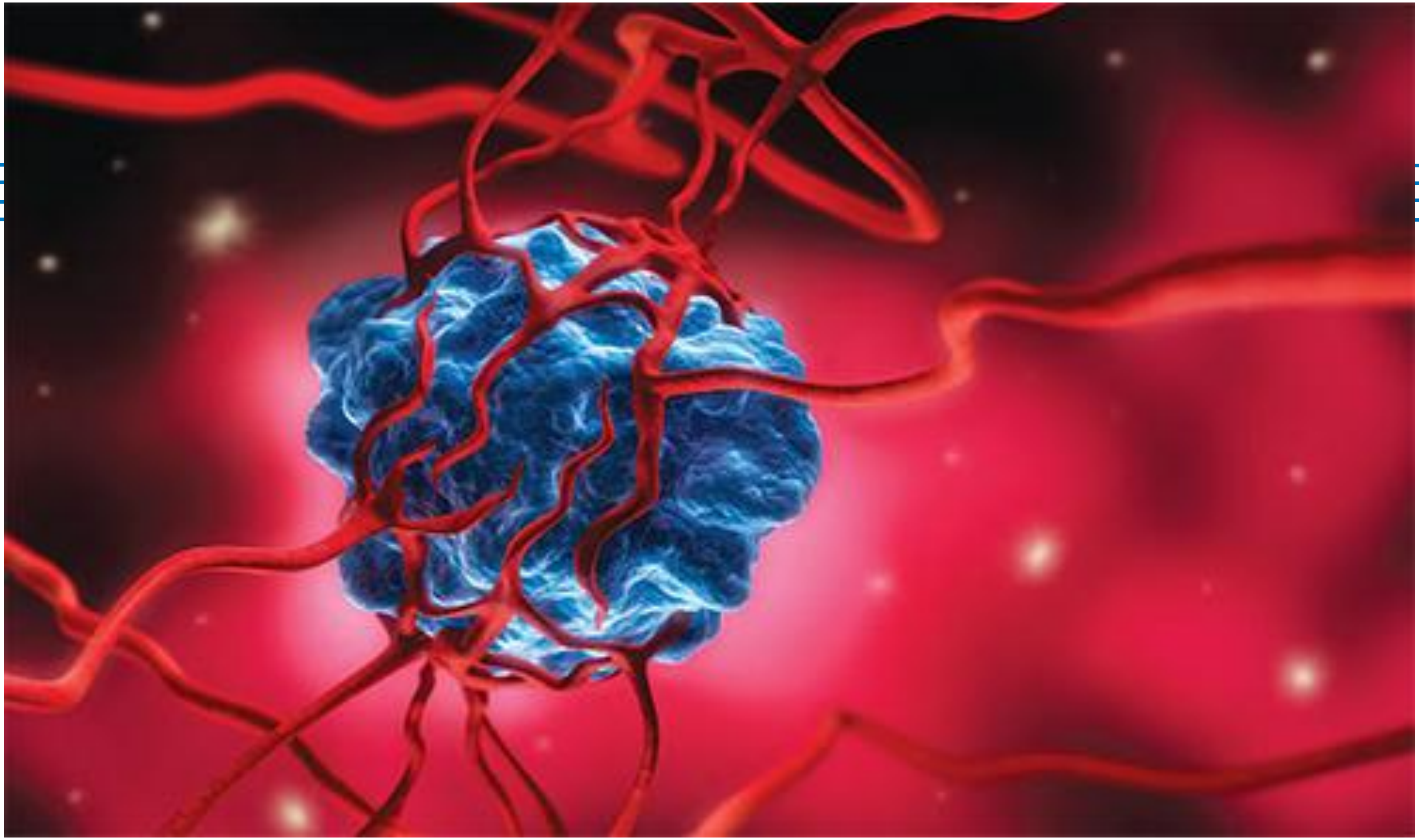
15 Multiple Integrals



15.6 Triple Integrals

Context

- Triple Integrals over Rectangular Boxes
- Triple Integrals over General Regions
- Changing the Order of Integration
- Applications of Triple Integrals

Triple Integrals (1 of 1)

We have defined single integrals for functions of one variable and double integrals for functions of two variables, so we can define triple integrals for functions of three variables.

Triple Integrals over Rectangular Boxes

Triple Integrals over Rectangular Boxes (1 of 6)

Let's first deal with the simplest case where *f* is defined on a rectangular box:

$$B = \{(x,\ y,\ z) \mid a \le x \le b,\ c \le y \le d,\ r \le z \le s\} \qquad \mathbf{1}$$

The first step is to divide *B* into sub-boxes.

We do this by dividing the interval $[a, b]$ into l subintervals $[x_{i-1}, x_i]$ of equal width Δx, dividing $[c, d]$ into m subintervals of width Δy, and dividing $[r, s]$ into n subintervals of width Δz.

Triple Integrals over Rectangular Boxes (2 of 6)

The planes through the endpoints of these subintervals parallel to the coordinate planes divide the box *B* into *lmn* sub-boxes

$$B_{ijk} = [x_{i-1}, \ x_i] \times [y_{j-1}, \ y_j] \times [z_{k-1}, \ z_k]$$

which are shown in Figure 1. Each sub-box has volume $\Delta V = \Delta x \, \Delta y \, \Delta z$.

