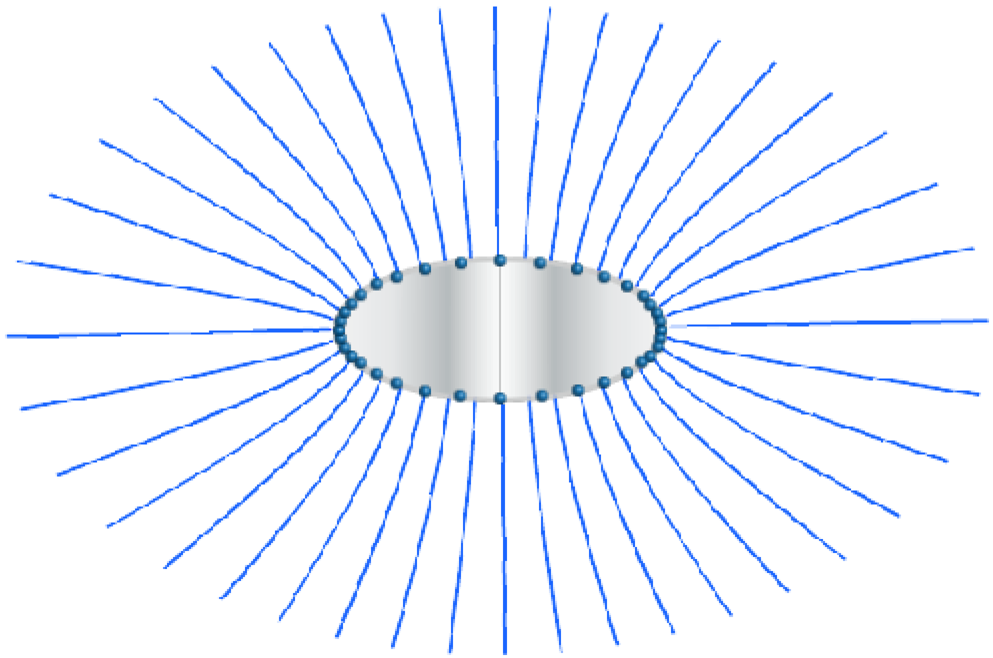
# Physics 212
# Lecture 7

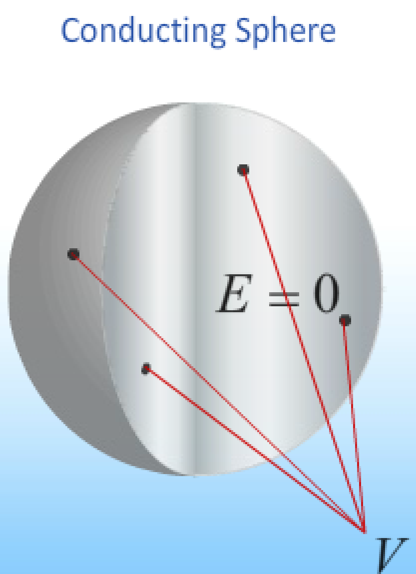
Today's Concept: (Applications of Gauss, E and V)

A) Conductors

B) Capacitance

# Main Point 1: (Conductors)



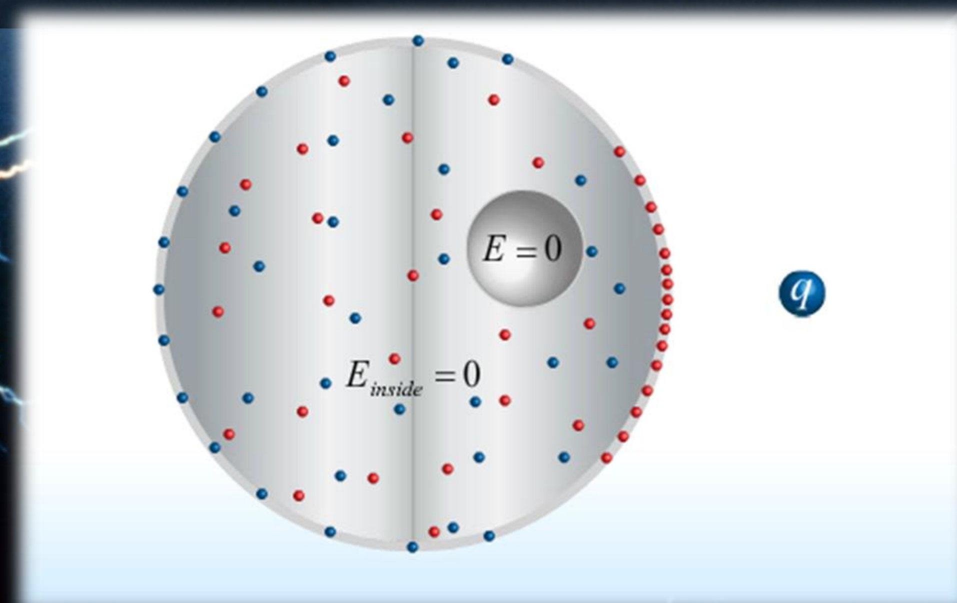



- Charges are free to move
- E = 0 in a conductor
- Surface = Equipotential
- E at surface perpendicular to surface

