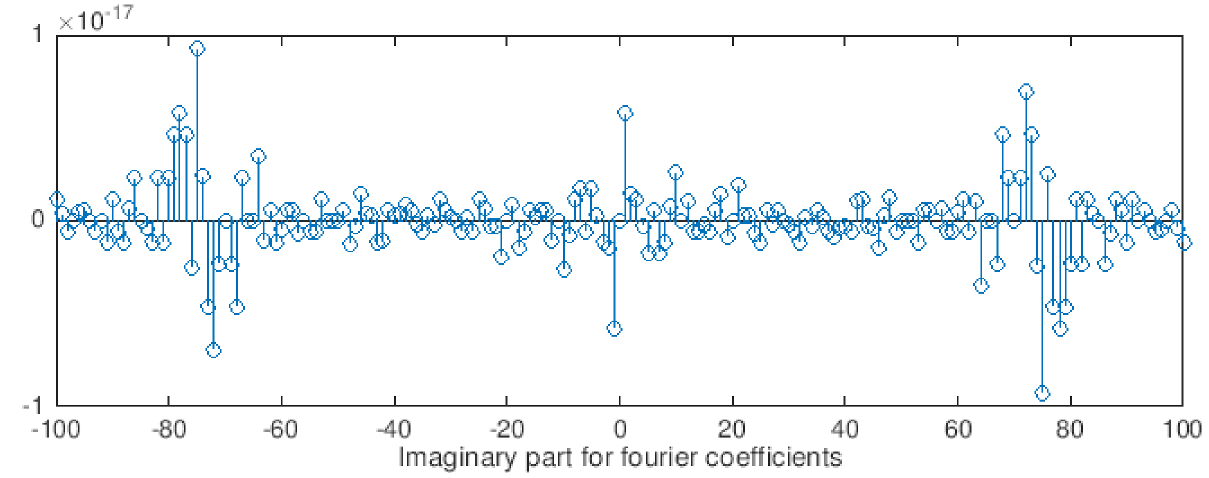
Computer Assignment - 03 - Spring 2019
Signals & Systems
- P . Venkatesh
- S20180010135

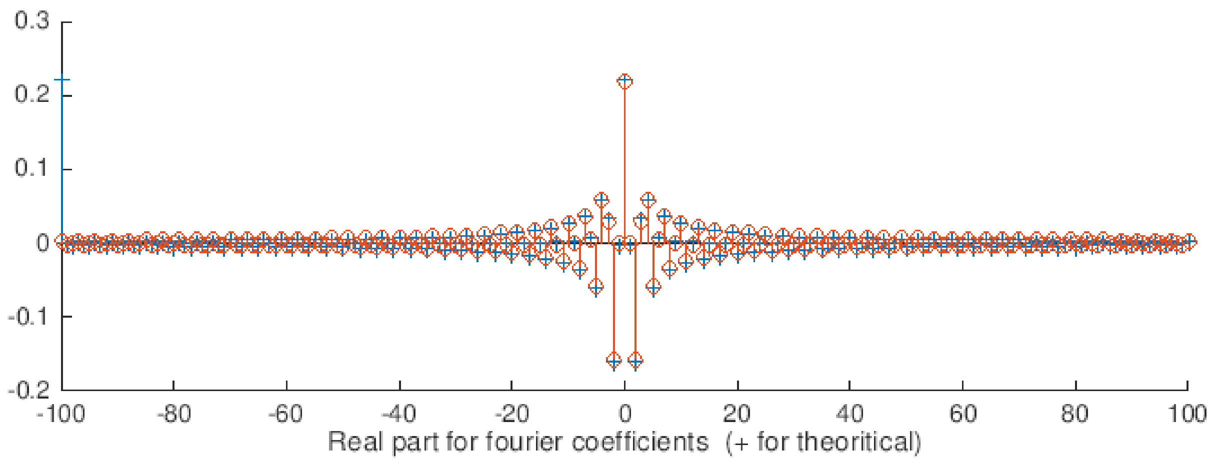
Analysis and Synthesis & Convergence

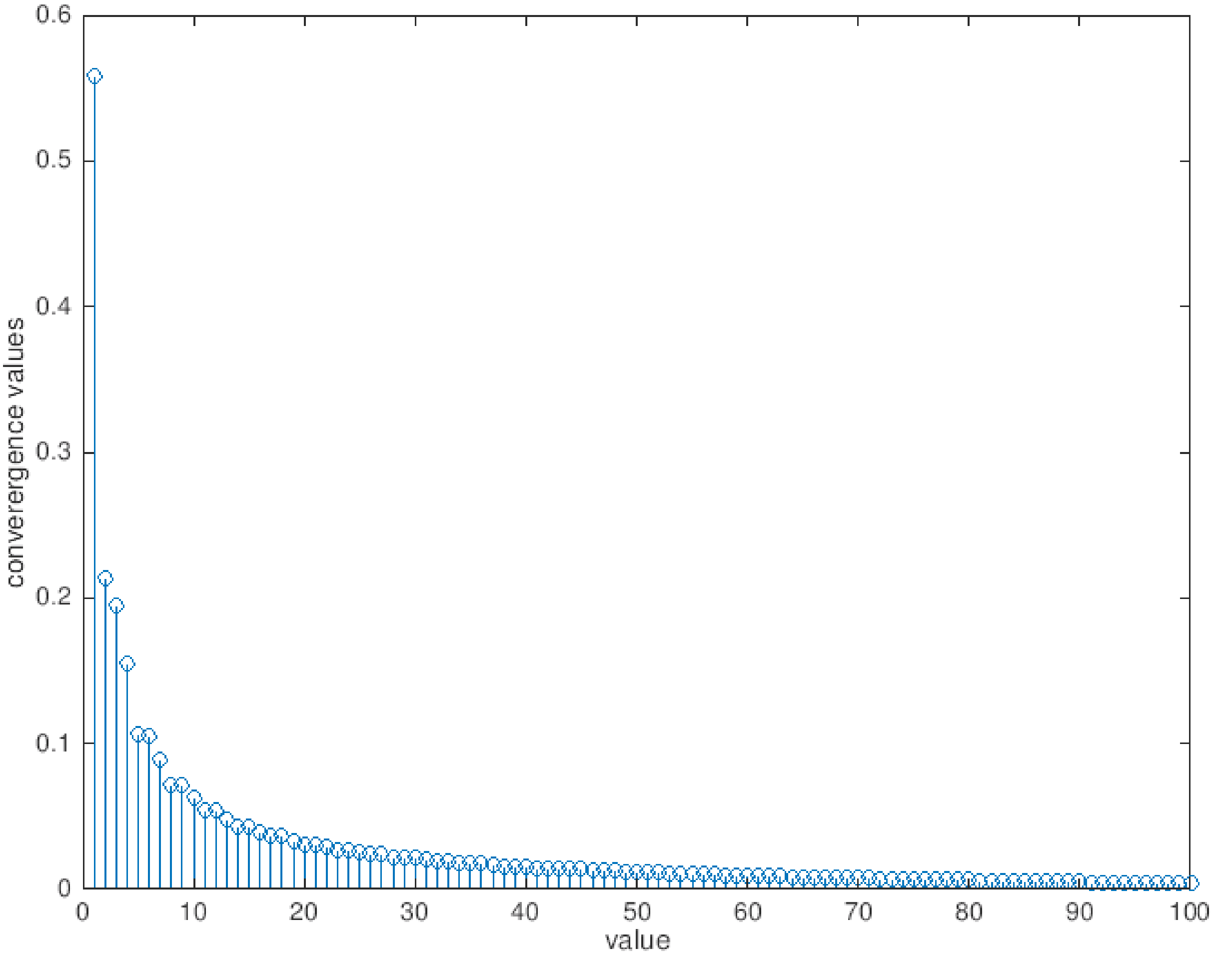
1a & 2a) **$x(t) = t^2$ $|t| < 1$**

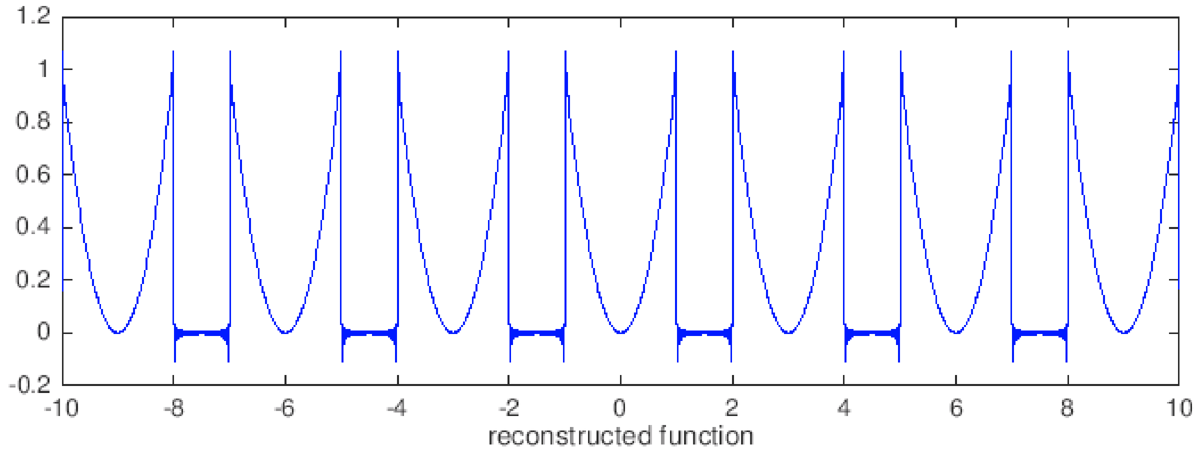
```
clear;
clc;
close all
%%
global W0 tval T0;
T0 = 3;
W0 = 2*pi/T0;
tval = -10:0.01:10;
k = -100:100;
c = f_coeff(abs(k(1)));
N=floor(abs(tval(1))/T0);
x = zeros(size(tval));
for i = -N:N
x = x + xin(tval+i*T0);
end
theor = 1/T0*((2*sin(k*W0)./(k*W0))-(4*sin(k*W0)./(k.^3*W0.^3))+(4*cos(k*W0)./
(k.^2*W0.^2)));
theor((size(k)+1)/2)=2/9;
figure(1)
subplot(2,1,1)
stem(k,imag(c));
xlabel('Imaginary part for fourier coefficients');
subplot(2,1,2)
hold on
stem(k,theor,'+')
stem(k,real(c),'o');
xlabel('Real part for fourier coefficients (+ for theoritical)');
for i=1:(length(k)-1)/2
conver(i) = (1/T0)*trapz(tval,(x-re_con(i)).^2);
end
figure(2)
subplot(2,1,1)
plot(tval,re_con(abs(k(1))),'b');
xlabel('reconstructed function');
subplot(2,1,2)
plot(tval,x,'r');
xlabel('original function');
figure(3)
stem(1:(length(k)-1)/2,conver);
xlabel('value');
ylabel('converergence values');
function x = xin(t)
x = zeros(size(t));
x(t>-1 & t<1) = t(t>-1 & t<1).^2;
```

