ZheJiang Univ-**University of Illinois** Fall **2024**

ECE 313: Problem Set 3: Problems

Due: Sat, Oct 12th at 11:59:00 p.m. **Reading:**
ECE 313 Course Notes, Sections 2.4.3 - 2.7

Note on reading: For most sections of the course notes there are short answer questions at the end of the chapter. We recommend that after reading each section you try answering the short answer questions. Do not hand in; answers to the short answer questions are provided in the appendix of the notes.

Note on turning in homework: Homework is assigned on a weekly basis , . You must upload handwritten homework to BB. No typeset homework will be accepted. No late homework will be accepted. Please write on the top right corner of the first page:
NAME AS IT APPEARS ON Canvas
NETID
SECTION
PROBLEM SET #
Page numbers are encouraged but not required. Five points will be deducted for improper headings.

1. **[Triple your money in five weeks?]**
A New Yorker runs an investment management service that has the stated goal of doubling the value of his clients' investments in a week via day trading. His brochure boasts that, "On average, my clients triple their money in five weeks!" After poring over back issues of the Wall Street Journal you learn the truth: at the end of any week, the investments of his clients will have doubled with probability 0.5, and will have decreased by 50% with probability 0.5. Thus, at the end of the first week, an initial investment of \$32 will be worth either \$64 or \$16, each with probability 0.5. Performance in any week is independent of performance during the other weeks. Anxious to apply your new skills in probability theory, you decide to invest \$32, and to let that investment ride for five weeks (in fact, you decide not to even look at the stock prices until the five weeks are over). Let the random variable X denote the value in dollars of your investment at the end of a five week period.

   (a) What are the possible values of X?

   (b) What is the pmf of the random variable X?

(c) What is the expected value of X? Is the brochure accurate?

(d) What is the probability that you will lose money on your investment?

2. **[Geometric Random Variable]**
During a bad economy, a graduating ECE student goes to career fair booths in the technology sector (e.g., Google, Apple, Qualcomm, Texas Instruments, Motorola, etc) - and his/her likelihood of receiving an off-campus interview invitation after a career fair booth visit depends on how well he/she did in ECE 313. Specifically, an A in 313 results in a probability $p = 0.95$ of obtaining an invitation, whereas a C in 313 results in a probability of $p = 0.15$ of an invitation.

   (a) Give the pmf for the random variable Y that denotes the number of career fair booth visits a student must make before his/her first invitation including the visit that results in the invitation. Express your answer in terms of p.

   (b) On average, how many booth visits must an A student make before getting an off-campus interview invitation? How about a C student?

   (c) Assuming that each student visits 5 booths during a typical career fair, find the probability that an A student in 313 **will not** get an off-campus interview invitation. Similarly, find the probability that a C student in 313 **will** get an invitation during a typical career fair.

(d) Assume an A student already visited 10 booths without getting an interview, and a C student already visited 5 booths and received 2 interviews. If they both go visit another 5 additional booths, who has a higher probability of landing an interview?

3. **[Rolling dice]**
Suppose you have a fair blue die and a fair red die, and define the following two random variables.

   (a) Let X denote the number of times one must roll the blue die until the outcome 4 has occurred 3 times. Find the pmf, the expected value and the variance of X.

   (b) Let Y denote the number of times one must roll the red die until the outcomes 1 or 2 have occurred four times. Find the pmf, the expected value and the variance of Y.

4. **[Customer support center]**
   Suppose the number of calls into a customer support center in any time interval is a Poisson random variable with mean 4 calls per minute.

   (a) What is the probability that there will be two calls in an interval of 3 minutes?

   (b) What is the probability that there is are at least three calls in an interval of one minute?

5. **[Overbooked flights]**
   Suppose that 105 passengers hold reservations for a 100-passenger flight from Chicago to Champaign, with each passenger showing up at the gate with probability 0.8, independently of any other passenger.

   (a) Find the probability that all the passengers that show up at the gate get a seat in the flight.

   (b) On average, how many passengers show up at the gate?

(c) Explain why the number of no-shows can be modeled as a binomial random variable Y with parameters $(n = 105, p = 0.2)$.

(d) Notice that the probability that everyone who shows up gets to go can also be expressed as $P\{Y \geq 5\}$. Use the Poisson approximation to compute $P\{Y \geq 5\}$ and compare your answer to the "more exact" answer that you found in part (a).