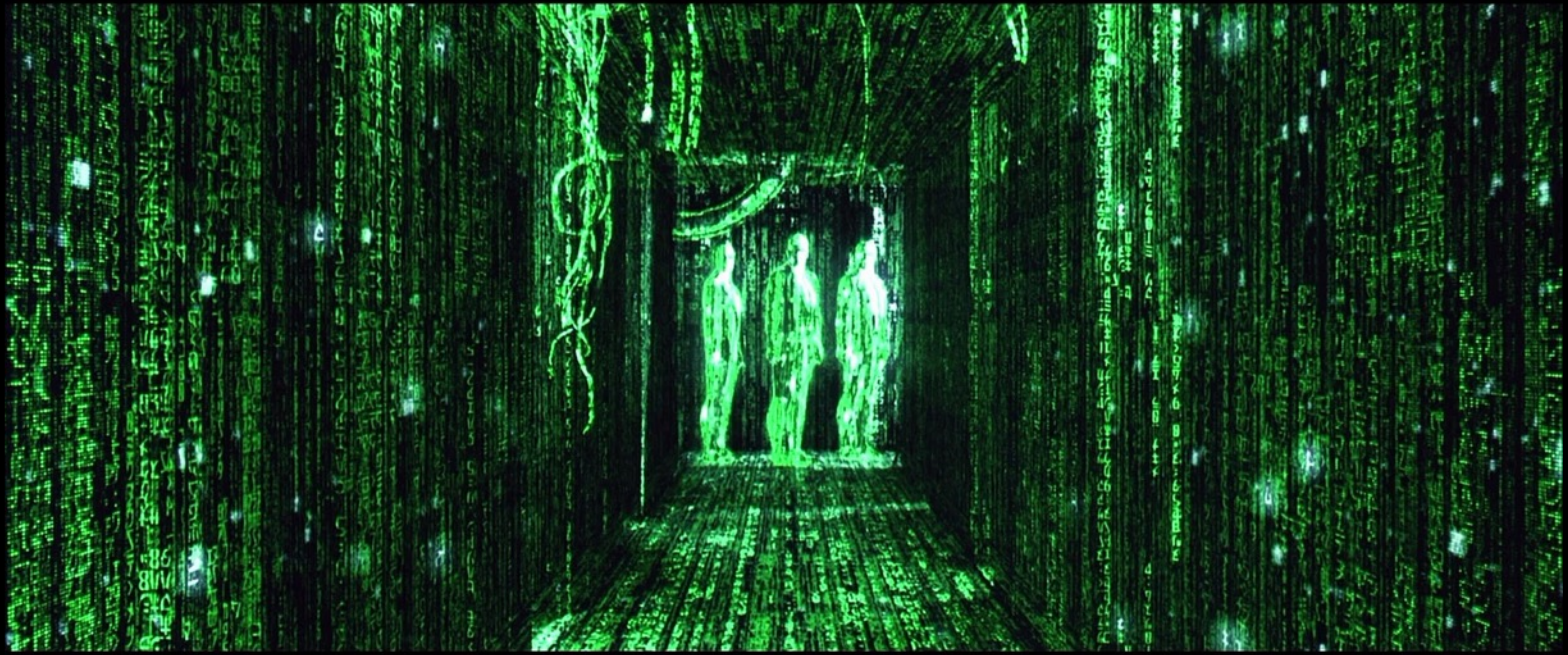
Matrix transformations.

Part 2



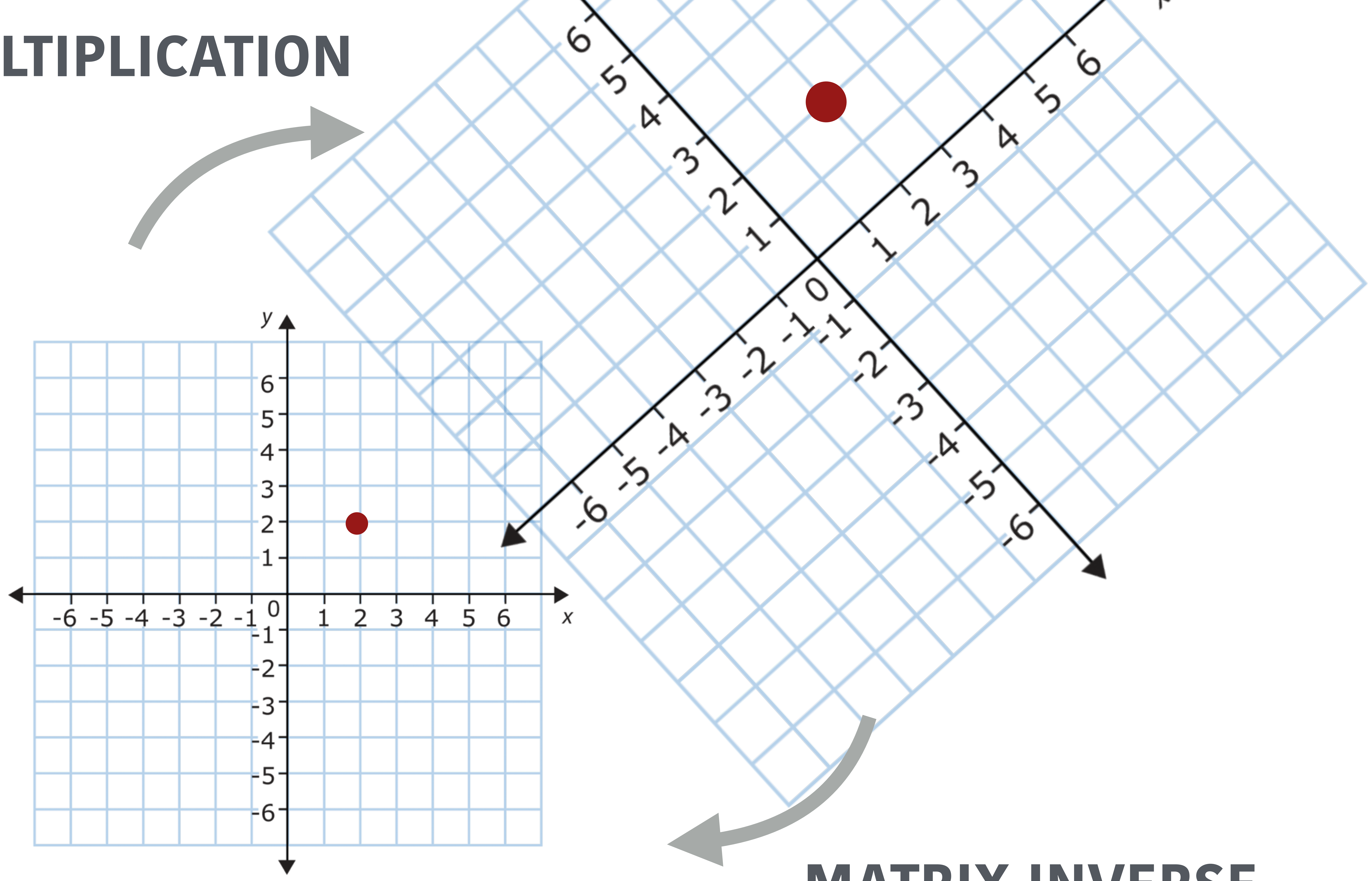
Inverse of a matrix.

Inverse of a matrix.

=

A **matrix** that “undoes” the transformation of the original matrix.

MATRIX MULTIPLICATION



MATRIX INVERSE

Inverse of a matrix.

- Scaled by **1/scale**

Inverse of a matrix.

- Scaled by **1/scale**
- Rotated by the **transpose of the rotation**.

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Inverse of a matrix.

- Scaled by **1/scale**
- Rotated by the **transpose of the rotation**.
- Translated by the **translation * -1**

The Inverse

```
Matrix Matrix::inverse() {
    float m00 = m[0][0], m01 = m[0][1], m02 = m[0][2], m03 = m[0][3];
    float m10 = m[1][0], m11 = m[1][1], m12 = m[1][2], m13 = m[1][3];
    float m20 = m[2][0], m21 = m[2][1], m22 = m[2][2], m23 = m[2][3];
    float m30 = m[3][0], m31 = m[3][1], m32 = m[3][2], m33 = m[3][3];

    float v0 = m20 * m31 - m21 * m30;
    float v1 = m20 * m32 - m22 * m30;
    float v2 = m20 * m33 - m23 * m30;
    float v3 = m21 * m32 - m22 * m31;
    float v4 = m21 * m33 - m23 * m31;
    float v5 = m22 * m33 - m23 * m32;

    float t00 = + (v5 * m11 - v4 * m12 + v3 * m13);
    float t10 = - (v5 * m10 - v2 * m12 + v1 * m13);
    float t20 = + (v4 * m10 - v2 * m11 + v0 * m13);
    float t30 = - (v3 * m10 - v1 * m11 + v0 * m12);

    float invDet = 1 / (t00 * m00 + t10 * m01 + t20 * m02 + t30 * m03);

    float d00 = t00 * invDet;
    float d10 = t10 * invDet;
    float d20 = t20 * invDet;
    float d30 = t30 * invDet;

    float d01 = - (v5 * m01 - v4 * m02 + v3 * m03) * invDet;
    float d11 = + (v5 * m00 - v2 * m02 + v1 * m03) * invDet;
    float d21 = - (v4 * m00 - v2 * m01 + v0 * m03) * invDet;
    float d31 = + (v3 * m00 - v1 * m01 + v0 * m02) * invDet;

    v0 = m10 * m31 - m11 * m30;
    v1 = m10 * m32 - m12 * m30;
    v2 = m10 * m33 - m13 * m30;
    v3 = m11 * m32 - m12 * m31;
    v4 = m11 * m33 - m13 * m31;
    v5 = m12 * m33 - m13 * m32;

    float d02 = + (v5 * m01 - v4 * m02 + v3 * m03) * invDet;
    float d12 = - (v5 * m00 - v2 * m02 + v1 * m03) * invDet;
    float d22 = + (v4 * m00 - v2 * m01 + v0 * m03) * invDet;
    float d32 = - (v3 * m00 - v1 * m01 + v0 * m02) * invDet;

    v0 = m21 * m10 - m20 * m11;
    v1 = m22 * m10 - m20 * m12;
    v2 = m23 * m10 - m20 * m13;
    v3 = m22 * m11 - m21 * m12;
    v4 = m23 * m11 - m21 * m13;
    v5 = m23 * m12 - m22 * m13;

    float d03 = - (v5 * m01 - v4 * m02 + v3 * m03) * invDet;
    float d13 = + (v5 * m00 - v2 * m02 + v1 * m03) * invDet;
    float d23 = - (v4 * m00 - v2 * m01 + v0 * m03) * invDet;
    float d33 = + (v3 * m00 - v1 * m01 + v0 * m02) * invDet;

    Matrix m2;

    m2.m[0][0] = d00;
    m2.m[0][1] = d01;
    m2.m[0][2] = d02;
    m2.m[0][3] = d03;
    m2.m[1][0] = d10;
    m2.m[1][1] = d11;
    m2.m[1][2] = d12;
    m2.m[1][3] = d13;
    m2.m[2][0] = d20;
    m2.m[2][1] = d21;
    m2.m[2][2] = d22;
    m2.m[2][3] = d23;
    m2.m[3][0] = d30;
    m2.m[3][1] = d31;
    m2.m[3][2] = d32;
    m2.m[3][3] = d33;

    return m2;
}
```

The matrix **class**.

