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SCT221-D1-0020/2024

1 (a)(i) Continuou random variable
- This is a type of variable that takes an infinite number of possible values within a given range.
- They are typically associated with measurements such as time, height, weight, or temperature.

Example: Ricky is 1.2m tall. Ricky's height is 1.2. The number is a continuous random variable.

(ii) Expectation
This refers to the long-run average value of repetitions of the experiment it represents. It gives the possible out comes.

b) Coin toss experiment.
- Sample Space = {(Heads, Tails)}
- Let X be the random variable

$$X = \begin{cases} 1 \text{ - If heads appears} \\ 0 \text{ - If tail appears} \end{cases}$$

The value of X can be 0 or 1

$P(X=1) = 0.5, \quad P(x=0) = 0.5$ Ans

c) Six-Sided Die Experiment.

$$X = \begin{cases} 2 & \text{if result} = 1 \\ 1 & \text{if result} = 6 \\ -1 & \text{if result} = \text{otherwise} \end{cases}$$

Distribution

	X	P(x)
1	2	$\frac{1}{6}$
6	1	$\frac{1}{6}$
2-5	-1	$\frac{4}{6}$

$$E(X) = 2 \cdot \frac{1}{6} + 1 \cdot \frac{1}{6} + (-1) \cdot \frac{4}{6}$$

$$= \frac{2}{6} + \frac{1}{6} - \frac{4}{6} = -\frac{1}{6}$$

$$= -\frac{1}{6}$$ Answer

d) The random variable X has probability density function

$$f(x) = \begin{cases} (3.6x - 2.4x^2) \\ 0 & \text{Otherwise} \end{cases}$$

Find the mean and median
Find $P(X > 0.5)$ and $Var(X)$

Mean = E(X)

Mean $\mu = E(X) = \int_0^1 x(3.6x - 2.4x^2)\,dx$

$= \int_0^1 (3.6x^2 - 2.4x^3)\,dx$

$= (3.6)\frac{1}{3} - (2.4)\frac{1}{4} = 1.2 - 0.6$

$= 0.6$ Answer

Variance $Var(\cancel{X}) = E(x^2) - [E(x)]^2$

$E(X^2) = \int_0^1 x^2(3.6x - 2.4x^2)\,dx$

$= \int_0^1 (3.6x^3 - 2.4x^4)\,dx$

$= 3.6\left(\frac{1}{4}\right) - 2.4\left(\frac{1}{5}\right)$

$= 0.9 - 0.48 = 0.42$

$Var(X) = 0.42 - (0.6)^2$

$= 0.42 - 0.36$

$= 0.06$ Answer

2

(a) Number of Seats = 100

Number of tickets sold = 110

Probability a passenger not showing up = 1 − 0.05 = 0.95

Let X be number shows up

$$X \sim \text{Binomial}(n = 110, p = 0.95$$

$$P(x > 100) = ?$$

Binomial Distribution

$$X \sim N(\mu, \sigma^2)$$

$$\mu = np = 110 \times 0.95 = 104.5$$

$$\sigma = \sqrt{np(1-p)} = \sqrt{110 \times 0.95 \times 0.05} = \sqrt{5.225} = 2.29$$

$$P(x > 100) = P\left(Z > \frac{100.5 - 104.5}{2.29}\right)$$

Approximation

$$= P\left(Z > \frac{-4}{2.29}\right) = P(Z > -1.75)$$

Standard normal table

$$P(Z > -1.75) = 1 - P(Z < -1.75)$$

$$= 1 - 0.0401$$

$$P = 0.9599$$ Answer

b) The number of claims $X \sim \text{Poisson}(\lambda)$

$P(X=2) = 3 \times P(X=4)$

Standard deviation of $X = ?$

$$P(X=k) = \frac{e^{-\lambda}\lambda^k}{k!}$$

Given

$$\frac{\cancel{e^{-\lambda}}\lambda^2}{2!} = 3\left(\frac{\cancel{e^{-\lambda}}\lambda^4}{4!}\right)$$

$$= \frac{\lambda^2}{2} = 3\left(\frac{\lambda^4}{24}\right)$$

$$= \frac{\lambda^2}{2} = \frac{3\lambda^4}{24} = \frac{\lambda^4}{8}$$

$$\frac{4\lambda^2 = \lambda^4}{\lambda^2 \quad \lambda^2} = 4 = \lambda^2$$

$$= 2$$

$$\sigma = \sqrt{\lambda} = \sqrt{2}$$

$$= 1.41$$ Answer

c)

Samp $n = 800$

Probability of defective: $p = 0.04$

$X = 75$

Binomial ($n = 800$ \$ $p = 0.04$)

