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STAGE II SEMESTER III EXAMINATIONS FOR THE DEGREE OF BACHELOR OF SCIENCE IN INFORMATION TECHNOLOGY

CAT I

STA 2200: Probability and Statistics II

Date: July, 2025 Due: 20th July, 2025 @11.59pm

Instructions:

1. Answer ALL questions, and clearly show your working
2. This work should be **handwritten and NOT typed. ANY Typed work will not be accepted for marking**
3. Your solutions should then be **scanned and saved as PDF document and submitted as a single document, with your registration number as the file name**

Question One.

a) i) Continuous Random Variable

This is a Variable that can assume an infinite number of possible values, this Variables are usually Obtained by measurement from a sample population. For example Weight of a person and the body temperature of a person.

ii) Expectation

The expected Value of a discrete random Variable is the average Value of the random Variable.

For example $x_1 \dots x_2 \dots x_n$ are different values of random Variable 'x' therefore the expectation is defined as

$E(x) = x_1 \cdot p(x_1) + x_2 \cdot p(x_n)$ denoted as

$E(x) = \mu$

b) Using a fair Coin that contains two side Head and Tail.

Experiment

Toss a fair coin 10 Times, Recording the number of Heads (H) and tails (T) each time.

Using a Random Variable, let's define a random Variable x = Number of Heads in 10 tosses.

The possible values is that x can take values from 0 to 10, depending on the number of heads

c) Let x be a random Variable, the amount of money won or lost in a single toss. x can take the following values

$x = 2$ (if the result is a "4")

$x = 1$ (if the result is a "6")

$x = -1$ if the result is any number 2, 3, 4, or 5

The probability distribution

$p(x=2) = 1/6$

$p(x=1)\ p(6) = 1/6$

$p(x=-1) = p\ (2, 3, 4, \text{ or } 5) = 4/6 = 2/3$

The expected value

$E(x) = \Sigma\ (x + p(x))$

$(2 \times 1/6) + (1 \times \frac{1}{6}) + (1 - 2/3)$

$= -1/6.$

d) i) Mean

$E(x) = \int_{-\infty}^{\infty} x \cdot f(x)\, dx$

$E(x) = \int_0^1 x\,(3.6x - 2.4x^2)\, dx$

$= \int_0^1 (3.6x^2 - 2.4x^3)\, dx$

$3.6 \cdot \frac{x^3}{3} - 2.4\,\frac{x^4}{4} \Big|_0^1$

$1.2 - 0.6 = 0.6$

ii) Median

$\int_0^x (3.6t - 2.4t^2)\, dt = 0.5$

$1.8t^2 - 0.8t^3 \Big|_0^x = 0.5$

$1.8x^2 - 0.8x^3 = 0.5$

Using expectations $p(x > 0.5)$

$\int_0^1 (3.6x - 2.4x^2)\, dx$

$1.8x^2 - 0.8x^3 \Big|_{0.5}^1$

$(1.8 - 0.8) - (1.8 \cdot 0.25) - (0.8 \times 0.125)$

$= 0.65$

Variance (x)

$$\int_0^1 x^2 (3.6x - 2.4x^2)\, dx$$

$$\int_0^1 (3.6x^3 - 2.4x^4)\, dx$$

$$3.6 \cdot \frac{x^4}{4} - 2.4 \cdot \frac{x^5}{5} \Big|_0^1$$

$$= 0.9 - 0.48$$

$$= 0.42$$

$$Var(x) = 0.42 - (0.6)^2$$

$$= 0.06$$

Question Two

a) $n = 110$

$p = 0.95$

$p(x > 100)$

$p(x = 101) + p(x = 102) + \ldots p(x = 110)$

$p(x = k) = \binom{n}{k} p^k (1-p)^{n-k}$ $\binom{n}{k} p^k (1-p)^{n-k}$

$$p(x > 100) = \sum_{k=101}^{110} \binom{110}{k} (0.95)^k (0.05)^{110-k}$$

$k = 104$ to $k = 110$

Using normal approximation to the Binomial

$p(x > 100)$

$$p(x > 100) = p\left(z > \frac{100.5 - 104.5}{2.286}\right) = p\left(z > \frac{-4}{2.286}\right)$$

$$p(z > -1.75) = 1 - p(z \leq -1.75) = 1 - 0.0401$$

$$= 0.96.$$

$$= 0.96.$$

b) $p(x=k) = \frac{e^{-\lambda}\lambda^{k}}{k!}$, k is the claim

$p = x = 2 = 3 \times p(x=4)$

$\frac{e^{-\lambda}\lambda^{2}}{2!} = 3 \times \frac{e^{-\lambda}\lambda^{4}}{4!}$ $\frac{e^{-\lambda}\lambda^{2}}{2!} = 3 \times \frac{e^{-\lambda}\lambda^{4}}{4!}$