Row major vs. **column major**.

$$\text{Row major} \quad \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \qquad \text{Column major} \quad \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$

We will use **column major** order as our standard.

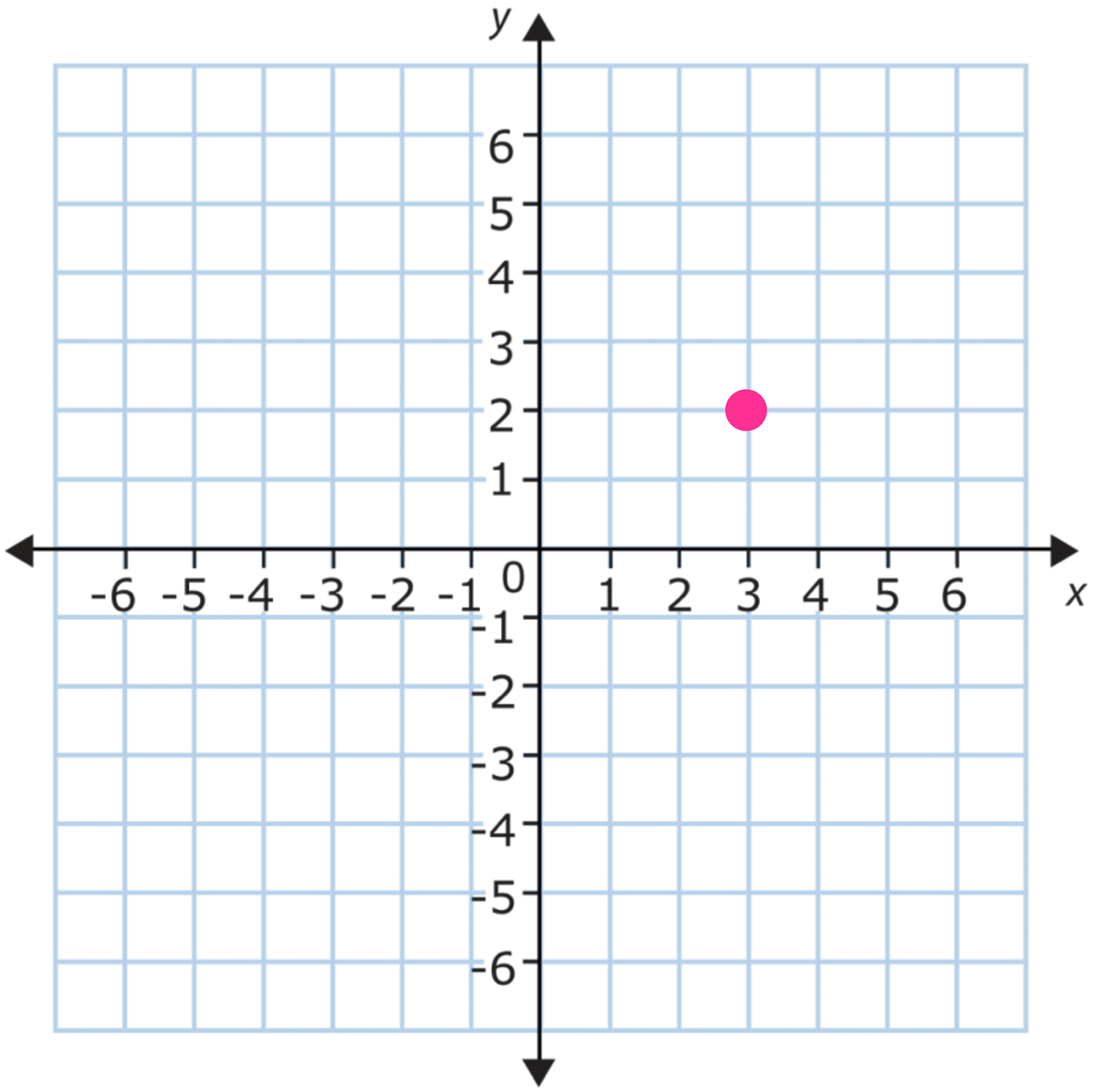
A **vector** class.

```
class Vector {
    public:

        Vector();
        Vector(float x, float y, float z);

        float length();
        void normalize();

        float x;
        float y;
        float z;
};
```

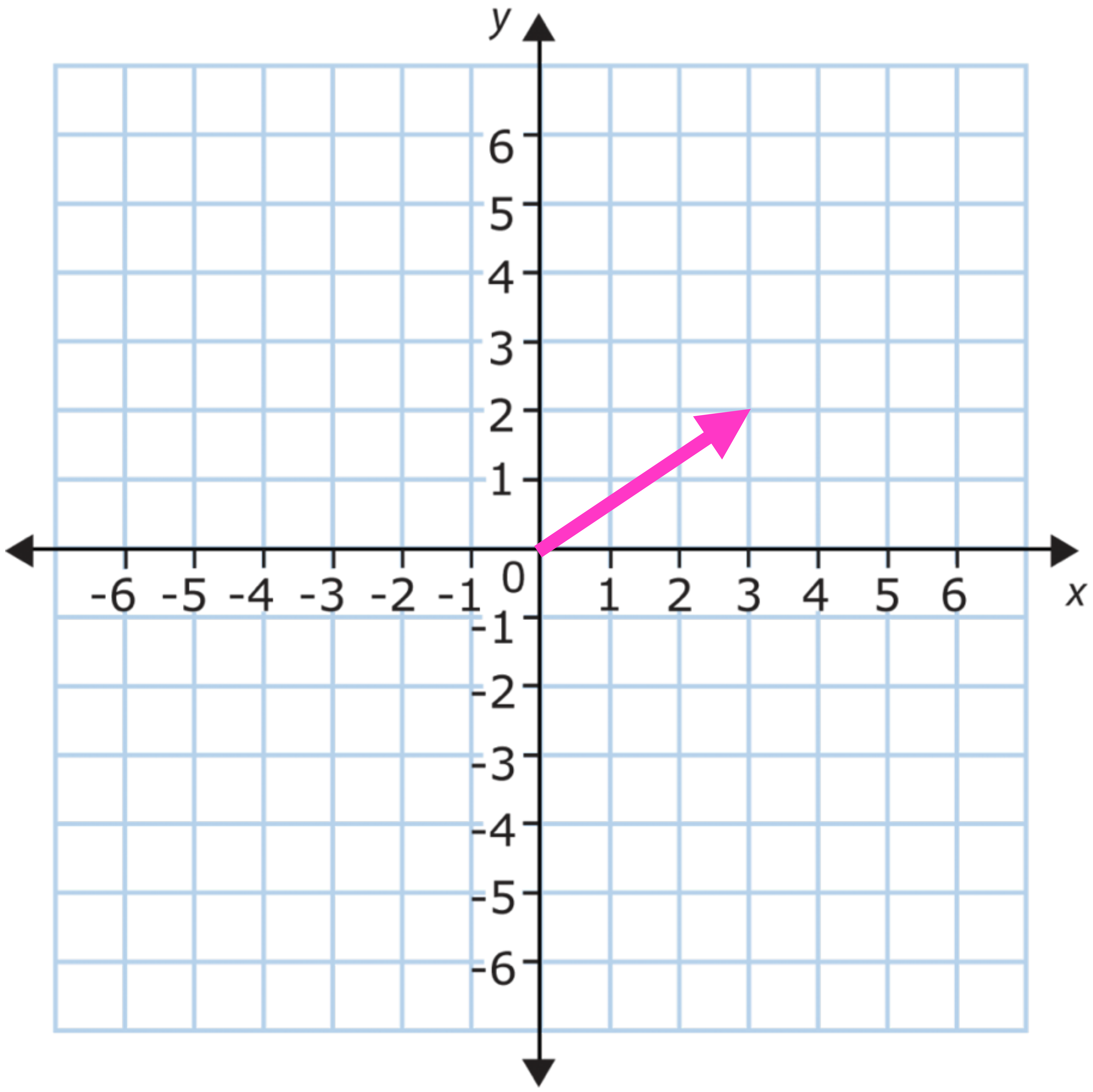
Vector **length**.

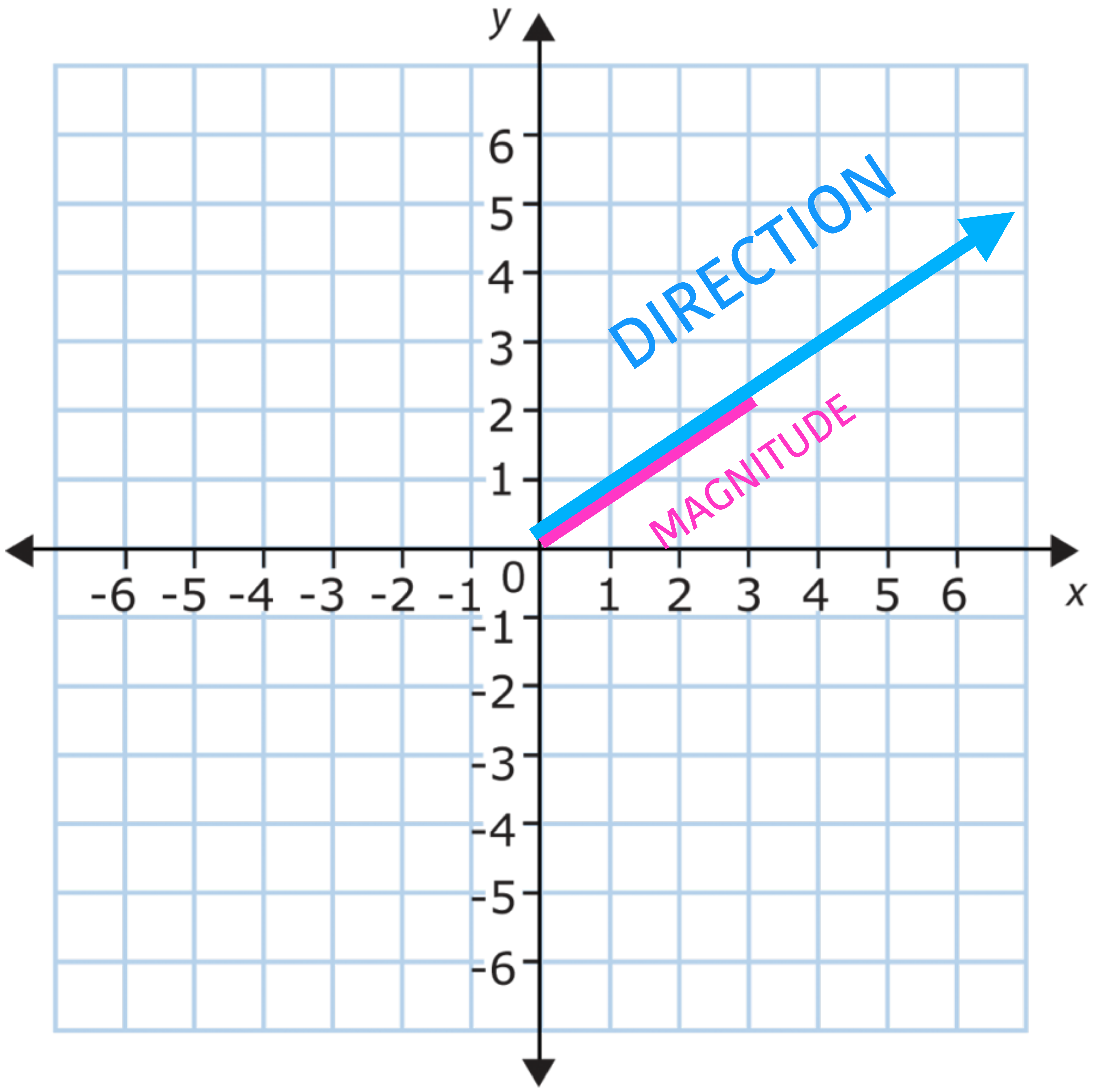
Use **Pythagorean Theorem** to get the **vector length**.

