Assignment Previewer

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EDIT ASSIGNMENT

INSTRUCTOR
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HW9 (Homework)

Current Score: – / 34 POINTS | 0.0 %

Scoring and Assignment Information

QUESTION	1	2	3	4	5	6	7	8	9	10	11	12
POINTS	– / 1	– / 13	– / 1	– / 1	– / 1	– / 3	– / 5	– / 1	– / 2	– / 1	– / 3	– / 2

Assignment Submission

For this assignment, you submit answers by question parts. The number of submissions remaining for each question part only changes if you submit or change the answer.

Assignment Scoring

Your best submission for each question part is used for your score.

1. [–/1 Points] DETAILS

SCalcET9M 15.2.053.

Use a computer algebra system to find the exact volume of the solid.

enclosed by $z = 16 - x^2 - y^2$ and $z = 0$

✕ 128π

Solution or Explanation

The two surfaces intersect in the circle $x^2 + y^2 = 16$, $z = 0$ and the region of integration is the disk D: $x^2 + y^2 \le 16$.

Using a CAS, the volume is $\iint_D \left(16 - x^2 - y^2\right) dA = \int_{-4}^{4} \int_{-\sqrt{16 - x^2}}^{\sqrt{16 - x^2}} \left(16 - x^2 - y^2\right) dy\, dx = 128\pi$.

2. [–/13 Points] DETAILS

SCalcET9M 15.1.001.MI.SA.

This question has several parts that must be completed sequentially. If you skip a part of the question, you will not receive any points for the skipped part, and you will not be able to come back to the skipped part.

Estimate the volume of the solid that lies below the surface $z = xy$ and above the following rectangle.

$$R = \left\{(x, y) \mid 9 \le x \le 15,\ 7 \le y \le 11\right\}$$

Exercise (a)

Use a Riemann sum with $m = 3$, $n = 2$, and take the sample point to be the upper right corner of each square.

Click here to begin!

Exercise (b)

Use the Midpoint Rule to estimate the volume of the solid.

Click here to begin!

3. [– / 1 Points] DETAILS

SCalcET9M 15.1.VE.001.

Watch the video below then answer the question.

Click **here** to view the transcript

$\int_0^x (2xy + 3)\, dy = x^3 + 3x.$

- True
- False

Solution or Explanation

True

4. [– / 1 Points] DETAILS

SCalcET9M 15.2.072.

Find the average value of *f* over the region *D*.

$f(x, y) = 3x \sin(y)$, D is enclosed by the curves $y = 0$, $y = x^2$, and $x = 6$

✕ $\frac{1}{48}(36 - \sin(36))$

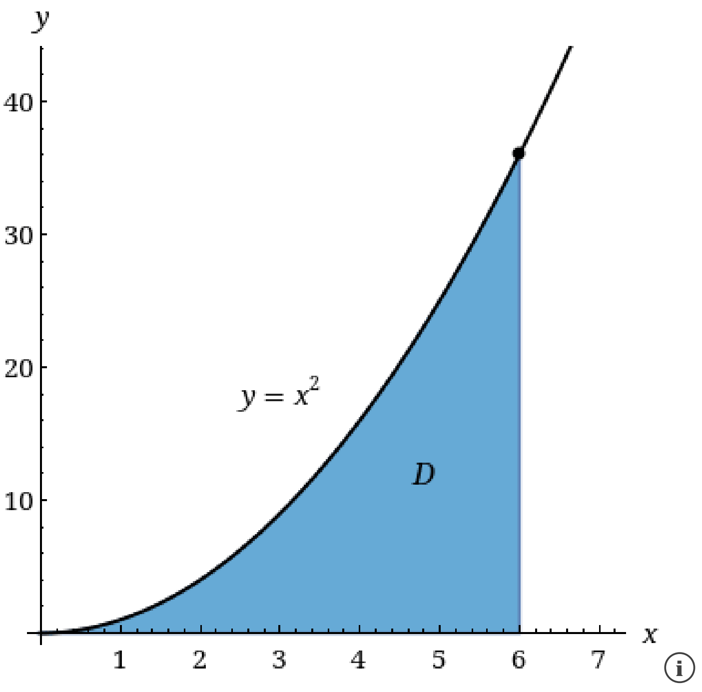
Solution or Explanation

Here $D = \left\{(x, y) \mid 0 \le x \le 6, 0 \le y \le x^2\right\}$, so $A(D) = \int_0^6 x^2\,dx = \frac{1}{3}x^3\Big]_0^6 = 72$ and

$$f_{avg} = \frac{1}{A(D)}\iint_D f(x, y)dA = \frac{1}{72}\int_0^6\int_0^{x^2} 3x\sin(y)\,dy\,dx$$

$$= \frac{1}{72}\int_0^6\Big[-3x\cos(y)\Big]_{y=0}^{y=x^2}dx = \frac{1}{24}\int_0^6\left[x - x\cos(x^2)\right]dx$$

$$= \frac{1}{24}\left[\frac{1}{2}x^2 - \frac{1}{2}\sin(x^2)\right]_0^6 = \frac{1}{24}\left(18 - \frac{1}{2}\sin(36) - 0\right) = \frac{1}{48}\left(36 - \sin(36)\right)$$



5. [– / 1 Points] DETAILS

SCalcET9M 15.2.079.

