1. (a) Let S be the quadratic surface in $\mathbb{R}^3$ with equation $x^2 + 2yz = 3$. Determine the tangent plane of S in $Q = (0, \frac{3}{2}, 1)$.

   (b) Find the linear approximation of the function $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$ at $(3, 2, 6)$ and use it to approximate the number $\sqrt{3.02^2 + 1.97^2 + 5.99^2}$

2. $u = e^{a_1x_1 + a_2x_2 + \cdots + a_nx_n}$, where $a_1^2 + a_2^2 + \cdots + a_n^2 = 1$, prove that

$$\frac{\partial^2 u}{\partial x_1^2} + \frac{\partial^2 u}{\partial x_2^2} + \cdots + \frac{\partial^2 u}{\partial x_n^2} = u$$

3. Let P, Q, R be real-valued C^1-functions on $D \subset \mathbb{R}^2$. Show:

   If there exists a C^3-function $f : D \to \mathbb{R}$ with

$$H_f(x, y) = \begin{pmatrix} f_{xx}(x, y) & f_{xy}(x, y) \\ f_{yx}(x, y) & f_{yy}(x, y) \end{pmatrix} = \begin{pmatrix} P(x, y) & Q(x, y) \\ Q(x, y) & R(x, y) \end{pmatrix} \qquad (\star)$$

   for all $(x, y) \in D$ then $\nabla Q = (P_y, R_x)$.