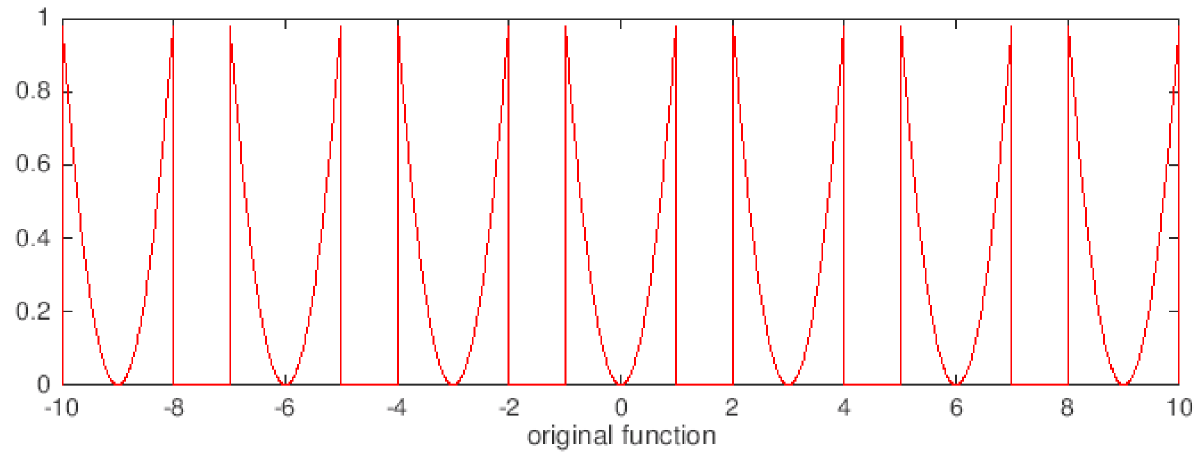
```
end
function x1 = re_con(m)
global W0 tval;
temp = f_coeff(m);
x1 = zeros(size(tval));
k = -m:m;
for i=1:2*m+1
x1 = x1+exp(1i*W0.*k(i)*tval)*temp(i);
end
end
function coeff = f_coeff(k)
global W0 tval T0;
k1 = -k:k;
coeff = zeros(size(k1));
for i = 1:length(k1)
basis = exp(-1i*W0.*k1(i)*tval);
coeff(i) = (1/T0)*trapz(tval,xin(tval).*basis);
end
end
```






converergence values
value


reconstructed function


original function

$$x(t) = t^2 \qquad |t| < 1$$

$$a_k = \frac{1}{T_0}\int x(t)\, e^{-jk\omega_0 t}\, dt$$

$$= \frac{1}{3}\int_{-1}^{1} t^2 e^{-jk\omega_0 t}\, dt$$

$$= \frac{1}{3}\left[\frac{t^2 e^{-jk\omega_0 t}}{-jk\omega_0} - \int_{-1}^{1} \frac{2t\, e^{-jk\omega_0 t}}{-jk\omega_0}\, dt\right]$$

$$= \frac{1}{3}\left[\frac{t^2 e^{-jk\omega_0 t}}{-jk\omega_0} - \left(\frac{2}{-jk\omega_0}\right)\int t\, e^{-jk\omega_0 t}\, dt\right]$$

$$= \frac{1}{3}\left[\frac{t^2 e^{-jk\omega_0 t}}{-jk\omega_0} + \frac{2}{jk\omega_0}\left[\frac{t\, e^{-jk\omega_0 t}}{-jk\omega_0}\right] - \frac{2}{jk\omega_0}\left[\int \frac{e^{-jk\omega_0 t}}{-jk\omega_0}\, dt\right]\right]$$

$$= \frac{1}{3}\left[\frac{t^2 e^{-jk\omega_0 t}}{-jk\omega_0} + \frac{t\, e^{-jk\omega_0 t}}{(k\omega_0)^2} + \frac{2}{(jk\omega_0)^2}\,\frac{e^{-jk\omega_0 t}}{-jk\omega_0}\right]$$

$$= \frac{1}{3}\left[\frac{1}{-jk\omega_0}\left[e^{-jk\omega_0} - e^{jk\omega_0}\right] + \frac{2}{(k\omega_0)^2}\left[e^{-jk\omega_0} + e^{jk\omega_0}\right] + \frac{2j}{(k\omega_0)^3}\left[e^{-jk\omega_0} - e^{jk\omega_0}\right]\right]$$

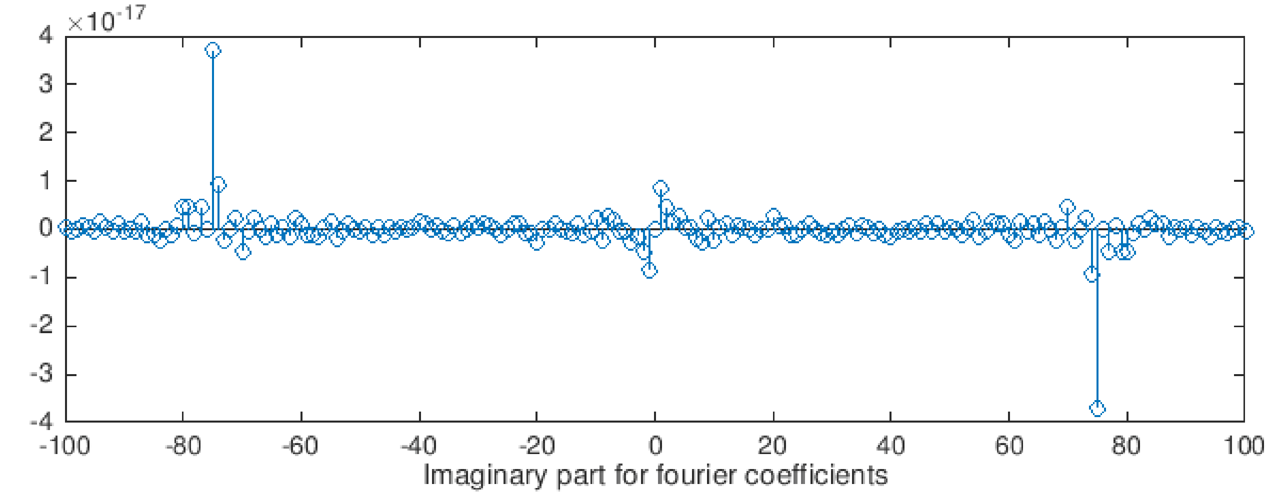
$$= \frac{1}{3}\left[\frac{j}{k\omega_0}\left[-2j\sin k\omega_0\right] + \frac{2}{(k\omega_0)^2}\left(2\cos k\omega_0\right) - \frac{2j}{(k\omega_0)^3}\left(-2j\sin k\omega_0\right)\right]$$

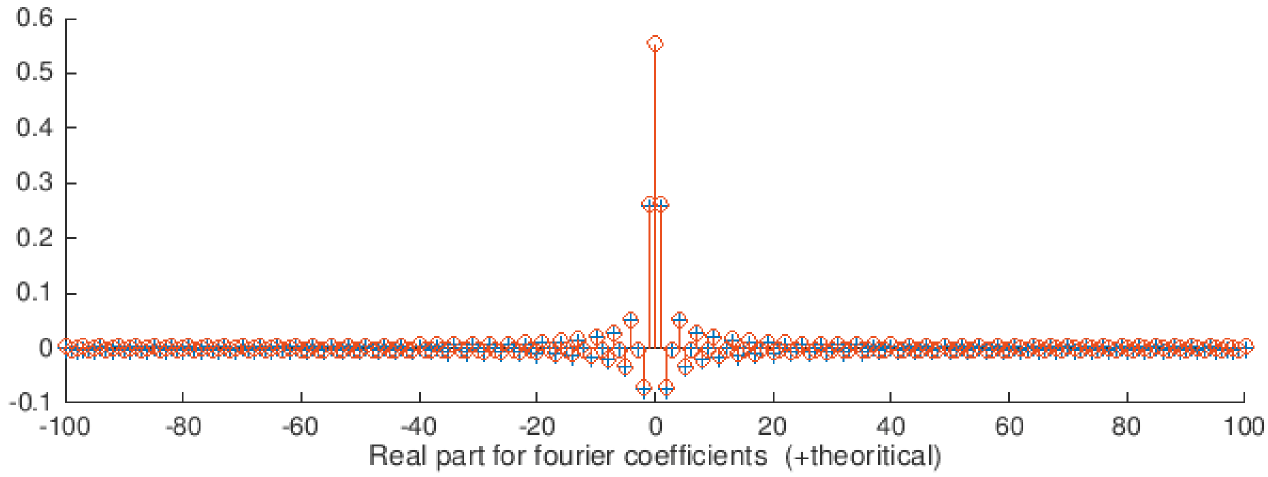
$$= \frac{1}{3}\left[\frac{2\sin k\omega_0}{k\omega_0} + \frac{4\cos k\omega_0}{(k\omega_0)^2} + \frac{4\sin k\omega_0}{(k\omega_0)^3}\right]$$

