ECE 313: Problem Set 8: Problems

Due: Saturday, Nov 16 at 11:59:00 p.m.
Reading: *ECE 313 Course Notes,* Sections 3.6.2 - 3.7

Note on reading: For most sections of the course notes there are short answer questions at the end of the chapter. We recommend that after reading each section you try answering the short answer questions. Do not hand in; answers to the short answer questions are provided in the appendix of the notes.

Note on turning in homework: You must upload handwritten homework to BB. No typeset homework will be accepted. No late homework will be accepted. Please write on the top right corner of the first page:
NAME AS IT APPEARS ON BB
NETID
SECTION
PROBLEM SET #
Page numbers are encouraged but not required. Five points will be deducted for improper headings.

1. **[Gaussian Distribution]**
   Suppose X is a $N(2, 16)$ random variable. Find
   $$P\{(X-3)(X+4) \geq 0\}.$$

2. **[Binary Communication in Gaussian Noise]**
   A wireless binary communication system consists of a transmitter and a receiver on which we transmit bits of information. In each transmission interval, the transmitter emits a radio signal corresponding to which bit is being transmitted, which is attenuated by the air and is received by the receiver together with noise. At the receiver, we perform an operation called "matched-filtering", which results in the output
   $$Y = \alpha\nu + X,$$
   where ν takes the value $+1$ if bit 1 is transmitted, and -1 if bit 0 is transmitted, α is the attenuation factor of the channel from the transmitter to the receiver, and the noise X is a Gaussian random variable with mean μ and variance σ^2.

   Now suppose that $\alpha = 0.1$, $\mu = 0$ and $\sigma^2 = 4$.

(a) Find the pdf of Y when bit 1 is transmitted.

(b) At the receiver, we decide that 1 was transmitted if $Y \geq 0$, and we decide that 0 was transmitted if $Y < 0$. Find the probability of error when a 1 is transmitted in terms of the Q function.

3. **[Gaussian versus Poisson Approximation for Binomial Distribution]**
A communication receiver recovers a block of $n = 10^5$ bits. It is known that each bit in the block can be in error with probability 10^{-4}; independently of whether other bits are in error. Let X be the number of bit errors.

(a) Write down an exact expression for $P\{X = 15\}$. You do not need to compute a numerical value for this probability.

(b) Determine an approximate value for $P\{X = 15\}$ via the Gaussian approximation with continuity correction.

(c) Solve part (b) using the Poisson approximation of a binomial distribution.

4. **[Gaussian Approximation]**
Suppose a *fair* coin is flipped 100 times, and

$$A = \{|\text{ (number of times heads shows) - (number of times tails shows) }| \geq 6\}.$$

(a) Let S denote the number of heads. Express A in terms of S.

(b) Using the Gaussian approximation with the continuity correction, express the approximate value of $P(A)$ in terms of the Q function.

5. **[Maximum Likelihood Estimation]**
Suppose that X has the following pdf:

$$f_\theta(u) = \begin{cases} \left(\frac{u}{\theta}\right) e^{-\frac{u^2}{2\theta}} & u \geq 0 \\ 0 & u < 0 \end{cases}$$

with parameter $\theta > 0$. Suppose the value of θ is unknown and it is observed that $X = 10$. Find the maximum likelihood estimate, $\widehat{\theta}_{ML}$, of θ.