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HW11 (Homework)

Current Score: – / 49 POINTS | 0.0 %

Scoring and Assignment Information

QUESTION	1	2	3	4	5	6	7	8	9	10	11
POINTS	– / 1	– / 1	– / 2	– / 3	– / 1	– / 2	– / 3	– / 3	– / 8	– / 19	– / 6

Assignment Submission

For this assignment, you submit answers by question parts. The number of submissions remaining for each question part only changes if you submit or change the answer.

Assignment Scoring

Your best submission for each question part is used for your score.

1. [– / 1 Points] DETAILS

SCalcET9M 15.5.019.

Find the exact area of the surface $z = 1 + 2x + 3y + 4y^2$, $1 \le x \le 12$, $0 \le y \le 1$.

✕ $\frac{165}{8}\sqrt{14}+\frac{55}{16}\ln\left(\frac{9}{5}+\frac{2}{5}\sqrt{14}\right)$

Solution or Explanation

$z = 1 + 2x + 3y + 4y^2$, so

$$A(S) = \iint_D \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\, dA = \int_1^{12}\int_0^1 \sqrt{1 + 4 + (3 + 8y)^2}\, dy\, dx = \int_1^{12}\int_0^1 \sqrt{14 + 48y + 64y^2}\, dy\, dx.$$

Using a CAS, we have $\int_1^{12}\int_0^1 \sqrt{14 + 48y + 64y^2}\, dy\, dx = \frac{165}{8}\sqrt{14} + \frac{55}{16}\ln\left(\frac{9}{5} + \frac{2}{5}\sqrt{14}\right)$

Resources

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2. [– / 1 Points] DETAILS

SCalcET9M 15.5.025.

Find the area of the finite part of the paraboloid $y = x^2 + z^2$ cut off by the plane $y = 16$. [*Hint:* Project the surface onto the xz-plane.]

✗ $\frac{\pi}{6}\left(65\sqrt{65}-1\right)$

Solution or Explanation

If we project the surface onto the xz-plane, then the surface lies "above" the disk $x^2 + z^2 \le 16$ in the xz−plane.

We have $y = f(x, y) = x^2 + z^2$ and, adapting the formula, the area of the surface is

$$A(S) = \iint_{x^2+z^2\le 16} \sqrt{[f_x(x, z)]^2 + [f_z(x, z)]^2 + 1}\, dA = \iint_{x^2+z^2\le 16} \sqrt{4x^2 + 4z^2 + 1}\, dA$$

Converting to polar coordinates $x = r\cos\theta$, $z = r\sin\theta$ we have

$$A(S) = \int_0^{2\pi}\int_0^4 \sqrt{4r^2+1}\, r\, dr\, d\theta = \int_0^{2\pi} d\theta \int_0^4 r(4r^2+1)^{1/2}\, dr = \Big[\theta\Big]_0^{2\pi}\left[\frac{1}{12}(4r^2+1)^{3/2}\right]_0^4 = \frac{\pi}{6}(65\sqrt{65}-1)$$

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3. [– / 2 Points] DETAILS

SCalcET9M 15.5.017.

(a) Use the Midpoint Rule for double integrals with four squares to estimate the surface area of the portion of the paraboloid $z = 7x^2 + 7y^2$ that lies above the square $[0, 1] \times [0, 1]$. (Round your answer to four decimal places.)

✗ 10.5397

(b) Use a computer algebra system to approximate the surface area in part (a) to four decimal places. Compare with the answer to part (a).

