1. **(total 4 points, 2 points each)**

   (a) Let $F(x,y,z) = x^2 + 2yz - 3$. Then S is the level surface $F(x,y,z) = 0$.

$$\nabla F(x,y,z) = \left(\frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}, \frac{\partial F}{\partial z}\right) = (2x, 2z, 2y)$$

   For $Q = (0, \frac{3}{2}, 1)$:

$$\nabla F(Q) = (0, 2, 3)$$

   Tangent plane equation:

$$0(x-0) + 2(y - \frac{3}{2}) + 3(z-1) = 0 \Rightarrow 2y + 3z = 6$$

   (b) For $f(x,y,z) = \sqrt{x^2+y^2+z^2}$, at point $(3,2,6)$:

$$f(3,2,6) = \sqrt{9+4+36} = \sqrt{49} = 7$$

   . The gradient:

$$\nabla f = \left(\frac{x}{\sqrt{x^2+y^2+z^2}}, \frac{y}{\sqrt{x^2+y^2+z^2}}, \frac{z}{\sqrt{x^2+y^2+z^2}}\right) = \left(\frac{3}{7}, \frac{2}{7}, \frac{6}{7}\right)$$

   The linear approximation is:

$$L(x,y,z) = 7 + \frac{3}{7}(x-3) + \frac{2}{7}(y-2) + \frac{6}{7}(z-6)$$

   For $\sqrt{3.02^2 + 1.97^2 + 5.99^2}$:

$$\Delta x = 0.02, \quad \Delta y = -0.03, \quad \Delta z = -0.01$$

$$L \approx 7 + \frac{3}{7}(0.02) + \frac{2}{7}(-0.03) + \frac{6}{7}(-0.01) \approx 6.9914$$

2. **(total 3 points)**

$$\frac{\partial u}{\partial x_i} = a_i e^{\sum a_j x_j} = a_i u$$

$$\frac{\partial^2 u}{\partial x_i^2} = a_i^2 u$$

Then:

$$\sum_{i=1}^{n} \frac{\partial^2 u}{\partial x_i^2} = u \sum_{i=1}^{n} a_i^2 = u$$

3. **total 3 points**

   Since $Q = f_{xy} = f_{yx}$, then:

$$\frac{\partial Q}{\partial x} = \frac{\partial f_{xy}}{\partial x} = f_{xyx} = f_{xxy} = \frac{\partial P}{\partial y} = P_y$$

$$\frac{\partial Q}{\partial y} = \frac{\partial f_{xy}}{\partial y} = f_{xyy} = f_{yyx} = \frac{\partial R}{\partial x} = R_x$$

   Therefore:

$$\nabla Q = (Q_x, Q_y) = (P_y, R_x)$$