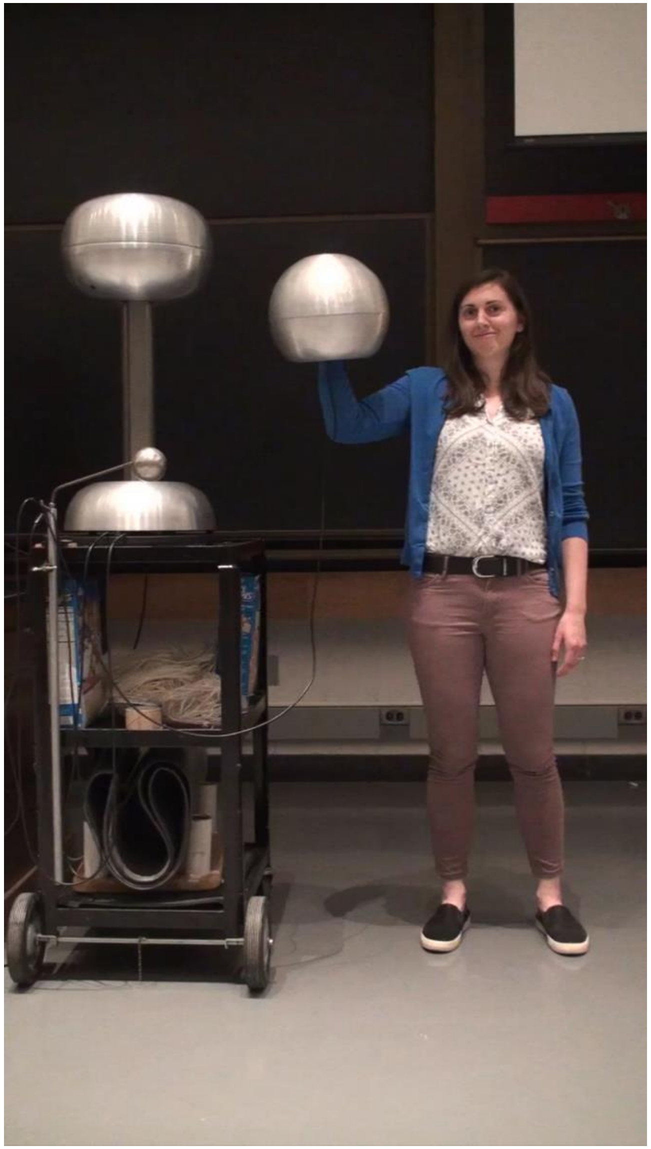
# Storm Safety

You are at the park when you see lightning. You decide to take shelter in a car, which car is safer, a (mainly steel) Volkswagen with thick rubber tires, or a (mainly fiberglass) Corvette with thin rubber tires

A) Corvette because it is fiberglass
B) Corvette because it is lower to ground
C) Volkswagen because it is steel
D) Volkswagen because tires are thicker
E) Neither—social distancing!

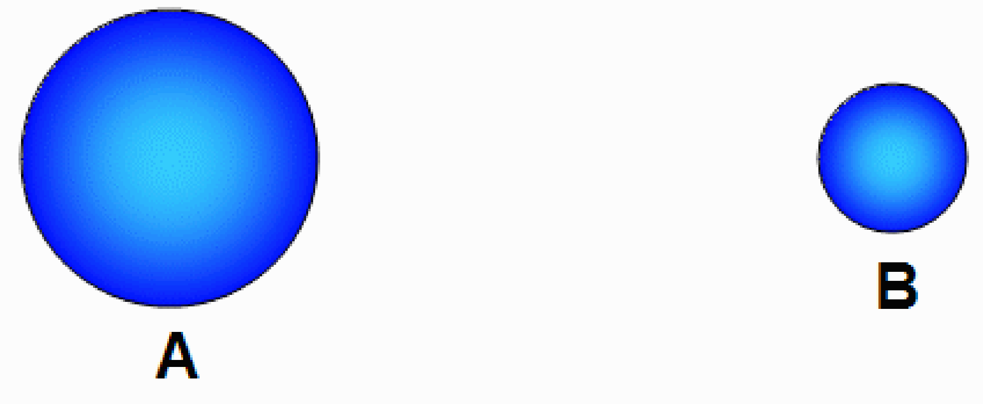



# Shocking!



# Check Point 1

Two spherical conductors are separated by a large distance. They each carry the same positive charge Q. Conductor A has a larger radius than conductor B

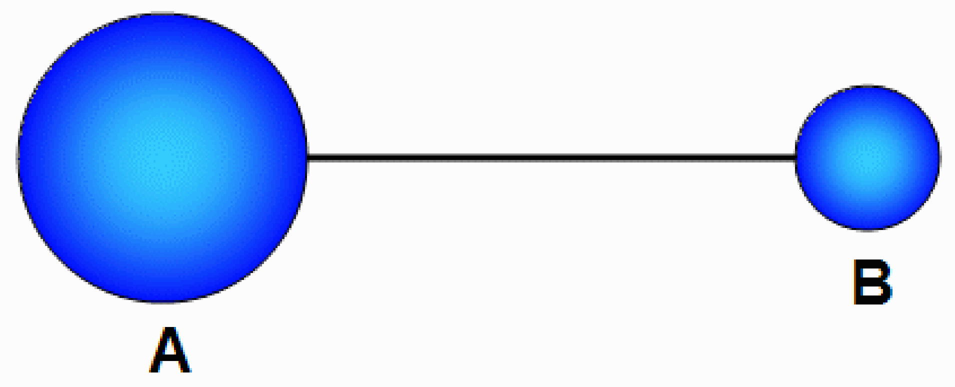



Compare the potential on surface A with the potential on surface B

A) $V_A > V_B$ B) $V_A = V_B$ C) $V_A < V_B$

# Check Point 2

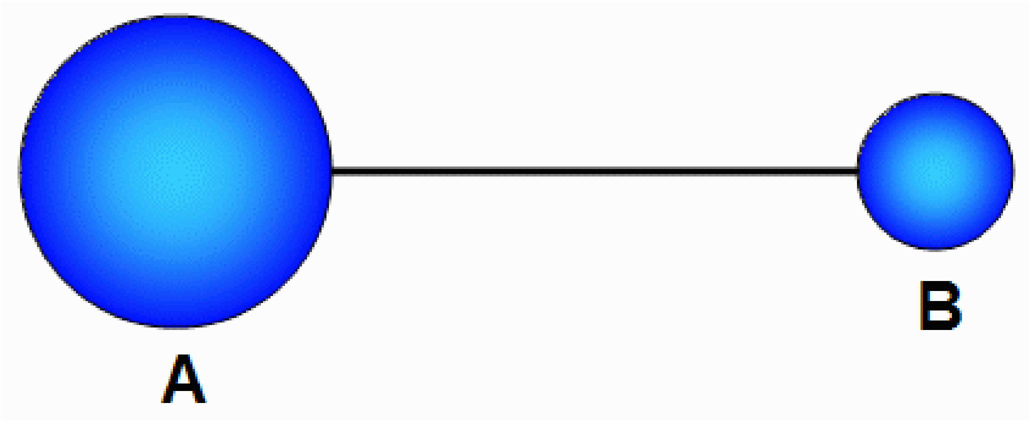
The two conductors are now attached by a conducting wire.