✗ 10.7731

Solution or Explanation

(a) The midpoints of the four squares are $\left(\frac{1}{4}, \frac{1}{4}\right), \left(\frac{1}{4}, \frac{3}{4}\right), \left(\frac{3}{4}, \frac{1}{4}\right)$, and $\left(\frac{3}{4}, \frac{3}{4}\right)$. Here $f(x, y) = 7x^2 + 7y^2$, so the Midpoint Rule gives the following.

$$A(S) = \iint_D \sqrt{\left[f_x(x, y)\right]^2 + \left[f_y(x, y)\right]^2 + 1}\, dA = \iint_D \sqrt{(14x)^2 + (14y)^2 + 1}\, dA$$

$$\approx \frac{1}{4}\left(\sqrt{\left[14\left(\frac{1}{4}\right)\right]^2 + \left[14\left(\frac{1}{4}\right)\right]^2 + 1} + \sqrt{\left[14\left(\frac{1}{4}\right)\right]^2 + \left[14\left(\frac{3}{4}\right)\right]^2 + 1}\right.$$

$$\left. + \sqrt{\left[14\left(\frac{3}{4}\right)\right]^2 + \left[14\left(\frac{1}{4}\right)\right]^2 + 1} + \sqrt{\left[14\left(\frac{3}{4}\right)\right]^2 + \left[14\left(\frac{3}{4}\right)\right]^2 + 1}\right)$$

$$= \frac{1}{4}\left(\sqrt{\frac{51}{2}} + 2\sqrt{\frac{247}{2}} + \sqrt{\frac{443}{2}}\right) \approx 10.5397$$

(b) A CAS estimates the integral to be $A(S) = \iint_D \sqrt{1 + (14x)^2 + (14y)^2}\, dA = \int_0^1\int_0^1 \sqrt{1 + 196x^2 + 196y^2}\, dy\, dx \approx 10.7731$. This agrees with the Midpoint estimate only in the value to the left of the decimal.

4. [– / 3 Points]

DETAILS

SCalcET9M 15.6.013.EP.

Evaluate each integral.

$$\int_{x-y}^{x+y} y\,dz =$$

✗ $2y^2$

$$\int_0^x \int_{x-y}^{x+y} y\,dz\,dy =$$

✗ $\frac{2x^3}{3}$

Now evaluate $\iiint_E y\,dV$, where $E = \left\{(x, y, z) \mid 0 \le x \le 6,\ 0 \le y \le x,\ x - y \le z \le x + y\right\}$.

✗ 216

Solution or Explanation

$$\iiint_E y\,dV = \int_0^6 \int_0^x \int_{x-y}^{x+y} y\,dz\,dy\,dx = \int_0^6 \int_0^x \Big[yz\Big]_{z=x-y}^{z=x+y} dy\,dx = \int_0^6 \int_0^x 2y^2\,dy\,dx$$
$$= \int_0^6 \left[\frac{2}{3}y^3\right]_{y=0}^{y=x} dx = \int_0^6 \frac{2}{3}x^3\,dx = \frac{1}{6}x^4\Big]_0^6 = 216$$

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5. [– / 1 Points]

DETAILS

SCalcET9M 15.6.017.

Evaluate the triple integral.

$\iiint_E 3xy\,dV$, where E lies under the plane $z = 1 + x + y$ and above the region in the xy-plane bounded by the curves $y = \sqrt{x}$, $y = 0$, and $x = 1$

✗ $\frac{65}{56}$

Solution or Explanation

Click to View Solution

Resources

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6. [– / 2 Points] DETAILS

SCalcET9M 15.6.044.

Find the mass and center of mass of the solid E with the given density function ρ.

E is bounded by the parabolic cylinder $z = 1 - y^2$ and the planes $x + 4z = 4$, $x = 0$, and $z = 0$; $\rho(x, y, z) = 5$.

$m =$

✗ $\boxed{16}$

$(\bar{x}, \bar{y}, \bar{z}) = (\quad)$

✗ $\boxed{\frac{10}{7}, 0, \frac{2}{7}}$

Solution or Explanation

Click to View Solution

7. [– / 3 Points] DETAILS

SCalcET9M 15.6.047.

Assume that the solid has constant density k.

Find the moments of inertia for a cube with side length L if one vertex is located at the origin and three edges lie along the coordinate axes.

