Assignment Previewer

This is a preview of the questions in your assignment, but it does not reflect all assignment settings. Open the student view to interact with the actual assignment.

Show answer key SHOW NEW RANDOMIZATION EDIT ASSIGNMENT

INSTRUCTOR
Qingchun Hou
International Campus Zhejiang University_CN

HW10 (Homework)

Current Score: – / 40 POINTS | 0.0 %

Scoring and Assignment Information

QUESTION	1	2	3	4	5	6	7	8	9	10	11	12
POINTS	– / 3	– / 1	– / 1	– / 1	– / 1	– / 16	– / 8	– / 1	– / 2	– / 2	– / 2	– / 2

Assignment Submission

For this assignment, you submit answers by question parts. The number of submissions remaining for each question part only changes if you submit or change the answer.

Assignment Scoring

Your best submission for each question part is used for your score.

1. [– / 3 Points]

DETAILS

SCalcET9M 14.4.003.EP.

Consider the following surface.

$$z = 2x^2 + y^2 - 9y$$

Let $z = f(x, y)$. Find $f_x(x, y)$ and $f_y(x, y)$.

$f_x(x, y) =$

✕ $4x$

$f_y(x, y) =$

✕ $2y - 9$

Find an equation of the tangent plane to the given surface at the point $(1, 4, -18)$.

✕ $z = 4x - y - 18$

Solution or Explanation

We have

$$z = f(x, y) = 2x^2 + y^2 - 9y \quad \Rightarrow \quad \begin{aligned} f_x(x, y) &= 4x, \\ f_y(x, y) &= 2y - 9, \end{aligned}$$

so $f_x(1, 4) = 4$, $f_y(1, 4) = -1$. By the equation

$$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0),$$

an equation of the tangent plane is

$$z - (-18) = f_x(1, 4)(x - 1) + f_y(1, 4)(y - 4) \quad \Rightarrow \quad z + 18 = 4(x - 1) + (-1)(y - 4)$$

or $z = 4x - y - 18$.

2. [– / 1 Points]

DETAILS

SCalcET9M 14.4.006.

Find an equation of the tangent plane to the given surface at the specified point.

$$z = y^2e^x, \quad (0, 5, 25)$$

✕ $z = 25x + 10y - 25$

Solution or Explanation

We have

$$z = f(x, y)y^2e^x \quad \Rightarrow \quad \begin{aligned} f_x(x, y) &= y^2e^x, \\ f_y(x, y) &= 2ye^x, \end{aligned}$$

so $f_x(0, 5) = 25$ and $f_y(0, 5) = 10$. Thus, an equation of the tangent plane is

$$z - 25 = f_x(0, 5)(x - 0) + f_y(0, 5)(y - 5) \quad \Rightarrow \quad z - 25 = 25x + 10(y - 5),$$

or $z = 25x + 10y - 25$.

3. [– / 1 Points] DETAILS

SCalcET9M 14.4.007.

Find an equation of the tangent plane to the given surface at the specified point.

$$z = \frac{5\sqrt{y}}{x}, \quad (-1, 1, -5)$$

✕ $z = -5x - \frac{5y}{2} - \frac{15}{2}$

Solution or Explanation

We have

$$z = f(x, y) \frac{5\sqrt{y}}{x} \quad \Rightarrow \quad f_x(x, y) = -\frac{5\sqrt{y}}{x^2},$$

$$f_y(x, y) = \frac{5}{2x\sqrt{y}},$$

so $f_x(-1, 1) = -5$ and $f_y(-1, 1) = -\frac{5}{2}$. Thus, an equation of the tangent plane is

$$z - (-5) = f_x(-1, 1)(x - (-1)) + f_y(-1, 1)(y - 1) \quad \Rightarrow \quad z + 5 = -5(x + 1) - \frac{5}{2}(y - 1),$$

or $z = -5x - \frac{5}{2}y - \frac{15}{2}$.

4. [– / 1 Points] DETAILS

SCalcET9M 14.4.048.

The pressure, volume, and temperature of a mole of an ideal gas are related by the equation $PV = 8.31T$, where P is measured in kilopascals, V in liters, and T in kelvins. Use differentials to find the approximate change in the pressure if the volume increases from 10 L to 10.3 L and the temperature decreases from 370 K to 365 K. (Note whether the change is positive or negative in your answer. Round your answer to two decimal places.)

✕ -13.38 kPa

Solution or Explanation

Click to View Solution

5. [– / 1 Points]

DETAILS

SCalcET9M 14.4.050.

