Question One BEATRICE CHEPKEMOI SCT221-D1-0054/2022

a) i) A continuous random variable is a type of random variable that can take any value within a given range or interval.

Examples: i) Temperature of a city on a day: Can be any real number like 32.5°C, 32.51°C

ii) Time taken to complete a task; time might be 5.3 minutes, 5.31 minutes

ii) The expectation of a random variable is essentially the long-run average or mean value it would take over many repetitions of an experiment.

Examples; Suppose X is the amount of time (in hours) a machine runs before maintenance and it follows an exponential distribution with:

$f(x) = \lambda e^{-\lambda x}, x \geq 0$

$E(X) = \int_0^\infty x \cdot \lambda e^{-\lambda x} dx = \frac{1}{\lambda}$

If $\lambda = 0.2$ then;

$E(X) = \frac{1}{0.2} = 5$ hours

b) Step 1: Define the Experiment; A fair coin has two sides: Head (H) and Tail (T)

Step 2: Assign values to the Outcomes

Let the random variable X represent the outcome:

$X = 1$ ~~of~~ if Heads

$X = 0$ if Tails

So; Outcome = Heads $\rightarrow X = 1$

Outcome = Tails $\rightarrow X = 0$

X is a random variable because it maps outcomes of experiment to real numbers based on chance.

Step 3: Check if X is a random variable

Yes because :- It's a function that assigns a number to each outcome.

- The value of X is not known in advance; it depends on the outcome of a random process
- It has a probability distribution:

Value of X	Meaning	Probability
1	Heads	0.5
0	Tails	0.5

c) The outcomes and winnings are: If result is 1, you win \$2

If result is 6, you win \$1

If result is 2, 3, 4, or 5 you lose \$1

Possible ~~num~~ values of XXX are: $X = 2X = 2X = 2$ (If die shows 1)

$X = 1X = 1X = 1$ (If die shows 6)

$X = 1X = -1X = -1$ (If die shows 2, 3, 4 or 5)

Probability of value of XXX:

d) If total area under PDF equals 1:

∫0·1 (3·6x − 2·4x2) dx = [1·8x2 − 0·8x3]0·1 = 1·8(1)2 − 0·8(1)3 = 1·8 − 0·8 = 1 \int ~~...~~
0^1 (3·6x − 2·4x^2)\, dx = \left[1·8x^2 − 0·8x^3\right]_0^1 = 1·8(1)^2 − 0·8(1)^3 = 1·8 − 0·8 = 1 ∫0,
(3·6x − 2·4x2)dx = [1·8x2 − 0·8x3]01 = 1·8(1)2 − 0·8(1)3 = 1·8 − 0·8 = 1

Mean

E[X] = ∫01x·f(x)dx = ∫01x(3·6x − 2·4x2) dx = ∫01 (3·6x2 − 2·4x3) dx\mathbb{E}[X] = \int_0^1 x
\cdot f(x)\, dx = \int_0^1 x (3·6x − 2·4x^2)\, dx = \int_0^1 (3·6x^2 − 2·4x^3)\, dx E[X] = ∫01x·f
(x)dx = ∫01 x(3·6x − 2·4x2) dx = ∫01(3·6x2 − 2·4x3)dx = [1·2x3 − 0·6x4]01 = 1·2 − 0·6 = 0·6 =
\left[1·2x^3 − 0·6x^4\right]_0^1 = 1·2 − 0·6 = \boxed{0·6} = [1·2x3 − 0·6x4]01 = 1·2 − 0·6 = 0·6

Mean = 0·6

Median

∫0mf(x)dx = 0·5\int_0^m f(x)\, dx = 0·5∫0mf(x)dx = 0·5∫0m(3·6x − 2·4x2)dx = 0·5\
int_0^m (3·6x − 2·4x^2)\, dx = 0·5∫0m(3·6x − 2·4x2)dx = 0·5[1·8x2 − 0·8x3]0m = 0·5 ⇒ 1·8m2
− 0·8m3 = 0·5\left[1·8x^2 − 0·8x^3\right]_0^m = 0·5\Rightarrow 1·8m^2 − 0·8m^3 =
0·5 [1·8x2 − 0·8x3]0m = 0·5 ⇒ 1·8m2 − 0·8m3 = 0·5

If m = 0·4m = 0·4m = 0·4:

0·8(0·4)3 − 1·8(0·4)2 + 0·5 = 0·0512 0·8(0·4)^3 − 1·8(0·4)^2 + 0·5 = 0·0512 0·8(0·4)
3 − 1·8(0·4)2 + 0·5 = 0·0512

If m = 1 = 0·42m = 0·42:

0·8(0·42)3 − 1·8(0·42)2 + 0·5 = −0·0007 0·8(0·42)^3 − 1·8(0·42)^2 + 0·5\approx −
0·0007 0·8(0·42)3 − 1·8(0·42)2 + 0·5 = −0·0007

median = ~~0·44~~ 0·415

P(X > 0·5)

