

# Assignment Previewer

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 INSTRUCTOR

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## HW2 (Homework)

Current Score: – / 57 POINTS | 0.0 %

Scoring and Assignment Information

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QUESTION	1	2	3	4	5	6	7	8	9	10	11	12	13	14	1
POINTS	– / 1	– / 2	– / 2	– / 1	– / 1	– / 3	– / 1	– / 6	– / 2	– / 1	– / 1	– / 1	– / 1	– / 3	– /



### Assignment Submission

For this assignment, you submit answers by question parts. The number of submissions remaining for each question part only changes if you submit or change the answer.

### Assignment Scoring

Your last submission is used for your score.

1. [- / 1 Points]

DETAILS

SCalcET9M 12.3.030.

Find the acute angle between the lines. Use degrees rounded to one decimal place.

$$8x - y = 6, \quad x + 6y = 30$$

   87.7 °

### Solution or Explanation

The line  $8x - y = 6 \Leftrightarrow y = 8x - 6$  has slope 8, so a vector parallel to the line is  $\mathbf{a} = \langle 1, 8 \rangle$ . The line  $x + 6y = 30 \Leftrightarrow y = -\frac{1}{6}x + 5$  has slope  $-\frac{1}{6}$ , so a vector parallel to the line is  $\mathbf{b} = \langle 6, -1 \rangle$ . The angle between the lines is the same as the angle  $\theta$  between the vectors. Here we have  $\mathbf{a} \cdot \mathbf{b} = 1(6) + 8(-1) = -2$ ,  $|\mathbf{a}| = \sqrt{1^2 + 8^2} = \sqrt{65}$ , and  $|\mathbf{b}| = \sqrt{6^2 + (-1)^2} = \sqrt{37}$ . Then  $\cos(\theta) = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|} = \frac{-2}{\sqrt{65}\sqrt{37}} = -\frac{2}{\sqrt{2405}}$  and  $\theta = \cos^{-1}\left(-\frac{2}{\sqrt{2405}}\right) \approx 92.3^\circ$ . Therefore, the acute angle between the two lines is approximately  $180^\circ - 92.3^\circ = 87.7^\circ$ .

SCalcET9M 12.3.040.

Find the scalar and vector projections of  $\mathbf{b}$  onto  $\mathbf{a}$ .

$$\mathbf{a} = \langle 1, 8 \rangle, \quad \mathbf{b} = \langle 3, 6 \rangle$$

scalar projection of  $\mathbf{b}$  onto  $\mathbf{a}$ 



$$\frac{51}{\sqrt{65}}$$

✗

vector projection of  $\mathbf{b}$  onto  $\mathbf{a}$ 



$$\left\langle \frac{51}{65}, \frac{408}{65} \right\rangle$$

✗

Solution or Explanation

$$|\mathbf{a}| = \sqrt{1^2 + 8^2} = \sqrt{65}$$

The scalar projection of  $\mathbf{b}$  onto  $\mathbf{a}$  is  $\text{comp}_{\mathbf{a}} \mathbf{b} = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|} = \frac{1 \cdot 3 + 8 \cdot 6}{\sqrt{65}} = \frac{51}{\sqrt{65}}$  and the vector projection of

$$\mathbf{b} \text{ onto } \mathbf{a} \text{ is } \text{proj}_{\mathbf{a}} \mathbf{b} = \left( \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|} \right) \frac{\mathbf{a}}{|\mathbf{a}|} = \frac{51}{\sqrt{65}} \cdot \frac{1}{\sqrt{65}} \langle 1, 8 \rangle = \left\langle \frac{51}{65}, \frac{408}{65} \right\rangle.$$

3. [- / 2 Points]

DETAILS

SCalcET9M 12.3.046.

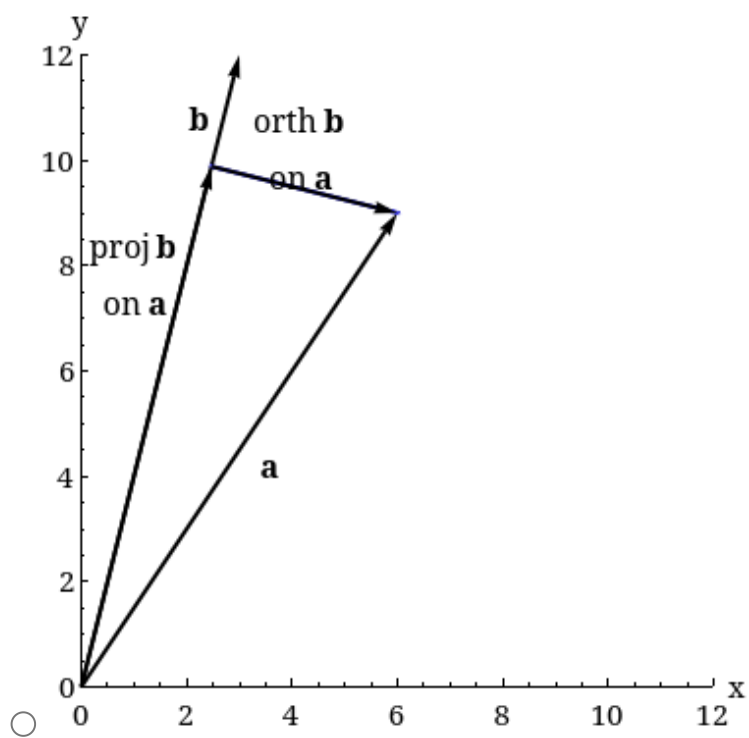
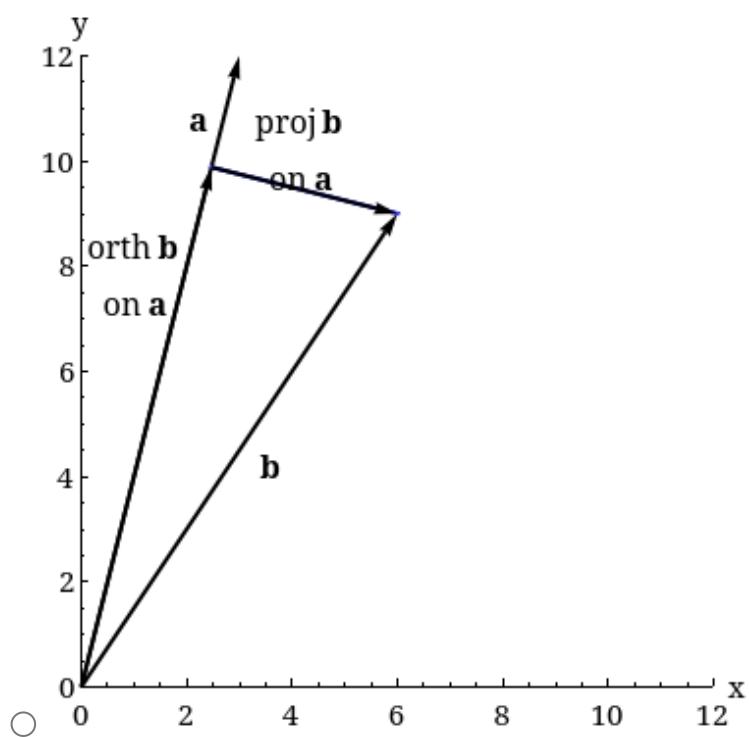
For the vectors  $\mathbf{a} = \langle 3, 12 \rangle$  and  $\mathbf{b} = \langle 6, 9 \rangle$ , find  $\text{orth}_{\mathbf{a}}\mathbf{b}$ .

