Electricity & Magnetism
Lecture 3

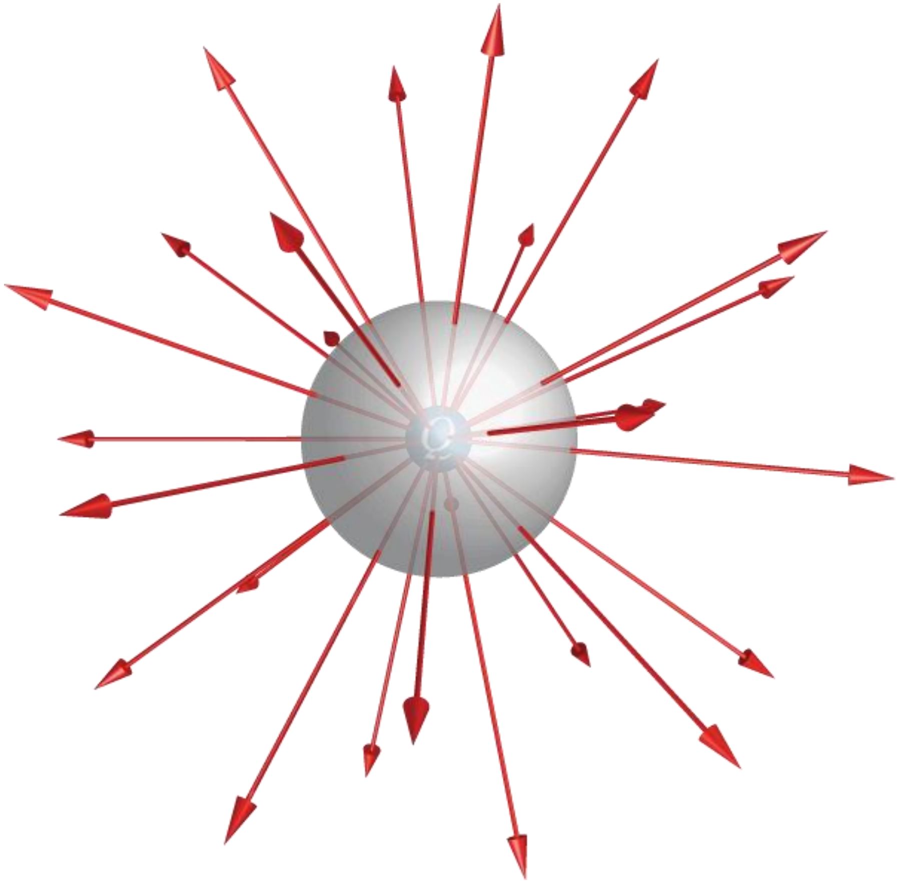
Today's Concepts:

