Physics 212
Lecture 8

Today's Concept:

Capacitors

(Capacitors in a circuits, Dielectrics, Energy in capacitors)

So what exactly does capacitance mean? What does a capacitor do in real life?

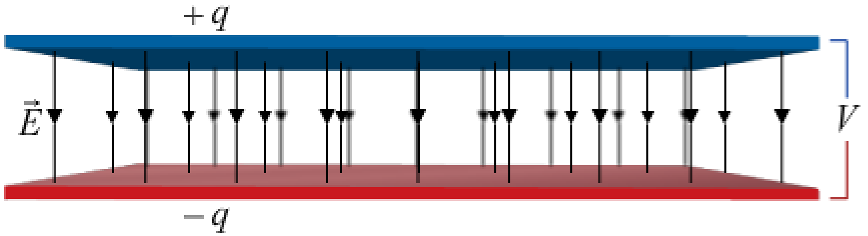
Energy in Capacitors (from lect 7)

Energy Stored in Capacitors

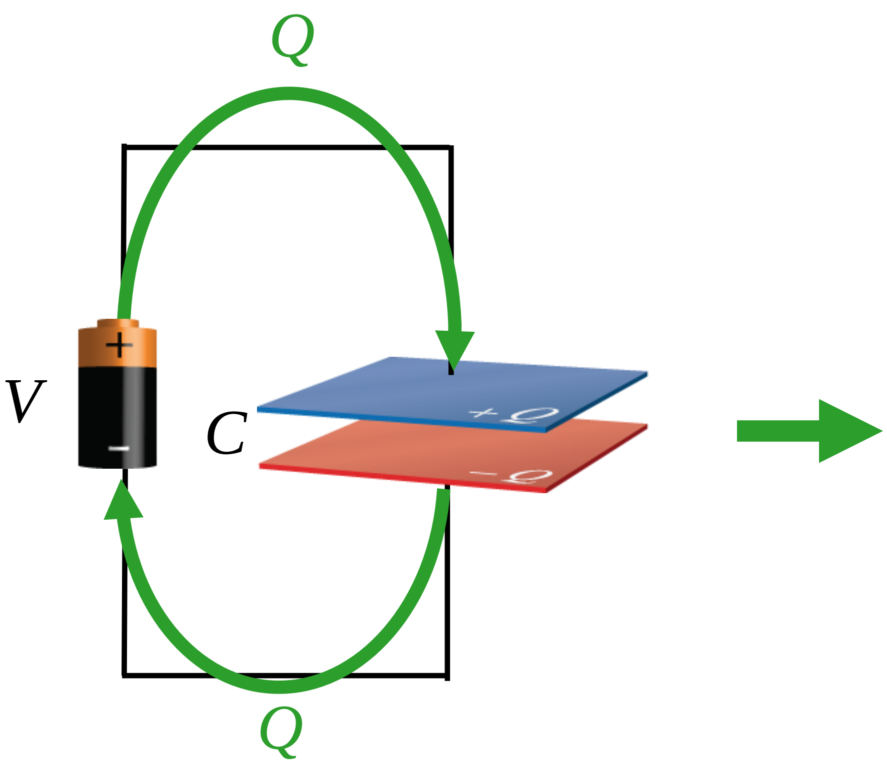
$$U = \frac{1}{2} QV$$ or $$U = \frac{1}{2}\frac{Q^2}{C}$$ or $$U = \frac{1}{2} CV^2$$

