Calculus III (Math 241)

Consider the curve C in $\mathbb{R}^3$ parametrized by

$$\gamma(t) = \left(t^3 + t^2 + 1, t^2 - t, -t^3 - t - 1\right), \quad t \in \mathbb{R}.$$

a) Is C contained in a plane? Justify your answer!

b) Determine the center and radius of the osculating circle and the TNB frame of C in $(1, 0, -1)$.

3. Consider the function $f\colon \mathbb{R}^2 \to \mathbb{R}$ given by

$$f(x, y) = \frac{xy^2}{\sqrt{x^2 + y^2}} \quad \text{for } (x, y) \neq \mathbf{0}$$

In this problem, you'll show $\lim_{\mathbf{h}\to\mathbf{0}} f(\mathbf{h}) = 0$.

(a) For $\epsilon = 1/2$, find some $\delta > 0$ so that when $0 < |\mathbf{h}| < \delta$ we have $|f(\mathbf{h})| < \epsilon$. Hint: As with the example in class, the key is to relate $|x|$ and $|y|$ with $|\mathbf{h}|$.

(b) Repeat with $\epsilon = 1/10$.

(c) Now show that $\lim_{\mathbf{h}\to\mathbf{0}} f(\mathbf{h}) = 0$. That is, given an arbitrary $\epsilon > 0$, find a $\delta > 0$ so that that when $0 < |\mathbf{h}| < \delta$ we have $|f(\mathbf{h})| < \epsilon$.

(d) Explain why the limit laws that you learned in class on Wednesday aren't enough to compute this particular limit.