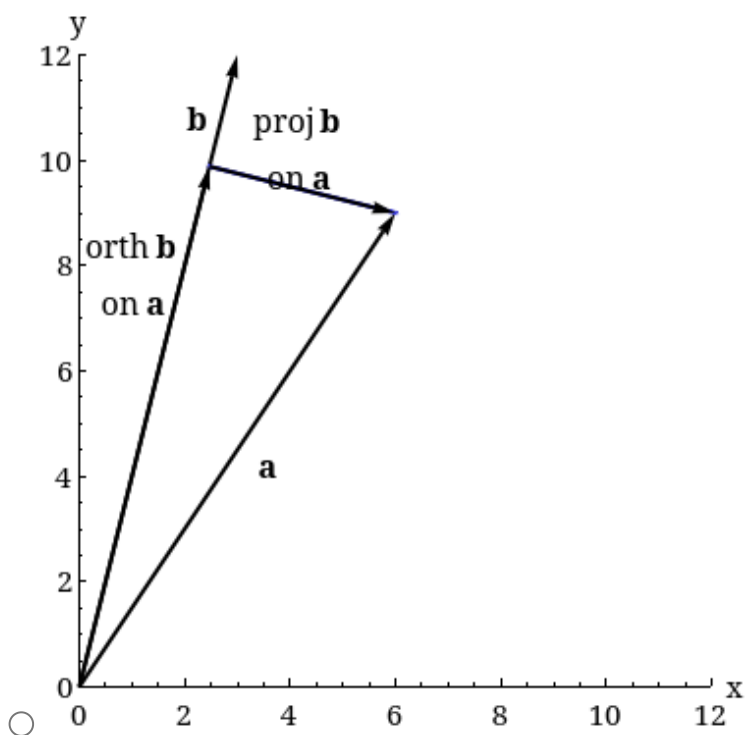
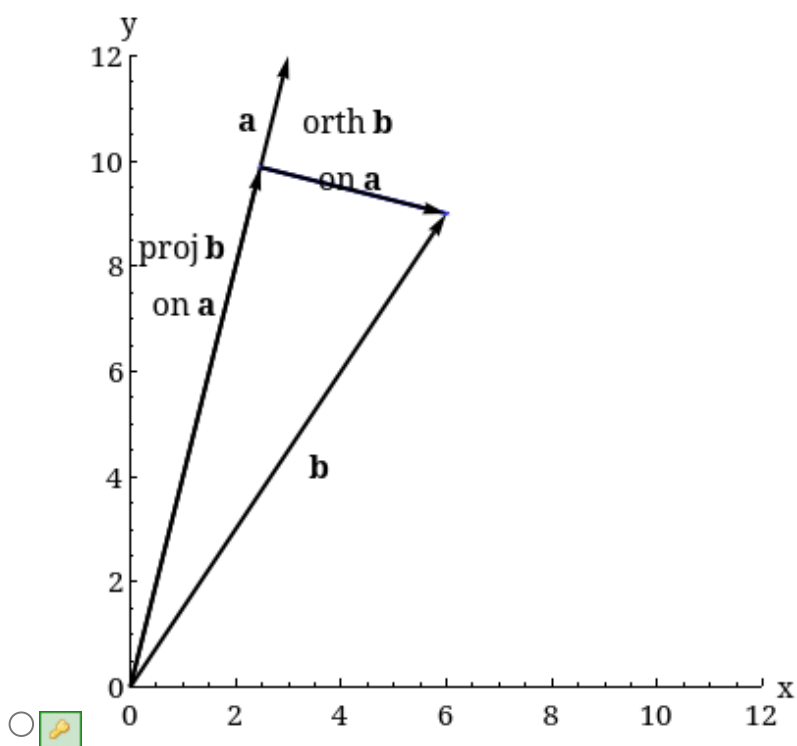
$\text{orth}_{\mathbf{a}}\mathbf{b} =$

✖

$$\left\langle \frac{60}{17}, -\frac{15}{17} \right\rangle$$

Illustrate by drawing the vectors  $\mathbf{a}$ ,  $\mathbf{b}$ ,  $\text{proj}_{\mathbf{a}}\mathbf{b}$ , and  $\text{orth}_{\mathbf{a}}\mathbf{b}$ .



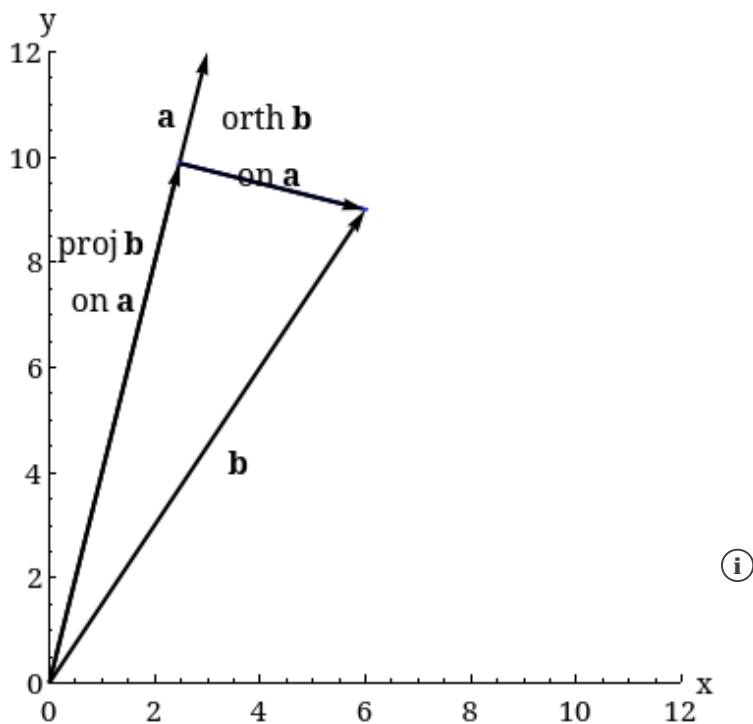


✗

Solution or Explanation

The vector projection of **b** onto **a** is  $\text{proj}_{\mathbf{a}} \mathbf{b} = \left( \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|} \right) \frac{\mathbf{a}}{|\mathbf{a}|} = \frac{126}{\sqrt{153}} \cdot \frac{1}{\sqrt{153}} \langle 3, 12 \rangle,$

$\text{orth}_{\mathbf{a}} \mathbf{b} = \mathbf{b} - \text{proj}_{\mathbf{a}} \mathbf{b} = \langle 6, 9 \rangle - \langle \frac{42}{17}, \frac{168}{17} \rangle = \langle \frac{60}{17}, -\frac{15}{17} \rangle.$



4. [- / 1 Points]

DETAILS

SCalcET9M 12.3.049.MI.

