

# 1 New Content

## 1.1 Monday, 13th April

1. Slope Fields are drawings with the slope at any given point, and can tell a lot about a differential equation without solving it.
2. A big reason slope fields are useful is that there are some differential equations we are unable to solve. The handout has some examples and rules.
3. Drawing out a slope field by hand is tedious and unpleasant, so don't do it unless you have to.

## 1.2 Tuesday, 14th April

1. Continuing from April Fools' Day:

$$\frac{dy}{dx} = 4xy \quad (1)$$

$$\int \frac{1}{y} dy = \int 4x dx \quad (2)$$

$$\ln y = 2x^2 + C_1 \quad (3)$$

$$y = Ce^{2x^2} \quad (4)$$

2.  $\frac{dy}{dt}$  is the same as a rate of change, but it's also a part of a differential equation.

$$\frac{dy}{dt} = ky \quad (1)$$

$$\int \frac{1}{y} dy = k dt \quad (2)$$

$$\ln y = kt + c \quad (3)$$

$$y = Ce^{kt} \quad (4)$$

3. **Growth and Decay** gives us the equation  $y = Ce^{kt}$  where  $C$  is the initial value,  $k$  is the constant of proportionality,  $e$  is  $e$ , and  $t$  is time. If  $k$  is positive, it is growing, and if negative, it is decaying.

4. In many problems using growth and decay, we want to first find  $k$ .

$$4 = Ce^{2k} \quad (1)$$

$$4 = 2e^{2k} \quad (2)$$

$$2 = e^{2k} \quad (3)$$

$$\ln 2 = \ln e^{2k} \quad (4)$$

$$\frac{1}{2} \ln 2 = k \quad (5)$$

5. Continuing on, using this fact:

$$y = 2e^{\frac{t}{2} \ln 2} \quad (6)$$

$$= e^{\ln 2^{\frac{t}{2}}} \quad (7)$$

$$= 2(2^{\frac{t}{2}}) \quad (8)$$

$$= 2(2^{\frac{3}{2}}) \quad (9)$$

6. This sort of thing can be applied as well to half-lives.

$$y = Ce^{24100k} \quad (1)$$

$$\frac{y}{c} = e^{24100k} \quad (2)$$

$$\ln \frac{1}{2} = 24100k \quad (3)$$

$$k = \frac{\ln \frac{1}{2}}{24100} \quad (4)$$

if  $k$  is half-life, then  $k = \frac{\ln \frac{1}{2}}{h}$  and if  $k$  is a constant of proportionality then  $h$  is  $\frac{\ln \frac{1}{2}}{k}$ .

## 2 AP Preparations

### 2.1 Monday, 20th April

3. From the 2000 AP Exam

- (a)  $x = -1$  is a relative minimum because  $f'$  goes from negative to positive.
- (b)  $x = -5$  is a relative maximum because  $f'$  goes from positive to negative.
- (c)  $(-7, -3)$   $(2, 3)$   $(3, 5)$
- (d) For this we check the maxima and endpoints.  $x = 7$  and  $x = -7$  are endpoints. It is  $x = 7$  since it's the best.

### 2.2 Wednesday, 22nd April

1. From the board at the time:

$$E(t) = \frac{15600}{(t^3 - 24t + 160)} \quad (1)$$

$$L(t) = \frac{9890}{t^2 - 38t + 370} \quad (2)$$

### 2.3 Wednesday, 29th April

- 1.  $\lim x \rightarrow 1^- = e$
- 2.  $\lim x \rightarrow 1^+ = 0$

3. The limit does not exist.

4.

$$x^2 = kx + 1$$

$$1 = k + 1$$

$$0 = k$$

5.  $\frac{1}{10}$

6.  $\frac{2}{\pi}$

7. 2

8. Basically just more L'hospital's review.

9.

$$\text{Given Rate: } \frac{dy}{dt} = -2 \quad (1)$$

$$\text{find } \frac{dx}{dt} \text{ when } x = \frac{-3}{2} \quad (2)$$

10.

$$k(t) = (t^2 - 9)^{4/5} \quad (1)$$

$$y = u^{4/5} \quad u = t^2 - 9 \quad (2)$$

$$y' = \frac{4}{5}u^{4/5} \quad (3)$$

$$(4)$$