Assignment Previewer

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HW4 (Homework)

Current Score: – / 27 POINTS | 0.0 %

Scoring and Assignment Information

QUESTION	1	2	3	4	5	6	7	8	9	10	11
POINTS	– / 1	– / 1	– / 2	– / 1	– / 1	– / 1	– / 3	– / 3	– / 2	– / 11	– / 1

Assignment Submission

For this assignment, you submit answers by question parts. The number of submissions remaining for each question part only changes if you submit or change the answer.

Assignment Scoring

Your best submission for each question part is used for your score.

1. [– / 1 Points]   DETAILS

SCalcET9M 13.2.050.

If $\mathbf{u}(t) = \langle \sin(8t), \cos(8t), t \rangle$ and $\mathbf{v}(t) = \langle t, \cos(8t), \sin(8t) \rangle$, use Formula 5 of this theorem to find

$$\frac{d}{dt}\left[\mathbf{u}(t) \times \mathbf{v}(t)\right].$$

✗ $\langle 8t\sin(8t) - \cos(8t) + 8\cos(16t),\ 2t - 8\sin(16t),\ 8t\sin(8t) - \cos(8t) + 8\cos(16t) \rangle$

Solution or Explanation
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2. [– / 1 Points]

DETAILS

SCalcET9M 13.3.005.

Find the length of the curve.

$\sqrt{2}\,t\,\mathbf{i} + e^{t}\mathbf{j} + e^{-t}\mathbf{k}, \quad 0 \le t \le 8$

✕ $\boxed{e^{8} - \frac{1}{e^{8}}}$

Solution or Explanation

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Resources

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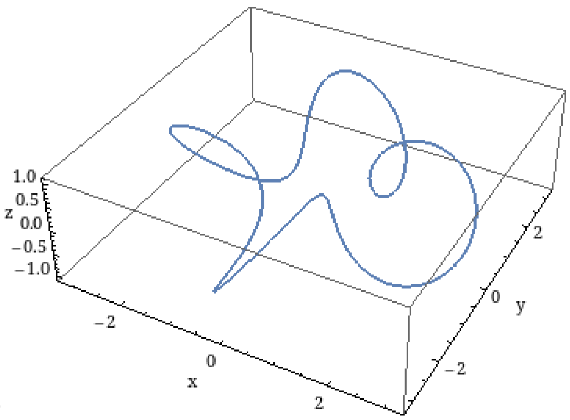
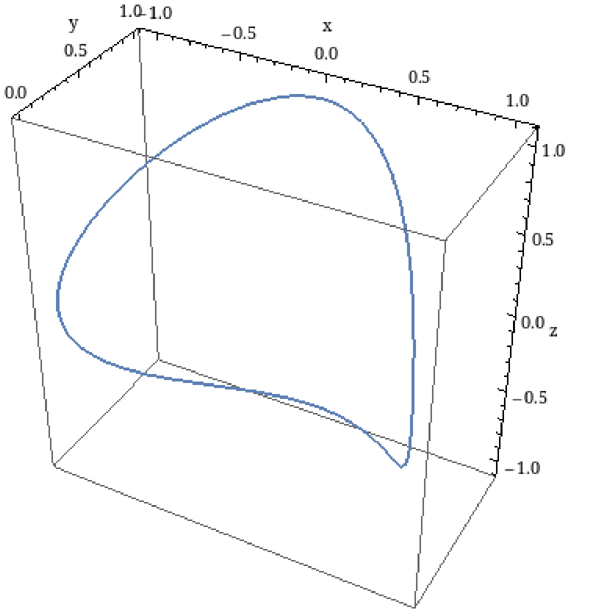
3. [– / 2 Points]

