Math 241 Calculus III (Prof. Honold) **Midterm Exam 1** 2021/10/20

Name: _____ Student ID: _____ Group A

For each of the following problems, find the correct answer (tick as appropriate!). No justifications are required. Each problem has exactly one correct solution, which is worth 1 mark. Incorrect solutions (including no answer, multiple answers, or unreadable answers) will be assigned 0 marks; there are no penalties.

1. The tangent to $f(t)=(t^5,t^3,t)$ in the point $(1,1,1)$ meets the plane $x-y+z=0$ in
☐ $(0,\frac{2}{3},\frac{2}{3})$ ☐ $(-\frac{2}{3},0,\frac{2}{3})$ ☐ $(\frac{2}{3},\frac{2}{3},0)$ ☐ $(\frac{2}{3},0,-\frac{2}{3})$ ☐ no point.

2. The arc length of the curve $g(t)=\big(t\sin(18t),t\cos(18t),4t^{3/2}\big),\ t\in[0,3]$ is
☐ 10 ☐ 36 ☐ 38 ☐ 84 ☐ 148

3. For a C^1-curve $\mathbf{u}\colon I\to\mathbb{R}^3\setminus\{\mathbf{0}\}$ and $t\in I$, the derivative $(\mathrm{d}/\mathrm{d}t)\,|\mathbf{u}(t)|^3$ is equal to
☐ $3\,|\mathbf{u}(t)|^2\,\mathbf{u}'(t)$ ☐ $3\,|\mathbf{u}(t)|^2$ ☐ $3\,|\mathbf{u}(t)|\,\mathbf{u}(t)\cdot\mathbf{u}'(t)$ ☐ $\frac{3}{2}\,|\mathbf{u}(t)|$ ☐ $3\,\mathbf{u}(t)\cdot\mathbf{u}'(t)$

4. The range of a parametric space curve $f\colon\mathbb{R}\to\mathbb{R}^3$ with nonzero curvature and $f''(t)=\mathbf{w}\in\mathbb{R}^3\setminus\{\mathbf{0}\}$ (i.e., a nonzero constant vector) is
☐ a line ☐ an ellipse ☐ a parabola ☐ a hyperbola ☐ non-planar

5. The volume of the pyramid ("tetrahedron") with vertices $(2,0,-1),(3,-1,0),(0,a,-1),(0,3,1)$ is equal to 1 for
☐ $a=-3$ ☐ $a=-2$ ☐ $a=-1$ ☐ $a=0$ ☐ $a=1$

6. The inverse matrix of $\begin{pmatrix}5&6\\6&7\end{pmatrix}$ is
☐ $\begin{pmatrix}-5&6\\6&-7\end{pmatrix}$ ☐ $\begin{pmatrix}5&-6\\-6&7\end{pmatrix}$ ☐ $\begin{pmatrix}-7&-6\\-6&-5\end{pmatrix}$ ☐ $\begin{pmatrix}-7&6\\6&-5\end{pmatrix}$ ☐ $\begin{pmatrix}7&-6\\-6&5\end{pmatrix}$

7. The matrix $\frac{1}{\sqrt{2}}\begin{pmatrix}1&1\\1&-1\end{pmatrix}$ describes a reflection whose axis in polar coordinates is given by
☐ $\theta=0°$ ☐ $\theta=22.5°$ ☐ $\theta=45°$ ☐ $\theta=72°$ ☐ $\theta=90°$

8. The maximum rank of $\mathbf{A}\in\mathbb{R}^{4\times5}$ with entries ±1 and exactly two entries equal to -1 is
☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5

9. The linear system $\begin{aligned}x_1+x_2+x_3&=b\\x_1-x_3&=-1\\3x_1-2x_2-7x_3&=1\end{aligned}$ has a solution if
☐ $b=0$ ☐ $b=-3$ ☐ $b=3$ ☐ $b=-1$ ☐ $b=1$

10. The distance from the point $(1,-1,1)$ to the line $x_1-x_2=2x_2-x_3=1$ is
☐ $\frac{1}{2}\sqrt{14}$ ☐ $\frac{11}{4}$ ☐ $\frac{1}{2}\sqrt{11}$ ☐ $\sqrt{5}$ ☐ $\frac{7}{2}$

Time allowed: 40 min CLOSED BOOK ***Good luck!***