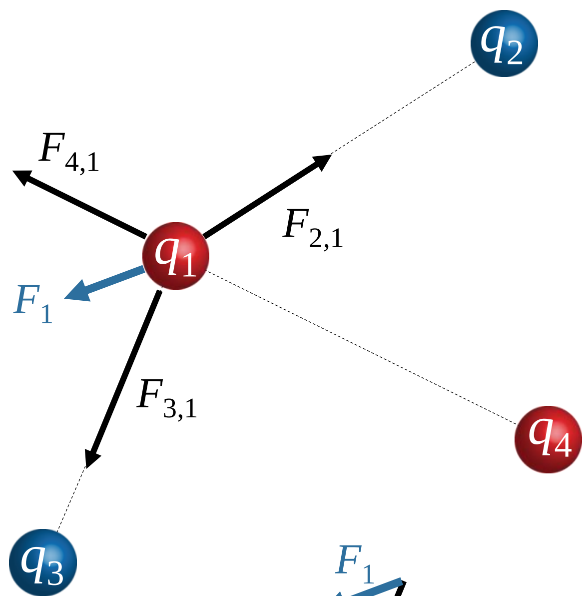
A) The Electric Field

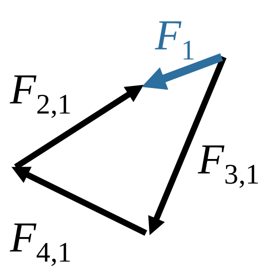
B) Continuous Charge Distributions

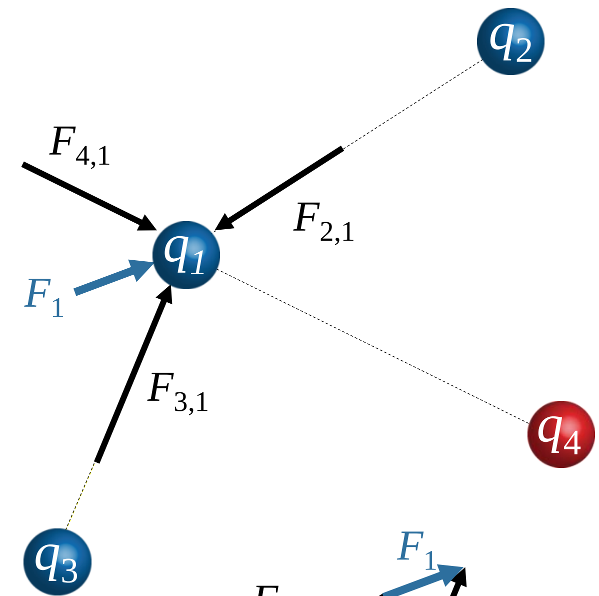
Reading: Ch. 5.4-5.5

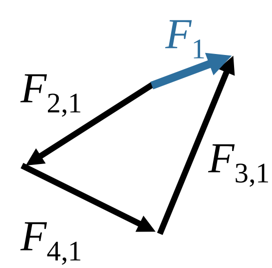
Coulomb's Law & superposition!

If there are more than two charges present, the total force on any given charge is just the vector sum of the forces due to each of the other charges:









$+q_1 \rightarrow -q_1$ -> direction reversed

MATH:

$$\vec{F}_1 = \frac{kq_1q_2}{r_{12}^2}\hat{r}_{12} + \frac{kq_1q_3}{r_{13}^2}\hat{r}_{13} + \frac{kq_1q_4}{r_{14}^2}\hat{r}_{14} \longrightarrow \vec{E} = \frac{\vec{F}_1}{q_1} = \frac{kq_2}{r_{12}^2}\hat{r}_{12} + \frac{kq_3}{r_{13}^2}\hat{r}_{13} + \frac{kq_4}{r_{14}^2}\hat{r}_{14}$$

Electric Field

The electric field E at a point in space is simply the force per unit charge at that point.

