

**Comments:** I figured out a way to use LaTeX a little better for my maths notes. Gummi (or emacs) allows for constant viewing of the pdf output so writing math is made a little less complex.

## 1 Notes - 15.02.10

1.

$$\frac{d}{dx} [\ln(8) + 3\ln(x)] = \frac{3}{x}$$

2.

$$\frac{40x(5x^2 - 3)^3}{(5x^2 - 3)^4} = \frac{40x}{5x^2 - 3}$$

3. **Reminder Vocab:** To find the derivative implicitly, you find  $\frac{dy}{dx}$  by taking the derivative with respect to  $x$  ( $\frac{d}{dx}$ )

4. **Logarithmic Differentiation** is a different way to differentiate by taking the natural log of both sides. Here is an example.

$$\begin{aligned} y &= \frac{x^2 \sqrt{5+x}}{x^3 + 4} \\ \ln(y) &= \ln \frac{x^2 \sqrt{5+x}}{x^3 + 4} \\ &= 2\ln(x) + \frac{1}{2}\ln(5+x) - \ln(x^3 + 4) \\ \frac{1}{y} \frac{dy}{dx} &= \frac{2}{x} + \frac{1}{2(5+x)} - \frac{3x^2}{x^3 + 4} \end{aligned}$$

## 2 Notes - 15.02.11

61. This problem is a (severe) compression of the one from last night, and I skipped the notation of all the horrible algebra.

$$\begin{aligned} y^{prime} &= \frac{-\frac{x}{x^2+1} * x + \sqrt{x^2+1}}{x^2} + \frac{1 + \frac{x}{\sqrt{x^2+1}}}{x + \sqrt{x^2+1}} \\ &= \frac{1+x^2}{x^2 \sqrt{x^2+1}} \end{aligned}$$

1. There are some interesting things with integrals. This set are indefinite.

$$\begin{aligned}\int \frac{x^2}{3-x^2} dx &= \int -\frac{1}{3} u(-1) du \\ u &= 3-x^3 \\ du &= -3x^2 dx \\ \int -\frac{1}{3} u(-1) du &= -\frac{1}{3} \ln|3-x^3| + c\end{aligned}$$

- 2.

$$\begin{aligned}\frac{d}{dx} [x+y-1] &= \ln(x^2+y^2) \\ 1 + \frac{dy}{dx} &= \frac{2x+2y \frac{dy}{dx}}{x^2+y^2}\end{aligned}$$

### 3 Notes - 15.02.12

1. **Long division** and rewriting are fun. Here is an example of why I am lying.

$$\int \frac{x^3 - 3x^2 + 5}{x-3} dx = \int \left( x^2 + \frac{5}{x-3} \right)$$

$$\int \frac{x^2 - 4}{x} dx = \int \left( x - \frac{4}{x} \right)$$

2. **Trig functions** are also just the coolest. 338/31 you are given csc and sec generally so you don't need to worry as much, just let  $u = 2x$

### 4 Notes - 15.02.18

1. **Inverse Functions** are functions found when one switches x and y and solves for y. These require the function they are based on to be 1:1, essentially passing the Horizontal Line Test. An interesting property is that  $f(f^{-1}(x)) = x$ . This also applies in reverse,  $f^{-1}(f(x)) = x$ . This is the **Definition of Inverse Functions**.

Interestingly enough, the graph of a function and its inverse is symmetric over  $y = x$ . The **Reflective Property of Inverse Functions** states that all points  $(a, b)$  on a function have a corresponding point  $(b, a)$  on its inverse. To figure this out, use the increasing from derivative thing.

The existence of an inverse function is based on its being a 1:1 function and its being strictly monotonic on its entire domain. **Monotonic** means that it doesn't change directions. If a function is monotonic, it is guaranteed to be 1:1 and thus will have an inverse.

2. **Guidelines for finding the inverse function:** In order to find if something has an inverse, figure out if it's 1:1 first and then figure out what its inverse is. Finding the inverse is based on swapping x and y.

$$f(x) = x^3 - 6x^2 + 12x$$

$$f'(x) = 3x^2 - 12x + 12$$

## 5 Notes - 15.02.25

Thus begins the reign of the great Marchi.