Normalizing a vector.



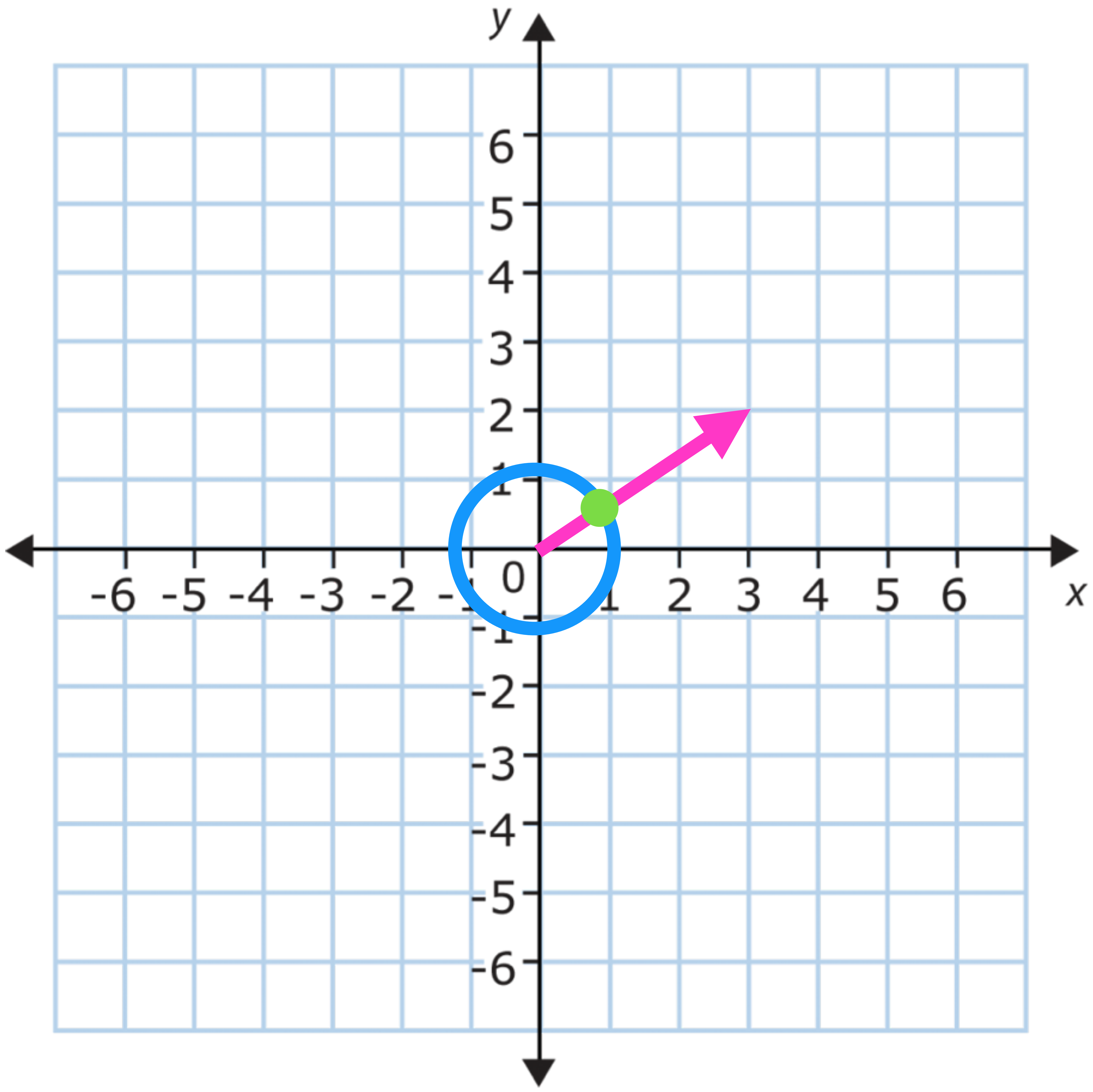
y
x
0
-6 -5 -4 -3 -2 -1
1 2 3 4 5 6
6 5 4 3 2 1
-1 -2 -3 -4 -5 -6


y
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
-6 -5 -4 -3 -2 -1
1 2 3 4 5 6
x



y
x
DIRECTION
MAGNITUDE
0
1
2
3
4
5
6
-1
-2
-3
-4
-5
-6

Divide each vector component by the vector length
(**just be careful if the length is 0!**).



y
x
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
-6 -5 -4 -3 -2 -1 1 2 3 4 5 6

Multiplying matrices and vectors.

```
Vector operator * (const Vector &v);
```

Creating final **entity model matrix**.

Identity matrix.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Scale matrix.

$$\begin{bmatrix} Sx & 0 & 0 & 0 \\ 0 & Sy & 0 & 0 \\ 0 & 0 & Sz & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Translate matrix.

$$\begin{bmatrix} 1 & 0 & 0 & Tx \\ 0 & 1 & 0 & Ty \\ 0 & 0 & 1 & Tz \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Z-axis **rotation** matrix.

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Building the final matrix.

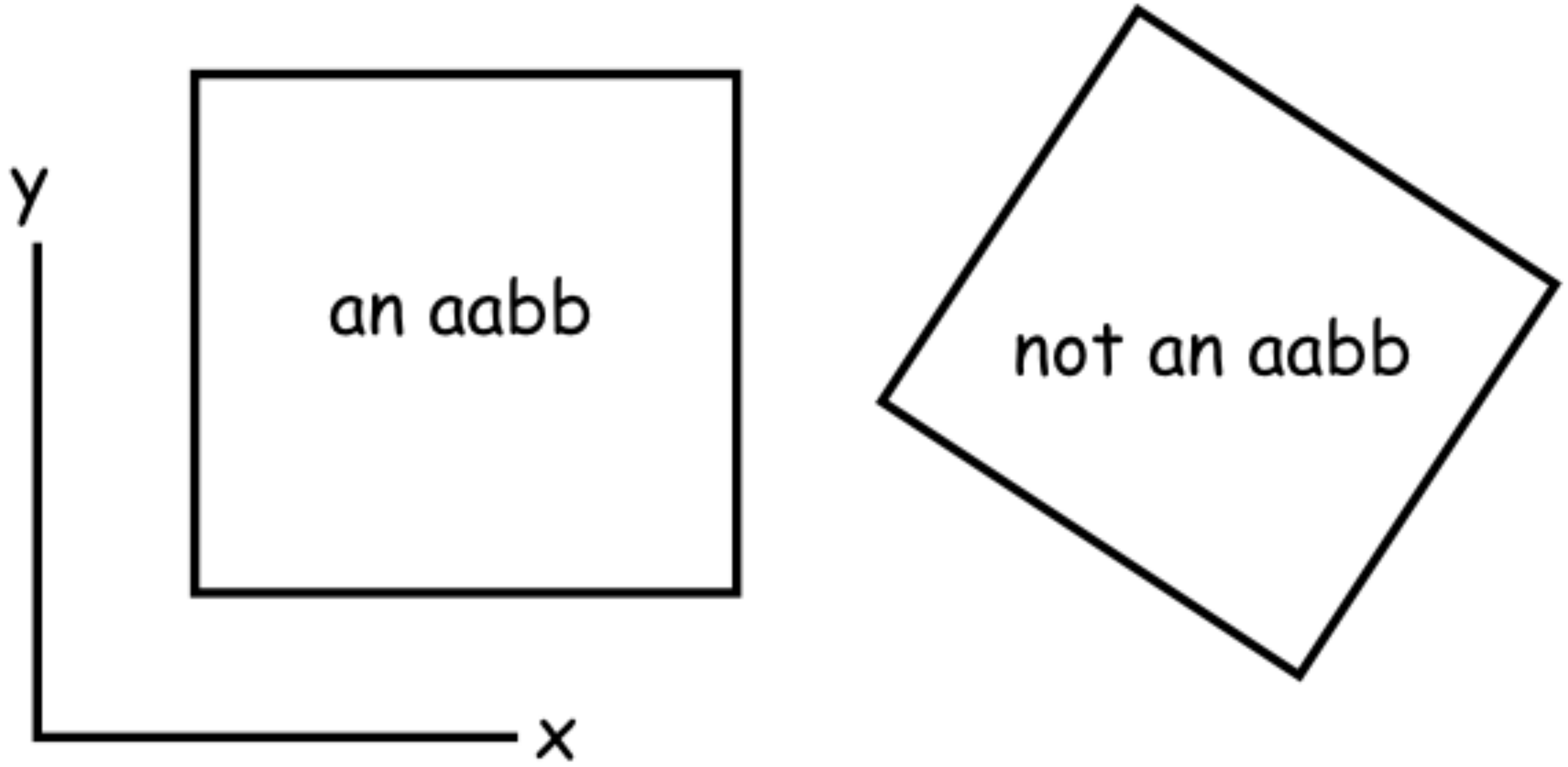
FINAL MATRIX = SCALE MATRIX * ROTATE MATRIX * TRANSLATE MATRIX

Remember that the order of matrix transform multiplication matters!

```
class Entity {
public:

    Matrix matrix;

    float x;
    float y;
    float scale_x;
    float scale_y;
    float rotation;


};
```

