Comments: I figured out a way to use LaTeX a little better for my maths notes. Gummi (or emacs) allows for constant viewing of the pdf output so writing math is made a little less complex. **Edit:** I now have vim macros, set up to be a more than competent replacement for any IDE.

1 Notes - 15.02.10

1.
$$\frac{d}{dx}\left[ln(8) + 3ln(x)\right] = \frac{3}{x}$$

2.
$$\frac{40x(5x^2-3)^3}{(5x^2-3)^4} = \frac{40x}{5x^2-3}$$

3. **Reminder Vocab:** To find the derivative implicitly, you find $\frac{dy}{dx}$ by taking the derivative with respect to x, which gives you $\left(\frac{dy}{dx}\right)$

4. **Logarithmic Differentiation** is a different way to differentiate by taking the natural log of both sides. Here is an example.

$$\begin{aligned} y &= \frac{x^2\sqrt{5+x}}{x^3+4} \\ ln(y) &= ln\frac{x^2\sqrt{5+x}}{x^3+4} \\ &= 2ln(x) + \frac{1}{2}ln\left(5+x\right) - ln\left(x^3+4\right) \\ \frac{1}{y}\frac{dy}{dx} &= \frac{2}{x} + \frac{1}{2(5+x)} - \frac{3x^2}{x^3+4}8 \end{aligned}$$

2 Notes - 15.02.11

61. This problem is a (severe) compression of the one from last night, and I skipped the notation of all the horrible algebra.

$$\begin{aligned} y^prime &= \frac{-\frac{x}{x^2+1} * x + \sqrt{x^2+1}}{x^2} + \frac{1+\frac{x}{\sqrt{x^2+1}}}{x+\sqrt{x^+1}} \\ &= \frac{1+x^2}{x^2\sqrt{x^2+1}} \end{aligned}$$

1. There are some interesting things with integrals. This set are indefinite.

$$\int \frac{x^2}{3-x^2}dx = \int -\frac{1}{3}u^(-1)du$$
$$u = 3 - x^3$$
$$du = -3x^2 dx$$
$$\int -\frac{1}{3}u^(-1)du = -\frac{1}{3}\ ln|3-x^3| + c$$

2.

$$\frac{d}{dx}\Big[x + y - 1 = ln(x^2 + y^2)$$
$$1 + \frac{dy}{dx} = \frac{2x + 2y\,\frac{dy}{dx}}{x^2 + y^2}$$

3 Notes - 15.02.12

1. **Long division** and rewriting are fun. Here is an example of why I am lying.

$$\int \frac{x^3 - 3x^2 + 5}{x - 3}\ dx = \int \left(x^2 + \frac{5}{x-3}\right)$$

$$\int \frac{x^2 - 4}{x}\ dx = \int \left(x - \frac{4}{x}\right)$$

2. **Trig functions** are also just the coolest. 338/31 you are given csc and sec generally so you don't need to worry as much, just let $u = 2x$

4 Notes - 15.02.18

1. **Inverse Functions** are functions found when one switches x and y and solves for y. These require the function they are based on to be 1:1, essentially passing the Horizontal Line Test. An interesting property is that $f(f^{-1}(x)) = x$. This also applies in reverse, $f^{-1}(f(x)) = x$. This is the **Definition of Inverse Functions**.

   Interestingly enough, the graph of a function and its inverse is symmetric over $y = x$. The **Reflective Property of Inverse Functions** states that all points (a, b) on a function have a corresponding point (b, a) on its inverse. To figure this out, use the increasing from derivative thing.

   The existence of an inverse function is based on its being a 1:1 function and its being strictly monotonic on its entire domain. **Monotonic** means that it doesn't change directions. If a function is monotonic, it is guaranteed to be 1:1 and thus will have an inverse.

2. **Guidelines for finding the inverse function:** In order to find if something has an inverse, figure out if it's 1:1 first and then figure out what its inverse is. Finding the inverse is based on swapping x and y.

$$f(x) = x^3 - 6x^2 + 12x$$
$$f'(x) = 3x^2 - 12x + 12$$

5 Notes - 15.02.25

Thus begins the reign of the great Marchi.

1. Trig functions should be mechanical. Even if the unit circle remains unmemorized, the 2 special triangles can be used to bring any special angle to bear.

2. The sine of an angle is the output, the arcsin of the output is an angle. Not tough.

3. Arcsin is not a function. In restricting the domain of it, or the range of $sin(x)$, one can make sin a 1:1 function and thus arcsin becomes a function. II-III and I-IV are continuous. Look at $sin(x)$ over the interval $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$.