$I_x =$

✗ $\boxed{\frac{2}{3}kL^5}$

$I_y =$

✗ $\boxed{\frac{2}{3}kL^5}$

$I_z =$

✗ $\boxed{\frac{2}{3}kL^5}$

Solution or Explanation

$$I_x = \int_0^L \int_0^L \int_0^L k(y^2 + z^2)\,dz\,dy\,dx = k\int_0^L \int_0^L \left(Ly^2 + \frac{1}{3}L^3\right)dy\,dx = k\int_0^L \frac{2}{3}L^4\,dx = \frac{2}{3}kL^5.$$

By symmetry, $I_x = I_y = I_z = \frac{2}{3}kL^5$.

8. [– / 3 Points] DETAILS

SCalcET9M 15.6.056.

Suppose X, Y, and Z are random variables with the joint density function
$f(x, y, z) = Ce^{-(0.5x + 0.2y + 0.1z)}$ if $x \geq 0$, $y \geq 0$, $z \geq 0$, and $f(x, y, z) = 0$ otherwise.

(a) Find the value of the constant C.

✕ 0.01

(b) Find $P(X \leq 1.125, Y \leq 0.75)$. (Round answer to five decimal places).

✕ 0.05993

(c) Find $P(X \leq 1.125, Y \leq 0.75, Z \leq 1)$. (Round answer to six decimal places).

✕ 0.005703

Solution or Explanation

Click to View Solution

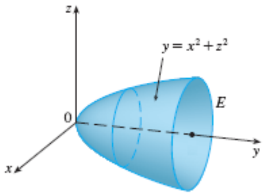
9. [– / 8 Points] DETAILS

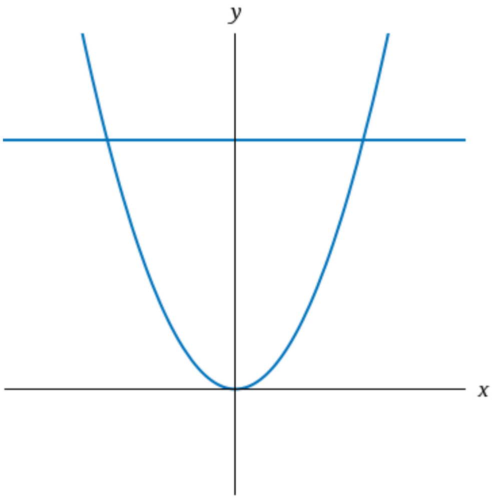
SCalcET9M 15.6.AE.003.

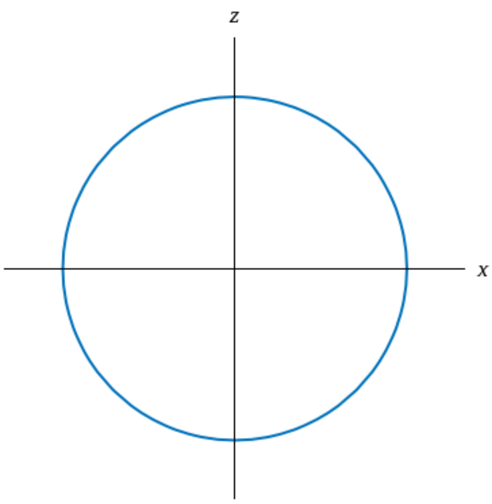
Example

Evaluate $\iiint_E \sqrt{x^2 + z^2}\,dV$, where E is the region bounded by the paraboloid $y = x^2 + z^2$ and the plane $y = 9$.

