

Assignment Previewer

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EDIT ASSIGNMENT

INSTRUCTOR

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HW12 (Homework)

Current Score: - / 39 POINTS | 0.0 %

Scoring and Assignment Information



QUESTION	1	2	3	4	5	6	7	8	9	10	11	12	13	14
POINTS	- / 1	- / 1	- / 1	- / 1	- / 1	- / 7	- / 13	- / 1	- / 1	- / 7	- / 1	- / 1	- / 2	- / 1

Assignment Submission

For this assignment, you submit answers by question parts. The number of submissions remaining for each question part only changes if you submit or change the answer.

Assignment Scoring

Your best submission for each question part is used for your score.

1. [- / 1 Points]

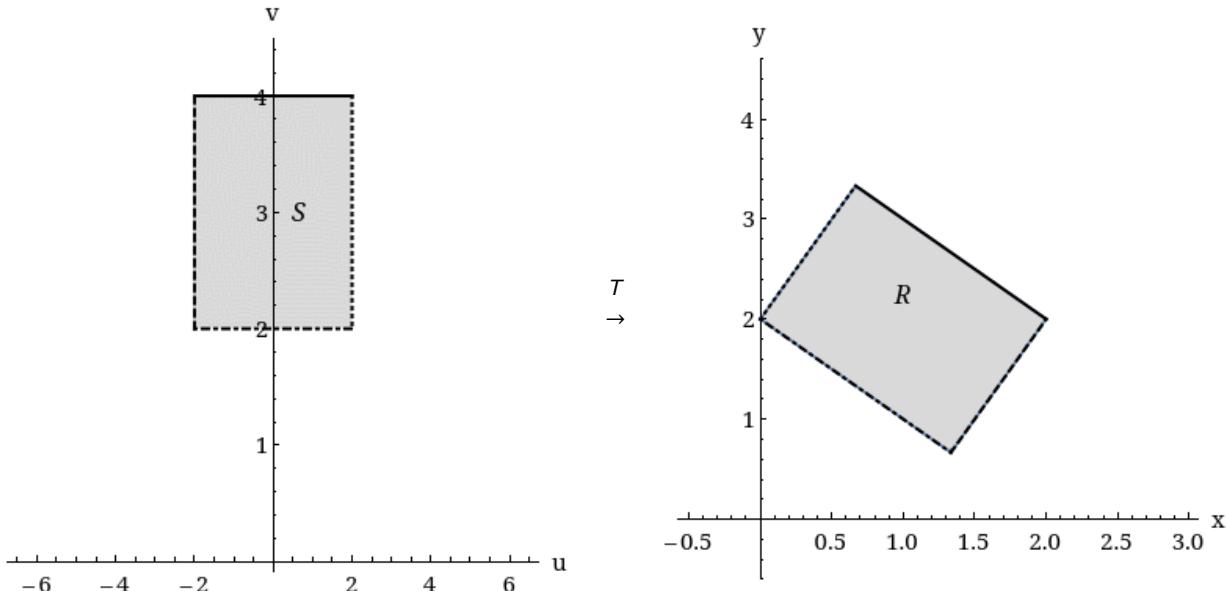
DETAILS

SCalcET9M 15.9.007.

A region R in the xy -plane is given. Find equations for a transformation T that maps a rectangular region S in the uv -plane onto R , where the sides of S are parallel to the u - and v -axes as shown in the figure below. (Enter your answers as a comma-separated list of equations.)