4. This brings us to the concept of "Principal Roots." Sin runs between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$, leaving us a 1:1 function.

   $$1 - cos^2(x) = sin(x)$$

   is something that helps a lot in dealing with Trig equations. See the following:

   $$\begin{aligned} 2sin^2(x) &= 2 + cos(x) \\ 2\left(1 - cos^2(x)\right) &= 2 + cos(x) \\ 2 - 2cos^2(x) &= 2 + cos(x) \\ -2cos^2(x) &= cos(x) \end{aligned}$$

5. It is important to check solutions. Some operations introduce false answers in finding roots (obviously.)

   $$\begin{aligned} 1 - cos(x) &= \sqrt{3}sin(x) \\ 1 - 2\cos x + \cos^2 x &= 3\sin^2 x \\ 1 - 3 - 2\cos x + \cos^2 x + 3\cos^2 x &= 0 \\ 4\cos^2 x - 2\cos x - 1 &= 0 \\ (2\cos x + 1)(\cos x - 1) &= 0 \end{aligned}$$

   However, one of the answers that this yields, $\frac{4\pi}{3} + k$, yields a false answer. This comes from the fact we had to square at the beginning to get anywhere with what we know. This highlights the importance of checking work.

6 Notes - 15.02.26

1. While the multiplicative inverse is the same as the reciprocal, the inverse and the reciprocal are different things.

2. The graph of csc is parabolae, or seemingly so, the domain of which being all reals except $0 + -\pi k$. Secant is the same but shifted, as expected of something cosine-related. Cotangent is fairly similar to tangent, but differently placed.

3. Evenness, oddness, etc: one can determine if something is even by whether $f(x) = f(-x)$. Symmetry over the y axis also applies. Odd functions follow $f(-x) = -f(x)$. Sin is odd, cos is even, obviously. Csc is odd, Sec is even. Tan is

$$\begin{aligned} f(t) &= |\csc t| \\ f(-t) &= |\csc -t| \end{aligned}$$

4. Think about Implicit differentiation, like this:

$$\begin{aligned} 9x^2 + 4y^2 &= 36 \\ 18x + 8y\frac{dy}{dx} &= 0 \\ \frac{dy}{dx} &= \frac{-9x}{4y} \end{aligned}$$

5. Trick for Implicit diff: numerator, treat y as though it were a constant. Denominator, treat x as though it were a constant. Ask someont to explain this a little more in depth at some point.

$$\begin{aligned} 4x^2y - 3y &= x^3 - 1 \\ \frac{dy}{dx} &= -\frac{8xy - 3x^2}{4x^2 - 3} \end{aligned}$$

6. Logarithmic differentiation: it is helpful to use both logarithms and implicit differentiation to deal with all of life's problems. For example:

$$\begin{aligned} y &= x^x \\ \ln y &= \ln x^2 \\ \ln y &= x \ln x \\ \frac{1}{y}\frac{dy}{dx} &= x\frac{1}{x} + \ln x \end{aligned}$$

7. Logarithmic differentiation offers an alternative to the quotient rule, as well. Instead of dealing with fractions that leave one wanting to vomit tremendously, the laws of logarithms can be applied to add and subtract things. So instead of the fraction:

$$y = \frac{(x-3)^4(x^2+1)}{(2x+5)^3}$$

You can have:

$$\ln y = \ln(x-3)^4 + \ln(x^2+1) - \ln(2x+5)^3$$

7 Notes - 15.02.27

1. You can graph $\frac{d}{dx}$ of a function on a graphing calculator. You can also do definite integrals. Mr. Marchi is very excited about some feature of graphing calculators he learned earlier (excited being a relative term, as I feel Marchi may only think in equations.) Integrals take a while to graph, because they're so tricky to calculate (as graphing calculators have the processing power of an analog wristwatch.) **Sto** button can be used to avoid using a rounded answer. Press Sto and then the alpha letter you want to represent it.
2. Storing values saves the problem of rounding, and can be helpful if a value is used again and again.
3. For programming Riemann Sums etc. use the list menu and take a sequence. It follows the syntax `seq(expression, variable, start, stop, scale)`, and can be stored as a list. Sto L1 lets you hit Stat then Edit to look at the lists. List then Sum(L1) will give you the value of the Riemann Sum you input.
4. There is a way to do that without storing it as a list, but storing it as a list makes it seem like you did the work by giving you the values. You can do L/R/Midpt Riemann sums with the same setup and different start/end values.
5. Trapezoid rule is trickier than the other one, because outside values are used only once. This runs into using two lists. Sequence is super useful.