Use geometry or symmetry, or both, to evaluate the double integral.

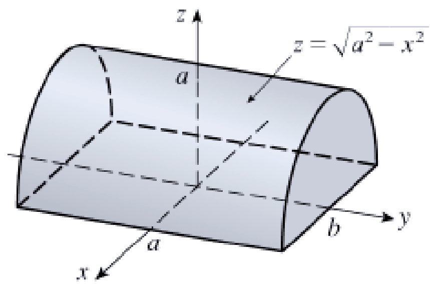
$$\iint_D (3ax^3 + 9by^3 + \sqrt{a^2 - x^2})\,dA, \quad D = [-a, a] \times [-b, b]$$

✗ $\pi a^2 b$

Solution or Explanation

$\iint_D (3ax^3 + 9by^3 + \sqrt{a^2 - x^2})\,dA = 3\iint_D ax^3\,dA + 9\iint_D by^3\,dA + \iint_D \sqrt{a^2 - x^2}\,dA$. Now ax^3 is odd with respect to x and by^3 is odd with respect to y, and the region of integration is symmetric with respect to both x and y, so $3\iint_D ax^3\,dA = 9\iint_D by^3\,dA = 0$. $\iint_D \sqrt{a^2 - x^2}\,dA$ represents the volume of the solid region under the graph of $z = \sqrt{a^2 - x^2}$ and above the rectangle D, namely a half circular cylinder with radius a and length $2b$ (see the figure) whose volume is $\frac{1}{2} \cdot \pi r^2 h = \frac{1}{2}\pi a^2(2b) = \pi a^2 b$. Thus

$$\iint_D (3ax^3 + 9by^3 + \sqrt{a^2 - x^2})\,dA = 0 + 0 + \pi a^2 b = \pi a^2 b.$$



Resources

Watch It

6. [– / 3 Points]

DETAILS

SCalcET9M 15.2.011.EP.

Consider the following.

$$\iint_D \frac{y}{x^2+1}\,dA, \quad D = \left\{(x, y) \mid 0 \le x \le 3,\ 0 \le y \le \sqrt{x}\right\}$$

Rewrite the above as an iterated integral.

$$\int_0^{\square\ ✗\ 3} \int_{\square\ ✗\ 0}^{\sqrt{x}} \frac{y}{x^2+1}\,dy\,dx$$

Evaluate the double integral.

✗ $\dfrac{\ln(10)}{4}$

Solution or Explanation

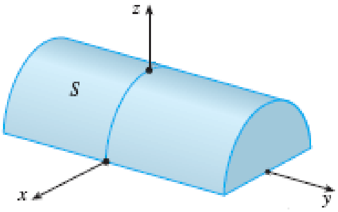
$$\iint_D \frac{y}{x^2+1}\,dA = \int_0^3 \int_0^{\sqrt{x}} \frac{y}{x^2+1}\,dy\,dx = \int_0^3 \left[\frac{1}{x^2+1} \cdot \frac{y^2}{2}\right]_{y=0}^{y=\sqrt{x}} dx = \frac{1}{2}\int_0^3 \frac{x}{x^2+1}\,dx$$

$$= \frac{1}{2}\left[\frac{1}{2}\ln(|x^2+1|)\right]_0^3 = \frac{1}{4}\left[\ln(x^2+1)\right]_0^3 = \frac{1}{4}(\ln(10) - \ln(1)) = \frac{1}{4}\ln(10)$$

7. [– / 5 Points]

DETAILS

SCalcET9M 15.1.AE.002.



Video Example

EXAMPLE 2 If $R = \{(x, y) \mid -1 \le x \le 1, -9 \le y \le 9\}$, evaluate the integral

$$\iint_R \sqrt{1 - x^2}\, dA.$$

SOLUTION It would be very difficult to evaluate this integral directly but, because $\sqrt{1 - x^2} \ge 0$, we can compute the integral by interpreting it as a volume. If $z = \sqrt{1 - x^2}$, then

$x^2 + z^2 =$ [ ] ✗ 1 and $z \ge 0$, so the given double integral represents the volume of the solid S that lies below the circular cylinder $x^2 + z^2 =$ [ ] ✗ 1 and above the rectangle R. (See the figure.) The volume of S is the area of a semicircle with radius [ ] ✗ 1 times the length of the cylinder. Thus

$$\iint_R \sqrt{1 - x^2}\, dA = \frac{1}{2}\pi(\ [\ ]\ ✗\ 1\ )^2 \cdot 18$$

= [ ]

✗ 9π.