R is bounded by $y = 2x - 2$, $y = 2x + 2$, $y = 2 - x$, $y = 4 - x$

✗ $x = \frac{v-u}{3}, y = \frac{1}{3}(u+2v)$

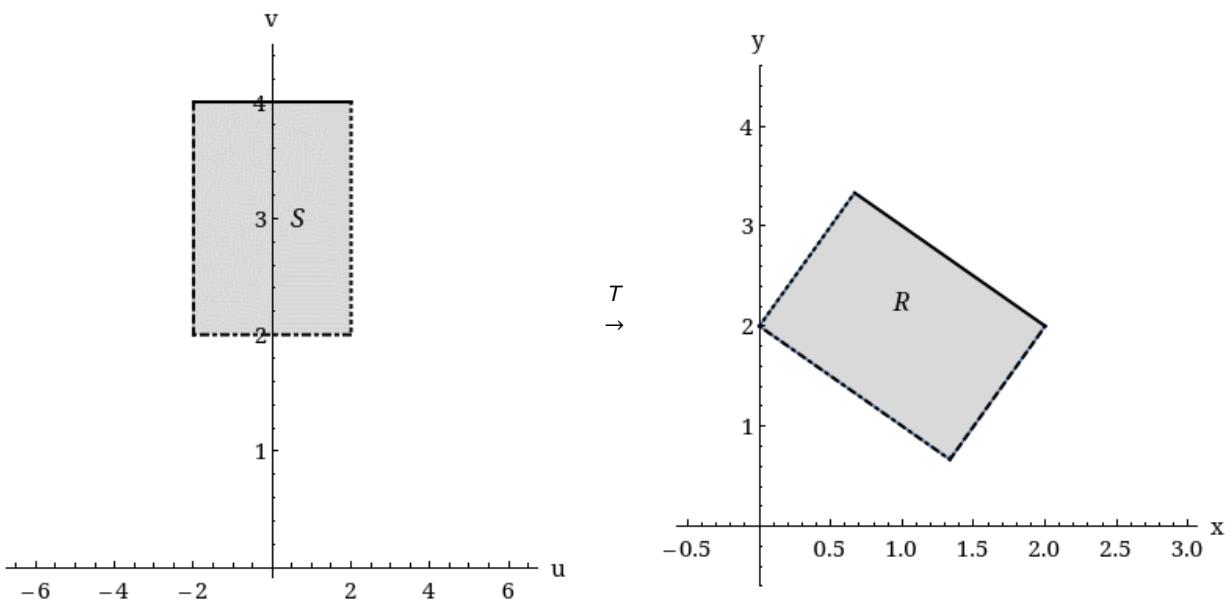


Solution or Explanation

R is a parallelogram enclosed by the parallel lines $y = 2x - 2$, $y = 2x + 2$ and the parallel lines $y = 2 - x$, $y = 4 - x$. The first pair of equations can be written as $y - 2x = -2$, $y - 2x = 2$. If we let $u = y - 2x$ then these lines are mapped to the vertical lines $u = -2$, $u = 2$ in the uv -plane. Similarly, the second pair of equations can be written as $x + y = 2$, $x + y = 4$, and setting $v = x + y$ maps these lines to the horizontal lines $v = 2$, $v = 4$ in the uv -plane. Boundary curves are mapped to boundary curves under a transformation, so here the equations $u = y - 2x$, $v = x + y$ define a transformation T^{-1} that maps R in the xy -plane to the square S enclosed by the lines $u = -2$, $u = 2$, $v = 2$, $v = 4$ in the uv -plane. To find the transformation T that maps S to R we solve

$u = y - 2x$, $v = x + y$ for x , y : Subtracting the first equation from the second gives $v - u = 3x \Rightarrow x = \frac{1}{3}(v - u)$ and adding twice the second equation to the first gives $u + 2v = 3y \Rightarrow y = \frac{1}{3}(u + 2v)$. Thus the transformation T is given by

$$x = \frac{1}{3}(v - u), y = \frac{1}{3}(u + 2v).$$



Resources

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2. [- / 1 Points]

DETAILS

SCalcET9M 15.9.009.

A region R in the xy -plane is given. Find equations for a transformation T that maps a rectangular region S in the uv -plane onto R , where the sides of S are parallel to the u - and v -axes. (Let u play the role of r and v the role of θ . Enter your answers as a comma-separated list of equations.)

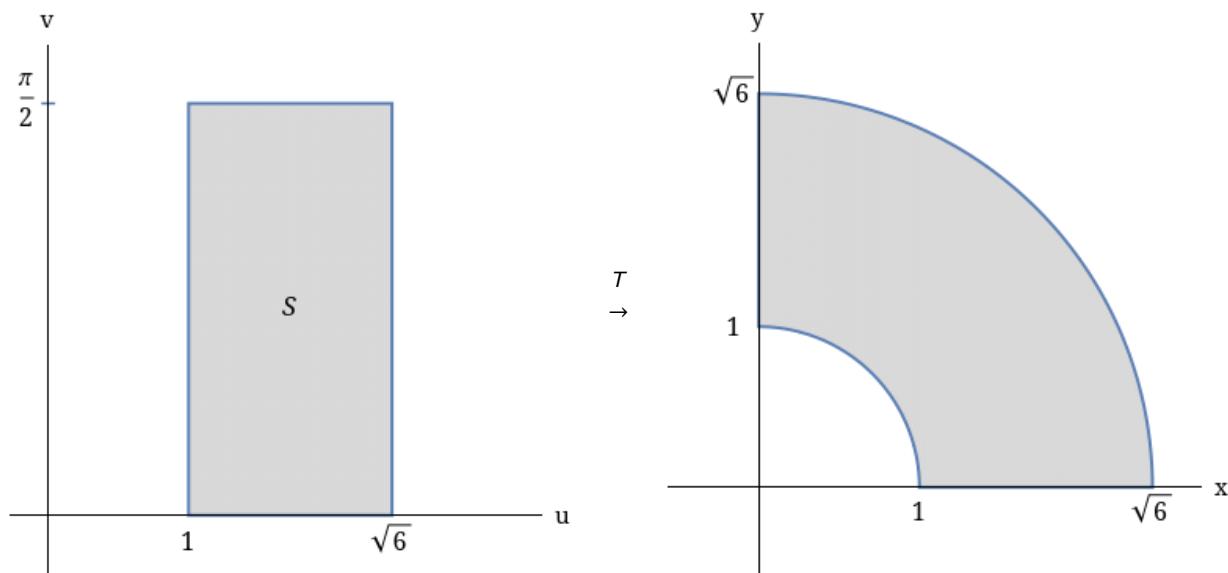
R lies between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 6$ in the first quadrant