Find the work done by a force  $\mathbf{F} = 4\mathbf{i} - 8\mathbf{j} + 7\mathbf{k}$  that moves an object from the point  $(0, 6, 4)$  to the point  $(2, 16, 24)$  along a straight line. The distance is measured in meters and the force in newtons.

✖ 🔑 68 J

Solution or Explanation

The displacement vector is  $\mathbf{D} = (2 - 0)\mathbf{i} + (16 - 6)\mathbf{j} + (24 - 4)\mathbf{k} = 2\mathbf{i} + 10\mathbf{j} + 20\mathbf{k}$ , so by the equation  $W = |\mathbf{F}||\mathbf{D}| \cos(\theta) = \mathbf{F} \cdot \mathbf{D}$ , the work done is

$W = \mathbf{F} \cdot \mathbf{D} = (4\mathbf{i} - 8\mathbf{j} + 7\mathbf{k}) \cdot (2\mathbf{i} + 10\mathbf{j} + 20\mathbf{k}) = 8 - 80 + 140 = 68$  joules.

### Resources

[Watch It Master It](#)

5. [- / 1 Points]

DETAILS

SCalcET9M 12.3.055.

Find the angle between a diagonal of a cube and one of its edges. (Round your answer to the nearest degree.)

   55 °

#### Solution or Explanation

For convenience, consider the unit cube positioned so that its back left corner is at the origin, and its edges lie along the coordinate axes. The diagonal of the cube that begins at the origin and ends at  $(1, 1, 1)$  has vector representation  $\langle 1, 1, 1 \rangle$ . The angle  $\theta$  between this vector and the vector of the edge which also begins at the origin and runs along the  $x$ -axis [that is,  $\langle 1, 0, 0 \rangle$ ] is given by

$$\cos(\theta) = \frac{\langle 1, 1, 1 \rangle \cdot \langle 1, 0, 0 \rangle}{|\langle 1, 1, 1 \rangle| |\langle 1, 0, 0 \rangle|} = \frac{1}{\sqrt{3}} \Rightarrow \theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) \approx 55^\circ.$$

#### Resources

[Watch It](#)

SCalcET9M 12.4.002.

Find the cross product  $\mathbf{a} \times \mathbf{b}$ .

$$\mathbf{a} = \langle 6, 5, -4 \rangle, \quad \mathbf{b} = \langle 2, -1, 1 \rangle$$



✗  $\mathbf{i} - 14\mathbf{j} - 16\mathbf{k}$

Verify that it is orthogonal to both  $\mathbf{a}$  and  $\mathbf{b}$ .

$$(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{a} = \text{[input]} \quad \text{✗} \quad \text{[key icon]} \quad 0$$

$$(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{b} = \text{[input]} \quad \text{✗} \quad \text{[key icon]} \quad 0$$

Solution or Explanation

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 & 5 & -4 \\ 2 & -1 & 1 \end{vmatrix} = \begin{vmatrix} 5 & -4 \\ -1 & 1 \end{vmatrix} \mathbf{i} - \begin{vmatrix} 6 & -4 \\ 2 & 1 \end{vmatrix} \mathbf{j} + \begin{vmatrix} 6 & 5 \\ 2 & -1 \end{vmatrix} \mathbf{k}$$

$$= (5 - 4)\mathbf{i} - [6 - (-8)]\mathbf{j} + (-6 - 10)\mathbf{k} = \mathbf{i} - 14\mathbf{j} - 16\mathbf{k}$$

$$\text{Now } (\mathbf{a} \times \mathbf{b}) \cdot \mathbf{a} = \langle 1, -14, -16 \rangle \cdot \langle 6, 5, -4 \rangle = 6 - 70 + 64 = 0 \text{ and}$$

$$(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{b} = \langle 1, -14, -16 \rangle \cdot \langle 2, -1, 1 \rangle = 2 + 14 - 16 = 0, \text{ so } \mathbf{a} \times \mathbf{b} \text{ is orthogonal to both } \mathbf{a} \text{ and } \mathbf{b}.$$

7. [- / 1 Points]

DETAILS

SCalcET9M 12.4.008.

If  $\mathbf{a} = \mathbf{i} - 7\mathbf{k}$  and  $\mathbf{b} = \mathbf{j} + \mathbf{k}$ , find  $\mathbf{a} \times \mathbf{b}$ .



✗  $7\mathbf{i} - \mathbf{j} + \mathbf{k}$


Solution or Explanation

$$\begin{aligned}\mathbf{a} \times \mathbf{b} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & -7 \\ 0 & 1 & 1 \end{vmatrix} \\ &= \begin{vmatrix} 0 & -7 \\ 1 & 1 \end{vmatrix} \mathbf{i} - \begin{vmatrix} 1 & -7 \\ 0 & 1 \end{vmatrix} \mathbf{j} + \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \mathbf{k} \\ &= 7\mathbf{i} - \mathbf{j} + \mathbf{k}\end{aligned}$$

SCalcET9M 12.4.013.


State whether each expression is meaningful. If not, explain why. If so, state whether it is a vector or a scalar.

(a)  $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$

- ☐ The expression is meaningful. It is a vector.
- ☐  The expression is meaningful. It is a scalar.
- ☐ The expression is meaningless. The cross product is defined only for two vectors.
- ☐ The expression is meaningless. The dot product is defined only for two vectors.




(b)  $\mathbf{a} \times (\mathbf{b} \cdot \mathbf{c})$

- ☐ The expression is meaningful. It is a vector.
- ☐ The expression is meaningful. It is a scalar.
- ☐  The expression is meaningless. The cross product is defined only for two vectors.
- ☐ The expression is meaningless. The dot product is defined only for two vectors.




(c)  $\mathbf{a} \times (\mathbf{b} \times \mathbf{c})$

- ☐  The expression is meaningful. It is a vector.
- ☐ The expression is meaningful. It is a scalar.
- ☐ The expression is meaningless. The cross product is defined only for two vectors.
- ☐ The expression is meaningless. The dot product is defined only for two vectors.