8. [– / 1 Points] DETAILS

SCalcET9M 15.1.055.

Use symmetry to evaluate the double integral.

$$\iint_R \frac{9xy}{1+x^{10}}\,dA, \quad R = \{(x, y) \mid -4 \le x \le 4, 0 \le y \le 1\}$$

✕ 0

Solution or Explanation

$\iint_R \frac{9xy}{1+x^{10}}\,dA = \int_{-4}^{4}\int_0^1 \frac{9xy}{1+x^{10}}\,dy\,dx = 9\int_{-4}^{4} \frac{x}{1+x^{10}}\,dx \int_0^1 y\,dy$ [by the equation] but $f(x) = \frac{x}{1+x^{10}}$ is an odd function so $\int_{-4}^{4} f(x)dx = 0$ by the equations. Thus $\iint_R \frac{9xy}{1+x^{10}}\,dA = 0 \cdot \int_0^1 y\,dy = 0$.

Resources

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9. [– / 2 Points] DETAILS

SCalcET9M 15.2.070.

Use the property to estimate the best possible bounds of the integral.

$$\iint_T 4\sin^4(x+y)\,dA,$$

T is the triangle enclosed by the lines $y = 0$, $y = 9x$, and $x = 3$.

✕ 0 $\le \iint_T 4\sin^4(x+y)\,dA \le$ ✕ 162

Solution or Explanation

Click to View Solution

10. [– / 1 Points]

DETAILS

SCalcET9M 15.1.054.

Find the average value of f over the given rectangle.

$$f(x, y) = 2e^y\sqrt{x + e^y}, \quad R = [0, 10] \times [0, 1]$$

f_{ave} =

✗ $\boxed{\frac{4}{75}\left((10+e)^{\frac{5}{2}} - (11)^{\frac{5}{2}} - e^{\frac{5}{2}} + 1\right)}$

Solution or Explanation

Click to View Solution

11. [– / 3 Points]

DETAILS

SCalcET9M 15.2.017.

Consider the following.

$$\iint_D x\,dA, \quad D \text{ is enclosed by the lines } y = x,\ y = 0,\ x = 2$$

Express D as a region of type I.

- $D = \{(x, y) \mid y \le x \le 2, 0 \le y \le x\}$
- $D = \{(x, y) \mid 0 \le x \le y, 0 \le y \le 2\}$
- $D = \{(x, y) \mid 0 \le x \le 2, 0 \le y \le x\}$
- $D = \{(x, y) \mid 0 \le x \le y, 0 \le y \le x\}$
- $D = \{(x, y) \mid 0 < x < y, 0 < y < x\}$

Express D as a region of type II.

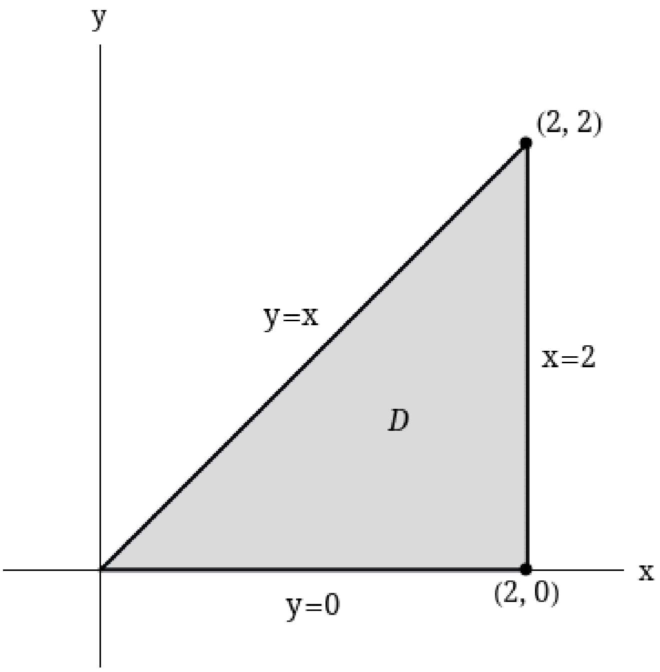
- $D = \{(x, y) \mid 0 \le y \le 2, 0 \le x \le 2\}$
- $D = \{(x, y) \mid 0 \le y \le x, 0 \le x \le y\}$
- $D = \{(x, y) \mid 0 \le y \le 2, 0 \le x \le y\}$
- $D = \{(x, y) \mid 0 \le y \le x, y \le x \le 2\}$
- $D = \{(x, y) \mid 0 \le y \le 2, y \le x \le 2\}$

Evaluate the double integral in two ways.

