

# Assignment Previewer

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 INSTRUCTOR

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## HW12 (Homework)

Current Score: – / 39 POINTS | 0.0 %

### Scoring and Assignment Information

QUESTION	1	2	3	4	5	6	7	8	9	10	11	12	13	14
POINTS	– / 1	– / 1	– / 1	– / 1	– / 1	– / 7	– / 13	– / 1	– / 1	– / 7	– / 1	– / 1	– / 2	– / 1

### Assignment Submission

For this assignment, you submit answers by question parts. The number of submissions remaining for each question part only changes if you submit or change the answer.

### Assignment Scoring

Your best submission for each question part is used for your score.

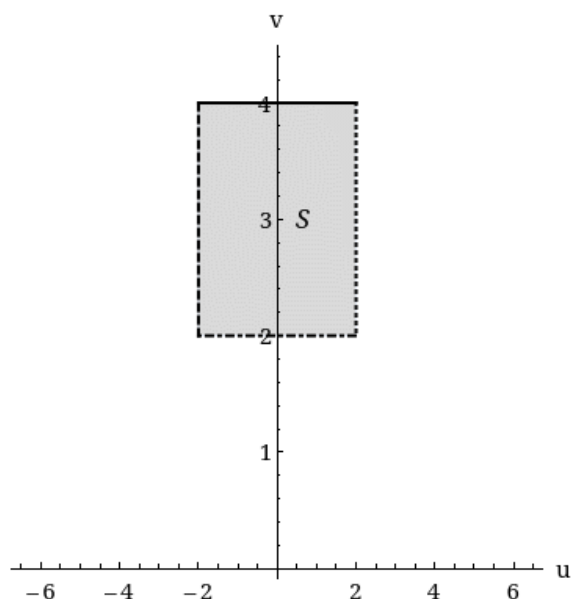
SCalcET9M 15.9.007.

A region  $R$  in the  $xy$ -plane is given. Find equations for a transformation  $T$  that maps a rectangular region  $S$  in the  $uv$ -plane onto  $R$ , where the sides of  $S$  are parallel to the  $u$ - and  $v$ -axes as shown in the figure below. (Enter your answers as a comma-separated list of equations.)

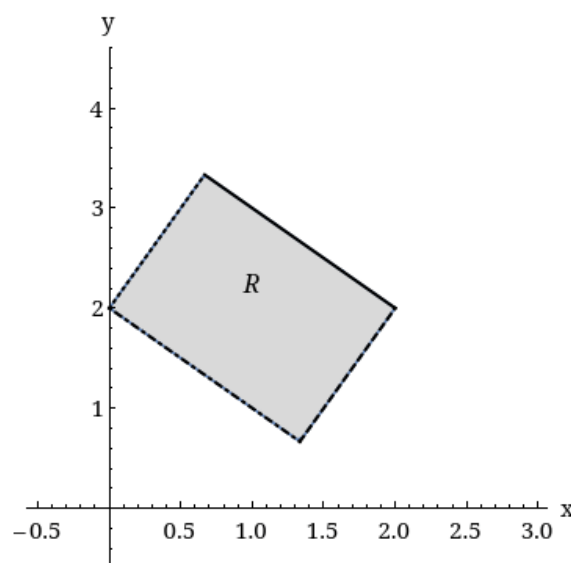
$R$  is bounded by  $y = 2x - 2$ ,  $y = 2x + 2$ ,  $y = 2 - x$ ,  $y = 4 - x$