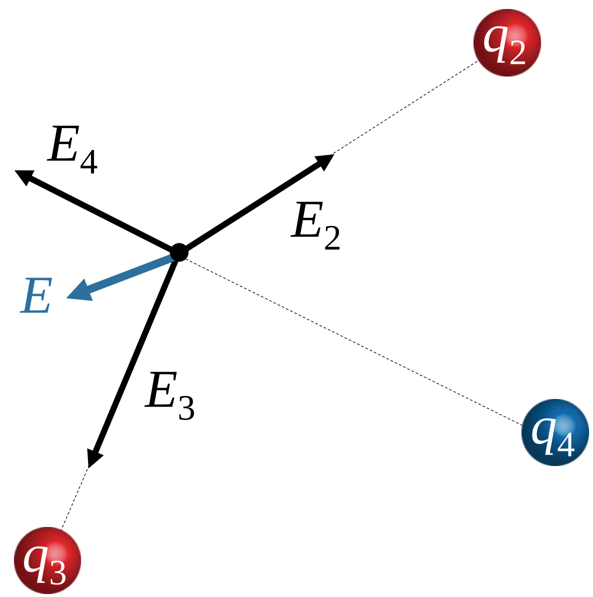
$$\vec{E} \equiv \frac{\vec{F}}{q}$$

Electric field due to a point charged particle

$$\vec{E} = k\frac{Q}{r^2}\hat{r}$$

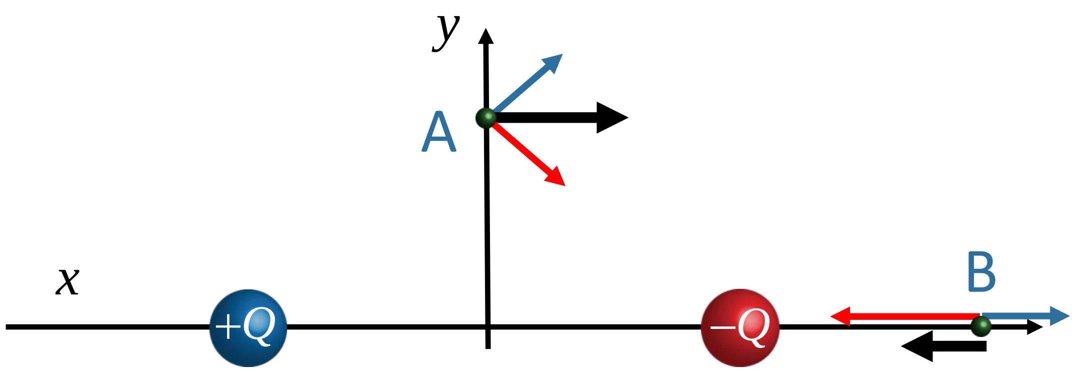
Superposition

$$\vec{E} = \sum_i k\frac{Q_i}{r_i^2}\hat{r}_i$$



Field points away from positive charges.
Field points toward negative charges.

Check Point 1



Two equal, but opposite charges are placed on the x axis. The positive charge is placed to the left of the origin and the negative charge is placed to the right, as shown in the figure above.