X $x = u \cos(v), y = u \sin(v)$

Solution or Explanation

R is a portion of an annular region (see the figure) that is easily described in polar coordinates as

$R = \{(r, \theta) \mid 1 \leq r \leq \sqrt{6}, 0 \leq \theta \leq \pi/2\}$. If we converted a double integral over R to polar coordinates the resulting region of integration is a rectangle (in the $r\theta$ -plane), so we can create a transformation T here by letting u play the role of r and v the role of θ . Thus T is defined by $x = u \cos v, y = u \sin v$ and T maps the rectangle $S = \{(u, v) \mid 1 \leq u \leq \sqrt{6}, 0 \leq v \leq \pi/2\}$ in the uv -plane to R in the xy -plane.



Resources

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3. [- / 1 Points]

DETAILS

SCalcET9M 15.9.014.

Find the Jacobian of the transformation.

$$x = 6pe^q, \quad y = 6qe^p$$

X 36e^{p+q}(1 - pq)

Solution or Explanation

$$x = 6pe^q, \quad y = 6qe^p$$

$$\frac{\partial(x, y)}{\partial(p, q)} = \begin{vmatrix} \frac{\partial x}{\partial p} & \frac{\partial x}{\partial q} \\ \frac{\partial y}{\partial p} & \frac{\partial y}{\partial q} \end{vmatrix} = \begin{vmatrix} 6e^q & 6pe^q \\ 6qe^p & 6e^p \end{vmatrix} = 36e^q e^p - 6pe^q \cdot 6qe^p = 36e^{p+q} - 36pqe^{p+q} = 36(1 - pq)e^{p+q}$$

4. [- / 1 Points]

DETAILS

SCalcET9M 15.9.015.

Find the Jacobian of the transformation.

$$x = 5uv, \quad y = 2vw, \quad z = 4wu$$

X 80uvw

Solution or Explanation

$$x = 5uv, \quad y = 2vw, \quad z = 4wu$$

$$\frac{\partial(x, y, z)}{\partial(u, v, w)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial w} \end{vmatrix} = \begin{vmatrix} 5v & 5u & 0 \\ 0 & 2w & 2v \\ 4w & 0 & 4u \end{vmatrix} = 5v \begin{vmatrix} 2w & 2v \\ 0 & 4u \end{vmatrix} - 5u \begin{vmatrix} 0 & 2v \\ 4w & 4u \end{vmatrix} + 0 \begin{vmatrix} 0 & 2w \\ 4w & 0 \end{vmatrix}$$

$$= 5v(8uw - 0) - 5u(0 - 8vw) + 0 = 40uvw + 40uvw = 80uvw$$

5. [- / 1 Points]

DETAILS

SCalcET9M 15.9.017.MI.

Use the given transformation to evaluate the integral.

$\iint_R (x - 8y) dA$, where R is the triangular region with vertices $(0, 0)$, $(7, 1)$, and $(1, 7)$. $x = 7u + v$, $y = u + 7v$

✗ -448

Solution or Explanation

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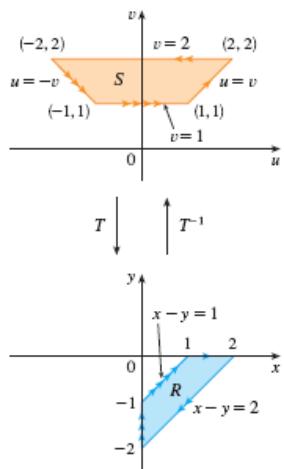
Resources

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6. [- / 7 Points]

DETAILS

SCalcET9M 15.9.AE.002.

[Video Example](#)

EXAMPLE 2 Using the change of variables $x = u^2 - v^2$, $y = 2uv$ to evaluate the integral $\iint_R y \, dA$, where R is the region bounded by the x -axis and the parabolas $y^2 = 2 - 2x$ and $y^2 = 2 + 2x$, $y \geq 0$.

