

Instructor

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- Assistant Professor / Researcher at ZJUI
- Algorithm Expert, Alistar, Senior Lecturer at Alibaba
- Ph.D at Tsinghua University
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Research Interest

- AI and Optimization: Learn to Optimize, Learn for Decision-making
- Power system
- Foundation model/ LLM
- E-commerce: Logistics and Digital supply chain

Services

- Supervisor and Reviewers for AI: ICLR, ICML, NeurIPS, AAAI, AISTATS etc.
- Supervisor and Reviewers for Power System: IEEE Transactions on Power Systems, etc.
- Supervisor and Reviewers for Optimization: IEEE Transactions on ITS, etc.

TA

- Jiashen Ren (任甲琿), Senior, EE
- Haoyu Zhu (朱皓宇), Senior, EE
- Zihan Zhou (周子涵), Senior, EE
- Xirui Yao (姚熙蕊), Senior, ECE
- Zhenbo Chen (陈桢博), Senior, EE
- Denghan Xiong (熊登翰), Senior, EE

Context

- Course Introduction
- RainClassRoom (雨课堂) Introduction
- Today's Lesson: Three-Dimensional Coordinate Systems
 - 3D Space
 - Surfaces and Solids
 - Distance and Spheres

Course Introduction

Basic Information

- Credits: 4
- Schedule: Tues/Thur 10:30-11:50 @ RC-2 101
- Discussion: Tues 18:00-19:50
- Instructor Office: ZJUI B312 | Email: qingchunhou@intl.zju.edu.cn
- Office hour: By appointment

Course Introduction

Basic Information

- Course Objectives:
 1. Understand the basic concepts and techniques of multivariable/vector calculus, including calculus of space curves, differentiation and integration of multivariable functions, and fundamental results of vector analysis.
 2. Solve problems using standard calculus techniques, focusing on modeling real-world problems in the language of multivariable/vector calculus.
- Key Topics: Analytic Geometry, Vector Functions, Differentiation, Integration, Vector Calculus

Course Introduction

Course Workflow & Evaluation

- 40 % final exam (closed book)
- 20 % midterm exam (closed book)
- 20 % homework
- 15 % discussion session work
- 5 % attendance
- 5 % in-class quiz bonus

Introduction to RainClassRoom (雨课堂)

RainClassRoom (雨课堂) : **An interactive classroom app developed by Tsinghua University**

Live Quizzes & Polls

- Embed quick questions (MCQ/True-False) into lectures to check understanding.
- Instant analytics show class-wide performance.

Attendance Tracking

- Auto-generated QR code for check-in via WeChat.
- Records participation in quizzes/polls as proof of attendance.

Interactive Tools

- Anonymous "Confusion Flags" for students to request clarification.
- "Bullet Chat" (live comments) to encourage questions/discussions.

Data Dashboard

- Export reports: attendance rates, quiz accuracy, individual engagement

Introduction to RainClassRoom (雨课堂)

Scan the QR code through Wechat

群聊: MATH241 25fall大群



该二维码7天内(9月20日前)有效, 重新进入将更新

Scan the QR code through Wechat

MATH241 - 2025Fall

班级容量500人



二维码有效期截至 2025-10-10

Vectors and the Geometry of Space

Lecture Notes for Calculus III

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ZJUI B312

12 Vectors and the Geometry of Space





12.1

Three-Dimensional Coordinate Systems

Three-Dimensional Coordinate Systems

To locate a point in a plane, two numbers are necessary.

We know that any point in the plane can be represented as an ordered pair (a, b) of real numbers, where a is the x -coordinate and b is the y -coordinate.

For this reason, a plane is called two-dimensional. To locate a point in space, three numbers are required.

We represent any point in space by an ordered triple (a, b, c) of real numbers.

