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HW8 (Homework)

Current Score: – / 59 POINTS | 0.0 %

Scoring and Assignment Information

QUESTION	1	2	3	4	5	6	7	8	9	10
POINTS	– / 1	– / 14	– / 1	– / 2	– / 1	– / 5	– / 10	– / 13	– / 1	– / 11

Assignment Submission

For this assignment, you submit answers by question parts. The number of submissions remaining for each question part only changes if you submit or change the answer.

Assignment Scoring

Your best submission for each question part is used for your score.

1. [– / 1 Points] DETAILS

SCalcET9M 14.7.051.

Find the volume of the largest rectangular box in the first octant with three faces in the coordinate planes and one vertex in the plane $x + 2y + 3z = 3$.

[ ] ✗ 1/6

Solution or Explanation
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Resources

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2. [– / 14 Points]

DETAILS

SCalcET9M 14.7.AE.005.

Video Example

EXAMPLE 5 Find the shortest distance from the point $(1, 0, -4)$ to the plane $x + 2y + z = 9$.

SOLUTION The distance from any point (x, y, z) to the point $(1, 0, -4)$ is

$$d = \sqrt{(\boxed{x-1})^2 + y^2 + (z+4)^2}$$

but if (x, y, z) lies on the plane $x + 2y + z = 9$, then

$z = \boxed{9 - x - 2y}$ and so we have

$$d = \sqrt{(\boxed{x-1})^2 + y^2 + (13 - x - 2y)^2}$$

. We can minimize d by minimizing the simpler expression

$$d^2 = f(x, y) = (\boxed{x-1})^2 + y^2 + (13 - x - 2y)^2.$$

By solving the equations

$$f_x = 2(x - 1) - 2(13 - x - 2y) = 4x + 4y - \boxed{}$$

$$f_y = 2y - 4(13 - x - 2y) = 4x + 10y - \boxed{} \quad 52$$

we find that the only critical point is $(x, y) = (\boxed{3, 4})$. Since $f_{xx} = 4$, $f_{xy} = 4$, and $f_{yy} = 10$, we have $D(x, y) = f_{xx}f_{yy} - (f_{xy})^2 = 24 > 0$ and $f_{xx} > 0$, so by the Second Derivatives Test f has a local minimum at $(x, y) = (\boxed{3, 4})$. Intuitively, we can see that this local minimum is actually an absolute minimum because there must be a point on the given plane that is closest to $(1, 0, -4)$. If $x = \boxed{}$ 3 and $y = \boxed{}$ 4, then

$$d = \sqrt{\left(\boxed{x-1}\right)^2 + y^2 + (13 - x - 2y)^2}$$

$$= \sqrt{\left(\boxed{2}\right)^2 + \left(4\right)^2 + \left(2\right)^2}$$

$$= \boxed{2\sqrt{6}}.$$

The shortest distance from $(1, 0, -4)$ to the plane $x + 2y + z = 9$ is

✕ $\boxed{2\sqrt{6}}$.

3. [–/1 Points] DETAILS

SCalcET9M 14.7.053.

Find the dimensions of a rectangular box of maximum volume such that the sum of the lengths of its 12 edges is a constant c. (Let x, y, and z be the dimensions of the rectangular box.)

$(x, y, z) = \left(\boxed{\frac{c}{12}, \frac{c}{12}, \frac{c}{12}}\right)$ ✕

Solution or Explanation

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Resources

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4. [– / 2 Points]

DETAILS

SCalcET9M 14.8.056.

Find the maximum and minimum volumes of a rectangular box whose surface area is 1500 cm^2 and whose total edge length is 200 cm.

maximum

✕ $\boxed{\frac{1}{27}\left(87,500 + 2500\sqrt{10}\right)}$

minimum

✕ $\boxed{\frac{1}{27}\left(87,500 - 2500\sqrt{10}\right)}$

Solution or Explanation

Click to View Solution

5. [– / 1 Points]

DETAILS

SCalcET9M 14.8.049.

Use Lagrange multipliers to find the volume of the largest rectangular box in the first octant with three faces in the coordinate planes and one vertex in the given plane.

$x + 5y + 8z = 15$

✕ $\boxed{\frac{25}{8}}$

Solution or Explanation

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Resources

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6. [– / 5 Points] DETAILS

SCalcET9M 14.7.EI.002.

