Assignment Previewer

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Show answer key SHOW NEW RANDOMIZATION EDIT ASSIGNMENT

INSTRUCTOR
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Hw5 (Homework)

Current Score: – / 57 POINTS | 0.0 %

Scoring and Assignment Information

QUESTION	1	2	3	4	5	6	7	8	9	10
POINTS	– / 6	– / 5	– / 4	– / 1	– / 8	– / 10	– / 7	– / 3	– / 4	– / 9

Assignment Submission

For this assignment, you submit answers by question parts. The number of submissions remaining for each question part only changes if you submit or change the answer.

Assignment Scoring

Your best submission for each question part is used for your score.

1. [– / 6 Points] DETAILS

SCalcET9M 13.4.001.

The table gives coordinates of a particle moving through space along a smooth curve.

t	x	y	z
0	2.6	9.9	3.7
0.5	3.3	7.7	3.3
1.0	4.3	6.0	3.1
1.5	5.3	6.8	2.8
2.0	7.5	7.6	2.7

(a) Find the average velocities over the time intervals [0, 1], [0.5, 1], [1, 2], and [1, 1.5]. (Round your answers to the nearest tenth.)

[0, 1]: $\mathbf{v}_{\text{ave}}$ = ✗ $1.7\mathbf{i} - 3.9\mathbf{j} - 0.6\mathbf{k}$

[0.5, 1]: $\mathbf{v}_{\text{ave}}$ = ✗ $2.0\mathbf{i} - 3.4\mathbf{j} - 0.4\mathbf{k}$

[1, 2]: $\mathbf{v}_{\text{ave}}$ = ✗ $3.2\mathbf{i} + 1.6\mathbf{j} - 0.4\mathbf{k}$

[1, 1.5]: $\mathbf{v}_{\text{ave}}$ = ✗ $2.0\mathbf{i} + 1.6\mathbf{j} - 0.6\mathbf{k}$

(b) Estimate the velocity and speed of the particle at t = 1. (Use the time intervals [0.5, 1] and [1, 1.5] to calculate your answer. Round the speed to two decimal places.)

$\mathbf{v}(1)$ = ✗ $2.0\mathbf{i} - 0.9\mathbf{j} - 0.5\mathbf{k}$

$|\mathbf{v}(1)|$ = ✗ 2.25

Solution or Explanation

Click to View Solution

Resources

Watch It

2. [– / 5 Points]

DETAILS

SCalcET9M 13.4.011.MI.SA.

This question has several parts that must be completed sequentially. If you skip a part of the question, you will not receive any points for the skipped part, and you will not be able to come back to the skipped part.

Tutorial Exercise

Find the velocity, acceleration, and speed of a particle with the given position function.

$$\mathbf{r}(t) = 7\sqrt{2}\,t\,\mathbf{i} + e^{7t}\mathbf{j} + e^{-7t}\mathbf{k}$$

Click here to begin!

3. [– / 4 Points] DETAILS

SCalcET9M 13.4.005.

Find the velocity, acceleration, and speed of a particle with the given position function.