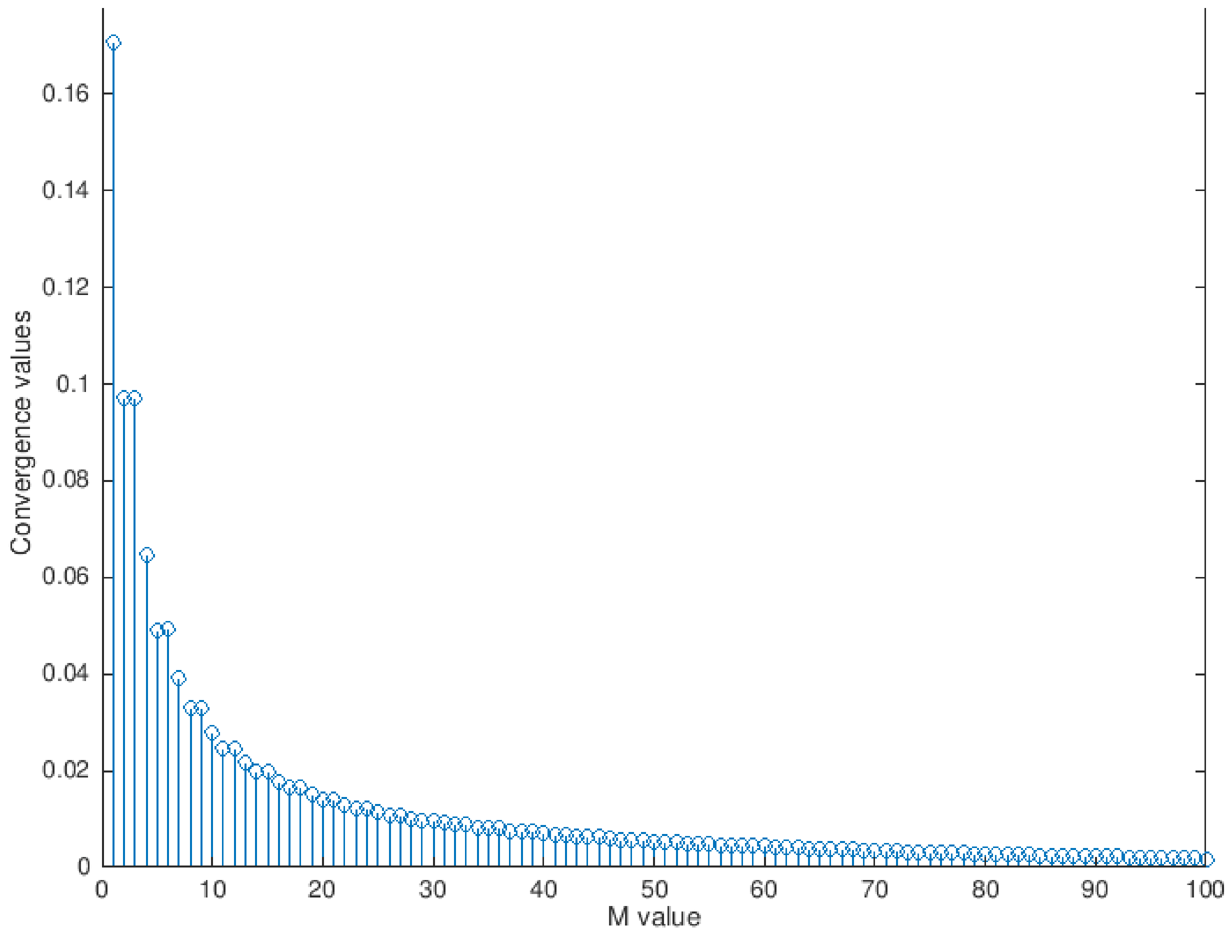
1b & 2b) **$x(t) = 1 - |t|/3$** **$|t| < 1$**

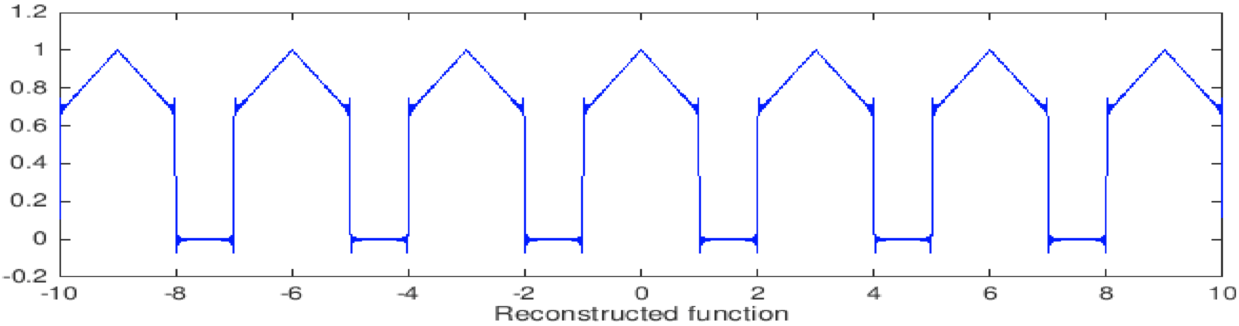
```
clear;
clc;
close all
%%
global W0 tval T0;
T0 = 3;
W0 = 2*pi/T0;
tval = -10:0.01:10;
k = -100:100;
c = f_coeff(abs(k(1)));
N=floor(abs(tval(1))/T0);
x = zeros(size(tval));
for i = -N:N
x = x + xin(tval+i*T0);
end
```

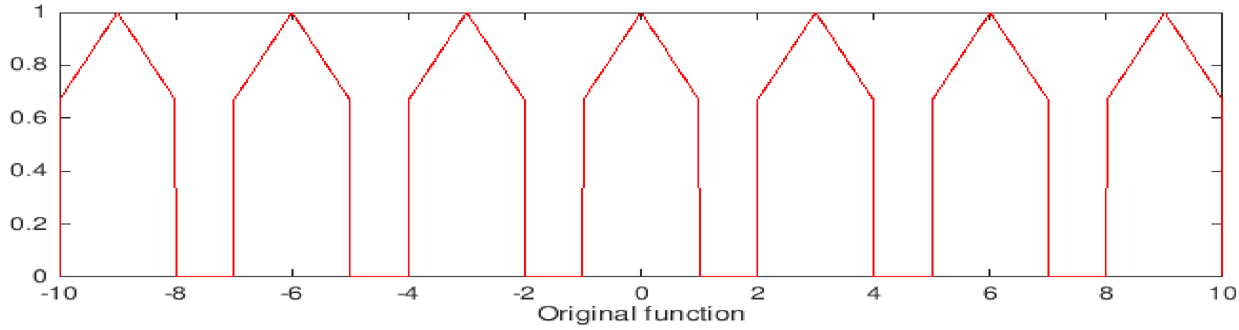
```
c_th = (1/T0)*((((4/3).*sin(k*W0))./(k*W0)) ...
        - (((2/3)*cos(k*W0))./((k*W0).^2)) ...
        + 2./(3*(k*W0).^2));
theor((size(k)+1)/2)=2/3;

figure(1)
subplot(2,1,1)
stem(k,imag(c));
xlabel('Imaginary part of fourier coefficients');
subplot(2,1,2)
hold on
stem(k,c_th,'+')
stem(k,real(c),'o');
xlabel('Real part of fourier coefficients +theoritical');
for i=1:(length(k)-1)/2
conv(i) = (1/T0)*trapz(tval,(x-re_con(i)).^2);
end
figure(2)
subplot(2,1,1)
plot(tval,re_con(abs(k(1))),'b');
xlabel('Regenerated function');
subplot(2,1,2)
plot(tval,x,'r');
xlabel('Original function');
figure(3)
stem(1:(length(k)-1)/2,conv);
xlabel('M value');
ylabel('Convergence values');
function x = xin(t)
x = zeros(size(t));
x(t>-1 & t<1) = 1-abs(t(t>-1 & t<1)./3);
end
function x1 = re_con(m)
global W0 tval;
temp = f_coeff(m);
x1 = zeros(size(tval));
k = -m:m;
for i=1:2*m+1
x1 = x1+exp(1i*W0.*k(i)*tval)*temp(i);
end
end
function coeff = f_coeff(k)
global W0 tval T0;
k1 = -k:k;
coeff = zeros(size(k1));
for i = 1:length(k1)
basis = exp(-1i*W0.*k1(i)*tval);
coeff(i) = (1/T0)*trapz(tval,xin(tval).*basis);
end
end
```


×10^-17
4
3
2
1
0
-1
-2
-3
-4
-100
-80
-60
-40
-20
0
20
40
60
80
100
Imaginary part for fourier coefficients


0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-100
-80
-60
-40
-20
0
20
40
60
80
100
Real part for fourier coefficients (+theoritical)


Convergence values
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0
10
20
30
40
50
60
70
80
90
100
M value





$$x(t) = 1 - \left|\frac{t}{3}\right| \qquad |t| < 1$$

$$a_k = \frac{1}{T_0} \int x(t)\, e^{-jk\omega_0 t}\, dt$$

$$= \frac{1}{T_0}\left[\int_{-1}^{0} \left(1+\frac{t}{3}\right) e^{-jk\omega_0 t}\, dt + \int_{0}^{1} \left(1-\frac{t}{3}\right) e^{-jk\omega_0 t}\, dt\right]$$

$$= \frac{1}{T_0}\left[\int_{-1}^{0} e^{-jk\omega_0 t}\, dt + \frac{1}{3}\int_{-1}^{0} t\, e^{-jk\omega_0 t}\, dt + \int_{0}^{1} e^{-jk\omega_0 t}\, dt - \frac{1}{3}\int_{0}^{1} t\, e^{-jk\omega_0 t}\, dt\right]$$

$$= \frac{1}{T_0}\left[\int_{0}^{1} \left(e^{jk\omega_0 t} + e^{-jk\omega_0 t}\right) dt - \frac{1}{3}\int_{0}^{1} t\left(e^{jk\omega_0 t} + e^{-jk\omega_0 t}\right) dt\right.$$

$$= \frac{1}{T_0}\left[2\int_{0}^{1} (\cos k\omega_0 t)\, dt - \frac{2}{3}\int_{0}^{1} t \cos(k\omega_0 t)\, dt\right]$$