$\frac{\lambda^{2}}{2} = \frac{3 \times \lambda^{4}}{24}$

$\frac{\lambda^{2}}{2} = \frac{\lambda^{4}}{8}$

$\lambda^{2}(\lambda^{2}-4) = 0$ $\lambda^{2}(\lambda^{2}-4) = 0$

$\sqrt{\lambda^{2}} = 4 \neq 0$

$(\sqrt{\lambda^{2}})^{2} = (4)^{2}$

$\sqrt{\lambda^{2}} = \sqrt{4}$

$\sqrt{\lambda} = \pm 2$, $\sqrt{\lambda} = 2$

c) $n = 800$, $p = 0.04$

$\lambda = np = 800 \times 0.04 = 32$

$p(x=k) = \frac{e^{-\lambda}\lambda^{k}}{k!}$, $k = 75$

$p(x=75) = \frac{e^{-32}(32)^{75}}{75!}$ $\frac{e^{-32}(32)^{75}}{75!}$

$= \frac{e^{-32}(32)^{75}}{75!}$

d) i)

Given $n = 100$, $p = 0.005$

Mean $(\lambda) = np = 400 \times 0.005 = 2$

$p(x=k) = \frac{e^{-\lambda}\lambda^{k}}{k!}$, $k = 1$

$p(x=1) = \frac{e^{-2}(2)^{1}}{1!} = 2e^{-2}$ $2e^{-2}$

$= 0.271$

ii) $p(x \le 2) = p(x=0) + p(x=1) + p(x=2)$

$= e^{-2} + 2e^{-2} + \frac{e^{-2}(2)^2}{2!}$

$= 5e^{-2}\ 5e^{-0}\ p(x \le 2)$

$= 0.68$

e) Mean = Claim x probability

$(20 \times 0.15) + (30 \times 0.10) + (40 \times 0.05) + (50 \times 0.20) + (60 \times 0.10)$
$(70 \times 0.10) + (80 \times 0.10)$

$= 55$

Variance = Claim size (mean) x probability

$= [(20-55)^2\ 20.15] + (30-55]^2\ 20.10 + (40-55) \times$
$20.05 + [50-55] \times 20.20 + (60-55) \times 20.10 + 70.55 \times 20.10$
$+ 80-55 \times 20.10$

$= 475$

$Sd = \sqrt{\text{variance}}$

$= \sqrt{475}$

$= 21.8$

Claims within range 40, 50, 60 and 70

$= 0.43$

$0.43 \times 100\%$

$= 43\%$

f) i) $E(x) = \frac{5}{p}$ $p = 0.05$, $E(x) = \frac{5}{p}(0.05) = 20$

ii) $p(x) = p(x=x) = 5(1-p)^2(x-1)$ for $x=3$ and
$p = 0.05$, $p(x=3) = 5(1-0.05)^{(3-1)} = 0.05 - 0.01$
$= 0.04$

iii) $Fx = 1 - (-p)^x$ for $x = 5$ and $p = 0.05$

$p(x \le 5) = 1-(1-0.05)^5 = 1-(0.95)^5$
$= 0.23$

iv) $p(x > 10)$
$= (1-p)^{10} = (0.95)^{10}$
$= 0.6$

g)(i) $x = 10$
$r = 3$
$p = 0.2$

$p(x=10) = \binom{10-1}{3-1}(0.2)^3(1-0.2)^{10-3}$

$\binom{9}{2}(0.2)^3(0.8)^7$

$p(x=10) = \frac{9!}{2!(9-2)!} = \frac{9 \times 8}{2} = 36$

$p(x=10) = 36 \times (0.2)^3 \times (0.8)^7$
$= 0.0604$

ii) $p(x \le 12) = \sum_{x=3}^{12} \binom{x-1}{2}(0.2)^3(0.8)^{x-3}$

$x = 3$ to $x = 12$

iii) $p(x \le 12)$
$\sum_{x=3}^{12} \binom{x-1}{2}(0.2)^3(0.8)^{x-3}$

$\binom{12-1}{2}(0.2)^3(0.8)^{12-3}$

$= \binom{11}{2}(0.008) \times (0.8)^9$

$= 0.6336$