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HW7 (Homework)

Current Score: – / 54 POINTS | 0.0 %

Scoring and Assignment Information

QUESTION	1	2	3	4	5	6	7	8	9	10
POINTS	– / 8	– / 5	– / 4	– / 5	– / 2	– / 6	– / 1	– / 11	– / 8	– / 4

Assignment Submission

For this assignment, you submit answers by question parts. The number of submissions remaining for each question part only changes if you submit or change the answer.

Assignment Scoring

Your best submission for each question part is used for your score.

1. [– / 8 Points] DETAILS

SCalcET9M 14.4.AE.004.

Video Example

EXAMPLE 4

(a) If $z = f(x, y) = x^2 + 5xy - y^2$, find the differential dz.

(b) If x changes from 2 to 2.01 and y changes from 3 to 2.93, compare the values of Δz and dz.

SOLUTION

(a) The definition of the differential gives

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$$

$$= \left(2x + 5y \right)dx + \left(5x - 2y \right)dy.$$

(b) Putting $x = 2$, $dx = \Delta x = 0.01$, $y = 3$, and $dy = \Delta y = -0.07$, we get

$$dz = \left[2(2) + 5(3) \right](0.01) + \left[5(2) - 2(3) \right]\left(- \right.$$

$$= -0.09.$$

The increment of z is as follows. (Round your final answer to two decimal places.)

$$\Delta z = f(2.01, 2.93) - f(2, 3)$$

$$= \left[\left(2.01 \right)^2 + 5(2.01)(2.93) - (2.93)^2 \right] - \left[2^2 + 5(2)(3) - \right.$$

$$= -0.10.$$

Notice that $\Delta z \approx dz$ is easier to compute.

2. [– / 5 Points] DETAILS

SCalcET9M 14.5.AE.002.

Video Example

EXAMPLE 2 The pressure P (in kilopascals), volume V (in liters), and temperature T (in kelvins) of a mole of an ideal gas are related by the equation $PV = 8.31T$. Find the rate at which the pressure is changing when the temperature is 400 K and increasing at a rate of 0.1 K/s and the volume is 100 L and increasing at a rate of 0.3 L/s.

SOLUTION If t represents the time elapsed in seconds, then at the given instant, we have $T = 400$, $dT/dt = 0.1$, $V = 100$, $dV/dt = 0.3$. Since

$$P = 8.31\frac{T}{V}$$

the Chain Rule gives the following. (Round your final answer to five decimal places.)

$$\frac{dP}{dt} = \frac{\partial P}{\partial T}\frac{dT}{dt} + \frac{\partial P}{\partial V}\frac{dV}{dt}$$

$$= \frac{8.31}{V}\frac{dT}{dt} - \left(\boxed{} \; \boxed{} \; ✗ \; \boxed{\frac{8.31T}{V^2}} \right)\frac{dV}{dt}$$

$$= \frac{8.31}{100}\left(\boxed{} ✗ \; \boxed{0.1} \right) - \frac{8.31(400)}{100^2}\left(\boxed{} ✗ \; \boxed{0.3} \right)$$

$$= \boxed{} ✗ \; \boxed{-0.09141}$$

The pressure is decreasing at a rate, rounded to three decimal places, of about [ ] ✗ 0.091 kPA/s.

3. [– / 4 Points] DETAILS

SCalcET9M 14.5.044.EP.

Consider Ohm's law, $V = IR$, where V is the voltage, I is the current, and R is the resistance. Solve the equation $V = IR$ for I.

$I =$

✗ $\boxed{\frac{V}{R}}$

Determine $\frac{dI}{dt}$.

$\frac{dI}{dt} = \Bigg($

✗ $\boxed{\frac{1}{R}}\Bigg)\frac{dV}{dt} + \Bigg($

✗ $\boxed{-\frac{V}{R^2}}\Bigg)\frac{dR}{dt}$

The voltage V in a simple electrical circuit is slowly decreasing as the battery wears out. The resistance R is slowly increasing as the resistor heats up. Use Ohm's law, $V = IR$, to find how the current I is changing (in A/s) at the moment when $R = 466\ \Omega$, $I = 0.03\ A$, $\frac{dV}{dt} = -0.01$ V/s, and $\frac{dR}{dt} = 0.02\ \Omega$/s. (Round your answer to six decimal places.)