✗  $x = \frac{v-u}{3}, y = \frac{1}{3}(u+2v)$

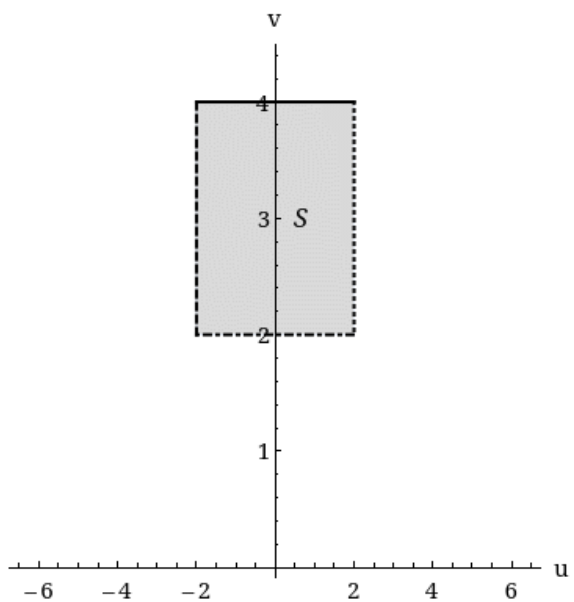


$T$   
→

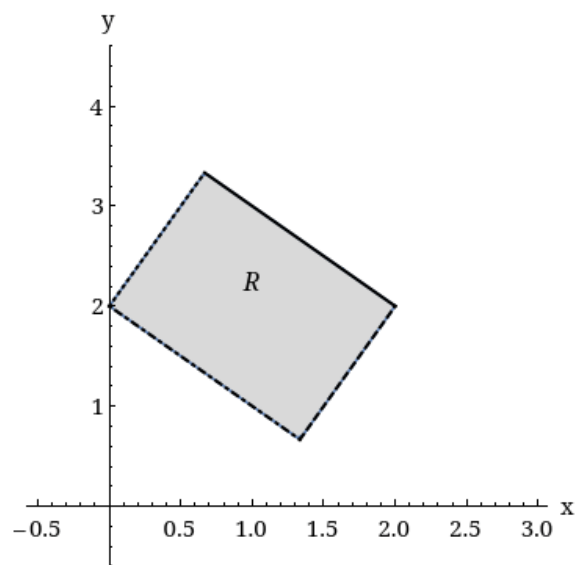


### Solution or Explanation

$R$  is a parallelogram enclosed by the parallel lines  $y = 2x - 2$ ,  $y = 2x + 2$  and the parallel lines  $y = 2 - x$ ,  $y = 4 - x$ . The first pair of equations can be written as  $y - 2x = -2$ ,  $y - 2x = 2$ . If we let  $u = y - 2x$  then these lines are mapped to the vertical lines  $u = -2$ ,  $u = 2$  in the  $uv$ -plane. Similarly, the second pair of equations can be written as  $x + y = 2$ ,  $x + y = 4$ , and setting  $v = x + y$  maps these lines to the horizontal lines  $v = 2$ ,  $v = 4$  in the  $uv$ -plane. Boundary curves are mapped to boundary curves under a transformation, so here the equations  $u = y - 2x$ ,  $v = x + y$  define a transformation  $T^{-1}$  that maps  $R$  in the  $xy$ -plane to the square  $S$  enclosed by the lines  $u = -2$ ,  $u = 2$ ,  $v = 2$ ,  $v = 4$  in the  $uv$ -plane. To find the transformation  $T$  that maps  $S$  to  $R$  we solve  $u = y - 2x$ ,  $v = x + y$  for  $x$ ,  $y$ : Subtracting the first equation from the second gives  $v - u = 3x \Rightarrow x = \frac{1}{3}(v - u)$  and adding twice the second equation to the first gives  $u + 2v = 3y \Rightarrow y = \frac{1}{3}(u + 2v)$ . Thus the transformation  $T$  is given by  $x = \frac{1}{3}(v - u)$ ,  $y = \frac{1}{3}(u + 2v)$ .



$T$   
 $\rightarrow$



### Resources

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SCalcET9M 15.9.009.

A region  $R$  in the  $xy$ -plane is given. Find equations for a transformation  $T$  that maps a rectangular region  $S$  in the  $uv$ -plane onto  $R$ , where the sides of  $S$  are parallel to the  $u$ - and  $v$ - axes. (Let  $u$  play the role of  $r$  and  $v$  the role of  $\theta$ . Enter your answers as a comma-separated list of equations.)

$R$  lies between the circles  $x^2 + y^2 = 1$  and  $x^2 + y^2 = 6$  in the first quadrant