Energy Density

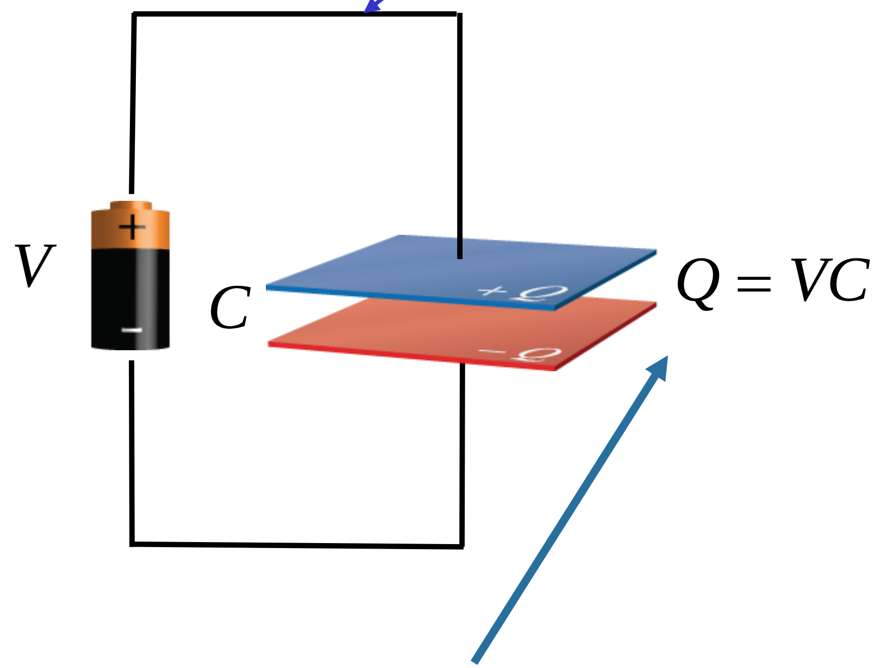
$$u = \frac{1}{2}\varepsilon_o E^2$$



Simple Capacitor Circuit

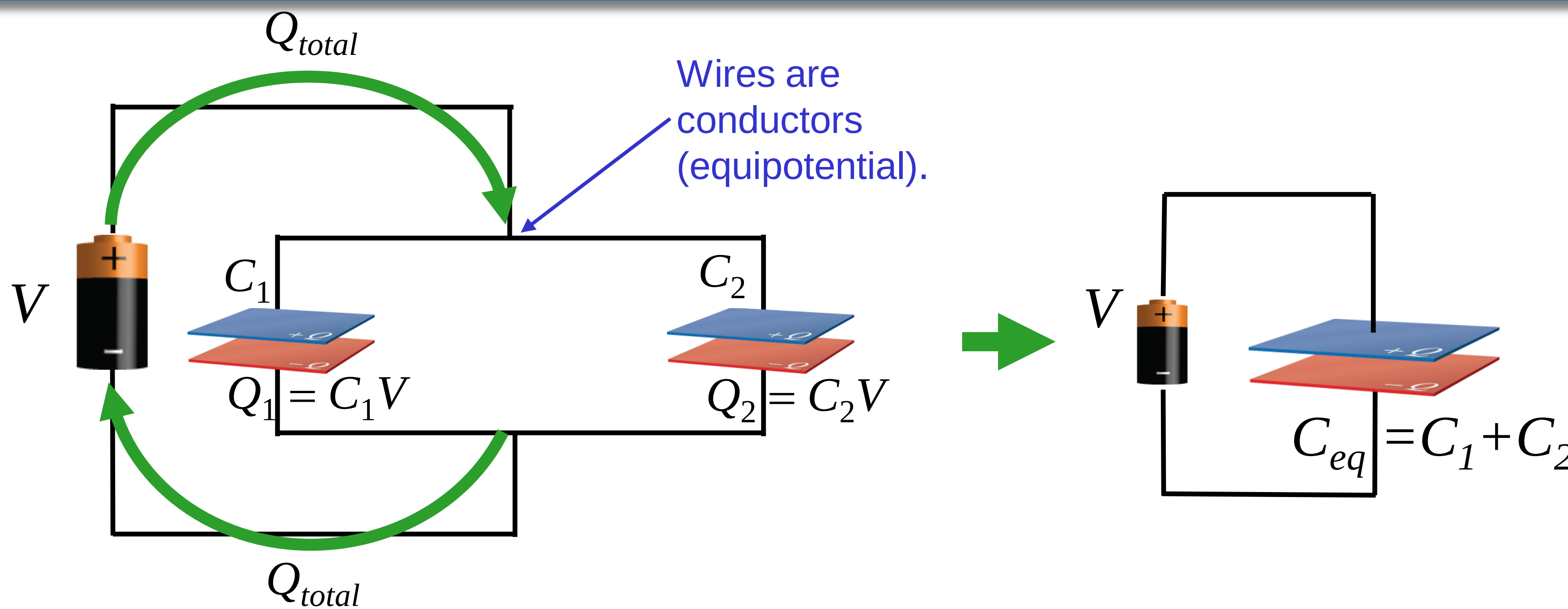


Wires are conductors (equipotential).



This "Q" really means that the battery has moved charge Q from one plate to the other, so that one plate holds $+Q$ and the other $-Q$.

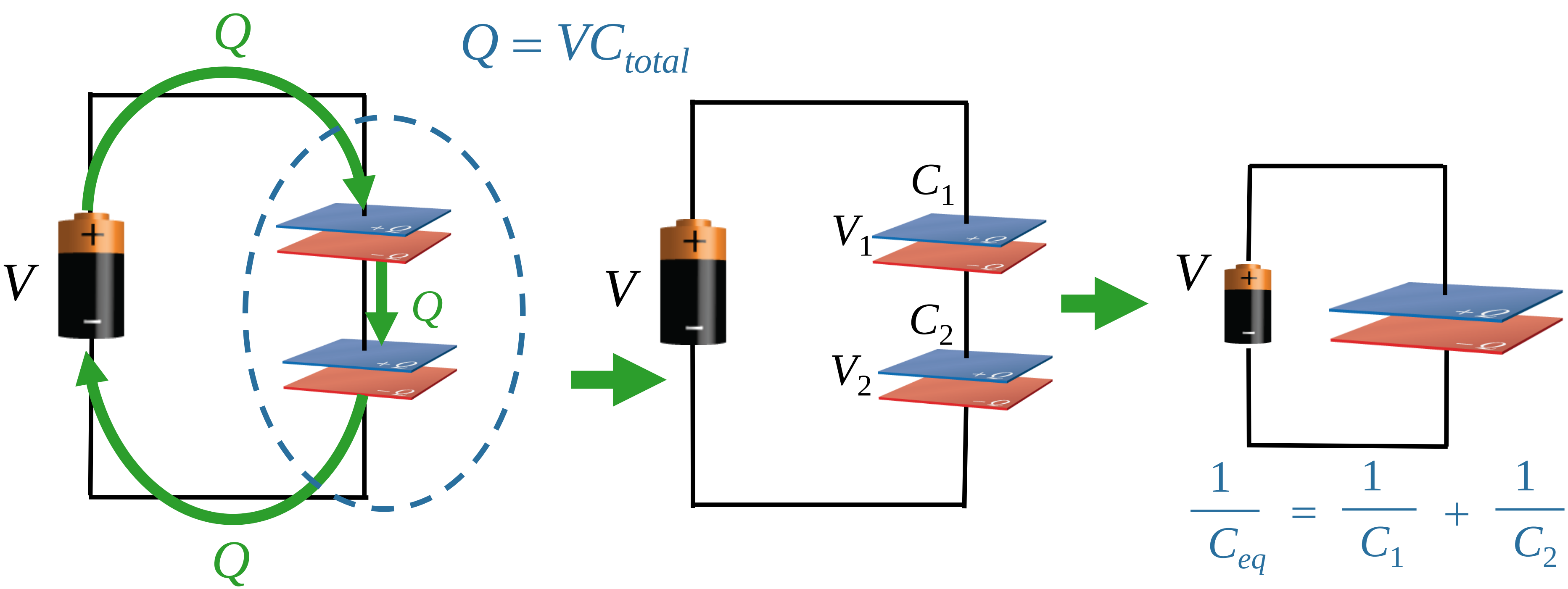
Parallel Capacitor Circuit



Key point: V is the same for both capacitors

Key Point: $Q_{total} = Q_1 + Q_2 = VC_1 + VC_2 = V\underbrace{(C_1 + C_2)}_{C_{total} = C_1 + C_2}$

