Question One

a) i) Continous Random Variable - Are Variables that can take any value within a given range including decimal values.

ii) Expectation - Is the average outcome we would expect from a random experiment if it were repeated several times.

b) - Random experiment - is an experiment whose outcome can not be predicted with certainity.

hence tossing a coin can't be predicted ~~by~~ whether it will be heads or tails

Sample space = $\{H, T\}$ = set of outcomes

Porobability of head = $PH = \frac{\text{favourable outcomes}}{\text{Total outcomes}} = \frac{1}{2}$

Propability of Tails = $PT = \frac{\text{favourable outcomes}}{\text{Total outcomes}} = \frac{1}{2}$

$= P(H) = \frac{1}{2}$

$= P(T) = \frac{1}{2}$

c)

Sample space = $(-1, 1, 2)$

Propability of loosing 1\$ $= \frac{4}{6} = \frac{2}{3}$

Propability of winning 1\$ $= \frac{1}{6}$

Propability of wining 2\$ $= \frac{1}{6}$

X	P(x)	xP(x)	x^2P(x)
-1	2/3	-0.667	0.667
1	1/6	0.167	0.167
2	1/6	0.333	0.667
	=1	=0.167	=1.560

$(x-\mu)^2$

$2 \times \frac{1}{6} = \frac{2}{6}$

$1 \times \frac{1}{6} = \frac{1}{6}$

$-1 \times \frac{4}{6} = \frac{-4}{6}$

$= \frac{2}{6} + \frac{1}{6} + \left(-\frac{4}{6}\right)$

$= \frac{2+1-4}{6} = \frac{-1}{6}$

hence expected value $= \frac{-1}{6}$

So answer is $-\frac{1}{6}$ or 0.1667

c) ii) Mean

$E(x) = \int_0^1 x(f(x))\,dx$

$= \int_0^1 x\,(3.6x - 2.4x^2)\,dx = 0.5$

$= (3.6x^2 - 2.4x)^3 dx$

$= 1.2x^3 - 0.6x^4$

$= \underline{0.6}$

<u>Median</u>

$= \int_0^1 (3.6x - 2.4x^2)\,dx = 0.5$

$= [1.8m^2 - 0.8m^3$

$= 0.8m^3 - 1.8m^2 + 0.5 = 0$

$= 0.9x - 0.875^3 = 0.5$

$= \underline{0.386}$

$P(x) 0.5$

$= E(x x^2) - [E(x)^2 =$

$= E(x^2) = [(x).x^2$

$= [(3.6x^3 - 2.4x^4).dx^2$

$= 0.9 - 0.75 - 0.8 \times 0.75 = 0.65$

$= 0.42$

Variance

$= Var(x) = E(x^2) - [E(x)]^2$

$E(x^2) = 0.92^3 - 0.42 - 0.36$

$= 0.06$

Question TWO

a) Let x be number of passengers who show up

$x\,(n = 110,\ P = 0.95$

$P(x > 106)$

mean. $\mu = np = 110 \times 0.95 = 104.5$

Variance $= np(1-p) = 110 \times 0.95 \times 0.05 = 5.225$

Standard deviation $= \sqrt{5.225} = 2.285$

$P(x > 100) = P(Y > 100.5)$

$Z = (100.5 - 04.5)/2.28 = -1.75$

$P(Z > -1.75) = 1 - P(Z \leq -1.75) = 1 - 0.0401 = 0.9599$

$= 0.9599 \times 100$

$= 95.9 = 96\%$

Answer $= 96\%$ chance of more passengers appearing.

$P(x > 100) = \binom{110}{101}(0.95)^{101}(0.05)^9 + \binom{110}{102}(0.95)^{102}(0.05)^8 + \ldots \binom{110}{110}(0.95)^{110}(0.05)^0$

$1 - 0.3076 = 0.6924$

$0.6924 \times 100 = 69\%$

2 b) let n be number of claims filled
n follows a poisson distribution with parameter λ
pmf of poisson distibution

$$P(N=2)=3\times P(N=4) = \frac{e^{-\lambda}\lambda^2}{2!} = 3\times\frac{e^{-\lambda}\lambda^4}{4!}$$

$$\lambda^2=4$$
$$\lambda=2$$

Since $\lambda=2$ then standard deviation $=\sqrt{2}=1.414$

$$=\underline{1.414}$$

c)

$$\lambda=800\times0.04=32$$
$$P(X=75)=e^{-\lambda}\frac{\lambda^{75}}{75!}$$

$$\cancel{P(X=1)=\binom{400}{1}(0.005)^1(0.995)^{399}} = \cancel{\text{accident}}$$
$$\lambda=800\times0.04=32$$