SOLUTION A similar region R is pictured in the figure. In Example 1, we discovered that $T(S) = R$, where S is the square $[0, 1] \times [0, 1]$. Indeed, the reason for making the change of variables to evaluate the integral is that S is a much simpler region than R . First we need to compute the Jacobian:

$$\begin{aligned} \frac{\partial(x, y)}{\partial(u, v)} &= \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} \\ &= \begin{vmatrix} 2u & -2v \\ 2v & 2u \end{vmatrix} \\ &= \begin{vmatrix} 4u^2 + 4v^2 & \end{vmatrix} \quad \text{✖ } 2u \\ &= \begin{vmatrix} 4u^2 + 4v^2 & \end{vmatrix} \quad \text{✖ } 4u^2 + 4v^2 \geq 0 \end{aligned}$$

Therefore by the theorem for the change of variables in a double integral

$$\begin{aligned} \iint_R y \, dA &= \iint_S 2uv \left| \frac{\partial(x, y)}{\partial(u, v)} \right| dA \\ &= \int_0^1 \int_0^1 (2uv)(\quad) du \, dv \\ &\quad \text{✖ } 4u^2 + 4v^2 \\ &= 8 \int_0^1 \int_0^1 (u^3v + uv^3) du \, dv \\ &\quad \text{ } \boxed{\frac{1}{4}u^4v + \frac{1}{2}u^2v^3} \\ &= 8 \int_0^1 \left[\quad \right]_0^1 dv \\ &\quad \text{ } \boxed{\frac{1}{4}v + \frac{1}{2}v^3} \\ &= 8 \int_0^1 (\quad) dv \\ &\quad \text{ } \boxed{\frac{1}{2}v^2} \\ &= 8 \left[\quad \right]_0^1 = \boxed{\frac{8}{2}} \quad \text{✖ } \end{aligned}$$

×

$$\frac{v^2}{8} + \frac{v^4}{8}$$

7. [- / 13 Points]

DETAILS

SCalcET9M 15.9.025.MI.SA.

This question has several parts that must be completed sequentially. If you skip a part of the question, you will not receive any points for the skipped part, and you will not be able to come back to the skipped part.

Tutorial Exercise

Evaluate the given integral by making an appropriate change of variables.

$$\iint_R \frac{x - 2y}{3x - y} dA, \text{ where } R \text{ is the parallelogram enclosed by the lines } x - 2y = 0, x - 2y = 8, 3x - y = 1, \text{ and } 3x - y = 9$$

Click here to begin!

8. [- / 1 Points]

DETAILS

SCalcET9M 15.9.026.

Evaluate the integral by making an appropriate change of variables.

$$\iint_R 3(x + y) e^{x^2 - y^2} dA, \text{ where } R \text{ is the rectangle enclosed by the lines } x - y = 0, x - y = 5, x + y = 0, \text{ and } x + y = 7$$

×

$$\frac{3}{10} (e^{35} - 36)$$

Solution or Explanation

[Click to View Solution](#)

9. [- / 1 Points]

DETAILS

SCalcET9M 16.1.038.

At time $t = 1$, a particle is located at position $(x, y) = (3, 1)$. If it moves in the velocity field

$$\mathbf{F}(x, y) = \langle xy - 2, y^2 - 10 \rangle$$

find its approximate location at time $t = 1.07$.

$$(x, y) = \left(\begin{array}{l} \boxed{} \\ \boxed{} \end{array} \right)$$

×

$$\boxed{3.07, 0.37}$$

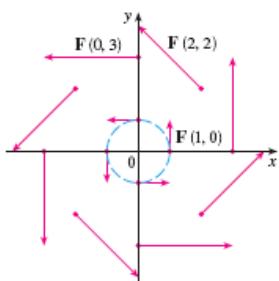
Solution or Explanation

[Click to View Solution](#)

10. [- / 7 Points]

DETAILS

SCalcET9M 16.1.AE.001.

[Video Example](#)

EXAMPLE 1 A vector field on \mathbb{R}^2 is defined by $\mathbf{F}(x, y) = -y\mathbf{i} + x\mathbf{j}$. Describe \mathbf{F} by sketching some of the vectors $\mathbf{F}(x, y)$ as in the figure.

$$\mathbf{j} = \langle$$

SOLUTION Since $\mathbf{F}(1, 0) = \mathbf{j}$, we draw the vector $\langle 0, 1 \rangle$ starting at the point $(1, 0)$ in the figure. Since $\mathbf{F}(0, 1) = -\mathbf{i}$, we draw the

$$(x, y) = \langle$$

vector $\langle -1, 0 \rangle$ with starting point $\langle 0, 1 \rangle$. Continuing in this way, we calculate several other representative values of $\mathbf{F}(x, y)$ in the table and draw the corresponding vectors to represent the vector field in the figure.

