Predicting Cognitive Biases with Prisoner's Dilemma Games

Jessica Sorrell

December 15, 2014

Abstract

This paper investigates a possible explanation for two ubiquitous cognitive biases: overconfidence bias and certainty bias. We use a prisoner's dilemma model of human interaction with variable parameters representing players' certainties to demonstrate that high degrees of certainty may confer a survival benefit in a competitive environment. The model used and the mechanics of the simulation are described, and the results are reported.

1 Introduction

An overconfidence bias has been well-documented in psychology literature. Adams & Adams (1960) [3] asked subjects to spell difficult words and for each word rate how confident they were that they had spelled the word correctly. They found that subjects overestimated their accuracy by 10% - 15% on average, meaning that they were correct less than 90% of the time they were 100% certain of being correct. Hsu et al. (2005) [1] replicated results from earlier studies [6] indicating that subjects preferred risky gambles to ambiguous gambles, even when the expected values of the gambles were equivalent. The subjects were given two opportunities for betting and told to choose one. In the first gamble, they were told they would win $5 if they correctly guessed the color of a randomly selected card from a deck of 40 cards in which 20 were red and 20 were blue. In the second case, they would win $5 if they correctly guessed the color of a randomly selected card from a deck of red and blue cards for which the proportion of red and blue cards was unknown. Subjects typically chose the first bet, indicating a preference for bets in which the risk is specified. These results have been consistently borne out by similar experiments [2].

There are many potential explanations for the prevalence of any cognitive bias. A bias may arise from normative social influence. It may also become established if it confers some survival benefit, consequently becoming a dominant pattern of behavior through natural selection. This project investigates the plausibility of the latter hypothesis. We model human interactions in a closed population with iterated prisoner's dilemma games, observing the conditions under which players with above-average certainty eventually dominate the environment.

1.1 Prisoner's Dilemma Games

A prisoner's dilemma game is a two-player game with the following constraints:

1. Each player is given an option to cooperate with the other player or defect against it.
2. Each player must make its decision without knowing what the other player has decided.
3. If both players choose to cooperate, they both suffer some small loss.

4. If both players choose to defect, they both suffer a moderate loss.
5. If one player cooperates and the other defects, the cooperating player suffers a large loss and the defecting player suffers no loss.

In iterated prisoner's dilemma games, players engage in several rounds of games and may modify their strategies based on the outcomes of previous games. For instance, if a player observes that other players are mostly cooperative, it may then elect to cooperate in the next round of games to minimize the group's loss, or it may elect to defect to minimize its own loss.

Our simulation consists of iterated prisoner's dilemma games played by a large number of players with different degrees of certainty in their decisions. A single round proceeds as follows:

1. Each player assesses the friendliness of its environment.
2. Each player competes in a prisoner's dilemma game with each of its neighbors, making decisions based on the friendliness of its environment.
3. Payoffs from these games are dealt out as damage to all players. Players who have suffered more than a specified amount of damage are removed from the board and replaced with a player that is identical or very similar to a nearest neighbor, chosen at random.
4. Players' memories of their environment are updated to reflect the most recent round of games.

If players with a particular attribute tend to dominate the population after several rounds of this simulation, we might infer that the shared attribute is responsible for that dominance. We may then further infer that this attribute increases the fitness of a player under natural selection.

2 The Model

In order to model human behavior, we make many of the same assumptions made in Axelrod (1981) [4], namely that general human interactions are similar to those in the standard prisoner's dilemma. This assumption is based on the observation that humans are social animals and that in the Paleolithic environment there were likely many opportunities for cooperation and defection that would yield payoffs similar to those in prisoner's dilemma games. For instance, a decision to "defect" and its corresponding payoff may be a generalization of an early human's decision to steal food from another human. Similarly, two people cooperating in order to hunt prey may decrease their chances of going hungry, but sharing the meal may leave neither individual well-fed.