$\frac{8}{3}$

Solution or Explanation

As a type I region, D lies between the lower boundary $y = 0$ and the upper boundary $y = x$ for $0 \le x \le 2$, so $D = \{(x, y) \mid 0 \le x \le 2, 0 \le y \le x\}$. If we describe D as a type II region, D lies between the left boundary $x = y$ and the right boundary $x = 2$ for $0 \le y \le 2$, so $D = \{(x, y) \mid 0 \le y \le 2, y \le x \le 2\}$.



Thus $\iint_D x\, dA = \int_0^2 \int_0^x x\, dy\, dx = \int_0^2 \left[xy\right]_{y=0}^{y=x} dx = \int_0^2 x^2\, dx = \left[\frac{1}{3}x^3\right]_0^2 = \frac{1}{3}(8 - 0) = \frac{8}{3}$ or

$\iint_D x\, dA = \int_0^2 \int_y^2 x\, dx\, dy = \int_0^2 \left[\frac{1}{2}x^2\right]_{x=y}^{x=2} dy = \frac{1}{2}\int_0^2 (4 - y^2) dy = \frac{1}{2}\left[4y - \frac{1}{3}y^3\right]_0^2 = \frac{1}{2}\left[\left(4 \cdot 2 - \frac{1}{3} \cdot 8\right) - 0\right]$

$= \frac{8}{3}.$

Resources

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12. [– / 2 Points] DETAILS

SCalcET9M 15.1.001.MI.

Estimate the volume of the solid that lies below the surface $z = xy$ and above the following rectangle.

$$R = \{(x, y) \mid 10 \le x \le 16,\ 0 \le y \le 4\}$$

(a) Use a Riemann sum with $m = 3$, $n = 2$, and take the sample point to be the upper right corner of each square.

1008

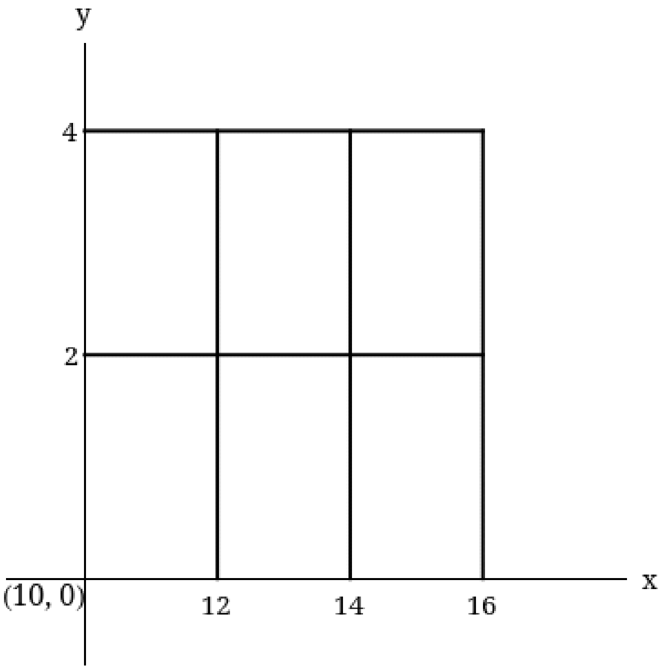
(b) Use the Midpoint Rule to estimate the volume of the solid.

624

Solution or Explanation

(a) The subrectangles are shown in the figure. The surface is the graph of $f(x,y) = xy$ and $\Delta A = 4$, so we estimate

$$V \approx \sum_{i=1}^{3}\sum_{j=1}^{2} f(x_i, y_j)\Delta A = f(12, 2)\Delta A + f(12, 4)\Delta A + f(14, 2)\Delta A + f(14, 4)\Delta A + f(16, 2)\Delta A + f(16, 4)\Delta A$$
$$= 24(4) + 48(4) + 28(4) + 56(4) + 32(4) + 64(4) = 1008$$



(b) $$V \approx \sum_{i=1}^{3}\sum_{j=1}^{2} f(\bar{x}_i, \bar{y}_j)\Delta A = f(11, 1)\Delta A + f(11, 3)\Delta A + f(13, 1)\Delta A + f(13, 3)\Delta A + f(15, 1)\Delta A + f(15, 3)\Delta A$$
$$= 11(4) + 33(4) + 13(4) + 39(4) + 15(4) + 45(4) = 624$$

Resources

Master It