A) Electric Flux

B) Field Lines

→ Gauss' Law

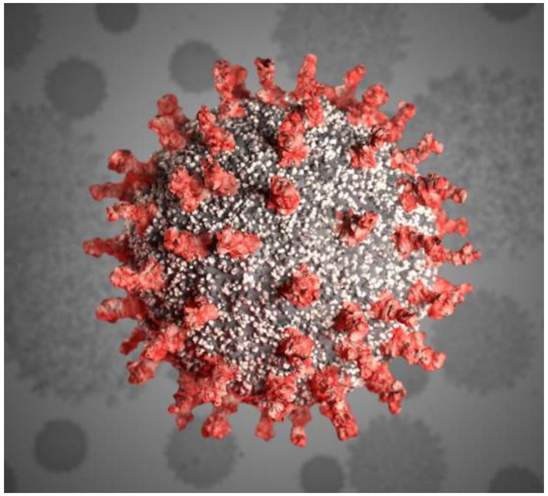
Electric Field Lines

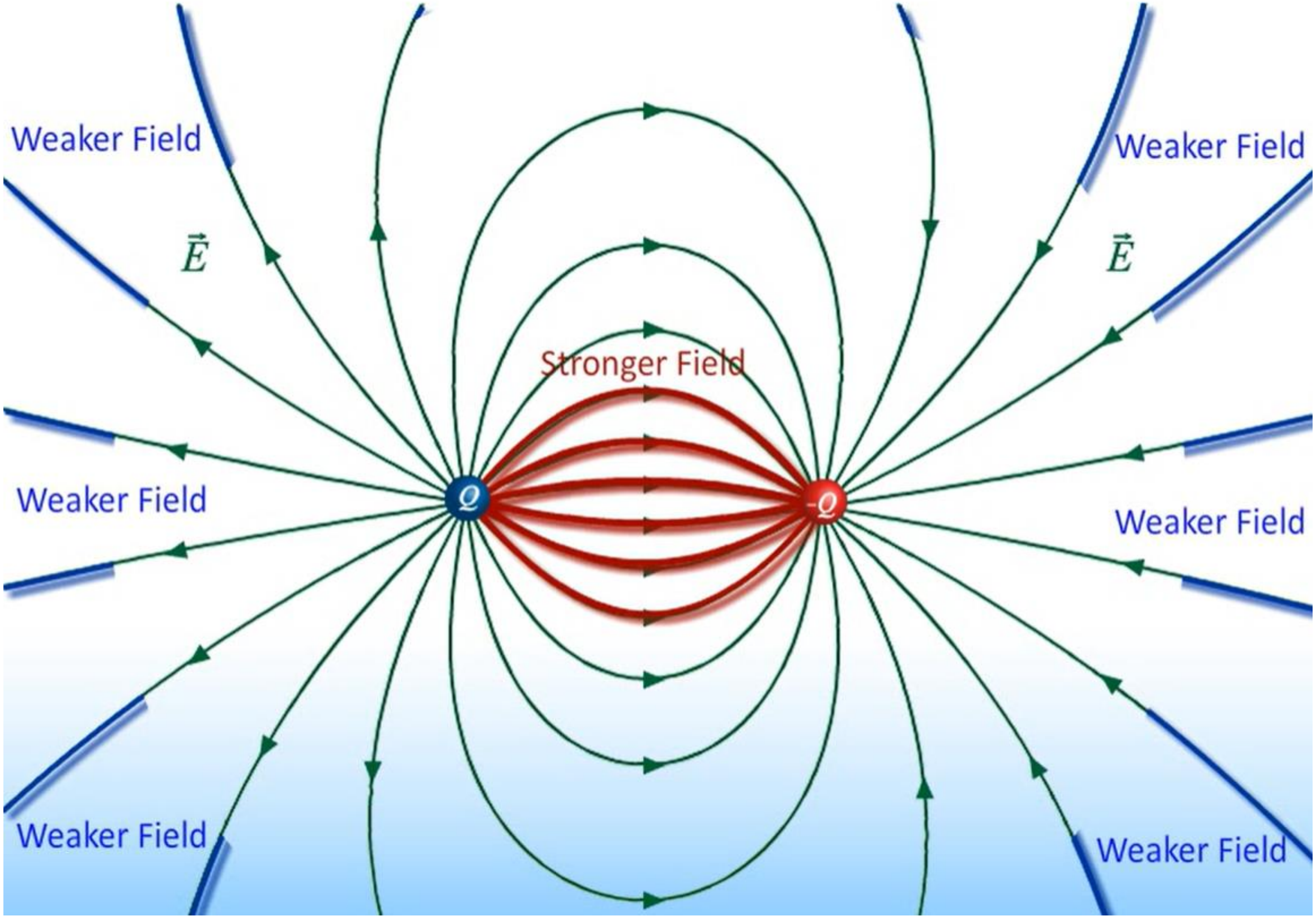


Direction & Density of Lines
represent
Direction & Magnitude of E

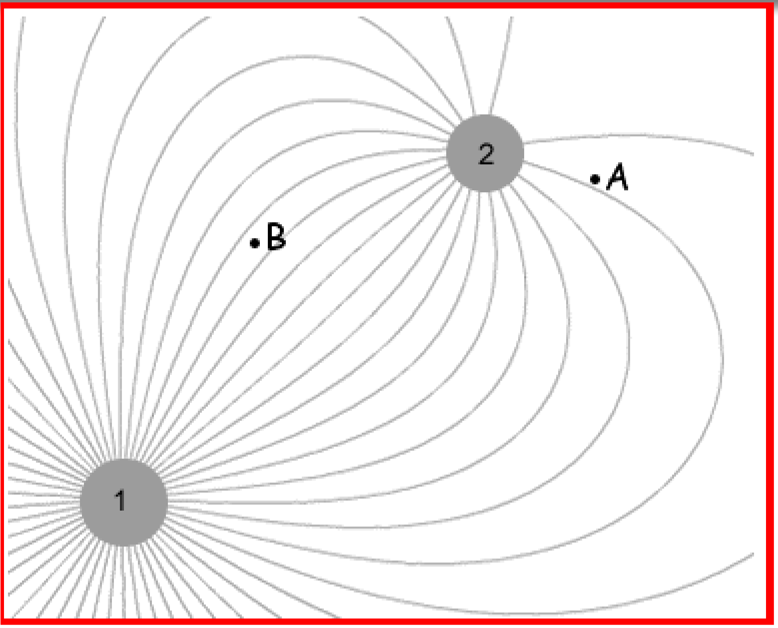
Point Charge:
    Direction is radial
    Density $\propto 1/R^2$

Why does this look familiar?




Weaker Field
Weaker Field
$\vec{E}$
$\vec{E}$
Stronger Field
Weaker Field
Weaker Field
Q
$-Q$
Weaker Field
Weaker Field

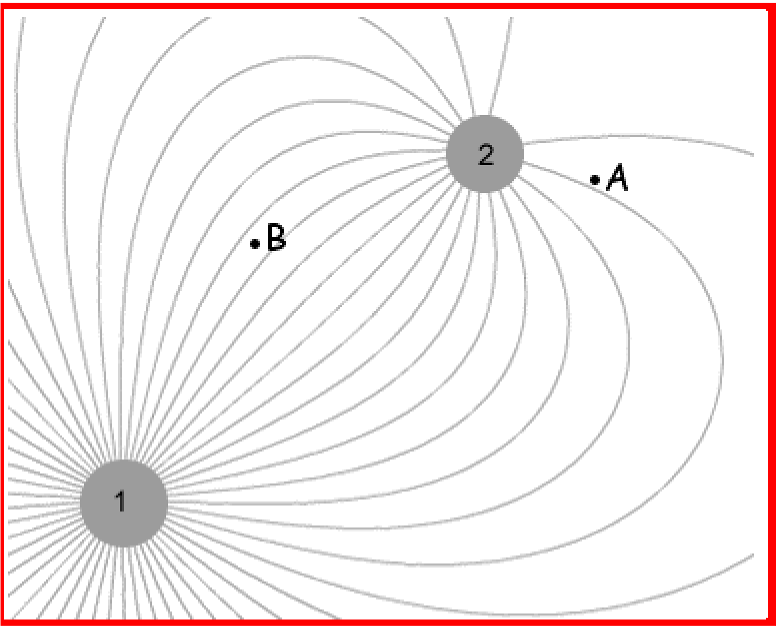
Check Point 1



A. Q_1 and Q_2 have the same sign
B. Q_1 and Q_2 have opposite signs
C. Not enough info

"They are connected by field lines and that can only happen if the charges are opposites."