2.1 Decision Function

To gain insight into how attributes of interest will ultimately affect survival rates, we use a sigmoid function to determine players' decisions [5].

$$f(x) = \frac{1}{1 + e^{a-bx}}$$

Here a and b are constant parameters specific to each player. This is an odd function, and when plotted on the Cartesian plane, will have odd symmetry about $x = a/b$ and the line $y = 0.5$. Refer to figure 1 for a comparison of plots for various a, b values.

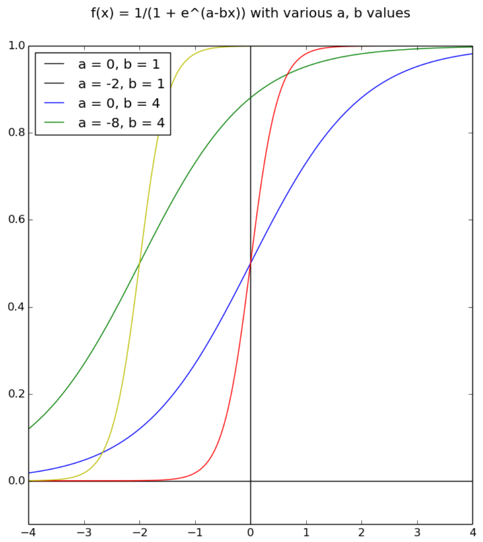

Figure 1: Comparison of decision functions

A decision function dictates a player's behavior in each round. Before a player engages in a round of prisoner's dilemma games, a value q is calculated by that player. This value q represents the player's assessment of the friendliness of other nearby players. Each time a player engages in a game for that round, it runs a random number generator on the interval [0,1]. If the number generated is less than that player's $f(q)$, it will cooperate in the current game. However, if the random number is greater than that player's decision function evaluated at q, the player will defect. This allows for a player to make different decisions in each of its games for a given round, depending upon its function and its environment.

If we consider what the parameters a and b of the decision function imply about a given player's decision algorithm, we see that a can be interpreted as a player's misanthropy, and b can be interpreted as a player's certainty in its decisions to cooperate or defect. Players with negative values of a will, on average, cooperate more than they defect. The parameter b gives the steepness of the logistic curve's slope. Large values of b, then, will yield clear distinctions between the values of q for which a player is likely to cooperate, and values of q for which a player is likely to defect. If a player has a comparatively small value for b, then there is a larger interval of q values for which a player is not significantly more likely to defect than cooperate. This may be interpreted as a player being uncertain about its decisions.

2.2 The Environment

We model the environment with an m x n grid where the values m and n may be determined at runtime. Each player will play a prisoner's dilemma game with any player to its North, South, East, and West. Though the grid is represented by a 2-dimensional array, we allow players that reside at the boundaries of the grid (i.e. at some index (i, 0), (0, i), (i, m-1), or (n-1, i) to play games with the appropriate player at the other edge (i.e. at the corresponding index (i, m-1), (n-1, i), (i, 0), or (0, 1) respectively). This ensures that each player plays the same number of games.

2.3 Memories

The value q, which represents the friendliness of a player's neighbors, is calculated by each player at the beginning of each round. Each player remembers the outcomes of games that it has played with its neighbors, up to some number of rounds that is specified by a "time horizon" parameter. The time horizon may be determined at runtime. If one of a player's four neighbors defected in the last game the player engaged in with this neighbor, the player's memory of this game is encoded as -1. If the neighbor cooperated, this outcome is encoded as $+1$. The player's four memory values for that round are then summed, yielding some integer between -4 and 4, which is the value of the memory for that round. In order to calculate q, we determine the weighted average of all previous memories, weighting recent memories more heavily than earlier memories. This weighted average is q, and will determine the x-value at which the decision function is evaluated.