(x, y)	$\mathbf{F}(x, y)$	(x, y)	$\mathbf{F}(x, y)$
$(1, 0)$	$\langle 0, 1 \rangle$	$(-1, 0)$	$\langle 0, -1 \rangle$
$(2, 2)$	<div style="border: 1px solid black; width: 150px; height: 20px; margin-top: 5px;"></div> <div style="border: 1px solid black; width: 150px; height: 20px; margin-top: 5px;"></div> $\times \langle -2, 2 \rangle$	$(-2, -2)$	$\langle 2, -2 \rangle$
$(3, 0)$	$\langle 0, 3 \rangle$	$(-3, 0)$	$\langle 0, -3 \rangle$
$(0, 1)$	$\langle -1, 0 \rangle$	<div style="border: 1px solid black; width: 150px; height: 20px; margin-top: 5px;"></div> <div style="border: 1px solid black; width: 150px; height: 20px; margin-top: 5px;"></div> $\times \langle 0, -1 \rangle$	$\langle 1, 0 \rangle$
$(-2, 2)$	$\langle -2, -2 \rangle$	$(2, -2)$	$\langle 2, 2 \rangle$
$(0, 3)$	<div style="border: 1px solid black; width: 150px; height: 20px; margin-top: 5px;"></div> <div style="border: 1px solid black; width: 150px; height: 20px; margin-top: 5px;"></div> $\times \langle -3, 0 \rangle$	$(0, -3)$	$\langle 3, 0 \rangle$

It appears from the figure that each arrow is tangent to a circle with center the origin. To confirm this, we take the dot product of the position vector $\mathbf{x} = x\mathbf{i} + y\mathbf{j}$ with the vector $\mathbf{F}(\mathbf{x}) = \mathbf{F}(x, y)$:

$$(\mathbf{x}\mathbf{i} + y\mathbf{j}) \cdot (-y\mathbf{i} + x\mathbf{j}) =$$

$$\mathbf{x} \cdot \mathbf{F}(\mathbf{x}) =$$

$$\times \langle -xy \rangle + yx$$

$$\begin{array}{l} \boxed{} \\ = \boxed{} \\ \times \boxed{0} \end{array}$$

This shows that $\mathbf{F}(x, y)$ is perpendicular to the position vector $\langle x, y \rangle$ and is therefore tangent to a circle with center the origin and radius

$$|\mathbf{x}| = \sqrt{x^2 + y^2}. \text{ Notice also that}$$

$$|\mathbf{F}(x, y)| = \sqrt{(-y)^2 + (x)^2} = \sqrt{x^2 + y^2} = |\mathbf{x}|$$

so the magnitude of the vector $\mathbf{F}(x, y)$ is equal to the radius of the circle.

11. [- / 1 Points]

DETAILS

SCalcET9M 16.1.026.

Find the gradient vector field ∇f of f .

$$f(s, t) = \sqrt{7s + 8t}$$

$$\nabla f(s, t) =$$

$$\begin{array}{l} \boxed{} \\ \boxed{} \end{array}$$

$$\times \boxed{\frac{7}{2\sqrt{7s+8t}}\mathbf{i} + \frac{4}{\sqrt{7s+8t}}\mathbf{j}}$$

Solution or Explanation

$$\begin{aligned} f(s, t) = \sqrt{7s + 8t} \Rightarrow \nabla f(s, t) &= f_s(s, t)\mathbf{i} + f_t(s, t)\mathbf{j} \\ &= \left[\frac{1}{2}(7s + 8t)^{-1/2} \cdot 7 \right] \mathbf{i} + \left[\frac{1}{2}(7s + 8t)^{-1/2} \cdot 8 \right] \mathbf{j} \\ &= \frac{7}{2\sqrt{7s+8t}}\mathbf{i} + \frac{4}{\sqrt{7s+8t}}\mathbf{j} \end{aligned}$$

12. [- / 1 Points]

DETAILS

SCalcET9M 16.1.028.

Find the gradient vector field ∇f of f .

$$f(x, y, z) = x^4 y e^{y/z}$$

$$\nabla f(x, y, z) =$$

$$\times \boxed{4x^3 y e^{y/z} \mathbf{i} + x^4 e^{y/z} \left(\frac{y}{z} + 1\right) \mathbf{j} - \frac{x^4 y^2}{z^2} e^{y/z} \mathbf{k}}$$

Solution or Explanation

$$f(x, y, z) = x^4 y e^{y/z} \Rightarrow$$

$$\begin{aligned}\nabla f(x, y, z) &= f_x(x, y, z) \mathbf{i} + f_y(x, y, z) \mathbf{j} + f_z(x, y, z) \mathbf{k} \\&= 4x^3 y e^{y/z} \mathbf{i} + x^4 \left[y \cdot e^{yz} \left(\frac{1}{z} \right) + e^{y/z} \cdot 1 \right] \mathbf{j} + \left[x^4 y e^{y/z} \left(-\frac{y}{z^2} \right) \right] \mathbf{k} \\&= 4x^3 y e^{y/z} \mathbf{i} + x^4 e^{y/z} \left(\frac{y}{z} + 1 \right) \mathbf{j} - \frac{x^4 y^2}{z^2} e^{y/z} \mathbf{k}\end{aligned}$$

13. [- / 2 Points]

DETAILS

SCalcET9M 16.1.029.