(d)  $\mathbf{a} \cdot (\mathbf{b} \cdot \mathbf{c})$

- ☐ The expression is meaningful. It is a vector.
- ☐ The expression is meaningful. It is a scalar.
- ☐ The expression is meaningless. The cross product is defined only for two vectors.
- ☒  The expression is meaningless. The dot product is defined only for two vectors.




(e)  $(\mathbf{a} \cdot \mathbf{b}) \times (\mathbf{c} \cdot \mathbf{d})$

- ☐ The expression is meaningful. It is a vector.
- ☐ The expression is meaningful. It is a scalar.
- ☒  The expression is meaningless. The cross product is defined only for two vectors.
- ☐ The expression is meaningless. The dot product is defined only for two vectors.



(f)  $(\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{c} \times \mathbf{d})$

- ☐ The expression is meaningful. It is a vector.
- ☒  The expression is meaningful. It is a scalar.
- ☐ The expression is meaningless. The cross product is defined only for two vectors.
- ☐ The expression is meaningless. The dot product is defined only for two vectors.



#### Solution or Explanation

- (a) Since  $\mathbf{b} \times \mathbf{c}$  is a vector, the dot product  $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$  is meaningful and is a scalar.
- (b) We have that  $\mathbf{b} \cdot \mathbf{c}$  is a scalar, so  $\mathbf{a} \times (\mathbf{b} \cdot \mathbf{c})$  is meaningless, as the cross product is defined only for two *vectors*.
- (c) Since  $\mathbf{b} \times \mathbf{c}$  is a vector, the cross product  $\mathbf{a} \times (\mathbf{b} \times \mathbf{c})$  is meaningful and results in another vector.
- (d) We have that  $\mathbf{b} \cdot \mathbf{c}$  is a scalar, so the dot product  $\mathbf{a} \cdot (\mathbf{b} \cdot \mathbf{c})$  is meaningless, as the dot product is defined only for two vectors.
- (e) Since  $(\mathbf{a} \cdot \mathbf{b})$  and  $(\mathbf{c} \cdot \mathbf{d})$  are both scalars, the cross product  $(\mathbf{a} \cdot \mathbf{b}) \times (\mathbf{c} \cdot \mathbf{d})$  is meaningless.
- (f) We have that  $\mathbf{a} \times \mathbf{b}$  and  $\mathbf{c} \times \mathbf{d}$  are both vectors, so the dot product  $(\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{c} \times \mathbf{d})$  is meaningful and is a scalar.

## Resources

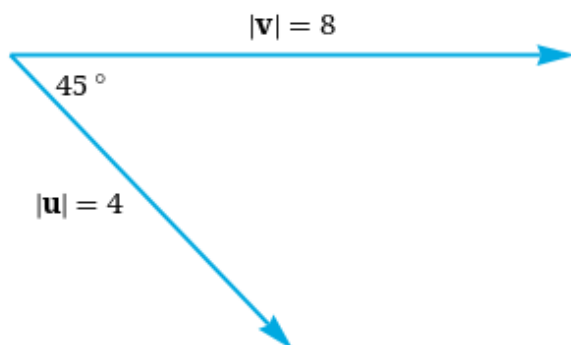
[Watch It](#)

9. [- / 2 Points]

DETAILS

SCalcET9M 12.4.014.


Find  $|\mathbf{u} \times \mathbf{v}|$ .



$|\mathbf{u} \times \mathbf{v}| =$

✗  $16\sqrt{2}$

Determine whether  $\mathbf{u} \times \mathbf{v}$  is directed into the screen or out of the screen.

- ☐  $\mathbf{u} \times \mathbf{v}$  is directed into the screen.
- ☐   $\mathbf{u} \times \mathbf{v}$  is directed out of the screen.

✗

### Solution or Explanation

If  $\theta$  is the angle between  $\mathbf{a}$  and  $\mathbf{b}$  (so  $0 \leq \theta \leq \pi$ ), then  $|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin(\theta)$ .

Using the theorem above, we have  $|\mathbf{u} \times \mathbf{v}| = |\mathbf{u}||\mathbf{v}| \sin(\theta) = (4)(8) \sin(45^\circ) = 32 \cdot \frac{\sqrt{2}}{2} = 16\sqrt{2}$ . By the right-hand rule,  $\mathbf{u} \times \mathbf{v}$  is directed out of the screen.

10. [- / 1 Points]

DETAILS

SCalcET9M 12.4.028.

Find the area of the parallelogram with vertices  $P(2, 1, 2)$ ,  $Q(5, 4, 3)$ ,  $R(7, 8, 12)$ , and  $S(4, 5, 11)$ .


✖

 $\sqrt{1190}$ 

Solution or Explanation

By plotting the vertices, we can see that the parallelogram is determined by the vectors  $\overrightarrow{PQ} = \langle 3, 3, 1 \rangle$  and  $\overrightarrow{PS} = \langle 2, 4, 9 \rangle$ .

Graph

Graph Description

i

Thus the area of parallelogram  $PQRS$  is given.

$$|\overrightarrow{PQ} \times \overrightarrow{PS}| = \left| \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 3 & 1 \\ 2 & 4 & 9 \end{vmatrix} \right| = |(23)\mathbf{i} - (25)\mathbf{j} + (6)\mathbf{k}| = |23\mathbf{i} - 25\mathbf{j} + 6\mathbf{k}| = \sqrt{1190} \approx 34.496$$

11. [- / 1 Points]

DETAILS

SCalcET9M 12.4.035.

Find the volume of the parallelepiped with adjacent edges  $PQ$ ,  $PR$ , and  $PS$ .

$$P(-2, 1, 0), \quad Q(2, 2, 2), \quad R(1, 4, -1), \quad S(3, 6, 1)$$

   24 cubic units

Solution or Explanation

We have  $\mathbf{a} = \overrightarrow{PQ} = \langle 4, 1, 2 \rangle$ ,  $\mathbf{b} = \overrightarrow{PR} = \langle 3, 3, -1 \rangle$ , and  $\mathbf{c} = \overrightarrow{PS} = \langle 5, 5, 1 \rangle$ .

Also,

$$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \begin{vmatrix} 4 & 1 & 2 \\ 3 & 3 & -1 \\ 5 & 5 & 1 \end{vmatrix} = 4 \begin{vmatrix} 3 & -1 \\ 5 & 1 \end{vmatrix} - 1 \begin{vmatrix} 3 & -1 \\ 5 & 1 \end{vmatrix} + 2 \begin{vmatrix} 3 & 3 \\ 5 & 5 \end{vmatrix} = 32 - 8 + 0 = 24,$$

so the volume of the parallelepiped is 24 cubic units.