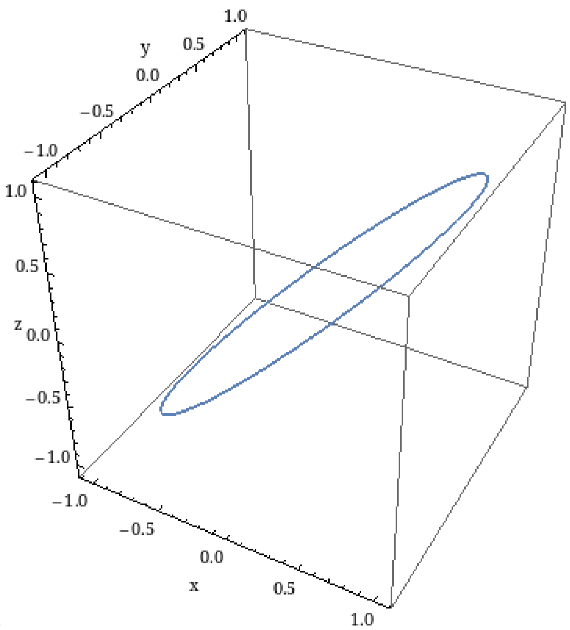
DETAILS

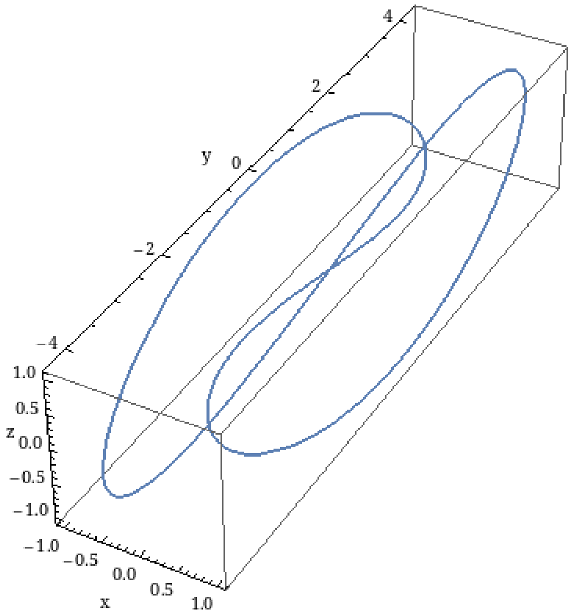
scalcet9m 13.3.012.nva

Graph the curve with parametric equations $x = \sin(t)$, $y = 4\sin(2t)$, $z = \sin(3t)$.









Find the total length of this curve correct to four decimal places.

35.8467

Solution or Explanation

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4. [– / 1 Points]

DETAILS

SCalcET9M 13.3.036.

Find an equation of a parabola that has curvature 4 at the origin.

$y(x)$ =

✖ $2x^2$

Solution or Explanation

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5. [– / 1 Points]

DETAILS

SCalcET9M 13.3.003.MI.

Find the length of the curve.

$\mathbf{r}(t) = \langle 7t, 3\cos(t), 3\sin(t)\rangle, \quad -4 \le t \le 4$

✖ $8\sqrt{58}$

Solution or Explanation

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Resources

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6. [– / 1 Points] DETAILS

SCalcET9M 13.3.027.

Use the theorem given below to find the curvature of $\mathbf{r}(t) = \sqrt{6}\,t^2\,\mathbf{i} + 2t\,\mathbf{j} + 2t^3\,\mathbf{k}$.

Theorem: The curvature of the curve given by the vector function $\mathbf{r}$ is $\kappa(t) = \dfrac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$.

$\kappa(t) =$

✕ $\dfrac{\sqrt{\frac{3}{2}}}{(3t^2+1)^2}$

Solution or Explanation

We have the following.

$$\mathbf{r}(t) = \sqrt{6}\,t^2\,\mathbf{i} + 2t\,\mathbf{j} + 2t^3\,\mathbf{k} \quad \Rightarrow \quad \mathbf{r}'(t) = 2\sqrt{6}\,t\,\mathbf{i} + 2\,\mathbf{j} + 6t^2\,\mathbf{k}$$

$$\mathbf{r}''(t) = 2\sqrt{6}\,\mathbf{i} + 12t\,\mathbf{k}$$

$$|\mathbf{r}'(t)| = \sqrt{24t^2 + 4 + 36t^4} = \sqrt{4(9t^4 + 6t^2 + 1)} = \sqrt{4(3t^2 + 1)^2} = 2(3t^2 + 1)$$

$$\mathbf{r}'(t) \times \mathbf{r}''(t) = 24t\,\mathbf{i} - 12\sqrt{6}\,t^2\,\mathbf{j} - 4\sqrt{2}\,\mathbf{k}$$

$$|\mathbf{r}'(t) \times \mathbf{r}''(t)| = \sqrt{576t^2 + 864t^4 + 96} = \sqrt{96(9t^4 + 6t^2 + 1)} = \sqrt{96(3t^2 + 1)^2} = 4\sqrt{6}\,(3t^2 + 1)$$