$\mathbf{r}(t) = 3\cos(t)\mathbf{i} + 2\sin(t)\mathbf{j}$

$\mathbf{v}(t)$ =

✖ $-3\sin(t)\mathbf{i} + 2\cos(t)\mathbf{j}$

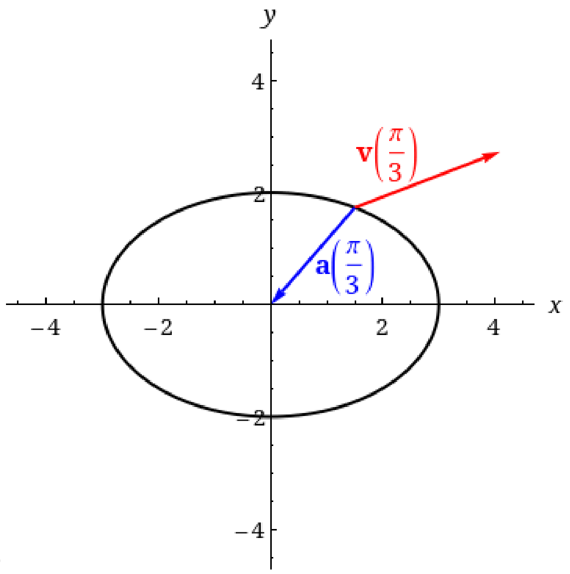
$\mathbf{a}(t)$ =

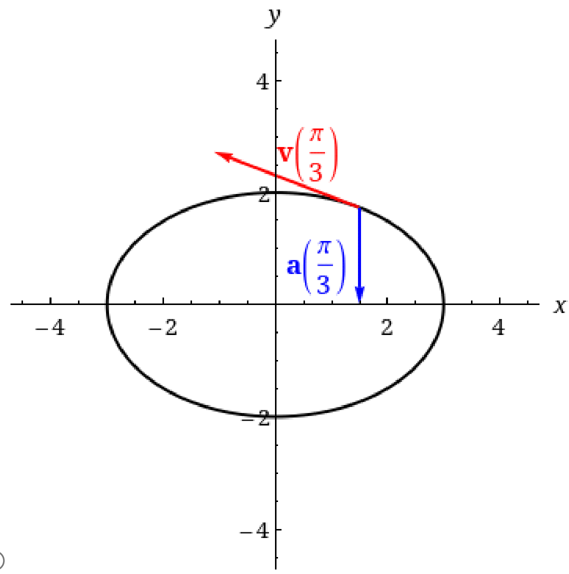
✖ $-3\cos(t)\mathbf{i} - 2\sin(t)\mathbf{j}$

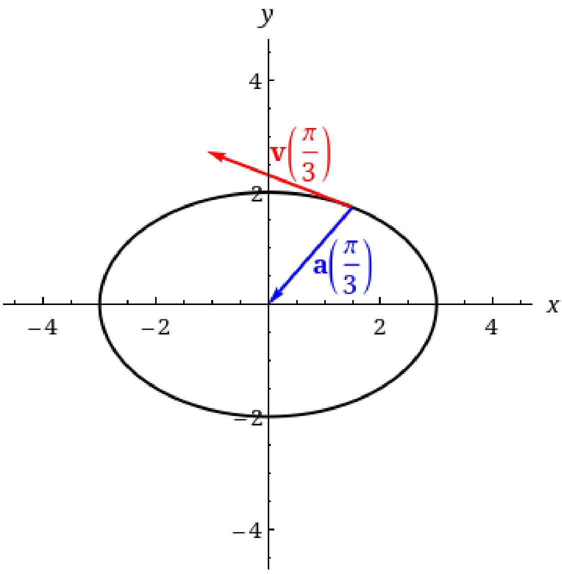
$|\mathbf{v}(t)|$ =

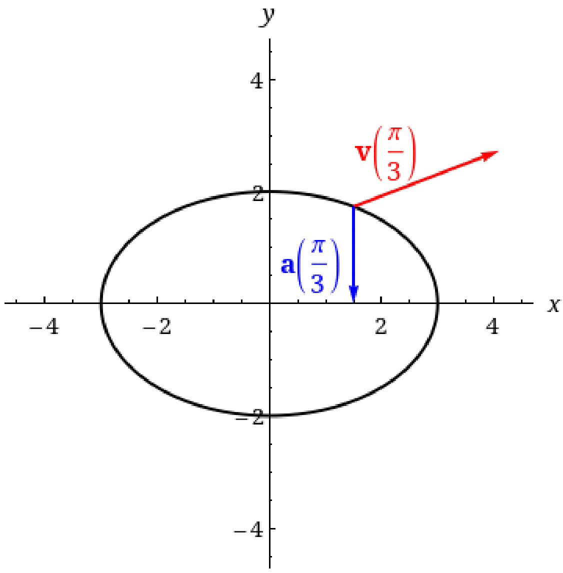
✖ $\sqrt{5\sin^2(t) + 4}$

Sketch the path of the particle and draw the velocity and acceleration vectors for $t = \frac{\pi}{3}$.