Series Capacitor Circuit



Key point: Q is the same for both capacitors

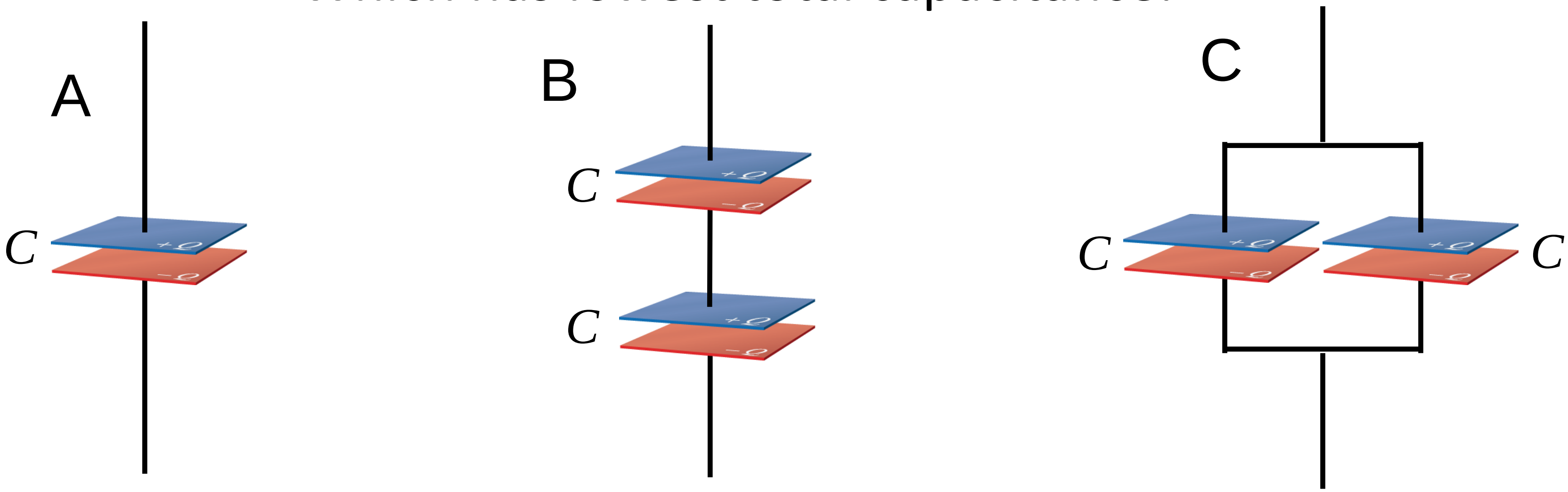
Key point: $Q = VC_{total} = V_1 C_1 = V_2 C_2$

Also: $V = V_1 + V_2$ $\longrightarrow$ $Q/C_{total} = Q/C_1 + Q/C_2$

$$\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2}$$

Check Point 1

Which has lowest total capacitance:

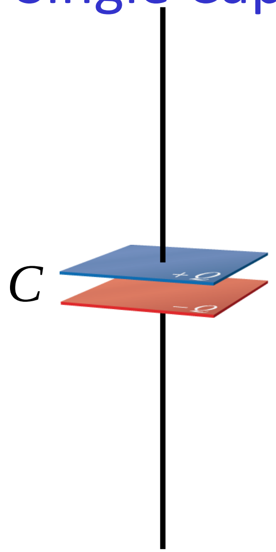


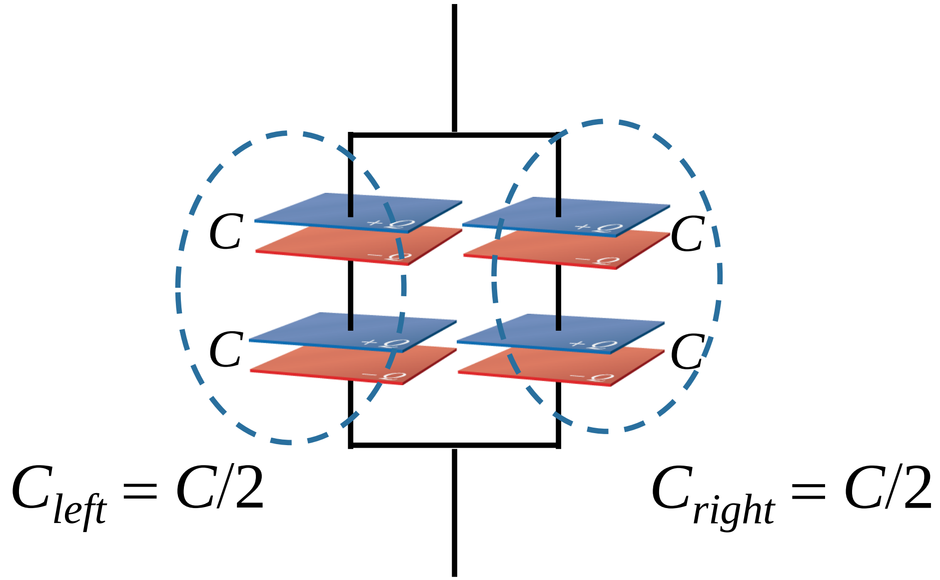
Check Point 2

Which has lowest total capacitance?