Then $\kappa(t) = \dfrac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3} = \dfrac{4\sqrt{6}\,(3t^2 + 1)}{8(3t^2 + 1)^3} = \dfrac{\sqrt{6}}{2(3t^2 + 1)^2}$.

7. [– / 3 Points]

DETAILS

SCalcET9M 13.3.023.

Consider the following vector function.

$$\mathbf{r}(t) = \left\langle 8t, \frac{1}{2}t^2, t^2 \right\rangle$$

(a) Find the unit tangent and unit normal vectors **T**(t) and **N**(t).

T(t) =

✗ $\frac{1}{\sqrt{64+5t^2}}\langle 8, t, 2t\rangle$

N(t) =

✗ $\frac{1}{\sqrt{320+25t^2}}\langle -5t, 8, 16\rangle$

(b) Use this formula to find the curvature.

$\kappa(t)$ =

✗ $\frac{8\sqrt{5}}{(64+5t^2)^{\frac{3}{2}}}$

Solution or Explanation

Click to View Solution

8. [– / 3 Points] DETAILS

SCalcET9M 13.3.004.EP.

Consider the following.

$$\mathbf{r}(t) = \left\langle 4t, t^2, \frac{1}{6}t^3 \right\rangle$$

Find $\mathbf{r}'(t)$.

$\mathbf{r}'(t) =$

✗ $\left\langle 4, 2t, \frac{t^2}{2} \right\rangle$

Find $|\mathbf{r}'(t)|$ for $0 \le t \le 1$.

$|\mathbf{r}'(t)| =$

✗ $\frac{t^2}{2} + 4$

Find the length of the curve.

$$\mathbf{r}(t) = \left\langle 4t, t^2, \frac{1}{6}t^3 \right\rangle, \quad 0 \le t \le 1$$

✗ $\frac{25}{6}$

Solution or Explanation

We have

$$\mathbf{r}(t) = \left\langle 4t, t^2, \frac{1}{6}t^3 \right\rangle \;\Rightarrow\; \mathbf{r}'(t) = \left\langle 4, 2t, \frac{1}{2}t^2 \right\rangle$$

$$\Rightarrow\; |\mathbf{r}'(t)| = \sqrt{4^2 + (2t)^2 + \left(\frac{1}{2}t^2\right)^2} = \sqrt{16 + 4t^2 + \frac{1}{4}t^4}$$

$$= \sqrt{\left(4 + \frac{1}{2}t^2\right)^2} = 4 + \frac{1}{2}t^2 \quad \text{for } 0 \le t \le 1.$$

Then using the formula $L = \int_a^b |\mathbf{r}'(t)|\,dt$, we have the following.

$$L = \int_0^1 |\mathbf{r}'(t)|\,dt = \int_0^1 \left(4 + \frac{1}{2}t^2\right) dt = \left[4t + \frac{1}{6}t^3\right]_0^1 = \frac{25}{6}$$

9. [–/2 Points]

DETAILS

SCalcET9M 13.2.055.

Show that if **r** is a vector function such that $\mathbf{r}''$ exists, then

$$\frac{d}{dt}\left[\mathbf{r}(t) \times \mathbf{r}'(t)\right] = \mathbf{r}(t) \times \mathbf{r}''(t).$$

By the formula $\frac{d}{dt}\left[\mathbf{u}(t) \times \mathbf{v}(t)\right] = \mathbf{u}'(t) \times \mathbf{v}(t) + \mathbf{u}(t) \times \mathbf{v}'(t)$, we get the following.

