1. **[Gambling with Dice]**
You roll a fair die. If you roll a 1, you win 12 dollars. If you roll a 2 or 3, you win 6 dollars. If you roll a 4, 5, or 6, you lose m dollars. Let X denote the amount of money you win (a negative amount indicates a loss). Note that X can take 3 values: 12, 6, and $-m$.

   (a) Find the pmf of the random variable X. Note that the pmf will depend on m.

   (b) If $E[X] = -1$, find m.

   (c) Fixing the value of m to the value you found in part (b), find the $\mathrm{Var}(X)$.

2. **[Illini T-Shirts]**
A bag contains 4 orange and 6 blue t-shirts. Two t-shirts are chosen from the bag at random. Let X denote the number of blue t-shirts chosen.

   (a) Find the pmf of X.

   (b) Find $E[(X + 1)(X + 2)]$.

3. **[Matching cards to boxes]**
Three boxes are placed on a table, with the i-th box containing a card with the number i, for $i = 1, 2, 3$. The cards are then removed from the boxes, randomly shuffled and placed back at random into the three boxes; all possibilities of which card is placed in which box are equally likely. Let X denote the number of boxes that get get back their original card.

   (a) Describe a suitable sample space Ω to describe the experiment. How many elements does Ω have?

   (b) Find the pmf of X. Hint: First argue that X can only take the values 0, 1, and 3.

   (c) Find $E[X]$.

   (d) Find $\mathrm{Var}(X)$.

. **[To diet or not to diet]**
Cookie Monster wants to eat a cookie but needs to go on a diet. So he decides to toss a fair coin to decide whether or not to eat the cookie. He will eat the cookie if the toss is heads. If the toss is tails, he does not give up immediately, but decides to flip the coin three more times. If the subsequent three tosses are all heads, then he eats the cookie. If not, he is good and refrains from eating the cookie.

   (a) Find the probability that Cookie Monster eats the cookie.

   (b) What is the probability that the first toss is heads, given that Cookie Monster eats the cookie.

5. **[Faulty solar cells]**
Suppose n solar cells are in a linear array, in positions 1 through n, for some $n \geq 4$. Suppose exactly two of the cells fail, with all possible choices of which two being equally likely.

   (a) What is the probability the two cells that fail are next to each other? Simplify your answer.

   (b) Given that at least one of the two failures is among the first four cells in the array, what is the conditional probability that both failures were among the first four cells in the array? Simplify your answer as much as possible.

6. **[Karnaugh Puzzle]**
Consider three events, A,B, and C in a probability space. Let $P(AB) = 0.25$, $P(AB^c) = 0.25$, $P(C) = 0.2$, and $P(ABC) = 0.1$. It is known that A and B are independent, and that A^c and C are mutually exclusive.

   (a) Exploit the independence of A and B to fill in the probabilities for all events in a 2-event Karnaugh map involving events A and B. Show all your steps clearly.

   (b) Now use the fact that A^c and C are mutually exclusive to fill in the probabilities for all events in a 3-event Karnaugh map involving the events A, B, and C. Show your steps clearly.

   (c) Find the numerical value of $P(A \cup B \cup C)$.