Check Point 2

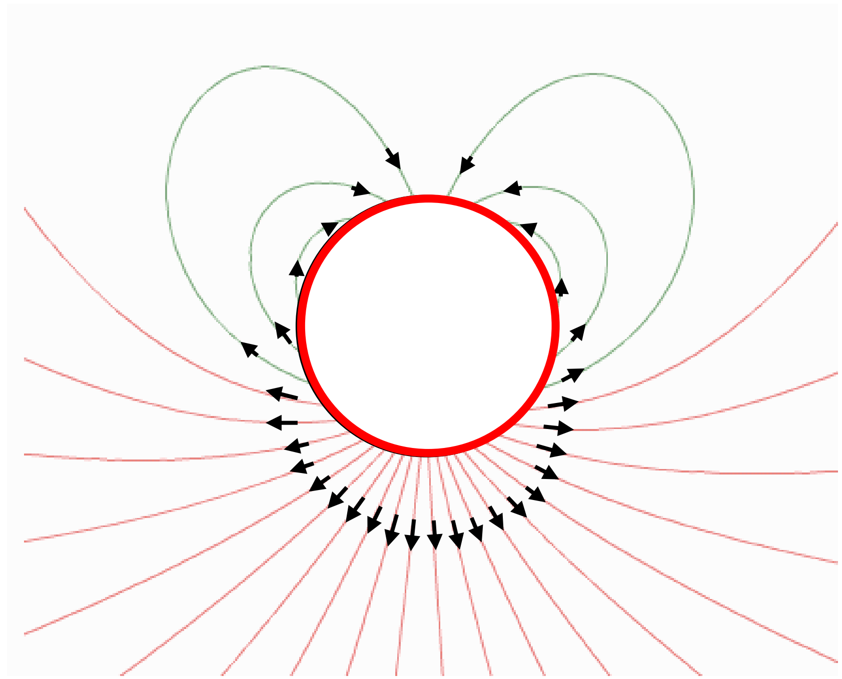


A. $|E_A| > |E_B|$
B. $|E_A| = |E_B|$
C. $|E_A| < |E_B|$
D. Not enough info

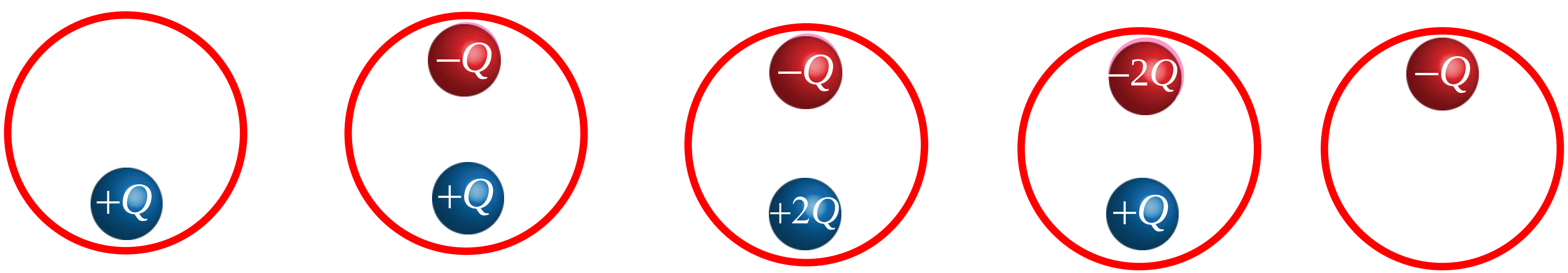
"The density of field lines is greater at point B which means the magnitude of the field is greater.."

Point Charges

Which configuration of charges inside the red circle match electric field pattern show?



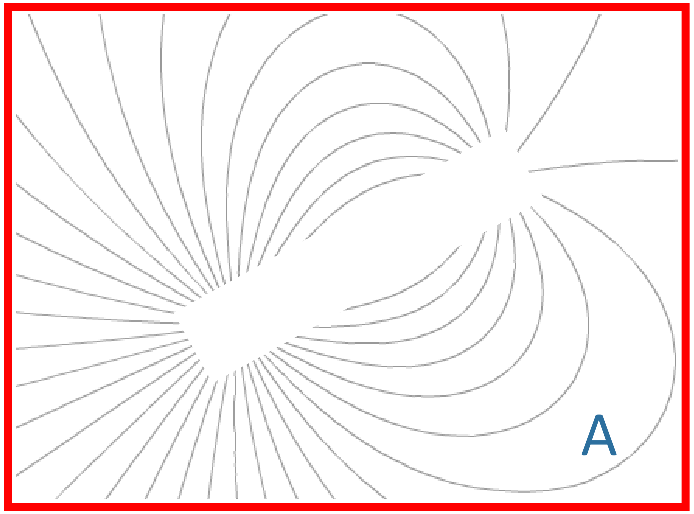
What charges are inside the red circle?