Concept

Three functions are graphed along with their contour maps. For each surface, use the contour map to identify the approximate locations of critical points. Then look at the graph to classify each critical point as a local maximum or minimum or saddle point. Afterward, you can verify your results analytically.

Instructions

Simulation

Exercise

Solution or Explanation

(a) The critical points are (0, 0) and $\left(0, \frac{1}{2}\right)$, where (0, 0) is a local minimum and $\left(1, \frac{1}{2}\right)$ is a saddle point.

The level curves near (0, 0) are oval in shape which indicates that this point is a local maximum or minimum. By looking at the graph of the function, we see that it is a local minimum. The level curves near $\left(1, \frac{1}{2}\right)$ resemble hyperbolas. Therefore, $\left(1, \frac{1}{2}\right)$ is a saddle point.

(b) The estimates in part (a) were found accurately. In order to find the critical points of the function $z = 2x^2 - x^3 + 2y^2 - 2xy$, we have to find the solutions to the following equations:

$\frac{d}{dx}z = 4x - 3x^2 - 2y = 0, \quad \frac{d}{dy}z = 4y - 2x = 0.$

The solutions are called critical points, which are (0, 0) and $\left(1, \frac{1}{2}\right)$. The next step is to evaluate the function

$D = z_{xx}z_{yy} - (z_{xy})^2 = (4 - 6x)(4) - (-2)^2 = 12 - 24x$ for all the critical points.

Note that $D(0, 0) = 12 > 0$ and $z_{xx}(0, 0) = 4 > 0$. Therefore, by the second derivative test, (0, 0) is a local minimum. At the point $\left(1, \frac{1}{2}\right)$ we have $D\left(1, \frac{1}{2}\right) = -12 < 0$. Therefore, $\left(1, \frac{1}{2}\right)$ is a saddle point.

7. [– / 10 Points] DETAILS

SCalcET9M 14.8.AE.005.

Video Example

EXAMPLE 5 Find the maximum value of the function $f(x, y, z) = x + 2y + 7z$ on the curve of intersection of the plane $x - y + z = 1$ and the cylinder $x^2 + y^2 = 1$.

SOLUTION We maximize the function $f(x, y, z) = x + 2y + 7z$ subject to the constraints $g(x, y, z) = x - y + z = 1$ and $h(x, y, z) = x^2 + y^2 = 1$. The Lagrange condition is $\nabla f = \lambda \nabla g + \mu \nabla h$, so we solve the equations

(1) $1 = \lambda + 2x\mu$

(2) $2 = -\lambda + 2y\mu$

(3) ✗ 7 $= \lambda$

(4) $x - y + z = 1$

(5) $x^2 + y^2 = 1$.

Putting $\lambda =$ ✗ 7 [from (3)] in (1), we get $2x\mu =$ ✗ -6, so $x = -3/\mu$. Similarly, (2) gives $y = 9/(2\mu)$. Substitution in (5) then gives

$\frac{9}{\mu^2} +$ ✗ $\frac{81}{4\mu^2}$ $= 1$

and so $\mu^2 =$ ✗ 117/4, $\mu = \pm\sqrt{117}/2$. Then

$x = \mp$ ✗ $\frac{2}{\sqrt{13}}$, $y = \pm$ ✗ $\frac{3}{\sqrt{13}}$, and, from (4),

$z = 1 - x + y = 1 \pm$ ✗ $\frac{5}{\sqrt{13}}$. The corresponding values of f are

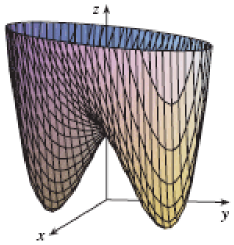
$\mp\frac{6}{\sqrt{117}} + 2\left(\pm\frac{9}{\sqrt{117}}\right) + 7\left(1 \pm \frac{15}{\sqrt{117}}\right) = 7 \pm$ ✗ $3\sqrt{13}$.

Therefore the maximum value of f on the given curve is

✗ $7 + 3\sqrt{13}$.

8. [– / 13 Points] DETAILS

SCalcET9M 14.7.AE.003.



Video Example

EXAMPLE 3 Find the local maximum and minimum values and saddle points of $f(x, y) = x^4 + y^4 - 4xy + 1$.

SOLUTION We first locate the critical points:

$f_x =$

✗ $4x^3 - 4y$ $f_y = 4y^3 - 4x$.

Setting these partial derivatives equal to 0, we obtain the equations

✗ $x^3 - y$ $= 0$

and

$y^3 - x = 0$.