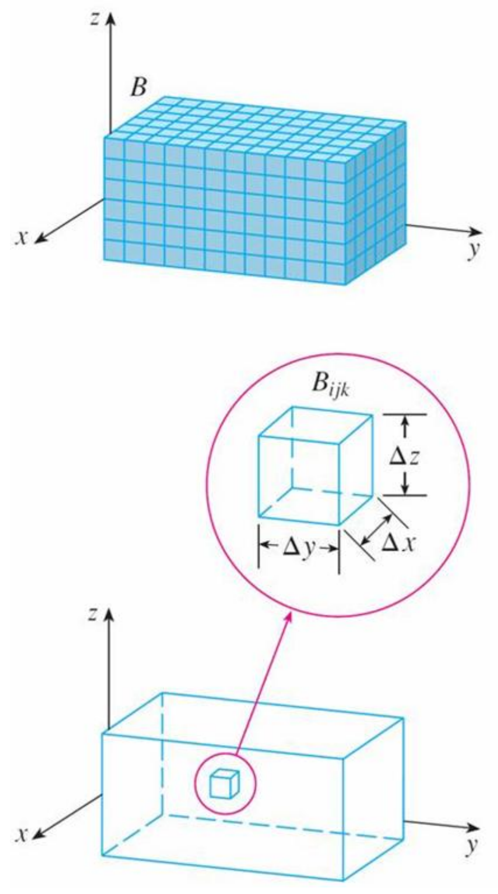


Figure 1

Triple Integrals over Rectangular Boxes (3 of 6)

Then we form the **triple Riemann sum**

$$\textbf{2} \qquad \sum_{i=1}^{l} \sum_{j=1}^{m} \sum_{k=1}^{n} f(x_{ijk}^*, y_{ijk}^*, z_{ijk}^*)\, \Delta V$$

where the sample point $(x_{ijk}^*, y_{ijk}^*, z_{ijk}^*)$ is in B_{ijk}.

By analogy with the definition of a double integral, we define the triple integral as the limit of the triple Riemann sums in (2).

Triple Integrals over Rectangular Boxes (4 of 6)

3 Definition The **triple integral** of *f* over the box *B* is

$$\iiint_B f(x,y,z)\,dV = \lim_{l,m,n\to\infty} \sum_{i=1}^{l}\sum_{j=1}^{m}\sum_{k=1}^{n} f(x_{ijk}^*,\ y_{ijk}^*,\ z_{ijk}^*)\,\Delta V$$

if this limit exists.

Again, the triple integral always exists if *f* is continuous.

We can choose the sample point to be any point in the sub-box, but if we choose it to be the point (x_i, y_j, z_k) we get a simpler-looking expression for the triple integral:

$$\iiint_B f(x,y,z)\,dV = \lim_{l,m,n\to\infty} \sum_{i=1}^{l}\sum_{j=1}^{m}\sum_{k=1}^{n} f(x_i,\ y_j,\ z_k)\,\Delta V$$

Triple Integrals over Rectangular Boxes (5 of 6)

Just as for double integrals, the practical method for evaluating triple integrals is to express them as iterated integrals as follows.

4 Fubini's Theorem for Triple Integrals If f is continuous on the rectangular box $B = [a, b] \times [c, d] \times [r, s]$, then

$$\iiint_B f(x, y, z)\, dV = \int_r^s \int_c^d \int_a^b f(x, y, z)\, dx\, dy\, dz$$

The iterated integral on the right side of Fubini's Theorem means that we integrate first with respect to x (keeping y and z fixed), then we integrate with respect to y (keeping z fixed), and finally we integrate with respect to z.

Triple Integrals over Rectangular Boxes (6 of 6)

There are five other possible orders in which we can integrate, all of which give the same value.

For instance, if we integrate with respect to *y*, then *z*, and then *x*, we have

$$\iiint\limits_B f(x,\ y,\ z)\ dV = \int_a^b \int_r^s \int_c^d f(x,\ y,\ z)\ dy\ dz\ dx$$

Example 1

Evaluate the triple integral $\iiint_B xyz^2 \; dV$, where B is the rectangular box given by

$$B = \{(x,\ y,\ z) \mid 0 \le x \le 1,\ \ -1 \le y \le 2,\ 0 \le z \le 3\}$$

Solution:

We could use any of the six possible orders of integration.

If we choose to integrate with respect to x, then y, and then z, we obtain

$$\iiint\limits_B xyz^2 \; dV = \int_0^3 \int_{-1}^2 \int_0^1 xyz^2 \; dx \; dy \; dz$$

Example 1 – Solution

$$= \int_0^3 \int_{-1}^2 \left[\frac{x^2 y z^2}{2} \right]_{x=0}^{x=1} dy\ dz$$

$$= \int_0^3 \int_{-1}^2 \frac{y z^2}{2}\ dy\ dz$$

$$= \int_0^3 \left[\frac{y^2 z^2}{4} \right]_{y=-1}^{y=2} dz$$

$$= \int_0^3 \frac{3z^2}{4} dz$$

$$= \left. \frac{z^3}{4} \right]_0^3 = \frac{27}{4}$$

多选题 1分

设置

MAQ: In the rectangular-box setting, which statements are True?