What is direction at point A

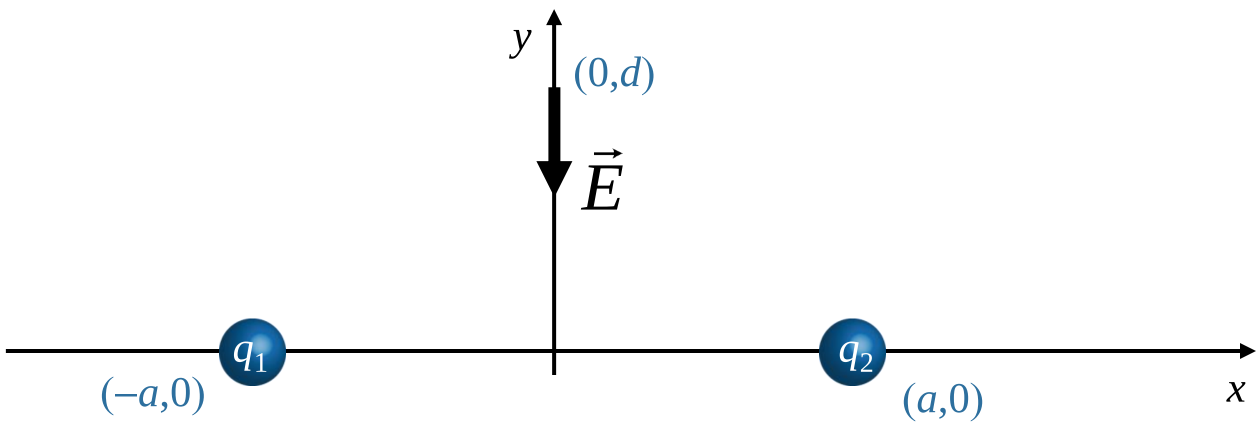
a) Up b) down c) Left d) Right e) zero

What is direction at point B

a) Up b) down c) Left d) Right e) zero

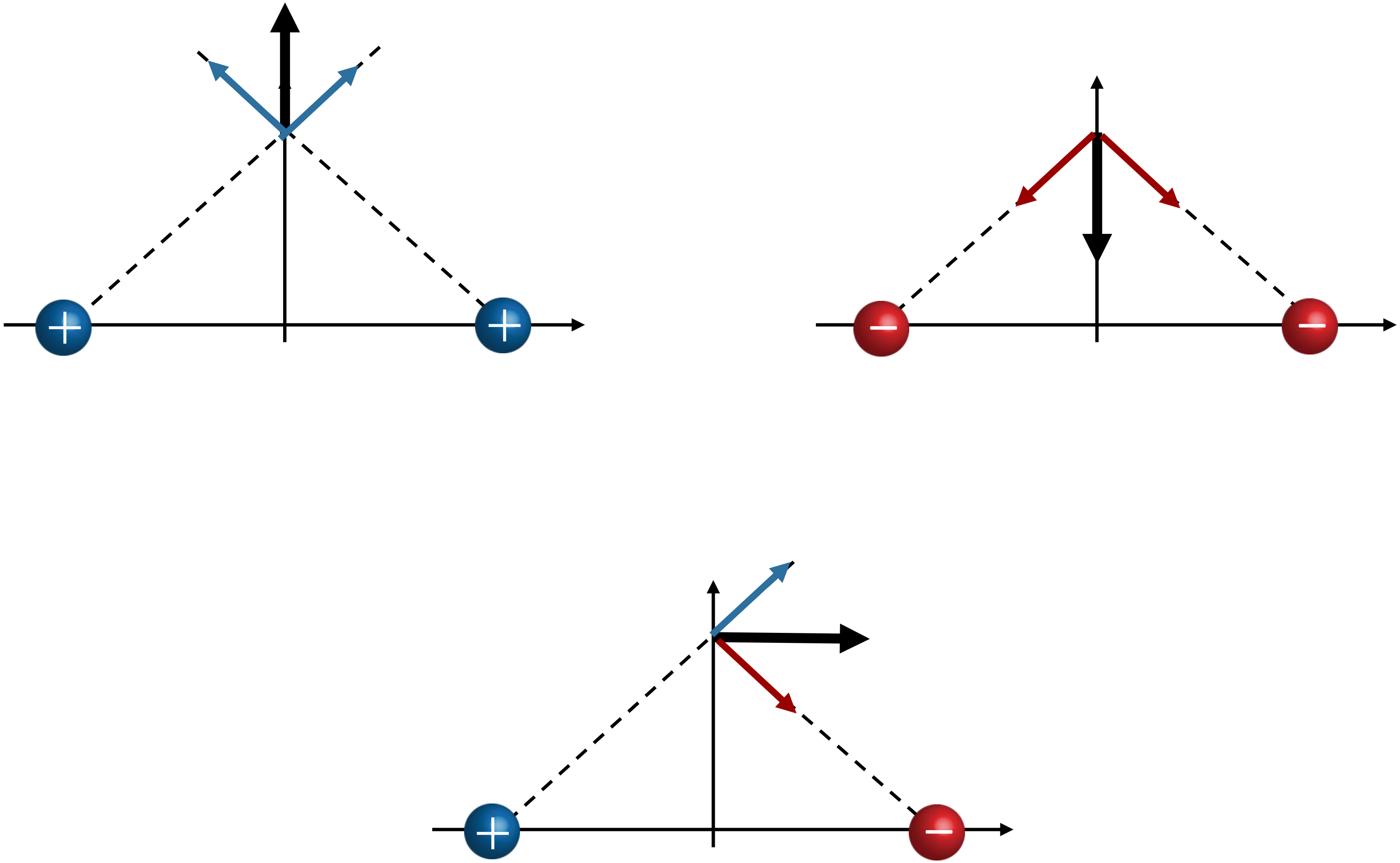
Two Charges

Two charges q_1 and q_2 are fixed at points $(-a,0)$ and $(a,0)$ as shown. Together they produce an electric field at point $(0,d)$ which is directed along the negative *y*-axis.

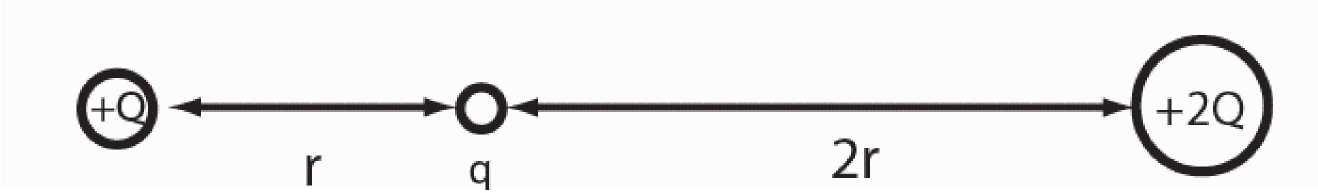


Which of the following statements is true:

A) Both charges are negative
B) Both charges are positive
C) The charges are opposite
D) There is not enough information to tell how the charges are related



Check Point 2



A positive test charge q is released from rest at distance r away from a charge of +Q and a distance 2r away from a charge of +2Q.