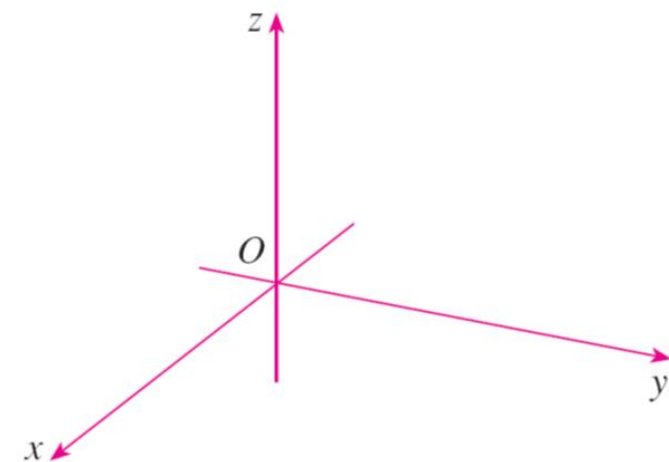


3D Space

3D Space (1 of 10)

In order to represent points in space, we first choose a fixed point O (the origin) and three directed lines through O that are perpendicular to each other, called the **coordinate axes** and labeled the x -axis, y -axis, and z -axis.

Usually we think of the x - and y -axis as being horizontal and the z -axis as being vertical, and we draw the orientation of the axes as in Figure 1.

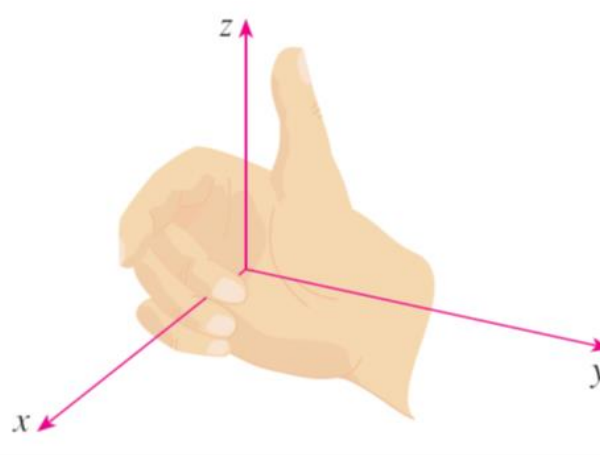


Coordinate axes

Figure 1

3D Space (2 of 10)

The direction of the z-axis is determined by the **right-hand rule** as illustrated in Figure 2:

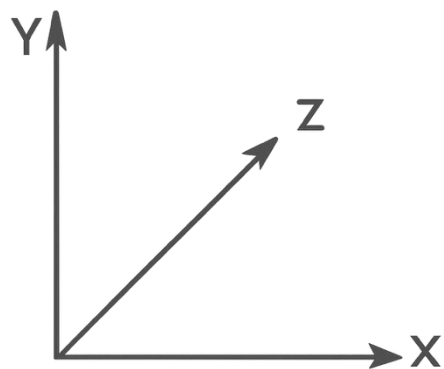


Right-hand rule

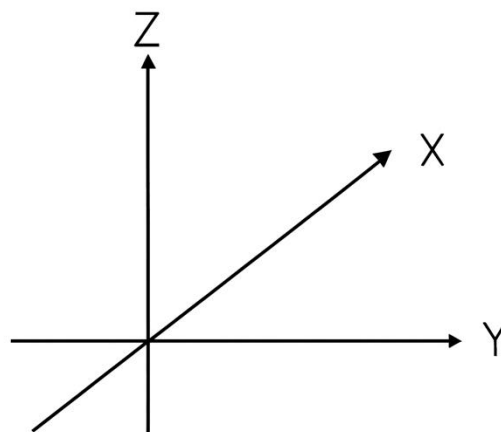
Figure 2

If you curl the fingers of your right hand around the z-axis in the direction of a 90° counterclockwise rotation from the positive x-axis to the positive y-axis, then your thumb points in the positive direction of the z-axis.