Compare the potential on surface A with the potential on surface B

A) $V_A > V_B$ B) $V_A = V_B$ C) $V_A < V_B$

# Check Point 3




What happens to the charge on sphere A when the wire is attached

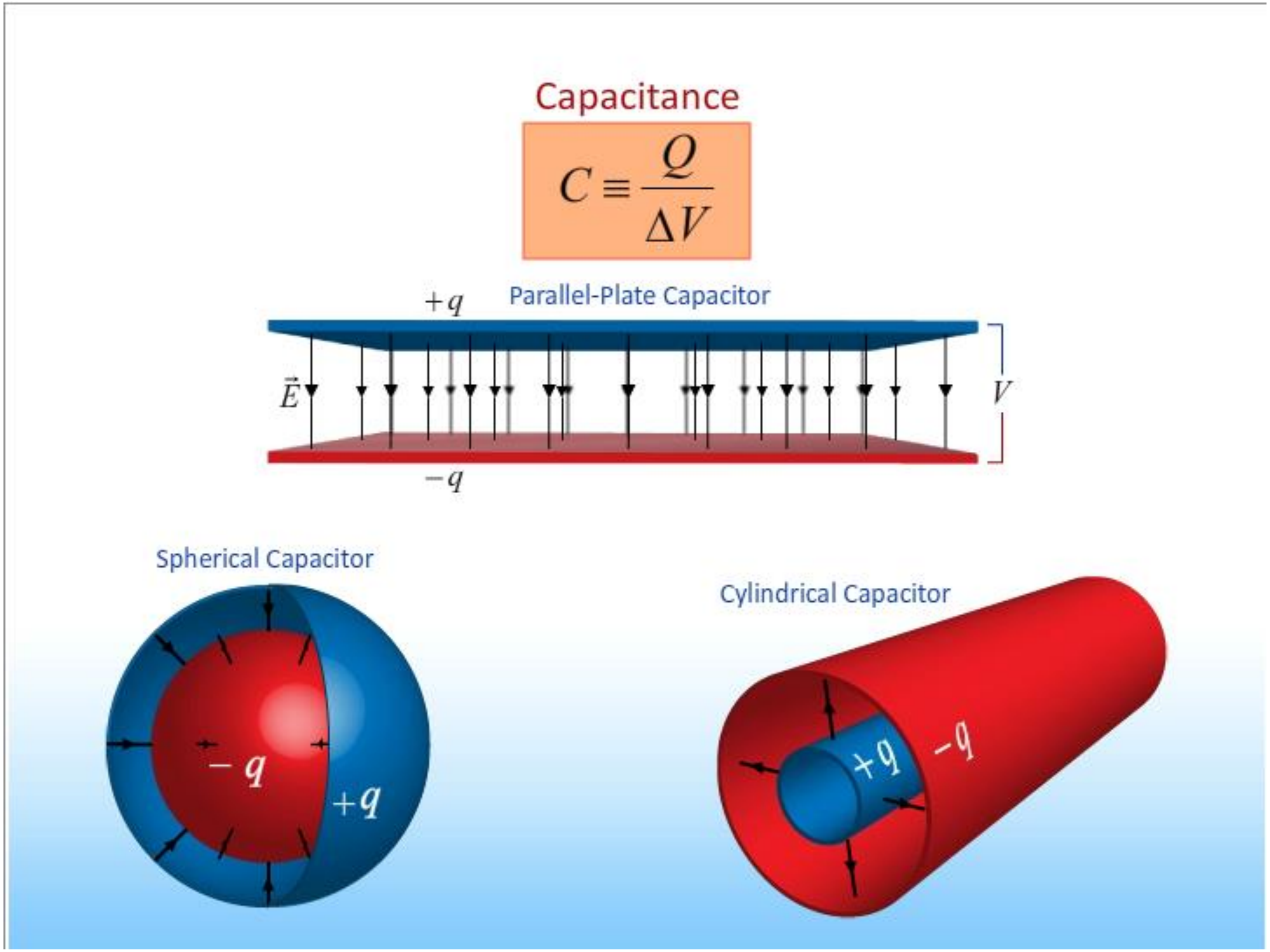
A) $Q_A$ increases B) $Q_A$ decreases C) $Q_A$ does not change

$$\frac{kQ_A}{R_A} = \frac{kQ_B}{R_B}$$

$$\Rightarrow \quad Q_A = Q_B \frac{R_A}{R_B}$$

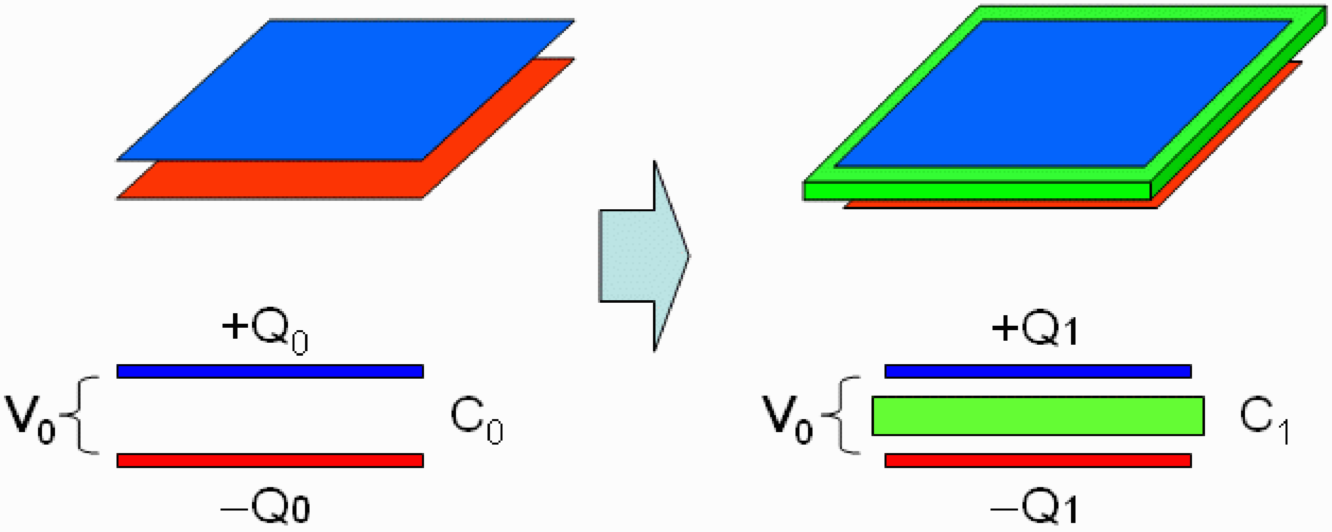
"Since the potential is greater on sphere B, the charge will flow from B to A and will increase the charge on sphere A."

# Main Point 2: Capacitance = Q/V



# Parallel Plate Capacitor

Two parallel plates of area carry equal and opposite charge $Q_0$. The potential difference between the two plates is measured to be $V_0$. An uncharged conducting plate (the green thing in the picture below) is slipped into the space between the plates without touching either one.The charge on the plates is adjusted to a new value $Q_1$ such that the potential difference between the plates remains the same as before.




THE CAPACITOR QUESTIONS WERE TOUGH!