Large = n

Small = p

moderate = $np = \lambda$

$\lambda = np = 800 \times 0.04 = 32$

Approx ~~$\lambda = np$~~ $X \sim$ Poisson $\lambda = 32$

$P(X = 75)$

$P(X = k) = \frac{e^{-\lambda}\lambda^{k}}{k!} =$

$= P(X = 75) = \frac{e^{-32} \times 32^{75}}{75!}$

$= P(X = 75) \approx 1.46 \times 10^{-14}$ Ans

d) A day probability of Accident = $P = 0.05$
No. of Days $= 400$

let No. of days $\bar{c}$ accidents be X

$X \sim$ Binomial $(n = 400, P = 0.005)$

By Poisson Approx.

$n = 400$ (large)
$p = 0.005$ (small)

$\lambda = np = 400 \times 0.05 = 2$

Approx. $= 2$.

(i) Probability of accident on only one day

$$P(X=1) = \frac{e^{-2} \times 2^1}{1!} = 2e^{-2}$$

$$P(X=1) \simeq 2 \times 0.1353 = 0.2707$$
$$= 0.2707 \text{ Answer}$$

(ii) Probability of at most two accident days

$$P(0) = \frac{e^{-2} \times 2^0}{0!} = e^{-2} \simeq 0.1353$$

$$P(1) = 2e^{-2} \simeq 0.2707$$

$$P(2) = \frac{e^{-1} \times 2^2}{2!} = \frac{4e^{-2}}{2} = 2e^{-2} = 0.2707$$

$$P(X \leq 2 = 0.1353 + 0.2707 + 0.2707$$
$$= 0.6767 \text{ Answer}$$

e) $\mu = \sum x \times P(x)$

$\mu = 20(0.15) + 30(0.10) + 40(0.0.05) + 50(0.20) + 60(0.10) + 70(0.10) + 80(0.30)$

$= 3.00 + 3.00 + 2.00 + 10.00 + 6.00 + 7.00 + 24.00$

$= 55.00$

Variance $\sigma = \Sigma(x-\mu)^2 \times P(x)$

$(20-55)^2 \times 0.15 = 122.25 \ldots$

$122.25 + 62.5 + 11.25 + 5.0 + 2.5 + 22.5 + 187.5$

$= 413.5$

$\sigma = \sqrt{413.5} \approx 20.33$

Ranges $34.67 — 75.33$ is $40, 50, 60, 70$

$P(40) + P(50) + P(60) + P(70) =$

$= 0.05 + 0.20 + 0.10 + 0.10 = 0.45$

$= 45\%$ Ans.

8) Let $X \sim$ Geometric $(p = 0.05)$

(i) Expected number of light bulbs tested

$$E(X) = \frac{1}{p} = \frac{1}{0.05} = \underline{\underline{20}} \text{ Ans}$$

(ii) Probability that the first defective bulb is found on the 3rd test

$$P(\cancel{X} = x) = (1-p)^{x-1} \times p$$

if $x = 3$

$$P(X=3) = (0.95)^2 \times 0.05 = 0.9025 \times 0.05$$
$$= \underline{\underline{0.0451}} \text{ Ans}$$

(iii) Probability that the first ~~de~~fective bulb is found within the first 5 tests

$$P(X \leq 5) = 1 - (1-P)^5 = 1 - (0.95)^5$$

$$= 1 - 0.7738 = \underline{\underline{0.2262}} \text{ Ans}$$

(iv) Probability that no defective bulb is found within the first 10 tests

$$P(X > 10) = (1-p)^{10} = (0.95)^{10}$$

$$\approx \underline{\underline{0.5987}} \text{ Ans}$$

(2)

(7) $r = 3$ - successful resolution

$p = 0.2$ prob of successful resolution per call

X = number of calls until the 3rd resolution

(i) Probability that the 3rd customer issue is resolved on the 10th call

$$P(X=x) = \binom{x-1}{r-1} p^r (1-p)^{x-r}$$

$$P(X=10 = \binom{9}{2} \times (0.2)^3 \times (0.8)^7$$

$$\approx 0.0604$$ Answer

(ii) The Probability that the call center resolves 3 issues within the first 12 calls

$$x = 12$$

$$P(X \leq 12) \sum_{x=3}^{12} P(X=x) = P(X \leq 12$$

$$= 0.834$$ Answer

(iii) Variance in the number of calls needed to resolve 3 customer issues

$$Var(X) = \frac{r(1-p)}{p^2} = \frac{3(0.8)}{0.2^2} = \frac{2.4}{0.04}$$

$$= 60$$ Answer