1 New Content

1.1 Monday, 13th April

1. Slope Fields are drawings with the slope at any given point, and can tell a lot about a differential equation without solving it.

2. A big reason slope fields are useful is that there are some differential equations we are unable to solve. The handout has some examples and rules.

3. Drawing out a slope field by hand is tedious and unpleasant, so don't do it unless you have to.

1.2 Tuesday, 14th April

1. Continuing from April Fools' Day:

$$\frac{dy}{dx} = 4xy \tag{1}$$
$$\int \frac{1}{y}dy = \int 4xdx \tag{2}$$
$$\ln y = 2x^2 + C_1 \tag{3}$$
$$y = Ce^{2x^2} \tag{4}$$

2. $\frac{dy}{dt}$ is the same as a rate of change, but it's also a part of a differential equation.

$$\frac{dy}{dt} = ky \tag{1}$$
$$\int \frac{1}{y}dy = kdt \tag{2}$$
$$lny = kt + c \tag{3}$$
$$y = Ce^{kt} \tag{4}$$

3. **Growth and Decay** gives us the equation $y = Ce^{kt}$ where C is the initial value, k is the constant of proportionality, e is e, and t is time. If k is positive, it is growing, and if negative, it is decaying.

4. In many problems using growth and decay, we want to first find k.

$$4 = Ce^{2k} \tag{1}$$
$$4 = 2e^{2k} \tag{2}$$
$$2 = e^{2k} \tag{3}$$
$$\ln 2 = \ln e^{2k} \tag{4}$$
$$\frac{1}{2}\ln 2 = k \tag{5}$$

5. Continuing on, using this fact:

$$y = 2e^{\frac{t}{2}\ln 2} \tag{6}$$
$$= e^{\ln 2^{\frac{t}{2}}} \tag{7}$$
$$= 2(2^{\frac{t}{2}} \tag{8}$$
$$= 2(2^{\frac{3}{2}}) \tag{9}$$

6. This sort of thing can be applied as well to half-lives.

$$y = Ce^{24100k} \tag{1}$$
$$\frac{y}{c} = e^{24100k} \tag{2}$$
$$\ln\frac{1}{2} = 24100k \tag{3}$$
$$k = \frac{\ln\frac{1}{2}}{24100} \tag{4}$$

if k is half-life, then $k = \frac{ln\frac{1}{2}}{h}$ and if k is a constant of proportionality then h is $\frac{ln\frac{1}{2}}{k}$.