### Resources


[Watch It](#)

12. [- / 1 Points]

DETAILS

SCalcET9M 12.4.037.

Use the scalar triple product to determine if the vectors  $\mathbf{u} = \mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$ ,  $\mathbf{v} = 3\mathbf{i} - \mathbf{j}$ , and  $\mathbf{w} = 6\mathbf{i} + 7\mathbf{j} - 6\mathbf{k}$  are coplanar.

- ☐ Yes, they are coplanar.
- ☐  No, they are not coplanar.



Solution or Explanation

We have

$$\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = \begin{vmatrix} 1 & 4 & -3 \\ 3 & -1 & 0 \\ 6 & 7 & -6 \end{vmatrix} = 1 \begin{vmatrix} -1 & 0 \\ 7 & -6 \end{vmatrix} - 4 \begin{vmatrix} 3 & 0 \\ 6 & -6 \end{vmatrix} + (-3) \begin{vmatrix} 3 & -1 \\ 6 & 7 \end{vmatrix} = 6 + 72 - 81 \neq 0,$$

which says that the volume of the parallelepiped determined by  $\mathbf{u}$ ,  $\mathbf{v}$ , and  $\mathbf{w}$  is *not* 0, and thus these three vectors are *not* coplanar.

### Resources

[Watch It](#)

13. [- / 1 Points]

DETAILS

SCalcET9M 12.4.043.

If  $\mathbf{a} \cdot \mathbf{b} = 12\sqrt{3}$  and  $\mathbf{a} \times \mathbf{b} = \langle 4, 8, 8 \rangle$ , find the angle (in degrees) between  $\mathbf{a}$  and  $\mathbf{b}$ .

  30 °

Solution or Explanation

We have the following theorem.

If  $\theta$  is the angle between  $\mathbf{a}$  and  $\mathbf{b}$  (so  $0 \leq \theta \leq \pi$ ), then the length of the cross product  $\mathbf{a} \times \mathbf{b}$  is given by  $|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin(\theta)$ .

From the theorem above, we have  $|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin(\theta)$ , where  $\theta$  is the angle between  $\mathbf{a}$  and  $\mathbf{b}$ .

We have another theorem.

If  $\theta$  is the angle between the vectors  $\mathbf{a}$  and  $\mathbf{b}$ , then  $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos(\theta)$ .

From the theorem above, we have  $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos(\theta) \Rightarrow |\mathbf{a}||\mathbf{b}| = \frac{\mathbf{a} \cdot \mathbf{b}}{\cos(\theta)}$ .

Substituting the second equation into the first gives  $|\mathbf{a} \times \mathbf{b}| = \frac{\mathbf{a} \cdot \mathbf{b}}{\cos(\theta)} \sin(\theta)$ , so  $\frac{|\mathbf{a} \times \mathbf{b}|}{\mathbf{a} \cdot \mathbf{b}} = \tan(\theta)$ . Here

$$|\mathbf{a} \times \mathbf{b}| = |\langle 4, 8, 8 \rangle| = \sqrt{16 + 64 + 64} = 12, \text{ so } \tan(\theta) = \frac{|\mathbf{a} \times \mathbf{b}|}{\mathbf{a} \cdot \mathbf{b}} = \frac{12}{12\sqrt{3}} = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30^\circ.$$

SCalcET9M 12.4.053.

Suppose that  $\mathbf{a} \neq \mathbf{0}$ .

(a) If  $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c}$ , does it follow that  $\mathbf{b} = \mathbf{c}$ ?

- ☐ Yes  
☒ No



(b) If  $\mathbf{a} \times \mathbf{b} = \mathbf{a} \times \mathbf{c}$ , does it follow that  $\mathbf{b} = \mathbf{c}$ ?

- ☐ Yes  
☒ No



(c) If  $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c}$ , and  $\mathbf{a} \times \mathbf{b} = \mathbf{a} \times \mathbf{c}$ , does it follow that  $\mathbf{b} = \mathbf{c}$ ?

- ☒ Yes  
☐ No



#### Solution or Explanation

- (a) No. If  $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c}$ , then  $\mathbf{a} \cdot (\mathbf{b} - \mathbf{c}) = 0$ , so  $\mathbf{a}$  is perpendicular to  $\mathbf{b} - \mathbf{c}$ , which can happen if  $\mathbf{b} \neq \mathbf{c}$ . For example, let  $\mathbf{a} = \langle 1, 1, 1 \rangle$ ,  $\mathbf{b} = \langle 1, 0, 0 \rangle$  and  $\mathbf{c} = \langle 0, 1, 0 \rangle$ .
- (b) No. If  $\mathbf{a} \times \mathbf{b} = \mathbf{a} \times \mathbf{c}$ , then  $\mathbf{a} \times (\mathbf{b} - \mathbf{c}) = \mathbf{0}$ , which implies that  $\mathbf{a}$  is parallel to  $\mathbf{b} - \mathbf{c}$ , which of course can happen if  $\mathbf{b} \neq \mathbf{c}$ .
- (c) Yes. Since  $\mathbf{a} \cdot \mathbf{c} = \mathbf{a} \cdot \mathbf{b}$ ,  $\mathbf{a}$  is perpendicular to  $\mathbf{b} - \mathbf{c}$ , by part (a). From part (b),  $\mathbf{a}$  is also parallel to  $\mathbf{b} - \mathbf{c}$ . Thus, since  $\mathbf{a} \neq \mathbf{0}$  but is both parallel and perpendicular to  $\mathbf{b} - \mathbf{c}$ , we have  $\mathbf{b} - \mathbf{c} = \mathbf{0}$ , so  $\mathbf{b} = \mathbf{c}$ .

#### Resources

[Watch It](#)

Determine whether each statement is true or false in  $\mathbb{R}^3$ .

(a) Two lines parallel to a third line are parallel.

- ☐ True  
☐ False



(b) Two lines perpendicular to a third line are parallel.

- ☐ True  
☒ False



(c) Two planes parallel to a third plane are parallel.

- ☐ True  
☐ False



(d) Two planes perpendicular to a third plane are parallel.

- ☐ True  
☒ False



(e) Two lines parallel to a plane are parallel.

- ☐ True  
☒ False



(f) Two lines perpendicular to a plane are parallel.

- ☐ True  
☐ False



(g) Two planes parallel to a line are parallel.

☐ True
   
☒ False

✗

(h) Two planes perpendicular to a line are parallel.

☒ True
   
☐ False

✗

(i) Two planes either intersect or are parallel.

☒ True
   
☐ False

✗

(j) Two lines either intersect or are parallel.

☐ True
   
☒ False

✗

(k) A plane and a line either intersect or are parallel.

