ZJU-UIUC Institute

Fall Semester 2025
Worksheet 8

Calculus III (Math 241)

Problem 1. Consider the function

$$f(x, y) = x^2 + 4y^2 - 4xy + 2.$$

(a) Find all critical points of f and show that there are infinitely many of them.

(b) Compute

$$D(x, y) = f_{xx}(x, y)f_{yy}(x, y) - \big(f_{xy}(x, y)\big)^2$$

and verify that $D = 0$ at every critical point.

(c) Show that f has a local *and* absolute minimum at each critical point.

Problem 2. Consider the problem of minimizing the function

$$f(x, y) = x$$

subject to the constraint

$$g(x, y) = y^2 + x^4 - x^3 = 0,$$

which describes a curve called a *piriform.*

(a) Use Lagrange multipliers formally to try to solve the problem. (Set up and solve the equations coming from $\nabla f = \lambda \nabla g$.)

(b) Show directly that the minimum value of f on the curve is $f(0, 0) = 0$, but that the Lagrange multiplier condition

$$\nabla f(0, 0) = \lambda \nabla g(0, 0)$$

cannot hold for any λ.

(c) Explain why the method of Lagrange multipliers fails to find the minimum value in this case.