1 New Content

1.1 Monday, 13th April

1. Slope Fields are drawings with the slope at any given point, and can tell a lot about a differential equation without solving it.

2. A big reason slope fields are useful is that there are some differential equations we are unable to solve. The handout has some examples and rules.

3. Drawing out a slope field by hand is tedious and unpleasant, so don't do it unless you have to.

1.2 Tuesday, 14th April

1. Continuing from April Fools' Day:

$$\frac{dy}{dx} = 4xy \tag{1}$$
$$\int \frac{1}{y} dy = \int 4x dx \tag{2}$$
$$\ln y = 2x^2 + C_1 \tag{3}$$
$$y = Ce^{2x^2} \tag{4}$$

2. $\frac{dy}{dt}$ is the same as a rate of change, but it's also a part of a differential equation.

$$\frac{dy}{dt} = ky \tag{1}$$
$$\int \frac{1}{y} dy = kdt \tag{2}$$
$$lny = kt + c \tag{3}$$
$$y = Ce^{kt} \tag{4}$$

3. **Growth and Decay** gives us the equation $y = Ce^{kt}$ where C is the initial value, k is the constant of proportionality, e is e, and t is time. If k is positive, it is growing, and if negative, it is decaying.

4. In many problems using growth and decay, we want to first find k.

$$4 = Ce^{2k} \tag{1}$$
$$4 = 2e^{2k} \tag{2}$$
$$2 = e^{2k} \tag{3}$$
$$\ln 2 = \ln e^{2k} \tag{4}$$
$$\frac{1}{2} \ln 2 = k \tag{5}$$

5. Continuing on, using this fact:

$$
\begin{aligned}
y &= 2e^{\frac{t}{2}\ln 2} && (6)\\
&= e^{\ln 2^{\frac{t}{2}}} && (7)\\
&= 2(2^{\frac{t}{2}} && (8)\\
&= 2(2^{\frac{3}{2}}) && (9)
\end{aligned}
$$

6. This sort of thing can be applied as well to half-lives.

$$
\begin{aligned}
y &= Ce^{24100k} && (1)\\
\frac{y}{c} &= e^{24100k} && (2)\\
\ln\frac{1}{2} &= 24100k && (3)\\
k &= \frac{\ln\frac{1}{2}}{24100} && (4)
\end{aligned}
$$

if k is half-life, then $k = \frac{ln\frac{1}{2}}{h}$ and if k is a constant of proportionality then h is $\frac{ln\frac{1}{2}}{k}$.

2 AP Preparations

2.1 Monday, 20th April

3. From the 2000 AP Exam

   (a) $x = -1$ is a relative minimum because $f\prime$ goes from negative to positive.

   (b) $x = -5$ is a relative maximum because $f\prime$ goes from positive to negative.

   (c) $(-7, -3)$ $(2, 3)$ $(3, 5)$

   (d) For this we check the maxima and endpoints. $x = 7$ and $x = -7$ are endpoints. It is $x = 7$ since it's the best.

2.2 Wednesday, 22nd April

1. From the board at the time:

$$
\begin{aligned}
E(t) &= \frac{15600}{(t^3 - 24t + 160)} && (1)\\
L(t) &= \frac{9890}{t^2 - 38t + 370} && (2)
\end{aligned}
$$

2.3 Wednesday, 29th April

1. $\lim x \to 1^- = e$

2. $\lim x \to 1^+ = 0$

3. The limit does not exist.

4.

$$\begin{aligned} x^2 &= kx+1 \\ 1 &= k+1 \\ 0 &= k \end{aligned}$$

5. $\frac{1}{10}$

6. $\frac{2}{\pi}$

7. 2

8. Basically just more L'hopital's review.

9.

$$\text{Given Rate: } \frac{dy}{dt} = -2 \tag{1}$$
$$\text{find } \frac{dx}{dt} \text{ when } x = \frac{-3}{2} \tag{2}$$

10.

$$k(t) = (t^2-9)^{4/5} \tag{1}$$
$$y = u^{4/5} \qquad u = t^2-9 \tag{2}$$
$$y' = \frac{4}{5}u^{4/5} \tag{3}$$
$$\tag{4}$$

2.4 Friday, 1 May

The AP test is soon. Be afraid.

1.

$$h(x) = x^2 arcsecx \tag{1}$$
$$h'(x) = x^2\left(\frac{1}{x\sqrt{x^2-1}}\right) \tag{2}$$
$$= \frac{x}{\sqrt{x^2-1}} + 2xarcsec(x) \tag{3}$$

2.

$$y = \ln(x^2+4) - \frac{1}{2}arctan\frac{x}{2} \tag{1}$$
$$y' = \frac{2x}{x^2+4} - \frac{1}{2} \times \frac{\frac{1}{2}}{\frac{x^2}{4}+1} \tag{2}$$
$$\tag{3}$$

Solve from here for practice, since O'connor did the favor of erasing it for us.

3. Rolle's theorem explains that where the derivative of both endpoints is the same the second derivative has to be 0 at some point in the thing.

4. Mean value theorem is the broader variation on Rolle's theorem. Both assume it is differentiable (and thus continuous as a requirement, as a function is differentiable iff it is continuous.)