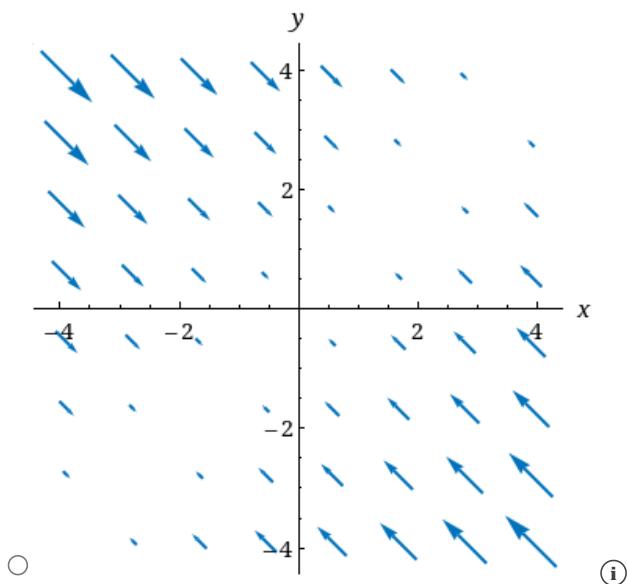
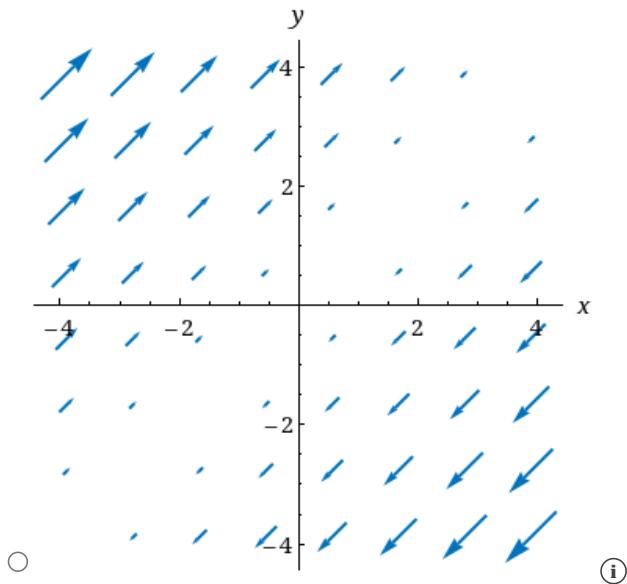
Find the gradient vector field ∇f of f .