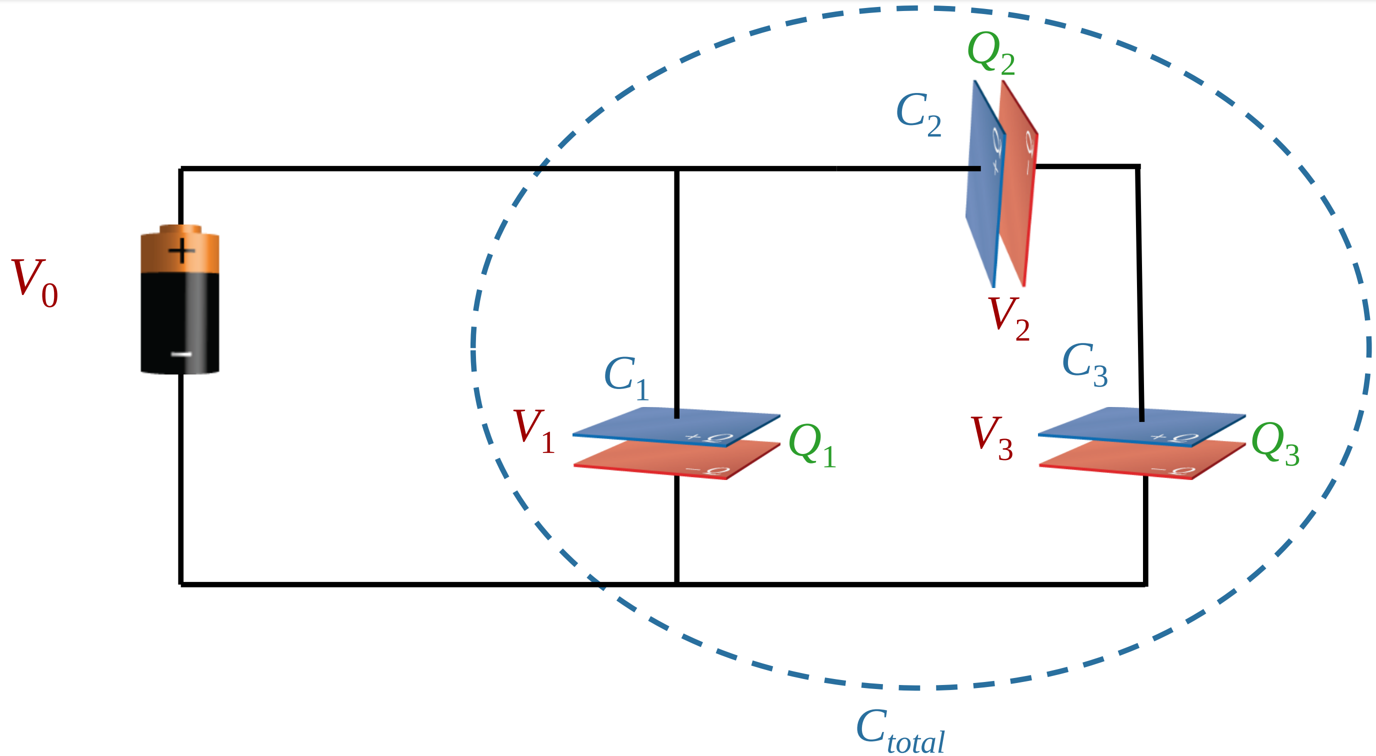
A) Single Capacitor B) 4 Capacitors C) Same

:





Similar to CheckPoint 3

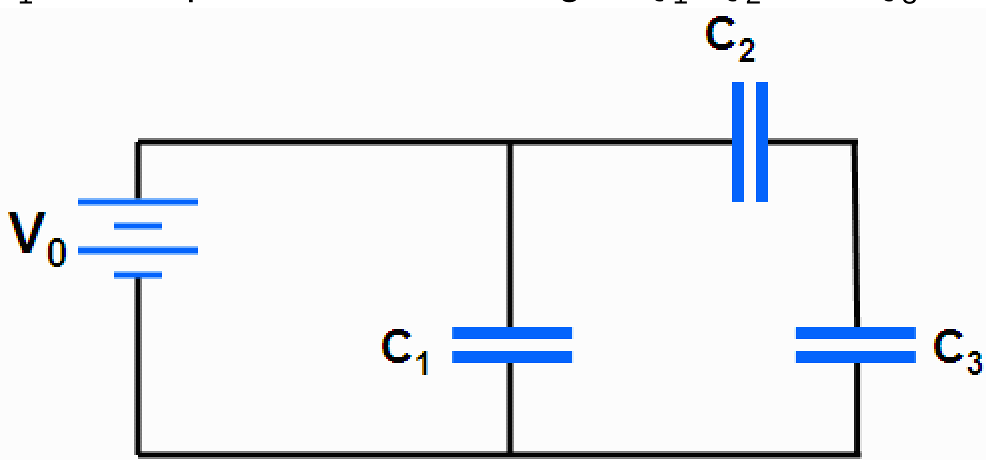


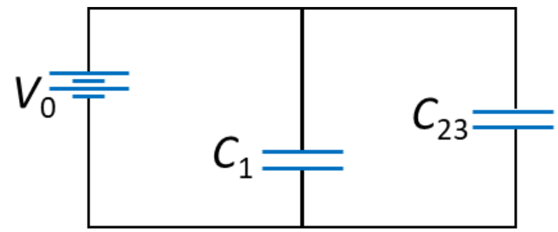
Which of the following is NOT necessarily true:

A) $V_0 = V_1$
B) $C_{total} > C_1$
C) $V_2 = V_3$
D) $Q_2 = Q_3$
E) $V_1 = V_2 + V_3$

Check Point 3

A circuit consists of three unequal capacitors C_1, C_2, and C_3 which are connected to a battery of voltage V_0. The capacitance of C_2 is twice that of C_1. The capacitance of C_3 is three times that of C_1. The capacitors obtain charges Q_1, Q_2, and Q_3.





A. $Q_1 > Q_3 > Q_2$ **B.** $Q_1 > Q_2 > Q_3$ **C.** $Q_1 > Q_2 = Q_3$ **D.** $Q_1 = Q_2 = Q_3$ **E.** $Q_1 < Q_2 = Q_3$

1. : $Q_2 = Q_3$ (capacitors in series)

