Homework 3 Data Mining *

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March 7, 2012

1 Hierarchical Clustering

A: Mean link probably works the best on large data, since you don't have to compute pairwise norms. It is hard to say which worked "best", since that depends on your measurement (*i.e.*, inter-cluster similarity, extra-cluster similarity, whether it looks "good", etc.). That said, it is likely that the min link clustering produced the best-looking results, since the points that should be clustered together tend to be closest to the clusters they should be in.

Single-Link Clustering:

```
CLUST
('m', (14.02, 5.03))
('n', (16.05, 5.01))
CLUST
('a', (4.01, 15.021))
('b', (3.02, 14.031))
('c', (2.99, 12.02))
('d', (3.107, 10.04))
('e', (3.08, 8.05))
('f', (3.04, 7.03))
('g', (4.01, 6.06))
('h', (5.07, 5.06))
('i', (6.02, 4.03))
('j', (8.06, 4.09))
('k', (10.02, 4.08))
('l', (12.01, 4.07))
CLUST
('o', (12.54, 12.51))
('p', (12.03, 12.04))
('q', (11.52, 11.57))
('r', (11.03, 11.09))
('s', (10.51, 10.532))
('t', (10.01, 10.01))
('u', (12.5, 15.52))
('v', (12.06, 15.1))
('w', (11.55, 14.57))
```

*CS 6955 Data Mining; Spring 2012 Instructor: Jeff M. Phillips, University of Utah

```
('x', (11.08, 14.3))
('y', (10.52, 13.53))
('z', (10.03, 13.008))
```

Complete-Link Clustering:

```
CLUST
('e', (3.08, 8.05))
('f', (3.04, 7.03))
('g', (4.01, 6.06))
('h', (5.07, 5.06))
('i', (6.02, 4.03))
CLUST
('j', (8.06, 4.09))
('k', (10.02, 4.08))
('l', (12.01, 4.07))
('m', (14.02, 5.03))
('n', (16.05, 5.01))
CLUST
('a', (4.01, 15.021))
('b', (3.02, 14.031))
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('w', (11.55, 14.57))
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('y', (10.52, 13.53))
('z', (10.03, 13.008))
```

Average-Link Clustering:

```
CLUST
('o', (12.54, 12.51))
('p', (12.03, 12.04))
('q', (11.52, 11.57))
('r', (11.03, 11.09))
('s', (10.51, 10.532))
('t', (10.01, 10.01))
('u', (12.5, 15.52))
('v', (12.06, 15.1))
```

```
('w', (11.55, 14.57))
('x', (11.08, 14.3))
('y', (10.52, 13.53))
('z', (10.03, 13.008))
CLUST
('j', (8.06, 4.09))
('k', (10.02, 4.08))
('l', (12.01, 4.07))
('m', (14.02, 5.03))
('n', (16.05, 5.01))
CLUST
('a', (4.01, 15.021))
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('h', (5.07, 5.06))
('i', (6.02, 4.03))
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Average-Link Clustering:

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('q', (11.52, 11.57))
('r', (11.03, 11.09))
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('t', (10.01, 10.01))
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('w', (11.55, 14.57))
('x', (11.08, 14.3))
('y', (10.52, 13.53))
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CLUST
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```