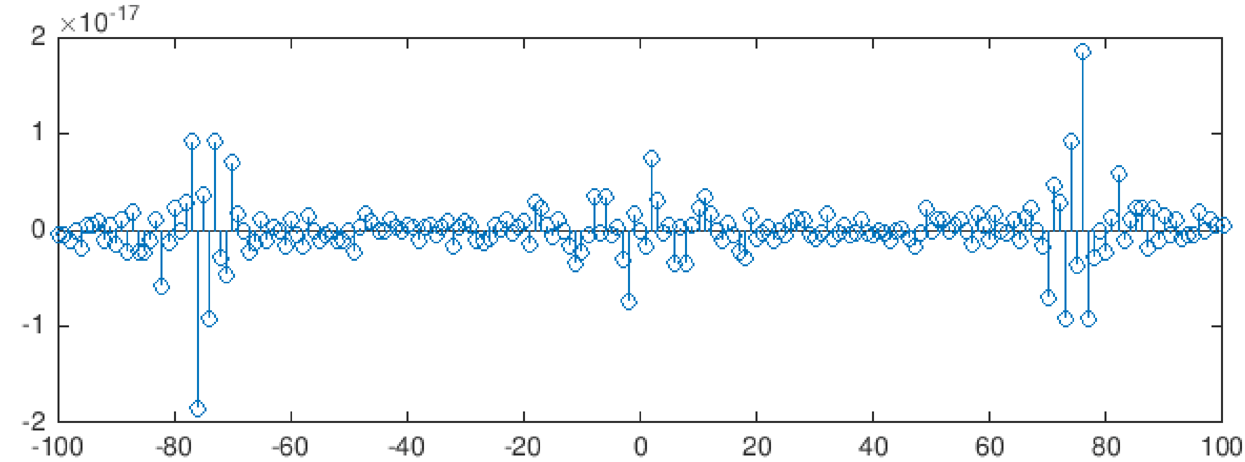
$$= \frac{1}{T_0}\left[\frac{2 \sin k\omega_0}{k\omega_0} - \frac{2}{3}\left[\frac{\sin k\omega_0}{k\omega_0} - \int_{0}^{1} \frac{\sin k\omega_0 t}{k\omega_0}\, dt\right]\right]$$

$$= \frac{1}{T_0}\left[\frac{2 \sin k\omega_0}{k\omega_0} - \frac{2}{3}\frac{\sin k\omega_0}{k\omega_0} - \frac{2\cos(k\omega_0)}{3(k\omega_0)^2} + \frac{2}{3(k\omega_0)^2}\right]$$

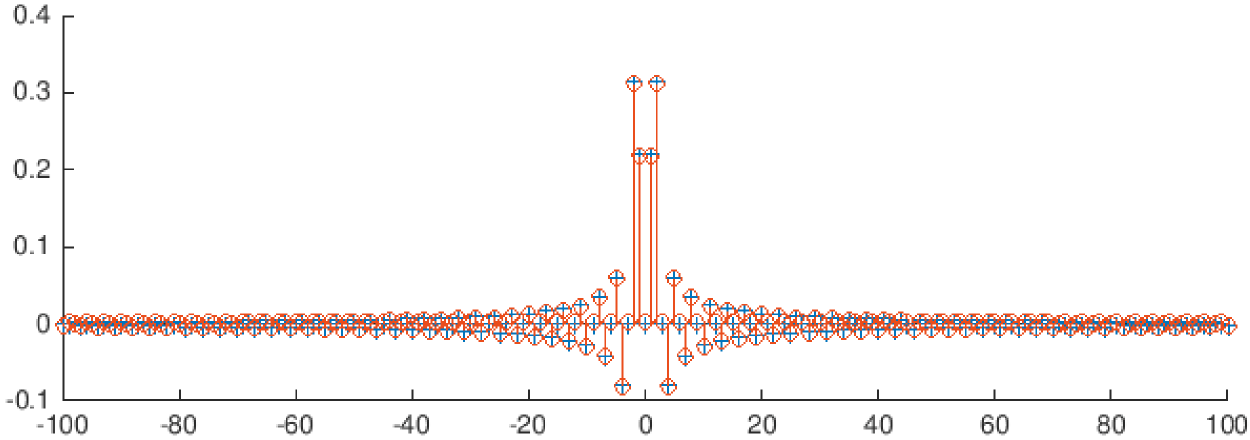
1c & 2c) **$x(t) = \cos(\pi t)$** **$|t| < 1$**

```
clear;
clc;
close all
%%
global W0 tval T0;
T0 = 3;
W0 = 2*pi/T0;
tval = -10:0.01:10;
k = -100:100;
c = f_coeff(abs(k(1)));
N=floor(abs(tval(1))/T0);
x = zeros(size(tval));
for i = -N:N
x = x + xin(tval+i*T0);
end
theor = ((2/3)*(sin(k*W0).*(k*W0)./(pi^2-(k*W0).^2)));
theor(size((k)+1)/2)=0;
figure(1)
subplot(2,1,1)
stem(k,imag(c));
xlabel('Imaginary part of fourier coefficients');
subplot(2,1,2)
hold on
stem(k,theor,'+')
stem(k,real(c),'o');
xlabel('Real part of fourier coefficients + theoritical');
for i=1:(length(k)-1)/2
conv(i) = (1/T0)*trapz(tval,(x-re_con(i)).^2);
end
figure(2)
subplot(2,1,1)
plot(tval,re_con(abs(k(1))),'b');
xlabel('Regenerated function');
subplot(2,1,2)
plot(tval,x,'r');
xlabel('Original function');
figure(3)
stem(1:(length(k)-1)/2,conv);
xlabel('M value');
ylabel('Convergence values');
function x = xin(t)
x = zeros(size(t));
x(t>-1 & t<1) = cos(pi*t(t>-1 & t<1));
end
function x1 = re_con(m)
global W0 tval;
temp = f_coeff(m);
x1 = zeros(size(tval));
```

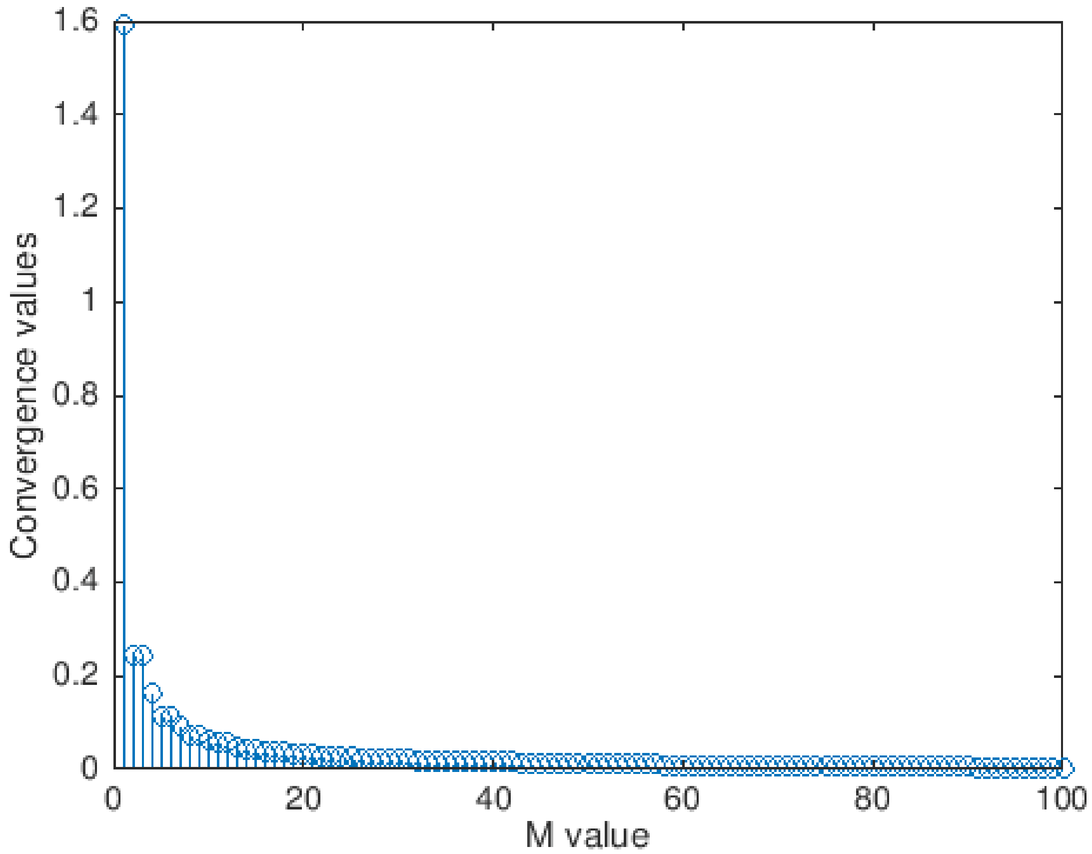
```
k = -m:m;
for i=1:2*m+1
x1 = x1+exp(1i*W0.*k(i)*tval)*temp(i);
end
end
function coeff = f_coeff(k)
global W0 tval T0;
k1 = -k:k;
coeff = zeros(size(k1));
for i = 1:length(k1)
basis = exp(-1i*W0.*k1(i)*tval);
coeff(i) = (1/T0)*trapz(tval,xin(tval).*basis);
end
end
```



