Problem 1

The introduced technique may have been introduced in ECE313, when normalizing the Gauss distribution, To avoid the usage of polar coordinate transformation, we use another famous example.

Prove Dirichlet Integral $I = \int_0^\infty \frac{\sin(x)}{x} dx = \frac{\pi}{2}$ in following step:

(a) Prove when x > 0, $\frac{1}{x} = \int_0^\infty e^{-xy}\, dy$

(b) Use the result of (a), rewrite the Dirichlet Integral into iterated integral

(c) Use Fubini-Tonelli theorem, calculate the iterated integral

(d) (optional), justify the usage of Fubini-Tonelli theorem

Problem 2

A question from previous final (around 2022)

Question 2 (ca. 12 marks)

Let $f\colon \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x, y) = x^4 - x^3y - xy + y^2.$$

a) Which obvious symmetry property does f have? What can you conclude from this about the graph and the contours of f?

b) Determine all critical points of f and their types.
Hint: There are 5 critical points.

c) Does f have a global extremum?

d) Determine the extrema of f on the unit square $Q = \{(x, y) \in \mathbb{R}^2; 0 \le x, y \le 1\}$.

(Hint for (d), Extrema located in Q must be critical points, besides we need to plug in values and study the situation of 4 planes)