○ 

○ 





Solution or Explanation

$\mathbf{r}(t) = 3\cos(t)\mathbf{i} + 2\sin(t)\mathbf{j} \Rightarrow$

$\mathbf{v}(t) = -3\sin(t)\mathbf{i} + 2\cos(t)\mathbf{j}$

$\mathbf{a}(t) = -3\cos(t)\mathbf{i} - 2\sin(t)\mathbf{j}$

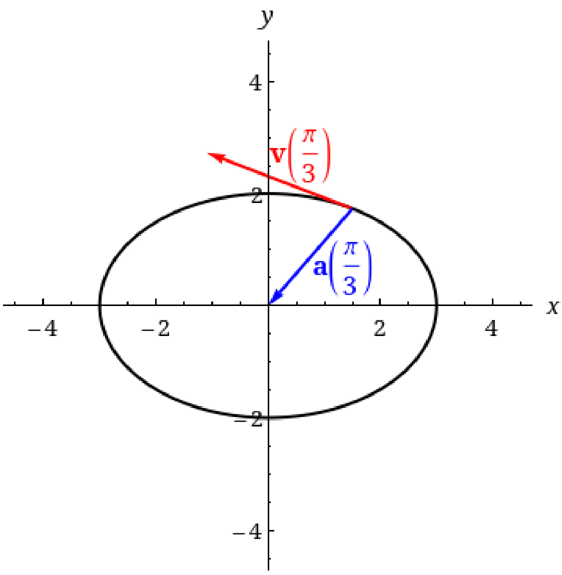
$|\mathbf{v}(t)| = \sqrt{9\sin^2(t) + 4\cos^2(t)} = \sqrt{4 + 5\sin^2(t)}$

At $t = \frac{\pi}{3}$:

$\mathbf{v}\left(\frac{\pi}{3}\right) = -\frac{3\sqrt{3}}{2}\mathbf{i} + \mathbf{j}$

$\mathbf{a}\left(\frac{\pi}{3}\right) = -\frac{3}{2}\mathbf{i} - \sqrt{3}\mathbf{j}$

Notice that $\frac{x^2}{9} + \frac{y^2}{4} = \sin^2(t) + \cos^2(t) = 1,$ so the path is an ellipse.



Resources

Watch It

4. [– / 1 Points]

DETAILS

SCalcET9M 13.4.034.

Water traveling along a straight portion of a river normally flows fastest in the middle, and the speed slows to almost zero at the banks. Consider a long straight stretch of river flowing north, with parallel banks 40 m apart. If the maximum water speed is 3 m/s, we can use the sine function, $f(x) = 3\sin(\pi x/40)$, as a basic model for the rate of water flow x units from the west bank. Suppose a boater would like to pilot the boat to land at the point B on the east bank directly opposite point A. If the boat maintains a constant heading and a constant speed of 5 m/s, determine the angle at which the boat should head. (Round your answer to one decimal place.)

[ ] ✗ 22.5 ° south of east

Solution or Explanation

Click to View Solution

5. [– / 8 Points]

DETAILS

SCalcET9M 13.4.025.MI.SA.

This question has several parts that must be completed sequentially. If you skip a part of the question, you will not receive any points for the skipped part, and you will not be able to come back to the skipped part.

Tutorial Exercise

A ball is thrown at an angle of 45° to the ground. If the ball lands 70 m away, what was the initial speed of the ball?

Click here to begin!

6. [– / 10 Points]

DETAILS

SCalcET9M 13.4.AE.003.

Video Example

EXAMPLE 3 A moving particle starts at an initial position **r**(0) = ⟨1, 0, 0⟩ with initial velocity **v**(0) = **i** - **j** + **k**. Its acceleration **a**(t) = 4t**i** + 8t**j** + **k**. Find its velocity and position at time t.

SOLUTION Since **a**(t) = **v**'(t), we have

v(t) = ∫**a**(t)dt = ∫(4t**i** + 8t**j** + **k**)dt

=

✗ $2t^2$ **i** +

✗ $4t^2$ **j** + t**k** + **C**

To determine the value of the constant vector **C**, we use the fact that **v**(0) = **i** - **j** + **k**. The preceding equation gives **v**(0) = **C**, so **C** = **i** - **j** + **k** and

v(t) = $2t^2$**i** + $4t^2$**j** + t**k** + **i** - **j** + **k**

= (

✗ $2t^2+1$)**i** + (

✗ $4t^2-1$)**j** + (t + 1)**k**

Since **v**(t) = **r**'(t), we have

r(t) = ∫**v**(t)dt

= ∫[($2t^2$ + 1)**i** + ($4t^2$ - 1)**j** + (t + 1)**k**]dt

= (

✗ $\frac{2}{3}t^3+t$)**i** + (

✗ $\frac{4}{3}t^3-t$)**j** + (

✗ $\boxed{\frac{1}{2}t^2 + t}$)**k** + **D**

Putting t = 0, we find that **D** = **r**(0) = **i**, so the position at time t is given by

r(t) = (

✗ $\boxed{\frac{2}{3}t^3 + t + 1}$)**i** + (

✗ $\boxed{\frac{4}{3}t^3 - t}$)**j** + (

✗ $\boxed{\frac{1}{2}t^2 + t}$)**k**

7. [– / 7 Points]

DETAILS

SCalcET9M 13.4.AE.006.