✗ -0.000023 A/s

Solution or Explanation

$$I = \frac{V}{R} \quad \Rightarrow \quad \frac{dI}{dt} = \frac{\partial I}{\partial V}\frac{dV}{dt} + \frac{\partial I}{\partial R}\frac{dR}{dt} = \frac{1}{R}\frac{dV}{dt} - \frac{V}{R^2}\frac{dR}{dt} = \frac{1}{R}\frac{dV}{dt} - \frac{I}{R}\frac{dR}{dt}$$

$$= \frac{1}{466}(-0.01) - \frac{0.03}{466}(0.02) \approx -0.000023 \text{ A/s}$$

4. [– / 5 Points] DETAILS

SCalcET9M 14.6.064.

Show that every normal line to the sphere $x^2 + y^2 + z^2 = r^2$ passes through the center of the sphere.

Construct a proof for the statement by selecting sentences from the following scrambled list and putting them in the correct order.

- Equivalently, this statement can be transformed to 1 = 1 = 1, which is true.
- The normal line is given by $\dfrac{x - x_0}{2x_0} = \dfrac{y - y_0}{2y_0} = \dfrac{z - z_0}{2z_0}$.
- Then $\nabla F(x, y, z) = \langle 2x, 2y, 2z \rangle$ and $\nabla F(x_0, y_0, z_0) = \langle 2x_0, 2y_0, 2z_0 \rangle$.
- Let (x_0, y_0, z_0) be a point on the sphere, such that $x_0{}^2 = y_0{}^2 = z_0{}^2 = r^2$ and $F(x, y, z) = x^2 + y^2 + z^2$.
- The normal line is given by $\dfrac{x^2 - x_0{}^2}{2x_0} = \dfrac{y^2 - y_0{}^2}{2y_0} = \dfrac{z^2 - z_0{}^2}{2z_0}$.
- For the center (0, 0, 0) to be on the line, we need $-\dfrac{x_0}{2x_0} = -\dfrac{y_0}{2y_0} = -\dfrac{z_0}{2z_0}$
- Let (x_0, y_0, z_0) be a point on the sphere and $F(x, y, z) = x^2 + y^2 + z^2$.

Proof:

1. ---Select---
   Let (x_0, y_0, z_0) be a point on the sphere $F(x, y, z) = x^2 + y^2 + z^2$.
2. ---Select---
   Then $\nabla F(x, y, z) = <2x, 2y, 2z>$ and $\nabla F(x_0, y_0, z_0) = <2x_0, 2y_0, 2z_0>$.
3. ---Select---
   The normal line is given by $(x - x_0)/(2x_0) = (y - y_0)/(2y_0) = (z - z_0)/(2z_0)$.
4. ---Select---
   For the center of the sphere (0, 0, 0) to be on the line, we need $-(x_0)/(2x_0) = -(y_0)/(2y_0) = -(z_0)/(2z_0)$.
5. ---Select---
   Equivalently, this statement can be transformed to 1 = 1 = 1, which is true.
6. Thus, the normal line through the point (x_0, y_0, z_0) on the sphere passes through the center of the sphere.

Solution or Explanation

Let (x_0, y_0, z_0) be a point on the sphere and $F(x, y, z) = x^2 + y^2 + z^2$. Then $\nabla F(x, y, z) = \langle 2x, 2y, 2z \rangle$ and $\nabla F(x_0, y_0, z_0) = \langle 2x_0, 2y_0, 2z_0 \rangle$, so the normal line is given by

$$\frac{x - x_0}{2x_0} = \frac{y - y_0}{2y_0} = \frac{z - z_0}{2z_0}.$$

For the center (0, 0, 0) to be on the line, we need $-\dfrac{x_0}{2x_0} = -\dfrac{y_0}{2y_0} = -\dfrac{z_0}{2z_0}$ or equivalently 1 = 1 = 1, which is true.

5. [–/2 Points] DETAILS

SCalcET9M 14.6.030.