2.4 Life Cycle

To reflect the effects of cooperation and defection on a player's lifespan, all players begin the simulation with an equal number of "life points". The payoffs of each prisoner's dilemma game are subtracted from a player's life points at the end of each round. When a player's life points fall below 1, that player is removed from the board and replaced with a new player whose parameters are a copy of those from a randomly chosen neighboring player, with a small probability of perturbation to the certainty parameter. For the purposes of this model, the probability of a perturbation to the certainty value is 0.2, and the change in certainty is chosen from a uniform random distribution on the interval [-0.05, 0.05]. The new player is also given a new set of memories, determined by an "optimism" parameter specified at runtime. All starting memories will be set to the value of this parameter.

2.5 Parameters

Since the goal of this project is to model the effect of certainty on survival, the altruism parameter is held constant for all players, but certainty varies between players. If certainty confers some survival benefit, we might expect to see populations approaching an equilibrium of players with large values of b. After some number of iterations, the games will be stopped and the distribution of b values of our population will be observed and interpreted.

The table below provides a list of the model's parameters and their definitions.

Parameter	Definition
m	the number of rows in the players' grid
n	the number of columns in the players' grid
life points	the number of points a player has to lose before it is removed from the board
misanthropy	the value that determines the x-value at which the curve is centered
certainty	the steepness of the decision curve
optimism	the value with which a newborn player's memory array is populated
rounds	the number of iterations that take place before the game is ended
time horizon	the number of prior rounds a player remembers

Figure 2: Table of parameters

3 Results

The simulation detailed above was tested with various parameter values that describe differing social environments. As this simulation did not investigate the outcomes of positive values of misanthropy, the misanthropy parameter will be referred to here as "altruism", indicating negative values of misanthropy. The four simulated environments are defined by

1. No altruism / No optimism (hereafter referred to as the "Hobbes environment"
2. No altruism / Some optimism (hereafter referred to as the "Rand environment")
3. Some altruism / No optimism (hereafter referred to as the "Nietzsche environment")
4. Some altruism / Some optimism (hereafter referred to as the "Tolstoy environment")

In the first definition of the Hobbes environment, all players had altruism and optimism parameters set to 0. In the first Rand environment, all players had altruism parameters set to 0 and optimism set to 2.0. In the first Nietzsche environment, all players had altruism parameters set to 2.0 and optimism parameters set to 0. In the first Tolstoy environment, all players had altruism parameters set to 2.0 and optimism parameters set to 2.0. The survival of players in these environments was analyzed for 3 different values of lifespan: 10, 50, and 100.

When run only a handful of times, the simulation gave very unpredictable results. This is not surprising given the probabilistic nature of the simulation. Small statistical anomalies occurring at the beginning of the simulation can have a profound effect on the end state, and these effects are quite pronounced for a small population. In order to mitigate this difficulty in interpreting results, a testing program was written to run a large number of trials for constant initial conditions. The program tested five distinct initial scenarios within each environment:

1. The certainty values of all players are uniformly random in the range 1.0 and 2.0.

2. The certainty values of most players are uniformly random in the range 1.0 and 2.0, but 25% of players have large certainty values. The deviating certainty values are uniformly random in the range 2.5 to 3.5.

3. The certainty values of most players are uniformly random in the range 1.0 and 2.0, but 25% of players have small certainty values. The deviating certainty values are uniformly random in the range 0.0 to 1.0.

4. The certainty values of half of the players are uniformly random in the range 1.0 and 2.0, but 25% of players have large certainty values, distributed as in the second scenario, and 25% of players have small certainty values distributed, distributed as in the third scenario.

5. The certainty values of half of the players are uniformly random in the range 2.5 to 3.5. The other half of the players have certainty values uniformly distributed in the range 0.0 to 1.0.

The test program then began a set of 100 games with the above parameters for each scenario, for a total of 500 games. The test program collected relevant statistics on the start and end states of each game, and reported a summary of the statistics for a given scenario when all games in that scenario had concluded.

