BEATRICE CHEPKEMOI SCT221-D1-0054/2022

Question One

a) i) A continuous random variable is a type of random variable that can take any value within a given range or interval.

Examples: i) Temperature of a city on a day: Can be any real number like 32.5°C, 32.51°C

ii) Time taken to complete a task; ~~time~~ might be 5.3 minutes, 5.31 minutes

ii) The expectation of a random variable is essentially the long-run average or mean value it would take over many repetitions of an experiment.

Examples: Suppose X is the amount of time (in hours) a machine runs before maintenance and it follows an exponential distribution with:

$f(x) = \lambda e^{-\lambda x}, x \geq 0$

$E(X) = \int_0^{\infty} x \cdot \lambda e^{-\lambda x} dx = \frac{1}{\lambda}$

If $\lambda = 0.2$ then:

$E(X) = \frac{1}{0.2} = 5$ hours

b) Step 1: Define the Experiment: A fair coin has two sides: Head (H) and Tail (T)

Step 2: Assign values to the Outcomes

Let the random variable X represent the outcome:

$X = 1$ ~~of~~ if Heads

$X = 0$ if Tails

So: Outcome = Heads → $X = 1$

Outcome = Tails → $X = 0$

What's the experiment.

X is a random variable because it maps outcomes of experiment to real numbers based on chance.

Step 3: Check if X is a random variable

Yes because: - It's a function that assigns a number to each outcome.

- The value of X is not known in advance; it depends on the outcome of a random process

- It has a probability distribution:

Value of X	Meaning	Probability
1	Heads	0.5
0	Tails	0.5

c) The outcomes and winnings are: If result is 1, you win \$2

If result is 6, you win \$1

If result is 2, 3, 4 or 5 you lose \$1

Possible ~~numb~~ values of XXX are: $X = 2X = 2X = 2$ (If die shows 1)

$X = 1X = 1X = 1$ (If die shows 6)

$X = 1X = -1X = -1$ (If die shows 2, 3, 4 or 5)

Probability of value of XXX:

d) If total area under PDF equals 1:

$\int 0 \cdot 1 (3.6x - 2.4x2)\, dx = [1.8x2 - 0.8x3] 0 \cdot 1 = 1.8(1)2 - 0.8(1)3 = 1.8 - 0.8 = 1$ \int ~~...~~
0^1 (3.6x - 2.4x^2)\,dx = \left[1.8x^2 - 0.8x^3\right]_0^1 = 1.8(1)^2 - 0.8(1)^3 = 1.8 - 0.8 = 1 $\int 0 1$
$(3.6x - 2.4x2)dx = [1.8x2 - 0.8x3] 01 = 1.8(1)2 - 0.8(1)3 = 1.8 - 0.8 = 1$

Mean

E[X] = ∫01 x · f(x) dx = ∫01 x(3.6x − 2.4x2) dx = ∫01 (3.6x2 − 2.4x3) dx \mathbb{E}[X] = \int_0^1 x
\cdot f(x)\, dx = \int_0^1 x (3.6x - 2.4x^2)\, dx = \int_0^1 (3.6x^2 - 2.4x^3)\, dx E[X] = ∫01 x · f
(x) dx = ∫01 x(3.6x − 2.4x2) dx = ∫01 (3.6x2 − 2.4x3) dx = [1.2x3 − 0.6x4] 01 = 1.2 − 0.6 = 0.6 =
\left[1.2x^3 - 0.6x^4\right]_0^1 = 1.2 - 0.6 = \boxed{0.6} = [1.2x3 − 0.6x4] 01 = 1.2 − 0.6 = 0.6

Mean = 0.6

Median

∫0m f(x) dx = 0.5 \int_0^m f(x)\,dx = 0.5 ∫0m f(x) dx = 0.5 ∫0m (3.6x − 2.4x2) dx = 0.5 \
int_0^m (3.6x - 2.4x^2)\,dx = 0.5 ∫0m (3.6x − 2.4x2) dx = 0.5 [1.8x2 − 0.8x3] 0m = 0.5 ⇒ 1.8m2
− 0.8m3 = 0.5 \left[1.8x^2 - 0.8x^3\right]_0^m = 0.5 \Rightarrow 1.8m^2 - 0.8m^3 =
0.5 [1.8x2 − 0.8x3] 0m = 0.5 ⇒ 1.8m² − 0.8m³ = 0.5

If m = 0.4 m = 0.4 m = 0.4:

0.8(0.4)3 − 1.8(0.4)2 + 0.5 = 0.0512 0.8(0.4)^3 − 1.8(0.4)^2 + 0.5 = 0.0512 0.8(0.4)
3 − 1.8(0.4)2 + 0.5 = 0.0512

If m = 1 = 0.42 m = 0.42:

0.8(0.42)3 − 1.8(0.42)2 + 0.5 = −0.0007 0.8(0.42)^3 − 1.8(0.42)^2 + 0.5 \approx −
0.0007 0.8(0.42)3 − 1.8(0.42)2 + 0.5 = −0.0007

median = ~~0.44~~ 0.415

P(X > 0.5)