2. How about Q_1 vs. Q_2 and Q_3? Calculate C_{23} first.

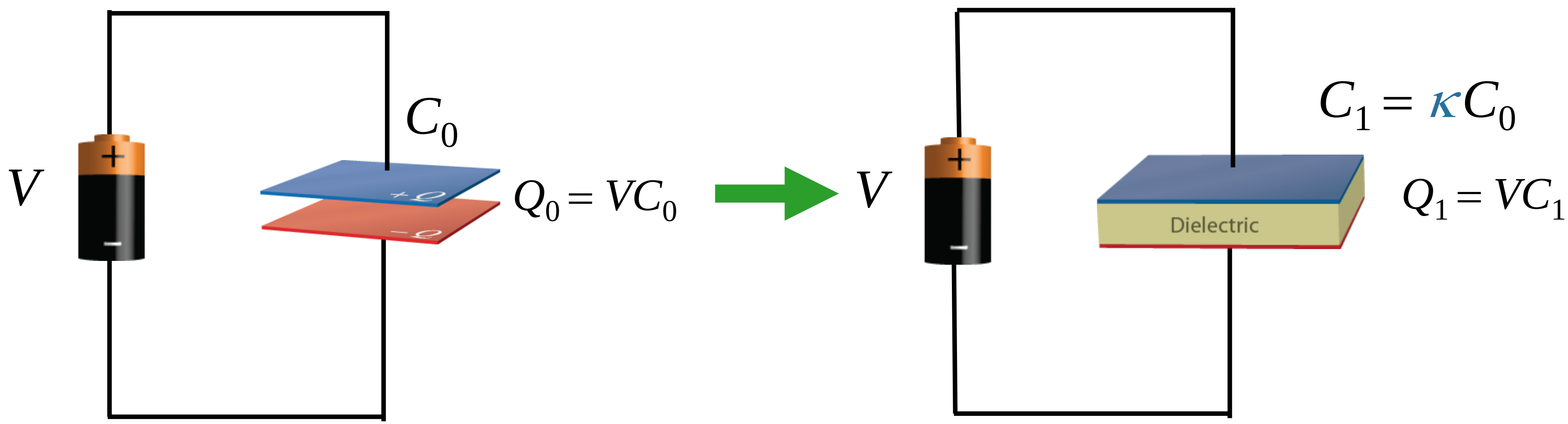
$$\frac{1}{C_{23}} = \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{2C_1} + \frac{1}{3C_1} = \frac{5}{6C_1}$$

$$\boxed{C_{23} = \frac{6}{5} C_1}$$

$$Q_1 = C_1 V_0$$

$$Q_{23} = Q_2 = Q_3 = C_{23} V_0 = \frac{6}{5} C_1 V_0$$

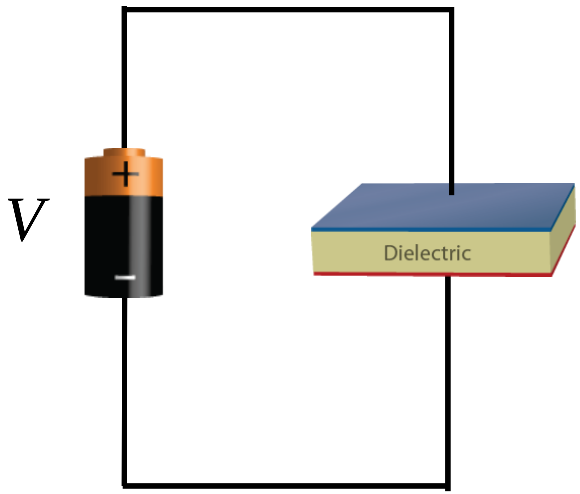
Dielectrics



By adding a dielectric, you are just making a new capacitor with larger capacitance (factor of κ)

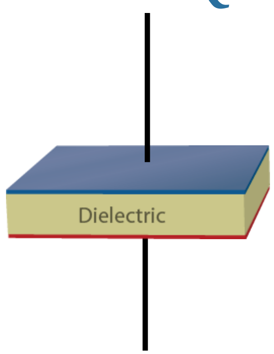
Messing with Capacitors

If connected to a battery V stays constant



$$V_1 = V$$
$$C_1 = \kappa C$$
$$\rightarrow Q_1 = C_1 V_1 = \kappa C V = \kappa Q$$

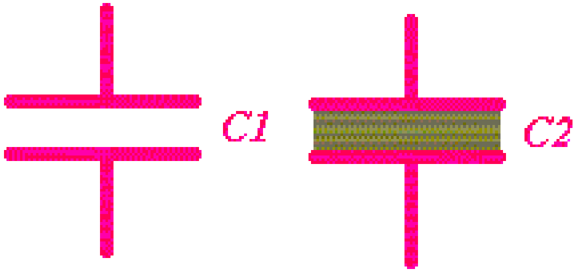
If isolated, then total Q stays constant



$$Q_1 = Q$$
$$C_1 = \kappa C$$
$$\rightarrow V_1 = Q_1 / C_1 = Q/\kappa C = V/\kappa$$

Check Point 4a

Two identical parallel plate capacitors are given the same charge Q, after which they are disconnected from the battery. Then, a dielectric is placed between the plates of C_2



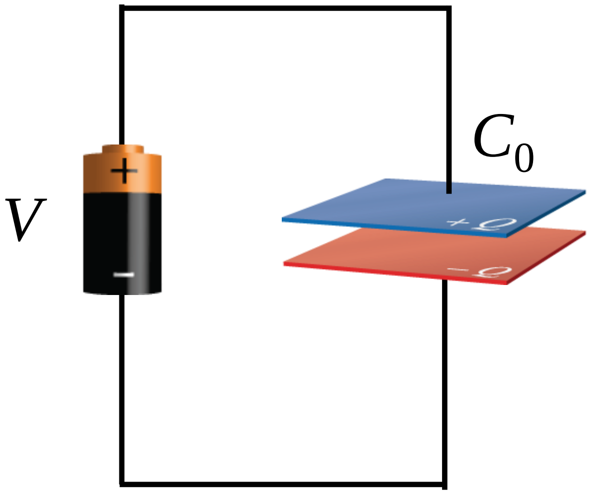
Compare the voltages of the two capacitors.

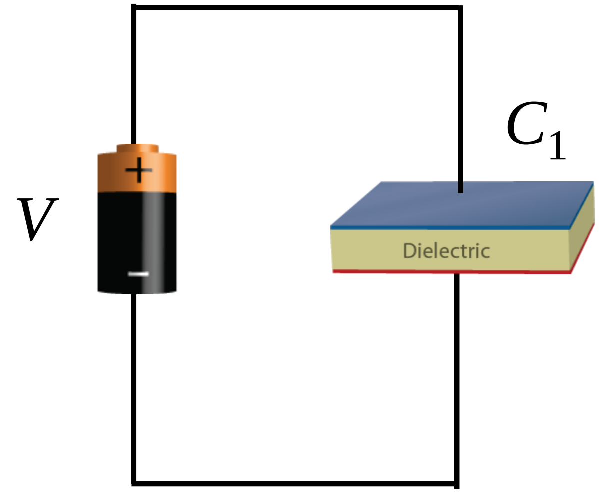
A $V_1 > V_2$ B $V_1 = V_2$ C $V_1 < V_2$

"Q is constant, C increases, C = Q/V."