How will the charge q accelerate immediately after it is released?

Left Right Still Other

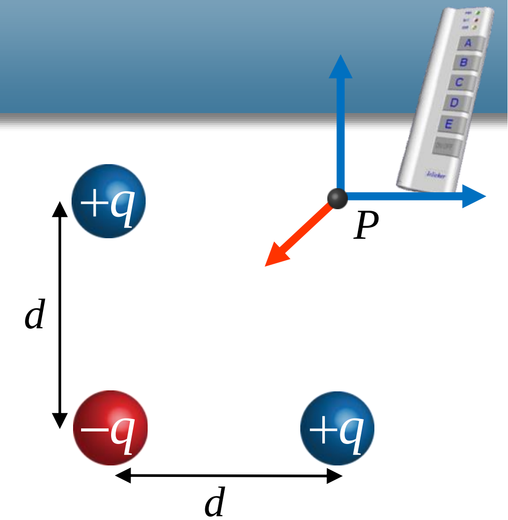
A B C D

(A Left) According to coulomb's law, distance is inverse squared while the charge is linear so the force enacted by the 2Q charge is less than the one from Q charge.

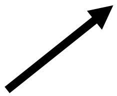
(B Right) The electric field of +Q is greater than that of +2Q as the electric field is related to the inverse SQUARE of distance and only directly related to charge.

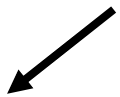
(C Still) I would say it will stay still since it is closer to the q charge but then the 2q+ charge will have about the same force but opposite direction.

Example



What is the direction of the electric field at point P, the unoccupied corner of the square?

A) 

B) 

C) $E = 0$

D) Need to know d

E) Need to know d & q

Calculate E at point P.

$$\vec{E} = \sum_i k \frac{Q_i}{r_i^2} \hat{r}_i$$

$$E_x = k\left(\frac{q}{d^2} - \frac{q}{\left(\sqrt{2}d\right)^2}\cos\frac{\pi}{4}\right)$$

$$E_y = k\left(\frac{q}{d^2} - \frac{q}{\left(\sqrt{2}d\right)^2}\sin\frac{\pi}{4}\right)$$

Charge Density

Linear $(\lambda = Q/L)$ Coulombs/meter

Surface $(\sigma = Q/A)$ Coulombs/meter^2

Volume $(\rho = Q/V)$ Coulombs/meter^3

Some Geometry

$$A_{sphere} = 4\pi R^2 \qquad A_{cylinder} = 2\pi RL$$

$$V_{sphere} = \tfrac{4}{3}\pi R^3 \qquad V_{cylinder} = \pi R^2 L$$

What has more net charge?.

A) A sphere w/ radius 4 meters and volume charge density ρ = 2 C/m^3

B) A sphere w/ radius 4 meters and surface charge density σ = 2 C/m^2

C) Both A) and B) have the same net charge.

$$Q_A = \rho V = \frac{4}{3}\pi R^3 \rho = \frac{4}{3}\pi 4^3 (2)$$

$$Q_B = \sigma A = 4\pi R^2 \sigma = 4\pi 4^2 (2) = \pi 4^3 (2)$$

Continuous Charge Distributions

Summation becomes an integral (be careful with vector nature)

$$\vec{E} = \sum_i k \frac{Q_i}{r_i^2} \hat{r}_i \quad \Longrightarrow \quad \vec{E} = \int k \frac{dq}{r^2} \hat{r}$$

WHAT DOES THIS MEAN ?

Integrate over all charges (dq)

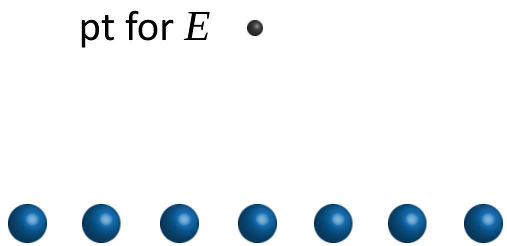
r is vector from dq to the point at which E is being calculated