A model for the surface area of a human body is given by $S = 0.0072w^{0.425}h^{0.725}$, where w is the weight (in kilograms), h is the height (in centimeters), and S is measured in square feet. If the errors in measurement of w and h are at most 6%, use differentials to estimate the maximum percentage error in the calculated surface area.

[ ] ✕ 6.9 %

Solution or Explanation

We have $S = 0.0072w^{0.425}h^{0.725}$. The errors in measurement are at most 6%, so $\left|\frac{\Delta w}{w}\right| \le 0.06$ and $\left|\frac{\Delta h}{h}\right| \le 0.06$. The relative error in the calculated surface area is

$$\frac{\Delta S}{S} \approx \frac{dS}{S} = \frac{0.0072(0.425w^{0.425-1})h^{0.725}\,dw + 0.0072w^{0.425}(0.725h^{0.725-1})\,dh}{0.0072w^{0.425}h^{0.725}}$$

$$= 0.425\frac{dw}{w} + 0.725\frac{dh}{h}$$

To estimate the maximum relative error, we use $\frac{dw}{w} = \left|\frac{\Delta w}{w}\right| = 0.06$ and

$$\frac{dh}{h} = \left|\frac{\Delta h}{h}\right| = 0.06 \quad\Rightarrow\quad \frac{dS}{S} = 0.425(0.06) + 0.725(0.06) = 0.069.$$

Thus, the maximum percentage error is 6.9%.

Resources

[Watch It]

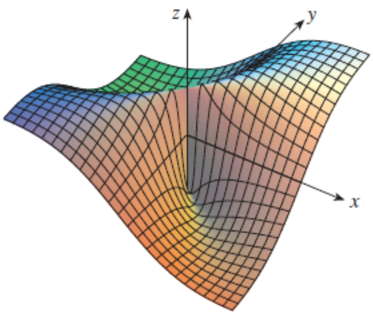
6. [– / 16 Points] DETAILS

SCalcET9M 14.4.054.

(a) The function

$$f(x, y) = \begin{cases} \dfrac{xy}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

is graphed in the figure below.



Show that $f_x(0, 0)$ and $f_y(0, 0)$ both exist but f is not differentiable at (0, 0). [*Hint:* Use the theorem that states if f is a function of two variables that is differentiable at (a, b), then f is continuous at (a, b) **(1)**.]

We have the following.

$$\lim_{h\to 0} \frac{f(h, 0) - f(0, 0)}{h} = \lim_{h\to 0} \frac{0 - 0}{h} = \boxed{}$$ ✗ 0

$$\lim_{h\to 0} \frac{f(0, h) - f(0, 0)}{h} = \lim_{h\to 0} \frac{0 - 0}{h} = \boxed{}$$ ✗ 0

Thus, $f_x(0, 0) = f_y(0, 0) =$ [ ] ✗ 0 . To show that f is not differentiable at (0, 0) we need only show that f is not continuous at (0, 0) and apply the contrapositive of **(1)**.

As $(x, y) \to (0, 0)$ along the x-axis $f(x, y) = \dfrac{0}{x^2} =$ [ ] ✗ 0 for $x \neq 0$ so as $(x, y) \to (0, 0)$ along the x-axis, $f(x, y) \to$ [ ] ✗ 0 . But as $(x, y) \to (0, 0)$ along the line $y = x$, $f(x, x) = \dfrac{x^2}{(2x^2)} =$ [ ] ✗ 1/2 for $x \neq 0$, so as $(x, y) \to (0, 0)$ along this line, $f(x, y) \to$ [ ] ✗ 1/2 .

Thus, $\lim_{(x, y)\to(0, 0)} f(x, y)$ ---Select--- ✗ does not exist , so f is ---Select--- ✗ discontinuous at (0, 0) and thus not differentiable there.

(b) Explain why f_x and f_y are not continuous at (0, 0).

$f_x(x, y) =$

For $(x, y) \neq (0, 0)$, ✗ $\dfrac{y\left(y^2 - x^2\right)}{\left(x^2 + y^2\right)^2}$.

$f_x(x, y) = f_x(0, y) =$

If we approach (0, 0) along the y-axis, then ✗ $\dfrac{1}{y}$, so as $(x, y) \to (0, 0)$, then $f_x(x, y) \to$ ---Select--- ✗ +/−∞ .