The results of these simulations suggest that higher degrees of certainty may confer some survival benefit within the Hobbes environment, but that uncertainty confers some survival benefit in more altruistic or optimistic environments.

In each of the five scenarios and for each of the three lifespans analyzed in the Hobbes environment, the median certainty of players at the end of the game was greater than the median certainty of players at the start. These results were especially pronounced for lifespans of 10 life points and the scenario in which 50% of players had high degrees of certainty, while the other 50% has small certainties. When players in the Hobbes environment had only 10 life points and were assigned certainties with this bi-modal distribution, by the end of the simulation the statistical summary of the end states of all games showed that on average nearly the entire board had a certainty level greater than the starting mean certainty.

In all other environments the end state certainties were lower than the median certainties in each scenario, but most markedly so in the Tolstoy environment where both altruism and optimism had positive values. For the bi-modal distribution scenario within this environment, and for all life spans in this scenario, on average at least 95% of the end state board had a certainty value smaller than the starting mean.

Once we had verified that optimism and altruism play a significant role in determining the ultimate end state equilibrium of the board, the boundaries of these end states were investigated. The approximate minimum values of altruism and/or optimism necessary for about 50% of simulations to reach high-certainty equilibrium and 50% to reach low-certainty equilibrium were found and are given below in figure 3. More results are given in the appendix, as is a link to a site containing results for a wider array of parameter values than those discussed in the paper.

Environment	Altruism	Optimism	Scenario	Starting Mean/Median	Final Mean/Median
Hobbes	0.0	0.0			
			1	1.50/1.50	1.57/1.55
			2	1.64/1.83	1.80/2.05
			3	1.36/1.28	1.51/1.44
			4	1.50/1.62	1.89/2.06
			5	1.00/1.75	2.9/2.52
Rand	0.0	0.15			
			1	1.50/1.50	1.45/1.46
			2	1.65/1.83	1.50/1.58
			3	1.36/1.28	1.3/1.24
			4	1.50/1.62	1.27/1.29
			5	1.00/1.75	.68/1.06
Nietzsche	0.15	0.0			
			1	1.50/1.50	1.45/1.47
			2	1.64/1.83	1.49/1.56
			3	1.36/1.28	1.29/1.23
			4	1.50/1.63	1.22/1.24
			5	1.00/1.75	.64/.97
Tolstoy	0.05	0.05			
			1	1.50/1.50	1.47/1.48
			2	1.64/1.83	1.55/1.67
			3	1.36/1.28	1.32/1.25
			4	1.50/1.62	1.36/1.41
			5	1.00/1.75	.77/1.33

Figure 3: Table of critical statistics

4 Conclusion

Through analysis of these combinations of scenarios and environments, it appears that certainty confers a survival benefit in neutral and unfriendly environments. The model, then, might provide a plausible explanation for the prevalence of overconfidence and certainty biases. If we assume that humans during the Paleolithic era were not inherently altruistic nor had a naturally positive outlook on their environment, it seems that certainty would in fact become a dominant trait in human interaction. However, in environments that more closely resemble the contemporary western world, where civilization has codified some expectation of altruism in social norms, it appears that uncertainty is more likely to become the dominant trait.

Since the results of this model and its simulation appear to match with observed human behavior, it may be worthwhile to continue developing similar models to help guide research in the field of evolutionary psychology. Though prisoner's dilemma models may elide many of the nuances of social interaction and its effect on fitness for survival, they offer a relatively simple framework in which to test plausible explanations for cognitive biases.

References

[1] Hsu, M., Bhatt, M., Adolphs, R., Tranel, D., Camerer, C. Neural Systems Responding to Degrees of Uncertainty in Decision Making. *Science* 9 (Dec 2005) pp.1680-1683. Retrieved from http://www.sciencemag.org/content/310/5754/1680.full on 24 Sept, 2014.

[2] Camerer, C., Weber, M. Recent Developments in Modeling Preferences: Uncertainty and Ambiguity. Journal of Risk and Uncertainty (1992) pp. 325-370.

