MATH 241 - Quiz 5 Solution

Question 1: total 4 points, 2 points each

(a) $f(x, y) = \frac{1}{x^2+y^2-1} = 2$, so $x^2 + y^2 = \frac{3}{2}$ is a circle

(b) The 0-contour of f is the union of the three lines $x = 0$, $y = 0$, $x + y = 6$

Question 2: (total 6 points, 3 points each) Find the limit if it exists, or show that the limit does not exist:

(a) By the Mean Value Theorem of one-variable calculus, there exists ξ between x and y such that $\sin x - \sin y = \cos(\xi)(x - y)$. It follows that

$$\frac{\sin x - \sin y}{x^2 - y^2} = \frac{\cos \xi}{x + y} \rightarrow \frac{\cos 1}{2} \quad \text{for } (x, y) \rightarrow (1, 1),$$

since $x \rightarrow 1$ and $y \rightarrow 1$ obviously implies $\xi \rightarrow 1$.

(b) On the curve $y = x^2$ we have

$$\frac{x^5 + y^5}{x^2 y^2} = \frac{x^5 + x^{10}}{x^6} = \frac{1}{x} + x^4 \rightarrow \pm\infty + 0 \quad \text{for } x \rightarrow 0,$$

i.e., the limit is $+\infty$ if we approach $(0, 0)$ from the right and $-\infty$ if we approach $(0, 0)$ from the left.

Conclusion: The limit doesn't exist.