Math 231 Test #2

Your Name:______________________

Please circle your discussion group (2 pt)

1 Yang Yihong A410	4 Zhang Junwei A404	7 Liu Yuanzhe A424
2 Xu Yixiao A425	5 Liang Jun A408	8 Jaden Peterson Wen A421
3 Dai Ruiqi A426	6 She Yuxuan A423	

- No notes, books or electronics during the exam.
- Do not open this test booklet until a proctor says start.
- For all free response questions, show work that justifies your answer.
- Raise your hand if you have a clarification question.
- Scratch paper is provided. You can ask for more if needed.
- Do not leave early: this disturbs others. If you finish your test early, check your work or just relax.
- Quit working when the test ends and hand your test booklet to proctors.

1. (15 points, 3 points each) Determine whether the statement is true or false. Circle the right answer.

   (a) Every bounded and monotonic sequence is convergent. (True or False)

   (b) If $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are two divergent series, $c_n = a_n \times b_n$, then $\sum_{n=1}^{\infty} c_n$ must be divergent. (True or False)

   (c) If $f(x) = 2x - x^2 + \frac{1}{3}x^3 - \cdots$ converges for all x, then $f'''(0) = 2$ (True or False)

   (d) Every function has power series representation at any point. (True or False)

   (e) If $a_n > 0$ and $\lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} < 1$, then $a_n = 0$. (True or False)

Question	1	2	3	4	5	6	7	8	9	Total
Points	15	10	15	10	10	12	8	6	14	100
Score										

2. (10 points) (1) If nth partial sum of a series $\sum_{n=1}^{\infty} a_n$ is $s_n = 2 - n6^{-n}$, find a_n and the value of $\sum_{n=1}^{\infty} a_n$

(2) Find the sum of the series $\sum_{n=1}^{\infty} \frac{1}{2^n} + \frac{(-1)^n}{3^n}$

3. (15 points) Test the convergence of the following series. Please state out the name of the test you used.

(a) $\frac{1000}{1!} + \frac{1000}{2!} + \frac{1000}{3!} + \cdots + \frac{1000}{n!} + \cdots$

(b) $\frac{1000}{1}+\frac{1000\cdot 1001}{1\cdot 3}+\frac{1000\cdot 1001\cdot 1002}{1\cdot 3\cdot 5}+\cdots$

(c) $\sum_{n=2}^{\infty}\frac{1}{\mathrm{nln}\, n}$

4. (10 points) (1) Test the convergence of the series $\sum_{n=2}^{\infty}\frac{1}{\ln(n!)}$. Please state out the name of the test you used. $(n! = 1\times 2\times 3\times \ldots (n-1)\times n.)$

(2) find p to make $\sum_{n=2}^{\infty}\frac{1}{\mathrm{nln^p}\, n}$ converge

5. (10 points) Use series to approximate $\int_0^1 \sqrt{1+x^4}\,dx$, the error with in 0.005.

6. (12 points) Find a **power series** representation for each of the following functions. Use summation notation and give the radius of convergence as $|x-a|<R$. (please simplify your answer as the $\sum c_n(x-a)^n$)

(a) $\frac{x}{(1+2x)^2}$

(b) $ln(1+3x^2)$

7. (8 points) Find a **power series** representation for $f(x) = e^{-6x}$ about $x = -4$. Use summation notation and give the radius of convergence as $|x - a| < R$..

8. (6 points) Calculate the series sum:

(a) $\frac{1}{0!} - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \cdots$

(b) $\sum_{n=0}^{\infty} \frac{3^n}{5^n n!}$

9. (14 points) For the parametric equation $x = 3t^2, y = 2t^3$,
(a) Find dy/dx and d^2y/dx^2.

(b) Find the area enclosed by the curve and x- $axix$ for $0 \leq t \leq 2$.

(c) Find the length of the curve, for $0 \leq t \leq 2$. Simplify the integral but do not need to evaluate the integral.

Function	Maclaurin Series Expansion
e^x	$\sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$
$\sin x$	$\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$
$\cos x$	$\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$
$\ln(1+x)$	$\sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots \quad (\text{for } \lvert x\rvert < 1)$
$(1+x)^k$	$\sum_{n=0}^{\infty} \binom{k}{n} x^n = 1 + kx + \frac{k(k-1)}{2!} x^2 + \cdots \quad (\text{for } \lvert x\rvert < 1)$