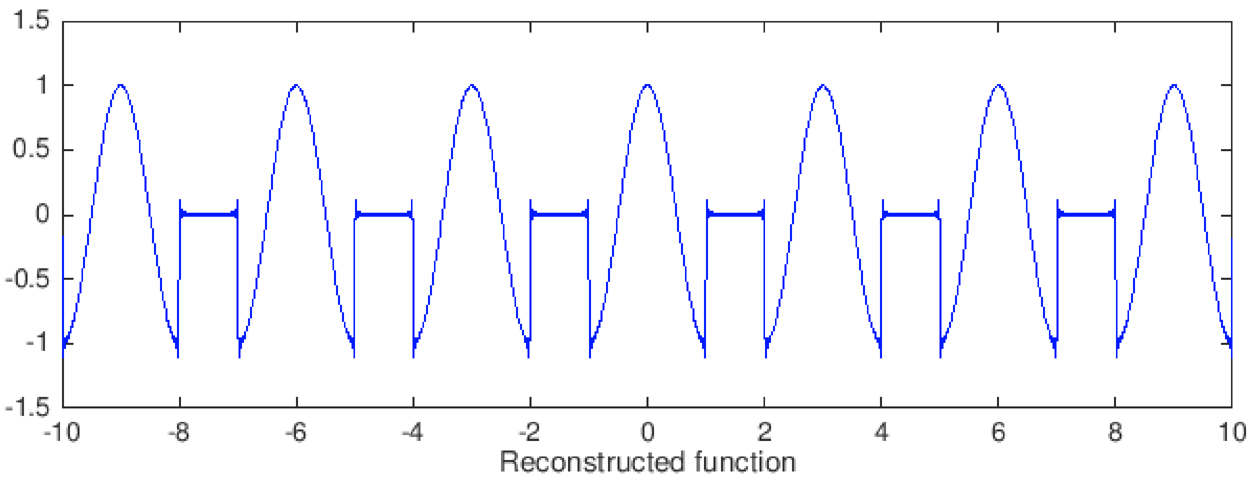
Imaginary part for fourier coefficients



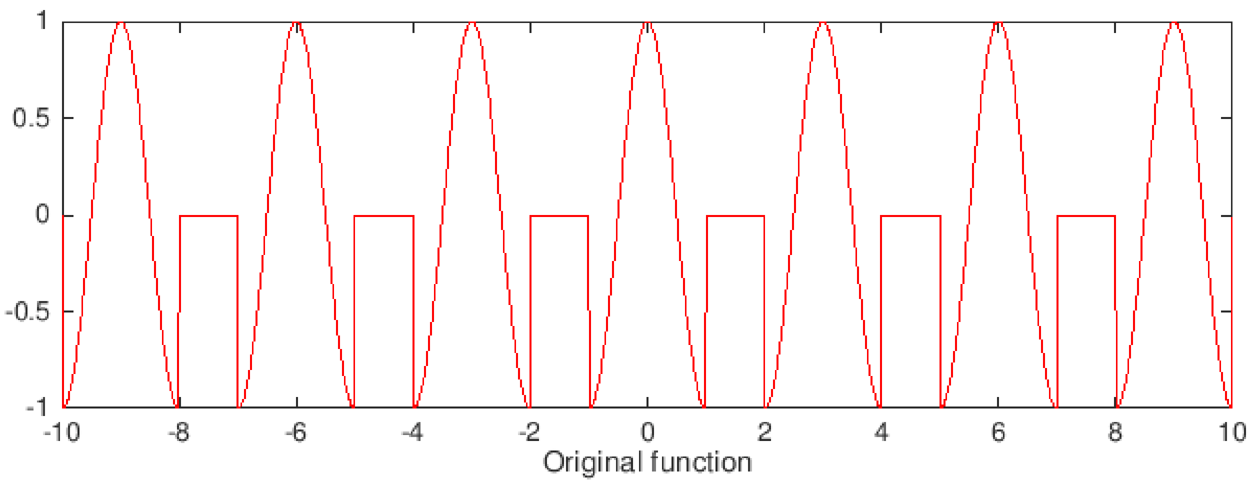
Real part for fourier coefficients (+ theoritical)



Convergence values
M value
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
20
40
60
80
100



1.5
1
0.5
0
-0.5
-1
-1.5
-10
-8
-6
-4
-2
0
2
4
6
8
10
Reconstructed function



1
0.5
0
-0.5
-1
-10
-8
-6
-4
-2
0
2
4
6
8
10
Original function

$x(t) = \cos(\pi t) \qquad |t| < 1$

$$a_k = \frac{1}{T_0}\int x(t)\, e^{-jk\omega_0 t}\, dt$$

$$= \frac{1}{T_0}\int_{-1}^{1} \cos(\pi t)\, e^{-jk\omega_0 t}\, dt$$

$$= \frac{1}{T_0}\int_{-1}^{1} \left(\frac{e^{j\pi t} + e^{-j\pi t}}{2}\right) e^{-jk\omega_0 t}\, dt$$

$$= \frac{1}{T_0}\int_{-1}^{1} \frac{e^{jt(\pi - k\omega_0)} + e^{-jt(\pi + k\omega_0)}}{2}\, dt$$

$$= \frac{1}{2T_0}\int_{-1}^{1} e^{jt(\pi - k\omega_0)}\, dt + \int_{-1}^{1} e^{-jt(\pi + k\omega_0)}\, dt$$

$$= \frac{1}{2T_0}\left[\left[\frac{e^{jt(\pi - k\omega_0)}}{j(\pi - k\omega_0)}\right]_{-1}^{1} + \left[\frac{e^{-jt(\pi + k\omega_0)}}{-j(\pi + k\omega_0)}\right]_{-1}^{1}\right]$$

$$= \frac{1}{2T_0}\left[\left[\frac{e^{j(\pi - k\omega_0)} - e^{-j(\pi - k\omega_0)}}{j(\pi - k\omega_0)}\right] + \left[\frac{e^{-j(\pi + k\omega_0)} - e^{j(\pi + k\omega_0)}}{-j(\pi + k\omega_0)}\right]\right]$$

$$= \frac{1}{2T_0}\left(\frac{2j\sin(\pi - k\omega_0)}{j(\pi - k\omega_0)} + \frac{(-2j)\sin(\pi + k\omega_0)}{-j(\pi + k\omega_0)}\right)$$

$$= \frac{1}{T_0}\left(\frac{\sin k\omega_0}{\pi - k\omega_0} - \frac{\sin k\omega_0}{\pi + k\omega_0}\right)$$

$$= \frac{\sin k\omega_0}{T_0}\left(\frac{\cancel{\pi} + k\omega_0 - \cancel{\pi} + k\omega_0}{\pi^2 - k^2\omega_0^2}\right)$$

$$= \frac{\sin k\omega_0}{T_0}\, \frac{2k\omega_0}{\pi^2 - k^2\omega_0^2}$$

$$\boxed{a_k = \frac{2}{3}\left[\frac{\sin(k\omega_0)\, k\omega_0}{\pi^2 - k^2\omega_0^2}\right]}$$

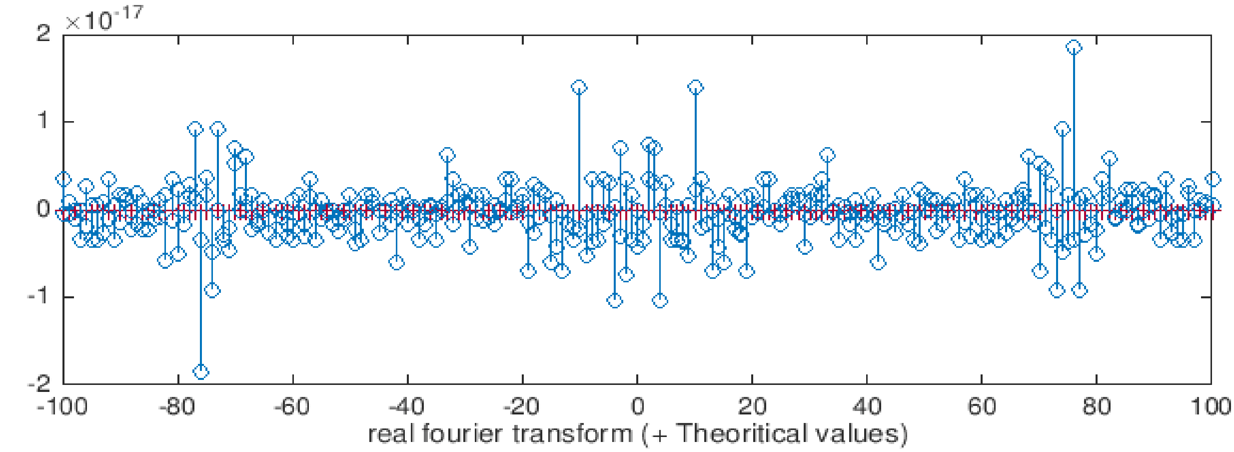
$$a_0 = \frac{1}{3}\int_{-1}^{1} \cos \pi t\, dt$$

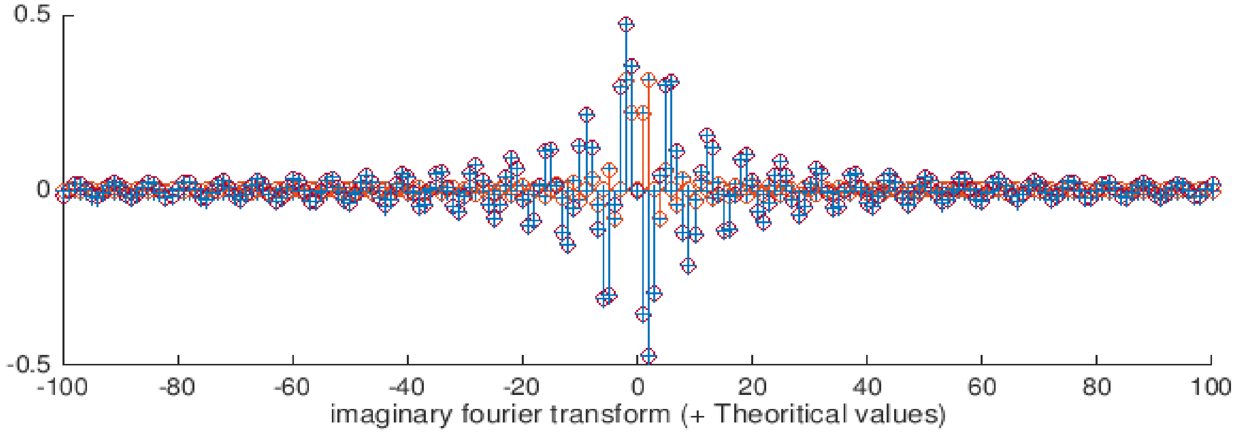
$$= \frac{1}{3}\left[\frac{\sin \pi t}{\pi}\right]_{-1}^{1}$$