8 Notes On Chapter 5

8.1 Section 5.5 - Bases other than e

1. Exponential Functions are functions that take the form a^x. Any function taking the form of a^x can also be written as $e^{\ln a^x}$. This means $a^x = e^{a \ln x}$. This means any exponential can be written in terms of the natural logarithmic function.
2. The **Change of Base** formula is as follows:

$$\log ax = \frac{\ln x}{\ln a}$$

3. Properties of Inverse Functions are thus:
   (a) $y = a^x$ if and only if $x = \log ay$
   (b) $a^{logax} = x$ for all $x > 0$
   (c) $\log aa^x = x$ for all x
4. The general formula for exponentials to a power u is:
$$\frac{d}{dx}a^u = u'(\ln a)a^u$$
5. The above is a very useful shortcut for the use of actual rules in problems. Do not view it as a plain formula. Here are some other formulae useful in similar things:
   (a) $\frac{d}{dx}[a^x] = (\ln a)a^x$
   (b) $\frac{d}{dx}[\log ax] = \frac{1}{(\ln a)x}$
   (c) $\frac{d}{dx}[\log au] = \frac{1}{(\ln a)x}$

8.2 Defining e

1. Interesting way to define e:
$$\lim x \to \infty \left(1 + \frac{1}{x}\right)^x = \lim x \to \infty \left(\frac{x+1}{x}\right)^x$$
   The book calls this "A limit involving e." It's on page 364.

8.3 Inverse Trig Functions

1. No trig function is 1:1, but any one of them can be limited in order to talk about its inverse functions.
2. Arcsin can be understood as "The sin of what is equal to...?" or some similar concept. In this sense, arcsin is to sin as ln is to e^x.
   (a) $\arctan 1 = \frac{\pi}{4}$
   (b) $\operatorname{arccsc} \frac{2}{\sqrt{3}} = \frac{\pi}{3}$
   (c) $\arcsin -\frac{\sqrt{3}}{2} = -\frac{\pi}{3}$
   (d) $\arctan(-1) = -\frac{\pi}{4}$
   (e) $\operatorname{arccsc}(-2) = -\frac{\pi}{6}$
   (f) $\arctan(-\sqrt{3}) = x$
   (g) $\arccos \frac{1}{2} = \frac{\pi}{3}$
   (h) $\arccos \frac{\sqrt{3}}{2} = \frac{\pi}{6}$
   (i) $\arccos -\frac{\sqrt{3}}{2} = \frac{5\pi}{6}$

   You get the idea, but essentially this is the principle of it; Inverse trig functions aren't that difficult.
3. In trig, just draw triangles. Do it. That's all you need. It really works wonders.

8.4 ...And How to Use Them.

1. $\frac{d}{dx}[y = \arcsin(x)] = \frac{d}{dx}[\sin y = x]$, hence $\cos y \frac{dy}{dx} = 1$ so

$$\frac{dy}{dx} = \frac{1}{\cos y} = \frac{1}{\sqrt{\sin y} = \frac{1}{\sqrt{\arcsin x}}}$$

2. Similarly, here's arccos.

$$\frac{d}{dx} \arccos x = \frac{d}{dx}\left[cosy = x\right] \tag{1}$$
$$-\sin y \frac{dy}{dx} = 1 \tag{2}$$
$$\frac{dy}{dx} = \frac{-1}{\sqrt{1-x^2}} \tag{3}$$

3. Here are the rules for all derivatives of inverse trig functions:

$$\frac{d}{dx} \arcsin u = \frac{u'}{\sqrt{1-u^2}} \tag{1}$$
$$\frac{d}{dx} \arccos u = \frac{u'}{\sqrt{1-u^2}} \tag{2}$$
$$\frac{d}{dx} \arctan u = \frac{u'}{u^2+1} \tag{3}$$
$$\frac{d}{dx} \operatorname{arccot} u = \frac{-u'}{u^2+1} \tag{4}$$
$$\frac{d}{dx} \operatorname{arcsec} u = \frac{u'}{|u|\sqrt{u^2-1}} \tag{5}$$
$$\frac{d}{dx} \operatorname{arccsc} u = \frac{-u'}{|u|\sqrt{u^2-1}} \tag{6}$$

   Notice how $\frac{d}{dx \arccos u} = -\frac{d}{dx} \arcsin u$ and the rest of those statements apply here.

