

## Calculus III (Math 241)

**W1** Show that the function  $g: \mathbb{R}^2 \rightarrow \mathbb{R}$  defined by  $g(0,0) = 0$  and

$$g(x,y) = \frac{xy^2}{x^2 + y^2} \quad \text{for } (x,y) \neq (0,0)$$

has directional derivatives at  $(0,0)$  in every direction but is not differentiable at  $(0,0)$ .

**W2** *From a previous midterm*

Consider the function  $u: \mathbb{R}^2 \setminus \{(0,0)\} \rightarrow \mathbb{R}$  defined by

$$u(x,y) = x + \frac{x}{x^2 + y^2}.$$

- a) Which symmetry properties does  $u$  have?
- b) Describe the behaviour of  $u(x,y)$  for  $(x,y) \rightarrow (0,0)$ ,  $x \geq 0$ .  
*Hint:* Use polar coordinates.
- c) Show that  $u$  is differentiable and determine the differential  $du$ .
- d) Show that there exists exactly one point  $(x_0, y_0)$  with  $x_0 \geq 0$  at which  $du$  vanishes.
- e) Sketch the contour of  $u$  through  $(x_0, y_0)$ .