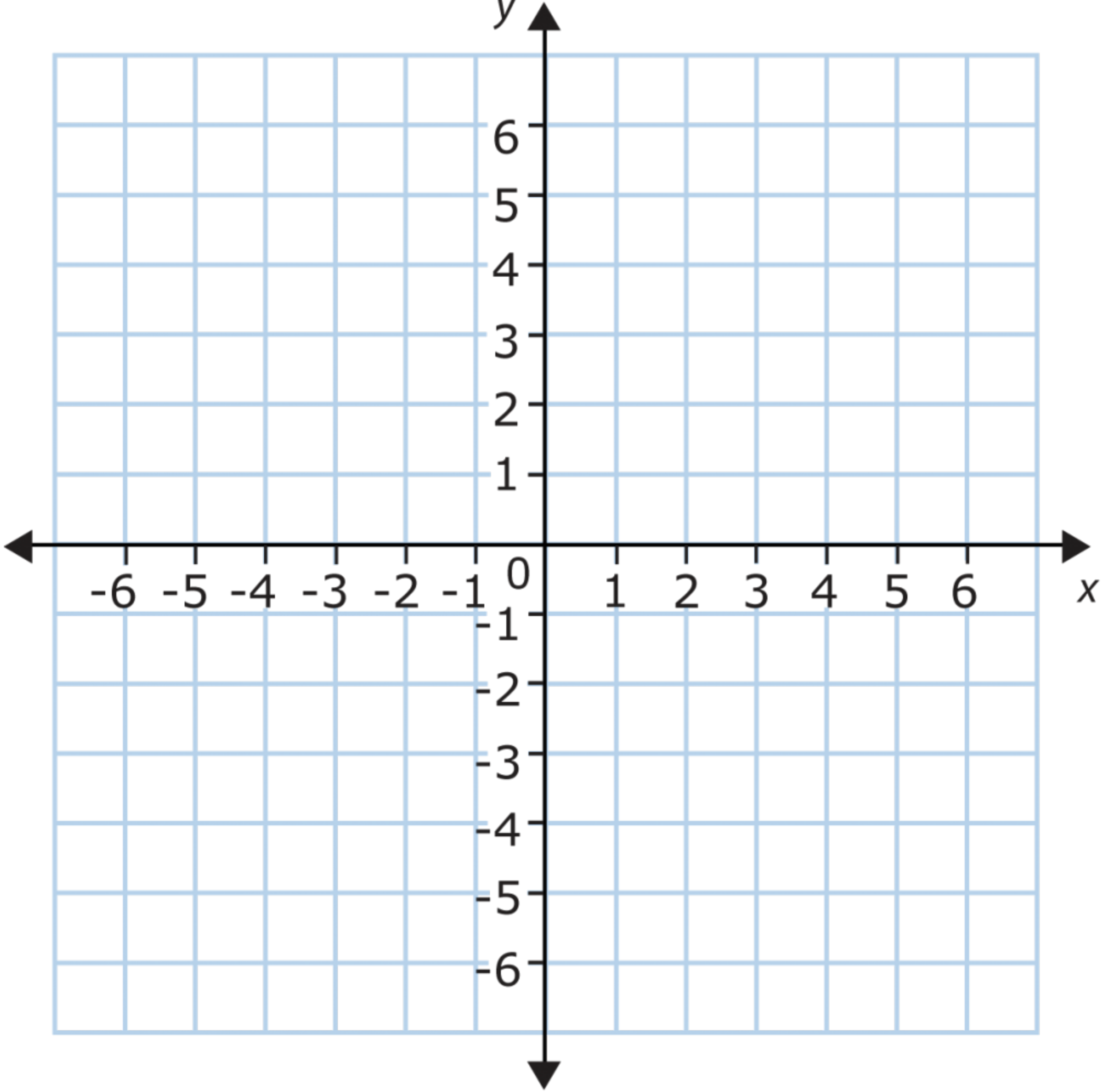
Benefits of storing the transformation matrix.

Non-**axis aligned** bounding boxes.

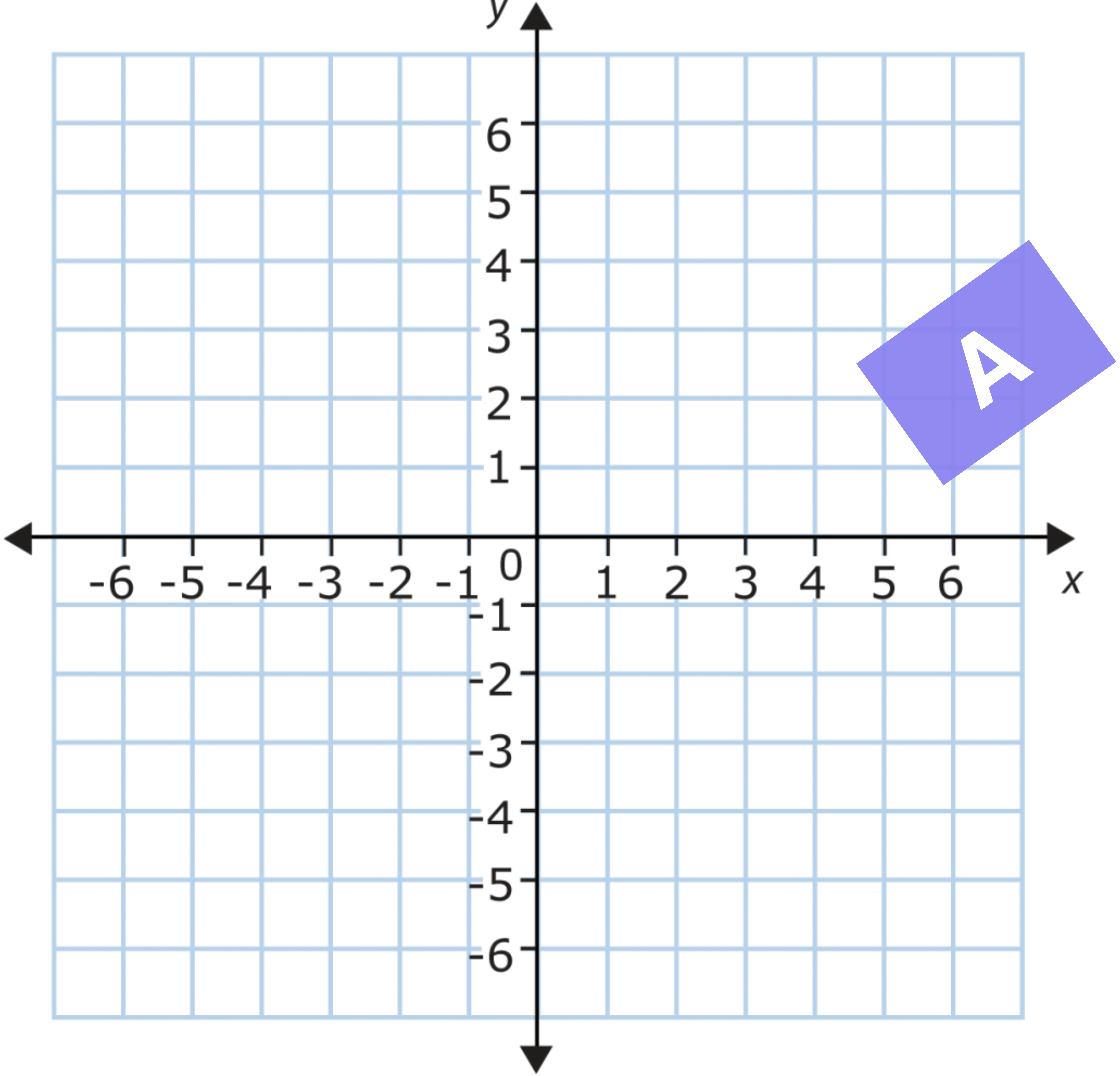

y
an aabb
x
not an aabb

Transforming between coordinate spaces.

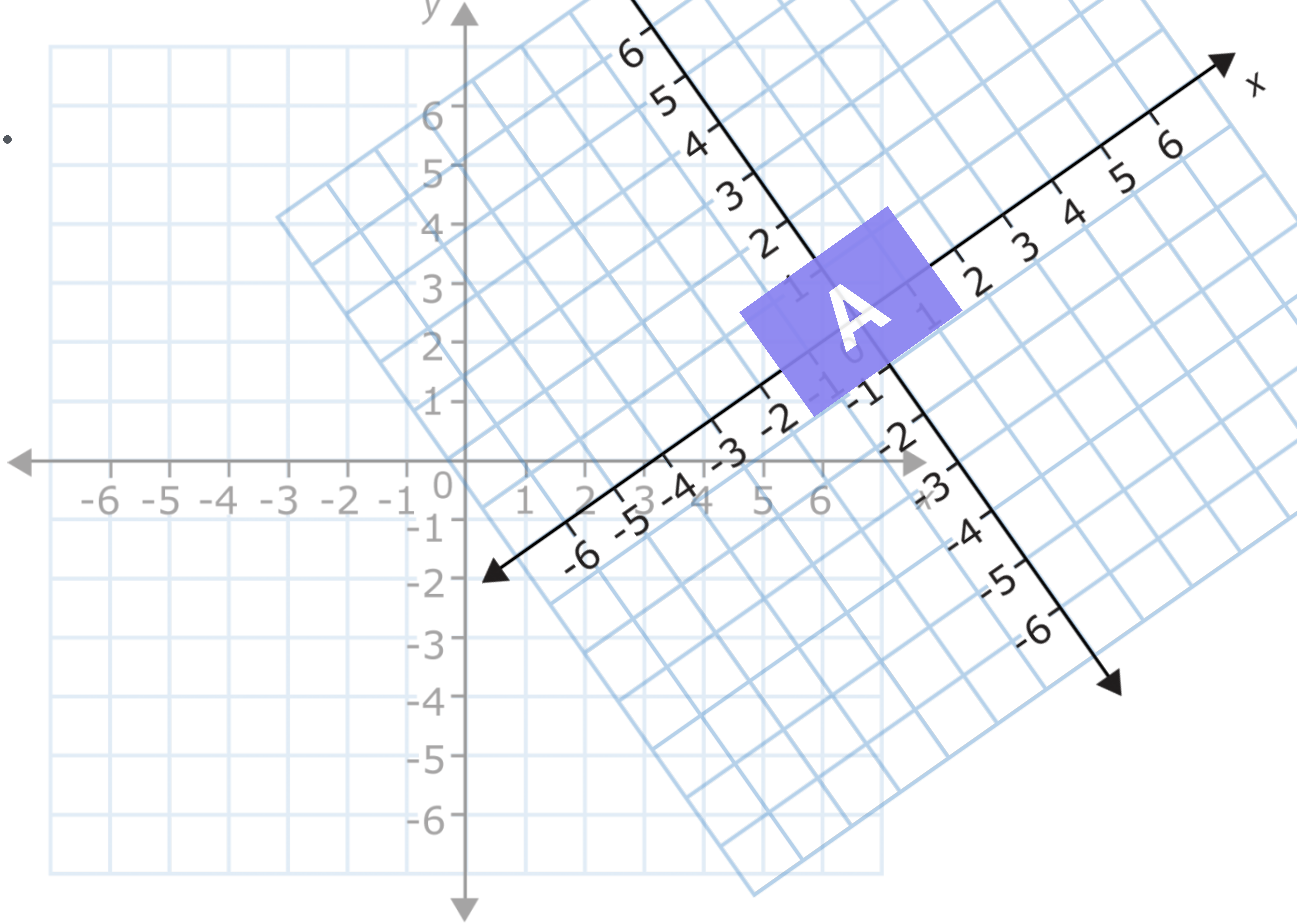
World space.