4. And here is some practice:

$$y = \arcsin x + x\sqrt{1-x^2} \tag{1}$$
$$= \frac{2-2x^2}{\sqrt{1-x^2}} \tag{2}$$
$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} + \frac{-x^2}{\sqrt{1-x^2}} + \sqrt{1-x^2} \tag{3}$$
$$= 2\sqrt{1-x^2} \tag{4}$$

8.5 Completing the Square, Separating the fraction, and integrating.

1.

$$\int \frac{dx}{\sqrt{20-8x-x^2}} = \int \frac{dx}{\sqrt{36-(x+4)^2}} \tag{1}$$

2. Found on p. 383 is a list of all basic integration rules we have. Here, we have work or something:

3. We can do problems where the degree of the numerator is less than the degree of the denominator, if the problem is nice and simple there. The issue boils down to what you need to do u-substitution.

4. The test, this Friday, will cover 5.6 and 5.7.

9 Notes on Chapter 7

9.1 Area between two curves

1. Make sure to typeset the notes from Monday.

2. Finding the integral statement of two variables is fairly easy.

3. Important is that the area between two curves will **Always be a positive number.**

9.2 Volume of a solid generated by revolving a region around a line

1. The volume of a solid generated by revolving a region about a line is easily found by treating it as a series of disks; this can be seen in the formula:

$$V = \pi \int_a^b [R(x)]^2 \, dx$$

This is known as the "disc method" because it's formed by discs about the x axis.

2. There is another method of dealing with volume of a revolved equation. This is known as the "washer method."
The washer method is good when you are revolving about the axis but it isn't the edge of the area. The formula is:

$$\pi \int_a^b \left[f(x)^2 - g(x)^2\right] dx$$

This isn't really a new method, it's just an application of the area between two curves to the disc method, **but you must square before you subtract.**

9.3 A summary, perhaps

1. The problem is as follows:

$$y = \sqrt{x} \qquad y = 0 \qquad x = 4$$

About the x-axis, the volume is:

$$V = \pi \int_0^4 \sqrt{x}^2 dx = 8\pi$$

About the line $y = 2$, the outer and inner radii switch. This also yields a hole. The volume is:

$$V = \pi \int_0^4 (2-0)^2 - (2-\sqrt{x})^2 dx$$

About the line $y = 3$:

$$V = \pi \int_0^4 \left[(3-0)^2 - (3-\sqrt{x})^2\right] dx$$

And about the line $y = -1$

$$V = \pi \int_0^4 \left[(\sqrt{x}+1)^2 - (0+1)^2\right]$$

2. If a negative area is yielded, flip the order of subtraction.

3. Then, working about the y-axis, we must put functions in terms of x. The integral is:

$$V = \pi \int_0^2 (4^2 - y^4) dy$$

Then about the line $x = 4$:

$$V = \pi \int_0^2 (4 - y^2)^2 dy$$

And about the line $x = 6$:

$$V = \pi \int_0^2$$

9.4 The Method of Cylinders

1. Think if the "slice" you were using was, rather than parallel to the axis of rotation, perpendicular to it. This could be solved by working with respect to a different variable (example switching from $y =$ to $x =$) but can also be solved with the "Method of Cylinders."

2. It is as thus:

$$V = 2\pi \int_a^b x f(x) dx$$

3. This method is pretty damn useful. It doesn't have a squaring of a term, which is nice. In this case x is the distance from the axis of rotation, $f(x)dx$ is essentially the height and the thickness of the cylinder.

10 Differential Equations

10.1 Solving a Differential Equation

1. Here is how you solve a Differential Equation:

$$\frac{dy}{dx} = 3x^4 - 5x + 7 \tag{1}$$
$$\int dy = \int (3x^4 - 5x + 7)dx \tag{2}$$
$$y = \frac{3}{5}x^5 - \frac{5}{2}x^2 + 7x + c \tag{3}$$
$$2x(y+1) = \frac{dy}{dx} \tag{1}$$
$$\int 2xdx = \int \frac{1}{y+1}dy \tag{2}$$
$$x^2 + c = ln|y+1| \tag{3}$$
$$e^{x^2+c} = |y+1| \tag{4}$$
$$\frac{dy}{dt} = ky \tag{1}$$
$$\int \frac{1}{y}dy = \int kdt \tag{2}$$
$$\ln y = kt + c \tag{3}$$
$$y = Ce^{kt} \tag{4}$$

That is how you solve a differential equation. Fear them.