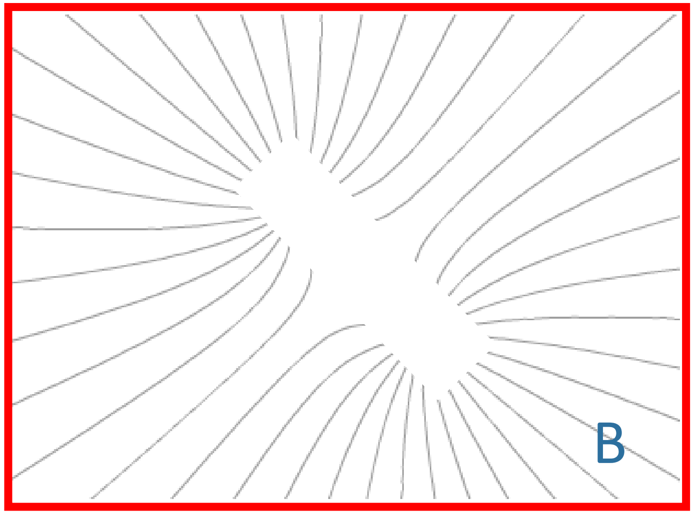


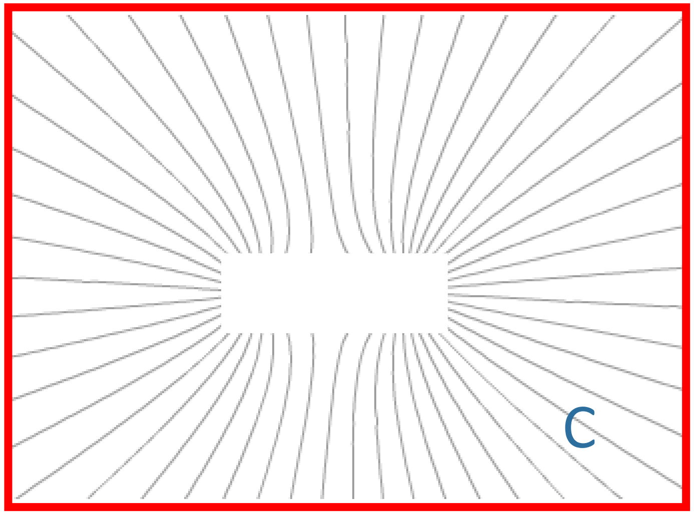
A B C D E

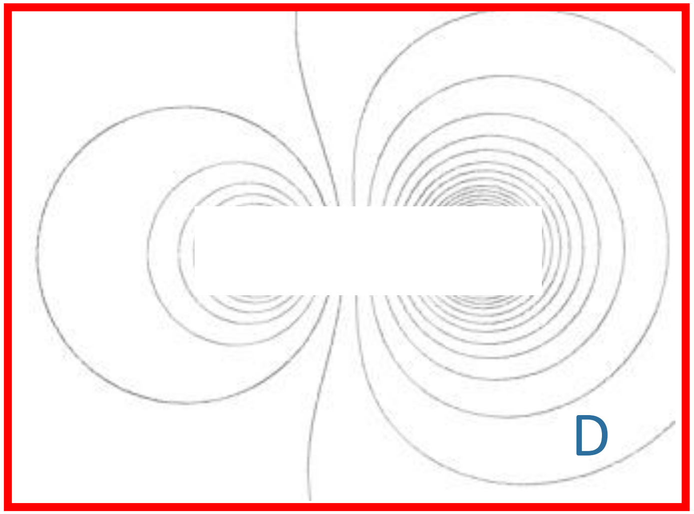
Electric Field lines

Which of the following field line pictures best represents the electric field from two charges that have the same sign but different magnitudes?









Electric Flux "Counts Field Lines"

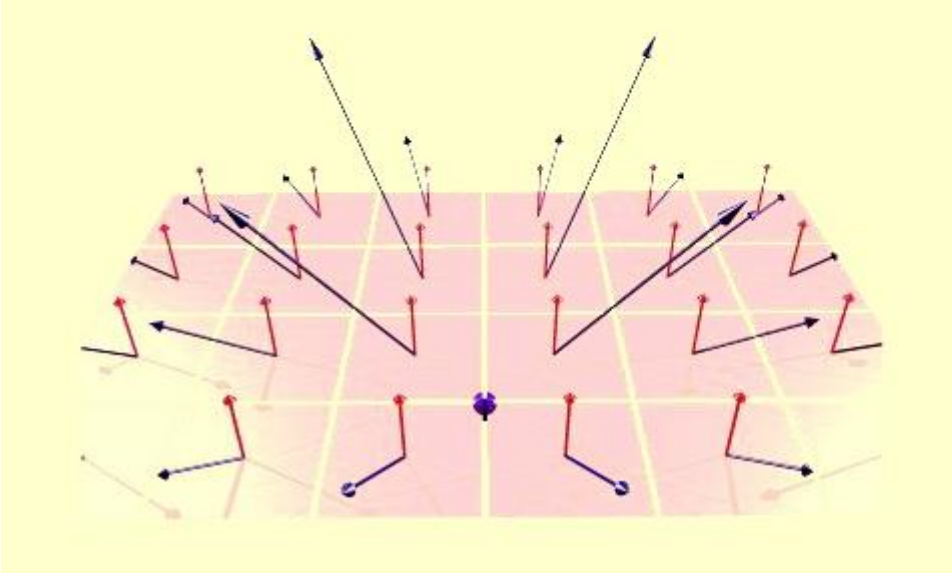
Can you give us a clear, simple definition of what flux is?

$$\Phi_S \equiv \int_S \vec{E} \cdot d\vec{A}$$

Representing the area of a surface as a vector in order to take the dot product.

Flux through surface S

Integral of $\vec{E} \cdot d\vec{A}$ on surface S



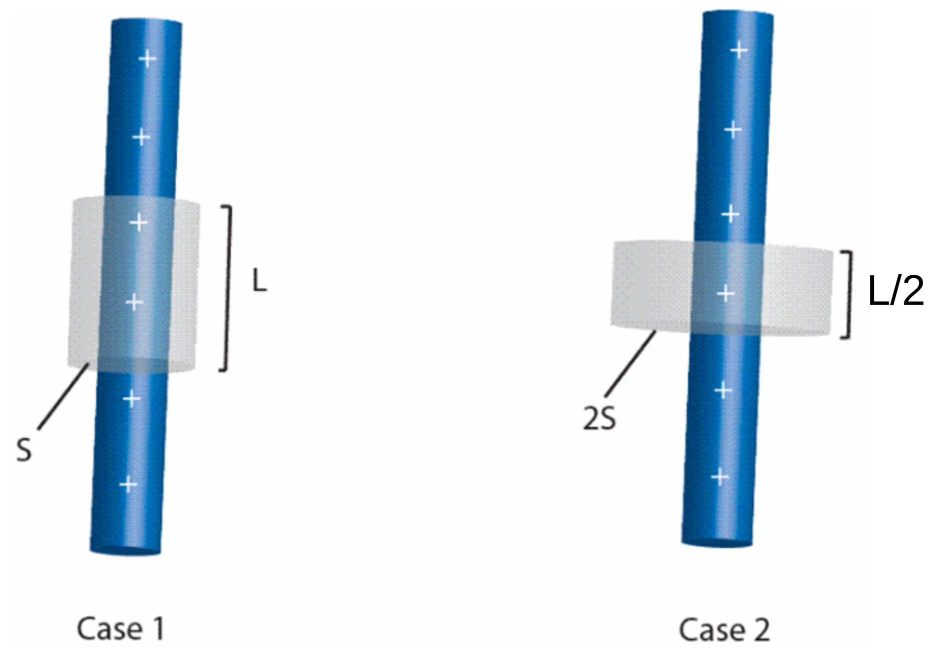
Check Point 3

A) The field lines travel horizontally through the rod, so more of them will pass through a taller cylinder.