THE PLAN:

We'll work through the example in the prelecture and then do the checkpoint questions.

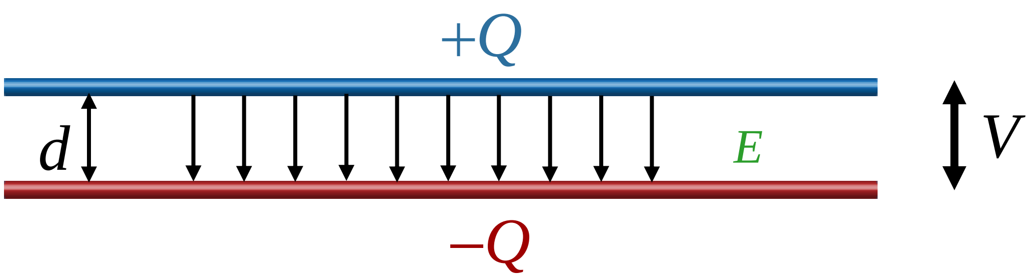
# Capacitance

Capacitance is defined for any pair of spatially separated conductors.

$$C \equiv \frac{Q}{V}$$

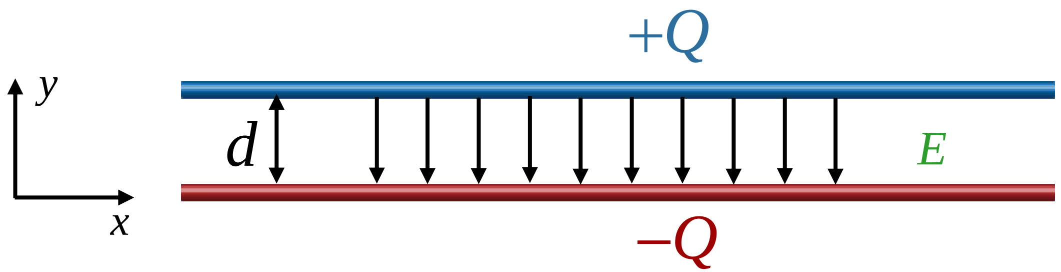
How do we understand this definition ?

- Consider two conductors, one with excess charge = $+Q$ and the other with excess charge = $-Q$




- These charges create an electric field in the space between them
- We can integrate the electric field between them to find the potential difference between the conductor
- This potential difference should be proportional to $Q$ !
  - The ratio of $Q$ to the potential difference is the capacitance and only depends on the geometry of the conductors

# Example (done in Prelecture 7)




$$E = \frac{\sigma}{\varepsilon_o}$$

What is σ ?

$$\sigma = \frac{Q}{A}$$

$A$ = area of plate

Second, integrate $E$ to find the potential difference $V$

$$V = -\int_0^d \vec{E} \cdot d\vec{y} \longrightarrow V = -\int_0^d (-E dy) = E \int_0^d dy = \frac{Q}{\varepsilon_o A} d$$

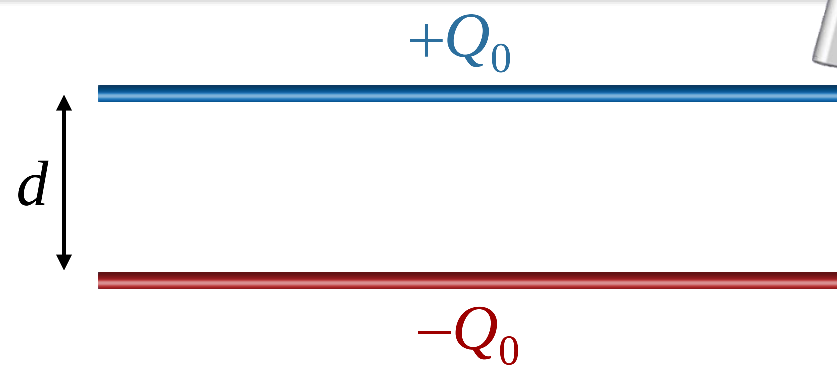
As promised, $V$ is proportional to $Q$ !

$$C \equiv \frac{Q}{V} = \frac{\cancel{Q}}{\cancel{Q}d / \varepsilon_o A} \longrightarrow C = \frac{\varepsilon_0 A}{d}$$

$C$ determined by geometry !

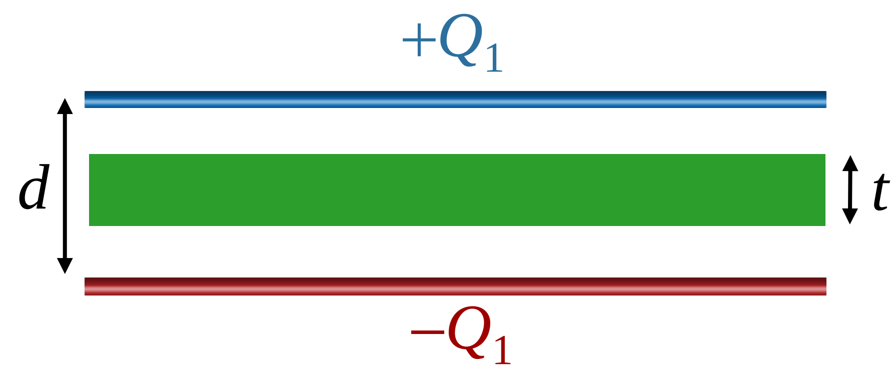
# Question Related to CheckPoint

Initial charge on capacitor $= Q_0$




Insert uncharged conductor

Charge on capacitor now $= Q_1$



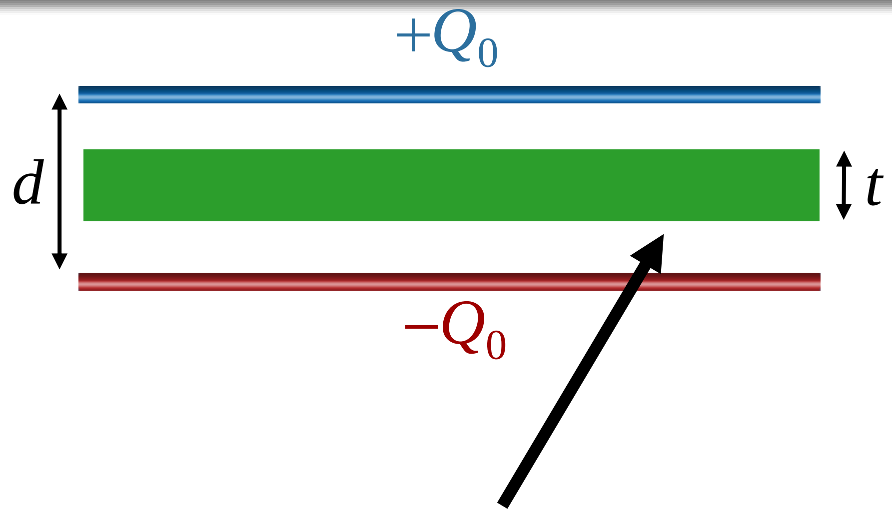

How is $Q_1$ related to $Q_0$ ?