Video Example

EXAMPLE 6 A projectile is fired with muzzle speed 190 m/s and an angle of elevation 45° from a position 10 m above ground level. Where does the projectile hit the ground and with what speed?

SOLUTION If we place the origin at ground level, then the initial position of the projectile is (0, 10) and so we need to adjust the parametric equations of the trajectory by adding 10 to the expression for y. With $v_0 = 190$ m/s, $\alpha = 45°$, and $g = 9.8$ m/s^2, we have

$190\cos(\pi/4)t =$

$x =$

✗ $95\sqrt{2}t$

$10 + 190\sin(\pi/4)t - \frac{1}{2}(9.8)t^2 =$

$y =$

✗ $10 + 95\sqrt{2}t - 4.9t^2$.

Impact occurs when $y = 0$, that is $4.9t^2 - 95\sqrt{2}t - 10 = 0$. Solving this quadratic equation (and using only the positive value of t), we get the following. (Round your answer to two decimal places.)

$$t = \frac{95\sqrt{2} + \sqrt{18{,}050 + 196}}{9.8} \approx \quad ✗ \quad 27.49 .$$

Then $x \approx 95\sqrt{2}(27.49) \approx$ ✗ 3694 (rounded to the nearest whole number), so the projectile hits the ground about ✗ 3694 m away. The velocity of the projectile is

$\mathbf{v}(t) = \mathbf{r}'(t) =$

✗ $95\sqrt{2}\mathbf{i} + (95\sqrt{2} - 9.8t)\mathbf{j}$.

So its speed at impact (rounded to the nearest whole number) is

$$|\mathbf{v}(27.49)| = \sqrt{(95\sqrt{2})^2 + (95\sqrt{2} - 9.8 \cdot 27.49)^2} \approx$$

✗ 191 m/s.

8. [– / 3 Points]

DETAILS

SCalcET9M 13.4.010.

Find the velocity, acceleration, and speed of a particle with the given position function.

$\mathbf{r}(t) = \langle 9\cos(t), 5t, 9\sin(t)\rangle$

$\mathbf{v}(t)$ =

✗ $\langle -9\sin(t), 5, 9\cos(t)\rangle$

$\mathbf{a}(t)$ =

✗ $\langle -9\cos(t), 0, -9\sin(t)\rangle$

$|\mathbf{v}(t)|$ =

✗ $\sqrt{106}$

Solution or Explanation

Click to View Solution

9. [– / 4 Points] DETAILS

SCalcET9M 13.4.006.

Find the velocity, acceleration, and speed of a particle with the given position function.

