Harvard University
Computer Science 121

Problem Set 6

Due Friday, November 5, 2010 at 1:20 PM.
Submit a single PDF (lastname+ps6.pdf) of your solutions to cs121+ps6@seas.harvard.edu
Late problem sets may be turned in until Monday, November 8, 2010 at 1:20 PM with a 20% penalty.
See syllabus for collaboration policy.

Name

Problem set by !!! Your Name Here !!!

with collaborator !!! Collaborators' names here !!!!

Note that recursive is another word for Turing-decidable, and recursively enumerable (r.e.) is another term for Turing-recognizable.

There are several possible levels of formalism for describing Turing machines:

Formal description: You can write out a formal 7-tuple representation and use either a state diagram or a table to describe the transition function, as done in Sipser 3.9.

Implementation description: You can describe *clearly* how the tape and head of the TM work without specifying the states or the transition function, as done in Sipser 3.11 and 3.12.

High-level description: You can give a still higher level description, as done in Sipser 3.23.

PROBLEM 1

(A) Write a general grammar that generates $\{ww : w \in \{a,b\}^*\}$. Explain in words what each rule in your grammar does.
(B) Demonstrate a derivation of this grammar for the string *abbabb*.

PROBLEM 2

For this problem, give an implementation-level description for any Turing machine you construct.

Define $\textsc{Prefix}(L) = \{x : xy \in L \text{ for some } y \in \Sigma^*\}$.

Show that if L is r.e., then $\textsc{Prefix}(L)$ is r.e.

PROBLEM 3

Show that every infinite r.e. language has an infinite recursive subset. (*Hint:* apply an ordering to Σ^*.)

PROBLEM 4

From this point on, give a high-level description for any Turing machine you construct.

(A) Let $L = \{\langle D, k\rangle : D \text{ is a DFA that accepts exactly } k \text{ strings, where } k \in \mathbb{N} \cup \{\infty\}\}$. Show that L is recursive.

(*Hint:* Show how to find a p such that if D accepts any string of length at least p, then D accepts infinitely many strings.)

(B) Let $L = \{\langle M\rangle : M \text{ is a TM and } L(M) \text{ contains a string with no } a\text{'s}\}$. Show that L is r.e.

PROBLEM 5

Consider $L = \{\langle M\rangle : M \text{ is a TM that accepts no strings shorter than 42 characters in length}\}$.

(A) Prove that L is not recursive.

(B) Prove that L is not r.e. (*Hint:* use the result of part A.)

PROBLEM 6 (Challenge!! 3 points)

Show that there is an infinite co-r.e. set that has no infinite r.e. subset. Recall that a language L is co-r.e. if its complement is r.e.