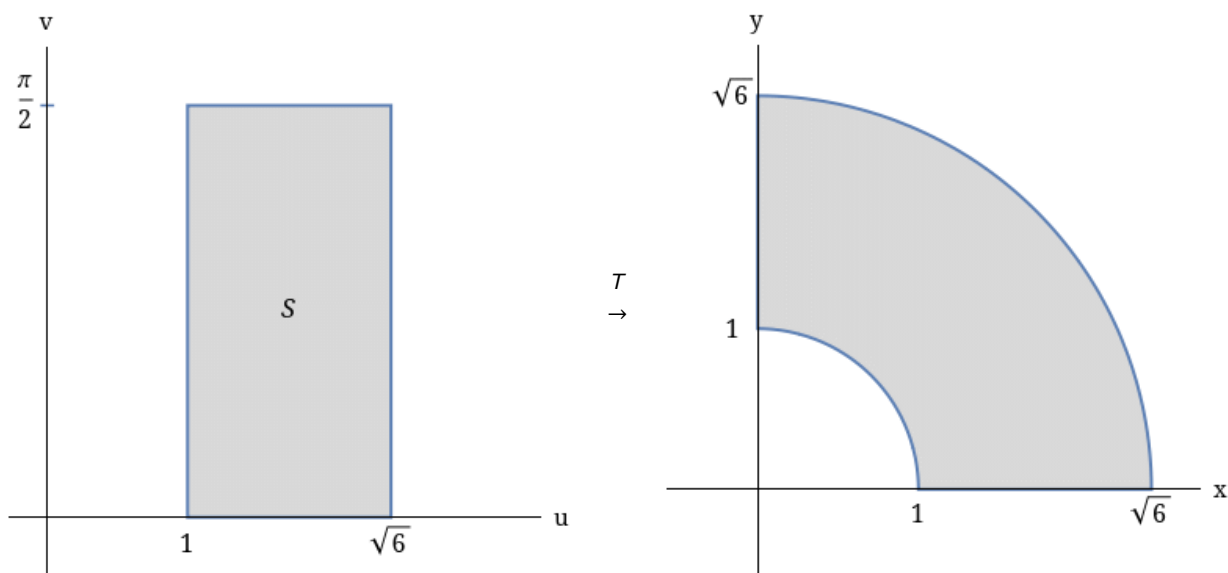


✗  $x = u \cos(v), y = u \sin(v)$

#### Solution or Explanation

$R$  is a portion of an annular region (see the figure) that is easily described in polar coordinates as

$R = \{(r, \theta) \mid 1 \leq r \leq \sqrt{6}, 0 \leq \theta \leq \pi/2\}$ . If we converted a double integral over  $R$  to polar coordinates the resulting region of integration is a rectangle (in the  $r\theta$ -plane), so we can create a transformation  $T$  here by letting  $u$  play the role of  $r$  and  $v$  the role of  $\theta$ . Thus  $T$  is defined by  $x = u \cos v$ ,  $y = u \sin v$  and  $T$  maps the rectangle  $S = \{(u, v) \mid 1 \leq u \leq \sqrt{6}, 0 \leq v \leq \pi/2\}$  in the  $uv$ -plane to  $R$  in the  $xy$ -plane.



#### Resources

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3. [- / 1 Points]

DETAILS

SCalcET9M 15.9.014.

Find the Jacobian of the transformation.

$$x = 6pe^q, \quad y = 6qe^p$$



✗  $36e^{p+q}(1-pq)$

Solution or Explanation

$$x = 6pe^q, \quad y = 6qe^p$$

$$\frac{\partial(x, y)}{\partial(p, q)} = \begin{vmatrix} \frac{\partial x}{\partial p} & \frac{\partial x}{\partial q} \\ \frac{\partial y}{\partial p} & \frac{\partial y}{\partial q} \end{vmatrix} = \begin{vmatrix} 6e^q & 6pe^q \\ 6qe^p & 6e^p \end{vmatrix} = 36e^qe^p - 6pe^q \cdot 6qe^p = 36e^{p+q} - 36pqe^{p+q} = 36(1-pq)e^{p+q}$$

4. [- / 1 Points]

DETAILS

SCalcET9M 15.9.015.

Find the Jacobian of the transformation.

$$x = 5uv, \quad y = 2vw, \quad z = 4wu$$



✗  $80uvw$

Solution or Explanation

$$x = 5uv, \quad y = 2vw, \quad z = 4wu$$

$$\begin{aligned} \frac{\partial(x, y, z)}{\partial(u, v, w)} &= \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial w} \end{vmatrix} = \begin{vmatrix} 5v & 5u & 0 \\ 0 & 2w & 2v \\ 4w & 0 & 4u \end{vmatrix} = 5v \begin{vmatrix} 2w & 2v \\ 0 & 4u \end{vmatrix} - 5u \begin{vmatrix} 0 & 2v \\ 4w & 4u \end{vmatrix} + 0 \begin{vmatrix} 0 & 2w \\ 4w & 0 \end{vmatrix} \\ &= 5v(8uw - 0) - 5u(0 - 8vw) + 0 = 40uvw + 40uvw = 80uvw \end{aligned}$$

5. [- / 1 Points]

DETAILS

SCalcET9M 15.9.017.MI.

Use the given transformation to evaluate the integral.

$$\iint_R (x - 8y) \, dA, \text{ where } R \text{ is the triangular region with vertices } (0, 0), (7, 1), \text{ and } (1, 7). \, x = 7u + v, \, y = u + 7v$$

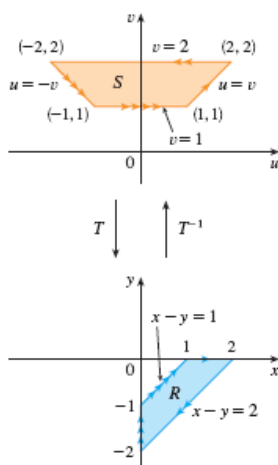
✗

Solution or Explanation

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### Resources

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[Video Example](#)

**EXAMPLE 2** Using the change of variables  $x = u^2 - v^2$ ,  $y = 2uv$  to evaluate the integral  $\iint_R y \, dA$ , where  $R$  is the region bounded by the  $x$ -axis and the parabolas  $y^2 = 2 - 2x$  and  $y^2 = 2 + 2x$ ,  $y \geq 0$ .