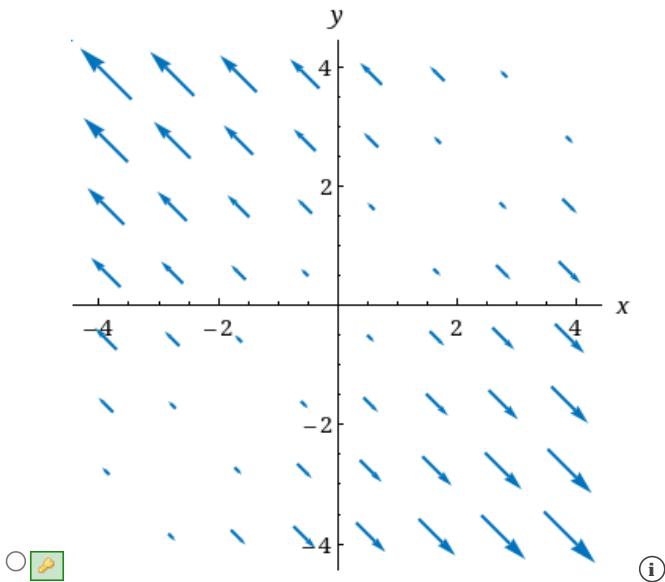
$$f(x, y) = \frac{1}{3}(x - y)^2$$

$$\nabla f(x, y) =$$

$$\times \quad \frac{2}{3}(x - y)\mathbf{i} + \frac{2}{3}(y - x)\mathbf{j}$$

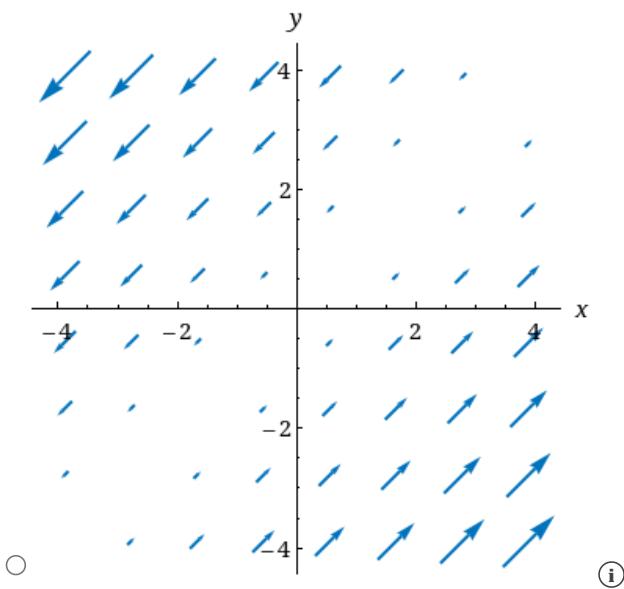
Sketch the gradient vector field.





○

①



○

①

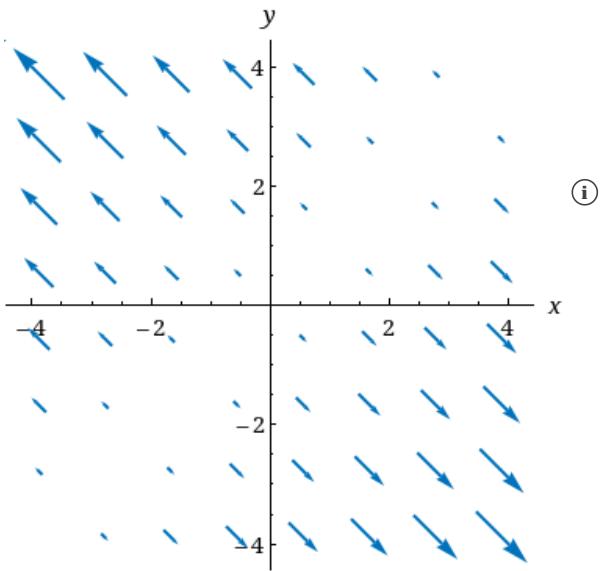
X

Solution or Explanation

We have the following.

$$\begin{aligned} f(x, y) = \frac{1}{3}(x - y)^2 &\Rightarrow \nabla f(x, y) = \frac{2}{3}(x - y)(1)\mathbf{i} + \frac{2}{3}(x - y)(-1)\mathbf{j} \\ &= \frac{2}{3}(x - y)\mathbf{i} + \frac{2}{3}(y - x)\mathbf{j} \end{aligned}$$

The length of $\nabla f(x, y)$ is $\sqrt{\left(\frac{2}{3}(x - y)\right)^2 + \left(\frac{2}{3}(y - x)\right)^2} = \frac{2\sqrt{2}}{3}|x - y|$. The vectors are $\mathbf{0}$ along the line $y = x$. Elsewhere the vectors point away from the line $y = x$ with length that increases as the distance from the line increases.



