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STA 2200: Probability and Statistics

Question One

(a) Explain the following

(i) Continuous random variable => It is a variable that can take on many value within the specified range or interval.

Example

The height of students in a class is a continuous random variable because it can take any value within a certain range (e.g between 150cm and 200cm

(ii) Expectation => It is the longrun average value of repetitions of the experiment it represents

Example

The expected value ~~within~~ when rolling a fair die is 3.5, calculated as $(1+2+3+4+5+6)/6$.

(b) Given a fair coin, perform an experimenting using the coin, and hence show that the outcome of your experiment is a random variable

A fair coin has outcomes; Heads (H) or Tails (T)

Perform an experiment: Toss the coin 5 times

Sample result: H, T, H, H, T.

Let X = number of Heads → X can be 0, 1, 2, 3, 4 or 5. So X is a random variable because its value depends on the outcome of a random experiment (the coin toss).

(c) Die Tossing Game

Win \$2 if you get a '1'

Win \$1 if you get a '6'

Lose \$1 for 2, 3, 4, 5.

X	$p(X)$
2	$1/6$
1	$1/6$
-1	$4/6$

$E(X) = \sum x P(x) = (2)\frac{1}{6} + (-1)\frac{4}{6} + (1)\frac{1}{6} = \frac{2-4+1}{6} = -\frac{1}{6}$

$= \$0.17$ average per game

(d) The Random Variable X has probability density function

$$f(x) = \begin{cases} (3.6x - 2.4x^2) & 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the mean and Median of the distribution. Also find $P(X > 0.5)$ and $Var(X)$.

Mean

$E(X) = \int x \cdot f(x)\,dx$ from 0 to 1 $= \int (3.6x^2 - 2.4x^3)\,dx$ from 0 to 1 $= [1.2x^3 - 0.6x^4]$ from 0 to 1 $= (1.2 - 0.6) - (0 - 0) = 0.6$

Median

$$\int_0^m f(x)\,dx = 0.5$$

$$\int_0^M (3.6x - 2.4x^2)\,dx = 0.5$$

$$\rightarrow [1.8x^2 - 0.8x^3]_0^M = 0.5 \Rightarrow$$

$$1.8m^2 - 0.8m^3 = 0.5$$

$P(X)$

$P(X > 0.5)$:

$P(X > 0.5) = \int_{0.5}^{1} (3.6x - 2.4x^2)\,dx = [1.8x^2 - 0.8x^3]_{0.5}^{1} =$

$(1.8 - 0.8) - (0.45 - 0.1) =$

$1 - 0.35 = 0.65$

Variance

$E(X^2) = \int_0^1 x^2(3.6x - 2.4x^2)\,dx = \int_0^1 (3.6x^3 - 2.4x^4)\,dx$

$= [0.9x^4 - 0.48x^5]_0^1 = 0.9 - 0.48 = 0.42$

$Var(X) = E(X^2) - (E(X))^2$

$0.42 - (0.6)^2 = 0.42 - 0.36 = 0.06$

Question two

2 (a) Calculate the probability that more passengers show up for the flight than there are seats available

$n = 110, p = 0.95$

$P(X > 100) = 1 - P(X \leq 100)$

$\mu = np = 110 \times 0.95 = 104.5$, $\sigma^2 = np(1-p) = 110 \times 0.95 \times 0.05 = 5.225 \Rightarrow \sigma = 2.29$.

$P(X > 100) = P(X \geq 101) = P\left(Z > \frac{100.5 - 104.5}{2.29}\right) =$

$P(Z > -1.75) = 0.96$ = 96% chance more than 100 show up.

2 (b) Poisson claims

Given $P(2 \text{ claims}) = 3 \times P(4 \text{ claims})$

For Poisson: $P(x) = (e^{-m} m^x / x!$

$\frac{\lambda^2 e^{-\lambda}}{2!} = 3 \times \frac{\lambda^4 e^{-\lambda}}{4!} = \frac{\lambda^2}{2} = 3 \times \frac{\lambda^4}{24} = 12\lambda^2$

$= 3\lambda^4 \Rightarrow \lambda^2 = 4 \Rightarrow \lambda = 2$

Standard Deviation $= \sqrt{\lambda} = \sqrt{2} = 1.414$

(c) Using the Poisson approximation to the binomial.

Binomial ($n = 800$, $p = 0.04$) $\Rightarrow$ $\lambda = 800 \times 0.04 = 32$

Poisson ($\lambda = 32$), find $P(X = 75)$

$$P(X = 75) = \frac{32^{75} e^{-32}}{75!} = \underline{0}$$

(d) Industrial Accidents

(i) Will be an accident on only one day?

Binomial $n = 400$, $p = 0.005$

$$P(X = 1) = \frac{2^1 e^{-2}}{1!} = 2e^{-2} = 0.2707$$

(ii) Are at most two days with an accident?

$$P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2)$$

$$(0.995)^{400} + 400(0.005)(0.995)^{399} + C(400, 2)(0.005)^2 (0.995)^{398} = 0.1353 + 0.2707 + 0.2707 = \underline{\underline{0.6767}}$$

(e) What Percentage of the claims

Mean

$E(X) = \sum x P(x) = 20 \times 0.15 + 30 \times 0.10 + 80 \times 0.3$

$= 20(0.15) + 30(0.10) + 40(0.05) + 50(0.20)$
$+ 60(0.10) + 70(0.10) \pm 3 + 3 + 2 + 10 + 6 + 7 + 24$
$= 55$

$E(X^2)$.
$= (400 \times 0.15) + (900 \times 0.10) + (1600 + 0.05) + (2500 \times 0.20)$
$+ (3600 \times 0.10) + (4900 \times 0.10) + (6400 \times 0.30)$
$= 60 + 90 + 80 + 500 + 360 + 490 + 1920 = 3500$

Variance

$Var(X) = E(X^2) - [E(X)]^2 = 3500 - 3025 = 475$

$\sigma = \sqrt{475} = 21.79$

Range within one standard deviation

$55 \pm 21.79 \Rightarrow (33.21, 76.79)$

Claims in this range:
40, 50, 60, 70

Prob = $0.05 + 0.20 + 0.10 + 0.10 = 0.45$ or 45%

(f) Geometric Distribution

(1) Expected number of tests

$E(X) = 1/p = 1/0.05 = 20$

2(f)(ii) First defective on 3rd test
$P(X=3) = (1-p)^2 p = (0.95)^2 (0.05) = \underline{0.045125}$

2f(iii) First defective within first 5 tests
$P(X \leq 5) = 1-(1-p)^5 = (0.95)^5 = 1-0.7738 = \underline{0.2262}$

2f(iv) No defective in first 10 tests
$P(X>10) = (1-p)^{10} = (0.95)^{10} = 0.5987$

(g) Negative Binomial Distribution
(i) 3rd success on 10th call
$P(X=10) = C(9,2)(0.2)^3(0.8)^7 = 36 \times 0.008 \times 0.2097 = 0.0604.$

(ii) 3 successes within first 12 calls
$P(X \leq 12) =$ Sum from $k=3$ to 12 of $C(k-1,2)(0.2)^3(0.8)^{k-3} = \underline{0.602}$

(iii) Variance
$Var(X) = r(1-p)/p^2 = 3(0.8)/(0.2)^2 = 2.4/0.04 = \underline{60}$