Calculus III (Math 241)

Q1 Find the limit, if it exists, or show that the limit does not exist.

a) $\lim\limits_{(x,y)\to(1,-1)} \mathrm{e}^{-xy}\cos(x+y)$;

b) $\lim\limits_{(x,y)\to(0,0)} \dfrac{5y^4\cos^2 x}{x^4+y^4}$;

c) $\lim\limits_{(x,y)\to(1,0)} \dfrac{xy-y}{(x-1)^2+y^2}$;

d) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^3-y^3}{x^2+y^2}$.

Q2 Using the ε-δ definition of the limit $\lim_{\mathbf{x}\to\mathbf{x}_0} f(\mathbf{x})$ for functions $f\colon D\to\mathbb{R}^m$, $D\subseteq\mathbb{R}^n$, and $\mathbf{x}_0\in D'$, show that

$$\lim_{\mathbf{x}\to\mathbf{x}_0} f(\mathbf{x})=\mathbf{a} \wedge \lim_{\mathbf{x}\to\mathbf{x}_0} f(\mathbf{x})=\mathbf{b} \quad \text{implies} \quad \mathbf{a}=\mathbf{b}.$$