Messing with Capacitors Clicker Question

Two identical parallel plate capacitors are connected to identical batteries. Then a dielectric is inserted between the plates of capacitor C_1. Compare the energy stored in the two capacitors.





A) $U_1 < U_0$

B) $U_0 = U_1$

C) $U_1 > U_0$

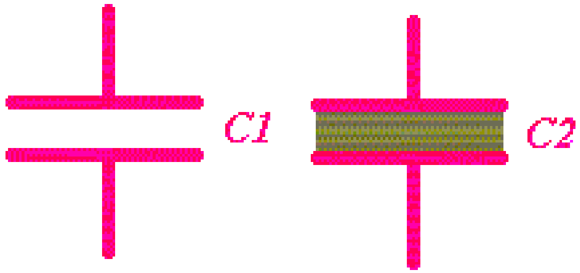
Compare using $U = {}^1/_2 CV^2$

$$U_1/U_0 = \kappa$$

Potential Energy goes UP

CheckPoint 4b

Two identical parallel plate capacitors are given the same charge Q, after which they are disconnected from the battery. Then, a dielectric is placed between the plates of C_2

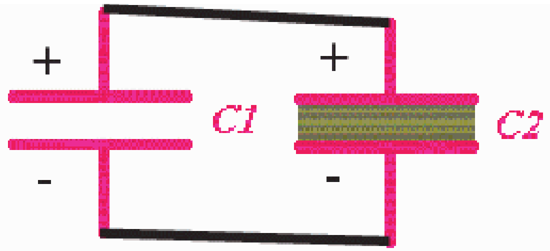


Compare the potential energy stored by the two capacitors.

A) $U_1 > U_2$ B) $U_1 = U_2$ C) $U_1 < U_2$

CheckPoint 4c

Two identical parallel plate capacitors are given the same charge Q, after which they are disconnected from the battery. After C_2 has been charged and disconnected, it is filled with a dielectric. **The two capacitors are now connected to each other by wires as shown. How will the charge redistribute itself, if at all?**



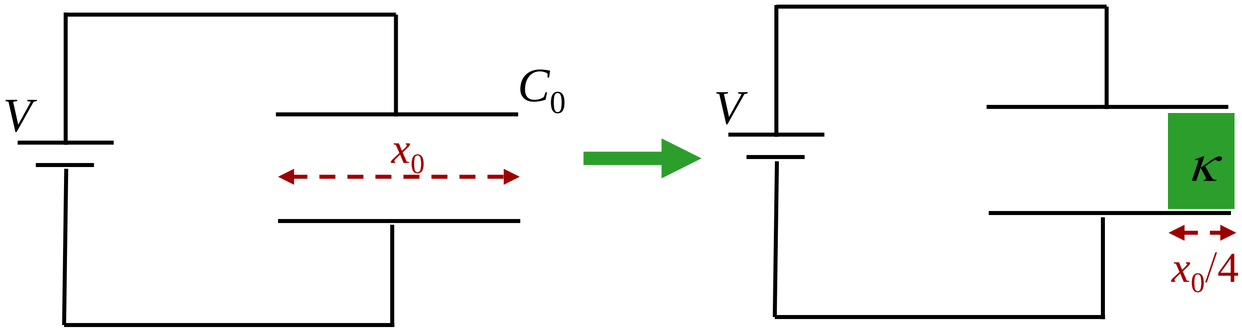
A. The charges will flow so that the charge on C_1 will become equal to the charge on C_2.
B. The charges will flow so that the energy stored in C_1 will become equal to the energy stored in C_2.
C. The charges will flow so that the potential difference across C_1 will become the same as the potential difference across C_2.
D. No charges will flow. The charge on the capacitors will remain what it was before they were connected.

V must be the same !!

Q: $$\frac{Q_1}{C_1} = \frac{Q_2}{C_2} \quad \Rightarrow \quad \boxed{Q_1 = \frac{C_1}{C_2} Q_2}$$

U: $$U_1 = \tfrac{1}{2} C_1 V^2 \qquad U_2 = \tfrac{1}{2} C_2 V^2 \quad \Rightarrow \quad \boxed{U_1 = \frac{C_1}{C_2} U_2}$$

Calculation



An air-gap capacitor, having capacitance C_0 and width x_0 is connected to a battery of voltage V.

A dielectric (κ) of width $x_0/4$ is inserted into the gap as shown.

What is Q_f, the final charge on the capacitor?

Conceptual Analysis:

$$C \equiv \frac{Q}{V}$$

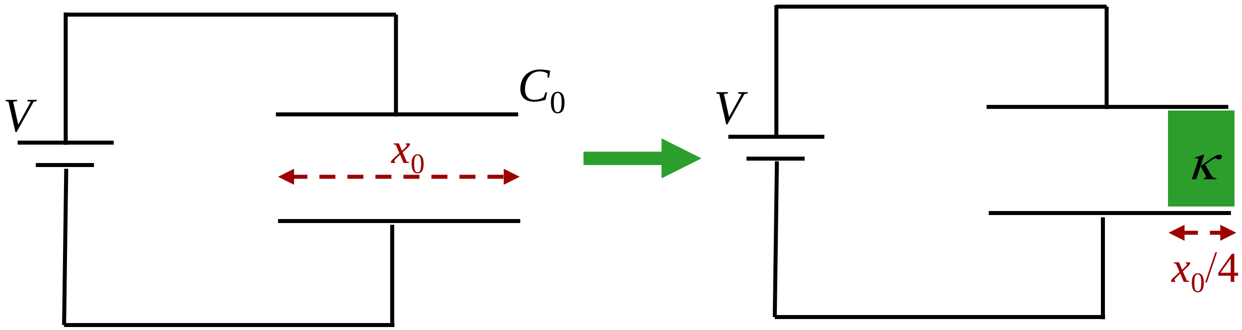
What changes when the dielectric added?