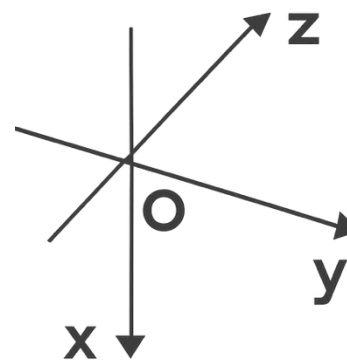
MAQ: Choose all incorrect 3-D coordinate system according to the right-hand rule.



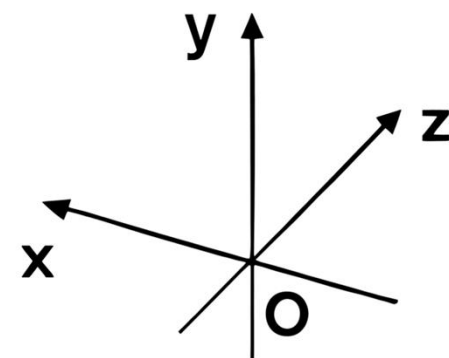
A



B



C



D

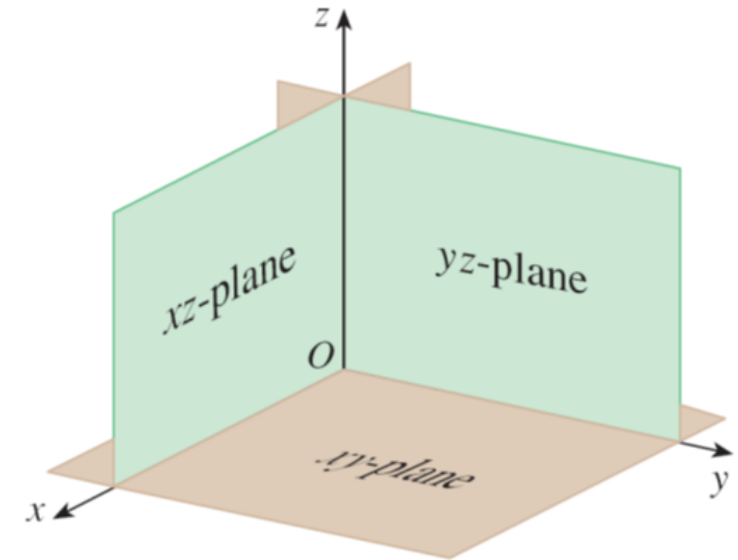
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3D Space (3 of 10)

The three coordinate axes determine the three **coordinate planes** illustrated in Figure 3(a).

The xy -plane is the plane that contains the x - and y -axes; the yz -plane contains the y - and z -axes; the xz -plane contains the x - and z -axes.

These three coordinate planes divide space into eight parts, called **octants**. The **first octant**, in the foreground, is determined by the positive axes.



Coordinate planes

Figure 3(a)

3D Space (4 of 10)

Because many people have some difficulty visualizing diagrams of three-dimensional figures, you may find it helpful to do the following [see Figure 3(b)].

Look at any bottom corner of a room and call the corner the origin.

The wall on your left is in the xz -plane, the wall on your right is in the yz -plane, and the floor is in the xy -plane.

