

Your Name: \_\_\_\_\_

Please circle your discussion group (1 pt)

1 Yang Yihong A410	4 Zhang Junwei A404	7 Liu Yuanzhe A424
2 Xu Yixiao A425	5 Liang Jun A408	8 Jaden Peterson Wen A421
3 Dai Ruiqi A426	6 She Yuxuan A423	

- No notes, books or electronics during the exam.
- Do not open this test booklet until a proctor says start.
- For all free response questions, show work that justifies your answer.
- Raise your hand if you have a clarification question.
- Scratch paper is provided. You can ask for more if needed.
- Do not leave early: this disturbs others. If you finish your test early, check your work or just relax.
- Quit working when the test ends and hand your test booklet to proctors.

1. (15 points, 3 points each) Determine whether the statement is true or false. Circle the right answer.

(a) Suppose  $f$  and  $g$  are continuous on  $[a, \infty)$  and  $f(x) \geq g(x)$ , if  $\int_0^\infty f(x)dx$  is convergent, then  $\int_0^\infty g(x)dx$  is convergent. (True or False)

(b) Approximate integration is a numerical method to calculate the value of definite integral. (True or False)

(c)  $3ts = e^t + e^t \frac{ds}{dt}$  is a linear differential equation. (True or False)

(d) The areas of two regions are the same in the first quadrant of  $xy$  plane, if we rotate them about  $x$  axis, the volumes we get must be the same. (True or False)

(e)  $\int_0^2 \frac{dx}{(x-1)^2} = -2$  (True or False)

Question	1	2	3	4	5	6	7	8	9	Total
Points	15	10	10	10	8	10	10	14	12	99
Score										

2. (10 points) If  $\int_0^{\pi/4} \tan^6 x \sec x = I$ , express the value of  $\int_0^{\pi/4} \tan^8 x \sec x \, dx$  in terms of  $I$ .

2. (10 points) Evaluate the following integral

$$\int \frac{x+5}{x^2-6x+13} dx$$

3. (10 points) If  $a \neq 0$  and  $n$  is a positive integer, find the partial fraction decomposition of  $f(x) = \frac{1}{x^n(x-a)}$  and calculate the coefficient.

Hint: first find the coefficient of  $\frac{1}{x-a}$ . Then subtract the resulting term and simplify what is left.

$$x^n - a^n = (x - a)(x^{n-1} + x^{n-2}a + x^{n-3}a^2 + \cdots + xa^{n-2} + a^{n-1})$$

4. (8 points) Determine whether the integral is convergent or divergent.

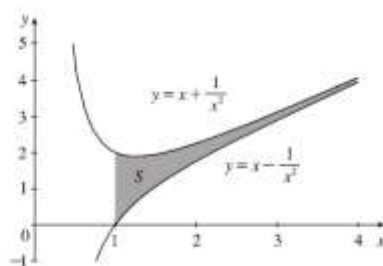
(a)  $\int_3^\infty \frac{x^3}{x^3 + e^x + \cos^2 x} dx$

- ☐ Convergent  
☐ Divergent

(b)  $\int_0^1 \frac{\sin x}{\sqrt{x}} dx$

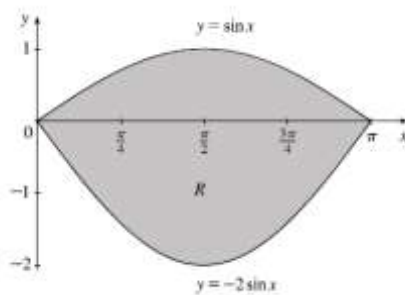
- ☐ Convergent  
☐ Divergent

5. (10 points) Consider the region  $S$  bounded by the curves  $y = x + \frac{1}{x^2}$  and  $y = x - \frac{1}{x^2}$  for  $x \geq 1$ .



- (a) Is the area of  $S$  finite or infinite? If finite, find the area.  
 (b) Now suppose we rotate  $S$  around the  $x$  axis. Is the volume of the resulting solid finite or infinite?

6. (10 points) Let  $R$  be the region shaded below. Find the coordinates of the centroid of  $R$ .



8. (14 points)  $y = 1 - e^{-x}$ ,  $0 \leq x \leq 2$ , find the exact arc length for this function.

9. (12 points) Solve the differential equation.

(a)

$$\frac{dz}{dt} + e^{t+z} = 0$$

(b)  $y' = x - y$