A If f is continuous on B, then $\iiint_B f \ dV$ exists

B If f has discontinuities in B, then the triple integral definitely does not exist

C In the rectangular box + continuous case, “the triple integral exists” and “it can be written as an iterated integral” are effectively equivalent claims

D If f is not continuous, then among the six possible orders of integration, at least one must fail

提交

Example

Solution:

A, C is true by definition in PPT.

For B, see example in D so it’s false

For D, a classic conceptual counterexample:

$$f(x,y,z) = \begin{cases} 1, & (x,y,z) = (0,0,0) \\ 0, & \text{otherwise.} \end{cases}$$

On any box containing the origin f is discontinuous at a single point yet the triple integral exists and equals 0.

Triple Integrals over General Regions

Triple Integrals over General Regions (1 of 10)

Now we define the **triple integral over a general bounded region *E*** in three-dimensional space (a solid) by much the same procedure that we used for double integrals.

We enclose *E* in a box *B* of the type given by Equation 1. Then we define *F* so that it agrees with *f* on *E* but is 0 for points in *B* that are outside *E*.

By definition,

$$\iiint_E f(x, y, z)\, dV = \iiint_B F(x, y, z)\, dV$$

This integral exists if *f* is continuous and the boundary of *E* is “reasonably smooth.”

Triple Integrals over General Regions (2 of 10)

The triple integral has essentially the same properties as the double integral.

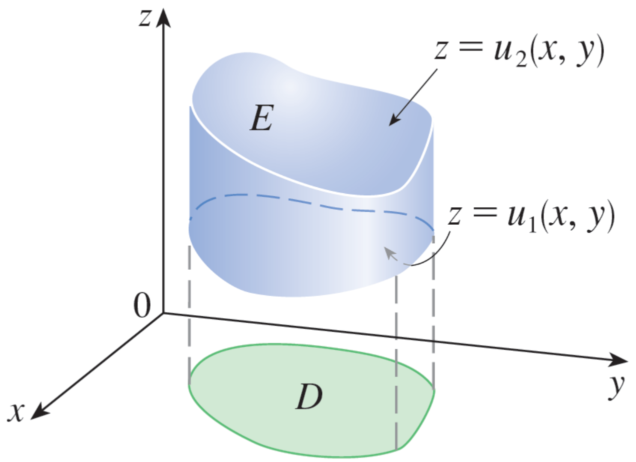
We restrict our attention to continuous functions f and to certain simple types of regions.

Triple Integrals over General Regions (3 of 10)

A solid region E is said to be of **type 1** if it lies between the graphs of two continuous functions of x and y, that is,

5 $$E = \{(x, y, z) \mid (x, y) \in D,\ u_1(x, y) \le z \le u_2(x, y)\}$$

where D is the projection of E onto the xy-plane as shown in Figure 2.



A type 1 solid region

Figure 2

Triple Integrals over General Regions (4 of 10)

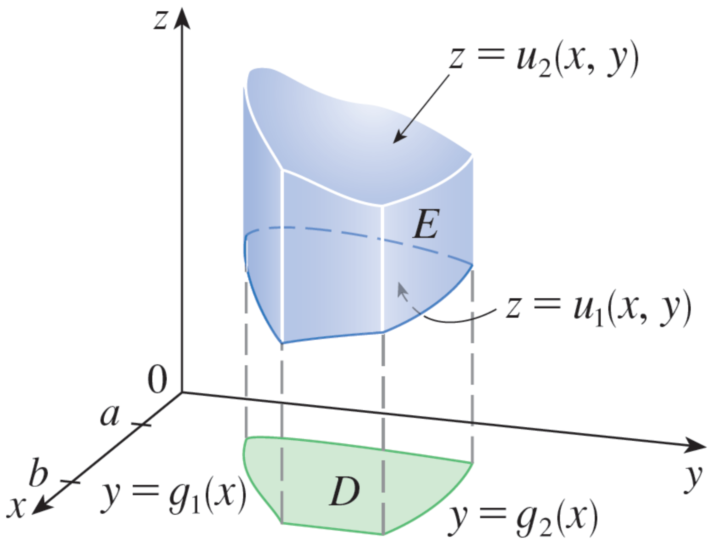
Notice that the upper boundary of the solid E is the surface with equation $z = u_2(x, y)$, while the lower boundary is the surface $z = u_1(x, y)$.

By the same sort of argument, it can be shown that if E is a type 1 region given by Equation 5, then

6 $$\iiint_E f(x, y, z)\, dV = \iint_D \left[\int_{u_1(x,y)}^{u_2(x,y)} f(x, y, z)\, dz \right] dA$$

The meaning of the inner integral on the right side of Equation 6 is that x and y are held fixed, and therefore $u_1(x, y)$ and $u_2(x, y)$ are regarded as constants, while $f(x, y, z)$ is integrated with respect to z.

In particular, if the projection *D* of *E* onto the *xy*-plane is a type I plane region (as in Figure 3).



