

Problem1

**31.** (b) (i)  $e^{-0.2} \approx 0.8187$   
 (ii)  $1 + e^{-1.8} - e^{-0.8} - e^{-1} \approx 0.3481$     (c) 2, 5

Problem2

**34** a)  $K$  is closed because it's defined using ' $\leq$ ' and the functions involved are continuous;  $K$  is bounded, since  $(x,y,z) \in K$  implies  $z(1-z) \geq 0$  and hence  $0 \leq z \leq 1$ , and then further  $x^2 + y^2 \leq 1$ , i.e.,  $|x|, |y| \leq 1$ .

b) The set  $K$  is rotation-invariant with respect to the  $z$ -axis; the  $z$ -section  $K_z = \{(x,y) \in \mathbb{R}^2; (x,y,z) \in K\}$  is empty for  $z < 0$  or  $z > 1$  and is a circle of radius  $r(z) = z(1-z)$  for  $0 \leq z \leq 1$ .

$$\begin{aligned}\implies \text{vol}(K) &= \pi \int_0^1 r(z)^2 dz \\ &= \pi \int_0^1 z^2(1-z)^2 dz \\ &= \pi \int_0^1 z^2 - 2z^3 + z^4 dz \\ &= \pi \left[ \frac{z^3}{3} - \frac{z^4}{2} + \frac{z^5}{5} \right]_0^1 \\ &= \pi \left( \frac{1}{3} - \frac{1}{2} + \frac{1}{5} \right) \\ &= \frac{\pi}{30}.\end{aligned}$$