$\mathbf{r}(t) = e^t\mathbf{i} + e^{2t}\mathbf{j}$

$\mathbf{v}(t)$ =

✕ $e^t\mathbf{i} + 2e^{2t}\mathbf{j}$

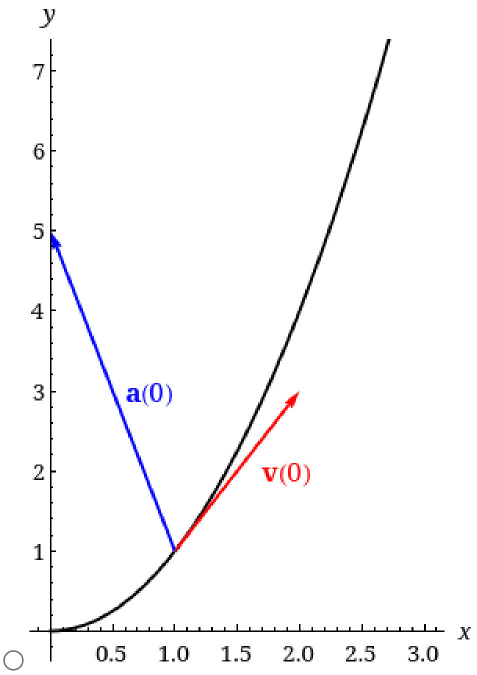
$\mathbf{a}(t)$ =

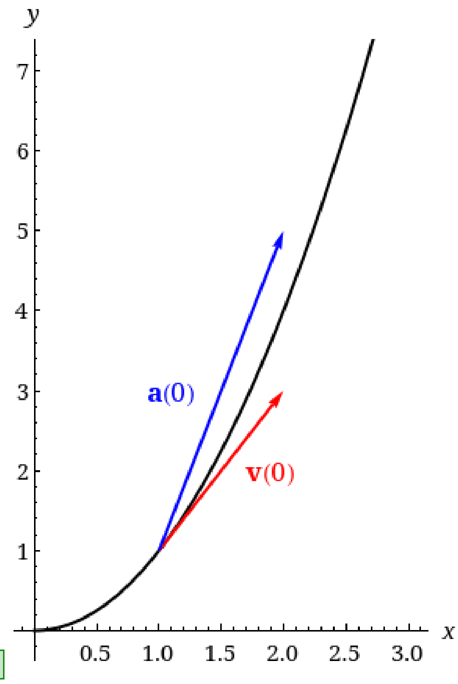
✕ $e^t\mathbf{i} + 4e^{2t}\mathbf{j}$

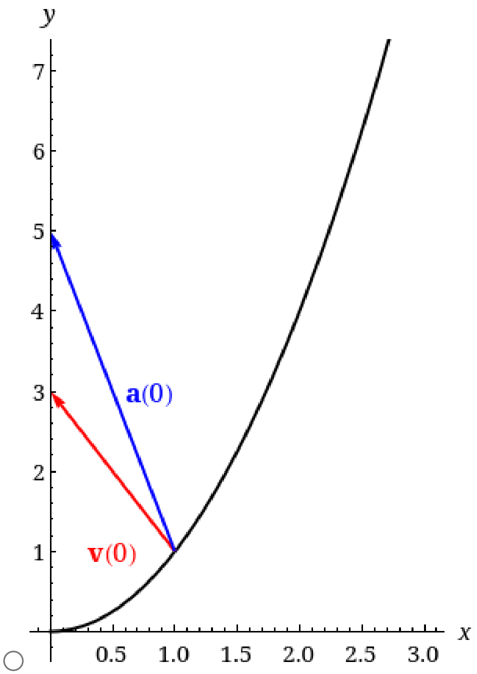
$|\mathbf{v}(t)|$ =

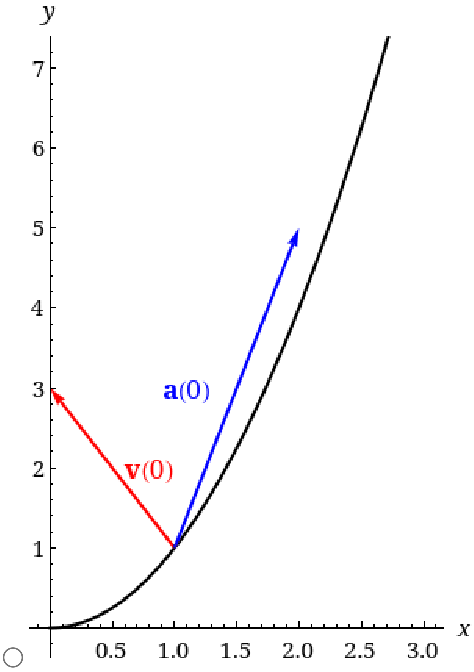
✕ $e^t\sqrt{4e^{2t}+1}$

Sketch the path of the particle and draw the velocity and acceleration vectors for $t = 0$.









Solution or Explanation

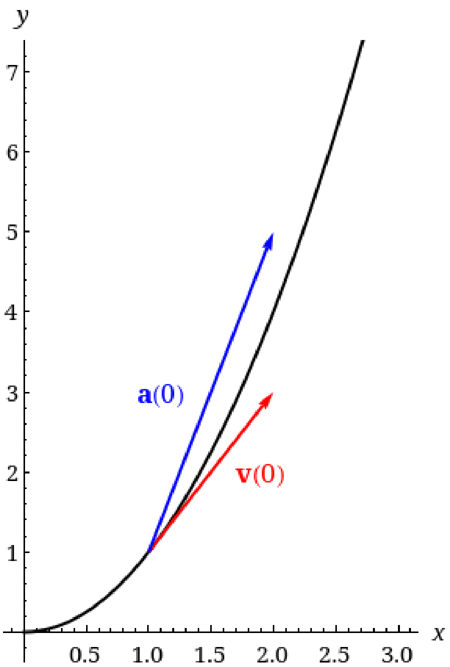
$$\mathbf{r}(t) = e^t\mathbf{i} + e^{2t}\mathbf{j} \quad \Rightarrow \qquad \text{At } t = 0:$$