A type 1 solid region where the projection *D* is a type I plane region

Figure 3

Triple Integrals over General Regions (6 of 10)

Then

$$E = \{(x,\ y,\ z) \mid a \le x \le b,\ g_1(x) \le y \le g_2(x),\ u_1(x,\ y) \le z \le u_2(x,\ y)\}$$

and Equation 6 becomes

$$\textbf{7} \qquad \iiint_E f(x,\ y,\ z)\, dV = \int_a^b \int_{g_1(x)}^{g_2(x)} \int_{u_1(x,y)}^{u_2(x,y)} f(x,\ y,\ z)\, dz\, dy\, dx$$

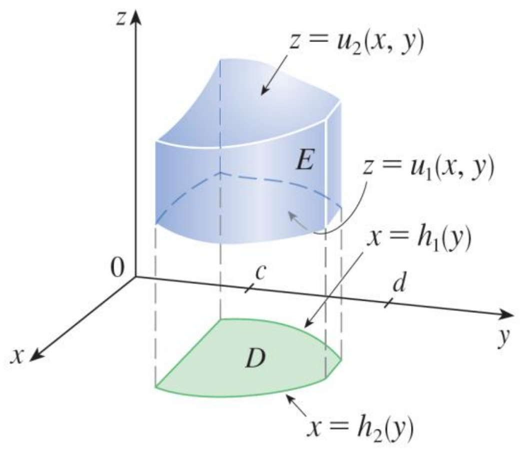
Triple Integrals over General Regions (7 of 10)

If, on the other hand, D is a type II plane region (as in Figure 4), then

$$E = \{(x,\ y,\ z) \mid c \le y \le d,\ h_1(y) \le x \le h_2(y),\ u_1(x,\ y) \le z \le u_2(x,\ y)\}$$

and Equation 6 becomes

$$\textbf{8} \qquad \iiint_E f(x,\ y,\ z)\, dV = \int_c^d \int_{h_1(y)}^{h_2(y)} \int_{u_1(x,y)}^{u_2(x,y)} f(x,\ y,\ z)\, dz\, dx\, dy$$



A type 1 solid region with a type II projection

Figure 4

Example 2

Evaluate $\iiint_E z\, dV$ where E is the solid in the first octant bounded by the surface $z = 12xy$ and the planes $y = x$, $x = 1$.

Solution:

When we set up a triple integral it's wise to draw *two* diagrams: one of the solid region E (Figure 5) and, for a type 1 region, one of its projection D onto the xy-plane (Figure 6).

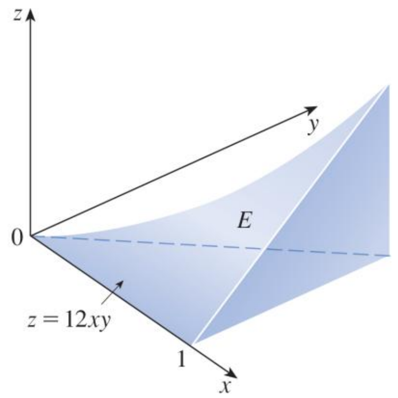


Figure 5

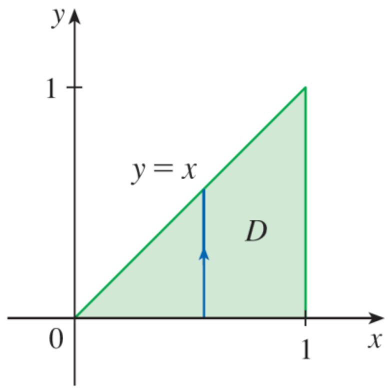


Figure 6

Example 2 – Solution (1 of 2)

The lower boundary of the solid E is the plane $z = 0$ and the upper boundary is the surface $z = 12xy$, so we use $u_1(x, y) = 0$ and $u_2(x, y) = 12xy$ in Formula 7.

Notice that the projection of E onto the xy-plane is the triangular region shown in Figure 6, and we have

9 $$E = \{(x, y, z) \mid 0 \le x \le 1, 0 \le y \le x, 0 \le z \le 12xy\}$$

Example 2 – Solution (2 of 2)

This description of *E* as a type 1 region enables us to evaluate the integral as follows:

$$\iiint_E z\ dV = \int_0^1 \int_0^x \int_0^{12xy} z\ dz\ dy\ dx = \int_0^1 \int_0^x \left[\frac{z^2}{2}\right]_{z=0}^{z=12y} dy\ dx$$

$$= \frac{1}{2}\int_0^1 \int_0^x (12xy)^2\ dy\ dx = 72\int_0^1 \int_0^x x^2 y^2\ dy\ dx$$

$$= 72\int_0^1 \left[x^2\, \frac{y^3}{3}\right]_{y=0}^{y=x} dx = 24\int_0^1 x^5 dx = 24\left[\frac{x6}{6}\right]_{x=0}^{x=1} = 4$$

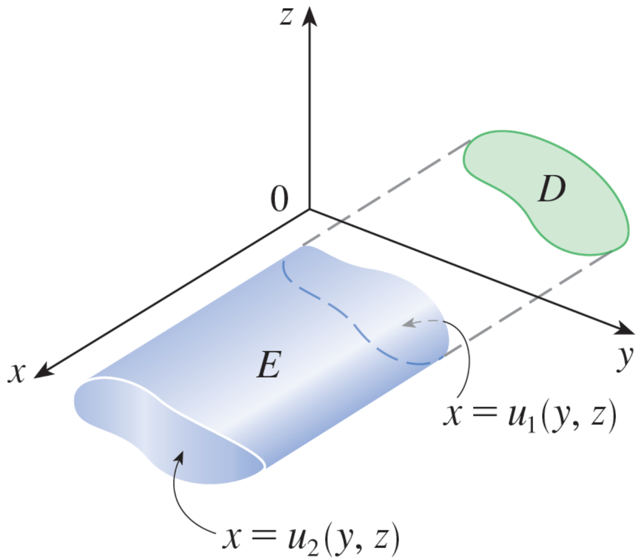
A solid region E is of **type 2** if it is of the form

$$E = \{(x,\ y,\ z) \mid (y,\ z) \in D,\ u_1(y,\ z) \le x \le u_2(y,\ z)\}$$

where, this time, D is the projection of E onto the yz-plane (see Figure 8).