B) There is twice the charge, but half of the surface area so they are equal

C) Twice the radius means 4x the Surface area of the base. 1/2 L turns this into 2x the flux in case 2.

An infinitely long charged rod has uniform charge density λ and passes through a cylinder (gray). The cylinder in Case 2 has twice the radius and half the length compared with the cylinder in Case 1.



$\Phi_1 = 2\Phi_2$	$\Phi_1 = \Phi_2$	$\Phi_1 = 1/2\Phi_2$	none
(A)	(B)	(C)	(D)

Check Point (Hard way shown in prelecture)

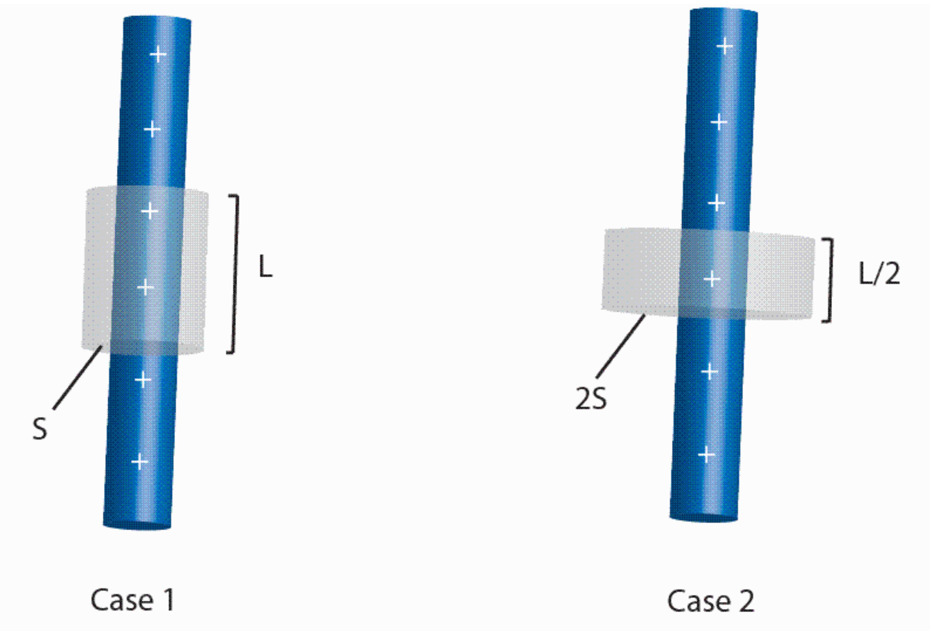
Definition of Flux:

$$\Phi_S \equiv \int_S \vec{E} \cdot d\vec{A}$$

E constant on barrel of cylinder
E perpendicular to barrel surface
(E parallel to dA)

$$\Phi_S = E \int_{barrel} d\vec{A}$$

$$= E A_{barrel}$$



$\Phi_1 = 2\Phi_2$	$\Phi_1 = \Phi_2$	$\Phi_1 = 1/2\Phi_2$	none
(A)	(B)	(C)	(D)

RESULT: GAUSS' LAW
Φ proportional to charge enclosed !

Case 1

$$A_{barrel} = 2\pi L s$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 s}$$