To solve these equations we substitute $y = x^3$ from the first equation into the second one. This gives

$0 = x^9 - x = x(x^8 - 1)$

$= x($ ✗ $x^4 - 1$ $)(x^4 + 1)$

$= x($ ✗ $x^2 - 1$ $)(x^2 + 1)(x^4 + 1)$

so there are three real roots: $x = 0, 1,$ ✗ -1 . The three critical points are $(0, 0)$, $(1, 1)$, and $(-1, -1)$. Next we calculate the second partial derivatives and $D(x, y)$:

f_{xx} =

✗ $12x^2$

f_{xy} =

✗ -4

f_{yy} =

✗ $12y^2$

$f_{xx}f_{yy} - (f_{xy})^2$ =

$D(x, y)$ =

✗ $144x^2y^2 - 16$

Since $D(0, 0) = -16 < 0$, it follows from the Second Derivative Test that the origin is a saddle point; that is, f has no local maximum or minimum at $(0, 0)$.

Since $D(1, 1)$ = ✗ 128 > 0 and $f_{xx}(1, 1) = 12 > 0$, we see that $f(1, 1) = -1$ is a local ---Select--- ✗ minimum .

Similarly, we have $D(-1, -1)$ = ✗ 128 > 0 and $f_{xx}(-1, -1) = 12 > 0$, so $f(-1, -1) = -1$ is also a local ---Select--- ✗ minimum .

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9. [–/1 Points] DETAILS

SCalcET9M 14.7.054.

The base of an aquarium with given volume V is made of slate and the sides are made of glass. If slate costs nine times as much (per unit area) as glass, find the dimensions of the aquarium that minimize the cost of the materials. (Let x, y, and z be the dimensions of the aquarium. Enter your answer in terms of V.)

$(x, y, z) = \Bigg($

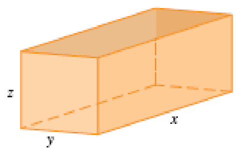
✗ $\dfrac{\sqrt[3]{2}\sqrt[3]{V}}{3^{2/3}}, \dfrac{\sqrt[3]{2}\sqrt[3]{V}}{3^{2/3}}, \dfrac{3\sqrt[3]{3}\sqrt[3]{V}}{2^{2/3}}\Bigg)$

Solution or Explanation

Click to View Solution

10. [–/11 Points] DETAILS

SCalcET9M 14.7.AE.006.



Video Example

EXAMPLE 6 A rectangular box without a lid is to be made from 48 m^2 of cardboard. Find the maximum volume of such a box.

SOLUTION Let the length, width, and height of the box (in meters) be x, y, and z, as shown in the figure. Then the volume of the box is

$V = xyz.$

We can express V as a function of just two variables, x and y by using the fact that the area of the four sides and the bottom of the box is

$2xz +$ ✗ $2yz$ $+ xy = 48.$

Solving this equation for z, we get $z =$ ✗ $\dfrac{48 - xy}{2(x+y)}$, so the expression for V becomes

$V(x, y) =$ ✗ $\dfrac{48xy - x^2y^2}{2(x+y)}$.

We compute the partial derivatives.

$\dfrac{\partial V}{\partial x} =$ ✗ $\dfrac{y^2(48 - 2xy - x^2)}{2(x+y)^2}$

$\dfrac{\partial V}{\partial y} =$ ✗ $\dfrac{x^2(48 - 2xy - y^2)}{2(x+y)^2}$

If V is a maximum, then $\partial V/\partial x = \partial V/\partial y = 0$, but $x = 0$ or $y = 0$ gives $V =$ ✗ 0, so we must solve the equations

$48 - 2xy - x^2 = 0$

$48 - 2xy - y^2 = 0.$

These imply that $x^2 = y^2$ and so $x = y$. (Note that x and y must both be positive in this problem.) If we put $x = y$ in either equation we get $48 - 3x^2 = 0$, which gives $x = 4$, $y = 4$, and $z = (48 - 4 \cdot 4)/[2(4 + 4)] =$ ✗ 2. We could use the Second Derivatives Test to show that this gives a local maximum of V, so it must occur when $x = 4$, $y = 4$, and $z =$ ✗ 2.

Then $V = 4 \cdot 4 \cdot$ ✗ 2 $=$ ✗ 32, so the maximum volume of the box is ✗ 32 m^3.