The back surface is $x = u_1(y, z)$, the front surface is $x = u_2(y, z)$, and we have

10 $$\iiint_E f(x,\ y,\ z)\ dV = \iint_D \left[\int_{u_1(y,z)}^{u_2(y,z)} f(x,\ y,\ z)\ dx \right] dA$$



A type 2 region

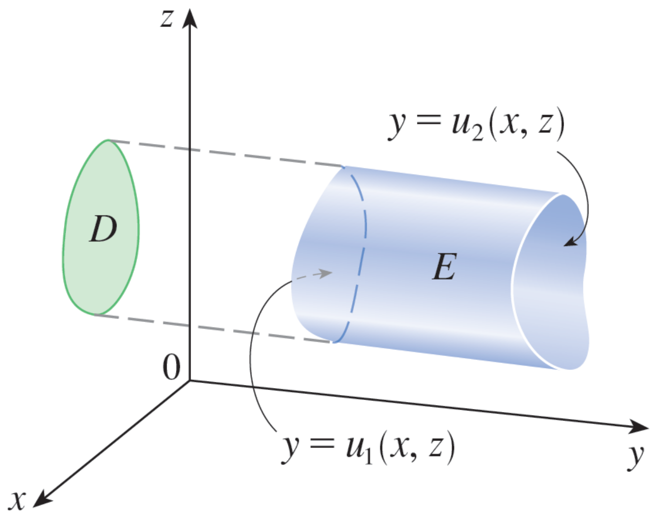
Figure 8

Triple Integrals over General Regions (9 of 10)

Finally, a **type 3** region is of the form

$$E = \{(x, y, z) \mid (x, z) \in D,\ u_1(x, z) \le y \le u_2(x, z)\}$$

where D is the projection of E onto the xz-plane, $y = u_1(x, z)$ is the left surface, and $y = u_2(x, z)$ is the right surface (see Figure 9).



A type 3 region

Figure 9

Triple Integrals over General Regions (10 of 10)

For this type of region we have

$$\textbf{11} \qquad \iiint_E f(x, y, z)\, dV = \iint_D \left[\int_{u_1(x,z)}^{u_2(x,z)} f(x, y, z)\,dy \right] dA$$

In each of Equations 10 and 11 there may be two possible expressions for the integral depending on whether *D* is a type I or type II plane region (and corresponding to Equations 7 and 8).

多选题 1分

设置

Let $E = \{(x, y, z) \in \mathbb{R}^3 | x \geq 0,\ y \geq 0,\ x^2 + y^2 \leq z \leq 4\}$
Consider $I = \iiint_E x\ dV$. Which of the following statements are correct?

A The region E can be written as $E = \left\{(x, y, z) | (y, z) \in D_{yz}, 0 \leq x \leq \sqrt{z - y^2}\right\}$, where $D_{yz} = \{(y, z) \mid 0 \leq y \leq 2, y^2 \leq z \leq 4\}$ and it's type 2

B The region E can be written as $E = \left\{(x, y, z) | (x, z) \in D_{xz}, 0 \leq \mathrm{y} \leq \sqrt{z - x^2}\right\}$, where $D_{xz} = \{ (x, z) \mid 0 \leq x \leq 2, x^2 \leq z \leq 4 \}$ and it's type 3

C The integral I can be expressed as $I = \int_0^2 \int_0^{4-y^2} \int_0^{\sqrt{z-y^2}} x\ dx\, dz\, dy$

D $I = \dfrac{16}{5}$

提交

Example

Solution:

By definition, A and B is true.

For C, it is from A but the correct form is $I = \int_0^2 \int_{y^2}^4 \int_0^{\sqrt{z-y^2}} x \; dx\, dz\, dy$, it's false.

For D, it is false. Use the natural Type 1 setup:

$$I = \iint\limits_{D_{xy}} \int_{x^2+y^2}^{4} x \; dzdxdy = \iint\limits_{D_{xy}} \int_{x^2+y^2}^{4} x \; dzdxdy = \iint\limits_{D_{xy}} x(4 - x^2 - y^2)\, dxdy$$

$$= \left(\int_0^{\frac{\pi}{2}} \cos\theta \; d\theta \right) \left(\int_0^2 (4r^2 - r^4) dr \right) = \frac{64}{15}$$

Changing the Order of Integration

Changing the Order of Integration (1 of 1)

Fubini's Theorem for Triple Integrals allows us to express a triple integral as an iterated integral, and there are six different orders of integration in which we can do this.

Given an iterated integral, it may be advantageous to change the order of integration because evaluating an iterated integral in one order may be simpler than in another.

In the next example we investigate equivalent iterated integrals using different orders of integration.