$f_y(x, y) =$

$\boxed{\dfrac{x(x^2 - y^2)}{(x^2 + y^2)^2}}$

Thus, $\lim_{(x, y) \to (0, 0)} f_x(x, y)$ ---Select--- ✗ does not exist and $f_x(x, y)$ is not continuous at (0, 0). Similarly, ✗ for $(x, y) \neq (0, 0)$, and if we approach (0, 0) along the x-axis, then

$f_y(x, y) = f_x(x, 0) =$ ✗ $\boxed{\dfrac{1}{x}}$.

Thus, $\lim_{(x, y) \to (0, 0)} f_y(x, y)$ ---Select--- ✗ does not exist and $f_y(x, y)$ is not continuous at (0, 0).

Solution or Explanation

(a) We have the following.

$$\lim_{h \to 0} \frac{f(h, 0) - f(0, 0)}{h} = \lim_{h \to 0} \frac{0 - 0}{h} = 0$$

$$\lim_{h \to 0} \frac{f(0, h) - f(0, 0)}{h} = \lim_{h \to 0} \frac{0 - 0}{h} = 0$$

Thus, $f_x(0, 0) = f_y(0, 0) = 0$. To show that f is not differentiable at (0, 0) we need only show that f is not continuous at (0, 0) and apply the contrapositive of the theorem below.

If f is a function of two variables that is differentiable at (a, b), then f is continuous at (a, b).

As $(x, y) \to (0, 0)$ along the x-axis $f(x, y) = \dfrac{0}{x^2} = 0$ for $x \neq 0$ so $f(x, y) \to 0$ as $(x, y) \to (0, 0)$ along the x-axis. But as $(x, y) \to (0, 0)$ along the line $y = x$, $f(x, x) = \dfrac{x^2}{(2x^2)} = \dfrac{1}{2}$ for $x \neq 0$ so $f(x, y) \to \dfrac{1}{2}$ as $(x, y) \to (0, 0)$ along this line.

Thus, $\lim_{(x, y) \to (0, 0)} f(x, y)$ does not exist, so f is discontinuous at (0, 0) and thus not differentiable there.

(b) For $(x, y) \neq (0, 0)$,

$$f_x(x, y) = \frac{(x^2 + y^2)y - xy(2x)}{(x^2 + y^2)^2} = \frac{y(y^2 - x^2)}{(x^2 + y^2)^2}.$$

If we approach (0, 0) along the y-axis, then $f_x(x, y) = f_x(0, y) = \dfrac{y^3}{y^4} = \dfrac{1}{y}$, so $f_x(x, y) \to \pm\infty$ as $(x, y) \to (0, 0)$.

Thus, $\lim_{(x, y) \to (0, 0)} f_x(x, y)$ does not exist and $f_x(x, y)$ is not continuous at (0, 0). Similarly,

$$f_y(x, y) = \frac{(x^2 + y^2)x - xy(2y)}{(x^2 + y^2)^2} = \frac{x(x^2 - y^2)}{(x^2 + y^2)^2}$$

for $(x, y) \neq (0, 0)$, and if we approach (0, 0) along the x-axis, then

$$f_y(x, y) = f_x(x, 0) = \frac{x^3}{x^4} = \frac{1}{x}.$$

Thus, $\lim_{(x, y) \to (0, 0)} f_y(x, y)$ does not exist and $f_y(x, y)$ is not continuous at (0, 0).

7. [– / 8 Points] DETAILS

SCalcET9M 14.4.AE.004.

Video Example

EXAMPLE 4

(a) If $z = f(x, y) = x^2 + 4xy - y^2$, find the differential dz.

(b) If x changes from 2 to 2.02 and y changes from 3 to 2.94, compare the values of Δz and dz.

SOLUTION

(a) The definition of the differential gives

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$$

$$= \Big(\ \boxed{2x+4y}\ \Big)dx + \Big(\ \boxed{4x-2y}\ \Big)dy.$$

(b) Putting $x = 2$, $dx = \Delta x = 0.02$, $y = 3$, and $dy = \Delta y = -0.06$, we get

$dz = \Big[\ \boxed{\ }\ \boxed{2}\ (2) + 4(3)\Big](0.02) + \Big[4(2) - 2(3)\Big]\Big(\ \boxed{\ }\ \boxed{-}$

$= \boxed{\ }\ \boxed{0.2}$.

The increment of z is as follows. (Round your final answer to two decimal places.)