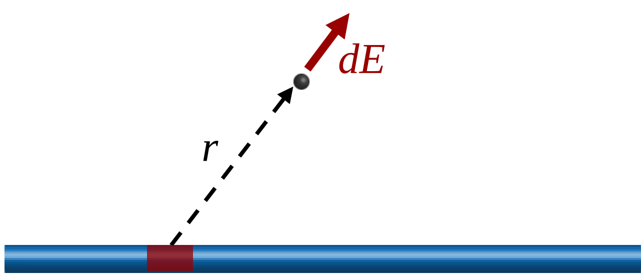
Linear Example:

$$\lambda = Q/L$$

pt for E

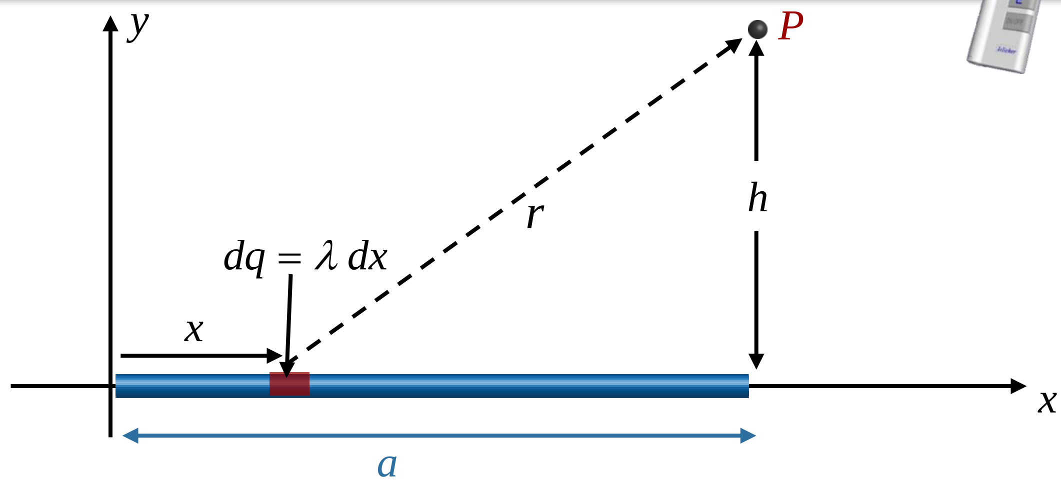
charges





$dq = \lambda\, dx$

Calculation



Charge is uniformly distributed along the x-axis from the origin to $x = a$. The charge density is λ C/m. What is the x-component of the electric field at point P: $(x,y) = (a,h)$?

We know:

$$\vec{E} = \int k \frac{dq}{r^2} \hat{r}$$

What is $\dfrac{dq}{r^2}$?

A) $\dfrac{dx}{x^2}$

B) $\dfrac{dx}{a^2 + h^2}$

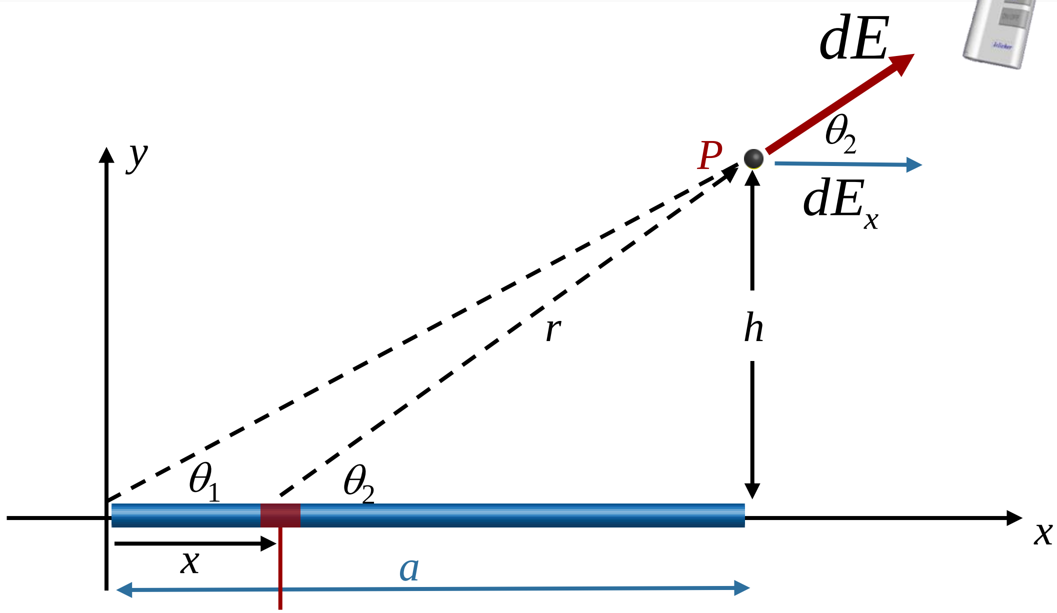
C) $\dfrac{\lambda dx}{a^2 + h^2}$

D) $\dfrac{\lambda dx}{(a - x)^2 + h^2}$

E) $\dfrac{\lambda dx}{x^2}$

Calculation

Charge is uniformly distributed along the x-axis from the origin to $x = a$. The charge density is λ C/m. What is the x-component of the electric field at point P: $(x,y) = (a,h)$?