**SOLUTION** A similar region  $R$  is pictured in the figure. In Example 1, we discovered that  $T(S) = R$ , where  $S$  is the square  $[0, 1] \times [0, 1]$ . Indeed, the reason for making the change of variables to evaluate the integral is that  $S$  is a much simpler region than  $R$ . First we need to compute the Jacobian:

$$\begin{aligned} \frac{\partial(x, y)}{\partial(u, v)} &= \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} \\ &= \begin{vmatrix} 2u & -2v \\ 2v & 2u \end{vmatrix} \\ &= 4u^2 + 4v^2 \geq 0 \end{aligned}$$

Therefore by the theorem for the change of variables in a double integral

$$\begin{aligned} \iint_R y \, dA &= \iint_S 2uv \left| \frac{\partial(x, y)}{\partial(u, v)} \right| dA \\ &= \int_0^1 \int_0^1 (2uv) (4u^2 + 4v^2) \, du \, dv \\ &= 8 \int_0^1 \int_0^1 (u^3v + uv^3) \, du \, dv \\ &= 8 \int_0^1 \left[ \frac{1}{4}u^4v + \frac{1}{2}u^2v^3 \right]_0^1 \, dv \\ &= 8 \int_0^1 \left( \frac{1}{4}v + \frac{1}{2}v^3 \right) \, dv \\ &= 8 \left[ \frac{1}{8}v^2 + \frac{1}{8}v^4 \right]_0^1 = 1 \end{aligned}$$

✗

$$\frac{v^2}{8} + \frac{v^4}{8}$$

7. [- / 13 Points]

DETAILS

SCalcET9M 15.9.025.MI.SA.

This question has several parts that must be completed sequentially. If you skip a part of the question, you will not receive any points for the skipped part, and you will not be able to come back to the skipped part.

**Tutorial Exercise**

Evaluate the given integral by making an appropriate change of variables.

$$\iint_R \frac{x - 2y}{3x - y} dA, \text{ where } R \text{ is the parallelogram enclosed by the lines } x - 2y = 0, x - 2y = 8, 3x - y = 1, \text{ and } 3x - y = 9$$

[Click here to begin!](#)

8. [- / 1 Points]

DETAILS

SCalcET9M 15.9.026.

Evaluate the integral by making an appropriate change of variables.

$$\iint_R 3(x + y) e^{x^2 - y^2} dA, \text{ where } R \text{ is the rectangle enclosed by the lines } x - y = 0, x - y = 5, x + y = 0, \text{ and } x + y = 7$$

✗

$$\frac{3}{10} (e^{35} - 36)$$

Solution or Explanation

[Click to View Solution](#)

9. [- / 1 Points]

DETAILS

SCalcET9M 16.1.038.

At time  $t = 1$ , a particle is located at position  $(x, y) = (3, 1)$ . If it moves in the velocity field

$$\mathbf{F}(x, y) = \langle xy - 2, y^2 - 10 \rangle$$

find its approximate location at time  $t = 1.07$ .

$$(x, y) = ($$

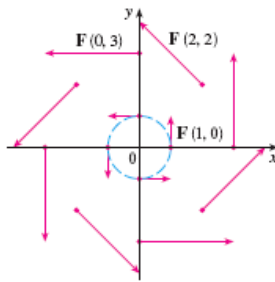
✗

$$3.07, 0.37)$$

Solution or Explanation

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SCalcET9M 16.1.AE.001.


[Video Example](#)

**EXAMPLE 1** A vector field on  $\mathbb{R}^2$  is defined by  $\mathbf{F}(x, y) = -y\mathbf{i} + x\mathbf{j}$ . Describe  $\mathbf{F}$  by sketching some of the vectors  $\mathbf{F}(x, y)$  as in the figure.

$$\mathbf{j} = \left\langle \begin{array}{c} \text{ } \\ \text{ } \end{array} \right\rangle$$

**SOLUTION** Since  $\mathbf{F}(1, 0) = \mathbf{j}$ , we draw the vector  $\times \langle 0, 1 \rangle$  starting at the point  $(1, 0)$  in the figure. Since  $\mathbf{F}(0, 1) = -\mathbf{i}$ , we draw the

$$(x, y) = \left( \begin{array}{c} \text{ } \\ \text{ } \end{array} \right)$$

vector  $\langle -1, 0 \rangle$  with starting point  $\times \langle 0, 1 \rangle$ . Continuing in this way, we calculate several other representative values of  $\mathbf{F}(x, y)$  in the table and draw the corresponding vectors to represent the vector field in the figure.