$$\boxed{\Phi_1 = \frac{\lambda L}{\varepsilon_0}}$$

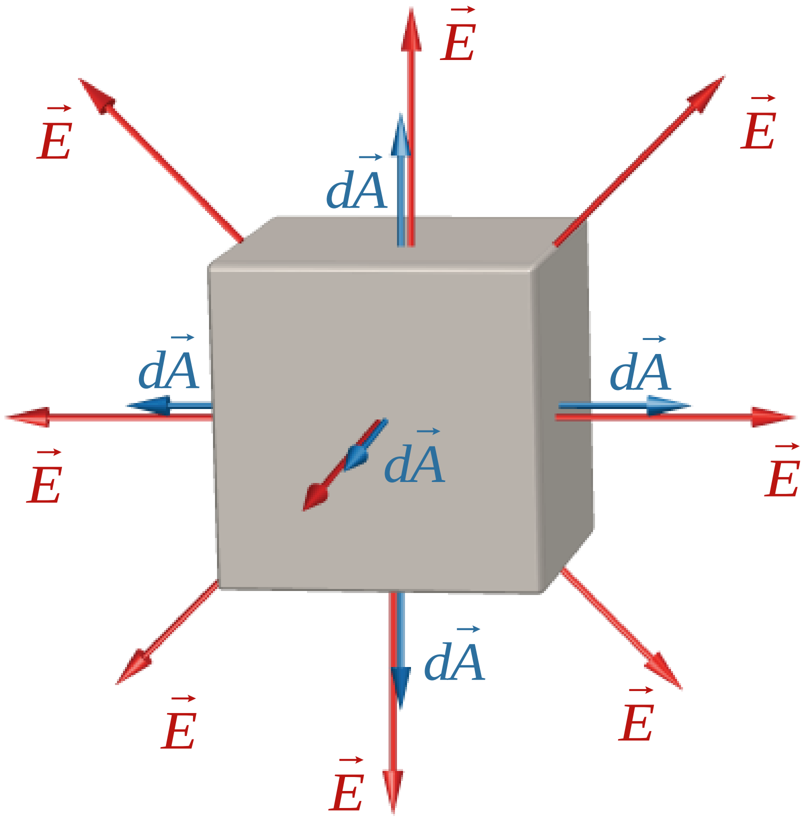
Case 2

$$A_2 = 2\pi(2s)\frac{L}{2}$$

$$E_2 = \frac{\lambda}{2\pi\varepsilon_0 2s}$$

$$\boxed{\Phi_2 = \frac{\lambda \frac{L}{2}}{\varepsilon_0}}$$

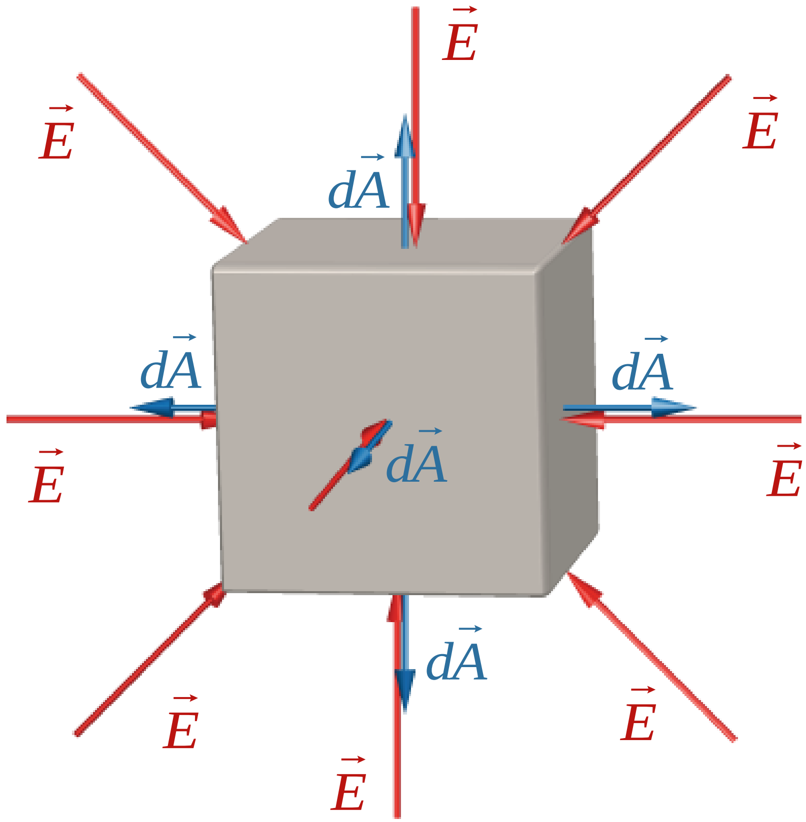
Direction Matters:



For a closed surface, $d\vec{A}$ points outward

$$\Phi_S = \int_S \vec{E} \cdot d\vec{A} > 0$$

Direction Matters:



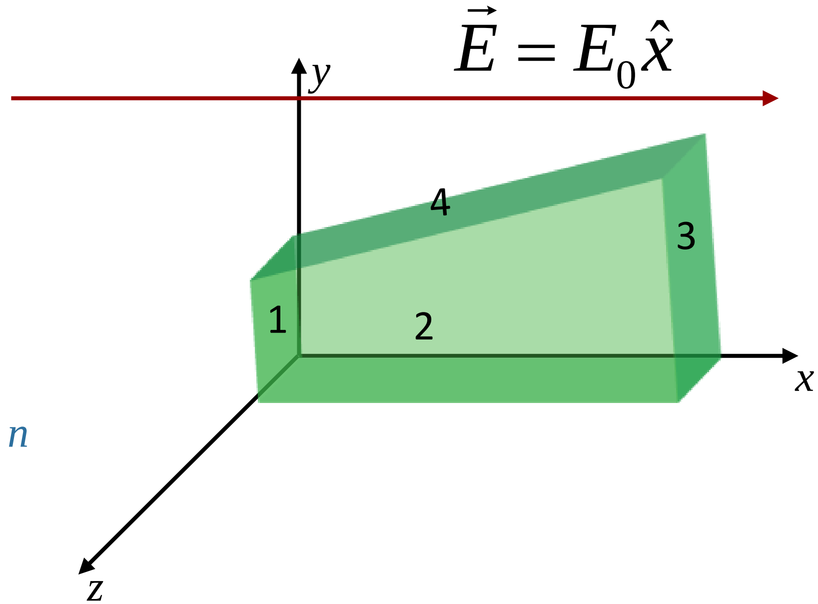
For a closed surface, $d\vec{A}$ points outward

$$\Phi_S = \int_S \vec{E} \cdot d\vec{A} < 0$$

Trapezoid in Constant Field

Label faces:

1: $x = 0$
2: $z = +a$
3: $x = +a$
4: slanted



Define $\Phi_n =$ Flux through Face n

Q1

A) $\Phi_1 < 0$

B) $\Phi_1 = 0$

C) $\Phi_1 > 0$

Q2

A) $\Phi_2 < 0$

B) $\Phi_2 = 0$

C) $\Phi_2 > 0$

Q3

A) $\Phi_3 < 0$

B) $\Phi_3 = 0$

C) $\Phi_3 > 0$

Q4

A) $\Phi_4 < 0$

B) $\Phi_4 = 0$

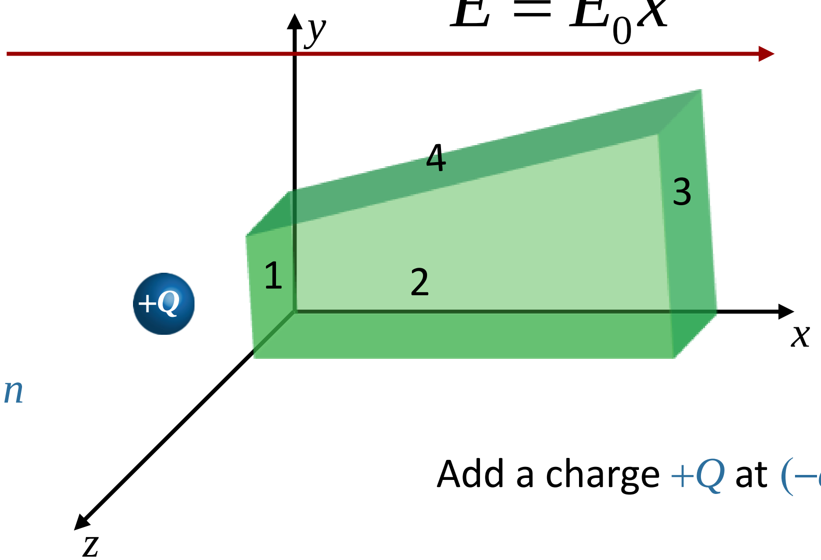
C) $\Phi_4 > 0$

Trapezoid in Constant Field + Q

$\vec{E} = E_0 \hat{x}$

Label faces:

1: $x = 0$
2: $z = +a$
3: $x = +a$
4: slanted



Define $\Phi_n =$ Flux through Face n
$\Phi =$ Flux through Trapezoid

Add a charge $+Q$ at $(-a, a/2, a/2)$

How does Flux change?
Note (-6 < -4) sign matters

A) Φ_1 increases

B) Φ_1 decreases

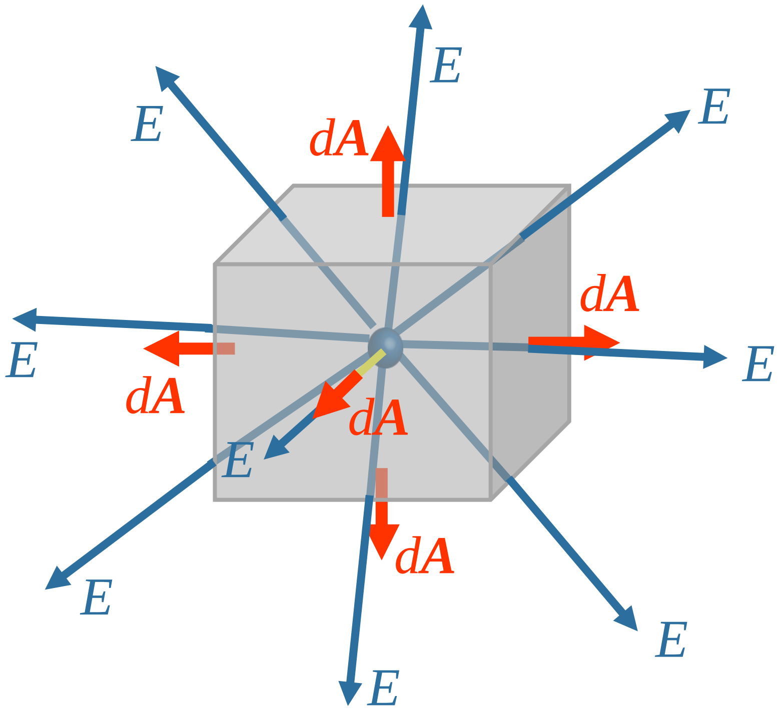
C) Φ_1 remains same

A) Φ_3 increases

B) Φ_3 decreases

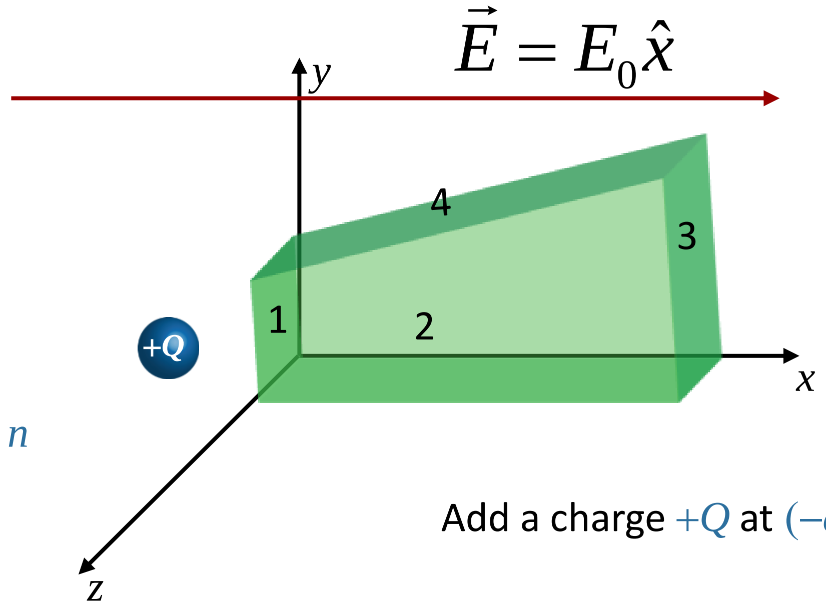
C) Φ_3 remains same

Gauss Law



$$\int_{closed-surface} \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$

Trapezoid in Constant Field + Q



Label faces:

1: $x = 0$
2: $z = +a$
3: $x = +a$
4: slanted

Define Φ_n = Flux through Face n
Φ = Flux through Trapezoid

Add a charge $+Q$ at $(-a, a/2, a/2)$

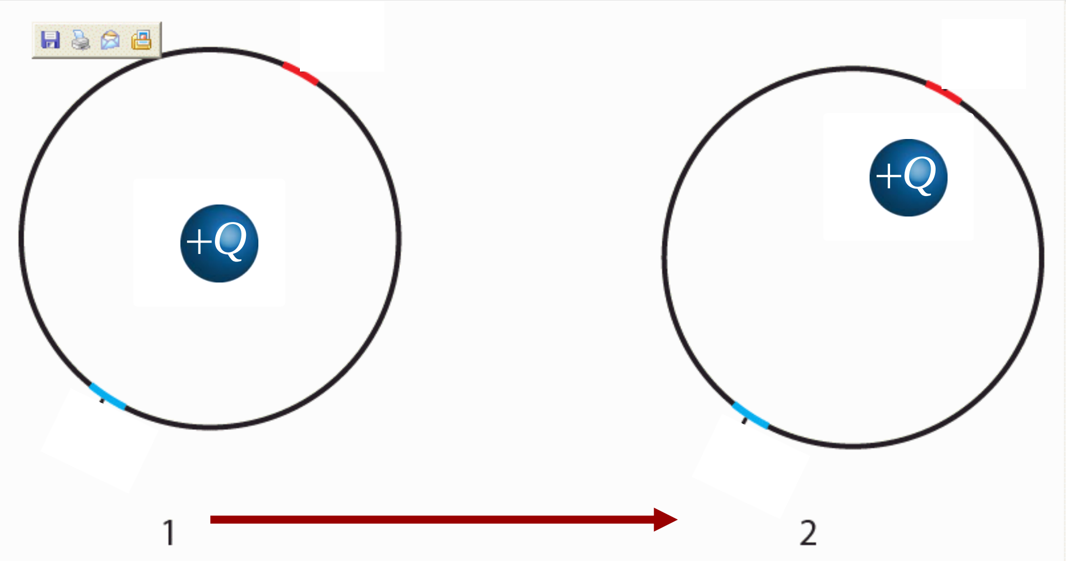
How does Flux change?
Note (-6 < -4) sign matters

A) Φ_1 increases
B) Φ_1 decreases
C) Φ_1 remains same

A) Φ_3 increases
B) Φ_3 decreases
C) Φ_3 remains same

A) Φ increases
B) Φ decreases
C) Φ remains same

Check Point 4



How does flux through entire surface change as charge is moved from center toward edge?

