STANLEY KIPROP LAGAT.
STA 2200 ~~ASSIGNMENT~~ CAT 1. JULY 2025.
PROBABILITY & STATISTICS II.

Question 1.

a).
(i) Continuous random variable is a type of random variable that can take assume any value in an interval of the real number line. It assumes an uncounted number of values.

(ii) Expectation is the mean value of a random variable based on its probability distribution. It gives a measure of central tendency for the random variable.

b). Perform an experiment using a fair coin. Toss and observe the outcomes of a single toss.
- Flip the coin into the air with enough force for it to rotate several times. Allow the coin to land on a flat surface and record the outcome of the side facing up.
The possible outcomes are Head (H) and Tail (T).
Let the outcome of head be 1 and tail be 0 this is a discrete random variable because it takes distinct values of 0 or 1.
The probability $P_{(x=1)} = 0.5$ and $P_{(x=0)} = 0.5$
Because the outcome of the experiment is uncertain and follows a probability distribution, X is a random variable.

c).

If the result is '1', $x = +2 \rightarrow$ win \$2.
If the result is '6', $X = +1 \rightarrow$ win \$1
If the result is '2', '3', '4' or '5', $X = -1 \rightarrow$ lose \$1.

Probability Distribution:

x	$P(X=x)$
-1	$\frac{1}{6} \times 4 = \frac{4}{6}$
1	$\frac{1}{6}$
2	$\frac{1}{6}$

Expected value:

$E(X) = (2 \times \frac{1}{6}) + (1 \times \frac{1}{6}) + (-1 \times \frac{4}{6}) = \frac{2}{6} + \frac{1}{6} - \frac{4}{6} = -\frac{1}{6}$

$E(X) = \$0.167$.

Expected value $= \$-\frac{1}{6} \approx \$0.167$

The ~~average~~ is lose $0.167 per game.

d).

$f(x) = \begin{cases} (3.6x - 2.4x^2) & 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$

(i) Mean.

$E(X) \int_0^1 x f(x)\,dx = \int_0^1 x(3.6x - 2.4x^2)\,dx = \int_0^1 (3.6x^2 - 2.4x^3)\,dx$

$= \left[\frac{3.6}{3}x^3 - \frac{2.4}{4}x^4\right]_0^1 = (1.2 - 0.6) - (0) = \underline{0.6}$

(ii) Median.

$\int_0^m f(x)\,dx = \frac{1}{2}$.

$\int_0^m (3.6x - 2.4x^2)\,dx = 0.5$

$[1.8x^2 - 0.8x^3]_0^m = \cancel{1.8m^2}\ 1.8m^2 - 0.8m^3 = 0.5$

$0.8m^3 - 1.8m^2 + 0.5 = 0$

Using a calculator $m \approx \underline{0.39}$

(iii) $P(X \geq 0.5)$

$P(X > 0.5) = \int_{0.5}^1 (3.6x - 2.4x^2)\,dx = [1.8x^2 - 0.8x^3]_{0.5}^1$

$= [1.8(1) - 0.8(1)] - [1.8(0.5)^2 - 0.8(0.5)^3] = (1.8 - 0.8) - (0.45 - 0.1)$

$= 1 - 0.35 = \underline{0.65}$

ii) Var(X)

$Var(X) = E(X^2) - (E(X))^2$

$E(X^2) = \int_0^1 x^2(3.6x - 2.4x^2)dx = \int_0^1 3.6x^3 - 2.4x^4 dx.$

$= [0.9x^4 - 0.48x^5]_0^1 = 0.9 - 0.48 = 0.42.$

$Var(X) = E(X^2) - (E(X))^2 = 0.42 - (0.6)^2 = 0.42 - 0.36 = \underline{0.06}$

Question 2.

(a) Let the number of passenger who showed up be X.

$n = 110 \rightarrow$ number of ticket sold.

Probability of each passenger showing up $\Rightarrow p = 0.95$

Number of seat = 100.

$P(X \geq 100)$

$X \sim$ Binomial $(n = 110, p = 0.95)$.

$$P(X \geq 100) = \sum_{K=101}^{110} \binom{110}{K} \times (0.95)^K \times (0.05)^{110-K}.$$

$\mu = np = 110 \times 0.95 = 104.5$

$\sigma = \sqrt{np(1-p)} = \sqrt{110 \times 0.95 \times 0.05} = \sqrt{5.225} = 2.286.$

$P(X > 100) \approx P\left(Z > \frac{100.5 - 104.5}{2.286}\right) = P\left(Z > \frac{-4}{2.286}\right)$

$P(Z > -4.75)$

$P(Z > -4.75) = 1 - P(Z < -4.75) = 1 - 0.0401$

$\simeq 0.9599$

Probability that more than 100 passengers show up is $\underline{95.99\%}$

(b) $P(k) = \frac{e^{-\lambda}\lambda^k}{k!}$

$P(2) = \frac{e^{-\lambda}\lambda^2}{2!}$

$P(4) = \frac{e^{-\lambda}\lambda^4}{4!}$.

