

Notes on Chapter 8

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8 May 2015

1 Integration by Parts, Example problems.

2 Trigonometric Integrals

2.1 Problems in the lesson

1.

$$\int \sin x \cos x dx = \frac{1}{2} \sin^2 x + c \quad (1)$$

$$u = \sin x \quad du = \cos x dx \quad (2)$$

2.

$$\int \sin^2 x \cos x dx = \frac{1}{3} \sin^3 x + c \quad (1)$$

$$u = \sin x \quad du = \cos x dx \quad (2)$$

3.

$$\int \sin^3 x \cos^2 x dx = \quad (1)$$

$$\int (1 - u^2) u^2 du = \quad (2)$$

$$-\frac{1}{3} \cos^3 x + \frac{1}{5} \cos^5 x + c \quad (3)$$

$$u = \cos x \quad du = -\sin x dx \quad (4)$$

4. In cases where there is a function taking the form $\int \sin^m x \cos^n x dx$ where one is odd and one is even, let u = the one with the even power.

$$\int \sin^3 x \cos^3 x dx = \quad (1)$$

$$\int u^3(1 - u^2)du = \quad (2)$$

$$\int (u^3 - u^5)du = \quad (3)$$

$$\frac{1}{4} \sin^4 x - \frac{1}{6} \sin^6 x + c \quad (4)$$

5. In cases where both are even, use the fact that:

$$\sin^2 x = \frac{1 - \cos 2x}{2} = \frac{1}{2} \cos 2x$$

6. So then in the case:

$$\int \sin^2 x dx = \int \left(\frac{1}{2} - \frac{1}{2} \cos 2x \right) dx \quad (1)$$

$$= \frac{1}{2}x - \frac{1}{4} \sin 2x + c \quad (2)$$

2.2 Problems from the Homework

- 9.

$$\int \sin^5 x \cos^2 x dx$$

Let $u = \cos x$ and $du = -\sin x dx$.

$$\int u^2 \sin^4 x dx = \int u^2(1 - u^2)^2 du$$

Resulting in:

$$\frac{1}{3} \cos^3 x + \frac{2}{5} \cos^5 x - \frac{1}{7} \cos^7 x + c$$

- 11.

$$\int \cos^3 \theta \sqrt{\sin \theta} d\theta$$

let $u = \sin \theta$ and $du = \cos \theta d\theta$

2.3 Double Angle Formulae

These come in handy in this sort of work. $\sin 2x = 2 \sin x \cos x$ so $\int (\sin x \cos x)^2 dx = \int (\frac{1}{2} \sin 2x)^2 dx$.

2.4 More Trig

- Secant to an odd power implies integration by parts, at least if it is by itself.
- Tangent raised to a power greater than one means at some point is going to be secant x.