Example 4

Express the iterated integral $\int_0^1 \int_0^{x^2} \int_0^y f(x,y,z)\, dz\, dy\, dx$ as a triple integral and then rewrite it as an iterated integral in the following orders.

(a) Integrate first with respect to *x*, then *z*, and then *y*.

(b) Integrate first with respect to *y*, then *x*, and then *z*.

Solution:

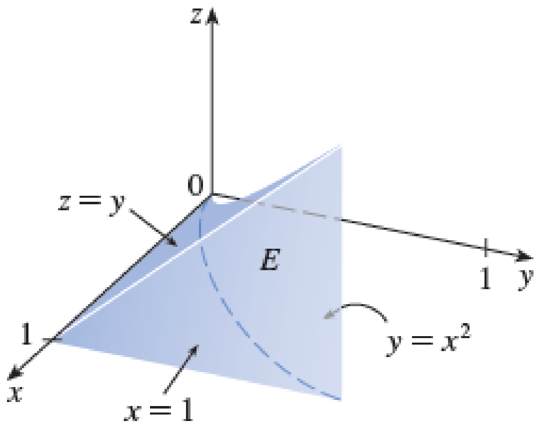
We can write

$$\int_0^1 \int_0^{x^2} \int_0^y f(x,y,z)\, dz\, dy\, dx = \iiint_E f(x,y,z)\, dv$$

where $E = \{(x,y,z) \mid 0 \le x \le 1, 0 \le y \le x^2, 0 \le z \le y\}$.

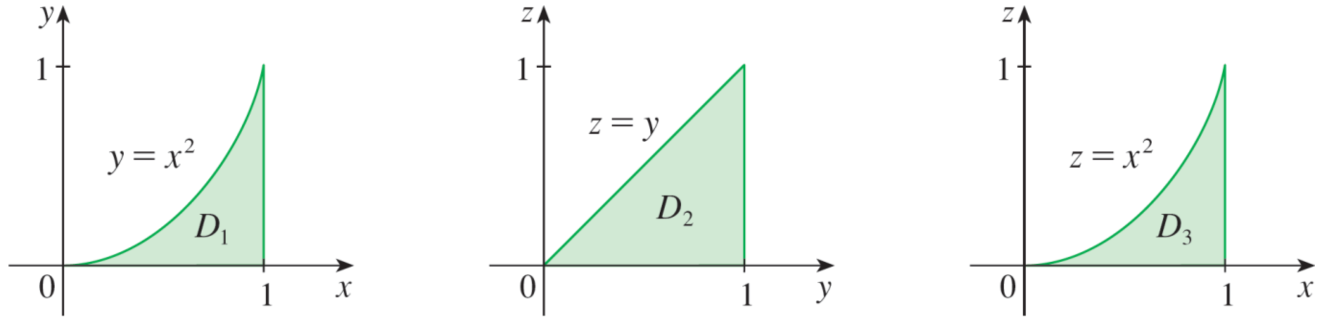
Example 4 – Solution (1 of 4)

From this description of E as a type 1 region we see that E lies between the lower surface $z = 0$ and the upper surface $z = y$, and its projection onto the xy-plane is $\{(x, y) \mid 0 \le x \le 1, 0 \le y \le x^2\}$, as shown in Figures 14 and 15.



The solid E

Figure 14



Projections of E

Figure 15

Example 4 – Solution (2 of 4)

So E is the solid enclosed by the planes $z = 0$, $x = 1$, $y = z$ and the parabolic cylinder $y = x^2$ (or $x = \sqrt{y}$).

Using Figure 14 as a guide, we can write projections onto the three coordinate planes as follows (see Figure 15):

onto the xy-plane:

$$D_1 = \{(x, y) \mid 0 \le x \le 1, 0 \le y \le x^2\}$$
$$= \{(x, y) \mid 0 \le y \le 1, \sqrt{y} \le x \le 1\}$$

onto the yz-plane:

$$D_2 = \{(y, z) \mid 0 \le y \le 1, 0 \le z \le y\}$$
$$= \{(y, z) \mid 0 \le z \le 1, z \le y \le 1\}$$

onto the xz-plane:

$$D_3 = \{(x, z) \mid 0 \le x \le 1, 0 \le z \le x^2\}$$
$$= \{(x, z) \mid 0 \le z \le 1, \sqrt{z} \le x \le 1\}$$

Example 4 – Solution (3 of 4)

(a) In order to integrate first with respect to *x*, then *z*, and then *y*, we need to consider *E* as a type 2 region where the back boundary is the surface $x = \sqrt{y}$ and the front boundary is the plane $x = 1$; the projection onto the *yz*-plane is D_2.

We describe *E* by

$$E = \{(x, y, z) \mid 0 \le y \le 1, 0 \le z \le y, \sqrt{y} \le x \le 1\}$$

and then

$$\iiint_E f(x, y, z)\, dv = \int_0^1 \int_0^y \int_{\sqrt{y}}^1 f(x, y, z)\, dx\, dz\, dy$$

Example 4 – Solution (4 of 4)

b) In order to integrate first with respect to y, then x, and then z, we need to consider E as a type 3 region where the left boundary is the plane $y = z$ and the right boundary is the surface $y = x^2$.

