NAME: IRON BUNDI

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Q (1) PROBABILITY STATISTIC 2.

a) i) A continuous random variable is one that can take any value within a given range.

ii) The expectation of a random variable is its average or mean value over many trials

b) $X = 1$ since $P(X=1) = 0.5$

$X = 0$ $P(X=0) = 0.5$

X is a random variable.

c)

Die roll	x	$P(x)$
1	+2	$\frac{1}{6}$
6	+1	$\frac{1}{6}$
2-5	-1	$\frac{4}{6}$

$E(x) = 2\times\frac{1}{6} + 1\frac{1}{6} + (-1)\frac{4}{6} = \frac{2+1-4}{6} = \frac{-1}{6}$

$-\frac{1}{6} = -0.17$

d) $f(x) = \begin{cases} 3.6x - 2.4x \\ 0 \end{cases}$ for $0 < x < 1$

$E(x) = \int_0^1 x(3.6x - 2.4x^2)\,dx = \int_0^1 3.6^2 - 2.4x^3)\,dx$

Mean $= [1.2x^3 - 0.6x^4]_0^1 = 1.2 - 0.6 = 0.6$

Variance $= E(x^2) = \int_0^1 x^2(3.6x - 2.4x^2)\,dx = \int_0^1 (3.6x^3 - 2.4x^4)$

$= 0.9x^4 - 0.48x^5]_0^1 = 0.9 - 0.48 = 0.42$

$Var(x) = E(x^2) - [E(x)]^2 = 0.42 - 0.6^2 = 0.42 - 0.36 =$

0.06 $P(X > 0.5)$:

$P(x > 0.5) = \int_{0.5}^1 (3.6x - 2.4x^2)\,dx$

$1.8x^2 - 0.8x^3]_{0.5}^1 = (1.8 - 0.8 - (1.8\times0.25 - 0.8\times0.125) =$

$1 - (0.45 - 0.1) = 1 - 0.35 = 0.65$

Median

$$\int_0^m (3.6x - 2.4x^2)\,dx = 0.5 \Rightarrow 1.8m^2 - 0.8m^3 = 0.5$$

$$1.8(0.62)^2 - 0.8(0.62)^3 = 0.52$$

Q2a) $n = 110 \quad p = 0.95, \quad P(X > 100) = 1 - P(X \leq 100$

Mean $= 110 \times 0.95 = 104.5$

Variance $= 110 \times 0.95 \times 0.05 = 5.225$

$SD = \sqrt{5.225} = 2.29.$

$$P(X > 100) = P\left(z > \frac{100.5 - 104.5}{2.29}\right)$$

$$P(z > -1.75) = \underline{0.96}$$

b) Let $P(X=2) = 3 \cdot P(X=4$

$$p(X = k) = \frac{e^{-\lambda}\lambda^k}{k!}$$

$$3 \Rightarrow \frac{\lambda^2}{2} \times \frac{24}{\lambda^4} = 3 \Rightarrow \frac{12}{\lambda^2} = 3 \Rightarrow \lambda^2 = 4 \Rightarrow \lambda = 2$$

$$\sqrt{2} = \underline{1.41}$$

c) $n = 800, \quad P = 0.04 \qquad \lambda = 800 \times 0.04 = 32.$

$$P(X = 75) = \frac{e^{-32} \times 32^{75}}{75}$$

Near 0

d) i) $\lambda = 400 \times 0.005 = 2$

$$p(X = 1) = \frac{e^{-2} \times 2^1}{1!} = 2e^{-2} = \underline{0.27}$$

ii) $p(X \leq 2) = P(0) + P(1) + P(2) = e^{-2}(1 + 2 + 2^2/2) =$

$e^{-2}(1+2+2) = 5e^{-2} = \underline{0.68}$

e) Let X = Claim amount.

$E(X) = \sum x \times P(x) = 20(0.15) + 30(0.1) + 80(0.3) = 55$

$E(X) = \sum x^2 \times P(x) = 20^2(0.15) + 80^2(0.30) = 3300$

$Var(X) = E(X)^2 - 3300 - 3025 = 275 \Rightarrow \sigma^2 = 275$

$\sqrt{275} = 16.6$

$55 - 16.6,\ 55 + 16.58 = 38.42,\ 71.58$

$0.05 + 0.2 + 0.1 + 0.1 = 0.45 = 45\%$

f) $p = 0.05$

i) $E(X) = \frac{1}{0.05} = 20.$

ii) $P(X = x) = (1 - p)^{x-1} \times p$

$(0.95)^2 \times 0.05 = 0.9025 \times 0.05 = 0.045.$

iii) $1 - (0.95)^5 = 1 - 0.7738 = 0.2262$

iv $(1 - p)^{10} = (0.95)^{10} = 0.5987$

g) i) $\binom{9}{2} \times (0.2)^3 \times (0.8)^7 = 36 \times 0.008 \times 0.2097 = 0.0604$

ii) $P(X \leq 12) = \sum_{x=3}^{12} P(X = x)$

0.8586

iii) Variance $= \frac{r(1-p)}{p^2} = \frac{3 \times 0.8}{0.2^2} = \frac{2.4}{0.04} = 60.$