P(X > 0.5) = ∫0.51 (3.6x − 2.4x2) dx P(X > 0.5) = \int_{0.5}^1 (3.6x − 2.4x^2)\,dx P
(X > 0.5) = ∫0.51 (3.6x − 2.4x2) dx = [1.8x2 − 0.8x3] 0.51 = (1.8 − 0.8) − (1.8 · 0.25 − 0.8 × 0.125) =
\left[1.8x^2 − 0.8x^3\right]_{0.5}^1 = (1.8 − 0.8) − \left(1.8 \cdot 0.25 − 0.8 \cdot 0.125 \right
= [1.8x2 − 0.8x3] 0.51 = (1.8 − 0.8) − (1.8 − 0.25 − 0.8 × 0.125) = 1.0 − (0.45 − 0.1) = 1.0 − 0.35 = 0.65 = 1.0
− (0.45 − 0.1) = 1.0 − 0.35 = \boxed{0.65} = 1.0 − (0.45 − 0.1) = 1.0 − 0.35 = 0.65

Variance

Var(X) = 0.42 − (0.6)2 = 0.42 − $\overset{0.36}{\cancel{0.3}}$ = 0.06 \text{Var}(X) = 0.42 − (0.6)^2 = 0.42 − 0.36
= 0.06 Var(X) = 0.42 − (0.6)2 = 0.42 − $\overset{0.36}{\cancel{0.36}}$ = 0.06

Question Two

a) P(x>100) = 1 - P(x ≤ 100) P(x>100) = 1 - P(x \ leq100) P(x>100) = 1 - P(x ≤ 100)
P(x>100) = P(Y>100.5), Y ~ N(104.5, 5.225) P(x>100) \ approx P \ left (Y > 100.5 \ right), \quad Y \ sim N(104.5, 5.225) P(x>100) = P(Y>100.5), Y ~ N (104.5, 5.225)

Z = 100.5 - 104.52.285 = -42.285 = -1.75 Z = \frac {100.5 - 104.5}{2.285} \ approx \frac{-4}{2.285} \approx -1.75 Z = 2.285100.5 - 104.5 = 2.285 - 4 = 1.75
P(X>100) = P(Z>-1.75) = 1 - P(Z<-1.75) = 1 - 0.0401 = 0.9599 P(X>100) \ approx P(Z>-1.75) = 1 - P(Z<1.75) = 1 - 0.0401 = 0.9599

So ~~0.96~~ P(more than 100 passengers show up) = 0.96

So = 96% chance that more passengers show up than seats available

~~b) Sample size : n = 800n~~
~~Probability a fan is defective : p = 0.04p~~
~~Exactly 75 fans are defective : P(x=75) P(x=75)~~
~~P(x=75) = e - λλ7575! = e - 32.327575 P(x=75) = \frac{e^{-\lambda}\lambda^{75}}~~

b)

c) Sample size : n=800n
Probability fan is defenstive : P = 0.04P
Exactly 75 fans are defective P(x=75) P(x=75)
P(x=75) = e-λλ 7575 = e -32.7575 P(x=75) = \frac{e^{-\lambda}\lambda^{75}} {75!} = \frac{e^{-32}\cdot 32^{75}}{75!} P(x=75) = 75!e - λλ75 = 75!e - 32.3275
P(x=75) = 1.78 x 10-15 \boxed{P(x=75) \approx 1.78 × 10^{-15}} P(x=75)
= 1.78x10-15
Chance of getting exactly 75 defective fans in a sample of 800 is virtually zero.

d) i) Let λ = np = 400×0.005 = 2 \lambda = np = 400 \times 0.005 = 2λ = np = 400...

Poisson(λ=2)

P(x=1) P(x=1) P(x=1)

P(x=1) = e−2.211 = 2e−2 P(x=1) = \frac{e^{2}} \cdot 2^1}{1!} = 2e^{-2} P(x=1) = ...

P(x=1) = e−2.211 = 2e−2 P(x=1) = \frac{e^{2}}{2} \cdot 2^1}{1!} ...

P(x=1) = e−2.211 = 2e−2 ... \approx 2 \cdot 0.1353 = 0.2707

2.21 = 2e−2 = ~~2.0.13~~ 2.01353 = 0.2707 \approx 2 \cdot 0.1353 = 0.2707

= 0.2707

ii) P(x ≤ 2) = P(0) + P(1) + P(2) P(X \leq 2) = P(0) + P(1) + P(2) P(X ≤ 2) = P(0) + P(1) + P(2)

P(0) = e−2.200!e = 2 = 0.1353 P(0) = \frac{e^{-2}} \cdot 2^0}{0!} = e^{-2} = 0.1353 P(0)

= e−2.20 = e−2 = 0.1353

P(1) = e−2.211 = 2e−2 = 0.2707 P(1) = \frac{e^{-2}} \cdot 2^1}{1!} = 2e^{-2} = 0.2707 P(1) =

1!e−2.21 = 2e−2 = 0.2707

P(2) = e−2.222 = 4e−22 = 2e−2 = 0.2707 P(2) = \frac{e^{-2}} \cdot 2^2}{2!} = \frac{4e^...