$$dq = \lambda\, dx$$

We know:

$$\vec{E} = \int k \frac{dq}{r^2}\hat{r}$$

$$\frac{dq}{r^2} = \frac{\lambda dx}{(a-x)^2 + h^2}$$

We want:

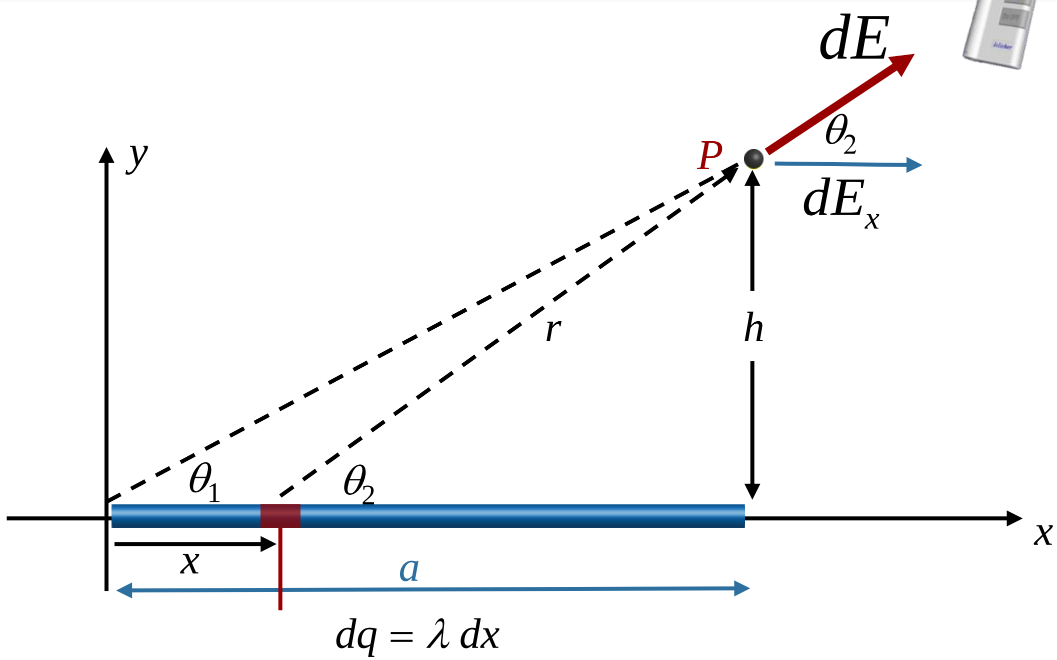
$$E_x = \int dE_x$$

What is correct expression for E_x ?

A) $\int \frac{\lambda k \cos\theta_1 dx}{(a-x)^2 + h^2}$ B) $\int \frac{\lambda k \cos\theta_2 dx}{(a-x)^2 + h^2}$ C) $\int \frac{\lambda k \sin\theta_1 dx}{(a-x)^2 + h^2}$ D) $\int \frac{\lambda k \sin\theta_2 dx}{(a-x)^2 + h^2}$

Calculation

Charge is uniformly distributed along the x-axis from the origin to $x = a$. The charge density is λ C/m. What is the x-component of the electric field at point P: $(x,y) = (a,h)$?



We know:

$$\vec{E} = \int k \frac{dq}{r^2} \hat{r}$$

$$\frac{dq}{r^2} = \frac{\lambda dx}{(a-x)^2 + h^2}$$

$$E_x = \int \frac{\lambda k \cos\theta_2 dx}{(a-x)^2 + h^2}$$

What is E_x ?