Solution







The solid E is shown in the top figure. If we regard it as a type 1 region, then we need to consider its projection D_1 onto the xy-plane, which is the parabolic region in the middle figure. (The trace of $y = x^2 + z^2$ in the plane $z = 0$ is the parabola $y =$

$z = \pm\Big($

✗ $\boxed{x^2}$.) From $y = x^2 + z^2$ we obtain ✗ $\boxed{\sqrt{y - x^2}}$ $\Big)$, so the lower boundary surface of E is $z = -\sqrt{y - x^2}$ and the upper boundary surface is $z =$

$\boxed{\sqrt{y-x^2}}$. Therefore the description of E as a type 1 region is

$$E = \left\{(x, y, z) \mid -3 \le x \le 3,\ x^2 \le y \le 9,\ -\sqrt{y - x^2} \le z \le \sqrt{y - x^2}\right\}$$

and so we obtain

$$\iiint \sqrt{x^2 + z^2}\, dV = \int_{-3}^{3} \int_{x^2}^{9} \int_{-\sqrt{y-x^2}}^{\sqrt{y-x^2}} \sqrt{x^2 + z^2}\, dz\, dy\, dx$$

Although this expression is correct, it is extremely difficult to evaluate. So let's instead consider E as a type 3 region. As such, its projection D_3 onto the xz-plane is the disk $x^2 + y^2 \le 9$ shown in the bottom figure. Then the left boundary of E is the paraboloid $y = x^2 + z^2$ and the boundary is the plane $y = 9$, so taking $\mu_1(x, z) = x^2 + z^2$ and $\mu_2(x, z) = 9$, we have

$$\iiint \sqrt{x^2 + z^2}\, dV = \iint \left[\int_{x^2+z^2}^{9} \sqrt{x^2 + z^2}\, dy\right] dA$$

$$= \iint \left(\boxed{\left(-x^2 - z^2 + 9\right)\sqrt{x^2+z^2}} \right) dA$$

Although this integral could be written as

$$\int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} (9 - x^2 - z^2)\sqrt{x^2 + z^2}\, dz\, dx$$

It's easier to convert to polar coordinates in the xz-plane: $x = r\cos(\theta)$, $z = r\sin(\theta)$. This gives the following.

$$\iiint \sqrt{x^2 + z^2}\, dV = \iint (9 - x^2 - z^2)\sqrt{x^2 + z^2}\, dA$$

$$= \int_0^{2\pi} \int_0^{3} \left(\boxed{r^2\left(9 - r^2\right)} \right) dr\, d\theta$$

$$= \int_0^{2\pi} d\theta \int_0^{3} \left(\boxed{9r^2 - r^4} \right) dr$$

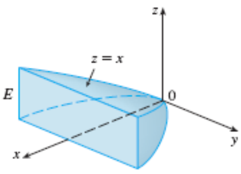
$$= 2\pi \left[\boxed{3r^3 - \frac{r^5}{5}} \right]_0^3$$

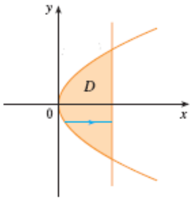
$$= \boxed{\frac{324\pi}{5}}$$

10. [– / 19 Points]

DETAILS

SCalcET9M 15.6.AE.006.





EXAMPLE 5 Find the center of mass of a solid of constant density that is bounded by the parabolic cylinder $x = 2y^2$ and the planes $x = z$, $z = 0$, and $x = 8$.

SOLUTION The solid E and its projection onto the xy-plane are shown in the figure. The lower and upper surfaces of E are the planes $z = 0$ and $z = x$, so we describe E as a type 1 region:

$E = \{(x, y, z) \mid$ ✗ -2 $\le y \le$ ✗ 2 , $2y^2 \le x \le 8$, $0 \le z \le x\}$.

Then, if the density is $\rho(x, y, z) = \rho$, the mass is

$$m = \iiint_D \rho \, dV = \int_{-2}^{2}\int_{2y^2}^{8}\int_0^x \rho \, dz \, dx \, dy$$

$$= \rho\int_{-2}^{2}\int_{2y^2}^{8}\left(\boxed{x}\right) dx \, dy$$

✗

$$= \rho\int_{-2}^{2}\left[\boxed{\frac{x^2}{2}}\right]_{x=2y^2}^{x=8} dy$$

✗

$$= \frac{\rho}{2}\int_{-2}^{2}\left(\boxed{64 - 4y^4}\right) dy$$

✗

$$= \rho\int_0^2 (64 - 4y^4) \, dy$$

$$= \rho\left[\boxed{64y - \frac{4y^5}{5}}\right]_0^2$$

✗

$$= \boxed{\frac{512\rho}{5}}.$$

✗

Because of the symmetry of E and ρ about the xz-plane, we can immediately say that $M_{xz} = 0$ and therefore $\overline{y} = 0$. The other moments are

