1 Section 5

1.1 377/5-12, 17-27 odds (Section 5.6, day 1)

5. Find the arcsin of $\frac{1}{2}$ without using a calculator.

$$\arcsin \frac{1}{2} = \frac{\pi}{6}$$

6. $\arcsin 0 = 0$

7. $\arccos \frac{1}{2} = \frac{\pi}{3}$

8. $\arccos 0 = \frac{\pi}{2}$

9. $\arctan \frac{\sqrt{3}}{3} = \frac{\pi}{6}$

10. $\operatorname{arccot}(-\sqrt{3}) = -\frac{\pi}{6}$

11. $\operatorname{arccsc}\left(-\sqrt{2}\right) = \arcsin \frac{1}{(-\sqrt{2})} = -\frac{\pi}{4}$

12. $\arccos\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$

17. Evaluate without using a calculator:

    (a)
    $$\sin\left(\arctan \frac{3}{4}\right) = \frac{3}{5}$$

    (b)
    $$\sec\left(\arcsin \frac{4}{5}\right) = \frac{5}{3}$$

21. Write in "Algebraic" form.
$$\cos\left(\arcsin 2x\right) = 2x$$

23. Write in "Algebraic" form.
$$\sin(\operatorname{arcsec} x) = \frac{\sqrt{x^2-1}}{|x|}$$

25. Write in "Algebraic" form.
$$\tan\left(\operatorname{arcsec} \frac{x}{3}\right) = \frac{\sqrt{x^2-9}}{3}$$

27. Write in "Algebraic" form.
$$\csc\left(\arctan \frac{x}{\sqrt{2}}\right) = \frac{\sqrt{x^2+2}}{x}$$

1.2 378/41-65 e.o.o. (Section 5.6, day 2)

41. Find the derivative.

$$f(x) = 2\arcsin(x-1) \tag{1}$$
$$\frac{dy}{dx} = \frac{2}{\sqrt{1-(x-1)^2}} \tag{2}$$
$$= \frac{2}{\sqrt{1-(x^2-2x+1)}} \tag{3}$$
$$= \frac{2}{\sqrt{2x-x}} \tag{4}$$

45. Find the derivative.

$$f(x) = \arctan\frac{x}{a} \tag{1}$$
$$\frac{dy}{dx} = \frac{\frac{1}{a}}{(\frac{x}{a})^2+1} \tag{2}$$
$$= \frac{1}{a\left(\frac{x}{a}\right)^2+a} \tag{3}$$
$$= \frac{1}{\frac{x^2}{a}+a} \tag{4}$$
$$= \frac{a}{x^2+a^2} \tag{5}$$

49. Find the derivative.

$$f(x) = \sin(\arccos t) \tag{1}$$
$$\frac{dy}{dx} = [\cos(\arccos t)]\left[\frac{-1}{\sqrt{1-t^2}}\right] \tag{2}$$
$$= \frac{-\cos(\arccos t)}{\sqrt{1-t^2}} \tag{3}$$
$$= \frac{-t}{\sqrt{1-t^2}} \tag{4}$$

53. Find the derivative.

$$y = \frac{1}{2}\left[\frac{1}{2}\ln\frac{x+1}{x-1} + \arctan x\right] \tag{1}$$
$$= \frac{1}{4}\ln\frac{x+1}{x-1} + \frac{1}{2}\arctan x \tag{2}$$
$$y = \frac{1}{4}\ln(x+1) - \frac{1}{4}\ln(x-1) + \frac{1}{2}\arctan x \tag{3}$$
$$\frac{dy}{dx} = 1\left(\frac{1}{4x+4}\right) - 1\left(\frac{1}{4x-4}\right) + \frac{1}{2x^2+2} \tag{4}$$
$$= \frac{1}{4x+4} - \frac{1}{4x-4} + \frac{1}{2x^2+2} \tag{5}$$

Notes: I had issue with these problems. Here's the solution as given in class:

53. Again.

$$y = \frac{1}{2}\left[\frac{1}{2}\ln\frac{x+1}{x-1} + \arctan x\right] \tag{1}$$

$$= \frac{1}{4}\ln(x+1) - \frac{1}{4}\ln(x-1) + \frac{1}{2}\arctan x \tag{2}$$

$$\frac{dy}{dx} = \frac{1}{4(x+1)} - \frac{1}{4(x-1)} + \frac{1}{2(x^2+1)} \tag{3}$$

$$= \frac{-x^2 - 1 + x^2 - 1}{2(x^2+1)(x^2-1)} \tag{4}$$

$$= \frac{-1}{x^4 - 1} \tag{5}$$

57.

$$y = \frac{1}{2}\left[\frac{1}{2}\ln\frac{x+1}{x-1} + \arctan x\right] \tag{1}$$

$$\frac{dy}{dx} = \frac{2}{\sqrt{1 - \left(\frac{x}{4}\right)}} - \frac{1}{2}\left[\sqrt{16 - x^2} + \frac{-2x^2}{2\sqrt{16-x^2}}\right] \tag{2}$$

$$= \frac{2}{\sqrt{\frac{16-x^2}{16}}} - \frac{1}{2}\left[\sqrt{16 - x^2 + \frac{-2x^2}{2\sqrt{16-x^2}}}\right] \tag{3}$$

$$= \frac{8}{\sqrt{16-x^2}} - \frac{\sqrt{16-x^2}}{2} + \frac{x^2}{2\sqrt{16-x^2}} \tag{4}$$

$$= \frac{16 - (16 - x^2) + x^2}{2\sqrt{16-x^2}} \tag{5}$$

$$= \frac{x^2}{\sqrt{16-x^2}} \tag{6}$$