Notes on Chapter 8

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1 Integration by Parts, Example problems.

Lorem Ipsum Dolor Sit Amet

2 Trigonometric Integrals

2.1 Problems in the lesson

1.

$$\int \sin x \cos x dx = \frac{1}{2}\sin^2 x + c \tag{1}$$

$$u = \sin x \qquad du = \cos x dx \tag{2}$$

2.

$$\int \sin^2 x \cos x dx = \frac{1}{3}\sin^3 x + c \tag{1}$$

$$u = \sin x \qquad du = \cos x dx \tag{2}$$

3.

$$\int \sin^3 x \cos^2 x dx = \tag{1}$$

$$\int (1-u^2)u^2 du = \tag{2}$$

$$-\frac{1}{3}\cos^3 x + \frac{1}{5}\cos^5 x + c \tag{3}$$

$$u = \cos x \qquad du = \sin x \tag{4}$$

4. In cases where there is a function taking the form $\int \sin^m x \cos^n x dx$ where one is odd and one is even, let $u =$ the one with the even power.

$$\int \sin^3 x \cos^3 x dx = \tag{1}$$

$$\int u^3(1-u^2)du = \tag{2}$$

$$\int (u^3 - u^5)du = \tag{3}$$

$$\frac{1}{4}\sin^4 x - \frac{1}{6}\sin^6 x + c \tag{4}$$

5. In cases where both are even, use the fact that:

$$\sin^2 x = \frac{1-\cos 2x}{2} = \frac{1}{2}\cos 2x$$

6. So then in the case:

$$\int \sin^2 x dx = \int \left(\frac{1}{2} - \frac{1}{2}\cos 2x\right) dx \tag{1}$$

$$= \frac{1}{2}x - \frac{1}{4}\sin 2x + c \tag{2}$$

2.2 Problems from the Homework

9.

$$\int \sin^5 x \cos^2 x dx$$

Let $u = \cos x$ and $du = \sin x dx$.

$$\int u^2 \sin^4 x dx = \int u^2(1-u^2)^2 du$$

Resulting in:

$$\frac{1}{3}\cos^3 x + \frac{2}{5}\cos^5 x - \frac{1}{7}\cos^7 x + c$$

11.

$$\int \cos^3\theta \sqrt{sin\theta} d\theta$$

let $u = sin\theta$ and $du = \cos\theta d\theta$

2.3 Double Angle Formulae

These come in handy in this sort of work. $\sin 2x = 2 \sin x \cos x$ so $\int (\sin x \cos x)^2 \, dx = \int \left(\frac{1}{2} \sin 2x\right)^2 dx$.

2.4 More Trig

- Secant to an odd power implies integration by parts, at least if it is by itself.
- Tangent raised to a power greater than one means at some point is going to be secant x.