Find the maximum rate of change of *f* at the given point and the direction in which it occurs.

$$f(x, y, z) = x \ln(yz), \quad \left(2, 5, \frac{1}{5}\right)$$

maximum rate of change

$\frac{2\sqrt{626}}{5}$

direction vector

$\left\langle 0, \frac{2}{5}, 10 \right\rangle$

Solution or Explanation

We have the following.

$$f(x, y, z) = x \ln(yz) \quad \Rightarrow \quad \nabla f(x, y, z) = \left\langle \ln(yz), x \cdot \frac{z}{yz}, x \cdot \frac{y}{yz} \right\rangle = \left\langle \ln(yz), \frac{x}{y}, \frac{x}{z} \right\rangle$$

$$\nabla f\left(2, 5, \frac{1}{5}\right) = \left\langle 0, \frac{2}{5}, 10 \right\rangle$$

Thus, the maximum rate of change is

$$\left|\nabla f\left(2, 5, \frac{1}{5}\right)\right| = \sqrt{0 + \frac{4}{25} + 100} = \sqrt{\frac{2504}{25}} = \frac{2\sqrt{626}}{5}$$

in the direction $\left\langle 0, \frac{2}{5}, 10 \right\rangle$, or equivalently, $\langle 0, 2, 50 \rangle$.

6. [–/6 Points] DETAILS

SCalcET9M 14.5.025.EP.

Consider the following.

$z = x^4 + x^2y, \quad x = s + 2t - u, \quad y = stu^2$

Find the following partial derivatives.

$\frac{\partial z}{\partial s}$ =

✗ $tu^2x^2 + 4x^3 + 2xy$

$\frac{\partial z}{\partial t}$ =

✗ $su^2x^2 + 2\left(4x^3 + 2xy\right)$

$\frac{\partial z}{\partial u}$ =

✗ $2stux^2 - 4x^3 - 2xy$

Find $\frac{\partial z}{\partial s}, \frac{\partial z}{\partial t}, \frac{\partial z}{\partial u}$ when $s = 3$, $t = 1$, $u = 5$.

$\frac{\partial z}{\partial s}$ =

✗ 0

$\frac{\partial z}{\partial t}$ =

✗ 0

$\frac{\partial z}{\partial u}$ =

✗ 0

Solution or Explanation

We have the following.

$$z = x^4 + x^2y,\ x = s + 2t - u,\ y = stu^2 \quad \Rightarrow \quad \frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s} = (4x^3 + 2xy)(1) + (x^2)(tu^2),$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t} = (4x^3 + 2xy)(2) + (x^2)(su^2),$$

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u} = (4x^3 + 2xy)(-1) + (x^2)(2stu)$$

When $s = 3$, $t = 1$, and $u = 5$ we have $x = 0$ and $y = 75$, so we get the following.

$$\frac{\partial z}{\partial s} = (0)(1) + (0)(25) = 0,$$

$$\frac{\partial z}{\partial t} = (0)(2) + (0)(75) = 0,$$

$$\frac{\partial z}{\partial u} = (0)(-1) + (0)(30) = 0$$

Resources

Watch It

7. [– / 1 Points]

DETAILS

SCalcET9M 14.6.023.

Find the directional derivative of $f(x, y) = \sqrt{xy}$ at $P(3, 3)$ in the direction from P to $Q(6, -1)$.

$D_{\mathbf{u}}f(3, 3) =$

✕ $\boxed{-\frac{1}{10}}$

Solution or Explanation

Click to View Solution

Resources

Watch It

8. [– / 11 Points] DETAILS

SCalcET9M 14.5.057.

A function f is called *homogeneous of degree n* if it satisfies the equation

$$f(tx, ty) = t^n f(x, y)$$

for all t, where n is a positive integer and f has continuous second-order partial derivatives.