$\Delta z = f(2.02, 2.94) - f(2, 3)$

$= \Big[\Big(\ \boxed{\ }\ \boxed{2.02}\ \Big)^2 + 4(2.02)(2.94) - (2.94)^2\Big] - \Big[2^2 + 4(2)(3) - \boxed{\ }$

$= \boxed{\ }\ \boxed{0.19}$.

Notice that $\Delta z \approx dz$ is easier to compute.

8. [– / 1 Points] DETAILS

SCalcET9M 14.5.044.

The voltage V in a simple electrical circuit is slowly decreasing as the battery wears out. The resistance R is slowly increasing as the resistor heats up. Use Ohm's Law, $V = IR$, to find how the current I is changing at the moment when $R = 385\ \Omega$, $I = 0.02$ A, $dV/dt = -0.05$ V/s, and $dR/dt = 0.06\ \Omega/s$. (Round your answer to six decimal places.)

$\boxed{\ }$ $\boxed{-0.000133}$ A/s

Solution or Explanation

Click to View Solution

9. [– / 2 Points] DETAILS

SCalcET9M 14.5.048.

If a sound with frequency f_s is produced by a source traveling along a line with speed v_s. If an observer is traveling with speed v_o along the same line from the opposite direction toward the source, then the frequency of the sound heard by the observer is

$$f_o = \left(\frac{c + v_o}{c - v_s}\right) f_s$$

where c is the speed of sound, about 332 m/s. (This is the **Doppler effect**.) Suppose that, at a particular moment, you are in a train traveling at 39 m/s and accelerating at 1.1 m/s^2. A train is approaching you from the opposite direction on the other track at 48 m/s, accelerating at 1.8 m/s^2, and sounds its whistle, which has a frequency of 475 Hz. At that instant, what is the perceived frequency that you hear? (Round your answer to one decimal place.)

[ ] ✗ 620.5 Hz

How fast is it changing? (Round your answer to two decimal places.)

[ ] ✗ 5.77 Hz/s

Solution or Explanation

Click to View Solution

10. [– / 2 Points] DETAILS

SCalcET9M 14.5.012.

Use the chain rule to find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$.

$$z = \tan^{-1}(x^3 + y^3), \quad x = s\ln(t), \quad y = te^s$$

$\frac{\partial z}{\partial s} =$ [ ]

[ ]

✗ $\frac{3\left(e^s t y^2 + x^2 \ln(t)\right)}{\left(x^3+y^3\right)^2+1}$

$\frac{\partial z}{\partial t} =$ [ ]

[ ]

✗ $\frac{3\left(\frac{sx^2}{t} + e^s y^2\right)}{\left(x^3+y^3\right)^2+1}$

Solution or Explanation

$z = \tan^{-1}(x^3 + y^3), \quad x = s\ln(t), \quad y = te^s \quad \Rightarrow$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s} = \frac{3x^2}{1+(x^3+y^3)^2}\cdot \ln(t) + \frac{3y^2}{1+(x^3+y^3)^2}\cdot te^s = \frac{3}{1+(x^3+y^3)^2}(x^2\ln(t) + y^2te^s)$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t} = \frac{3x^2}{1+(x^3+y^3)^2}\cdot \frac{s}{t} + \frac{3y^2}{1+(x^3+y^3)^2}\cdot e^s = \frac{3}{1+(x^3+y^3)^2}\left(\frac{x^2 s}{t} + y^2e^s\right)$$

11. [– / 2 Points] DETAILS

SCalcET9M 14.5.016.

Use the Chain Rule to find $\partial z/\partial s$ and $\partial z/\partial t$.

$z = \tan(u/v)$, $u = 8s + 2t$, $v = 2s - 8t$

$\frac{\partial z}{\partial s}$ = ✗ $\frac{(8v - 2u)\sec^2\left(\frac{u}{v}\right)}{v^2}$

$\frac{\partial z}{\partial t}$ = ✗ $\frac{(8u + 2v)\sec^2\left(\frac{u}{v}\right)}{v^2}$

Solution or Explanation

Click to View Solution

12. [– / 2 Points] DETAILS

SCalcET9M 14.5.020.

Suppose f is a differentiable function of x and y, and $g(r, s) = f(5r - s, s^2 - 3r)$. Use the table of values below to calculate $g_r(8, 2)$ and $g_s(8, 2)$.

	f	g	f_x	f_y
(38, −20)	1	4	7	5
(8, 2)	4	1	9	6

$g_r(8, 2)$ = ✗ 20

$g_s(8, 2)$ = ✗ 13

Solution or Explanation

Click to View Solution