$$\mathbf{v}(t) = e^t\mathbf{i} + 2e^{2t}\mathbf{j} \qquad \mathbf{v}(0) = \mathbf{i} + 2\mathbf{j}$$

$$\mathbf{a}(t) = e^t\mathbf{i} + 4e^{2t}\mathbf{j} \qquad \mathbf{a}(0) = \mathbf{i} + 4\mathbf{j}$$

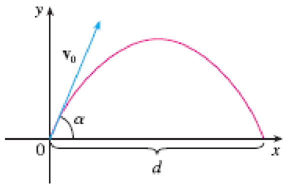
$$|\mathbf{v}(t)| = \sqrt{e^{2t} + 4e^{4t}} = e^t\sqrt{1 + 4e^{2t}}$$

Notice that $y = e^{2t} = (e^t)^2 = x^2$, so the particle travels along a parabola, but $x = e^t$, so $x > 0$.



10. [– / 9 Points] DETAILS

SCalcET9M 13.4.AE.005.



Video Example

EXAMPLE 5 A projectile is fired with an angle of elevation α and initial velocity $\mathbf{v}_0$. (See the figure.) Assuming that air resistance is negligible and the only external force is due to gravity, find the position function $\mathbf{r}(t)$ of the projectile. What value of α maximizes the range (the horizontal distance traveled)?

SOLUTION We set up the axes so that the projectile starts at the origin. Since the force due to gravity acts downward, we have

$$\mathbf{F} = m\mathbf{a} = -mg\mathbf{j}$$

where $g = |\mathbf{a}| \approx 9.8$ m/s^2. Thus

$$\mathbf{a} = -g\mathbf{j}.$$

Since $\mathbf{v}'(t) = \mathbf{a}$, we have

$\mathbf{v}(t) =$

✗ $\boxed{-gt\mathbf{j}}$ + **C**

where $\mathbf{C} = \mathbf{v}(0) = \mathbf{v}_0$. Therefore

$$\mathbf{r}'(t) = \mathbf{v}(t) = -gt\mathbf{j} + \mathbf{v}_0.$$

Integrating again, we obtain

$\mathbf{r}(t) = ($ ✗ $\boxed{-\frac{1}{2}gt^2\mathbf{j}}$ $)$ + $($ ✗ $\boxed{t}$ $)\mathbf{v}_0 + \mathbf{D}$.

But $\mathbf{D} = \mathbf{r}(0) = 0$, so the position vector of the projectile is given by

$\mathbf{r}(t) = ($ ✗ $\boxed{-\frac{1}{2}gt^2\mathbf{j}}$ $)$ + $($ ✗ $\boxed{t}$ $)\mathbf{v}_0$. (1)

If we write $|\mathbf{v}_0| = v_0$ (the initial speed of the projectile), then

$\mathbf{v}_0 = ($ ✗ $\boxed{v_0 \cos(\alpha)}$ $)\mathbf{i} + v_0 \sin(\alpha)\,\mathbf{j}$

and Equation (1) becomes

$$\mathbf{r}(t) = (v_0 \cos(\alpha))t\mathbf{i} + [(v_0 \sin(\alpha))t - \frac{1}{2}gt^2]\mathbf{j}.$$

The parametric equations of the trajectory are therefore

$$x = (v_0 \cos(\alpha))t \qquad y = (v_0 \sin(\alpha))t - \frac{1}{2}gt^2.$$

The horizontal distance d is the value of x when $y = 0$. Setting $y = 0$, we obtain $t = 0$ or $t = (2v_0 \sin(\alpha))/g$. This second value of t then gives

$x = \Big($

$d =$

$\boxed{v_0 \cos(\alpha)}\Big)\dfrac{2v_0 \sin(\alpha)}{g}$

$$= \dfrac{v_0^2\Big(\boxed{2\sin(\alpha)\cos(\alpha)}\Big)}{g} = \dfrac{v_0^2 \sin(2\alpha)}{g}.$$

Clearly, d has its maximum value when $\sin(2\alpha) = 1$, that is,

$\alpha =$

$\boxed{\dfrac{\pi}{4}}$.