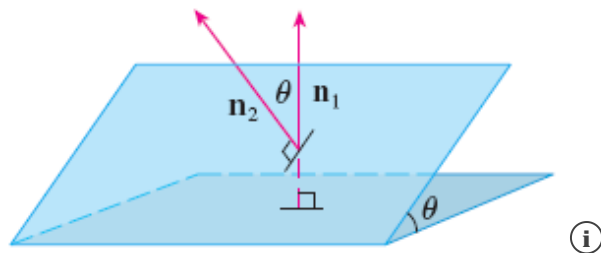
☒ True
   
☐ False

✗

#### Solution or Explanation

- (a) True; each of the first two lines has a direction vector parallel to the direction vector of the third line, so these vectors are each scalar multiples of the third direction vector. Then the first two direction vectors are also scalar multiples of each other, so these vectors, and hence, the two lines are parallel.
- (b) False; for example, the  $x$ - and  $y$ -axes are both perpendicular to the  $z$ -axis, yet the  $x$ - and  $y$ -axes are not parallel.
- (c) True; each of the first two planes has a normal vector parallel to the normal vector of the third plane, so these two normal vectors are parallel to each other and the planes are parallel.
- (d) False; for example, the  $xy$ - and  $yz$ -planes are not parallel, yet they are both perpendicular to the  $xz$ -plane.

- (e) False; the  $x$ - and  $y$ -axes are not parallel, yet they are both parallel to the plane  $z = 1$ .
- (f) True; if each line is perpendicular to a plane, then the lines' direction vectors are both parallel to a normal vector for the plane. Thus, the direction vectors are parallel to each other and the lines are parallel.
- (g) False; the planes  $y = 1$  and  $z = 1$  are not parallel, yet they are both parallel to the  $x$ -axis.
- (h) True; if each plane is perpendicular to a line, then any normal vector for each plane is parallel to a direction vector for the line. Thus, the normal vectors are parallel to each other and the planes are parallel.
- (i) True; see the figure.



- (j) False; they can be skew.
- (k) True. Consider any normal vector for the plane and any direction vector for the line. If the normal vector is perpendicular to the direction vector, the line and plane are parallel. Otherwise, the vectors meet at an angle  $\theta$ ,  $0^\circ \leq \theta < 90^\circ$ , and the line will intersect the plane at an angle  $90^\circ - \theta$ .

## Resources

[Watch It](#)

SCalcET9M 12.5.006.

Find parametric equations for the line. (Use the parameter  $t$ .)the line through the points  $(0, \frac{1}{2}, 1)$  and  $(3, 1, -3)$  $(x(t), y(t), z(t)) = ($ 

✗  $3t + 3, \frac{t}{2} + 1, -4t - 3$ )

Find the symmetric equations.

- ☐  $2x - 2 = \frac{y - 3}{3} = \frac{z + 3}{-4}$
- ☐  $3 + 3x = 1 + \frac{y}{2} = -3 - 4z$
- ☐  $x - 3 = 2y - 2 = z + 3$
- ☒  $\frac{x - 3}{3} = 2y - 2 = \frac{z + 3}{-4}$
- ☐  $\frac{x + 3}{-4} = 2y - 2 = \frac{z - 3}{3}$

✗

Solution or Explanation

The vector  $\mathbf{v} = \langle 3 - 0, 1 - \frac{1}{2}, -3 - 1 \rangle = \langle 3, \frac{1}{2}, -4 \rangle$  is parallel to the line. Letting  $P_0 = (3, 1, -3)$ , parametric equations are  $x = 3 + 3t$ ,  $y = 1 + \frac{1}{2}t$ , and  $z = -3 - 4t$ , while symmetric equations are  $\frac{x - 3}{3} = \frac{y - 1}{1/2} = \frac{z + 3}{-4}$  or  $\frac{x - 3}{3} = 2y - 2 = \frac{z + 3}{-4}$ .

## Resources

[Watch It](#)

SCalcET9M 12.5.013.EP.

Consider the line through the points  $(-4, -6, 1)$  and  $(-2, 0, -3)$ . Write a direction vector  $\mathbf{v}_1$  for this line.

 $\mathbf{v}_1 =$ 



✗  $\langle 2, 6, -4 \rangle$

Consider the line through the points  $(12, 20, 2)$  and  $(7, 5, 12)$ . Write a direction vector  $\mathbf{v}_2$  for this line.

 $\mathbf{v}_2 =$ 



✗  $\langle -5, -15, 10 \rangle$

Are  $\mathbf{v}_1$  and  $\mathbf{v}_2$  parallel?

☐  Yes

☐ No

✗

Is the line through  $(-4, -6, 1)$  and  $(-2, 0, -3)$  parallel to the line through  $(12, 20, 2)$  and  $(7, 5, 12)$ ?

☐  Yes

☐ No

✗

Solution or Explanation

Direction vectors of the lines are

$$\mathbf{v}_1 = \langle -2 - (-4), 0 - (-6), -3 - (1) \rangle = \langle 2, 6, -4 \rangle$$

and

$$\mathbf{v}_2 = \langle 7 - (12), 5 - (20), 12 - (2) \rangle = \langle -5, -15, 10 \rangle,$$

and since  $\mathbf{v}_2 = -\frac{5}{2}\mathbf{v}_1$ , the direction vectors and thus the lines are parallel.

## Resources

[Watch It](#)

18. [- / 2 Points]


DETAILS

SCalcET9M 12.5.019.

Determine whether the lines  $L_1$  and  $L_2$  are parallel, skew, or intersecting.


$$L_1: x = 9 + 6t, y = 12 - 3t, z = 3 + 9t$$

$$L_2: x = 4 + 16s, y = 12 - 8s, z = 16 + 20s$$

- ☐ parallel
- ☐  skew
- ☐ intersecting



If they intersect, find the point of intersection. (If an answer does not exist, enter DNE.)

 DNE

#### Solution or Explanation

Since the direction vectors  $\langle 6, -3, 9 \rangle$  and  $\langle 16, -8, 20 \rangle$  are not scalar multiples of each other, the lines aren't parallel. For the lines to intersect, we must be able to find one value of  $t$  and one value of  $s$  that produce the same point from the respective parametric equations. Thus we need to satisfy the following three equations:  $9 + 6t = 4 + 16s$ ,  $12 - 3t = 12 - 8s$ , and  $3 + 9t = 16 + 20s$ . Solving the last two equations we get  $t = \frac{26}{3}$ ,  $s = \frac{13}{4}$  and checking, we see that these values don't satisfy the first equation.

Thus, the lines aren't parallel and don't intersect, so they must be skew lines.

#### Resources

[Watch It](#)

19. [- / 1 Points]

DETAILS

SCalcET9M 12.5.023.