A) $Q_1 < Q_0$
B) $Q_1 = Q_0$
C) $Q_1 > Q_0$

Plates not connected to anything

CHARGE CANNOT CHANGE !

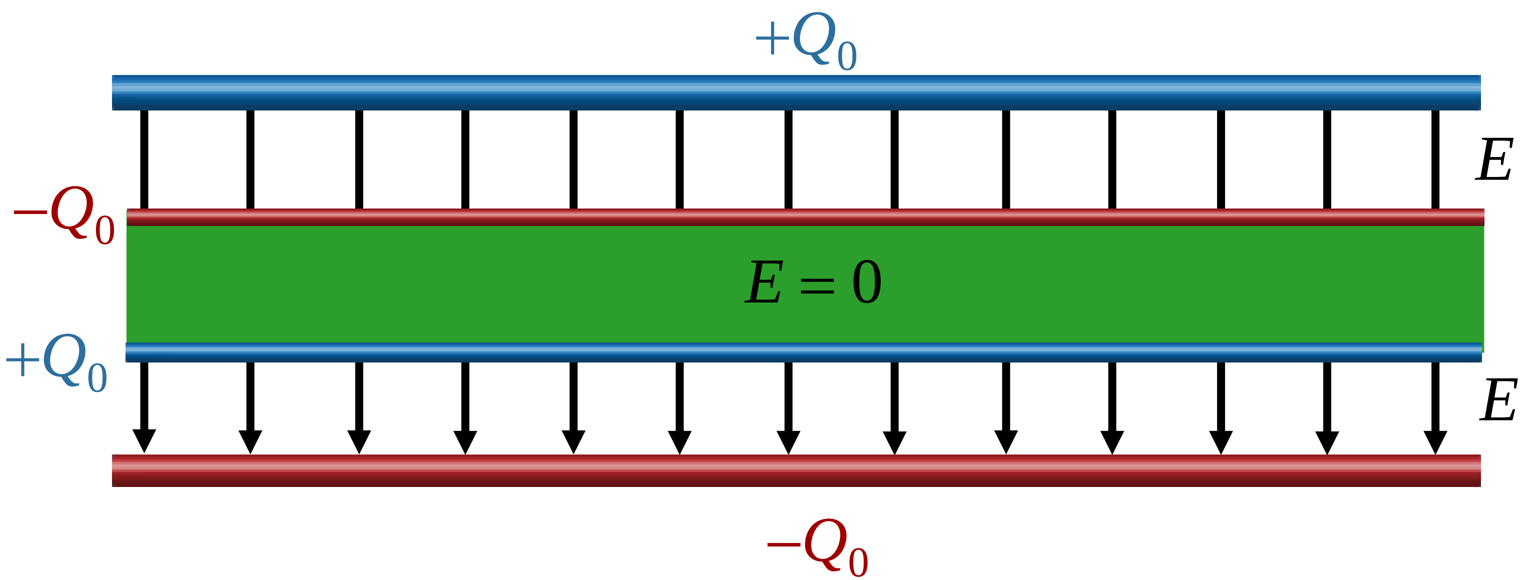
# Where to Start ?




What is the total charge induced on the bottom surface of the conductor?

A) $+Q_0$
B) $+Q_0/2$
C) 0
D) $-Q_0/2$
E) $-Q_0$

# Why ?




WHAT DO WE KNOW ?

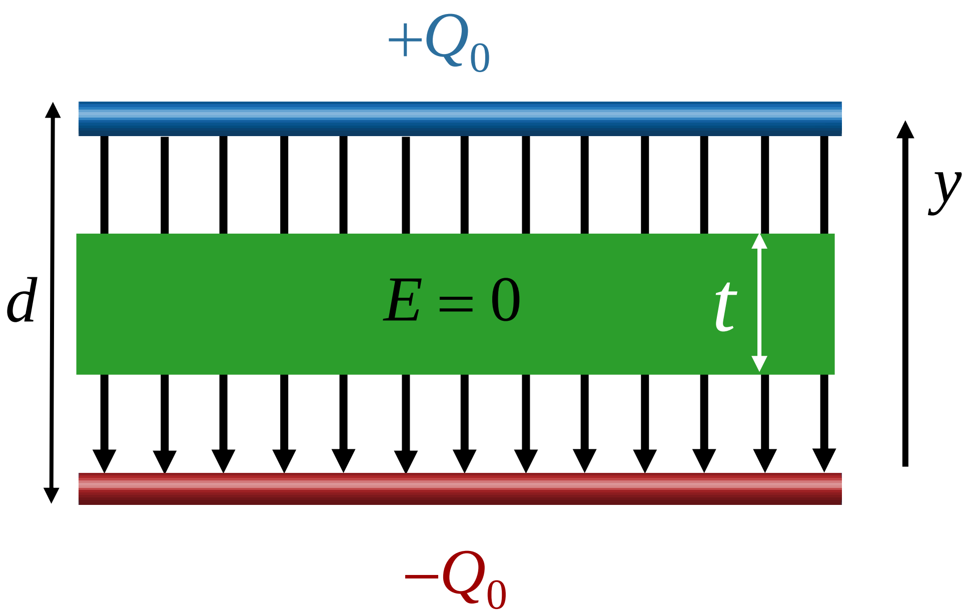
$E$ must be $= 0$ in conductor !

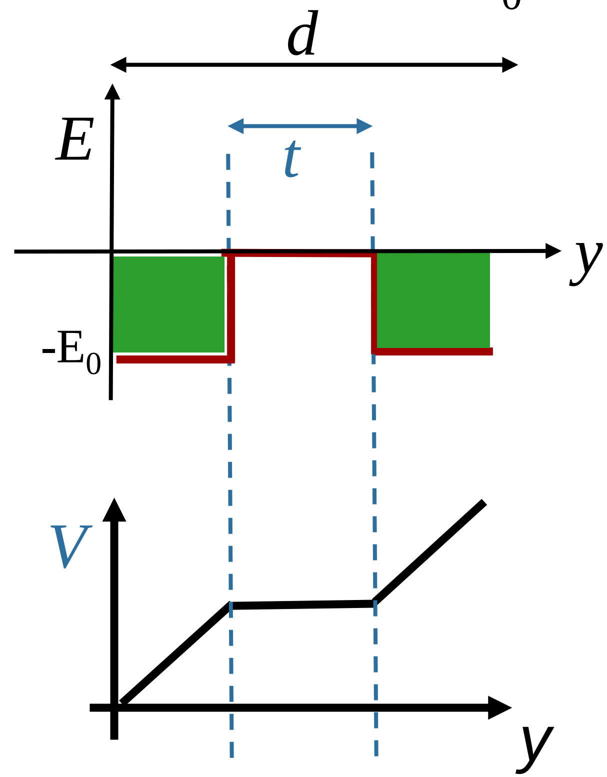
Charges inside conductor move to cancel $E$ field from top & bottom plates.

