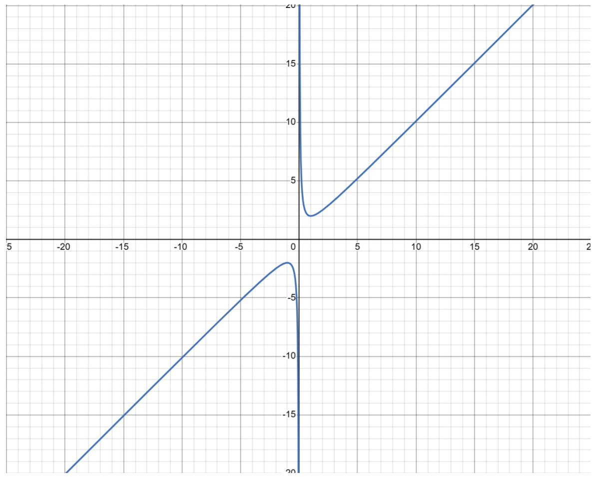
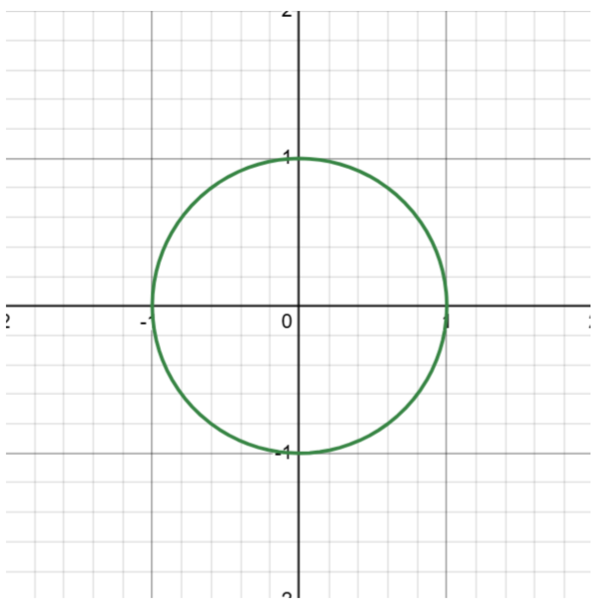
1. **(total 2 points: 0.5 point for expression and 0.5 point for graph)**
   (a) $(y, z) = (t^3, t^3 + t^{-3})$, $t \neq 0$. so $z = y + \frac{1}{y}$.



   (b) it's a circle, $x^2 + y^2 = 1$



2. (a) **(3 points)** Given the vector function: $\mathbf{r}(t) = \langle 2\cos t, 2\sin t, e^t \rangle$ we compute the derivative: $\mathbf{r}'(t) = \langle -2\sin t, 2\cos t, e^t \rangle$**(1 point)**.The tangent line to the curve is parallel to the plane when the curve's tangent vector is orthogonal to the plane's normal vector. Let the normal vector of the plane be $\langle \sqrt{3}, 1, 0 \rangle$**(0.5 point)**. Then we require: $\mathbf{r}'(t) \cdot \langle \sqrt{3}, 1, 0 \rangle = 0 \Rightarrow \langle -2\sin t, 2\cos t, e^t \rangle \cdot \langle \sqrt{3}, 1, 0 \rangle = 0 \Rightarrow -2\sqrt{3}\sin t + 2\cos t + 0 = 0 \Rightarrow \tan t = \frac{1}{\sqrt{3}} \Rightarrow t = \frac{\pi}{6}$ (since $0 \leq t \leq \pi$)**(1 point)**.Now evaluate

the position vector at $t = \frac{\pi}{6}$: $\mathbf{r}\left(\frac{\pi}{6}\right) = \langle\sqrt{3}, 1, e^{\pi/6}\rangle$, so the point is $(\sqrt{3}, 1, e^{\pi/6})$**(0.5 point)**.

(b) **(3 points)**Given current knowledge, a method is to consider edge cases: the plane is tangent to the ellipsoid.
Define the implicit function $g(x, y, z) = 4x^2 + y^2 + z^2 - 4$. Then the gradient (normal vector to the surface) is:

$$\nabla g = \left(\frac{\partial g}{\partial x}, \frac{\partial g}{\partial y}, \frac{\partial g}{\partial z}\right) = (8x, 2y, 2z)$$

(1 point) The given plane has normal vector $\vec{n} = (4, 2, 1)$. We want to find points on the surface where the tangent plane is parallel to the given plane, i.e., where:

$$(8x, 2y, 2z) = \lambda \cdot (4, 2, 1)$$

(1 point)

$$\begin{cases} 8x = 4\lambda \Rightarrow x = \frac{\lambda}{2} \\ 2y = 2\lambda \Rightarrow y = \lambda \\ 2z = \lambda \Rightarrow z = \frac{\lambda}{2} \end{cases}$$

Substitute into the ellipsoid equation:

$$4x^2 + y^2 + z^2 = 4 \Rightarrow 4\left(\frac{\lambda}{2}\right)^2 + \lambda^2 + \left(\frac{\lambda}{2}\right)^2 = 4 \Rightarrow \lambda^2\left(1 + 1 + \frac{1}{4}\right) = 4$$

$$\Rightarrow \lambda^2 \cdot \frac{9}{4} = 4 \Rightarrow \lambda^2 = \frac{16}{9} \Rightarrow \lambda = \pm\frac{4}{3}$$

Then:

$$(x, y, z) = \left(\pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{2}{3}\right)$$

Plug into the plane equation:

$$a = 4x + 2y + z$$

We get:

$$a \in [-6, 6]$$

(1 point)

3. **(total 2 points: 1 point for integral and 1 point for final answer)**

$$f'(t) = \int_0^t f''(s)\,ds + f'(0) = \left(\int_0^t 1\,ds, \int_0^t \cos s\,ds, \int_0^t \sin s\,ds\right) + (0, 1, -1)$$

$$= (t, \sin t, -\cos t + 1) + (0, 1, -1) = (t, \sin t + 1, -\cos t)$$

$$f(t) = \int_0^t f'(s)\,ds + f(0) = \left(\int_0^t s\,ds, \int_0^t (\sin s + 1)\,ds, \int_0^t -\cos s\,ds\right) + (0, 0, 0)$$

$$= \left(\frac{1}{2}t^2, -\cos t + 1 + t, -\sin t\right)$$

$$f(2\pi) = \left(\frac{1}{2}(2\pi)^2, -\cos(2\pi) + 1 + 2\pi, -\sin(2\pi)\right)$$

$$= \left(2\pi^2, -1 + 1 + 2\pi, 0\right) = (2\pi^2, 2\pi, 0)$$