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STA: 2200: Probability and statistic II

Question One.

(A) (i) Continuous random variable: is a variable that can take any real value within a given range.

(ii) Expectation. is the long-run average value of repititions of an experiments.

(B) Given a fair coin, perform an experment using the coin, and hence show that the outcome of your experment is a random ~~number~~ variable

Experiment: Toss a fair coin twice.

Sample space (S): [HH, HT, TH, TT]   H = Head, T = Tail.

Random Variable (X) it represents number of Heads

$S: [HH, HT, TH, TT] = X = (2, 1, 1, 0)$

$X=2 \quad X=1 \quad X=1 \quad X=0$

∴ ~~Since X assigns nu~~

X - takes values depending on the outcome of a random experiment.

⇒ Since the outcome is uncertain before the coin is tossed, X is a random variable.

∴ X - is a discrete variable.

(C) Dice Game - Define X, Distribution and Expectation.

Given: Win \$2 if die shows 1
Win \$1 if die shows 6
Else lose \$1 otherwise

A fair six sided toss outcomes.
[1, 2, 3, 4, 5, 6]

Hence the probability 1/6

Wine $= \frac{1}{6} + \frac{1}{6} = \frac{2}{6}$

loss $= \frac{4}{6}$ - Remaining.

The Expected Value.

$E(x) = \sum x_i P(x_i)$

$E(x) = 2 \times \frac{1}{6} + 1 \times \frac{1}{6} + (-1) \times \frac{4}{6}$

$\cancel{E(x) = \frac{2}{6} + \frac{1}{6} + \frac{4}{6}}$

$E(x) = \frac{2}{6} + \frac{1}{6} - \frac{4}{6}$.

$= -\frac{1}{6}$

∴ The expected value is $-\$0.17$

~~loosing \$0.17~~

loosing \$0.17 per ~~cent~~ roll or played this game.

(D) The random variable X has probability destiny function.

$$f(x) = \begin{cases} (3.6x - 2.4x^2) & 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the mean and median of the distribution.
Also find $P(X > 0.5)$ and $Var(X)$.

Find the Mean ($E(X)$):

$$E(X) = \int_{-\infty}^{\infty} x \cdot f(x)\, dx.$$

$$E(X) = \int_0^1 x(3.6x - 2.4x^2)\, dx$$

$$E(X) = \int_0^1 (3.6x^2 - 2.4x^3)\, dx$$

$$E(X) = \left[\frac{3.6x^3}{3} - \frac{2.4x^4}{4}\right]_0^1$$

$$- E(X) = \left[1.2x^3 - 0.6x^4\right]_0^1$$

$$- E(X) = (1.2(1)^3 - 0.6(1)^4) - (1.2(0)^3 - 0.6(0)^4)$$

$$- E(X) = (1.2 - 0.6) - (0)$$

$$E(X) = 0.6$$

$$\text{Mean} = 0.6.$$

Find Median (M) $P(X \leq m) = 0.5$

$$\int_0^M (3.6x - 2.4x^2)\, dx = 0.5$$

$$\left[1.8x^2 - 0.8x^3\right]_0^M = 0.5$$

$$1.8m^2 - 0.8m^3 = 0.5$$

Rearrange the equation $= 0.8m^3 - 1.8m^2 + 0.5 = 0.$

lets test around the mean 0.6

If $m = 0.6$: $0.8(0.6)^3 - 1.8(0.6)^2 + 0.5 = 0.1728 - 0.648 + 0.5 = 0.0248 \neq 0$

lets try $= 0.62$

$0.8(0.62)^3 - 1.8(0.62)^2 + 0.5 = 0.8(0.238328) - 1.8(0.3844) + 0.5 =$

$0.1906624 - 0.69192 + 0.5 = -0.0012576$ (very close to 0)

$\therefore$ The Median is approximately $\underline{0.62}$

$$= 0.62$$

(iii) Find $P(x > 0.5)$:

$P(x > 0.5) = \int_{0.5}^{1} (3.6x - 2.4x^2)\,dx$

$= [1.8x^2 - 0.8x^3]_{0.5}^{1}$

$= (1.8(1)^2 - 0.8(1)^3 - (1.8(0.5)^2 - 0.8(0.5)^3)$

$= (1.8 - 0.8) - (1.8(0.25) - 0.8(0.125))$

$= 1.0 - (0.45 - 0.1)$

$= 1.0 - 0.35$

$P(x > 0.5) = \underline{\underline{0.65}}.$

$= \underline{\underline{0.65}}$

(iv) find Variance. $Var(X)$

$Var(X) = E(X^2) - (E(X))^2$ find $E(x^2)$ - first

$E(X^2) = \int_0^1 x^2 \cdot f(x)\,dx = \int_0^1 x^2 (3.6x - 2.4x^2)\,dx$

$E(X^2) = \left[\frac{3.6x^3}{4} - \frac{2.4x^5}{5}\right]_0^1$

$E(X^2) = [0.9x^4 - 0.48x^5)_0^1$

$E(X^2) = (0.9(1)^4 - 0.48(1)^5) - (0.9(0)^4 - 0.48(0)^5)$

$= 0.9 - 0.48 - (0)$

$E(X^2) = 0.42$

substitute $E(X^2)$ and $E(X)$ into the variance formula.

$E(X) = 0.6$

$Var(X) = E(x^2) - (E(X))^2$

$Var(X) = 0.42 - (0.6)^2$

$Var(X) = 0.42 - 0.36$

$Var(X) = \underline{\underline{0.06}}.$

Question two:

(A) A computer plane has 100 seats. The probability that any particular passenger will not show up for a flight is 0.05, independent of other passengers. The airline sells 110 tickets for the flight calculate the probability that more passengers show up for the flight than there are seats available

Plane seat = 100
Ticket sold = 110
Probability of passenger showing up $p = 0.95$
let $X \sim \text{Binomial}(n = 110, p = 0.95)$

$$P(X > 100) = 1 - P(X \leq 100)$$

$$P(X > 100) = 1 - (100, 110, 0.95) = \underline{0.9508}$$

(b) Standard Deviation of numbers claimed to be filled.
let $X \sim \text{Passion}(\lambda)$
Given: $P(X=2) = 3 \times P(X=4)$
use Passion formula: $\frac{\lambda^2}{2!} \times e^{-\lambda} = 3 \cdot \frac{\lambda^4}{4!} \cdot e^{-\lambda}$
cance $e^{-\lambda}$ and simplify.

$$\frac{\lambda^2}{2} = 3 \times \frac{\lambda^4}{24} \Rightarrow \frac{\lambda^2}{2} = \frac{\lambda^4}{8} \Rightarrow \frac{1}{4}\lambda^4 \Rightarrow \lambda^2\left(1 - \frac{\lambda^2}{4}\right) = 0$$

$$\Rightarrow \lambda^2 = 4 \Rightarrow \lambda = 2$$

The Standard Deviation $= \sqrt{\lambda} = \sqrt{2} = \boxed{1.4142}$

(C) $n = 800$
$P = 0.04$
$\lambda = np = 800 \times 0.04 = 32$
We want find $P(X = 75)$ when $\lambda = 32$

$$P(X = 75) = \frac{32^{75} e^{-32}}{75!}$$

$P(X = 75) = \text{passion.pmf}(75, 32) = \underline{0.0}$

(d) Accidents occurs infrequently Probability 0.005
For a given period of 400:
(i) Probability that there will be an accident only on one day.
Find $P(X=1)$

$$P(X=1) = \frac{2^1 e^{-2}}{1!}$$

$$P(x=1) = 2 \times e^{-2}$$

$$P(x=1) = 2 \times 0.135335$$

$$P(x=1) = 0.27067.$$

$$= \underline{\underline{0.2707}}$$

(ii) Probability that there are at most two days with an accident.

$$P(x=0): \frac{2^0 e^{-2}}{0!} = \frac{1 \times e^{-2}}{1} = e^{-2} = 0.135335$$

$P(x=1) = 0.2707$ - from above.

$$P(x=2) = \frac{2^2 e^{-2}}{2!} = \frac{4e^{-2}}{2} = 2e^{-2} = 2 \times 0.135335 = 0.27067$$

Sum up = $P(X \leq 2) = 0.135335 + 0.27067 + 0.27067$

$$P(X \leq 2) = 0.676675$$

$$= \underline{\underline{0.6767}}$$

(e) What percentage of the claim is within one standard deviation of the mean claim size.
Calculate the Mean $(E(X))$

$$E(X) = (20 \times 0.15) + (30 \times 0.10) + (40 \times 0.05) + (50 \times 0.20) + (60 \times 0.10) + (70 \times 0.10) + (80 \times 0.30)$$