$(x, y)$	$\mathbf{F}(x, y)$	$(x, y)$	$\mathbf{F}(x, y)$
$(1, 0)$	$\langle 0, 1 \rangle$	$(-1, 0)$	$\langle 0, -1 \rangle$
$(2, 2)$	$\times \langle -2, 2 \rangle$	$(-2, -2)$	$\langle 2, -2 \rangle$
$(3, 0)$	$\langle 0, 3 \rangle$	$(-3, 0)$	$\langle 0, -3 \rangle$
$(0, 1)$	$\langle -1, 0 \rangle$	$(0, -1)$	$\times \langle 0, -1 \rangle$
$(-2, 2)$	$\langle -2, -2 \rangle$	$(2, -2)$	$\langle 2, 2 \rangle$
$(0, 3)$	$\times \langle -3, 0 \rangle$	$(0, -3)$	$\langle 3, 0 \rangle$

It appears from the figure that each arrow is tangent to a circle with center the origin. To confirm this, we take the dot product of the position vector  $\mathbf{x} = x\mathbf{i} + y\mathbf{j}$  with the vector  $\mathbf{F}(\mathbf{x}) = \mathbf{F}(x, y)$ :

$$\begin{aligned} \mathbf{x} \cdot \mathbf{F}(\mathbf{x}) &= (x\mathbf{i} + y\mathbf{j}) \cdot (-y\mathbf{i} + x\mathbf{j}) = \\ &= \begin{array}{c} \text{ } \\ \text{ } \end{array} \\ &= \times \langle -xy \rangle + yx \end{aligned}$$

$$= \boxed{0}$$

This shows that  $\mathbf{F}(x, y)$  is perpendicular to the position vector  $\langle x, y \rangle$  and is therefore tangent to a circle with center the origin and radius

$|\mathbf{x}| = \sqrt{x^2 + y^2}$ . Notice also that

$$|\mathbf{F}(x, y)| = \sqrt{(-y)^2 + (x)^2} = \sqrt{x^2 + y^2} = |\mathbf{x}|$$

so the magnitude of the vector  $\mathbf{F}(x, y)$  is equal to the radius of the circle.

11. [- / 1 Points]

DETAILS

SCalcET9M 16.1.026.

Find the gradient vector field  $\nabla f$  of  $f$ .

$$f(s, t) = \sqrt{7s + 8t}$$

$\nabla f(s, t) =$



$$\frac{7}{2\sqrt{7s+8t}}\mathbf{i} + \frac{4}{\sqrt{7s+8t}}\mathbf{j}$$

Solution or Explanation

$$\begin{aligned} f(s, t) = \sqrt{7s + 8t} &\Rightarrow \nabla f(s, t) = f_s(s, t)\mathbf{i} + f_t(s, t)\mathbf{j} \\ &= \left[ \frac{1}{2}(7s + 8t)^{-1/2} \cdot 7 \right] \mathbf{i} + \left[ \frac{1}{2}(7s + 8t)^{-1/2} \cdot 8 \right] \mathbf{j} \\ &= \frac{7}{2\sqrt{7s + 8t}}\mathbf{i} + \frac{4}{\sqrt{7s + 8t}}\mathbf{j} \end{aligned}$$

SCalcET9M 16.1.028.

Find the gradient vector field  $\nabla f$  of  $f$ .

$$f(x, y, z) = x^4 y e^{y/z}$$

 $\nabla f(x, y, z) =$ 



✗

$$4x^3 y e^{y/z} \mathbf{i} + x^4 e^{y/z} \left( \frac{y}{z} + 1 \right) \mathbf{j} - \frac{x^4 y^2}{z^2} e^{y/z} \mathbf{k}$$

Solution or Explanation

$$f(x, y, z) = x^4 y e^{y/z} \Rightarrow$$

$$\begin{aligned} \nabla f(x, y, z) &= f_x(x, y, z) \mathbf{i} + f_y(x, y, z) \mathbf{j} + f_z(x, y, z) \mathbf{k} \\ &= 4x^3 y e^{y/z} \mathbf{i} + x^4 \left[ y \cdot e^{y/z} \left( \frac{1}{z} \right) + e^{y/z} \cdot 1 \right] \mathbf{j} + \left[ x^4 y e^{y/z} \left( -\frac{y}{z^2} \right) \right] \mathbf{k} \\ &= 4x^3 y e^{y/z} \mathbf{i} + x^4 e^{y/z} \left( \frac{y}{z} + 1 \right) \mathbf{j} - \frac{x^4 y^2}{z^2} e^{y/z} \mathbf{k} \end{aligned}$$

Find the gradient vector field  $\nabla f$  of  $f$ .