A) Only C B) only Q C) only V D) C and Q E) C and V

Adding dielectric changes the physical capacitor → C changes

V does not change and C changes → Q changes

Calculation



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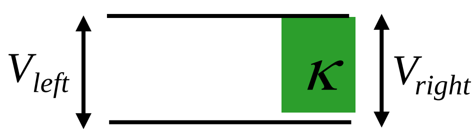
Strategic Analysis:

- Calculate new capacitance C
- Apply definition of capacitance to determine Q

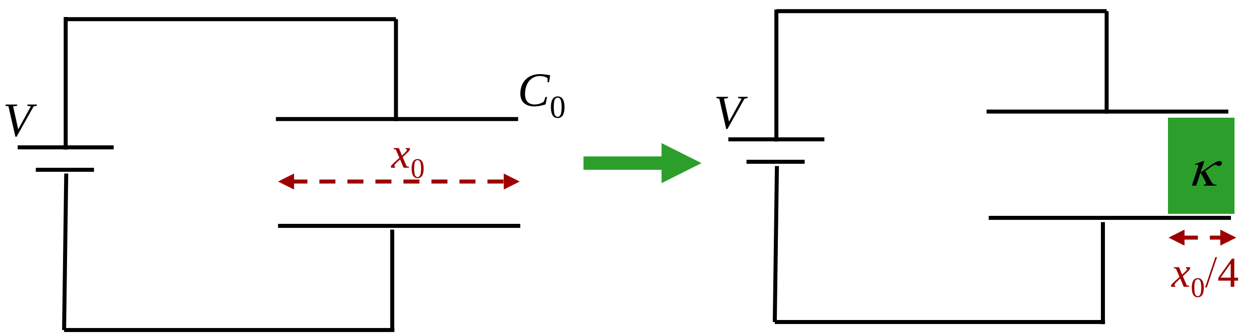
To calculate C, let's first look at:



A) $V_{left} < V_{right}$ B) $V_{left} = V_{right}$ C) $V_{left} > V_{right}$

The conducting plate is an equipotential !

Calculation



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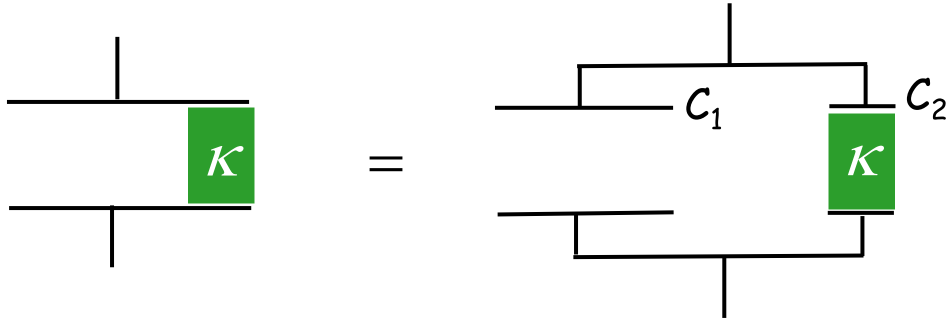
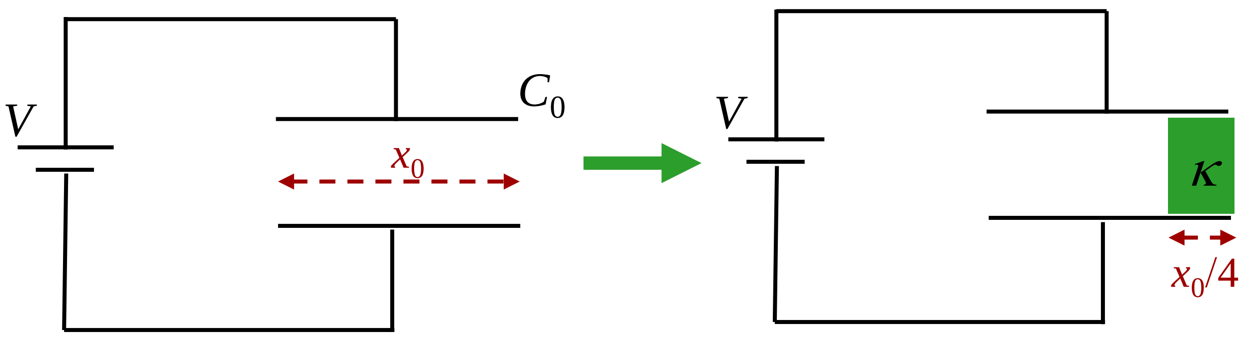
Can consider capacitor to be two capacitances, C_1 and C_2, in parallel



What is C_1 ?

A) $C_1 = C_0$ B) $C_1 = {}^3/_4 C_0$ C) $C_1 = {}^4/_3 C_0$ D) $C_1 = {}^9/_{16} C_0$

Calculation



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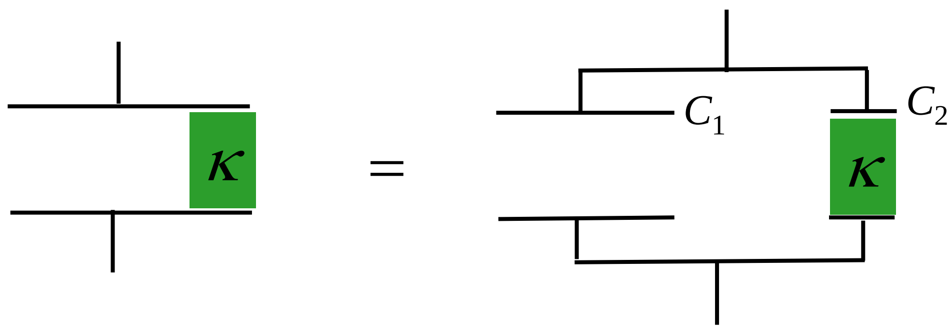