Find an equation of the plane.

the plane through the point  $(4, 8, 7)$  and with normal vector  $4\mathbf{i} + \mathbf{j} - \mathbf{k}$

✖  $4x + y - z = 17$

Solution or Explanation

We have that  $4\mathbf{i} + \mathbf{j} - \mathbf{k} = \langle 4, 1, -1 \rangle$  is a normal vector to the plane and  $(4, 8, 7)$  is a point on the plane, so setting  $a = 4$ ,  $b = 1$ ,  $c = -1$ ,  $x_0 = 4$ ,  $y_0 = 8$ , and  $z_0 = 7$  in equation  $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$  gives  $4(x - 4) + (y - 8) - (z - 7) = 0$  or  $4x + y - z = 17$  to be an equation of the plane.

### Resources

[Watch It](#)

20. [- / 1 Points]

DETAILS

SCalcET9M 12.5.046.

Find the point at which the line intersects the given plane.

$$x = t - 1, \quad y = 1 + 2t, \quad z = 3 - t; \quad 4x - y + 3z = 9$$

$(x, y, z) = ($

$\times \quad -6, -9, 8 \quad )$

#### Solution or Explanation

Substitute the parametric equations of the line into the equation of the plane:

$$4(t - 1) - (1 + 2t) + 3(3 - t) = 9 \Rightarrow -t + 4 = 9 \Rightarrow t = -5.$$

Therefore, the point of intersection of the line and the plane is given by

$$x = -5 - 1 = -6, \quad y = 1 + 2(-5) = -9, \quad \text{and} \quad z = 3 - (-5) = 8, \quad \text{that is, the point } (-6, -9, 8).$$

21. [- / 1 Points]

DETAILS

SCalcET9M 12.5.049.

Find direction numbers for the line of intersection of the planes  $x + y + z = 4$  and  $x + z = 0$ . (Enter your answers as a comma-separated list.)



✗ 1, 0, -1

#### Solution or Explanation

Setting  $x = 0$ , we see that  $(0, 4, 0)$  satisfies the equations of both planes, so they do in fact have a line of intersection.

We have that  $\mathbf{v} = \mathbf{n}_1 \times \mathbf{n}_2 = \langle 1, 1, 1 \rangle \times \langle 1, 0, 1 \rangle = \langle 1, 0, -1 \rangle$  is the direction of this line. Therefore, direction numbers of the intersecting line are 1, 0, -1.

#### Resources

[Watch It](#)

22. [- / 1 Points]

DETAILS

SCalcET9M 12.5.050.

Find the cosine of the angle between the planes  $x + y + z = 0$  and  $x + 4y + 3z = 5$ .



✗  $\frac{8}{\sqrt{78}}$

#### Solution or Explanation

The angle between the two planes is the same as the angle between their normal vectors. The normal vectors of the two planes are  $\langle 1, 1, 1 \rangle$  and  $\langle 1, 4, 3 \rangle$ . The cosine of the angle  $\theta$  between these two planes is

$$\cos(\theta) = \frac{\langle 1, 1, 1 \rangle \cdot \langle 1, 4, 3 \rangle}{\left| \langle 1, 1, 1 \rangle \right| \left| \langle 1, 4, 3 \rangle \right|} = \frac{1 + 4 + 3}{\sqrt{1 + 1 + 1} \sqrt{1 + 16 + 9}} = \frac{8}{\sqrt{78}}.$$

SCalcET9M 12.5.058.

Consider the following planes.

$$5x - 3y + z = 2, \quad 3x + y - 5z = 4$$

- (a) Find parametric equations for the line of intersection of the planes. (Use the parameter
- $t$
- .)

$$(x(t), y(t), z(t)) = ($$



$$\times \quad 14t + 1, 28t + 1, 14t \quad )$$

- (b) Find the angle between the planes. (Round your answer to one decimal place.)

✗



78.5 °

## Solution or Explanation

- (a) If we set  $z = 0$  then the equations of the planes reduce to  $5x - 3y = 2$  and  $3x + y = 4$ , and solving these two equations gives  $x = 1$ ,  $y = 1$ . Thus, a point on the line of intersection is  $(1, 1, 0)$ . A vector  $\mathbf{v}$  in the direction of this intersecting line is perpendicular to the normal vectors of both planes, so let  $\mathbf{v} = \mathbf{n}_1 \times \mathbf{n}_2 = \langle 5, -3, 1 \rangle \times \langle 3, 1, -5 \rangle = \langle 14, 28, 14 \rangle$ . By the equations  $x = x_0 + at$ ,  $y = y_0 + bt$ , and  $z = z_0 + ct$ , parametric equations for the line are  $x = 1 + 14t$ ,  $y = 1 + 28t$ , and  $z = 14t$ .

$$(b) \cos(\theta) = \frac{\mathbf{n}_1 \cdot \mathbf{n}_2}{\|\mathbf{n}_1\| \|\mathbf{n}_2\|} = \frac{15 - 3 - 5}{\sqrt{35} \sqrt{35}} = \frac{1}{5} \Rightarrow \theta = \cos^{-1}\left(\frac{1}{5}\right) \approx 78.5^\circ$$

24. [- / 1 Points]

DETAILS

SCalcET9M 12.5.062.

Find an equation for the plane consisting of all points that are equidistant from the points  $(-7, 1, 2)$  and  $(3, 3, 6)$ .

✗  $5x + y + 2z = 0$

Solution or Explanation

The distance from a point  $(x, y, z)$  to  $(-7, 1, 2)$  is  $d_1 = \sqrt{(x + 7)^2 + (y - 1)^2 + (z - 2)^2}$  and the distance from  $(x, y, z)$  to  $(3, 3, 6)$  is  $d_2 = \sqrt{(x - 3)^2 + (y - 3)^2 + (z - 6)^2}$ . The plane consists of all points  $(x, y, z)$  where

$$d_1 = d_2 \Rightarrow d_1^2 = d_2^2 \Leftrightarrow$$

$$(x + 7)^2 + (y - 1)^2 + (z - 2)^2 = (x - 3)^2 + (y - 3)^2 + (z - 6)^2 \Leftrightarrow$$

$$x^2 + 14x + y^2 - 2y + z^2 - 4z + 54 = x^2 - 6x + y^2 - 6y + z^2 - 12z + 54 \Leftrightarrow$$

$$20x + 4y + 8z = 0,$$


so an equation for the plane is  $20x + 4y + 8z = 0$  or equivalently,  $5x + y + 2z = 0$ .

SCalcET9M 12.5.067.

Which of the following four planes are parallel?



$$P_1: 9x + 15y - 9z = 36 \quad P_2: 4x - 12y + 8z = 5$$

$$P_3: 9y = 1 + 3x + 6z \quad P_4: 3z = 3x + 5y - 12$$

- ☐  $P_1$  and  $P_3$  are parallel and  $P_2$  and  $P_4$  are parallel.  
☐   $P_1$  and  $P_4$  are parallel and  $P_2$  and  $P_3$  are parallel.  
☐  $P_2$  and  $P_3$  are parallel.  
☐ All four planes are parallel.  
☐  $P_1$ ,  $P_2$ , and  $P_4$  are parallel.