[3] Adams, P., Adams, J. Confidence in the Recognition and Reproduction of Words Difficult to Spell. The American Journal of Psychology V.73, No. 4 (Dec, 1960) pp.544-552

[4] Axelrod, R., Hamilton, W. The Evolution of Cooperation. Science Vol. 211, No. 4489 (Mar, 1981) pp.1390-1396

[5] Harrald, P., Fogel, D. Evolving Continuous Behaviors in the Iterated Prisoner's Dilemma. BioSystems Vol. 37 (1996) pp.135-145

[6] Ellsberg, D. Risk, Ambiguity and the Savage Axioms. The Quarterly Journal of Economics Vol. 75, No. 4 (Nov, 1961) pp. 643 - 669.

A The Simulation

The simulation, written in Java, is publicly available at the following link:
https://github.com/hatgirl/PDGames

Additional results for the simulation run with various parameters may also be found at this link.

B Results

The following are the results of the test program for each of the environments, lifespans, and scenarios detailed in the paper.

Results for Hobbes Environment (alt = 0.0, opt = 0.0) with 10 Life Points

Summary of games for the following population:
Certainties ranging from .5 to 1.5, uniformly distributed
Average startstate median: 1.50
Average startstate mean: 1.50
Average startstate max: 2.00
Average startstate min: 1.00
Starting proportion greater than starting avg: 0.500
****
****
Certainties ranging from .5 to 1.5, uniformly distributed
Average endstate median: 1.57
Average endstate mean: 1.55
Average endstate max: 2.29
Average endstate min: 0.74
Final population proportion greater than starting median: 0.570
Final population proportion greater than starting average: 0.570
****
****

****
****
Summary of games for the following population:
0.250 very large certainty values
Average startstate median: 1.64
Average startstate mean: 1.83
Average startstate max: 3.50
Average startstate min: 1.00
Starting proportion greater than starting avg: 0.500
****
****
0.250 very large certainty values
Average endstate median: 1.80
Average endstate mean: 2.05
Average endstate max: 3.72
Average endstate min: 0.76
Final population proportion greater than starting median: 0.615
Final population proportion greater than starting average: 0.476
****
****

****
****
Summary of games for the following population:
0.250 very small certainty values
Average startstate median: 1.36
Average startstate mean: 1.28
Average startstate max: 2.00
Average startstate min: 0.00
Starting proportion greater than starting avg: 0.500
****
****
0.250 very small certainty values
Average endstate median: 1.51
Average endstate mean: 1.44
Average endstate max: 2.28
Average endstate min: 0.00
Final population proportion greater than starting median: 0.630
Final population proportion greater than starting average: 0.690
****
****

****
****
Summary of games for the following population:
Some very large certainty values, some small.
Average startstate median: 1.50
Average startstate mean: 1.62
Average startstate max: 3.50
Average startstate min: 0.00
Starting proportion greater than starting avg: 0.500

****
****
Some very large certainty values, some small.
Average endstate median: 1.89
Average endstate mean: 2.06
Average endstate max: 3.72
Average endstate min: 0.01
Final population proportion greater than starting median: 0.687
Final population proportion greater than starting average: 0.626
****
****

****
****
Summary of games for the following population:
All certainty values are very large or very small.
Average startstate median: 1.00
Average startstate mean: 1.75
Average startstate max: 3.50
Average startstate min: 0.00
Starting proportion greater than starting avg: 0.500
****
****
All certainty values are very large or very small.
Average endstate median: 2.90
Average endstate mean: 2.52
Average endstate max: 3.77
Average endstate min: 0.00
Final population proportion greater than starting median: 0.808
Final population proportion greater than starting average: 0.794
****
****

