JKUAT

Stage II Semester 3

B.Science Information Technology

CAT I

STA 2200: Probability and Statistics II

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Q 1.

(a) i) Continuous random variable - is avariable that can take an infinite number of values within a given range. for example, the height of individuals can be measured and can take any value within interval.

ii) Expectations - is avalue of a random variable is a measure of the central tendency of the distribution of the variable. It is calculated as the weighted average of all possible values that the random variable that can take, each multiplied by its probability.

(b) Random variable X, let X be defined as:

$X = 2$ if the results is a '1' (winning KSh. 2)

$X = 1$ if the results is a '6' (winning KSh. 1)

$X = 1$ for all other outcomes (Losing KSh. 1)

Probability distribution of X

When rolling a fair six-sided die are:

$P(X=2) = 1/6$ roll (1)

$P(X=1) = 1/6$ roll (6)

$P(X=-1) = 4/6 = 2/3$ rolling (2, 3, 4 or 5)

Expected value $E(X)$

$E(X) = 2 \cdot 1/6 + 1 \cdot 1/6 - 1 \cdot 2/3$

$= 2/6 + 1/6 - 2/3 = 2/6 + 1/6 - 4/6 = 3/6 - 4/6 = -1/6$.

(c) mean of the distribution

$$E(X) = \int_0^1 x f(x)\,dx = \int_0^1 x(3.6x - 2.4x^2)\,dx$$

$$E(X) = \int_0^1 (3.6x^2 - 2.4x^3)\,dx = \left[\frac{3.6x^3}{3} - \frac{2.4x^4}{4}\right]_0^1$$

$$= [1.2 - 0.6) = \underline{\underline{0.6}}$$

median of distribution:

$P(x \le m) = 0.5$

$$F(x) = \int_0^x (3.6t - 2.4t^2)\,dt = [1.8t^3]_0^x = 1.8x^2 - 0.8x^3$$

$F(m) = 0.5$.

$1.8m^2 - 0.8m^3 = \underline{0.5}$.

$P(X) > 0.5)$

$P(X > 0.5) = 1 - P(x \leq 0.5) = 1 - F(0.5) = 1 - (1.8(0.5)^2$
$- 0.8(0.5)^3)$

Variance $Var(x)$

$Var(X) = E(X^2) - (E(X))^2$

$E(x^2) = \int_0^1 x^2 f(x)\,dx = \int_0^1 x^2(3.6x - 2.4x^2)\,dr$
$= \int_0^1 (3.6x^3 - 2.4x^4)\,dx$

$Var(x) = 0.42 - (0.6)^2 = 0.42 - 0.36 = \underline{\underline{0.06}}$

mean = 0.6

median (M) -

$P(x > 0.5) = 0.65$

variance = 0.06

Question TWO

a) probability of passenger showing up $= 1 - 0.05 = 0.05$

$n = 110$, $P = 0.95$

$P(x > 100$

$P(x \leq 100$ - subract 1.

$u = np = 110 \times 0.95 = 104.5$; $\sigma =$

$\sqrt{(np(1-p))} = \sqrt{(110 \times 0.95 \times 0.05)} \approx 2.26$

$Z = (100 - 104.5) / 2.26 \approx -1.99$

$P = (Z \leq -1.99) \approx 0.0233$

$P = (x > 100 = 1 - P(X \leq 100) \approx 1 - 0.0233 =$

$\underline{\underline{0.9767}}$

b)

Let λ be the average ~~of~~ number of claims. $P(2 \text{ claims})$

$= 3 \times P(4 \text{ claims})$

poisson distribution formula: $(e-\lambda\lambda 2)/2! = 3 \times (e-\lambda\lambda 4)/4!$

$\therefore$ $12 = 3\lambda 2$, so $\lambda 2 = 4$, and $\lambda = 2$

$\sqrt{2} = \sqrt{2} \approx \underline{\underline{1.414}}$

c) $\lambda = nP = 800 \times 0.04 = 32$

$P(X = 75)$: $P(x = 75) = (e - 32 \times 3275) / 75!$

≈ 0

Q2.

d) (i) poisson distribution with $\lambda = 400 \times 0.005 = 2$

$$P(X=1) = \frac{(e-2 \times 21)}{1!} \simeq \underline{\underline{0.2707}}$$

(ii) $P(x \leq 2) = P(X=0) + P(X=1) + P(X=2)$

$P(X=0) = e-2 \simeq 0.1353$; $P(X=1) = 0.2707$ from d) i) $P(X=2) = \frac{(e-2 \times 22)}{2!}$

$$= \underline{\underline{0.2707}}$$

$P(X \leq \ ) = 0.1353 + 0.2707 = \underline{\underline{0.6767}}$

$$= \underline{\underline{0.6767}}$$

e). $u = \sum = 20(0.15) + 30(0.10) + 40(0.05) + 50(0.20) + 60(0.10) + 70(0.10) + 80(0.30) = 52$

$E(x^2) = \sum = 20^2(0.15) + 30^2(0.10) + 80^2(0.30) = 3100$

$(\sigma^2) = E(x^2) - u^2 = 3100 - 52^2 = 576$; $\sigma = \sqrt{576} = 24$

$52 - 24 = 28$, $52 + 24 = 76$

Q2

$P(28 \leq X \leq 76) = P(X=30) + P(X=40) + P(X=50) + P(X=60) + P(X=70) = 0.10 + 0.05 + 0.20 + 0.10 + 0.10 = \underline{\underline{0.55}}$

$= \underline{\underline{55\%}}$

(f) (i)

$E(X) = \frac{1}{p} = 1/0.05 = \underline{\underline{20}}$

(ii) $P(X=x) = (1-p)x-1p$

$P(X=3) = (1-0.05)\ 3-1 \times 0.05 = (0.95)^2 \times 0.05 =$

$\underline{\underline{0.0451}}$

(iii) $F(n) = 1-(1-p)n$

Subs $n=5$

$F(5) = 1-(1-p)^5 \quad = \underline{\underline{1-(1-p)^{\wedge 5}}}$

(iv) $1-P = (1-p)^{10}$

$\underline{\underline{(1-p)^{\wedge}(10)}}$

6.

Formula Q2

(g) i) $P(X=K) = \binom{K-1}{r-1} p^{r} (1-p)^{K-r}$

where $r=3$ and $K=10$

using $p=0.2$

$$P(X=10) = \binom{10-1}{3-1} (0.2)^3 (0.8)^{10-3}$$

$$P(X=10) = \binom{9}{2} (0.2)^3 (0.8)^7$$

~~(9) {2} (0.2) ^3 (0.8) ^7~~ $= \binom{9}{2} (0.2)^3 (0.8)^7$

(ii)

(ii) $K=3$ to $K=12$

$$P(X \leq 12) = \sum_{K-3}^{12} \binom{K-1}{2} (0.2)^3 (0.8)^{K-3}$$

$$\sum_{K-3}^{12} \binom{K-1}{2} (0.2)^3 (0.8)^{K-3}$$

iii) $Var(X) = \frac{r(1-p}{p^2}$

$r=3$ and $p=0.2$

$$Var(X) = \frac{3(1-0.2)}{(0.2)^2} = \underline{60}$$