i

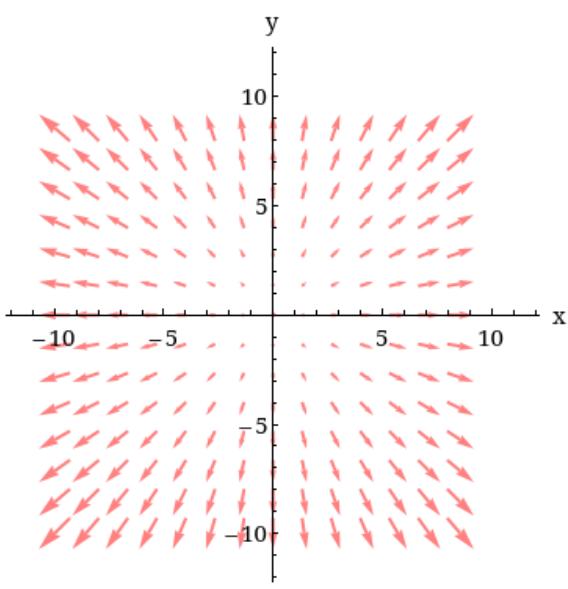
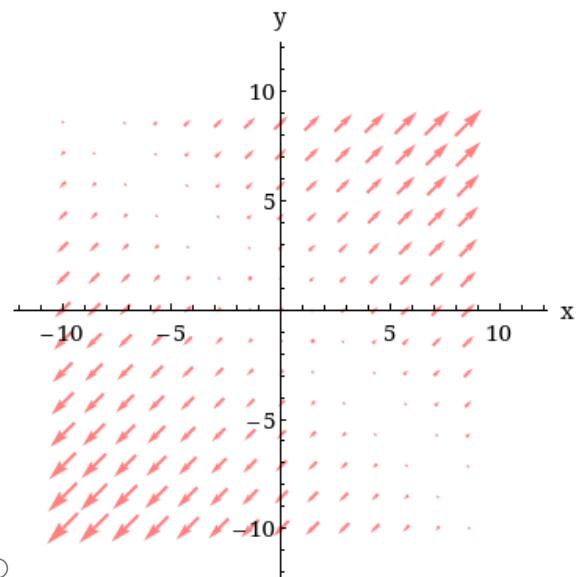
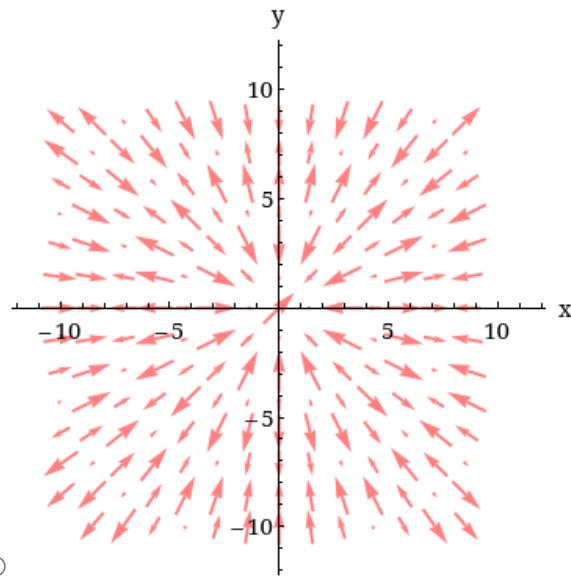
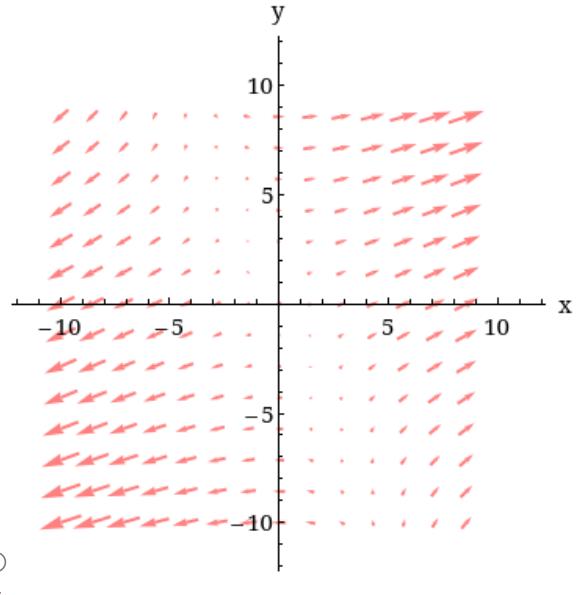
14. [- / 1 Points]

DETAILS

SCalcET9M 16.1.031.

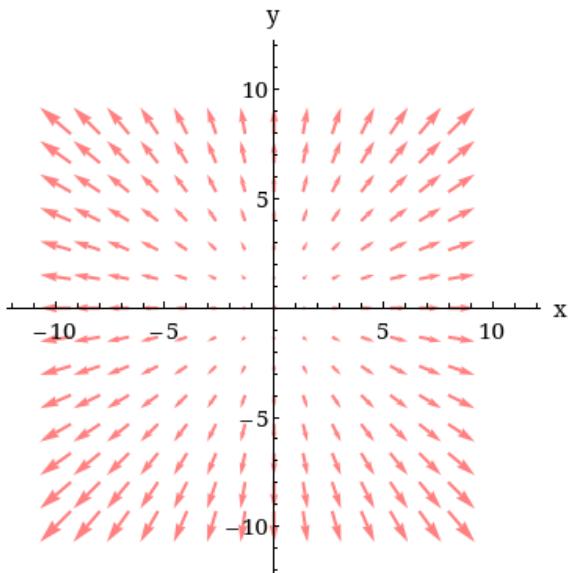
Match the function f with the correct gradient vector field plot.

$$f(x, y) = 10x^2 + 10y^2$$

Solution or Explanation

$f(x, y) = 10x^2 + 10y^2 \Rightarrow \nabla f(x, y) = 20xi + 20yj$. Thus, each vector $\nabla f(x, y)$ has the same direction and **twenty** times the length of the position vector of the point (x, y) , so the vectors all point directly away from the origin and their lengths increase as we move away from the origin. Hence, ∇f is the graph below.



Resources

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