$\Rightarrow \frac{e^{-\lambda}\lambda^2}{2!} = 3 \times \frac{e^{-\lambda}\lambda^4}{4!}$

$\frac{e^{-\lambda}\lambda^2}{2!} = \frac{3 \cdot e^{-\lambda}\lambda^4}{4!}$

$\frac{e^{-\lambda}\lambda^2}{2} = \frac{e^{-\lambda}\lambda^4}{8}$.

$\frac{\lambda^2}{2} = \frac{\lambda^4}{8}$.

$4\lambda^2 = \lambda^4 \Rightarrow \lambda^4 - 4\lambda^2 = 0. \Rightarrow \lambda^2(\lambda^2-4) = 0.$

$\lambda \neq 0 \quad \therefore \lambda = 2.$

$\sigma = \sqrt{\lambda} = \sqrt{2}$

(c)

$n = 800$

$p = 0.04$

$k = 75$.

$\lambda = np = 800 \times 0.04 = 32.$

poisson aproximation:

$P(X=k) = \frac{e^{-\lambda}\lambda^k}{k!}$

$= \frac{e^{-32} \times 32^{75}}{75!} \simeq 3.93 \times 10^{-11}$

(d) Let ~~a~~ number of accident in 400 days be X.

$p = 0.05$

$n = 400$

$\lambda = pn = 0.05 \times 400 = 2$.

$X \sim \text{poisson}(\lambda = 2)$.

(i) $P(X=1) = \frac{e^{-2} \times 2^1}{1!} = 2e^{-2} = 2 \times 0.1353 = 0.2706$

(ii) $P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$

$P(x=0) = \frac{e^{-2} \times 2^0}{0!} = e^{-2} = 0.1353$

$P(X=1) = \frac{e^{-2} \times 2^1}{1!} = 2 \times e^{-2} = 0.2706$

$P(X=2) = \frac{e^{-2} \times 2^2}{2!} = 2e^{-2} = 0.2706$

$P(X \leq 2) = 0.1353 + 0.2706 + 0.2706$.

$= 0.6765$

(e) $\mu = \sum(x \cdot P(x)) = 20 \times 0.15 + 30 \times 0.10 + 40 \times 0.05 + 50 \times 0.20 + 60 \times 0.10 + 70 \times 0.10 + 80 \times 0.30 = 55$.

$\sigma^2 = \sum[(x-\mu)^2 \times P(x)] = (20-55)^2 \times 0.15 + (30-55)^2 \times 0.10 + (40-55)^2 \times 0.05 + (50-55)^2 \times 0.20 + (60-55)^2 \times 0.10 + (70-55)^2 \times 0.10 + (80-55)^2 \times 0.30 = 475$

$\sigma = \sqrt{475} = 21.79$

Claims within one std deviation of mean between 55 ± 21.79

$\therefore$ between 33.21 and 75.79.

Total probability will be ~~30×0.1+40×0.05+50×0.20+60×0.10+70×0.10~~

$= 0.10 + 0.05 + 0.20 + 0.10 + 0.10 = 0.55$

$= 55\%$

(f) (i)

$E(X) = \frac{1}{p} = \frac{1}{0.05} = \underline{\underline{20}}$.

(ii) $P(X=x) = (1-p)^{x-1} \times p$

$P(X=3) = (1-0.05)^{3-1} \times 0.05 = (0.95)^2 \times 0.05 = \underline{\underline{0.0451}}$

(iii) $F(x) = 1-(1-p)^x$.

$F(5) = 1-(1-0.05)^5 = 1-(0.95)^5 = \underline{\underline{0.2262}}$

(iv) $P(X>10) = (1-p)^{10} = (0.95)^{10} = \underline{\underline{0.5987}}$

(9).

(i) $P(X=x) = \binom{x-1}{r-1} \cdot p^r \cdot (1-p)^{x-r}$.

$x=10$

$r=3$

$p=0.2$

$P(X=10) = \binom{9}{2} \times (0.2)^3 \times (0.8)^7$.

$= 36 \times 0.008 \times 0.2097 = \underline{\underline{0.0604}}$

(ii) $P(X \le 12) = \sum_{x=3}^{12} \binom{x-1}{2} \cdot (0.2)^3 \cdot (0.8)^{x-3}$.

x	$\binom{x-1}{2}$	$(0.8)^{x-3}$	$p(x)$
3	1	1	0.008
4	3	0.8	0.0192
5	6	0.64	0.03072
6	10	0.512	0.04096
7	15	0.4096	0.04915
8	21	0.32768	0.05532
9	28	0.262144	0.05889
10	36	0.2097152	0.06039
11	45	0.16777216	0.06040
12	55	0.13421773	0.05906

$$p(x \le 12) = \sum_{x=3}^{12} \binom{x-1}{2} \cdot (0.2)^3 \cdot (0.8)^{(x-3)}$$

$= 0.008 + 0.0192 + 0.03072 + 0.04096 + 0.04915 + 0.05532 + 0.05889 + 0.06039 + 0.06040 + 0.05906$

$= 0.4442$

(iii) $Var = \frac{r(1-p)}{p^2}$

$r = 3 \quad p = 0.2$

$Var = \frac{3 \times 0.8}{0.04} = \frac{2.4}{0.04} = 60$