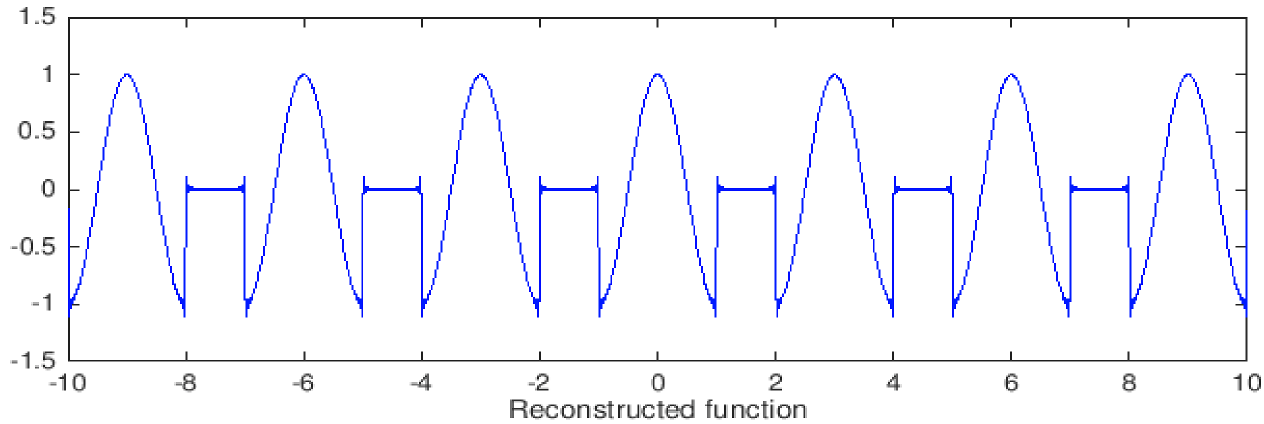
$$\boxed{a_0 = 0}$$

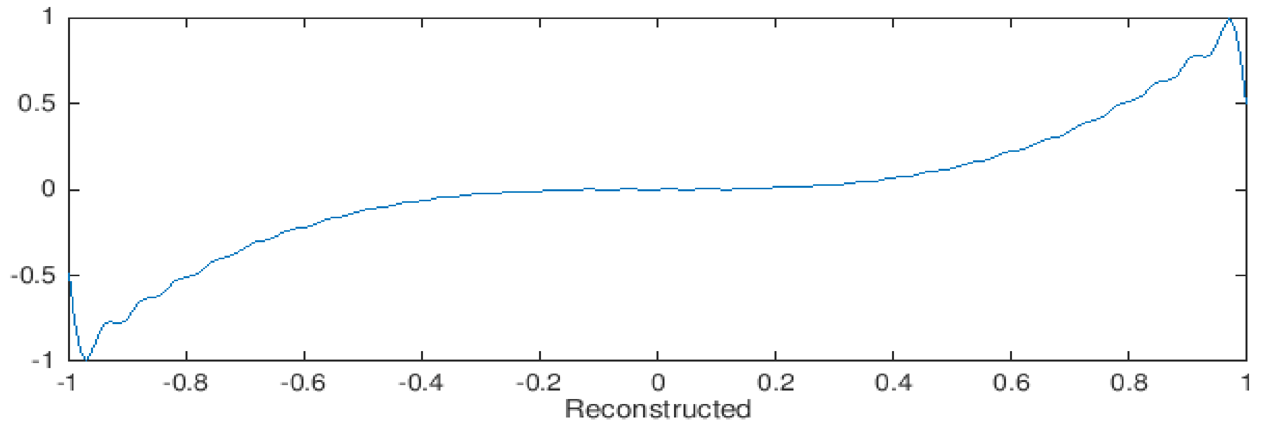
3a)

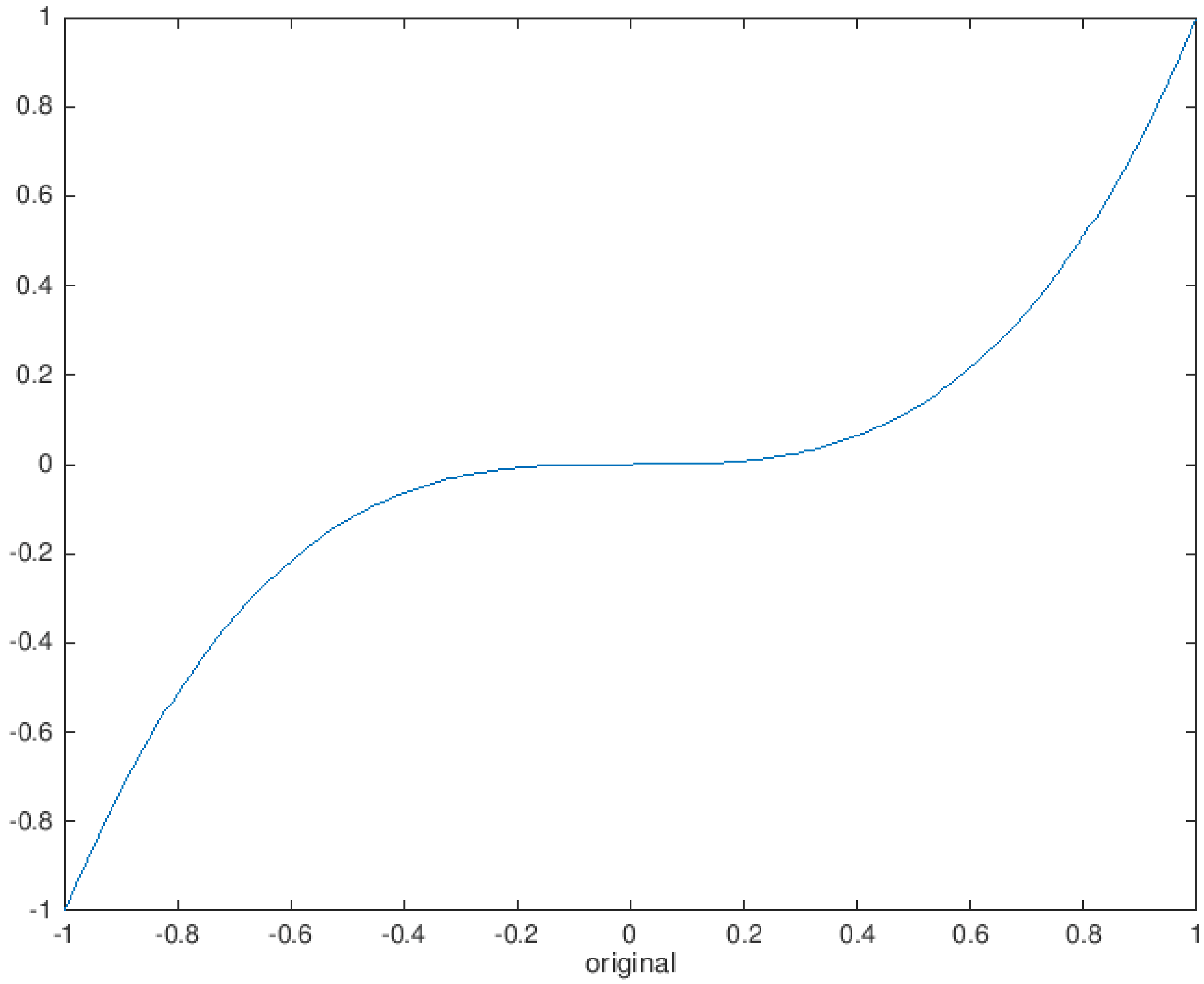
```
clc
clear
close all
%%
tvec = -1:0.01:1;
w = -100:100;
for i = 1:length(w)
xfwd(i) = trapz(tvec,xin(tvec).*exp(-1i*w(i)*tvec));
end
for i = 1:length(tvec)
xrev(i) = (1/(2*pi))*trapz(w,exp(1i*w*tvec(i)).*xfwd);
end
theor = 2*1i*cos(w)./(w) ...
- 6*1i*sin(w)./(w).^2 ...
-12*1i*cos(w)./(w).^3 ...
+12*1i*sin(w)./(w).^4;
theor((size(w)+1)/2) = 0;
figure(1)
subplot(2,1,1)
hold on
stem(w,real(theor),'+');
stem(w,real(xfwd),'o');
xlabel('real fourier transform (+ Theoritical values)');
subplot(2,1,2)
hold on
stem(w,imag(xfwd),'o');
stem(w,imag(theor),'+');
xlabel('imaginary fourier transform (+ Theoritical values)')
figure(2)
plot(tvec,xrev);
xlabel('Reconstructed');
figure(3)
plot(tvec,xin(tvec));
xlabel('original');
function x = xin(t)
x = t.^3;
end
```


real fourier transform (+ Theoritical values)


imaginary fourier transform (+ Theoritical values)


Reconstructed function


Reconstructed



3) a) $x(t) = t^3 \qquad |t|<1$

$$X(\omega) = \int_{-1}^{1} t^3 e^{-j\omega t}\,dt$$

$$= \left[\frac{t^3 \cdot e^{-j\omega t}}{-j\omega}\right]_{-1}^{1} - \frac{3}{-j\omega}\int_{-1}^{1} t^2 e^{-j\omega t}\,dt$$

$$= \frac{e^{-j\omega} - e^{j\omega}}{-j\omega} + \frac{3}{j\omega}\int_{-1}^{1} t^2 e^{-j\omega t}\,dt$$

$$= \frac{-2j\sin\omega}{-j\omega} + \frac{3}{j\omega}\int_{-1}^{1} t^2 e^{-j\omega t}\,dt$$