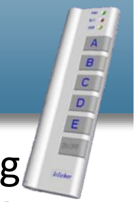
$C_1 = {}^3/_4 C_0$

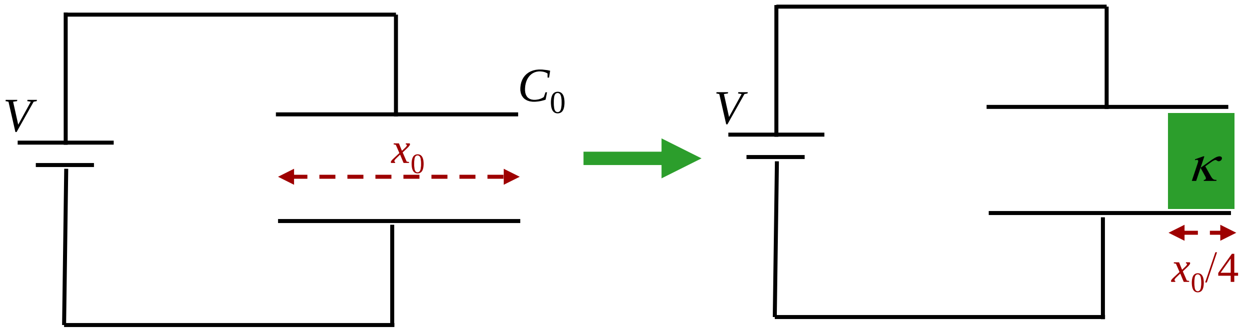
What is C_2 ?

A) $C_2 = \kappa C_0$ B) $C_2 = {}^3/_4\, \kappa C_0$ C) $C_2 = {}^4/_3\, \kappa C_0$ D) $C_2 = {}^1/_4\, \kappa C_0$

In general. For parallel plate capacitor filled with dielectric: $C = \kappa \varepsilon_0 A/d$

Calculation





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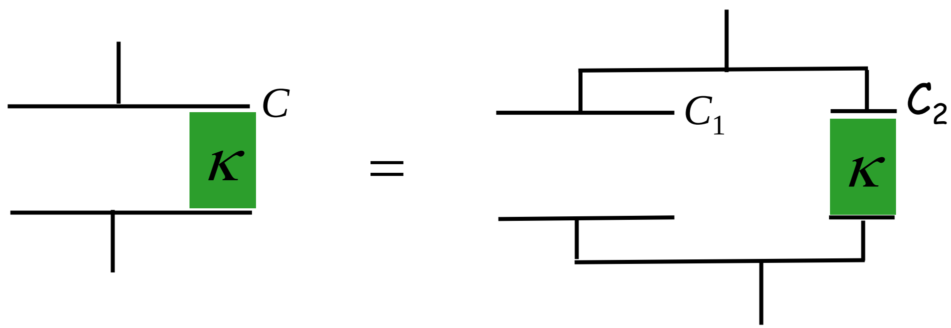


$C_1 = {}^3/_4 C_0$ $\qquad$ $C_2 = {}^1/_4\, \kappa C_0$

What is C?

A) $C = C_1 + C_2$

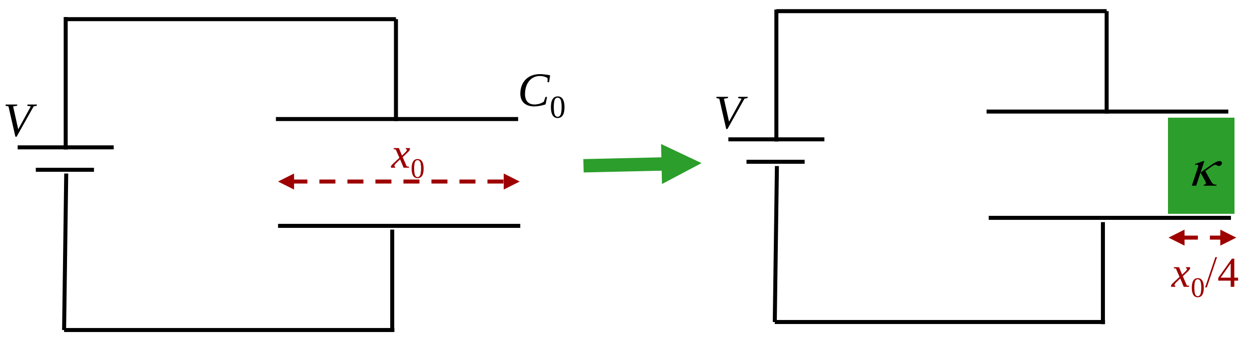
B)

C) $C = \left(\dfrac{1}{C_1} + \dfrac{1}{C_2}\right)^{-1}$

C = parallel combination of C_1 and C_2: $\quad C = C_1 + C_2$

$$C = C_0\,({}^3/_4 + {}^1/_4\, \kappa)$$

Calculation



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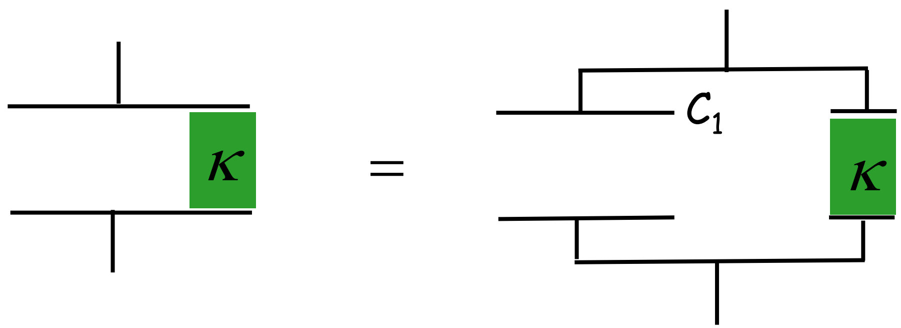

$C_1 = {}^3/_4 C_0$ $\qquad$ $C_2 = {}^1/_4\, \kappa C_0$

$$C = C_0\,({}^3/_4 + {}^1/_4\, \kappa)$$

What is Q?

$$C \equiv \frac{Q}{V} \quad \longrightarrow \quad Q = VC$$

$$Q_f = VC_0\left(\frac{3}{4} + \frac{1}{4}\kappa\right)$$