The projection onto the xz-plane is D_3 and

$$E = \{(x, y, z) \mid 0 \le z \le 1, \sqrt{z} \le x \le 1, z \le y \le x^2\}$$

Thus

$$\iiint_E f(x, y, z)\, dV = \int_0^1 \int_{\sqrt{z}}^1 \int_z^{x^2} f(x, y, z)\, dy\, dx\, dz$$

Applications of Triple Integrals

Applications of Triple Integrals (1 of 10)

We know that if $f(x) \geq 0$, then the single integral $\int_a^b f(x)\,dx$ represents the area under the curve $y = f(x)$ from a to b, and if $f(x, y) \geq 0$, then the double integral $\iint_D f(x, y)\,dA$ represents the volume under the surface $z = f(x, y)$ and above D.

The corresponding interpretation of a triple integral $\iiint_E f(x, y, z)\,dV$, where $f(x, y, z) \geq 0$, is not very useful because it would be the "hypervolume" of a four-dimensional object and, of course, that is very difficult to visualize. (Remember that E is just the *domain* of the function f; the graph of f lies in four-dimensional space.)

Applications of Triple Integrals (2 of 10)

Nonetheless, the triple integral $\iiint_E f(x, y, z)\, dV$ can be interpreted in different ways in different physical situations, depending on the physical interpretations of x, y, z and $f(x, y, z)$.

Let's begin with the special case where $f(x, y, z) = 1$ for all points in E. Then the triple integral does represent the volume of E:

12 $$V(E) = \iiint_E dV$$

Applications of Triple Integrals (3 of 10)

For example, you can see this in the case of a type 1 region by putting $f(x, y, z) = 1$ in Formula 6:

$$\iiint_E 1\, dV = \iint_D \left[\int_{u_1(x,y)}^{u_2(x,y)} dz \right] dA = \iint_D [u_2(x, y) - u_1(x, y)]\, dA$$

and we know this represents the volume that lies between the surfaces $z = u_1(x, y)$ and $z = u_2(x, y)$.

Example 5

Use a triple integral to find the volume of the tetrahedron T bounded by the planes $x + 2y + z = 2$, $x = 2y$, $x = 0$, and $z = 0$.

Solution:

The tetrahedron T and its projection D onto the xy-plane are shown in Figures 16 and 17.

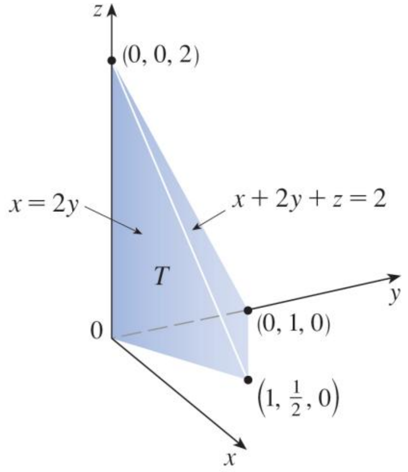


Figure 16

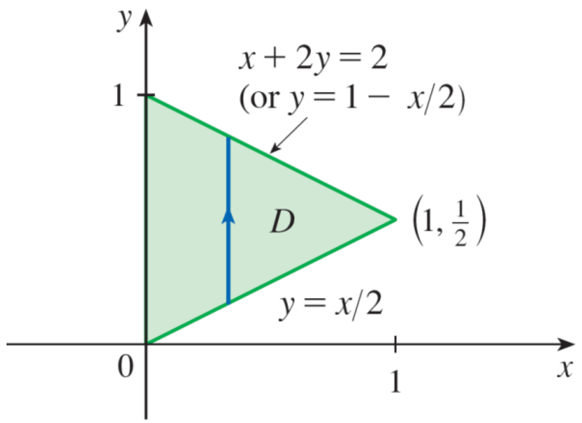


Figure 17

Example 5 – Solution

The lower boundary of T is the plane $z = 0$ and the upper boundary is the plane $x + 2y + z = 2$, that is, $z = 2 - x - 2y$.

Therefore we have

$$V(T) = \iiint_T dV$$

$$= \int_0^1 \int_{\frac{x}{2}}^{1-\frac{x}{2}} \int_0^{2-x-2y} dz\, dy\, dx$$

$$= \int_0^1 \int_{\frac{x}{2}}^{1-\frac{x}{2}} (2 - x - 2y)\, dy\, dx = \tfrac{1}{3}$$

(Notice that it is not necessary to use triple integrals to compute volumes. They simply give an alternative method for setting up the calculation.)

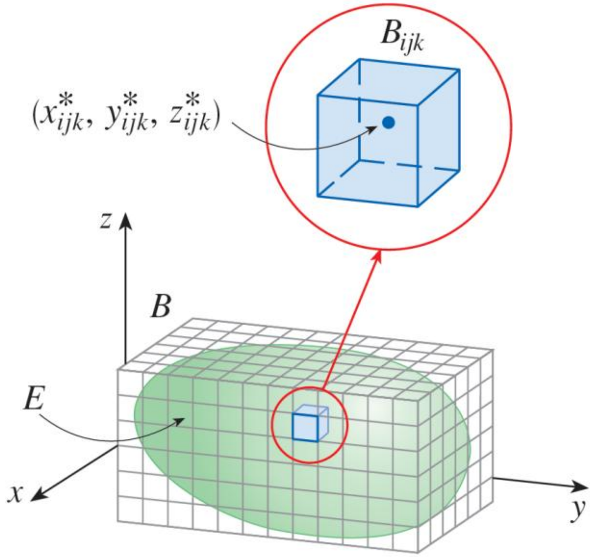
Applications of Triple Integrals (4 of 10)

All the applications of double integrals can be extended to triple integrals using analogous reasoning.