A) $\displaystyle\int_0^a \frac{\lambda k \cos\theta_2 dx}{(a-x)^2 + h^2}$

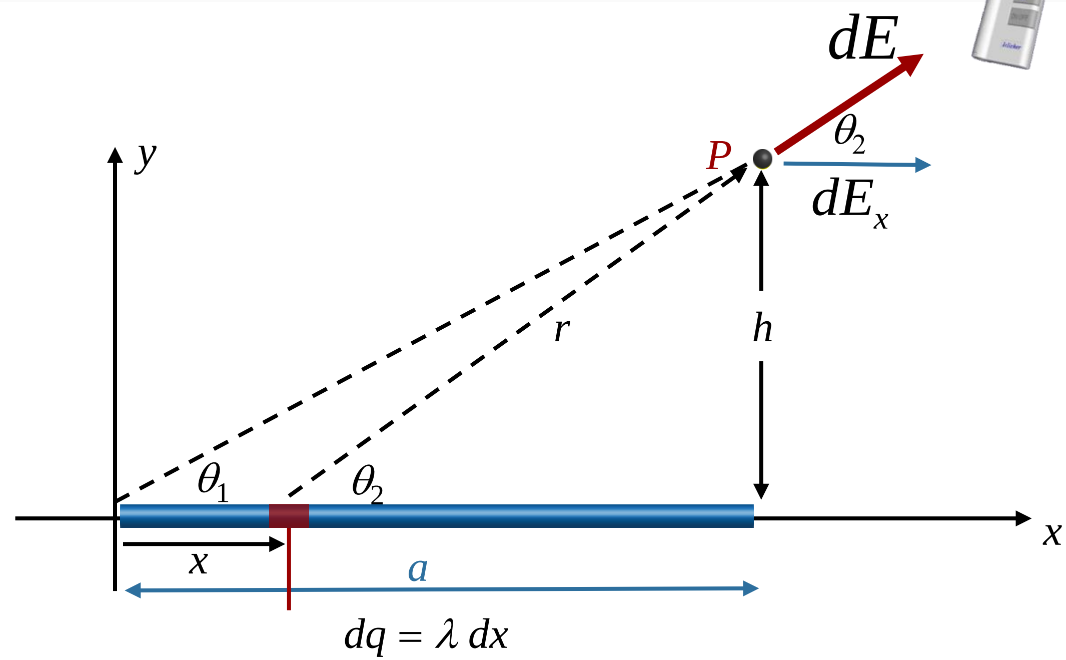
B) $\displaystyle\lambda k \cos\theta_2 \int_0^a \frac{dx}{h^2 + (x-a)^2}$

C) A and B are both OK

$\cos\theta_2$ DEPENDS ON x!

Calculation

Charge is uniformly distributed along the x-axis from the origin to $x = a$. The charge density is λ C/m. What is the x-component of the electric field at point P: $(x,y) = (a,h)$?



$$\vec{E} = \int k \frac{dq}{r^2} \hat{r}$$

We know:

$$\frac{dq}{r^2} = \frac{\lambda dx}{(a-x)^2 + h^2}$$

$$E_x = \int dE \cos\theta_2$$

What is $\cos\theta_2$?