Are any of them identical? (Select all that apply.)

- ☐   $P_1$   
☐  $P_2$   
☐  $P_3$   
☐   $P_4$



Solution or Explanation

Let  $P_i$  have normal vector  $\mathbf{n}_i$ . Then  $\mathbf{n}_1 = \langle 9, 15, -9 \rangle$ ,  $\mathbf{n}_2 = \langle 4, -12, 8 \rangle$ ,  $\mathbf{n}_3 = \langle 3, -9, 6 \rangle$ ,  $\mathbf{n}_4 = \langle 3, 5, -3 \rangle$ .

Now  $\mathbf{n}_1 = 3\mathbf{n}_4$ , so  $\mathbf{n}_1$  and  $\mathbf{n}_4$  are parallel, and hence  $P_1$  and  $P_4$  are parallel; similarly  $P_2$  and  $P_3$  are parallel because  $\mathbf{n}_2 = \frac{4}{3}\mathbf{n}_3$ .

However,  $\mathbf{n}_1$  and  $\mathbf{n}_2$  are not parallel (so not all four planes are parallel).

Notice that the point  $(4, 0, 0)$  lies on both  $P_1$  and  $P_4$ , so these two planes are identical.

The point  $(\frac{5}{4}, 0, 0)$  lies on  $P_2$  but not on  $P_3$ , so these are different planes.

SCalcET9M 12.5.069.

Let  $P$  be a point not on the line  $L$  that passes through the points  $Q$  and  $R$ . The distance  $d$  from the point  $P$  to the line  $L$  is

$$d = \frac{|\mathbf{a} \times \mathbf{b}|}{|\mathbf{a}|}$$

where  $\mathbf{a} = \overrightarrow{QR}$  and  $\mathbf{b} = \overrightarrow{QP}$ .

Use the above formula to find the distance from the point to the given line.

$$(4, 1, -2); \quad x = 2 + t, y = 1 - 3t, z = 3 - 3t$$

$d =$



✗  $\sqrt{\frac{262}{19}}$

#### Solution or Explanation

Let  $Q = (2, 1, 3)$  and  $R = (3, -2, 0)$ , points on the line corresponding to  $t = 0$  and  $t = 1$ . Let  $P = (4, 1, -2)$ . Then,  $\mathbf{a} = \overrightarrow{QR} = \langle 1, -3, -3 \rangle$ ,  $\mathbf{b} = \overrightarrow{QP} = \langle 2, 0, -5 \rangle$ . The distance is

$$d = \frac{|\mathbf{a} \times \mathbf{b}|}{|\mathbf{a}|} = \frac{|\langle 1, -3, -3 \rangle \times \langle 2, 0, -5 \rangle|}{|\langle 1, -3, -3 \rangle|} = \frac{|\langle 15, -1, 6 \rangle|}{|\langle 1, -3, -3 \rangle|} = \frac{\sqrt{15^2 + (-1)^2 + 6^2}}{\sqrt{1^2 + (-3)^2 + (-3)^2}} = \frac{\sqrt{262}}{\sqrt{19}} = \sqrt{\frac{262}{19}}.$$

#### Resources

[Watch It](#)

27. [- / 1 Points]

DETAILS

SCalcET9M 12.5.071.

Find the distance from the point to the given plane.

$$(1, -7, 7), \quad 3x + 2y + 6z = 5$$



✖

$$\frac{26}{7}$$

Solution or Explanation

By the equation  $D = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$ , the distance is

$$D = \frac{|3(1) + 2(-7) + 6(7) - 5|}{\sqrt{3^2 + 2^2 + 6^2}} = \frac{|26|}{\sqrt{49}} = \frac{26}{7}.$$

### Resources

[Watch It](#)

SCalcET9M 12.5.079.

Let  $L_1$  be the line through the origin and the point  $(7, 0, -1)$ . Let  $L_2$  be the line through the points  $(1, -1, 1)$  and  $(7, 1, 5)$ . Find the distance between  $L_1$  and  $L_2$ .



✖  $\frac{25}{\sqrt{339}}$

### Solution or Explanation

A direction vector for  $L_1$  is  $\mathbf{v}_1 = \langle 7, 0, -1 \rangle$  and a direction vector for  $L_2$  is  $\mathbf{v}_2 = \langle 6, 2, 4 \rangle$ . These vectors are not parallel so neither are the lines. Parametric equations for the lines are  $L_1: x = 7t, y = 0, z = -t$ , and  $L_2: x = 1 + 6s, y = -1 + 2s, z = 1 + 4s$ . No values of  $t$  and  $s$  satisfy these equations simultaneously, so the lines don't intersect and hence are skew. We can view the lines as lying in two parallel planes; a common normal vector to the planes is  $\mathbf{n} = \mathbf{v}_1 \times \mathbf{v}_2 = \langle 2, -34, 14 \rangle$ . Line  $L_1$  passes through the origin, so  $(0, 0, 0)$  lies on one of the planes, and  $(1, -1, 1)$  is a point on  $L_2$  and, therefore, on the other plane. Equations of the planes then are  $2x - 34y + 14z = 0$  and  $2x - 34y + 14z - 50 = 0$ . In a previous exercise, the distance between the planes  $ax + by + cz + d_1 = 0$  and  $ax + by + cz + d_2 = 0$  is given by

$$D = \frac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}}. \text{ By this exercise, the distance between two skew lines is}$$

$$D = \frac{|0 - (-50)|}{\sqrt{4 + 1156 + 196}} = \frac{25}{\sqrt{339}}.$$

There is an alternate solution (without reference to planes): direction vectors of the two lines are  $\mathbf{v}_1 = \langle 7, 0, -1 \rangle$  and  $\mathbf{v}_2 = \langle 6, 2, 4 \rangle$ . Then,  $\mathbf{n} = \mathbf{v}_1 \times \mathbf{v}_2 = \langle 2, -34, 14 \rangle$  is perpendicular to both lines. Pick any point on each of the lines, say  $(0, 0, 0)$  and  $(1, -1, 1)$ , and form the vector  $\mathbf{b} = \langle 1, -1, 1 \rangle$  connecting the two points. Then, the distance between the two skew lines is the absolute value of the scalar

projection of  $\mathbf{b}$  along  $\mathbf{n}$ , that is,  $D = \frac{|\mathbf{n} \cdot \mathbf{b}|}{|\mathbf{n}|} = \frac{|2 + 34 + 14|}{\sqrt{4 + 1156 + 196}} = \frac{25}{\sqrt{339}}.$

### Resources

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