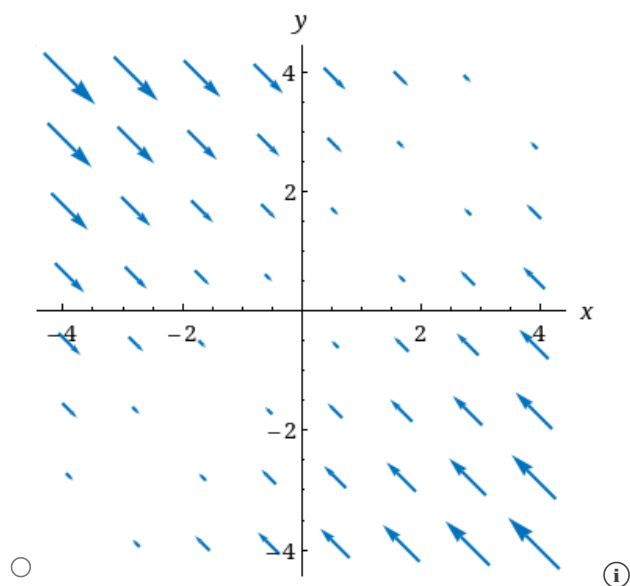
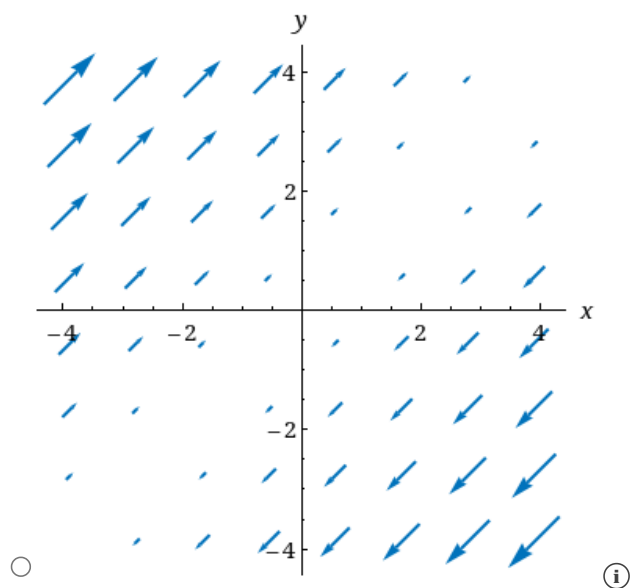
$$f(x, y) = \frac{1}{3}(x - y)^2$$

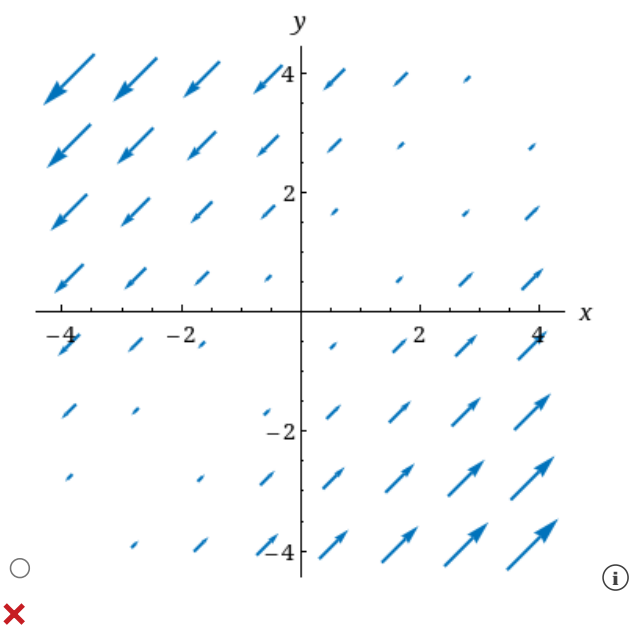
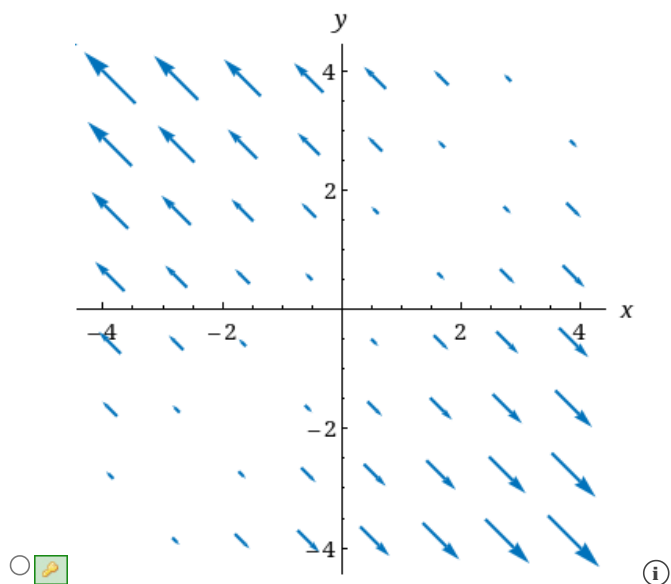
$\nabla f(x, y) =$