{-2} = 2e^{-2} = 0.2707 P(2) = 2!e−2.22 = 24e−2 = 2e−2 = 0.2707

P(x ≤ 2) = 0.1353 + 0.2707 + 0.2707 = 0.6767 P(X \leq 2)

= 0.1353 + 0.2707 + 0.2707 = 0.6767 P(X ≤ 2) = 0.1353 + 0.2707 + 0.2707

= 0.6767

e) Var(X) = E[X2] − (E[X] 2 \text{Var}(X) = E[X^2] − (E[X])^2 Var(X) = E[X2] − (E[X])2

Var(X) = 3500 − 552 = 3500 − 3025 = 475 \text{Var}(X) = 3500 − 55^2 = 3500

~~55~~ − 3025 = 475 Var(X) = 3500 − 552 = 3500 − 3025 = 475 σ = 475 = 21.79 \sigma = \sqrt{475} \approx \boxed{21.79} σ = 475 = 21.79

P(40) = 0.05, P(50) = 0.20, P(60) = 0.10, P(70) = 0.10, P(40) = 0.05, \quad P(50) =

0.20, \quad P(60) = 0.10, \quad P(70) = 0.10 P(40) = 0.05, P(50) = 0.20, P(60) = 0.10,

P(70) = 0.10

Total = 0.05 + 0.20 + 0.10 + 0.10 = 0.45

45% of claims are within one standard deviation of mean.

f) i) Probability of a defective bulb = p = 0.05p

Let XXX be number of bulbs tested until first defective one.

E(X) = 1p = 10.05 = 20 E(X) = \frac{1}{p} = \frac{1}{0.05} = \boxed{20} E(X) = p1 =

0.051 = 20

ii) P(X = 3) = (1 − p) 3−1 · p = (0.95)2·0.05 P(X = 3) = (1 − p)^{3−1} \cdot 0.05 P(X = 3) =

(1 − p) 3 − 1 p = (0.95)2·0.05 = 0.9025·0.05 = 0.0451 ~~= 0.9025 \cdot 0.05 =~~

iii) P(X ≤ 5) = 1 − (1 − p) 5 = 1 − (0.95)5 P(X \leq 5) = 1 − (1 − p)^5 = 1 − (0.95)^5 P(X ≤ 5) =

1 − (1 − p) 5 = 1 − 0.95)5 ≐ 1 − 0.07738 = 0.2262

iv) $P(X>10)=(1-p)10=(0.95)10$ ...
~~(0.95)^{10}~~

g) i) $P(Y=10)=(92).(0.2)3.(0.8)7P(Y=10)$ = \binom{9}{2}\cdot (0.2)^3 ... $P(Y=10)=(29).(0.2)3.(0.8)7 = 36 \cdot 0.008 \cdot 0.2097 = 0.060$
cdot (0.8)^7

$= 0.0604$

ii) $P(Y\le 12)=\sum k=312P(Y=k)P(Y$\leq 12) = \sum _{k=3}^{12} P(Y=k)P(Y ...
$k=3 \sum 12 P(Y=k)$
$P(Y\le 12)=0.816$

iii) $Var(Y)=r(1-p)P2=3(1-0.2)(0.2)2=$ ~~3.0.80.04~~ $3.0.80.04=2.4$ ... = \frac{3 \cdot (1-0.2)}{(0.2)^2} =
60\text{Var}(Y) = \frac{r(1-p)}{p^2} = \frac{3 \cdot (1-0.2)}{(0.2)^2} = \boxed{60} $Var(Y)=p2r(1-p)=(0.$ ...
cdot 0.8}{0.04} = \frac{2.4}{0.04} = \boxed{60} Var(Y) ...
$(1-0.2)=0.\cancel{04}2.4=60$

iv) P(x>10) = (1-p)10 = (0·95) 10 = 0·5987~~P(x>10) = (1-p)^{10} = (0.95)^{10}~~

g) i) P(Y=10) = (92)·(0·2)3·(0·8)7P(Y=10) = \binom{9}{2}\cdot (0·2)^3\cdot (0·8)^7P(Y=10) = (29)·(0·2)3·(0·8)7 = 36·0·008·0·2097 = 0·0604

= 0·0604

ii) P(Y≤12) = Σk=312P(Y=k)P(Y\leq12) = \sum_{k=3}^{12} P(Y=k)P(Y≤12) = k=3Σ12P(Y=k)

P(Y≤12) = 0·816

iii) Var(Y) = r(1-p)P2 = 3(1-0·2)(0·2)2 = ~~3·0·80·04~~ 3·0·80·04 = 2·40·04 = 60\text{Var}(Y) = \frac{r(1-p)}{P^2} = \frac{3\cdot (1-0·2)}{(0·2)^2} = \frac{3\cdot 0·8}{0·04} = \frac{2·4}{0·04} = \boxed{60}Var(Y) = P2r(1-p) = (0·2)23·(1-0·2) = 0·~~04~~2·4 = 60