Problem1

Suppose X and Y are random variables with joint density function

$$f(x, y) = \begin{cases} 0.1e^{-(0.5x+0.2y)} & \text{if } x \geqslant 0,\ y \geqslant 0 \\ 0 & \text{otherwise} \end{cases}$$

(a) Verify that f is indeed a joint density function.
(b) Find the following probabilities.
(i) $P(Y \geqslant 1)$ (ii) $P(X \leqslant 2, Y \leqslant 4)$
(c) Find the expected values of X and Y.

Problem2

From an old final exam

Let $K = \{(x,y,z) \in \mathbb{R}^3; \sqrt{x^2+y^2} \leq z(1-z)\}$.

a) Show that K is compact (i.e., closed and bounded).
b) Determine the volume $\text{vol}(K)$.