# Calculate V

Now calculate $V$ as a function of distance from the bottom conductor.

$$V(y) = -\int_0^y \vec{E} \cdot d\vec{y}$$




What is $\Delta V$ ?

A) $\Delta V = E_0 d$

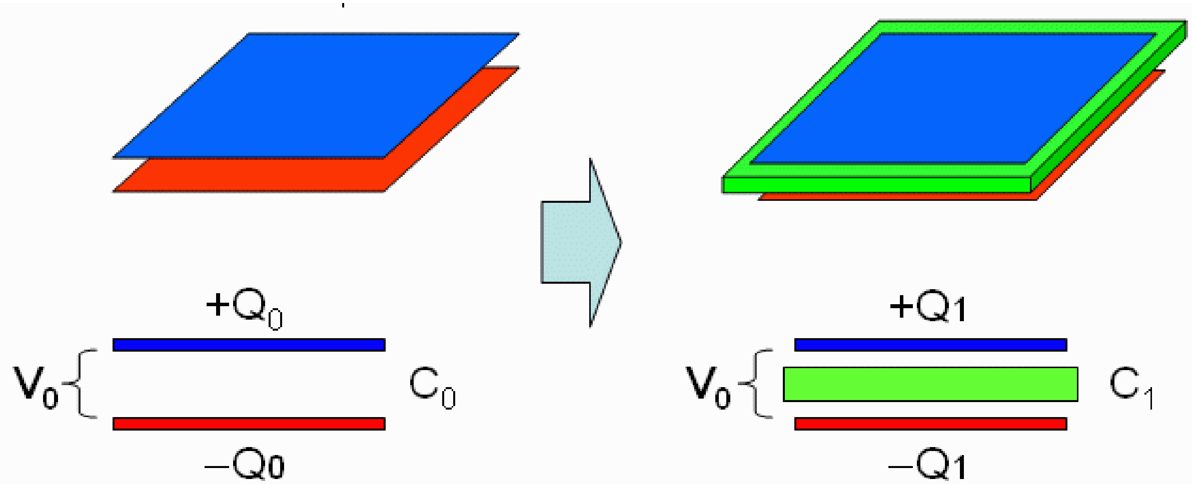
B) $\Delta V = E_0(d - t)$

C) $\Delta V = E_0(d + t)$

The integral = area under the curve

# Check Point 4

Two parallel plates are given a charge $Q_0$ such that the potential difference between the plates is $V_0$. If a conductor is slid between plates, does C change?




A) $C_1 > C_o$ B) $C_1 = C_o$ C) $C_1 < C_o$

We can determine $C$ from either case

same $V$ (preflight)

same $Q$ (lecture)

$C$ depends only on geometry !

$$E_0 = Q_0 / \varepsilon_0 A$$

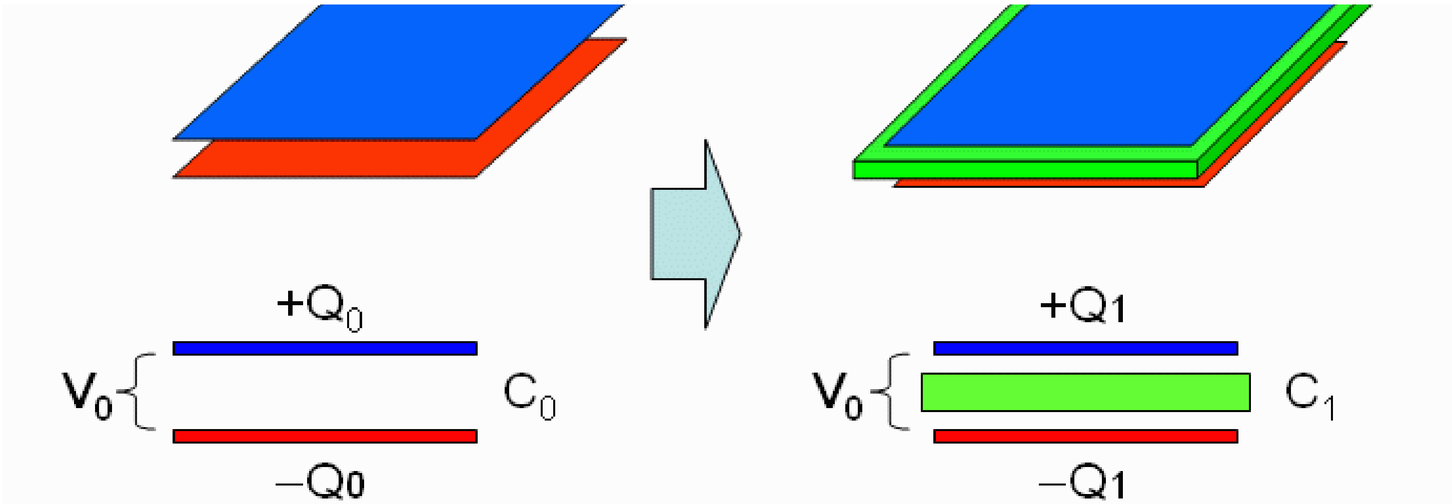
Same Q:

$$V_0 = E_0 d \quad \rightarrow \quad C_0 = Q_0 / E_0 d \quad \rightarrow \quad C_0 = \varepsilon_0 A / d$$

$$V_1 = E_0 (d - t) \quad \rightarrow \quad C_1 = Q_0 / (E_0 (d - t)) \quad \rightarrow \quad C_1 = \varepsilon_0 A / (d - t)$$

