

Problem1

31. (b) (i) $e^{-0.2} \approx 0.8187$

(ii) $1 + e^{-1.8} - e^{-0.8} - e^{-1} \approx 0.3481$ (c) 2, 5

Problem2

34 a) K is closed because it's defined using ' \leq ' and the functions involved are continuous; K is bounded, since $(x, y, z) \in K$ implies $z(1-z) \geq 0$ and hence $0 \leq z \leq 1$, and then further $x^2 + y^2 \leq 1$, i.e., $|x|, |y| \leq 1$.

b) The set K is rotation-invariant with respect to the z -axis; the z -section $K_z = \{(x, y) \in \mathbb{R}^2; (x, y, z) \in K\}$ is empty for $z < 0$ or $z > 1$ and is a circle of radius $r(z) = z(1-z)$ for $0 \leq z \leq 1$.

$$\begin{aligned}
 \Rightarrow \text{vol}(K) &= \pi \int_0^1 r(z)^2 \, dz \\
 &= \pi \int_0^1 z^2(1-z)^2 \, dz \\
 &= \pi \int_0^1 z^2 - 2z^3 + z^4 \, dz \\
 &= \pi \left[\frac{z^3}{3} - \frac{z^4}{2} + \frac{z^5}{5} \right]_0^1 \\
 &= \pi \left(\frac{1}{3} - \frac{1}{2} + \frac{1}{5} \right) \\
 &= \frac{\pi}{30}.
 \end{aligned}$$