

Calculus III (Math 241)

1. Consider the optimization problem

$$\text{Minimize } f(x, y, z) = x^2 + y^2 + z^2 \quad \text{subject to} \quad x + 2y + 3z = 1.$$

- Use the method of Lagrange multipliers to find the values of λ .
- Determine whether the critical point you found gives a minimum or maximum value of f .

2. For the following two functions, analyze their critical points and classify them using the second derivative test.

$$(a) \ f_1(x, y) = x^2 - 3xy + 2y^2.$$

$$(b) \ f_2(x, y) = x^3 - 6xy + 3y^2.$$

For each function:

- Find all critical points;
- Compute f_{xx} , f_{yy} , and f_{xy} ;
- Evaluate $D = f_{xx}f_{yy} - (f_{xy})^2$ and classify each critical point as a local maximum, minimum, or saddle point.