# Back to Check Point 4

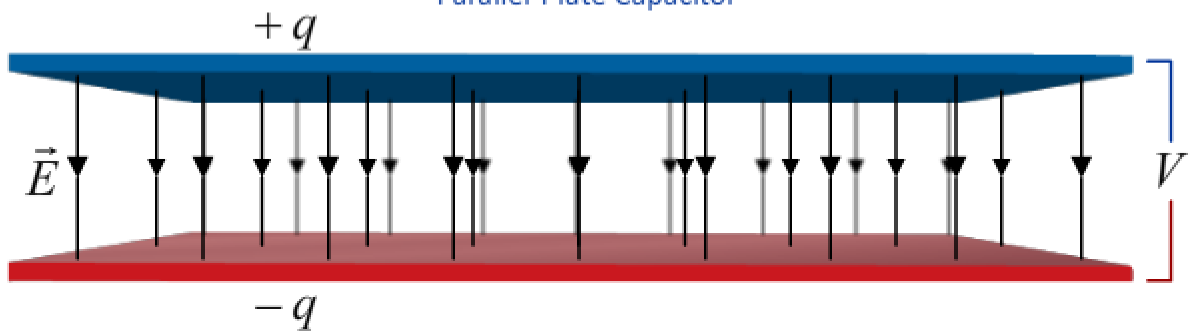
Two parallel plates are given a charge $Q_0$ such that the potential difference between the plates is $V_0$. If a conductor is slid between plates, <u>how would charge need to be adjusted to keep same potential difference?</u>




A) $Q_1 < Q_o$ B) $Q_1 = Q_o$ C) $Q_1 > Q_o$

"delta V= E*d, and d is smaller for the second plates as there is an uncharged conducting plate where E=0 inside. As a result, E has to be greater for the second plates, and so Q1 is greater than Q0."

# Main Point 3: Capacitors Store Energy in E

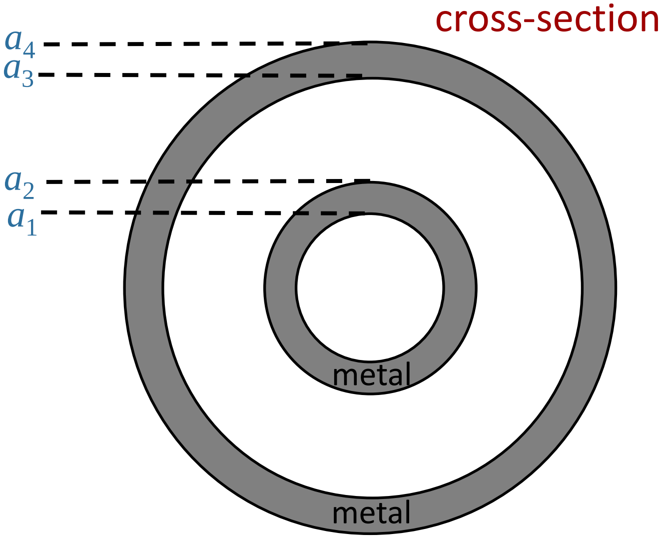



$$u = \frac{1}{2}\,\varepsilon_o E^2$$ Energy Density

Energy Stored in Capacitors

$$U = \frac{1}{2}QV$$ or $$U = \frac{1}{2}\frac{Q^2}{C}$$ or $$U = \frac{1}{2}CV^2$$

# Calculation




A capacitor is constructed from two conducting cylindrical tubes of radii $a_1$, $a_2$, $a_3$, and $a_4$ and length $L$ $(L >> a_i)$.

What is the capacitance $C$ of this capacitor ?

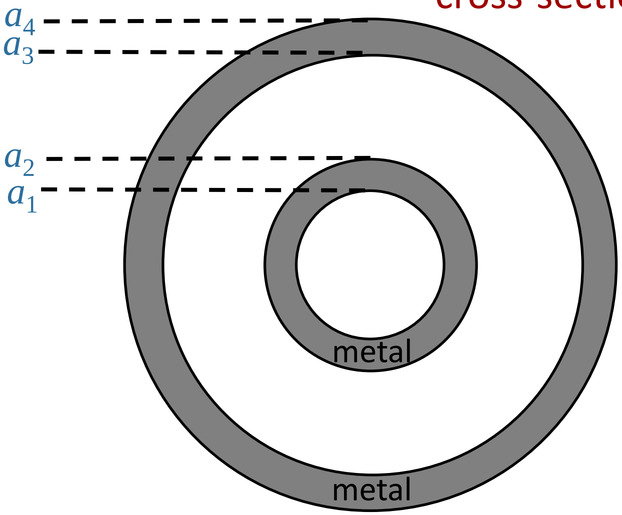
- **Conceptual Analysis:**

$$C \equiv \frac{Q}{V}$$

But what is $Q$ and what is $V$ ? They are not given?

- **Important Point:** $C$ is a property of the object! (concentric cylinders here)
  - Assume some $Q$ (i.e., $+Q$ on one conductor and $-Q$ on the other)
  - These charges create $E$ field in region between conductors
  - This $E$ field determines a potential difference $V$ between the conductors
  - $V$ should be proportional to $Q$; the ratio $Q/V$ is the capacitance.

# Calculation

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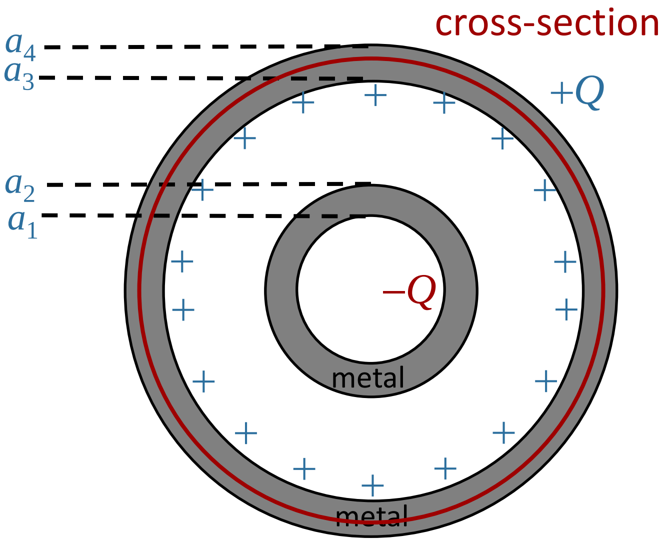
What is the capacitance $C$ of this capacitor ?

$$C \equiv \frac{Q}{V}$$

- Strategic Analysis:
  - Put $+Q$ on outer shell and $-Q$ on inner shell
  - Cylindrical symmetry: Use Gauss' Law to calculate $E$ everywhere
  - Integrate $E$ to get $V$
  - Take ratio $Q/V$: should get expression only using geometric parameters $(a_i, L)$

# Calculation

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What is the capacitance $C$ of this capacitor ?

$$C \equiv \frac{Q}{V}$$

Where is $+Q$ on outer conductor located?

