

Exercise 3

1. A random sample of 250 adults was taken, and they were asked whether they prefer watching sports or opera on television. The following table gives the two-way classification of these adults.

	Prefer Watching Sports	Prefer Watching Opera	Total
Male	96	24	120
Female	45	85	130
Total	141	109	250

- a. If one adult is selected at random from this group, find the probability that this adult
- prefers watching opera

$$P(\text{Opera}) = \frac{n(\text{Opera})}{n(\text{Total})} = \frac{109}{250} = 0.436$$

- prefers watching sports given that the adult is a female

$$P(\text{Sports}|\text{Female}) = \frac{n(\text{Sports and Female})}{n(\text{Female})} = \frac{45}{130} = 0.3462$$

- is a female and prefers watching opera

$$P(\text{Female and Opera}) = \frac{n(\text{Female and Opera})}{n(\text{Total})} = \frac{85}{250} = 0.34$$

- prefers watching sports or is a male

$$\begin{aligned} P(\text{Sports or Male}) &= P(\text{Sport}) + P(\text{Male}) - P(\text{Sports and Male}) \\ &= \frac{141}{250} + \frac{120}{250} - \frac{96}{250} = 0.66 \end{aligned}$$

Or,

$$P(\text{Sports or Male}) = \frac{96 + 24 + 45}{250} = 0.66$$

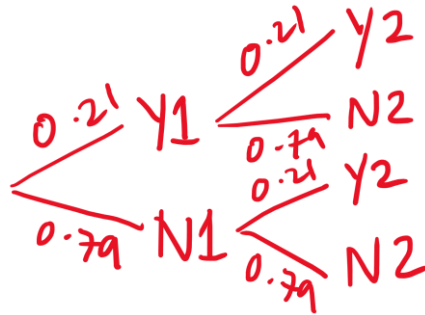
- b. Are the events female and prefers watching sports independent? Are they mutually exclusive? Explain why or why not.

The events female and prefers watching sports are not independent. This is because $P(\text{Female and Sports}) \neq P(\text{Female}) \times P(\text{Sports})$. You can calculate this.

The events female and prefers watching sports are not mutually exclusive because both can happen together. $n(\text{Female and Sports}) \neq 0$.

2. In a Gallup Annual Economy and Personal Finance poll, conducted April 3–6, 2014, 21% of adults aged 18 to 29 said that college costs and loans were the biggest financial problem their families were dealing with. Suppose two adults aged 18 to 29 are selected. Find the following probabilities.

a. Draw a tree diagram for this event.



- b. Both adults will say that college costs and loans are the biggest financial problem their families are dealing with.

These are independent events.

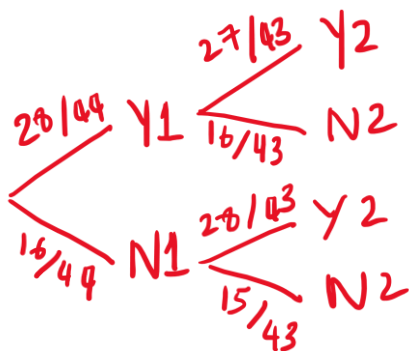
$$P(\text{both said so}) = P(Y1 \text{ and } Y2) = 0.21 \times 0.21 = 0.0441$$

- c. Exactly one adult will say that college costs and loans are the biggest financial problem their families are dealing with.

$$\begin{aligned} P(\text{exactly one said so}) &= P(Y1 \text{ and } N2) + P(N1 \text{ and } Y2) \\ &= 0.21 \times 0.79 + 0.79 \times 0.21 = 0.3318 \end{aligned}$$

3. A car rental agency currently has 44 cars available, 28 of which have a GPS navigating system. Two cars are selected at random from these 44 cars.

a. Draw a tree diagram for this event.



- b. Find the probability that both of these cars have GPS navigation systems.

These are dependent events. The second outcome depends on the first outcome.

$$P(\text{both has GPS}) = P(Y1 \text{ and } Y2) = P(Y1) \times P(Y2|Y1) = \frac{28}{44} \times \frac{27}{43} = 0.3996$$

- c. Find the probability that only one of these cars has GPS navigation systems.

$$P(\text{only one has GPS}) = P(Y1 \text{ and } N2) + P(N1 \text{ and } Y2) = \frac{28}{44} \times \frac{16}{43} + \frac{16}{44} \times \frac{28}{43} \\ = 0.4736$$

4. A certain state's auto license plates have three letters of the alphabet followed by a three-digit number.

- a. How many different license plates are possible if all three-letter sequences are permitted and any number from 000 to 999 is allowed?

$$\underline{26} \underline{26} \underline{26} \underline{10} \underline{10} \underline{10}$$

$$\text{Total way} = 26 \times 26 \times 26 \times 10 \times 10 \times 10 = 17576000$$

- b. Arnold witnessed a hit-and-run accident. He knows that the first letter on the license plate of the offender's car was a B, that the second letter was an O or a Q, and that the last number was a 5. How many of this state's license plates fit this description?

First letter is known. Second letter is either of two letters. Last number is known

$$\underline{1} \underline{2} \underline{26} \underline{10} \underline{10} \underline{1}$$

$$\text{Total way} = 1 \times 2 \times 26 \times 10 \times 10 \times 1 = 5200$$

5. A box contains 10 red marbles and 10 green marbles.

- a. Sampling at random from this box five times with replacement, you have drawn a red marble all five times. What is the probability of drawing a red marble the sixth time?

$$\frac{10}{20} = \frac{1}{2}$$

- b. Sampling at random from this box five times without replacement, you have drawn a red marble all five times. Without replacing any of the marbles, what is the probability of drawing a red marble the sixth time?

$$\frac{5}{15} = \frac{1}{3}$$

- c. You have tossed a fair coin five times and have obtained heads all five times. A friend argues that according to the law of averages, a tail is due to occur and, hence, the probability of obtaining a head on the sixth toss is less than .50. Is he right? Is coin

tossing mathematically equivalent to the procedure mentioned in part a or the procedure mentioned in part b above? Explain.

He is not right. The coin tosses follow procedure in part (a) where they are independent. (This is also known as gambler's fallacy where gamblers believe they are more likely to win after losing consecutively)

6. There are 5 boys and 7 girls in a class. 4 students will be selected to represent the class in a relay race where they will take turn running on the track and passing the baton to the next student. Suppose the team members are selected at random with equal probability.

- a. How many ways can the students be selected for the team?

$$\text{Total ways} = {}^{12}C_4 = 495$$

- b. Find the probability that the team consists of 4 boys or 4 girls only.

$$\text{Probability} = \frac{{}^5C_4 + {}^7C_4}{{}^{12}C_4} = 0.08081$$

- c. Suppose that a team of 2 boys and 2 girls is formed, and their positions on the track (first, second, third, or fourth runner) are chosen at random. Find the probability that the 2 boys run before the 2 girls in the team.

$$\text{Probability} = \frac{2 \times 2}{4!} = 0.1667$$