✗  $\frac{2}{3}(x - y)\mathbf{i} + \frac{2}{3}(y - x)\mathbf{j}$

Sketch the gradient vector field.



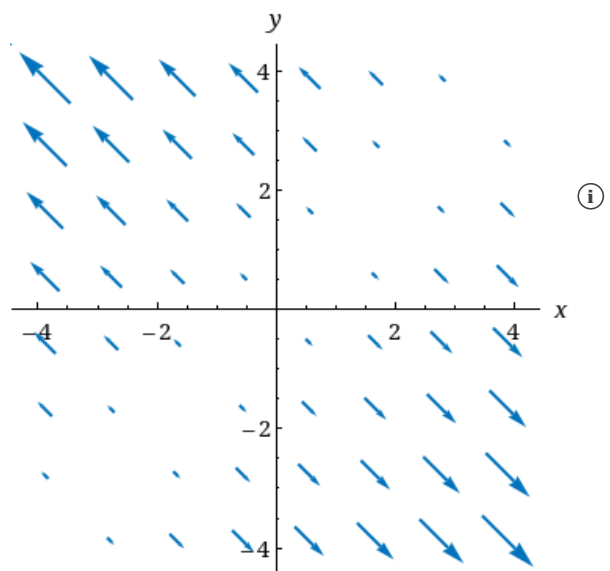


Solution or Explanation

We have the following.

$$\begin{aligned} f(x, y) &= \frac{1}{3}(x - y)^2 \Rightarrow \nabla f(x, y) = \frac{2}{3}(x - y)(1)\mathbf{i} + \frac{2}{3}(x - y)(-1)\mathbf{j} \\ &= \frac{2}{3}(x - y)\mathbf{i} + \frac{2}{3}(y - x)\mathbf{j} \end{aligned}$$