A) $\dfrac{x}{\sqrt{a^2+h^2}}$

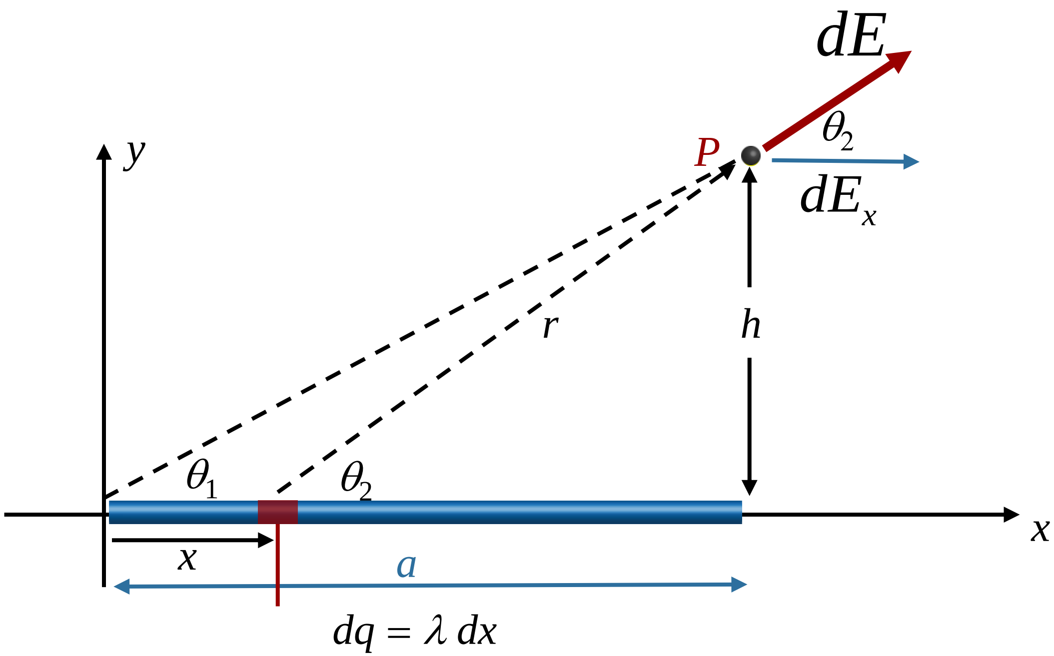
B) $\dfrac{a-x}{\sqrt{(a-x)^2+h^2}}$

C) $\dfrac{a}{\sqrt{a^2+h^2}}$

D) $\dfrac{a}{\sqrt{(a-x)^2+h^2}}$

Calculation

Charge is uniformly distributed along the x-axis from the origin to $x = a$. The charge density is λ C/m. What is the x-component of the electric field at point P: $(x,y) = (a,h)$?



We know: $$\vec{E} = \int k \frac{dq}{r^2}\hat{r}$$

$$\frac{dq}{r^2} = \frac{\lambda dx}{(a-x)^2 + h^2} \qquad E_x = \int dE \cos\theta_2 \qquad \cos\theta_2 = \frac{a-x}{\sqrt{(a-x)^2 + h^2}}$$

Putting it all together

$$E_x(P) = \lambda k \int_0^a \frac{a-x}{\left((a-x)^2 + h^2\right)^{3/2}} dx \quad \Longrightarrow \quad E_x(P) = \frac{\lambda k}{h}\left(1 - \frac{h}{\sqrt{h^2 + a^2}}\right)$$

Takeaways

Electric field

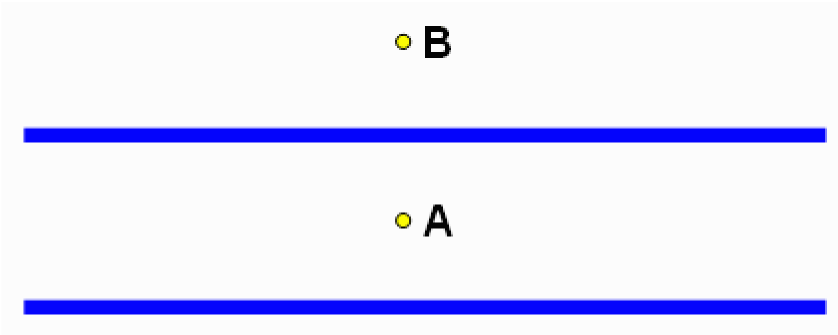
- Definition
- How to add electric fields due to multiple charges?
- Separating Ex and Ey

Concept of charge density

- Discrete --> continuous charge density
- Setting up integral for determining net electric field

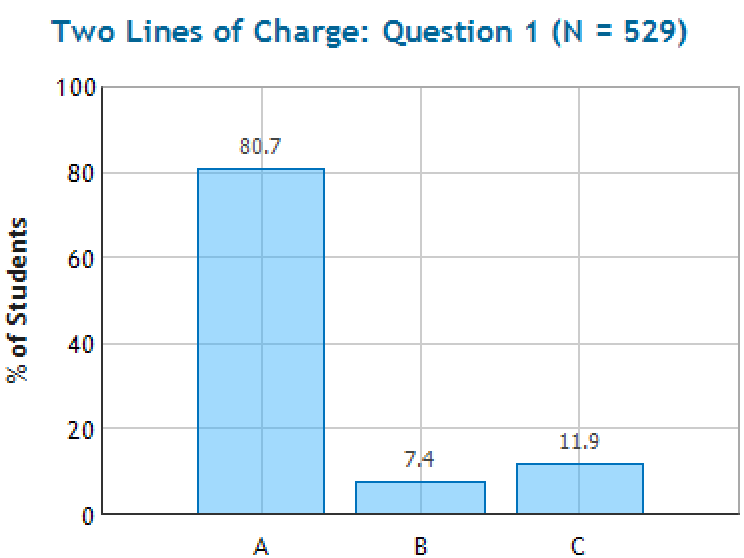
CheckPoint

Two infinite lines of charge are shown below.



Both lines have identical charge densities $+\lambda$ C/m. Point A is equidistant from both lines and Point B is located a above the top line as shown. How does E_A, the magnitude of the electric field at point A, compare to E_B, the magnitude of the electric field at point B?

- $E_A < E_B$
- $E_A = E_B$
- $E_A > E_B$



Electric Field at point A cancels out to be zero and electric field at point B experiences E field from both line to move upward.