$$M_{yz} = \iiint_E \left(\boxed{x}\right)\rho \, dV$$

✗

$$= \int_{-2}^{2}\int_{2y^2}^{8}\int_{0}^{x}\left(\boxed{x}\right)\rho\, dz\, dx\, dy$$

✗

$$= \rho\int_{-2}^{2}\int_{2y^2}^{8}\left(\boxed{x^2}\right) dx\, dy$$

✗

$$= \rho\int_{-2}^{2}\left[\frac{x^3}{3}\right]_{x=2y^2}^{x=8} dy$$

$$= \frac{2\rho}{3}\int_{0}^{2}\left(\boxed{512 - 8y^6}\right) dy$$

✗

$$= \frac{2\rho}{3}\left[\boxed{512y - \frac{8y^7}{7}}\right]_0^2$$

✗

$$= \boxed{\frac{4096}{7}\rho}$$

✗

$$M_{xy} = \iiint_E \left(\boxed{z}\right)\rho\, dV$$

✗

$$= \int_{-2}^{2}\int_{2y^2}^{8}\int_{0}^{x}\left(\boxed{z}\right)\rho\, dz\, dx\, dy$$

✗

$$= \rho\int_{-2}^{2}\int_{2y^2}^{8}\left[\boxed{\frac{z^2}{2}}\right]_{z=0}^{z=x} dx\, dy$$

✗

$$= \frac{\rho}{2}\int_{-2}^{2}\int_{2y^2}^{8} x^2\, dx\, dy$$

$\frac{\rho}{3}\int_0^{2}\Bigg($

$=$

✕ $\boxed{512 - 8y^6}\Bigg)\,dy$

$=$

✕ $\boxed{\frac{2048}{7}\rho}$.

Therefore the center of mass is

$(\bar{x}, \bar{y}, \bar{z}) = \left(\frac{M_{yz}}{m}, \frac{M_{xz}}{m}, \frac{M_{xy}}{m}\right)$

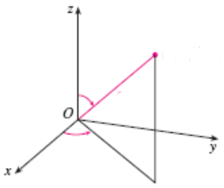
$= \Bigg($

✕ $\boxed{\frac{40}{7}, 0, \frac{20}{7}}\Bigg)$.

11. [– / 6 Points]

DETAILS

SCalcET9M 15.8.AE.001.



EXAMPLE 1 The point (2, $\pi/4$, $\pi/3$) is given in spherical coordinates. Plot the point and find its rectangular coordinates.

SOLUTION We plot the point in the figure. From these equations we have

$x = \rho \sin(\varphi)\cos(\theta) = 2\sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) = 2\left(\frac{\sqrt{3}}{2}\right)\left(\boxed{\frac{1}{\sqrt{2}}}\right)$ ✗

$= \boxed{\sqrt{\frac{3}{2}}}$ ✗

$y = \rho \sin(\varphi)\sin(\theta) = 2\sin\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right) = 2\left(\frac{\sqrt{3}}{2}\right)\left(\boxed{\frac{1}{\sqrt{2}}}\right)$ ✗

$= \boxed{\sqrt{\frac{3}{2}}}$ ✗

$z = \rho\cos(\varphi) = 2\cos\left(\frac{\pi}{3}\right) = 2\left(\frac{1}{2}\right)$

$= \boxed{1}$ ✗ .

Thus the point (2, $\pi/4$, $\pi/3$) is ✗ $(x, y, z) = \left(\boxed{\sqrt{\frac{3}{2}}, \sqrt{\frac{3}{2}}, 1}\right)$ in rectangular coordinates.