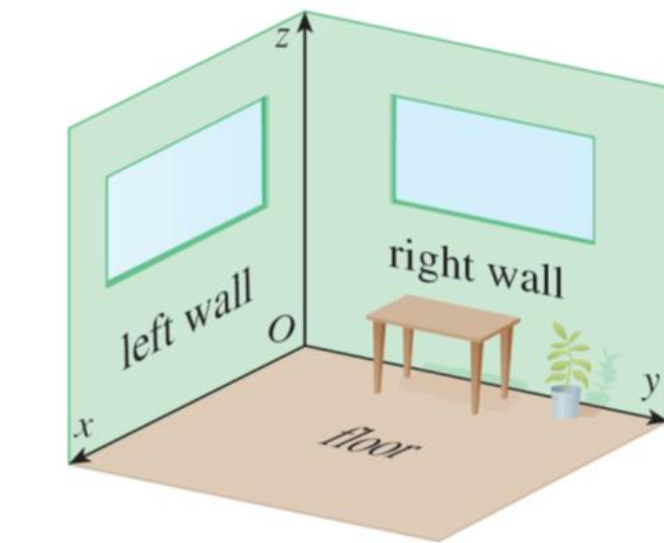


Figure 3(b)

3D Space (5 of 10)

The x -axis runs along the intersection of the floor and the left wall.

The y -axis runs along the intersection of the floor and the right wall.

The z -axis runs up from the floor toward the ceiling along the intersection of the two walls.

You are situated in the first octant, and you can now imagine seven other rooms situated in the other seven octants (three on the same floor and four on the floor below), all connected by the common corner point O .

3D Space (6 of 10)

Now if P is any point in space, let a be the (directed) distance from the yz -plane to P , let b be the distance from the xz -plane to P , and let c be the distance from the xy -plane to P .

We represent the point P by the ordered triple (a, b, c) of real numbers and we call a , b , and c the **coordinates** of P ; a is the x -coordinate, b is the y -coordinate, and c is the z -coordinate.

3D Space (7 of 10)

Thus, to locate the point (a, b, c) , we can start at the origin O and move a units along the x -axis, then b units parallel to the y -axis, and then c units parallel to the z -axis as in Figure 4.

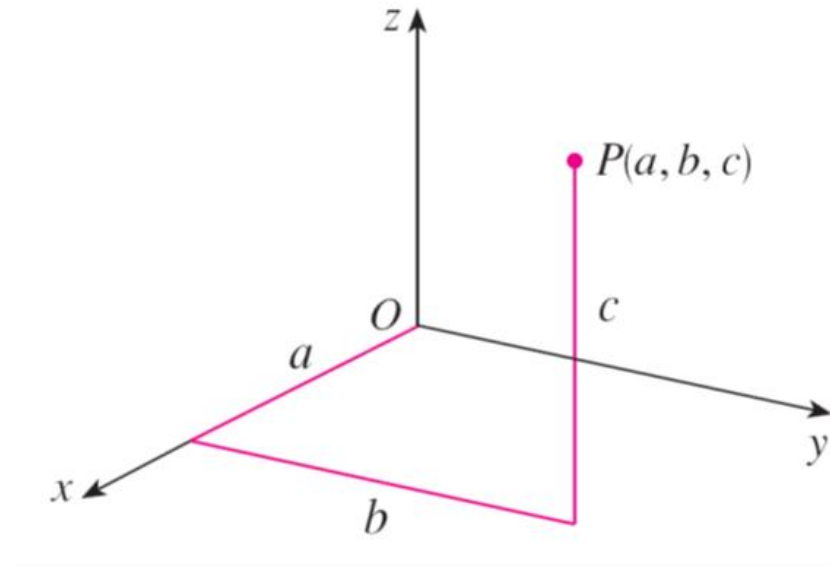


Figure 4

3D Space (8 of 10)

The point $P(a, b, c)$ determines a rectangular box as in Figure 5.

If we drop a perpendicular from P to the xy -plane, we get a point Q with coordinates $(a, b, 0)$ called the **projection** of P onto the xy -plane.

Similarly, $R(0, b, c)$ and $S(a, 0, c)$ are the projections of P onto the yz -plane and xz -plane, respectively.