Results for Rand Environment (alt = 0.0, opt = 0.15) with 10 Life Points

Summary of games for the following population:
Certainties ranging from .5 to 1.5, uniformly distributed
Average startstate median: 1.50
Average startstate mean: 1.50
Average startstate max: 2.00
Average startstate min: 1.00
Starting proportion greater than starting avg: 0.500
****
****
Certainties ranging from .5 to 1.5, uniformly distributed
Average endstate median: 1.45
Average endstate mean: 1.46
Average endstate max: 2.34
Average endstate min: 0.63
Final population proportion greater than starting median: 0.455
Final population proportion greater than starting average: 0.455

****
****

****
****
Summary of games for the following population:
0.250 very large certainty values
Average startstate median: 1.64
Average startstate mean: 1.83
Average startstate max: 3.50
Average startstate min: 1.00
Starting proportion greater than starting avg: 0.500
****
****
0.250 very large certainty values
Average endstate median: 1.50
Average endstate mean: 1.58
Average endstate max: 3.55
Average endstate min: 0.62
Final population proportion greater than starting median: 0.371
Final population proportion greater than starting average: 0.217
****
****

****
****
Summary of games for the following population:
0.250 very small certainty values
Average startstate median: 1.36
Average startstate mean: 1.28
Average startstate max: 2.00
Average startstate min: 0.00
Starting proportion greater than starting avg: 0.500
****
****
0.250 very small certainty values
Average endstate median: 1.30
Average endstate mean: 1.24
Average endstate max: 2.31
Average endstate min: 0.00
Final population proportion greater than starting median: 0.456
Final population proportion greater than starting average: 0.518
****
****

****
****
Summary of games for the following population:
Some very large certainty values, some small.
Average startstate median: 1.50

Average startstate mean: 1.62
Average startstate max: 3.50
Average startstate min: 0.00
Starting proportion greater than starting avg: 0.500
****
****
Some very large certainty values, some small.
Average endstate median: 1.27
Average endstate mean: 1.29
Average endstate max: 3.57
Average endstate min: 0.00
Final population proportion greater than starting median: 0.357
Final population proportion greater than starting average: 0.282
****
****

****
****
Summary of games for the following population:
All certainty values are very large or very small.
Average startstate median: 1.00
Average startstate mean: 1.75
Average startstate max: 3.50
Average startstate min: 0.00
Starting proportion greater than starting avg: 0.500
****
****
All certainty values are very large or very small.
Average endstate median: 0.68
Average endstate mean: 1.06
Average endstate max: 3.69
Average endstate min: 0.00
Final population proportion greater than starting median: 0.279
Final population proportion greater than starting average: 0.225
****
****

Results for Nietzsche Environment (alt = 0.15, opt = 0.00) with 10 Life Points

Summary of games for the following population:
Certainties ranging from .5 to 1.5, uniformly distributed
Average startstate median: 1.50
Average startstate mean: 1.50
Average startstate max: 2.00
Average startstate min: 1.00
Starting proportion greater than starting avg: 0.500
****
****
Certainties ranging from .5 to 1.5, uniformly distributed
Average endstate median: 1.45

Average endstate mean: 1.47
Average endstate max: 2.35
Average endstate min: 0.63
Final population proportion greater than starting median: 0.450
Final population proportion greater than starting average: 0.450
****
****

****
****
Summary of games for the following population:
0.250 very large certainty values
Average startstate median: 1.64
Average startstate mean: 1.83
Average startstate max: 3.50
Average startstate min: 1.00
Starting proportion greater than starting avg: 0.500
****
****
0.250 very large certainty values
Average endstate median: 1.49
Average endstate mean: 1.56
Average endstate max: 3.56
Average endstate min: 0.66
Final population proportion greater than starting median: 0.361
Final population proportion greater than starting average: 0.209
****
****

****
****
Summary of games for the following population:
0.250 very small certainty values
Average startstate median: 1.36
Average startstate mean: 1.28
Average startstate max: 2.00
Average startstate min: 0.00
Starting proportion greater than starting avg: 0.500
****
****
0.250 very small certainty values
Average endstate median: 1.29
Average endstate mean: 1.23
Average endstate max: 2.33
Average endstate min: 0.00
Final population proportion greater than starting median: 0.448
Final population proportion greater than starting average: 0.513
****
****

