0.1 Intro to Area

Finding the area under the curve is done through a sum of rectangles, where the number of rectangles is n and n goes to infinity.

0.2 Summations

The summation of two things can be broken apart if the things are added or subtracted. Similar to integrals.

From 1 to n summed is $\frac{n(n+1)}{2}$. From 1^2 to n^2 summed is $\frac{n(n+1)(2n+1)}{6}$. Cubed is $\left[\frac{n(n+1)}{2}\right]$.

1 Section 4.3

1.1 First Fundamental Theorem of Calculus

Let f be continuous on the closed interval $[a, b]$ and x be in (a, b).

$$\frac{d}{dx}\int_a^x f(t)dt = f(x)$$

This is fundamental because:

- Every continuous function is a derivative of another function.
- Every continuous function has an antiderivative.
- Integration and differentiation are reverse operands.

1.2 Comparison Property

If f and g are integrable on $[a, b]$ and if $f(x) <= g(x)$ for all x in $[a, b]$ then:

$$\int_a^b f(x)dx <= \int_a^b g(x)dx$$

1.3 Linearity of Definite Integrals

Important about definite integrals:

- Constants may be moved out front.
- Sums and differences may be moved into separate integrals.

1.4 Other Important Properties

- Integral from a to b is the same as the opposite of the integral from b to a
- Middle points may work.

$$\frac{d}{dx}\int_1^x 3t^2 dt = 3x^2$$

$$\frac{d}{dx}\int_x^1 2t dt = -2x$$

Then there's:

$$\frac{d}{dx}\int_1^x xt\ dt$$

which comes out to

$$\frac{d}{dx}\left(x\left(\frac{1}{2}x^2 - \frac{1}{2}\right)\right)$$

$$\frac{3}{2}x^2 - \frac{1}{2}$$

But what if, rather than from 1 to x, it goes from 1 to x^2? Short answer: u-substitution.

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$