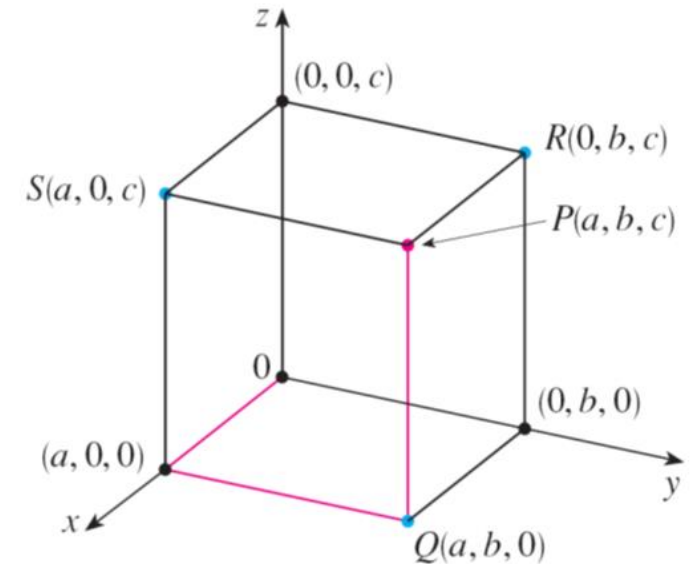


Figure 5

MCQ: Which of the following points is on the yz -plane.

- ☐ A $(3,0,5)$
- ☒ B $(0,3,5)$
- ☐ C $(3,5,0)$

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3D Space (9 of 10)

As numerical illustrations, the points $(-4, 3, -5)$ and $(3, -2, -6)$ are plotted in Figure 6.

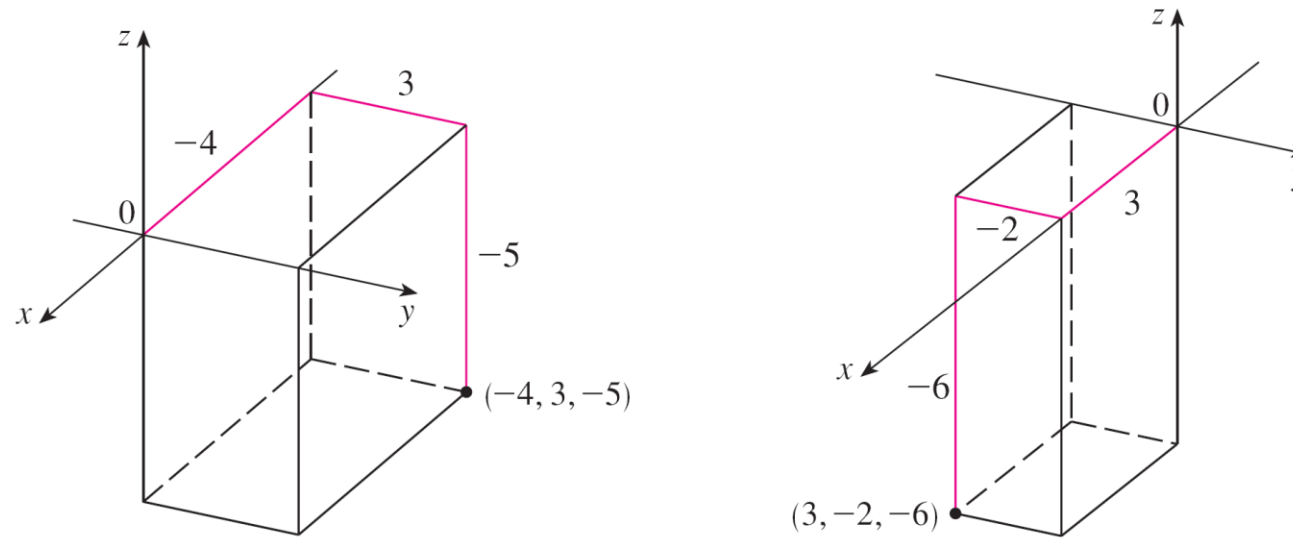


Figure 6

3D Space (10 of 10)

The Cartesian product $\mathbb{R} \times \mathbb{R} \times \mathbb{R} = \{(x, y, z) | x, y, z \in \mathbb{R}\}$ is the set of all ordered triples of real numbers and is denoted by \mathbb{R}^3 .