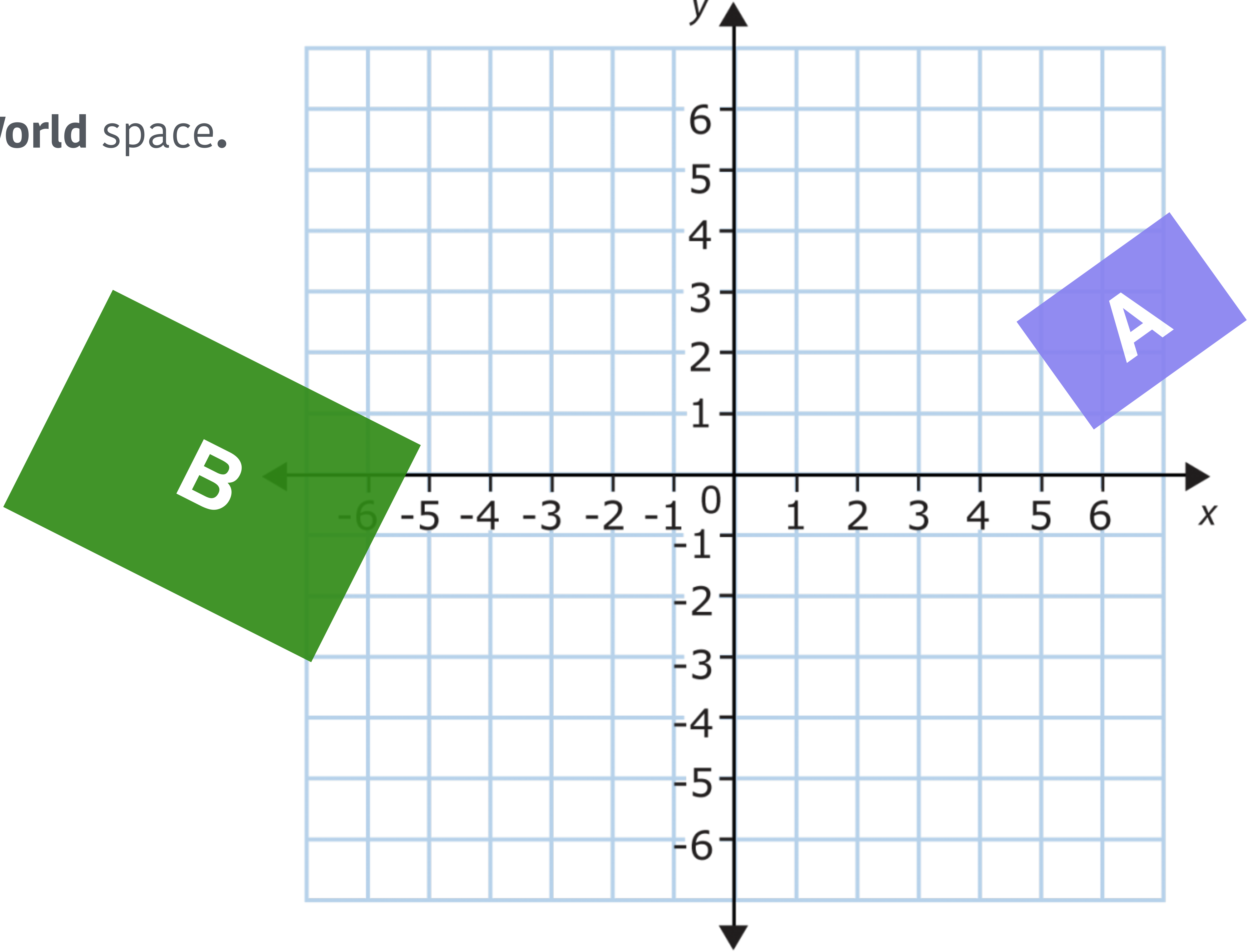
World space.

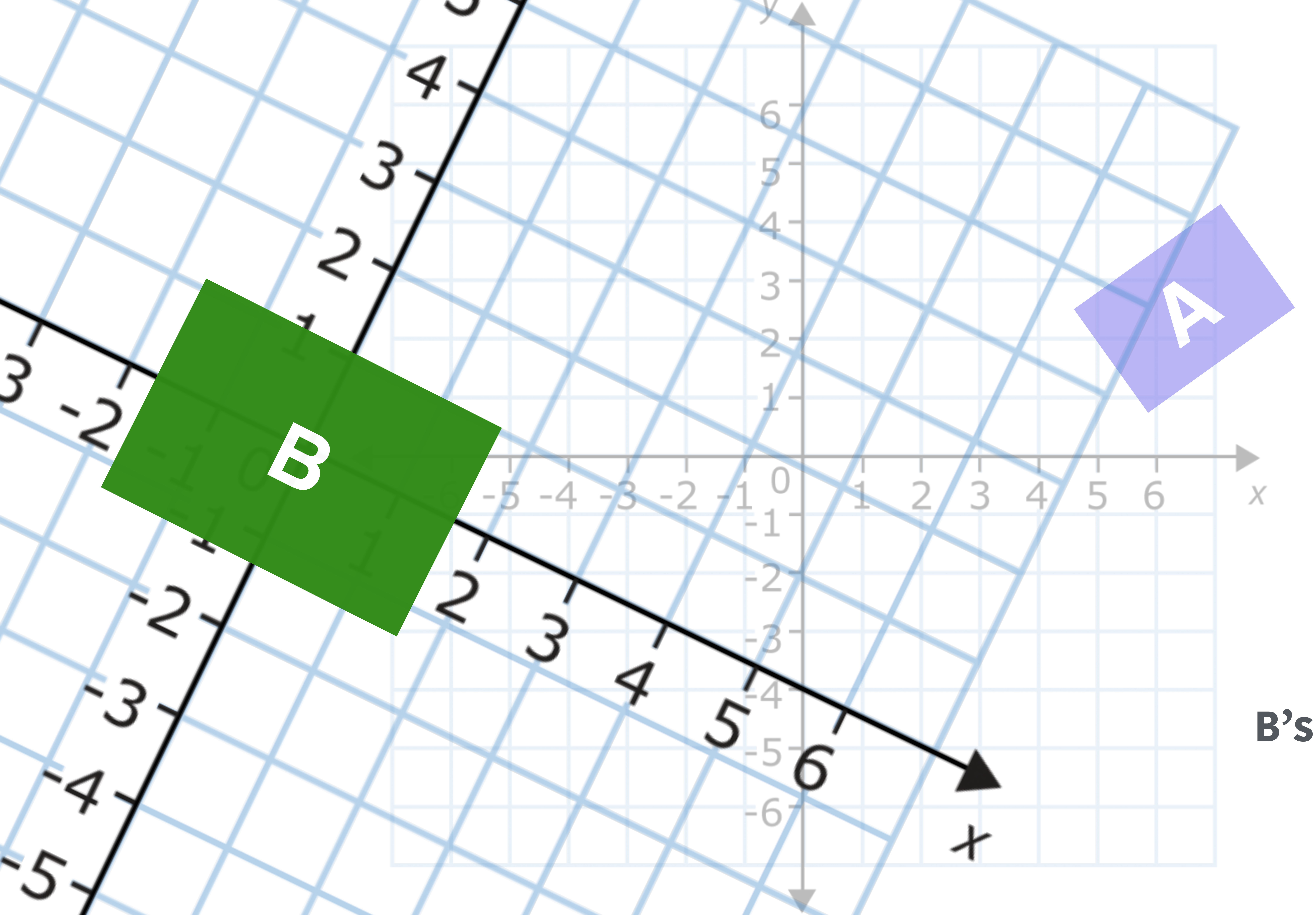


A's object space.



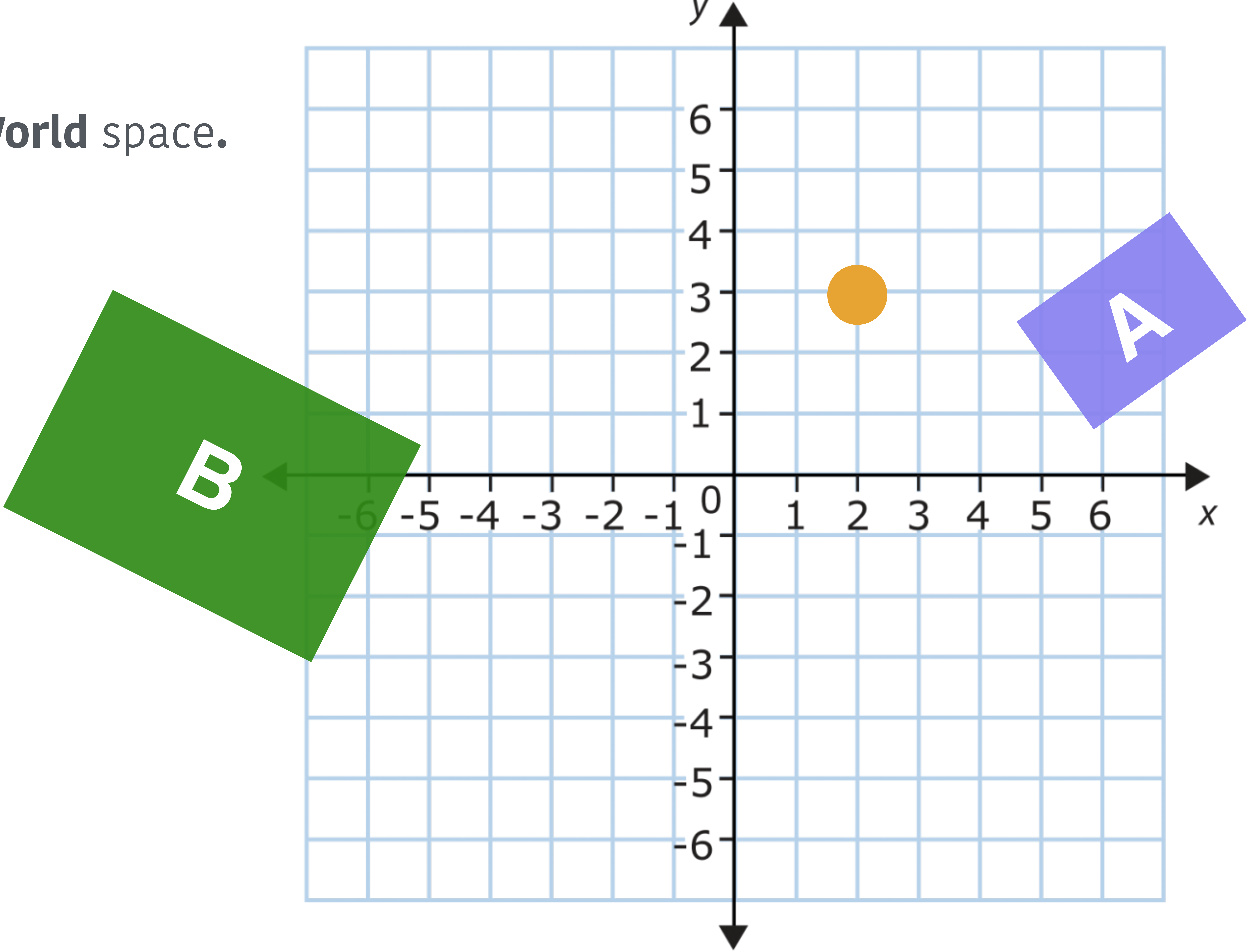
World space.