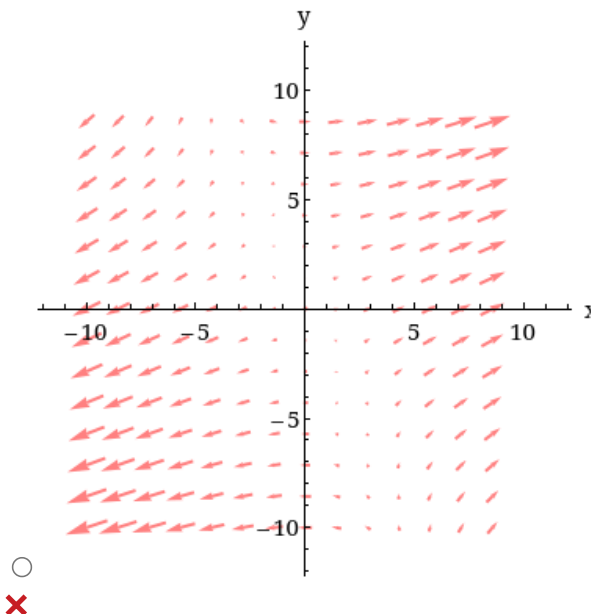
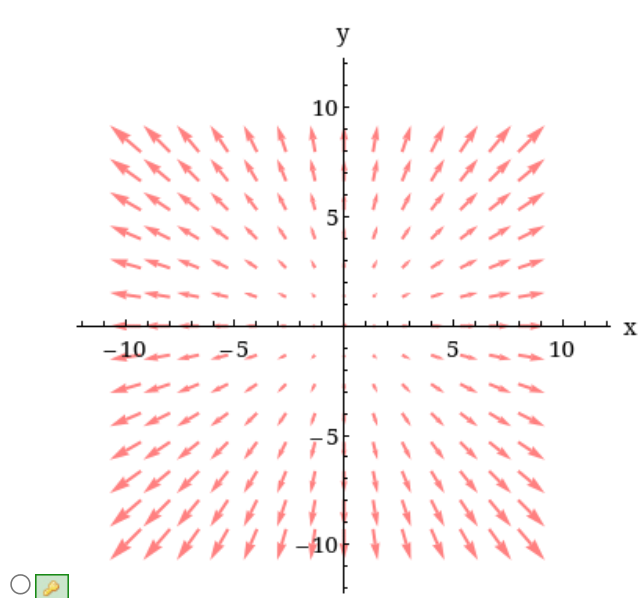
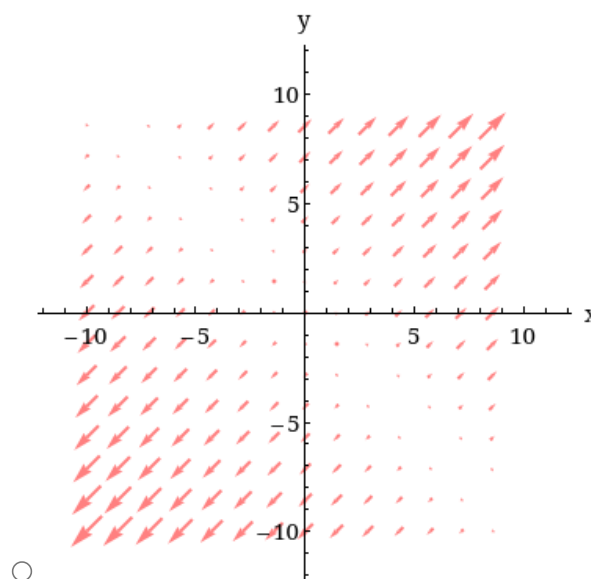
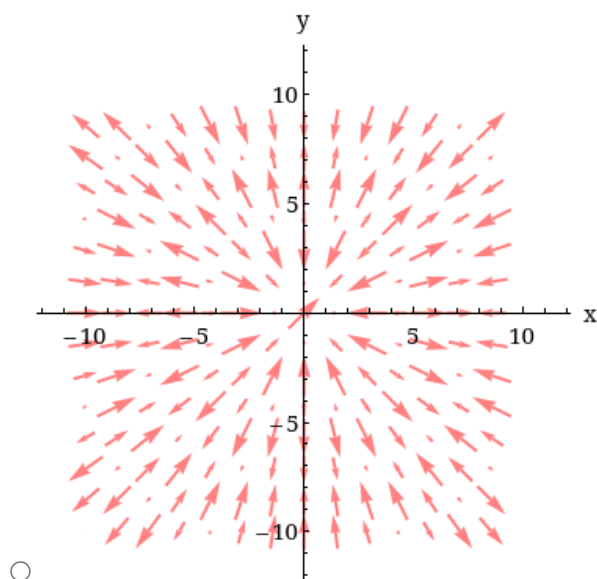
The length of  $\nabla f(x, y)$  is  $\sqrt{\left(\frac{2}{3}(x - y)\right)^2 + \left(\frac{2}{3}(y - x)\right)^2} = \frac{2\sqrt{2}}{3}|x - y|$ . The vectors are **0** along the line  $y = x$ . Elsewhere the vectors point away from the line  $y = x$  with length that increases as the distance from the line increases.



SCalcET9M 16.1.031.

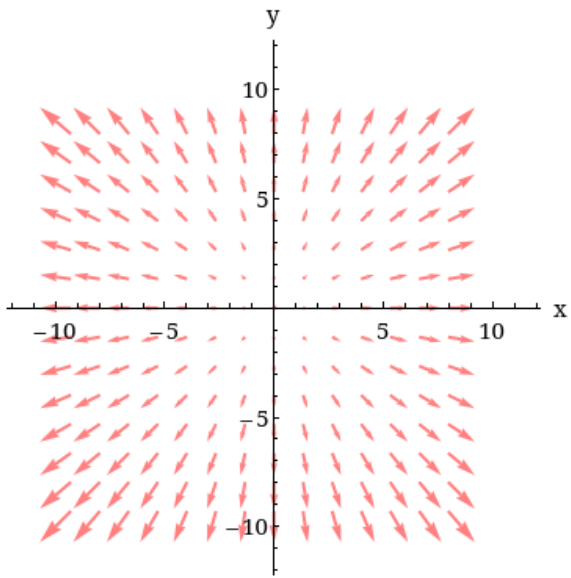
Match the function  $f$  with the correct gradient vector field plot.

$$f(x, y) = 10x^2 + 10y^2$$



Solution or Explanation

$f(x, y) = 10x^2 + 10y^2 \Rightarrow \nabla f(x, y) = 20x\mathbf{i} + 20y\mathbf{j}$ . Thus, each vector  $\nabla f(x, y)$  has the same direction and twenty times the length of the position vector of the point  $(x, y)$ , so the vectors all point directly away from the origin and their lengths increase as we move away from the origin. Hence,  $\nabla f$  is the graph below.



### Resources

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