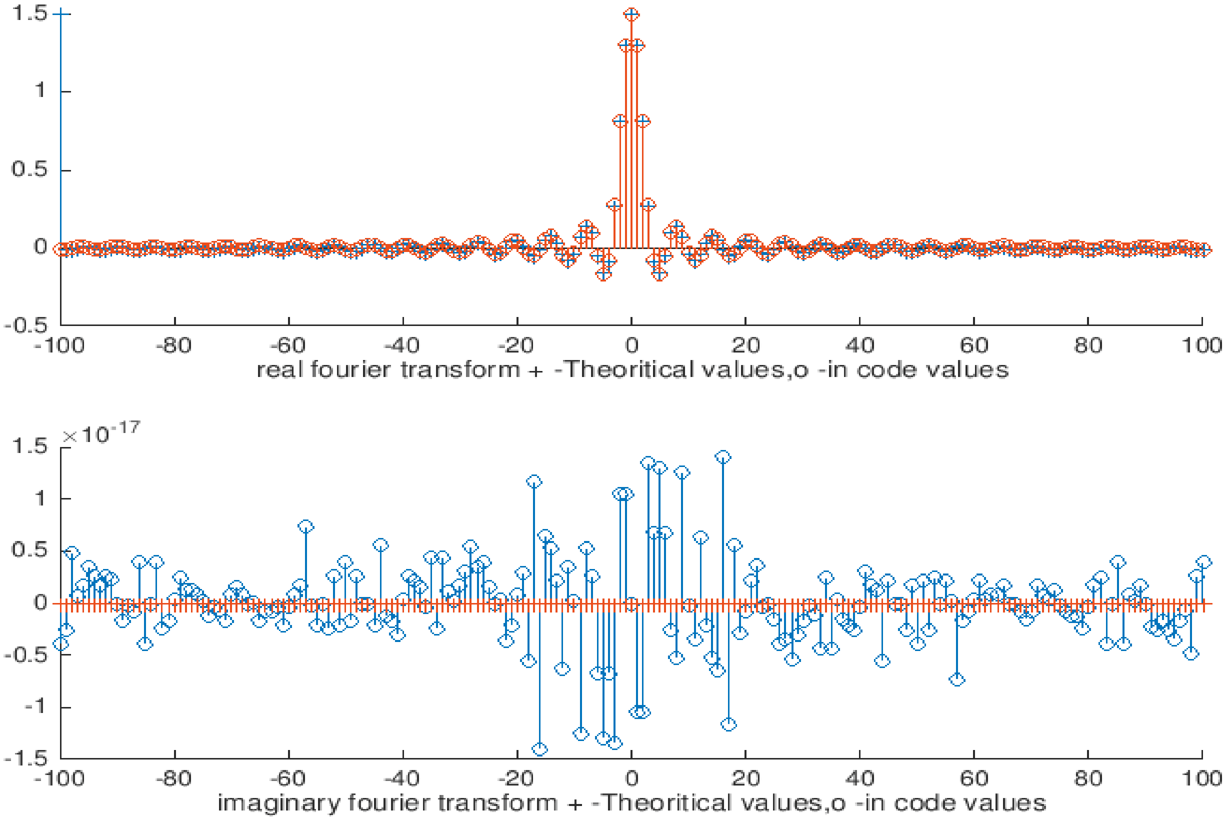
$$= \frac{2\sin\omega}{\omega} + \frac{3}{j\omega}\left[\frac{2\sin\omega}{\omega} + \frac{4\cos\omega}{\omega^2} - \frac{4\sin\omega}{\omega^3}\right]$$

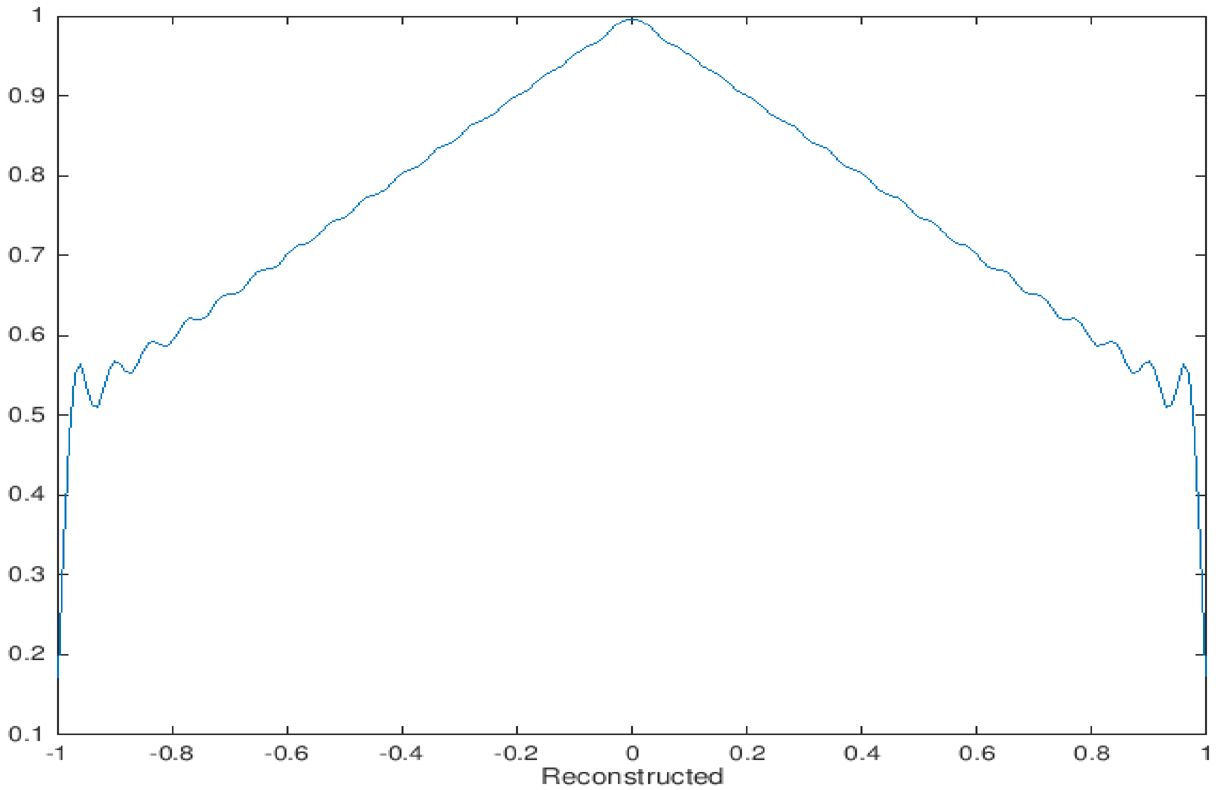
$$= \frac{2\sin\omega}{\omega} + \frac{6\sin\omega}{j\omega^2} + \frac{12\cos\omega}{\omega^3} - \frac{12\sin\omega}{j\omega^4}$$

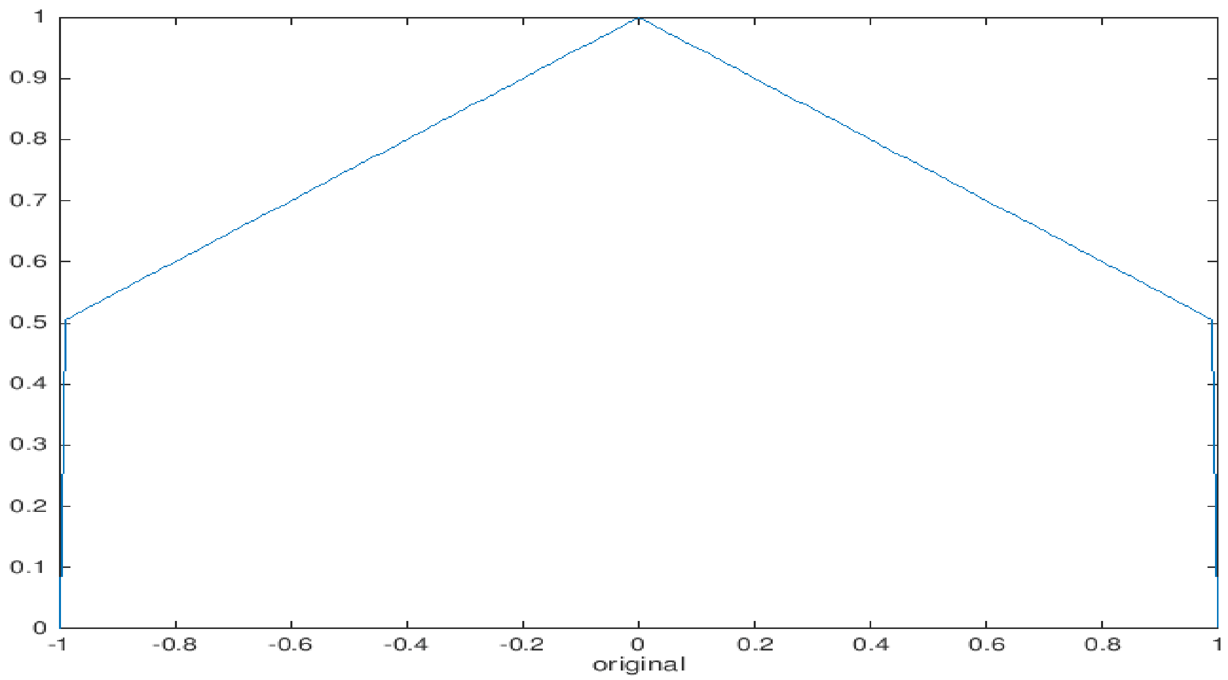
$$= \frac{2\sin\omega}{\omega} - j\left\{\frac{6\sin\omega}{\omega^2} + \frac{12\cos\omega}{\omega^3} - \frac{12\sin\omega}{\omega^4}\right]$$