P(X > 0·5) = ∫0·51(3·6x − 2·4x2)dx P(X > 0·5) = \int_{0·5}^1 (3·6x − 2·4x^2)\, dx P
(X > 0·5) = ∫0·51 (3·6x − 2·4x2)dx = [1·8x2 − 0·8x3]0·51 = (1·8 − 0·8) − (1·8·0·25 − 0·8×0·125) =
\left[1·8x^2 − 0·8x^3\right]_{0·5}^1 = (1·8 − 0·8) − \left(1·8\cdot 0·25 − 0·8\cdot 0·125\right
= [1·8x2 − 0·8x3]0·51 = (1·8 − 0·8) − (1·8 − 0·25 − 0·8×0·125) = 1·0 − (0·45 − 0·1) = 1·0 − 0·35 = 0·65 = 1·0
− (0·45 − 0·1) = 1·0 − 0·35 = \boxed{0·65} = 1·0 − (0·45 − 0·1) = 1·0 − 0·35 = 0·65

Variance

Var(X) = 0·42 − (0·6)2 = 0·42 − 0·36 = 0·06\text{Var}(X) = 0·42 − (0·6)^2 = 0·42 − 0·36
= 0·06 Var(X) = 0·42 − (0·6)2 = 0·42 − 0·36 = 0·06

Question Two

a) P(x>100) = 1 - P(x≤100) P(x>100) = 1 - P(x\ leq100) P(x>100) = 1 - P(x≤100)
P(x>100) = P(Y>100.5), Y~N(104.5, 5.225) P(x>100) \approx P \left(Y > 100.5\
right), \quad Y \sim N(104.5, 5.225) P(x>100) = P(Y>100.5), Y~N (104.5, 5.225)

Z = 100.5 - 104.5 2.285 = -4 2.285 = -1.75 Z = \frac{100.5 - 104.5}{2.285}\
approx\frac{-4}{2.285}\approx -1.75 Z = 2.285 100.5 - 104.5 = 2.285 - 4 = 1.75
P(X>100) = P(Z>-1.75) = 1 - P(Z<-1.75) = 1 - 0.0401 = 0.9599 P(X>100) \ approx
P(Z>-1.75) = 1 - P(Z<1.75) = 1 - 0.0401 = 0.9599

So ~~0 0.96~~ P(more than 100 passengers show up) = 0.96

So = 96 % chance that more passengers show up than seats available

~~b) Sample size : n = 800n~~
~~Probability a fan is defective : p = 0.04p~~
~~Exactly 75 fans are defective : P(x=75) P(x=75)~~
~~P(x=75) = e-λλ7575! = e-32.327575 P(x=75) = \frac{e^{-\lambda}\lambda^{75}}~~

b)

c) Sample size : n = 800n
Probability fan is defenstive : p = 0.04p
Exactly 75 fans are defective P(x=75) P(x=75)
P(x=75) = e-λλ7575 = e-32.7575 P(x=75) = \frac{e^{-\lambda}\lambda^{75}}
{75!} = \frac{e^{-32}\cdot 32^{75}}{75!} P(x=75) = 75!e - λλ75 = 75!e - 32.3275
P(x=75) = 1.78 × 10-15 \boxed{P(x=75)\approx 1.78 × 10^{-15}} P(x=75)
= 1.78 × 10 - 15
Chance of getting exactly 75 defective fans in a sample of 800 is virtually zero.

d i) Let λ = np = 400×0.005 = 2 \lambda = np = 400 \times 0.005 = 2λ = np = 400

Poisson(λ=2)

P(x=1) P(x=1) P(x=1)

P(x=1) = e-2.211 = 2e-2P(x=1) = \frac{e^{2}} \cdot 2^1}{1!} = 2e^{-2} P(x=1) =

P(x=1) = e-2.211 = 2e-2P(x=1) = \frac{e^{2}} \cdot 2^1}{1!} \approx 2 \cdot 0.1353 = 0.2707

2.21 = 2e-2 = ~~2 e 0 1 3~~ 2.01353 = 0.2707 \approx 2 \cdot 0.1353 = 0.2707

= 0.2707

ii) P(x ≤ 2) = P(0) + P(1) + P(2) P(X \leq 2) = P(0) + P(1) + P(2) P(X ≤ 2) = P(0) + P(1) + P(2)

P(0) = e-2.200!e = 2=0.1353P(0) = \frac{e^{2}} \cdot 2^0}{0} = e^{-2} = 0.1353P(0)

= e-2.20 = e-2 = 0.1353

P(1) = e-2.211 = 2e-2 = 0.2707P(1) = \frac{e^{2}} \cdot 2^1}{1!} = 2e^{-2} = 0.2707 P(1) =

1!e-2.21 = 2e-2 = 0.2707

P(2) = e-2.222 = 4e-22 = 2e-2 = 0.2707 P(2) = \frac{e^{-2}} \cdot 2^2}{0} = \frac{4e^