For example, suppose that a solid object occupying a region E has density $\rho(x, y, z)$, in units of mass per unit volume, at any given point (x, y, z), in E.

Applications of Triple Integrals (5 of 10)

To find the total mass m of E we divide a rectangular box B containing E into sub-boxes B_{ijk} of the same size (as in Figure 18), and consider $\rho(x, y, z)$ to be 0 outside E.



The mass of each sub-box B_{ijk} is approximated by $\rho\left(x^*_{ijk}, y^*_{ijk}, z^*_{ijk}\right) \Delta V$

Figure 18

Applications of Triple Integrals (6 of 10)

If we choose a point $\left(x_{ijk}^{*}, y_{ijk}^{*}, z_{ijk}^{*}\right)$ in B_{ijk}, then the mass of the part of E that

that occupies B_{ijk} is approximately $\rho\left(x_{ijk}^{*}, y_{ijk}^{*}, z_{ijk}^{*}\right) \Delta V$, where ΔV is the volume

of B_{ijk}.

We get an approximation to the total mass by adding the (approximate) masses of all the sub-boxes, and if we increase the number of sub-boxes, we obtain the total mass m of E as the limiting value of the approximations:

13 $$m = \lim_{l,m,n\to\infty} \sum_{i=1}^{l} \sum_{j=1}^{m} \sum_{k=1}^{n} \rho\left(x_{ijk}^{*}, y_{ijk}^{*}, z_{ijk}^{*}\right) \Delta V = \iiint_E \rho(x, y, z)\, dV$$

Applications of Triple Integrals (7 of 10)

Similarly, the **moments** of *E* about the three coordinate planes are

14 $$M_{yz} = \iiint_E x\rho(x, y, z)\, dV \qquad M_{xz} = \iiint_E y\rho(x, y, z)\, dV$$

$$M_{xy} = \iiint_E z\rho(x, y, z)\, dV$$

The **center of mass** is located at the point $(\bar{x}, \bar{y}, \bar{z})$, where

15 $$\bar{x} = \frac{M_{yz}}{m} \qquad \bar{y} = \frac{M_{xz}}{m} \qquad \bar{z} = \frac{M_{xy}}{m}$$

Applications of Triple Integrals (8 of 10)

If the density is constant, the center of mass of the solid is called the **centroid** of E.

The **moments of inertia** about the three coordinate axes are

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$$I_x = \iiint_E (y^2 + z^2)\rho(x, y, z)\, dV \qquad I_y = \iiint_E (x^2 + z^2)\rho(x, y, z)\, dV$$

$$I_z = \iiint_E (x^2 + y^2)\rho(x, y, z)\, dV$$

Applications of Triple Integrals (9 of 10)

The total **electric charge** on a solid object occupying a region E and having charge density $\sigma(x, y, z)$ is

$$Q = \iiint_E \sigma(x, y, z)\, dV$$

If we have three continuous random variables X, Y, and Z, their **joint density function** is a function of three variables such that the probability that (X, Y, Z) lies in E is

$$P((X, Y, Z) \in E) = \iiint_E f(x, y, z)\, dV$$

Applications of Triple Integrals (10 of 10)

In particular,

$$P(a \le X \le b,\ c \le Y \le d,\ r \le Z \le s) = \int_a^b \int_c^d \int_r^s f(x,\ y,\ z)\ dz\ dy\ dx$$

The joint density function satisfies

$$f(x,y,z) \ge 0 \qquad \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,\ y,\ z)\ dz\ dy\ dx = 1$$

多选题 1分

设置

MAQ: An insulating solid occupies a region E. Its charge density is

$$\sigma(x,y,z) = (x^2+x)(y-z)+2y$$

The region E is given in Type 2 form:

$$E = \{(x,y,z)|(y,z) \in D, -(1-y-z) \leq x \leq (1-y-z)\}$$

Where $D = \{(y,z) \mid y \geq 0, z \geq 0, y+z \leq 1\}$. Determine Q.

A $Q = \dfrac{1}{6}$

B $Q = \iiint\limits_E 2y\,dV$

C $Q = \iiint\limits_E (y+z)\,dV$

D $Q = \iiint\limits_E (x^2+x+2y)\,dV$

提交

Example

Solution:

$$\sigma(x,y,z) = (x^2 + x)(y - z) + 2y = (x^2 + x + 1)(y - z) + y + z$$

By symmetry, $y \leftrightarrow z$ $\quad Q = \iiint\limits_E (y+z)dV = \iiint\limits_E 2y dV$ B,C are true

$$Q = \iiint\limits_E 2y dV = \iint\limits_D 2y\big(2(1-y-z)\big)dA = \iint\limits_D 4y(1-y-z)dydz$$

$$Q = \int_0^1 \int_0^{1-y} 4y(1-y-z)dzdy = \int_0^1 2y\,(1-y)^2 dy = \frac{1}{6}$$ A is true

Recap

- Triple Integrals over Rectangular Boxes
- Triple Integrals over General Regions
- Changing the Order of Integration
- Applications of Triple Integrals