$$P(X=75)\approx1.61\times10^{-12}$$

$$P(X=75)=1.61\times10^{-12}$$

d) i) $n=400$
$p=0.005$

$$n\times p$$
$$=400\times0.005=2 \Rightarrow Y\sim \text{Poisson}(\lambda=2)$$

$$P(Y=1)=\frac{e^{-2}\cdot2^1}{1!}=e^{-2}\cdot2=0.2707$$
$$=0.2707$$

ii)

$$P(Y\le2)=P(Y=0)+P(Y=$$

$$P(0)=\frac{e^{-2}\cdot2^0}{0!}=e^{-2}=0.1353$$
$$P(1)=0.2707 \quad ([redacted])$$
$$P(2)=\frac{e^{-2}\cdot2^2}{2!}=e^{-2}\cdot2=0.2707$$
$\cancel{P(2}$

$$P(Y\le2)=0.1353+2707+0.2707$$
$$=0.6767$$

2. e)

$E x = 20(0.15) + 30(0.10) + 40(0.05) + 50(0.20) + 60(0.10) + 70(0.10) + 80(0.30)$

$E(x) = 55$

$\text{Variance} = E(x^2) - (E(x))^2$

$E(x^2) = 20^2(0.15) + 30^2(0.10) + 40^2(0.05) + 50^2(0.20) + 60^2(0.10) + 70^2(0.10) + 80^2(0.30)$

$= 400(0.5) + 900(0.10) + 1600(0.05) + 2500(0.20) + 3600(0.10) + 4900(0.10) + 6400(0.30)$

$E(x^2) = \underline{3500}$

$Var(x) = E(x^2) - (E(x))^2$

$= 3500 - (55)^2$

$= 3500 - 3025 \quad = Var(x) = 475$

The claim size within one standard deviation of the mean claim size of 55 are between ~~55 - 21 = 33.~~

$55 - 21.79 = 33.21$ and $55 + 21.79 = 76.79$

Those claim sizes are of 40, 50, 60, 70 and total claim size is

$= 0.05 + 0.20 + 0.10 + 0.10 = 0.45$

$= \underline{0.45}$

$0.45 \times 100 = 45\%$

$= 45\%$

2 f)

i) $E(x) = \frac{1}{p}$

$E(x) = \frac{1}{0.05}$

$E(x) = 20$

ii) $P(x=3) = (1-0.05)^{3-1} \, 0.05$

$= (0.95)^2 \, 0.05$

$= 0.9025 \times 0.05$

$P(x=3) = 0.045125$

$P(x=3) = 0.0451 \quad = \underline{0.0451}$

iii) $P(x \le 5) = F(5) = 1 - (1-p)^5$

$= 1 - (1 - 0.05)^5$

$= 1 - (0.95)^5$

$= 1 - 0.7737809375$

$= 0.2262190625$

$P(x \le 5) = F(5) = \underline{0.2262}$

2) f) iv) $p(x>10) = 1-p(x \le 10) = 1-(1-(1-p)^{10})$

$= (1-p)^{10}$

$= (1-0.05)^{10}$

$= (0.95)^{10}$

$= 0.598736$

$p(x>10) = \underline{\underline{0.5987}}$

2) g.) i) $p(x=k) = \binom{k-1}{r-1} p^r (1-p)^{k-r}$

$\binom{k-1}{r-1} = \binom{10-1}{3-1} = \binom{9}{2} = \frac{9 \times 8}{2 \times 1} = 36$

$p^r = (0.2)^3 = 0.008$

$(1-p)^{k-r} = (0.8)^{10-3} = (0.8)^7$

$(0.8)^7 = 0.2097152$

$p(x=10) = 36 \times 0.008 \times 0.2097152$

$36 \times 0.008 = 0.288$

$= 0.288 \times 0.2097152 = 0.060398$

$= \underline{\underline{0.060398}}$

ii) $p(x \le 12) = \sum_{k=3}^{12} p(x=k) = \sum_{k=3}^{12} \binom{k-1}{3-1} (0.2)^3 (0.8)^{k-3}$

$p(x=k) = \binom{k-1}{2} (0.008)(0.8)^{k-3}$

$p(x=3) = 1 \times 0.008 \times 1 = 0.008$ To $\underline{\underline{x=12}}$

$p(x=12) = 55 \times 0.008 \times 0.134217728 = 0.0590$

Total $= \underline{\underline{0.4017}}$

$= \underline{\underline{0.4017}}$

iii) $Var\ x = \frac{r(1-p)}{p^2}$

Here, $r=3$, $p=0.2$, $1-p=0.8$

$Var\ x = \frac{3 \times 0.8}{(0.2)^2} = \frac{3 \times 0.8}{0.04} = \frac{2.4}{0.04} = \underline{\underline{60}}$

$= \underline{\underline{60}}$