Lectures on Manifold (Work in Progress)

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1 Preliminaries

Topology

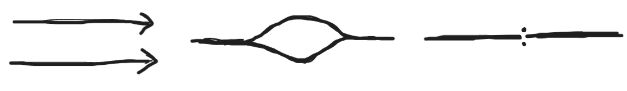
Definition 1.1 (Hausdorff Space). *A topological space* $(X, \mathcal{O})$ *is called Hausdorff space if, for any* $p, q \in X$*, there exists open sets* U, V *which include* p, q *respectively and which satisfy* $U \cap V = \emptyset$*.*

In other words, given two distinct points, they always have disjoint open neighbourhoods.

To see why Hausdorff condition is important, let us consider a Non-Hausdorff space first. Perhaps the easiest to visualise is the line with two origins which is just given by gluing two real lines together, point-for-point except for the origins of both lines. Let $\sqcup$ denotes disjoint union. More precisely, we define a space l to be

$$l = \mathbb{R}_1 \sqcup \mathbb{R}_2 / \sim$$

when $(x, 1) \sim (x, 2)$ for all $x \neq 0$. In the topological sense of convergence of sequences, we can say that a sequence of points in $l \backslash \{O_1, O_2\}$ converges to O_1 and also converges to O_2. This phenomena do not occur in Hausdorff spaces.



2 Smooth Manifold

Abstract

A smooth manifold is a space that locally looks like Euclidean space, and is equipped with a smooth structure that allows calculus to be done on the smooth functions on manifold.

Smooth manifolds arise naturally in many areas of mathematics and physics, and are essential in the study of geometry, topology, and mathematical physics like general relativity theory.

A topological manifold M of dimension n is a Hausdorff topological space which locally looks like the space $\mathbb{R}^n$.

References

[1] author, *title of a book*, publisher, year