1. $If\ z = e^x \sin y$,where $x = st^3$ and $y = s^3 t$, $find\ \frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$.

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s} = (e^x \sin y)(t^3) + (e^x \cos y)\ (3ts^2)$$
$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t} = (e^x \sin y)(3st^2) + (e^x \cos y)(s^3)$$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s} = \left(e^{st^3} \sin st^3\right)(t^3) + \left(e^{st^3} \cos s^3 t\right)(3st^2)$$
$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t} = \left(e^{st^3} \sin s^3 t\right)(3\mathrm{s}\mathrm{t}^2) + \left(e^{st^3} \cos s^3 t\right)(s^3)$$

1 points for partial derivative, 1 point for chain rule formula,1 point for substitute x,y into s,t

2. Let $F(x, y, z) = \ln(x) + zy + z^3 - 1$ and suppose $F(x, y, z) = 0$ defines $z = f(x, y)$ near the point $P(\frac{1}{e}, 1, 1)$, find $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$

1 point

$$F_z = y + 3z^2,\ F_y = z,\ F_x = \frac{1}{x}$$

1 point

$$F_z(P) = 4, F_y(P) = 1, F_x(P) = e$$

2 point,

$$\left.\frac{\partial z}{\partial x}\right|_P = -\frac{F_x}{F_z} = -\frac{e}{4} \qquad \left.\frac{\partial z}{\partial y}\right|_P = -\frac{F_y}{F_z} = -\frac{1}{4}$$

3. Let z = cos(xy), find gradient at point P = $(\frac{\sqrt{\pi}}{2}, \frac{\sqrt{\pi}}{2})$, and find the directional derivative in the direction of u (1,1)

for all $(x, y) \in D$ then $\nabla Q = (P_y, R_x)$.

$\boxed{\frac{\partial z}{\partial x} = -\sin(xy)\cdot y \qquad \frac{\partial z}{\partial y} = -\sin(xy)\cdot x}$

1 point

$\boxed{\nabla z(x,y) = (-\sin(xy)\, y,\ -\sin(xy)\cdot x) \qquad \nabla z\left(\frac{\sqrt{\pi}}{2}, \frac{\sqrt{\pi}}{2}\right) = \left\langle -\frac{\sqrt{2\pi}}{4},\ -\frac{\sqrt{2\pi}}{4}\right)}$

0.5 point

$u = (1,1)$, normalize $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$, 0.5 point.

$D_u z(P) = -\frac{\sqrt{2\pi}}{2}\cdot\frac{\sqrt{\pi}}{2} - \frac{\sqrt{2\pi}}{2}\cdot\frac{\sqrt{\pi}}{2} = -\frac{\sqrt{\pi}}{2}$

formula for directional derivative 0.5 point

answer 0.5 point