3b)
clear;

```
clc;
tvec = -1:0.01:1;
w = -100:100;
for i = 1:length(w)
xfwd(i) = trapz(tvec,xin(tvec).*exp(-1i*w(i)*tvec));
end
for i = 1:length(tvec)
xrev(i) = (1/(2*pi))*trapz(w,exp(1i*w*tvec(i)).*xfwd);
end
theor = sin(w)./w-cos(w)./w.^2+1./w.^2;
theor((size(w)+1)/2) = 1.5; %%since as k = 0 theor0 = infinite
figure(1)
subplot(2,1,1)
hold on
stem(w,real(theor),'+');
stem(w,real(xfwd),'o');
xlabel('real fourier transform + -Theoritical values,o -in code values')
subplot(2,1,2)
hold on
stem(w,imag(xfwd),'o');
stem(w,imag(theor),'+');
xlabel('imaginary fourier transform + -Theoritical values,o -in code values')
figure(2)
plot(tvec,xrev);
xlabel('Reconstructed');
figure(3)
plot(tvec,xin(tvec));
xlabel('original');
function x = xin(t)
x = zeros(size(t));
x(t<1 & t> -1)= 1-abs(t(t<1 & t> -1))/2;
end
```


real fourier transform + -Theoritical values,o -in code values
imaginary fourier transform + -Theoritical values,o -in code values


Reconstructed



3) b) $x(t) = 1 - \frac{|t|}{2} \qquad |t| < 1$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$= \int_{-1}^{1} \left(1 - \frac{|t|}{2}\right) e^{-j\omega t} dt$$

$$= \int_{-1}^{0} \left(1 + \frac{t}{2}\right) e^{-j\omega t} dt + \int_{0}^{1} \left(1 - \frac{t}{2}\right) e^{-j\omega t} dt$$

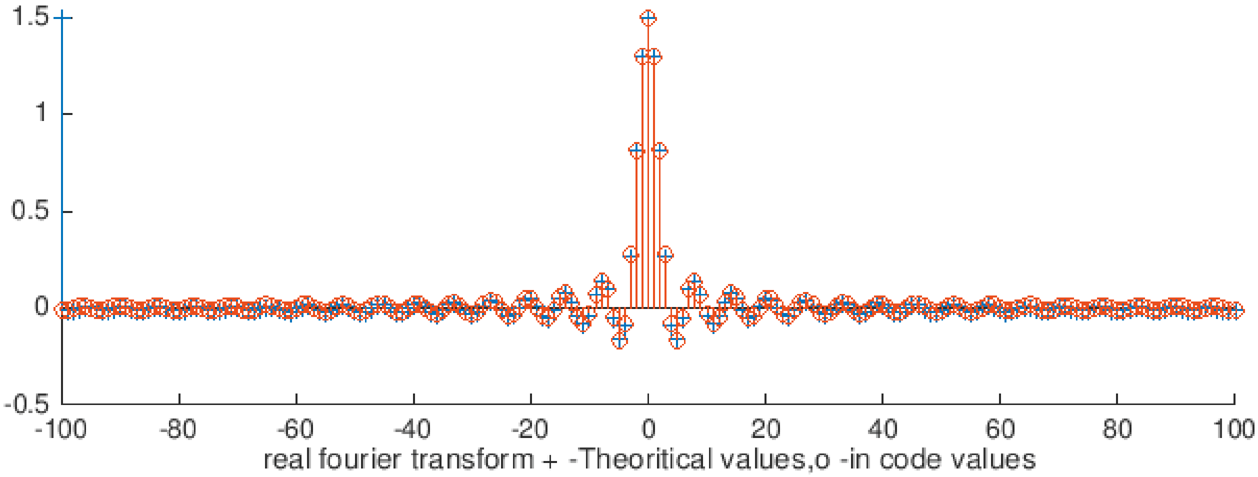
$$= 2 \int_{0}^{1} \cos(\omega t) dt - \frac{2}{2} \int_{0}^{1} \cos(\omega t) t dt$$

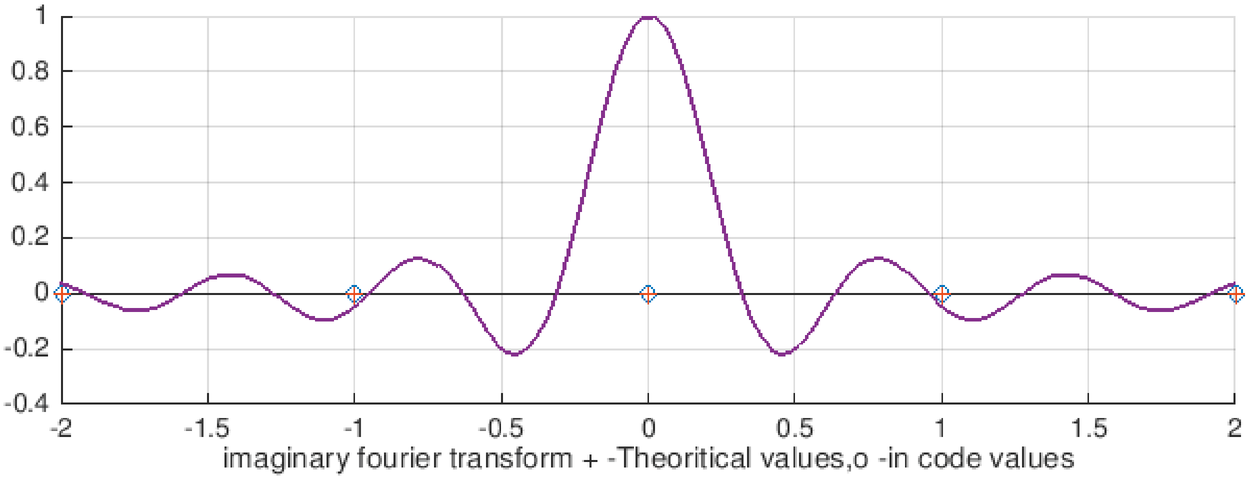
$$= 2 \frac{\sin\omega}{\omega} - \frac{\sin\omega}{\omega} - \frac{\cos\omega}{\omega^2} + \frac{1}{\omega^2}$$

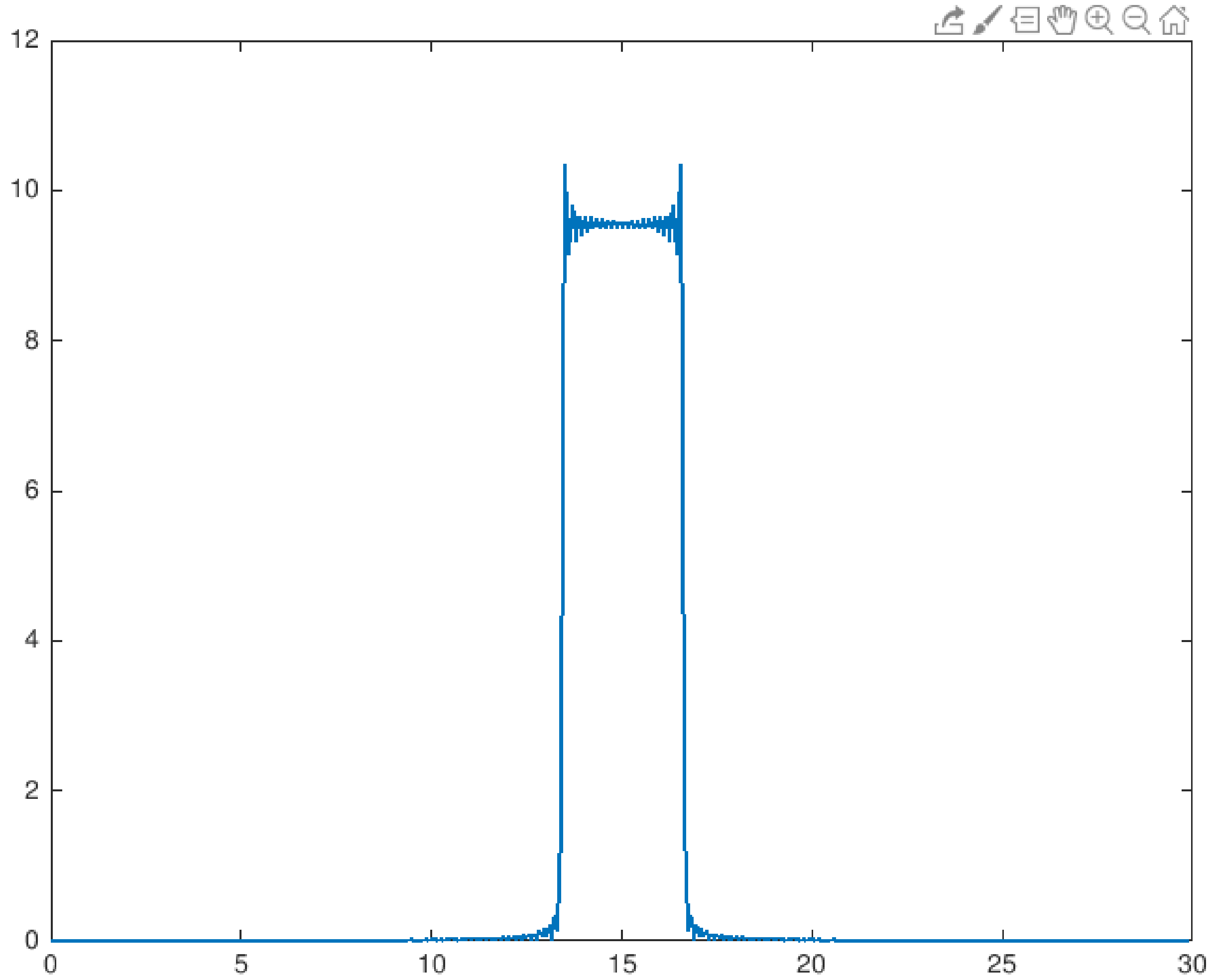
$$X(j\omega) = \frac{\sin\omega}{\omega} - \frac{\cos\omega}{\omega^2} + \frac{1}{\omega^2}$$

3c)

```
Fs=30;
Ts=1/Fs;
t=-2*pi:Ts:2*pi-Ts;
y=sinc(pi*t);
figure(1);
plot(t,y,'linewidth',1.5); grid on;
xlim([-2 2]);

N=600;
fy=(fft(y,N));
figure(2);
fr=(0:N-1)*Fs/N;
plot(fr,fftshift(abs(fy)),'linewidth',1.5);
```







12
10
8
6
4
2
0
0
5
10
15
20
25
30