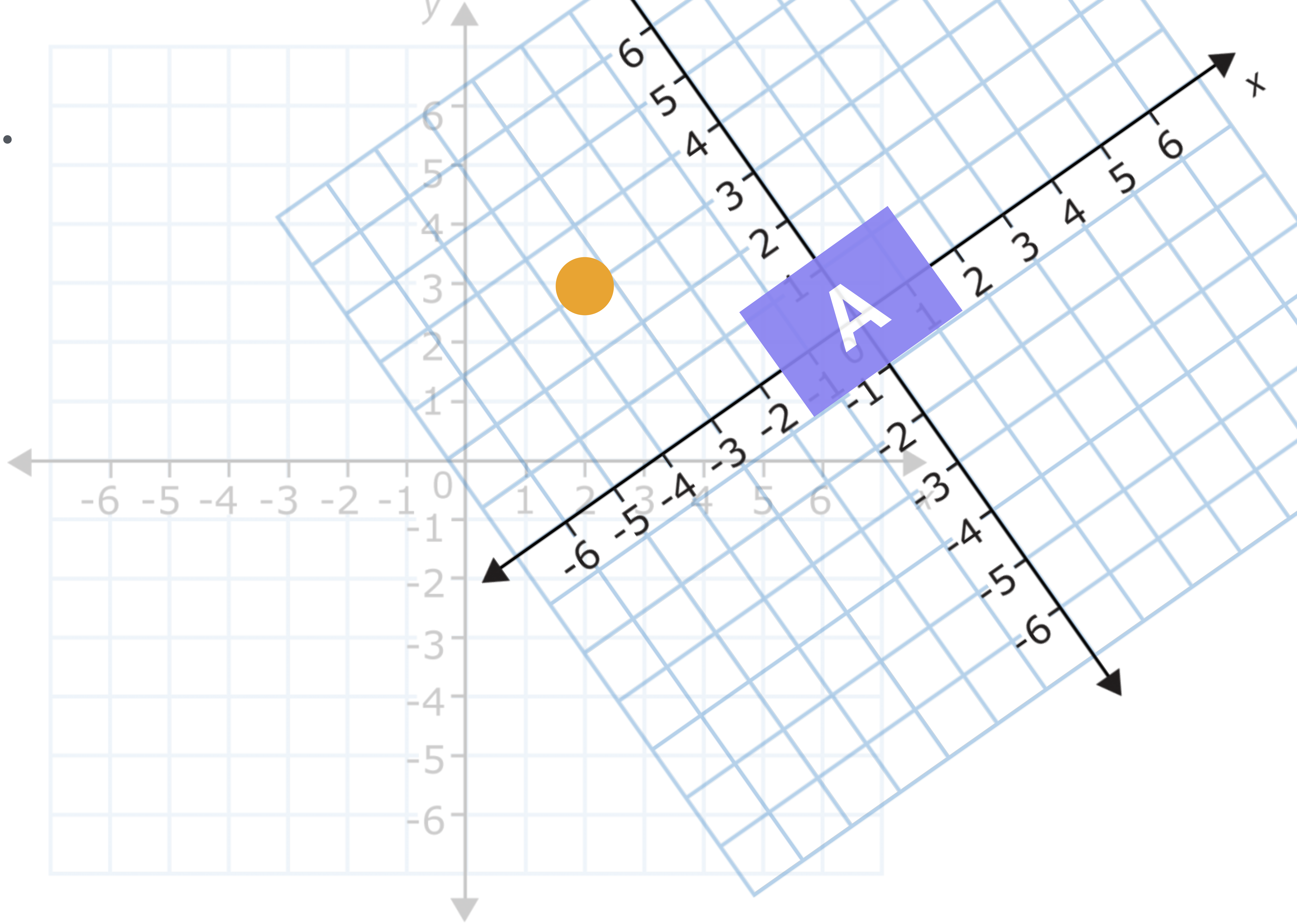


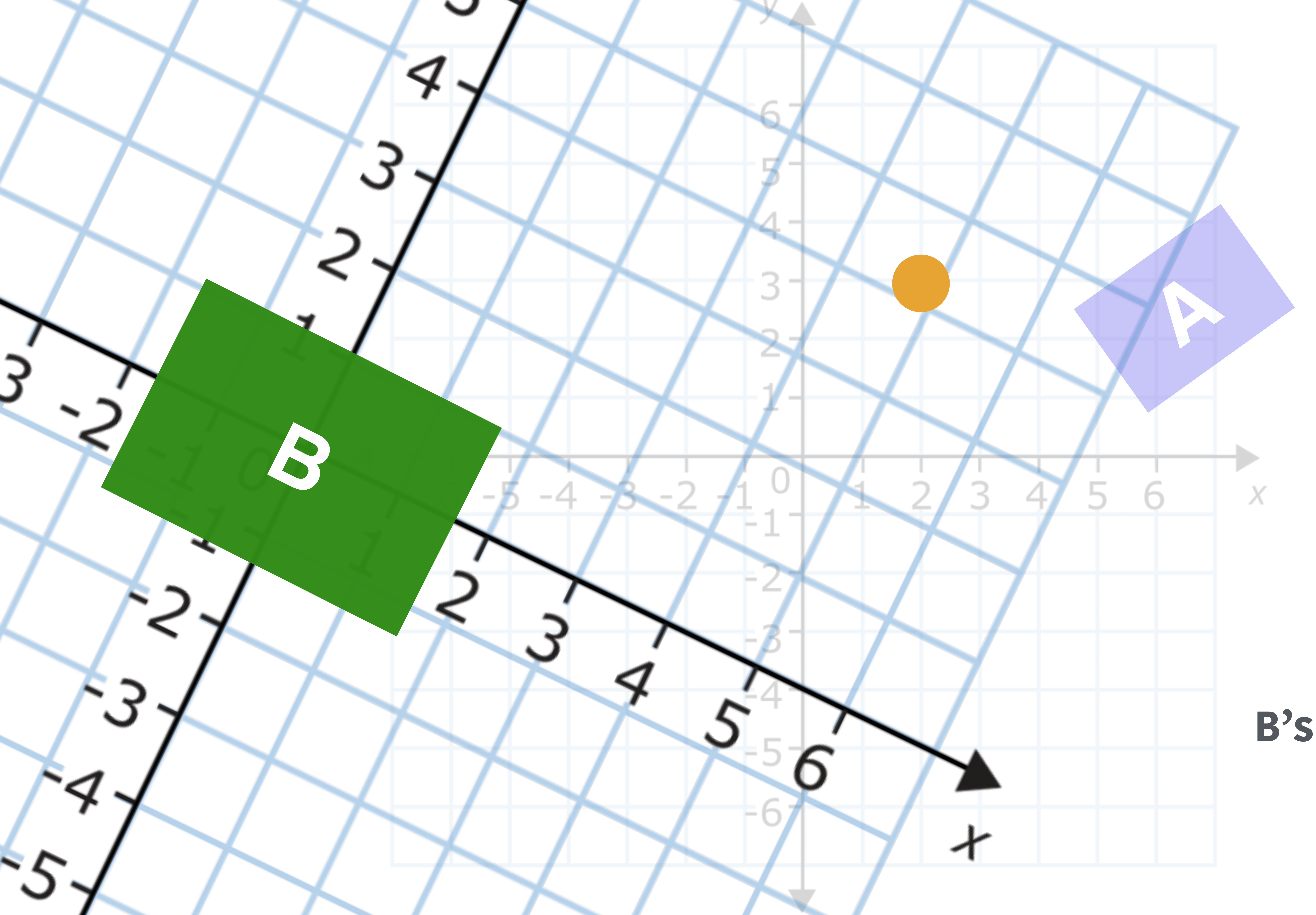
B's object space.

World space.

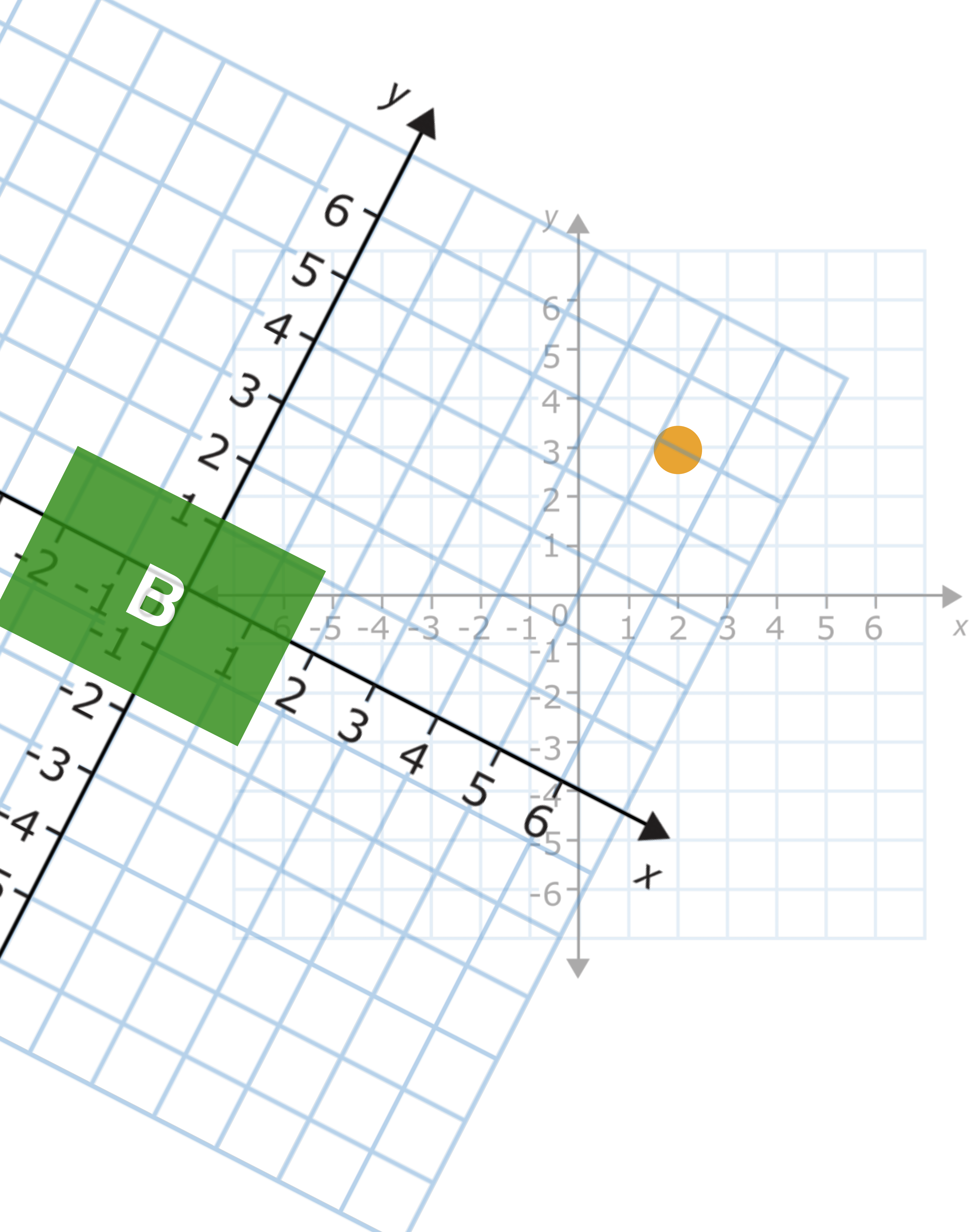


A's object space.