(a) Show that if f is homogeneous of degree n, then

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf(x, y).$$

Differentiating both sides of $f(tx, ty) = t^n f(x, y)$ with respect to t using the chain rule, we get the following.

$$\frac{\partial}{\partial t}f(tx, ty) = \frac{\partial}{\partial t}\left[t^n f(x, y)\right] \Leftrightarrow$$

$$\frac{\partial}{\partial(tx)}f(tx, ty)\cdot\frac{\partial(tx)}{\partial t} + \frac{\partial}{\partial(ty)}f(tx, ty)\cdot\frac{\partial(ty)}{\partial t} = x\left(\frac{\partial}{\partial(\ \boxed{tx}\ )}\right)f(tx, ty) + y\left(\frac{\partial}{\partial(\ \boxed{ty}\ )}\right)f(tx, ty) = n(\ \boxed{t}\ )^{n-1}f(x, y)$$

✗ ✗ ✗

Setting $t = 1$: $x\frac{\partial}{\partial x}f(x, y) + (\ \boxed{y}\ )\frac{\partial}{\partial y}f(x, y) = nf(x, y).$ ✗

(b) Show that if f is homogeneous of degree n, then

$$x^2\frac{\partial^2 f}{\partial x^2} + 2xy\frac{\partial^2 f}{\partial x\partial y} + y^2\frac{\partial^2 f}{\partial y^2} = n(n-1)f(x, y).$$

Differentiating both sides of $f(tx, ty) = t^n f(x, y)$ with respect to t using the chain rule, we get

$$\frac{\partial}{\partial(tx)}f(tx, ty)\cdot\frac{\partial(tx)}{\partial t}+\frac{\partial}{\partial(ty)}f(tx, ty)\cdot\frac{\partial(ty)}{\partial t} = x\left(\frac{\partial}{\partial\left(tx\right)}\right)f(tx, ty)$$

$$+\ y\left(\frac{\partial}{\partial\left(ty\right)}\right)f(tx, ty)$$

$$= n\left(t\right)^{n-1}f(x, y),$$

and differentiating again with respect to t gives

$$x\left[\frac{\partial^2}{\partial(tx)^2}f(tx, ty)\cdot\frac{\partial(tx)}{\partial t}+\frac{\partial^2}{\partial(ty)\partial(tx)}f(tx, ty)\cdot\frac{\partial\left(ty\right)}{\partial t}\right]$$

$$+\ y\left[\frac{\partial^2}{\partial(tx)\partial(ty)}f(tx, ty)\cdot\frac{\partial(tx)}{\partial t}+\frac{\partial^2}{\partial(ty)^2}f(tx, ty)\cdot\frac{\partial\left(ty\right)}{\partial t}\right] = n(n-1)\left(t\right)^{n-2}f(x, y).$$

Setting $t = 1$ and using the fact that $f_{yx} = f_{xy}$, we have $x^2f_{xx} + 2xyf_{xy} + \left(y^2\right)f_{yy} = n(n-1)f(x, y)$.

Solution or Explanation

(a) Differentiating both sides of $f(tx, ty) = t^nf(x, y)$ with respect to t using the chain rule, we get the following.

$$\frac{\partial}{\partial t}f(tx, ty) = \frac{\partial}{\partial t}\left[t^nf(x, y)\right] \quad\Leftrightarrow$$

$$\frac{\partial}{\partial(tx)}f(tx, ty)\cdot\frac{\partial(tx)}{\partial t}+\frac{\partial}{\partial(ty)}f(tx, ty)\cdot\frac{\partial(ty)}{\partial t} = x\frac{\partial}{\partial(tx)}f(tx, ty)+y\frac{\partial}{\partial(ty)}f(tx, ty) = nt^{n-1}f(x, y)$$

Setting $t = 1$: $x\frac{\partial}{\partial x}f(x, y)+y\frac{\partial}{\partial y}f(x, y) = nf(x, y)$.

(b) Differentiating both sides of $f(tx, ty) = t^nf(x, y)$ with respect to t using the chain rule, we get

$$\frac{\partial}{\partial(tx)}f(tx,ty)\cdot\frac{\partial(tx)}{\partial t}+\frac{\partial}{\partial(ty)}f(tx,ty)\cdot\frac{\partial(ty)}{\partial t}=x\frac{\partial}{\partial(tx)}f(tx,ty)+y\frac{\partial}{\partial(ty)}f(tx,ty)=nt^{n-1}f(x,y)$$

and differentiating again with respect to t gives

$$x\left[\frac{\partial^2}{\partial(tx)^2}f(tx,ty)\cdot\frac{\partial(tx)}{\partial t}+\frac{\partial^2}{\partial(ty)\partial(tx)}f(tx,ty)\cdot\frac{\partial(ty)}{\partial t}\right]$$
$$+\,y\left[\frac{\partial^2}{\partial(tx)\partial(ty)}f(tx,ty)\cdot\frac{\partial(tx)}{\partial t}+\frac{\partial^2}{\partial(ty)^2}f(tx,ty)\cdot\frac{\partial(ty)}{\partial t}\right]=n(n-1)t^{n-2}f(x,y).$$

Setting $t = 1$ and using the fact that $f_{yx} = f_{xy}$, we have $x^2f_{xx} + 2xyf_{xy} + y^2f_{yy} = n(n-1)f(x,y)$.