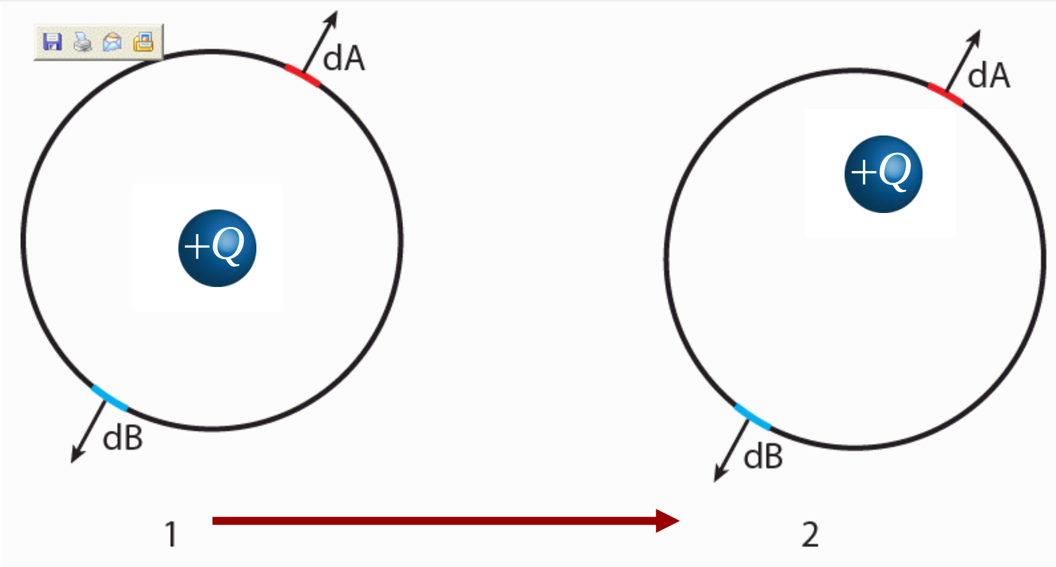
A
Φ_E increases

B
Φ_E decreases

C
Φ_E stays same

"The same number of lines exit the surface."

Check Point 5



How does flux through red surface element dA change as charge is moved from center toward edge?

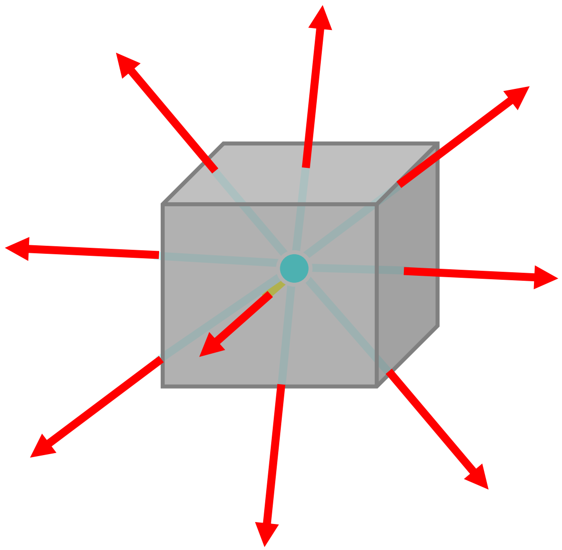
A
$d\Phi_A$ increases
$d\Phi_B$ decreases

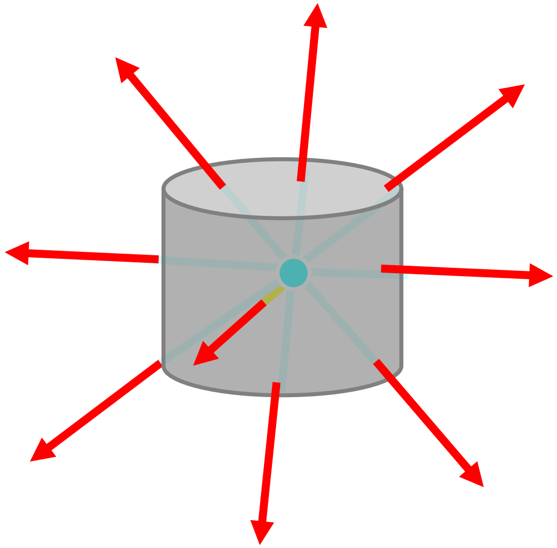
B
$d\Phi_A$ decreases
$d\Phi_B$ increases

C
$d\Phi_A$ stays same
$d\Phi_B$ stays same

"Since the area is the same, as the charge moves closer, the electric fields becomes stronger. Meaning larger electric flux for the point. "

Things to notice about Gauss Law





$$\Phi_{closed-surface} = \int_{closed-surface} \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$

If $Q_{enclosed}$ is the same, the flux has to be the same, which means that the integral must yield the same result for any surface.

Things to notice about Gauss Law

$$\int \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$

In cases of high symmetry, it may be possible to bring E outside the integral. In these cases, we can solve Gauss Law for E

$$E \int d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$

$$E = \frac{Q_{enclosed}}{A\varepsilon_0}$$

So - if we can figure out $Q_{enclosed}$ and the area of the surface A, then we know E!

This is the topic of the next lecture.

Takeaways

Electric field lines

- Direction and density
- Field distribution due to combination of charges

Concept of electric flux

- Definition, + or – ve
- Total flux through a closed surface

Gauss' law introduction

- Integral becomes simple for symmetrical distributions!