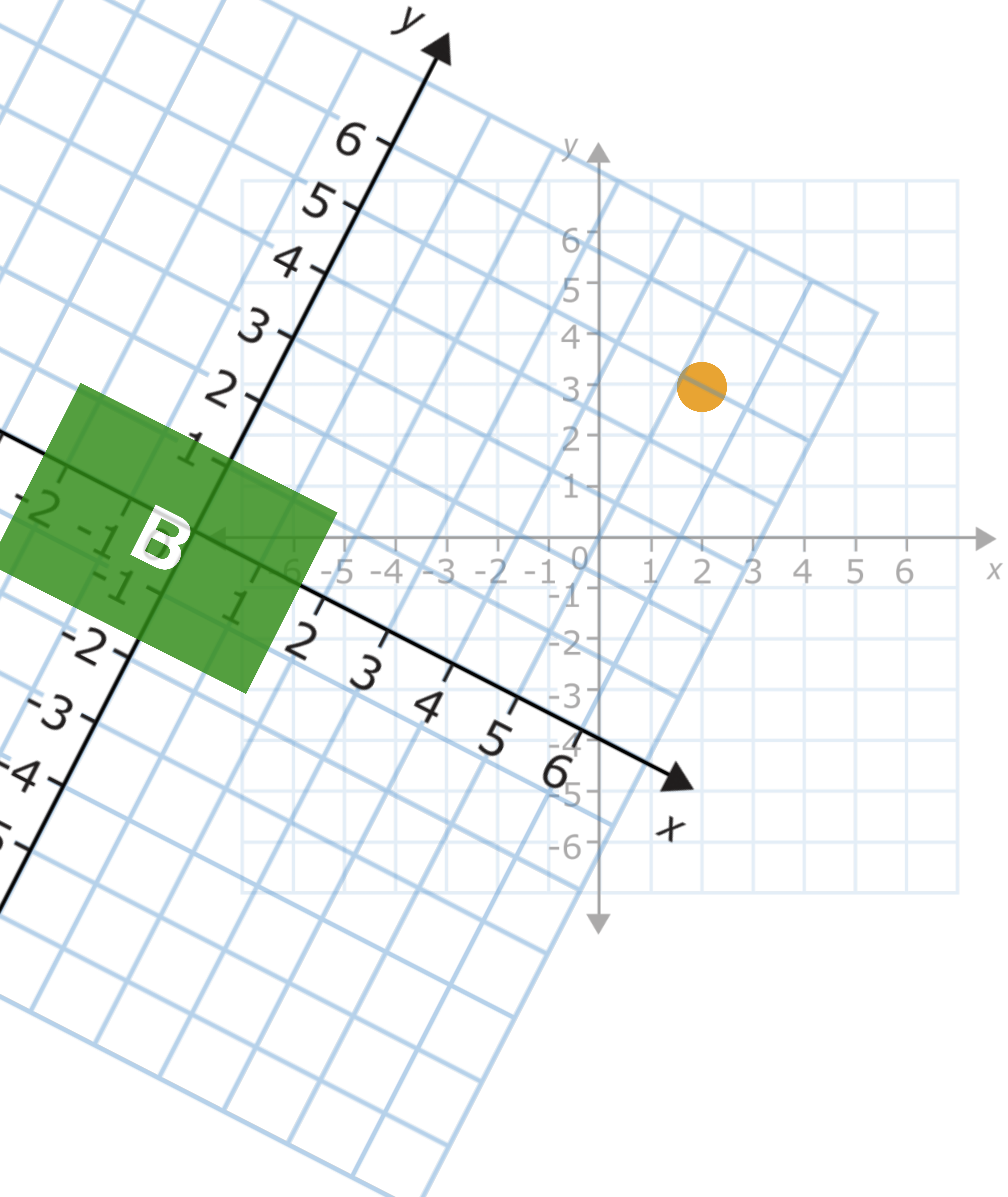




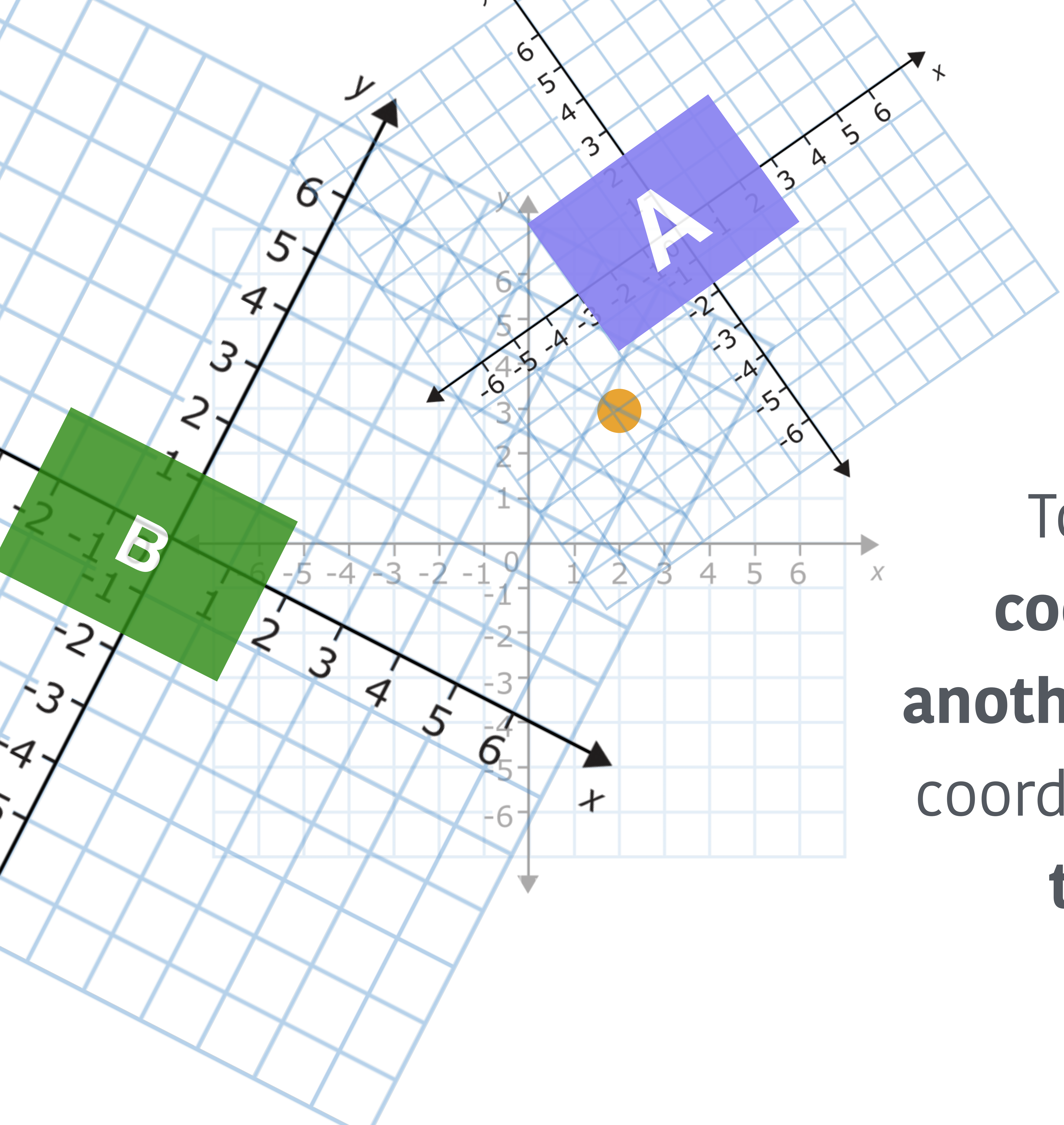
B's object space.



To express a **world space** coordinate vector in terms of an entity's **object space**, multiply the vector by the **inverse** of that entity's transform matrix.

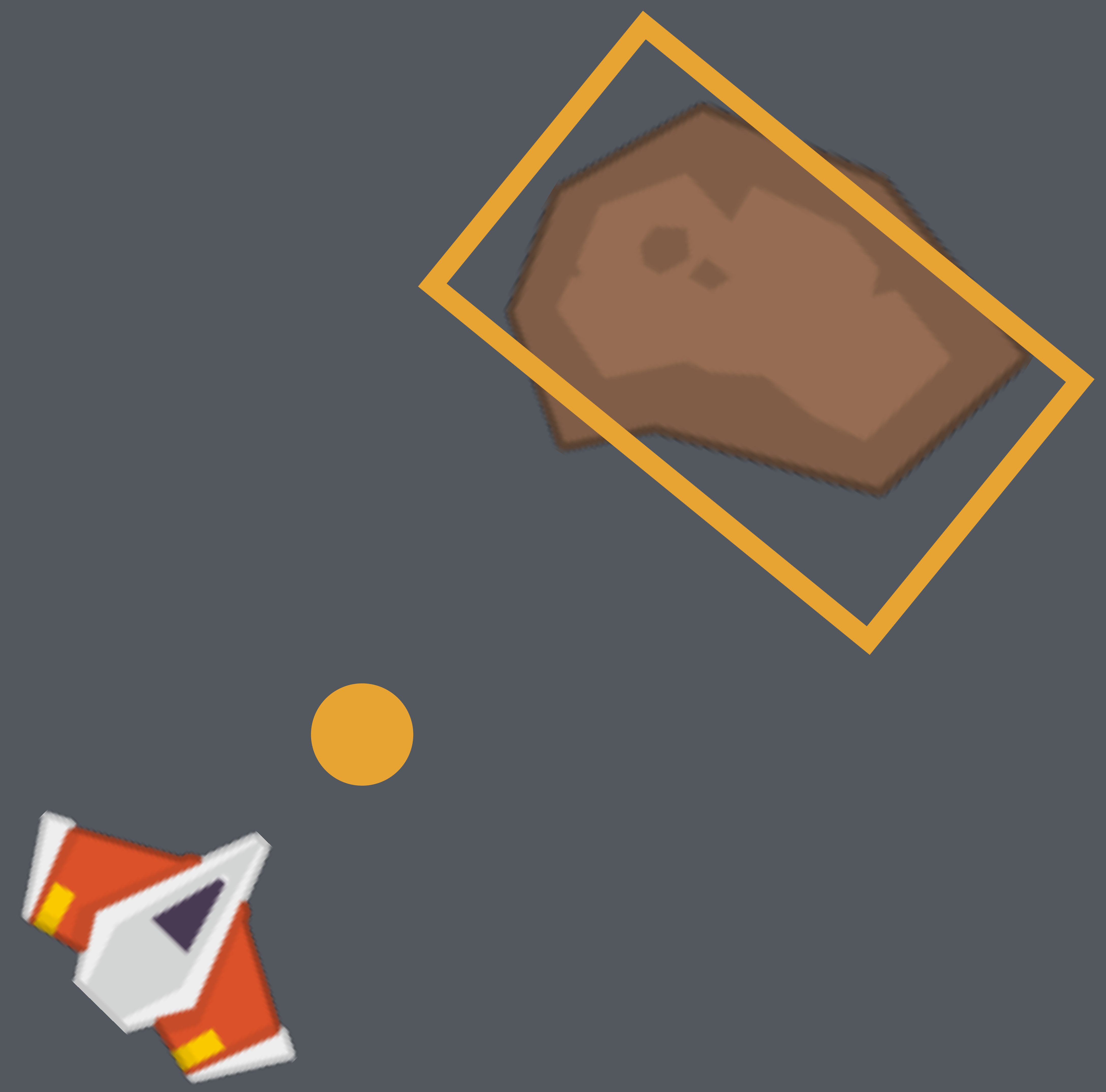


To express an **object space** coordinate vector in terms of **world space**, multiply the vector by that entity's **transform matrix**.

