Symmetric difference

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Definition 1. *A symmetric difference of the two sets A and B is*

$$A \triangle B = (A \backslash B) \cup (B \backslash A)$$

Show that $A \triangle B = (A \cup B) \backslash (A \cap B)$.

Proof. For any $x \in (A \backslash B) \cup (B \backslash A)$ we have

$$x \in A \backslash B \text{ or } x \in B \backslash A$$

$$\therefore (x \in A \text{ and } x \notin B) \text{ or } (x \in B \text{ and } x \notin A)$$

Observe that $x \in A$ and $x \notin B$ means that $x \notin A \cap B$. Similarity $x \in B$ and $x \notin A$ also means that $x \notin A \cap B$. Thus

$$(x \in A \text{ and } x \notin A \cap B) \text{ or } (x \in B \text{ and } x \notin A \cap B)$$

$$\therefore x \in A \cup B \text{ and } x \notin A \cap B$$

Hence $x \in (A \cup B) \backslash (A \cap B)$.

Conversely, for any $x \in (A \cup B) \backslash (A \cap B)$, we have

$$x \in A \cup B \text{ and } x \notin A \cap B$$

$$\therefore (x \in A \text{ or } x \in B) \text{ and } (x \notin A \text{ and } x \notin B)$$

$$\therefore (x \in A \text{ and } x \notin B) \text{ or } (x \in B \text{ and } x \notin A)$$

$$\therefore x \in A \backslash B \text{ or } x \in B \backslash A$$

Hence $x \in (A \backslash B) \cup (B \backslash A)$. $\square$

Draw the Venn diagrams for this proof.