A) at $r = a_4$ B) at $r = a_3$ C) both surfaces D) throughout tube

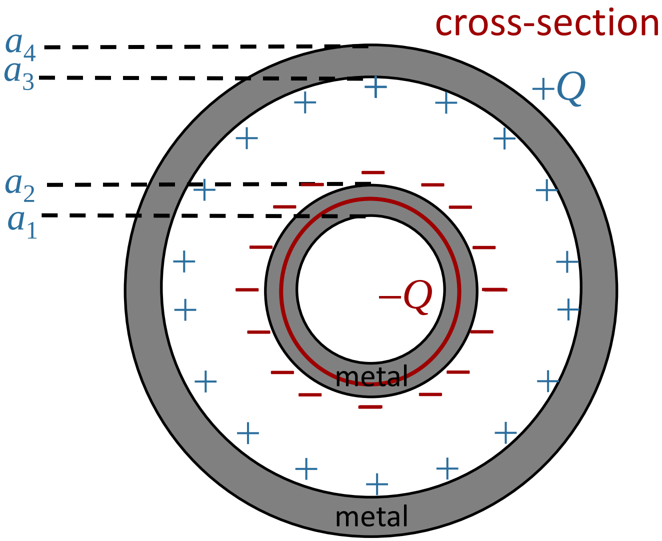
Why?

Gauss' law: $\int \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_o}$ → $Q_{enclosed} = 0$

We know that $E = 0$ in conductor (between $a_3$ and $a_4$)

$Q_{enclosed} = 0$ → $+Q$ must be on inside surface $(a_3)$, so that $Q_{enclosed} = +Q - Q = 0$

# Calculation




A capacitor is constructed from two conducting cylindrical tubes of radii $a_1$, $a_2$, $a_3$, and $a_4$ and length $L$ $(L >> a_i)$.

What is the capacitance $C$ of this capacitor ?

$$C \equiv \frac{Q}{V}$$

Where is –Q on inner conductor located?

A) at $r = a_2$ B) at $r = a_1$ C) both surfaces D) throughout tube

Why?

Gauss' law: $\int \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_o}$

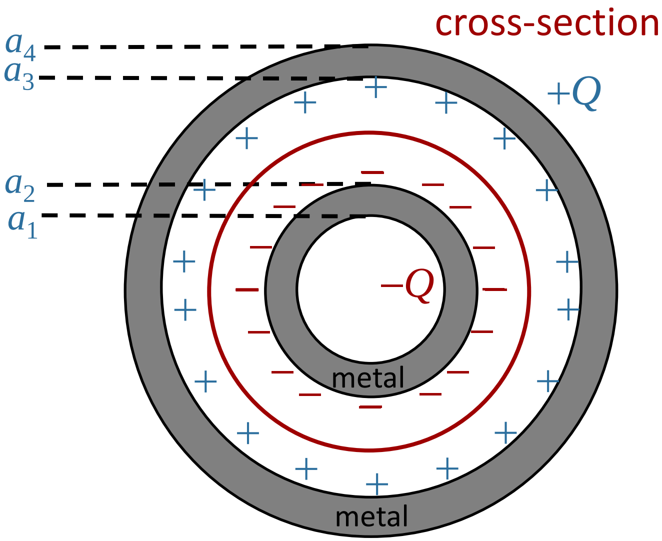
$$Q_{enclosed} = 0$$

We know that $E = 0$ in conductor (between $a_1$ and $a_2$)

$$Q_{enclosed} = 0$$

$+Q$ must be on outer surface $(a_2)$, so that $Q_{enclosed} = 0$

# Calculation




A capacitor is constructed from two conducting cylindrical tubes of radii $a_1$, $a_2$, $a_3$, and $a_4$ and length $L$ $(L >> a_i)$.

What is the capacitance $C$ of this capacitor ?

$$C \equiv \frac{Q}{V}$$

$a_2 < r < a_3$: What is $|E(r)|$?

A) $0$

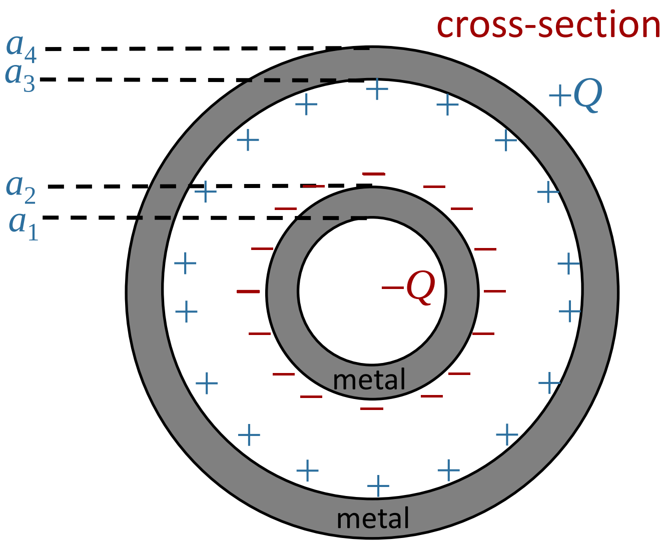
B) $\dfrac{1}{4\pi\varepsilon_o}\dfrac{Q}{r^2}$

C) $\dfrac{1}{2\pi\varepsilon_o}\dfrac{Q}{Lr}$

D) $\dfrac{1}{2\pi\varepsilon_o}\dfrac{2Q}{Lr}$

E) $\dfrac{1}{4\pi\varepsilon_o}\dfrac{2Q}{r^2}$

# Calculation




A capacitor is constructed from two conducting cylindrical tubes of radii $a_1$, $a_2$, $a_3$, and $a_4$ and length $L$ ($L >> a_i$).

What is the capacitance $C$ of this capacitor ?

$$C \equiv \frac{Q}{V} \qquad a_2 < r < a_3: \quad E = \frac{1}{2\pi\varepsilon_0}\frac{Q}{Lr}$$

What is $V \equiv V_{outer} - V_{inner}$?

(A) $\dfrac{Q}{2\pi\varepsilon_o L}\ln\dfrac{a_1}{a_4}$

(B) $\dfrac{Q}{2\pi\varepsilon_o L}\ln\dfrac{a_4}{a_1}$

(C) $\dfrac{Q}{2\pi\varepsilon_o L}\ln\dfrac{a_3}{a_2}$

(D) $\dfrac{Q}{2\pi\varepsilon_o L}\ln\dfrac{a_2}{a_3}$

# Voltage across a parallel plate capacitor