9. [– / 8 Points]

DETAILS

SCalcET9M 14.5.054.

Assume that all the given functions have continuous second-order partial derivatives. If $z = f(x, y)$, where $x = 2r\cos(\theta)$ and $y = 2r\sin(\theta)$, find the following.

(a) $\dfrac{\partial z}{\partial r}$

$\dfrac{\partial z}{\partial r} = \dfrac{\partial z}{\partial x}\Big($

✕ $2\cos(\theta)$ $\Big) + \dfrac{\partial z}{\partial y}\Big($

✕ $2\sin(\theta)$ $\Big)$

(b) $\dfrac{\partial z}{\partial \theta}$

$\dfrac{\partial z}{\partial \theta} = \dfrac{\partial z}{\partial x}\Big($

✕ $-2r\sin(\theta)$ $\Big) + \dfrac{\partial z}{\partial y}\Big($

✕ $2r\cos(\theta)$ $\Big)$

(c) $\dfrac{\partial^2 z}{\partial r\, \partial \theta}$

$\dfrac{\partial^2 z}{\partial r\, \partial \theta} = \Big($

✕ $2\cos(\theta)$ $\Big)\dfrac{\partial z}{\partial y} + \Big($

✕ $-2\sin(\theta)$ $\Big)\dfrac{\partial z}{\partial x} + \Big($

✕ $4r\cos(\theta)\sin(\theta)$ $\Big)\left(\dfrac{\partial^2 z}{\partial y^2} - \dfrac{\partial^2 z}{\partial x^2}\right) + \Big($

✕ $4r\left(\cos^2(\theta) - \sin^2(\theta)\right)$ $\Big)\dfrac{\partial^2 z}{\partial y\, \partial x}$

Solution or Explanation

Click to View Solution

10. [– / 4 Points] DETAILS

SCalcET9M 14.6.022.EP.

Consider the following.

$$f(x, y) = \frac{x}{y^2}$$

Find $\nabla f(x, y)$.

$\nabla f(x, y) =$

✕ $\left\langle \frac{1}{y^2}, -\frac{2x}{y^3} \right\rangle$

Determine $\nabla f(x, y)$ at the point $P = (3, -1)$.

$\nabla f(3, -1) =$

✕ $\langle 1, 6 \rangle$

Determine a unit vector in the direction of $\overrightarrow{PQ}$ where $P = (3, -1)$ and $Q = (-9, 15)$.

u =

✕ $\left\langle -\frac{3}{5}, \frac{4}{5} \right\rangle$

Find the directional derivative of the function at the point P in the direction of the point Q.

$$f(x, y) = \frac{x}{y^2}, \quad P(3, -1), \quad Q(-9, 15)$$

✕ $\frac{21}{5}$

Solution or Explanation

We have

$$f(x, y) = \frac{x}{y^2} \quad \Rightarrow \quad \nabla f(x, y) = \left\langle \frac{1}{y^2}, -\frac{2x}{y^3} \right\rangle,$$

so $\nabla f(3, -1) = \langle 1, 6 \rangle$. The unit vector in the direction of $\overrightarrow{PQ} = \langle -9 - 3, 15 - (-1) \rangle = \langle -12, 16 \rangle$ is

$$\mathbf{u} = \frac{1}{\sqrt{(-12)^2 + 16^2}} \langle -12, 16 \rangle = \left\langle -\frac{3}{5}, \frac{4}{5} \right\rangle,$$

so $D_{\mathbf{u}} f(3, -1) = \nabla f(3, -1) \cdot \mathbf{u} = \langle 1, 6 \rangle \cdot \left\langle -\frac{3}{5}, \frac{4}{5} \right\rangle = \frac{21}{5}$.