$$E(X) = 3.0 + 3.0 + 2.0 + 10.0 + 6.0 + 7.0 + 24.0$$

$$E(X) = \underline{\underline{55.0}}$$

Calculate the variance $(Var(X))$

$$Var(X) = E(X^2) - (E(X))^2$$

$$E(X^2) = (20^2 \times 0.15) + (30^2 \times 0.10) + (40^2 \times 0.05) + (50^2 \times 0.20) + (60^2 \times 0.10) + (70^2 \times 0.10) + (80^2 \times 0.30)$$

$$E(X^2) = 60 + 90 + 80 + 500 + 360 + 490 + 1920$$

$$E(X^2) = 3500 = 3500 - (55)^2 = 3500 - 3025 = 475$$

$$X = \sqrt{475}$$

The Standard Deviation is = $\underline{21.794}$

Claims within Range = 50, 60, 70, 40

$$= P(X=40) + P(X=50) + P(X=60) + P(X=70)$$

$$= 0.05 + 0.20 + 0.10 + 0.10$$

$$= 0.45 \times 100 = 45\%$$

$$= \underline{\underline{45\%}}$$

(f)· $P = 0.05$

(i) Given the information what is the expected number of lights bulbs the inspectors need to test until they find the 1st defective one?

$E(X) = 1/p$

$E(X) = 1/0.05$

$EX = 20$

The expected number of light bulbs the inspectors need to test until they find the first defective one is $\underline{20}$

(ii) Probability that the inspectors find the first defective bulb on 3rd test.

find $P(X=3)$

using the PMF: $P(X=3) = (1-p)^{3-1} \cdot p = (0.95)^2 \times 0.05$

$P(X=3) = 0.9025 \times 0.05$

$P(X=3) = 0.045125$

$= \underline{0.045125}$

(iii) What is the probability that the inspectors find the first defective bulb within the first 5 tests

$P(X \leq 5)$

$F(x) = P(X \leq x) = 1-(1-p)^x$

$P(X \leq 5)\ 1-(1-0.05)^5$

$P(X \leq 5) = 1-(0.95)^5$

$= 0.77378$

$P(X \leq 5) = \underline{0.22622}$

(iv) What is the probability that the inspectors will no find a defective bulb within the first 10 tests.

$P(X > 10)$ $P(X > x) = (1-p)^x$

For Geometric distribution,

$P(X > 10) = (1-0.05)^{10}$

$P(X > 10) = (0.95)^{10}$

$P(X > 10) = 0.59873$

$= \underline{0.59873}$

(9) A call center finds that the probability that a customer call center results in a resolution on first attempt is $P = 0.2$. The call center wants to determine the number of calls needed to successfully resolve 3 customer issues.

(i) What is the probability that the 3rd customer issue resolved on the 10th call.

$k = 10$ $r = 3$ - number of successes.

$P(X=10) = \binom{10-1}{3-1}(0.2)^3(1-0.2)^{10-3}$

$P(X=10) = \binom{9}{2}(0.2)^3(0.8)^7$ ~~2136~~

Calculate $\binom{9}{2}$

$\binom{9}{2} = \frac{9!}{2!(9-2)} = \frac{9!}{2!7!} = \frac{9\times8}{2\times1} = 36$

$P(X=10) = 36 \times (0.2)^3 \times (0.8)^7$

$P(X=10) = 36 \times 0.008 \times 0.2097152$

$P(X=10) = 0.06039$

$=$ ~~0.8006~~

$= \underline{0.06039}$

(ii) What is the probability that the call center resolves

let Y be the number of successful resolutions in 12 calls.

~~12~~ $P(X \leq 12)$

Binomial PMF: ~~P(Y)~~ $P(Y=k) = \binom{n}{k} p^k (1-p)^{n-k}$.

$P(Y=0)$ - on zero success $= 0.06872$

$P(Y=1)$ on One success $= 0.20616$

$P(Y=2)$ on 2 success trial $= 0.28347$

~~Calculate P(Y<3)~~:

Sum $= 0.28347 + 0.20616 + 0.06872$

$= \underline{0.55835}$

Calculate $P(Y \geq 3) = 1 - P(Y < 3)$

$P(Y \geq 3) = 0.44165$.

$= \underline{0.4417}$

(iii) What is the variance

$Var(X) = \frac{r(1-p)}{p^2}$

~~the~~ $r = 3$ and $P = 0.2$.

$var(X) = \frac{3(1-0.2)}{(0.2)^2}$

$var(X) = \frac{3 \times 0.8}{0.04}$

$var(X) = \frac{2.4}{0.04}$

$var(X) = 60$

The variance in the number of calls needed to resolve 3 customer issue is $\underline{60}$.