1. The curves $\mathbf{r}_1(t) = \langle t,\ 2t^2,\ t^3\rangle, \quad \mathbf{r}_2(t) = \langle \sin t,\ \sin 2t,\ 3t\rangle$ intersect at the origin. Find the cosine value of angle of intersection between the curves at the origin.

2. (a) Let $\mathbf{r}(t)$ be a differentiable vector-valued function such that $\mathbf{r}(t) \neq \mathbf{0}$. Show that: $\frac{d}{dt}\left|\mathbf{r}(t)\right| = \frac{1}{|\mathbf{r}(t)|}\,\mathbf{r}(t)\cdot\mathbf{r}'(t)$ (**Hint:** $\left|\mathbf{r}(t)\right|^2 = \mathbf{r}(t)\cdot\mathbf{r}(t)$)

   (b) Let $\mathbf{r} : I \to \mathbb{R}^3 \setminus \{\mathbf{0}\}$ be a C^2-curve with nonzero curvature, and let $t \in I$. Prove that the derivative $\frac{d}{dt}\frac{\mathbf{r}(t)}{|\mathbf{r}(t)|}$ is perpendicular to $\mathbf{r}(t)$

3. Show that the osculating plane at every point on the curve

$$\mathbf{r}(t) = \langle t+2,\ 1-t,\ \tfrac{1}{2}t^2\rangle$$

   is the same plane.