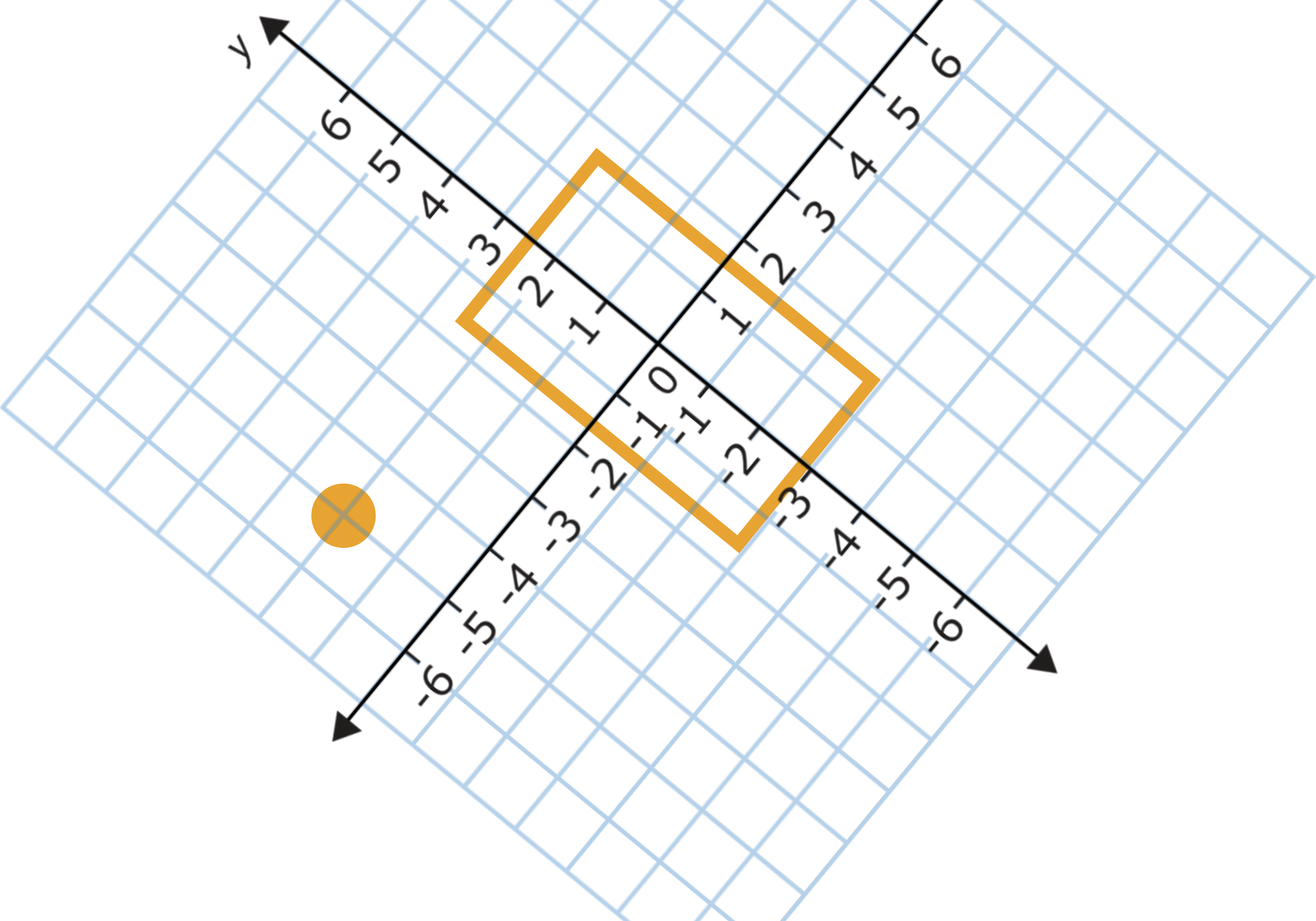


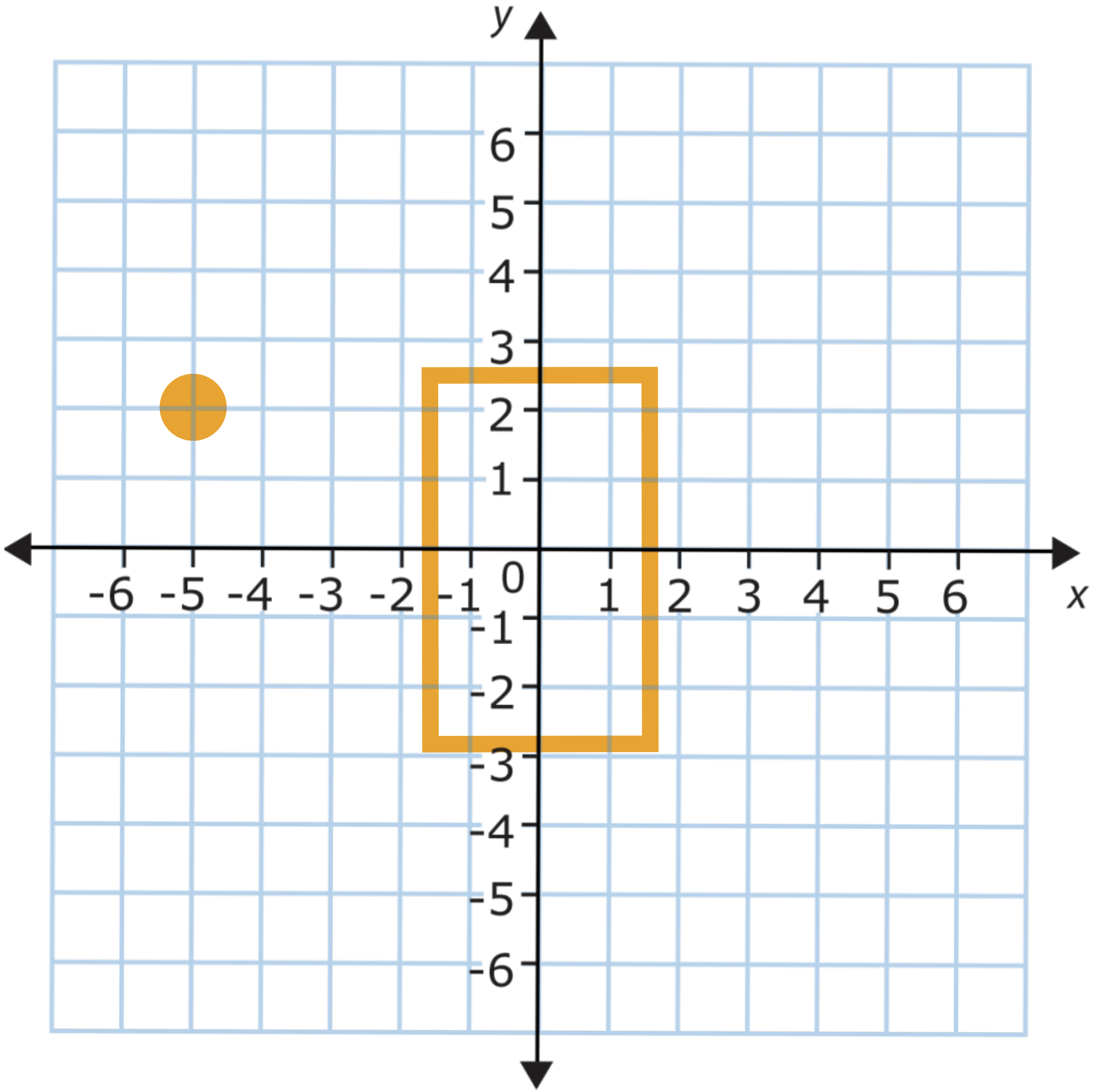
To express an **object space coordinate** vector in terms of **another object's space**, convert the coordinate **to world space**, then **to the other object's space**.

Point / rotated rectangle collision.



???


y
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
1
2
3
4
5
6
-1
-2
-3
-4
-5
-6

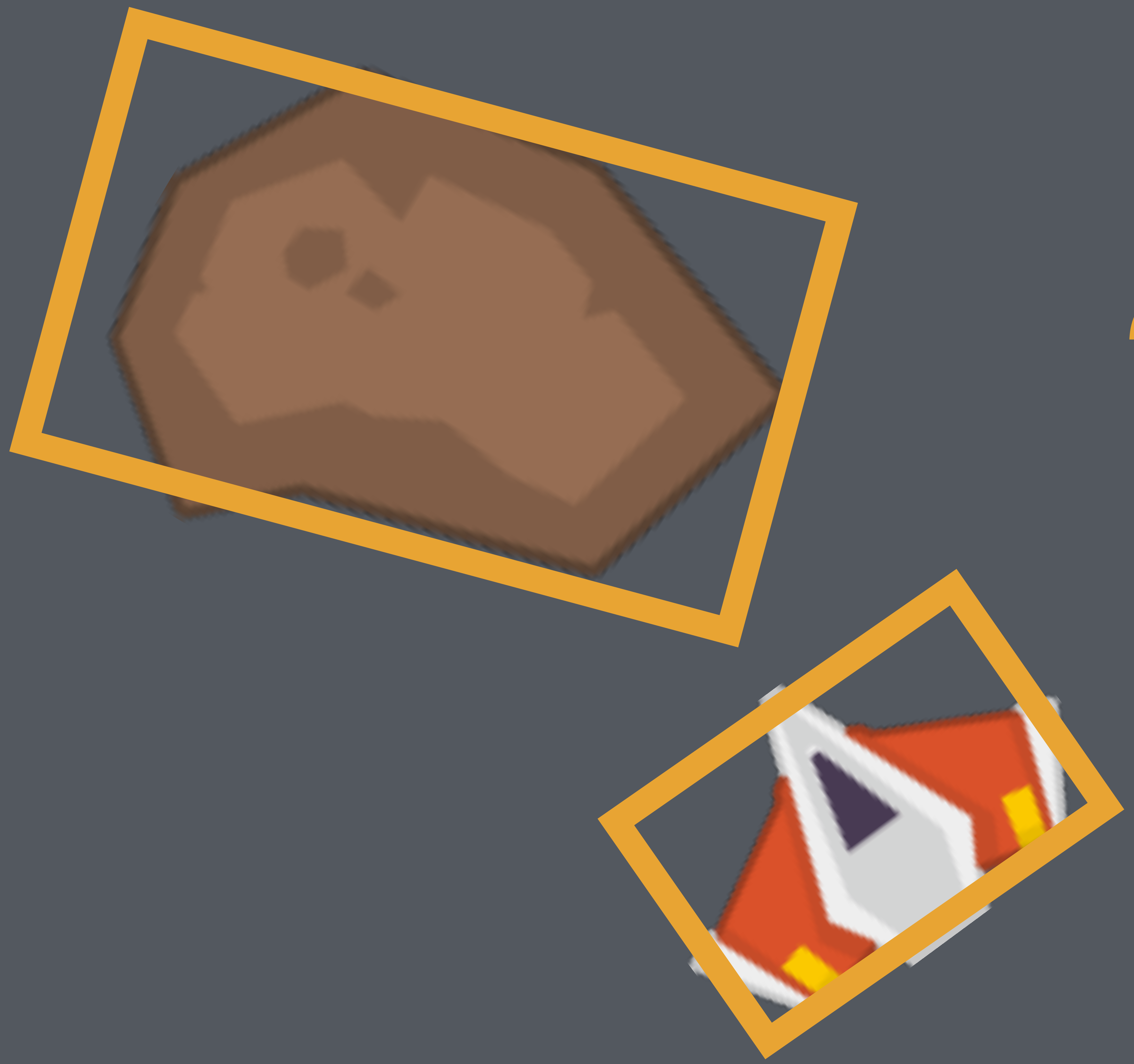

y
x
0
-6 -5 -4 -3 -2 -1 1 2 3 4 5 6
6 5 4 3 2 1 -1 -2 -3 -4 -5 -6

Point / rotated rectangle collision.

1. Multiply the point vector by the inverse of the entity's transform matrix.

2. Do regular point / rectangle check with the resulting coordinates.

Rotated rectangle / rotated rectangle collision.



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Separating axis theorem collision.