1. Trig functions should be mechanical. Even if the unit circle remains unmemorized, the 2 special triangles can be used to bring any special angle to bear.
2. The sine of an angle is the output, the arcsin of the output is an angle. Not tough.
3. Arcsin is not a function. In restricting the domain of it, or the range of  $\sin(x)$ , one can make sin a 1:1 function and thus arcsin becomes a function. II-III and I-IV are continuous. Look at  $\sin(x)$  over the interval  $[-\frac{\pi}{2}, \frac{\pi}{2}]$ .
4. This brings us to the concept of "Principal Roots." Sin runs between  $-\frac{\pi}{2}$  and  $\frac{\pi}{2}$ , leaving us a 1:1 function.

$$1 - \cos^2(x) = \sin(x)$$

is something that helps a lot in dealing with Trig equations. See the following:

$$2\sin^2(x) = 2 + \cos(x)$$

$$2(1 - \cos^2(x)) = 2 + \cos(x)$$

$$2 - 2\cos^2(x) = 2 + \cos(x)$$

$$-2\cos^2(x) = \cos(x)$$

5. It is important to check solutions. Some operations introduce false answers in finding roots (obviously.)

$$1 - \cos(x) = \sqrt{3}\sin(x)$$

$$1 - 2\cos x + \cos^2 x = 3\sin^2 x$$

$$1 - 3 - 2\cos x + \cos^2 x + 3\cos^2 x = 0$$

$$4\cos^2 x - 2\cos x - 1 = 0$$

$$(2\cos x + 1)(\cos x - 1) = 0$$

However, one of the answers that this yields,  $\frac{4\pi}{3} + k$ , yields a false answer. This comes from the fact we had to square at the beginning to get anywhere with what we know. This highlights the importance of checking work.

## 6 Notes - 15.02.26

1. While the multiplicative inverse is the same as the reciprocal, the inverse and the reciprocal are different things.
2. The graph of  $\csc$  is parabola, or seemingly so, the domain of which being all reals except  $0 + \pi k$ . Secant is the same but shifted, as expected of something cosine-related. Cotangent is fairly similar to tangent, but differently placed.
3. Evenness, oddness, etc: one can determine if something is even by whether  $f(x) = f(-x)$ . Symmetry over the y axis also applies. Odd functions follow  $f(-x) = -f(x)$ . Sin is odd, cos is even, obviously. Csc is odd, Sec is even. Tan is

$$\begin{aligned}f(t) &= |\csc t| \\f(-t) &= |\csc -t|\end{aligned}$$

4. Think about Implicit differentiation, like this:

$$\begin{aligned}9x^2 + 4y^2 &= 36 \\18x + 8y \frac{dy}{dx} &= 0 \\ \frac{dy}{dx} &= \frac{-9x}{4y}\end{aligned}$$

5. Trick for Implicit diff: numerator, treat y as though it were a constant. Denominator, treat x as though it were a constant. Ask someone to explain this a little more in depth at some point.

$$\begin{aligned}4x^2y - 3y &= x^3 - 1 \\ \frac{dy}{dx} &= -\frac{8xy - 3x^2}{4x^2 - 3}\end{aligned}$$

6. Logarithmic differentiation: it is helpful to use both logarithms and implicit differentiation to deal with all of life's problems. For example:

$$\begin{aligned}y &= x^x \\ \ln y &= \ln x^x \\ \ln y &= x \ln x \\ \frac{1}{y} \frac{dy}{dx} &= x \frac{1}{x} + \ln x\end{aligned}$$

7. Logarithmic differentiation offers an alternative to the quotient rule, as well. Instead of dealing with fractions that leave one wanting to vomit tremendously, the laws of logarithms can be applied to add and subtract things. So instead of the fraction:

$$y = \frac{(x-3)^4(x^2+1)}{(2x+5)^3}$$

You can have:

$$\ln y = \ln(x-3)^4 + \ln(x^2+1) - \ln(2x+5)^3$$