We have given a one-to-one correspondence between points P in space and ordered triples (a, b, c) in \mathbb{R}^3 . It is called a **three-dimensional rectangular coordinate system**.

Notice that, in terms of coordinates, the first octant can be described as the set of points whose coordinates are all positive.



Surfaces and Solids

Surfaces and Solids (1 of 2)

In two-dimensional analytic geometry, the graph of an equation involving x and y is a curve in \mathbb{R}^2 .

In three-dimensional analytic geometry, an equation in x , y , and z represents a *surface* in \mathbb{R}^3 .

Example 1

What surfaces in \mathbb{R}^3 are represented by the following equations?

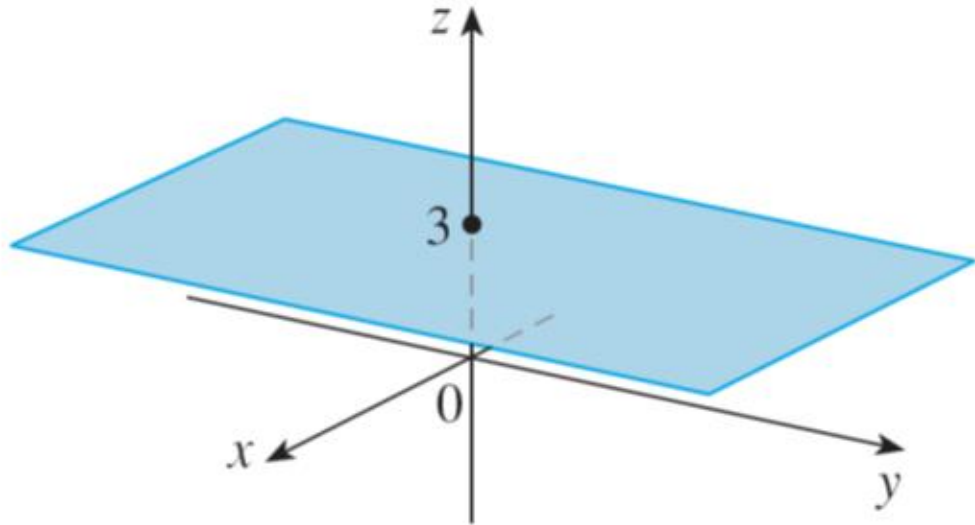
(a) $z = 3$ (b) $y = 5$

Solution:

(a) The equation $z = 3$ represents the set $\{(x, y, z) \mid z = 3\}$, which is the set of all points in \mathbb{R}^3 whose z -coordinate is 3. (x and y can each be any value).

Example 1 – Solution (1 of 2)

This is the horizontal plane that is parallel to the xy -plane and three units above it as in Figure 7(a).

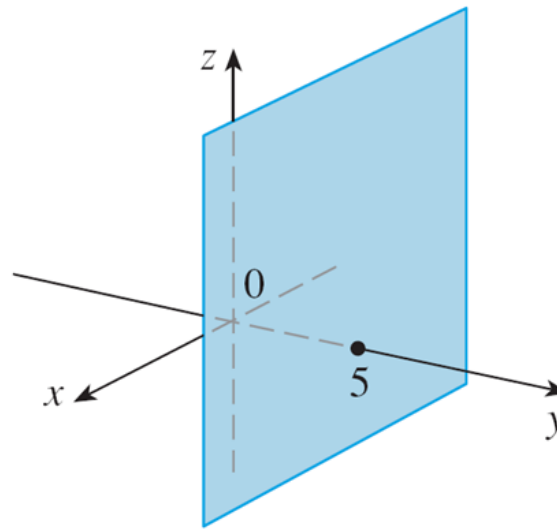


$z = 3$, a plane in \mathbb{R}^3

Figure 7(a)

Example 1 – Solution (2 of 2)

- (b) The equation $y = 5$ represents the set of all points in \mathbb{R}^3 whose y -coordinate is 5. This is the vertical plane that is parallel to the xz -plane and five units to the right of it as in Figure 7(b).



