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MTH 112 Midterm Exam Spring 2015

A 3x5 notecard is allowed. A non-graphing calculator is allowed.
The exam is worth **100** points.

1. (5 points) A bicycle with a 24-inch wheel (diameter) travels a distance of 400 feet. How many revolutions does the wheel make (to the nearest revolution)?

   (a) 58 revolutions

   (b) 64 revolutions

   (c) 122 revolutions

   (d) 128 revolutions

   (e) none of the above

2. (5 points) Find the exact value of $\sec\left(-\frac{2\pi}{3}\right)$.

   (a) $\frac{\sqrt{3}}{3}$

   (b) $-\frac{2\sqrt{3}}{3}$

   (c) $-\frac{\sqrt{3}}{2}$

   (d) 2

   (e) none of the above

   $-\frac{2\pi}{3} = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ ↑ cos

   $\sec = -2$

3. (5 points) Find the exact value of $\sin(\theta)$, if θ is an angle in standard position whose terminal side contains the point: $(-5, 9)$.

   (a) $\frac{5}{\sqrt{106}}$

   (b) $\frac{\sqrt{106}}{9}$

   (c) $\frac{9}{\sqrt{106}}$

   (d) $-\frac{9}{5}$

   (e) none of the above

   ↑ cos ↑ sin

   $-5^2 + 9^2 = 106$

   $c = \sqrt{106}$

   $\sin\theta = \frac{9}{\sqrt{106}}$

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4. (5 points) Which of the following are undefined for $\theta = \frac{\pi}{2}$.

(a) Both $\tan(\theta)$ and $\csc(\theta)$ are undefined
(b) Both $\cot(\theta)$ and $\sec(\theta)$ are undefined
(c) Both $\tan(\theta)$ and $\cot(\theta)$ are undefined
(d) Both $\cot(\theta)$ and $\csc(\theta)$ are undefined
(e) Both $\tan(\theta)$ and $\sec(\theta)$ are undefined

$(0, 1)$

$\sin = 1$

$\cos = 0$

$\csc = \frac{1}{1} = 1$

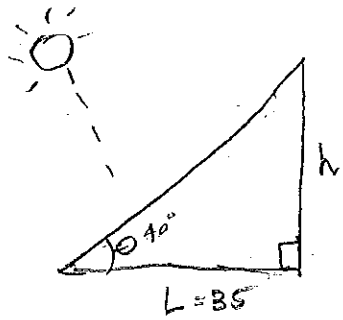
$\sec = \frac{1}{0} \rightarrow$ undefined

$\tan = \frac{1}{0} \rightarrow$ undefined

$\cot = \frac{0}{1} = 0$

5. (5 points) The sun casts a shadow of length L on a tree. The angle of elevation of the suns rays from the base of the tree is θ. What is the height of the tree?

(a) $L\cot(\theta)$
(b) $\frac{\tan(\theta)}{L}$
(c) $\frac{\cot(\theta)}{L}$
(d) $\frac{L}{\cot(\theta)}$.
(e) none of the above



$L \cdot \tan\theta = \frac{h}{L} \cdot L$

$h = \tan\theta \, L$

$h = \tan 10^\circ (35)$

$h = 29.3685$

$\frac{L}{\cot\theta} = \frac{35(\tan 10^\circ)}{\frac{1}{\tan 10^\circ}} \left(\tan 10^\circ\right)$

$L = 35(\tan 10^\circ)$

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6. (5 points) Suppose $u > 0$, simplify

$$\cos\left(\arctan(u)\right).$$

(Hint: Draw a picture.)

cos arctan(-1) = $\frac{-\sqrt{2}}{2}$

(a) $\sqrt{1-u^2}$

(b) $\dfrac{1}{\sqrt{1+u^2}}$

(c) $\sqrt{u^2-1}$

(d) $\dfrac{u}{\sqrt{1+u^2}}$

(e) none of the above

7. (5 points) If $\cot(\theta) > 0$ and $\sin(\theta) < 0$ then θ is in quadrant

(a) I

(b) II

(c) III

(d) IV

(e) cannot be determined from the given information

cot = $\frac{\cos}{\sin}$ →

$\dfrac{-\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = \sqrt{3}$

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8. Suppose θ is an angle in standard position in quadrant IV with terminal side on the line $14x + 10y = 0$.

(a) (5 points) Find $\sec(\theta)$.

$\sin = \frac{-7}{\sqrt{74}}$

$\cos = \frac{5}{\sqrt{74}}$

$\sec = \frac{\sqrt{74}}{5}$

$14x + 10y = 0$ $\quad mx+b=y$

IF $x = 5$, $y = -7$

$y = 14(5) + 10y$

$0 = 70 + 10y$

$10y \quad -10y$

$\frac{-10y}{-10} = \frac{70}{-10}$

$y = -7$

$5^2 + (-7)^2 = \sqrt{74}$

$c = \sqrt{74}$

(b) (5 points) Find $\tan(\theta)$.

$\frac{\frac{-7}{\sqrt{74}}}{\frac{5}{\sqrt{74}}} = -\frac{7}{5}$

(c) (5 points) Find the angle θ in radians.

$\frac{7\pi}{14}$

9. (15 points) Simplify $\sin(\theta)\cos^2(\theta)\csc(\theta)\tan(\theta)$.

$\sin\theta \cos^2\theta \frac{1}{\sin(x)} \cdot \theta \frac{\sin}{\cos\theta}$

$\left(\frac{\cos}{\cos}\right)\frac{\cos}{\sin(x)} \frac{\sin}{\cos} \cdot \theta\left(\frac{\sin}{\sin}\right) = \frac{\cos\theta \sin^2\theta}{\sin\cos}$

$= 1$

$\sin\theta \cos^2\theta \frac{\cos\theta \sin^2\theta}{\sin\cos} \rightarrow \sin\theta \frac{\cos\theta}{\sin\theta\cos\theta} = \sin\theta\csc\theta$

$[\sin\theta\csc\theta]$

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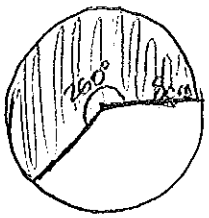
10. (15 points) Find the *exact* area of the sector of the circular wedge if the central angle is 260° and the radius is 8 cm.



$A = \frac{1}{2} r^2 \theta$

$A = \frac{1}{2}(8)^2 \left(\frac{260^\circ}{180}\pi\right)$

$A = \frac{1}{2}(8)^2 \left(\frac{13}{9}\pi\right)$

$A = 32\left(\frac{13\pi}{9}\right)$

$\left[A = \frac{416}{9}\ cm^2\ \pi\right] = 46\,cm^2\,\pi$

11. Let

$$f(x) = -2\sin\left(2x - \frac{\pi}{6}\right) - 4.$$

$f(x) = -2\sin 2\left(x - \frac{\pi}{12}\right) - 4$

(a) (5 points) Find the amplitude f.

Amplitude $= [2]$

(b) (5 points) Find the phase shift of f.

phase shift $= \left[\frac{\pi}{12}\right]$

(c) (5 points) Find the period of f.

$$\frac{2\pi}{b} = \frac{2\pi}{2} = [\pi]$$

(d) (5 points) What are the largest and smallest values of $f(x)$?

x can have $[-\infty, \infty]$ values.