ECE 313: Problem Set 11: Problems

Due: Saturday, Dec 7 at 11:59:00 p.m.
Reading: *ECE 313 Course Notes,* Sections 4.4 – 4.6

Note on reading: For most sections of the course notes there are short answer questions at the end of the chapter. We recommend that after reading each section you try answering the short answer questions. Do not hand in; answers to the short answer questions are provided in the appendix of the notes.

Note on turning in homework: Homework is assigned on a weekly basis on Mondays, and is due by 11:59 p.m. on the following Monday. You must upload handwritten homework to BB. No typeset homework will be accepted. No late homework will be accepted. Please write on the top right corner of the first page:
NAME AS IT APPEARS ON BB
NETID
SECTION
PROBLEM SET #
Page numbers are encouraged but not required. Five points will be deducted for improper headings.

1. **[Independent or not?]**
   Decide whether X and Y are independent for each of the following pdfs. The constant C in each case represents the value making the pdf integrate to one. Justify your answer.

   (a)
   $$f_{X,Y}(u,v) = \begin{cases} C(u^3+v^3), & \text{if } 0 \le u \le 1 \text{ and } 0 \le v \le 1; \\ 0, & \text{otherwise.} \end{cases}$$

   (b)
   $$f_{X,Y}(u,v) = \begin{cases} Cuv\, e^{-\frac{u^2}{2}}, & \text{if } 0 \le u \le 1 \text{ and } 1 \le v \le 2; \\ 0, & \text{otherwise.} \end{cases}$$

   (c)
   $$f_{X,Y}(u,v) = \begin{cases} C\exp(v-u), & \text{if } 0 \le u \le 1 \text{ and } 0 \le v \le 2u; \\ 0, & \text{otherwise.} \end{cases}$$

2. **[Sum of independent geometric random variables and Negative Binomial]**
   Let X and Y be independent geometric random variables with parameter p. Let $S = X+Y$.

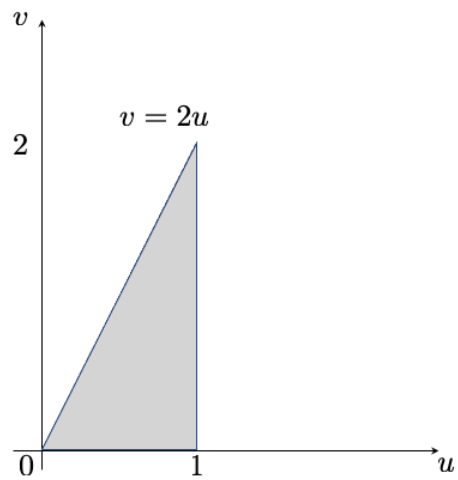

Figure 1: Support for Problem 1 (c).

(a) Find the pmf of S using convolution.

(b) Verify your answer to part (b) using the negative binomial distribution.

3. **[Sum of two independent continuous-type random variables]**
Suppose X and Y have the joint pdf

$$f_{X,Y}(u,v) = \begin{cases} 4uv, & \text{if } 0 \le u \le 1,\ 0 \le v \le 1; \\ 0, & \text{otherwise.} \end{cases}$$

Let $S = X + Y$. Find the pdf of S, i.e., find $f_S(s)$.

Hint: You may want to consider the cases $s < 0$, $0 \le s < 1$, $1 \le s \le 2$, and $s > 2$ separately.

4. **[Rayleigh fading and selection diversity]**
The signal strength received at a base-station from a cellular phone in a dense urban environment can be modeled quite well by a Rayleigh random variable. When a cellular phone user is around the midpoint between two base-stations, some systems use *selection diversity*, where the base station with the larger received signal strength is used to decode the user's message. Let X and Y denote the signal strengths received at the two base-stations. Then

$$f_X(u) = f_Y(u) = \begin{cases} ue^{-\frac{u^2}{2}}, & \text{if } u \geq 0; \\ 0, & \text{otherwise.} \end{cases}$$

Assume that X and Y are independent.

(a) Find the mean signal strength at each base-station, i.e., find $E[X]$.
Hint: Convert the integral required to compute $E[X]$ into one that corresponds to finding the variance of a $N(0,1)$ random variable.

(b) Find the pdf of the signal strength chosen by the selection diversity system, i.e., find the pdf of $Z = \max(X, Y)$.

(c) Find the mean signal strength after selection diversity, i.e., find $E[Z]$. Again, you may want to use the hint given in part (a).