****
****
Summary of games for the following population:
Some very large certainty values, some small.
Average startstate median: 1.50
Average startstate mean: 1.63
Average startstate max: 3.50
Average startstate min: 0.00
Starting proportion greater than starting avg: 0.500
****
****
Some very large certainty values, some small.
Average endstate median: 1.22
Average endstate mean: 1.24
Average endstate max: 3.55
Average endstate min: 0.00
Final population proportion greater than starting median: 0.333
Final population proportion greater than starting average: 0.265
****
****

****
****
Summary of games for the following population:
All certainty values are very large or very small.
Average startstate median: 1.00
Average startstate mean: 1.75
Average startstate max: 3.50
Average startstate min: 0.00
Starting proportion greater than starting avg: 0.500
****
****
All certainty values are very large or very small.
Average endstate median: 0.64
Average endstate mean: 0.97
Average endstate max: 3.68
Average endstate min: 0.00
Final population proportion greater than starting median: 0.245
Final population proportion greater than starting average: 0.184
****
****

Results for Tolstoy Environment (alt = 0.09, opt = 0.09) with 10 Life Points

Summary of games for the following population:
Certainties ranging from .5 to 1.5, uniformly distributed
Average startstate median: 1.50
Average startstate mean: 1.50
Average startstate max: 2.00
Average startstate min: 1.00

Starting proportion greater than starting avg: 0.500
****
****
Certainties ranging from .5 to 1.5, uniformly distributed
Average endstate median: 1.56
Average endstate mean: 1.54
Average endstate max: 2.28
Average endstate min: 0.75
Final population proportion greater than starting median: 0.562
Final population proportion greater than starting average: 0.562
****
****

****
****
Summary of games for the following population:
0.250 very large certainty values
Average startstate median: 1.64
Average startstate mean: 1.83
Average startstate max: 3.50
Average startstate min: 1.00
Starting proportion greater than starting avg: 0.500
****
****
0.250 very large certainty values
Average endstate median: 1.77
Average endstate mean: 1.99
Average endstate max: 3.71
Average endstate min: 0.76
Final population proportion greater than starting median: 0.591
Final population proportion greater than starting average: 0.448
****
****

****
****
Summary of games for the following population:
0.250 very small certainty values
Average startstate median: 1.36
Average startstate mean: 1.28
Average startstate max: 2.00
Average startstate min: 0.00
Starting proportion greater than starting avg: 0.500
****
****
0.250 very small certainty values
Average endstate median: 1.48
Average endstate mean: 1.42
Average endstate max: 2.27
Average endstate min: 0.00
Final population proportion greater than starting median: 0.603

Final population proportion greater than starting average: 0.669
****
****

****
****
Summary of games for the following population:
Some very large certainty values, some small.

Average startstate median: 1.50
Average startstate mean: 1.62
Average startstate max: 3.50
Average startstate min: 0.00
Starting proportion greater than starting avg: 0.500
****
****
Some very large certainty values, some small.
Average endstate median: 1.82
Average endstate mean: 1.99
Average endstate max: 3.72
Average endstate min: 0.00
Final population proportion greater than starting median: 0.666
Final population proportion greater than starting average: 0.603
****
****

****
****
Summary of games for the following population:
All certainty values are very large or very small.
Average startstate median: 1.00
Average startstate mean: 1.75
Average startstate max: 3.50
Average startstate min: 0.00
Starting proportion greater than starting avg: 0.500
****
****
All certainty values are very large or very small.
Average endstate median: 2.84
Average endstate mean: 2.38
Average endstate max: 3.75
Average endstate min: 0.00
Final population proportion greater than starting median: 0.760
Final population proportion greater than starting average: 0.743
****
****