Question One

a) i) Continuous Random Variable → A Continuous Random variable takes any real value within a range. e.g The height of students in a class is a Continuous variable because it can take any value like 168.5 cm, 170.1 cm etc.

ii) Expectation → Expectation is the average or mean value a random variable is expected to take. $E(X) = \sum x \cdot P(x)$ or for Continuous variables, $E(X) = \int x f(x)\,dx$

b) Flip a Coin ten times.
Possible outcomes Heads (H), Tails (T)
Let random variable X = 1 if H, 0 if T.
Suppose you get: H, T, H, H, T, T, H, T, H, H
X = 1, 0, 1, 1, 0, 0, 1, 0, 1, 1
You observe a variation in outcomes hence X is a random variable.

c) Define X:
$X = 2$ for rolling a 1 → $P = \frac{1}{6}$

$X = 1$ for rolling a 6 → $P = \frac{1}{6}$

$X = -1$ for all others → $P = \frac{4}{6}$

$E(X) = 2 \cdot \frac{1}{6} + 1 \cdot \frac{1}{6} + (-1) \cdot \frac{4}{6} = \frac{2 + 1 - 4}{6} = -\frac{1}{6}$

You lose $\frac{1}{6}$ of a unit per game

d) $f(x) = 3.6x - 2.4x^2$, for $0 < x < 1$
Mean:

$E(x) = \int_0^1 x(3.6x - 2.4x^2)\,dx = \int_0^1 (3.6x^2 - 2.4x^3)\,dx = [1.2x^3 - 0.6x^4]_0^1 = 1.2 - 0.6 = 0.6$

Median:
Solve $\int_0^m (3.6x - 2.4x^2)\,dx = 0.5$

$\int_0^m (3.6x - 2.4x^2)\,dx = [1.8x^2 - 0.8x^3]_0^m = 1.8m^2 - 0.8m^3 = 0.5$

$P(x > 0.5)$

$P(x > 0.5) = \int_{0.5}^1 (3.6x - 2.4x^2)\,dx = [1.8x^2 - 0.8x^3]_{0.5}^1 = (1.8 - 0.8) - (0.45 - 0.1) = 1 - 0.35$

$\Rightarrow 0.65$

Variance:
$E(x^2) = \int_0^1 x^2(3.6x - 2.4x^2)\,dx = \int_0^1 (3.6x^3 - 2.4x^4)\,dx = [0.9x^4 - 0.48x^5]_0^1 = 0.9 - 0...$

$= 0.42$

$Var(x) = E(x^2) - (E(x))^2 = 0.42 - (0.6)^2 = 0.42 - 0.36 = 0.06$

Question two.

a) Let $x \sim Bin(110, 0.95)$

Normal Approx:

$\mu = 104.5, \sigma = \sqrt{110 \cdot 0.95 \cdot 0.05} = 2.28$

$$P(x > 100) = P\left(z > \frac{100.5 - 104.5}{2.28}\right)$$

$$= P(z > -1.75)$$
$$= 0.96$$

b $\frac{\lambda^2}{2} = 9 \cdot \frac{\lambda^4}{24} = \lambda^2 = 4 = \lambda = 2$

Std dev $= \sqrt{2} = 1.41$

c) $n = 800, P = 0.04 = \lambda = 32$

$P(x = 75) = \frac{32^{75} e^{-32}}{75!} =$ Verysmall(≈ 0)

d) $x \sim Poisson(\lambda = 2)$

i) $P(x = 1) = \frac{2^1 e^{-2}}{1!} = 2e^{-2} = 0.2707$

ii) $P(x \leq 2) = P(0) + P(1) + P(2) = e^{-2} + 2e^{-2} + 2e^{-2} = 5e^{-2} = 0.6767$

e) $\mu = \sum x P(x) = 55 \quad \sigma = 20$

Range: $[35, 75]$

Claim sizes 40, 50, 60, 70, 80

Total: $0.05 + 0.20 + 0.10 + 0.10 + 0.30 = 0.75$ or 75%

f) $P = 0.05$

i) $E(x) = \frac{1}{P} = 20$

ii) $P(x = 3) = (1-P)^2 \cdot P = 0.95^2 \cdot 0.05 = 0.0451$

iii) $P(x \leq 5) = 1 - (1-P)^5 = 1 - 0.95^5 = 0.2262$

iv) $P(x > 10) = 0.95^{10} = 0.5987$

9) $r = 3,\ p = 0.2$

i) $P(X = 10) = \binom{9}{2}(0.2)^3(0.8)^7 = 36 \cdot 0.008 \cdot 0.2097 = 0.0605$

ii) $P(X \leq 12) = \sum_{x=3}^{12} P(X = x) = 0.8954$

iii) $Var(X) = \frac{r(1-p)}{p^2} = \frac{3 \cdot 0.8}{0.04} = 60$