$y = 5$, a plane in \mathbb{R}^3

Figure 7(b)

Surfaces and Solids (2 of 2)

In general, if k is a constant, then $x = k$ represents a plane parallel to the yz -plane, $y = k$ is a plane parallel to the xz -plane, and $z = k$ is a plane parallel to the xy -plane.

In Figure 5, the faces of the rectangular box are formed by the three coordinate planes $x = 0$ (the yz -plane), $y = 0$ (the xz -plane), and $z = 0$ (the xy -plane), and the planes $x = a$, $y = b$, and $z = c$.

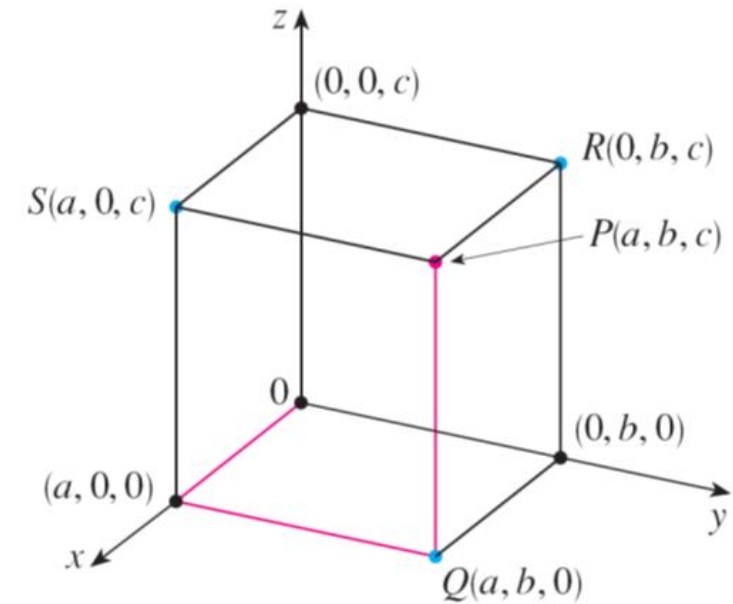


Figure 5

MCQ: What does equation $x^2 + y^2 \leq 1, 2 \leq z \leq 4$ represent in \mathbb{R}^3

- ☐ A A ring (doughnut-shaped region) with radius 1 along the z -axis, between $z=2$ and $z=4$.
- ☒ B A cylinder with radius 1, extending vertically between $z=2$ and $z=4$.
- ☐ C A cone with apex at the origin, extending from $z=2$ to $z=4$.
- ☐ D A sphere with radius 1, centered at the origin, limited between $z=2$ and $z=4$.

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Distance and Spheres

Distance and Spheres (1 of 2)

The familiar formula for the distance between two points in a plane is easily extended to the following three-dimensional formula.

Distance Formula in Three Dimensions The distance $|P_1P_2|$ between the points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ is

$$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Example 4

The distance from the point $P(2, -1, 7)$ to the point $Q(1, -3, 5)$ is

$$|PQ| = \sqrt{(1-2)^2 + (-3+1)^2 + (5-7)^2} = \sqrt{1+4+4} = 3$$

Distance and Spheres (2 of 2)

Equation of a Sphere An equation of a sphere with center $C(h, k, l)$ and radius r is

$$(x - h)^2 + (y - k)^2 + (z - l)^2 = r^2$$

In particular, if the center is the origin O , then an equation of the sphere is

$$x^2 + y^2 + z^2 = r^2$$

Recap

- Course Introduction
- RainClassRoom (雨课堂) Introduction
- Today's Lesson: Three-Dimensional Coordinate Systems
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