{-2} = 2e^{-2} = 0.2707P(2) = 2!e-2.22 = 24e-2 = 2e-2 = 0.2707

P(x ≤ 2) = 0.1353 + 0.2707 + 0.2707 = 0.6767P(X \leq 2)

= 0.1353 + 0.2707 + 0.2707 = 0.6767P(x ≤ 2) = 0.1353 + 0.2707 + 0.2707

= 0.6767

e) Var(X) = E[X2] - (E[X]2 \text{Var} (X) = E[X^2] - (E[X]^2 Var(X) = E[X2] - (E[X])2

Var(X) = 3500 - 552 = 3500 - 3025 = 475 \text{Var}(X) = 3500 - 55^2 = 3500

~~55~~ - 3025 = 475 Var(X) = 3500 - 552 = 3500 - 3025 = 475 σ = 475 = 21.79\

Sigma = \sq √475 \approx \boxed{21.79} σ = 475 = 21.79

P(40) = 0.05, P(50) = 0.20, P(60) = 0.10, P(70) = 0.10, P(40) = 0.05, \quad P(50) =

0.20, \quad P(60) = 0.10, \quad P(70) = 0.10P(40) = 0.05, P(50) = 0.20, P(60) = 0.10,

P(70) = 0.10

Total = 0.05 + 0.20 + 0.10 + 0.10 = 0.45

45% of claims are within one standard deviation of mean.

f) i) Probability of a defective bulb = p = 0.05p

Let XXX be number of bulbs tested until first defective one.

E(X) = 1p = 10.05 = 20E(X) = \frac{1}{p} = \frac{1}{0.05} = \boxed{20} E(X) = p1 =

0.051 = 20

ii) P(x = 3) = (1-p)3-1·p = (0.95)2·0.05P(X = 3) = (1-p)^{3-1} \cdot 0.05P(X = 3) =

(1-p)3-1p = (0.95)2·0.05 = 0.9025·0.05 = 0.0451 ~~= 0.9025 \cdot 0.05 =~~

iii) P(X ≤ 5) = 1-(1-p)5 = 1-(0.95)5P(X \leq 5) = 1 - (1-p)^5 = 1-(0.95)^5P(X ≤ 5) =

1-(1-p)5 = 1-0.95)5 ≐ 1-0.07738 = 0.2262

iv) P(x>10) = (1-p)10 =(0-95)10 ...
~~{0-95)^{10}~~

g) i) P(Y=10) = (92).(0-2)3.(0-8)7P(Y=10) = \binom{9}{2}\cdot (0-2)^3
cdot (0-8)^7P(Y=10) =(29).(0-2)3.(0-8)7 = 36-0-008-0-2097 = 0-060

=0-0604

ii) P(Y≤12) = Σk=312P(Y=k)P(Y\leq 12) = \sum_{k=3}^{12} P(Y=k)P(Y
k=3Σ12P(Y=k)
P(Y≤12) = 0-816

iii) Var(Y) = r(1-p)P2 = 3(1-0-2)(0-2)2 = ~~3-0-80-04~~ 3-0-80-04 = 2.4
= \frac{3\cdot (1-0-2)}{(0-2)^2} = \frac{r(1-p)}{p^2} = \frac{3\cdot (1-0-2)}{(0-2)^2} =
60\text{Var}(Y) = \frac{r(1-p)}{p^2} = \boxed{60} Var(Y) = p2r(1-p) = (0-
cdot 0-8}{0-04} = \frac{2.4}{0-04} = \boxed{60}
(1-0-2) = 0-~~04~~2.4 = 60

iv) P(X>10) = (1-p)10 = (0·95)10 = 0·5987 ~~P(X>10) = (1-p)^{10} = (0·95)^{10}~~

g) i) P(Y=10) = (92)·(0·2)3·(0·8)7P(Y=10) = \binom{9}{2}\cdot (0·2)^3\ cdot (0·8)^7P(Y=10) = (29)·(0·2)3·(0·8)7 = 36·0·008·0·2097 = 0·0604

= 0·0604

ii) P(Y≤12) = Σk=312P(Y=k)P(Y\leq12) = \sum_{k=3}^{12} P(Y=k)P(Y≤12) = k=3Σ12P(Y=k)

P(Y≤12) = 0·816

iii) Var(Y) = r(1-p)P2 = 3(1-0·2)(0·2)2 = ~~3·0·80·04~~ 3·0·80·04 = 2·40·04 = 60\text{Var}(Y) = \frac{r(1-p)}{P^2} = \frac{3\cdot (1-0·2)}{(0·2)^2} = \frac{3\cdot 0·8}{0·04} = \frac{2.4}{0·04} = \boxed{60}Var(Y) = P2r(1-p) = (0·2)23·(1-0·2) = 0·~~04~~2.4 = 60