- ○ $\frac{d}{dt}\left[\mathbf{r}(t) \times \mathbf{r}'(t)\right] = \mathbf{r}'(t) \times \mathbf{r}'(t) + \mathbf{r}(t) \times \mathbf{r}'(t)$
- ○ $\frac{d}{dt}\left[\mathbf{r}(t) \times \mathbf{r}'(t)\right] = \mathbf{r}(t) \times \mathbf{r}'(t) + \mathbf{r}(t) \times \mathbf{r}''(t)$
- ○ $\frac{d}{dt}\left[\mathbf{r}(t) \times \mathbf{r}'(t)\right] = \mathbf{r}'(t) \times \mathbf{r}(t) + \mathbf{r}(t) \times \mathbf{r}'(t)$
- ○ $\frac{d}{dt}\left[\mathbf{r}(t) \times \mathbf{r}'(t)\right] = \mathbf{r}(t) \times \mathbf{r}'(t) + \mathbf{r}(t) \times \mathbf{r}'(t)$
- ○ $\frac{d}{dt}\left[\mathbf{r}(t) \times \mathbf{r}'(t)\right] = \mathbf{r}'(t) \times \mathbf{r}'(t) + \mathbf{r}(t) \times \mathbf{r}''(t)$

✗

But $\mathbf{r}'(t) \times \mathbf{r}'(t) =$ ---Select--- ✗ 0 . Thus, $\frac{d}{dt}\left[\mathbf{r}(t) \times \mathbf{r}'(t)\right] = \mathbf{r}(t) \times \mathbf{r}''(t)$.

Solution or Explanation

By the formula $\frac{d}{dt}\left[\mathbf{u}(t) \times \mathbf{v}(t)\right] = \mathbf{u}'(t) \times \mathbf{v}(t) + \mathbf{u}(t) \times \mathbf{v}'(t)$, we get the following.

$$\frac{d}{dt}\left[\mathbf{r}(t) \times \mathbf{r}'(t)\right] = \mathbf{r}'(t) \times \mathbf{r}'(t) + \mathbf{r}(t) \times \mathbf{r}''(t)$$

But $\mathbf{r}'(t) \times \mathbf{r}'(t) = \mathbf{0}$. Thus, $\frac{d}{dt}\left[\mathbf{r}(t) \times \mathbf{r}'(t)\right] = \mathbf{r}(t) \times \mathbf{r}''(t)$.

10. [–/11 Points]

DETAILS

SCalcET9M 13.3.013.MI.SA.

This question has several parts that must be completed sequentially. If you skip a part of the question, you will not receive any points for the skipped part, and you will not be able to come back to the skipped part.

Tutorial Exercise

Let C be the curve of intersection of the parabolic cylinder $x^2 = 2y$, and the surface $3z = xy$. Find the exact length of C from the origin to the point $\left(1, \frac{1}{2}, \frac{1}{6}\right)$.

Click here to begin!

11. [– / 1 Points] DETAILS

SCalcET9M 13.3.032.

Use the formula $\kappa(x) = \dfrac{|f''(x)|}{\left[1 + (f'(x))^2\right]^{3/2}}$ to find the curvature.

$$y = 5\tan(x)$$

$\kappa(x) =$

✕ $\boxed{\dfrac{10\sec^2(x)\,|\tan(x)|}{(25\sec^4(x)+1)^{3/2}}}$

Solution or Explanation

We have

$$f(x) = 5\tan(x) \quad\Rightarrow\quad f'(x) = 5\sec^2(x) \quad\Rightarrow\quad f''(x) = 5(2\sec(x)\cdot\sec(x)\tan(x)) = 10\sec^2(x)\tan(x).$$

By the given formula, the curvature is

$$\kappa(x) = \frac{|f''(x)|}{\left[1 + (f'(x))^2\right]^{3/2}} = \frac{|10\sec^2(x)\tan(x)|}{\left[1 + (5\sec^2(x))^2\right]^{3/2}} = \frac{10\sec^2(x)\,|\tan(x)|}{(1 + 25\sec^4(x))^{3/2}}.$$