```
('f', (3.04, 7.03))
('g', (4.01, 6.06))
('h', (5.07, 5.06))
('i', (6.02, 4.03))
```

B: Begin with the definition of the average-link cluster:

$$\frac{1}{|S_1||S_2|} \sum_{(s_1,s_2) \in S_1 \times S_2} ||s_1 - s_2||_2 = \frac{1}{|S_1||S_2|} \sum_{(s_1,s_2) \in S_1 \times S_2} \sqrt{s_1^2 - s_2^2} \tag{1}$$

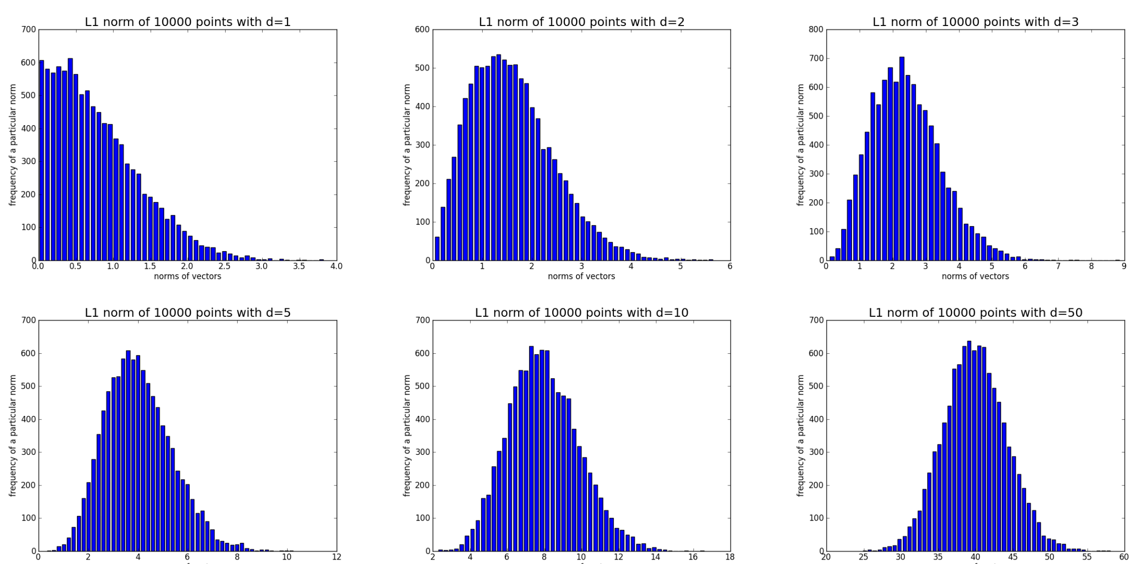
$$= \frac{1}{|S_1||S_2|} \sqrt{\sum_{s_1 \in S_1} s_1^2 - \sum_{s_2 \in S_2} s_2^2} \tag{2}$$

This is clearly equivalent to the mean cluster, as long as it's $\mathbb{R}^1$.

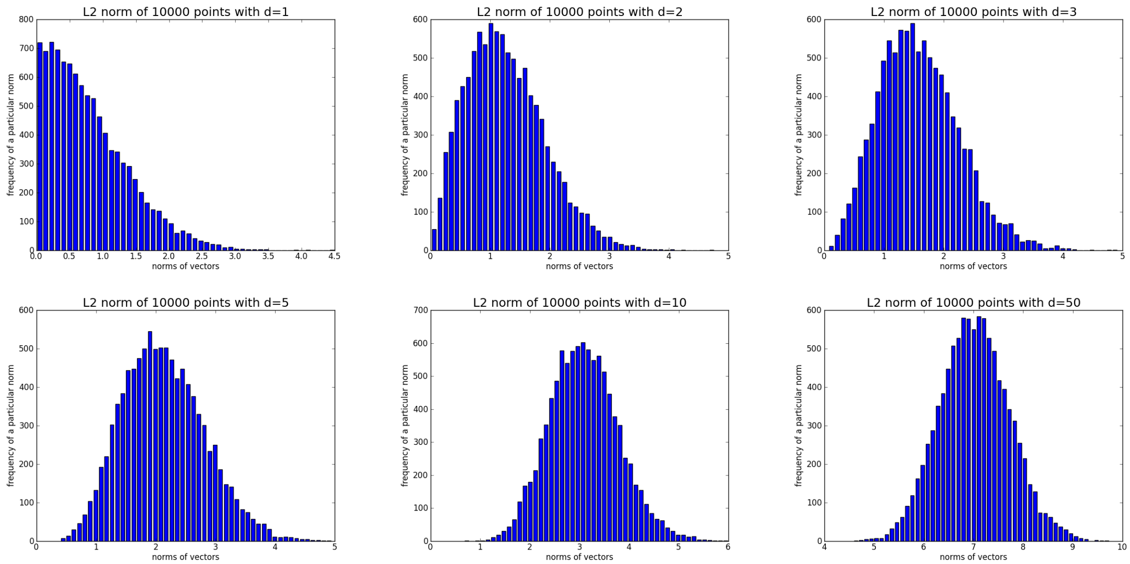
2 Point Assignment Clustering

3 High Dimensions

A: I used $n = 10,000$ for my histograms. **First**, the histograms for the L_1 norms for increadingly high dimensionality d:



And second, the histograms for the L_2 norms on increasingly high dimensionality d:



B: I estimated the probability by generating $n = 10,000$ samples, norming them, and calculating at the empirical $\mathbf{Pr}(L_P(\vec{x}) < 1)$ for all $\vec{x}$ in my generated observations.

The tables are broken apart by dimension d (*i.e.*, the first table is for $d = 1$, the second for $d = 2$, and so on). The *index* column is just the number of the experiment—it's useful because there should be 30 in all, which we can plainly see. The P column refers to which norm L_P we're using, *e.g.*, L_1, L_2, and so on.

Finally, (and most importantly) the last column refers to the empirical probability that the norm is less 1. We calculate this by taking the number of samples for which this condition is true, and dividing that number by the total.

For $d = 1$:

index	d	P in L_P	$\mathbf{Pr}(< 1)$
1	1	0.5	1.000000
2	1	1	1.000000
3	1	2	1.000000
4	1	3	1.000000
5	inf	3.0	1.000000

For $d = 2$:

index	d	P in L_P	$\mathbf{Pr}(< 1)$
6	2	0.5	0.166200
7	2	1	0.501400
8	2	2	0.785800
9	2	3	0.883900
10	inf	3.0	0.883900

For $d = 3$:

index	d	P in L_P	$\mathbf{Pr}(< 1)$
11	3	0.5	0.011600
12	3	1	0.158100
13	3	2	0.523600
14	3	3	0.706900
15	inf	3.0	0.706900

For $d = 5$:

index	d	P in L_P	$\mathbf{Pr}(< 1)$
16	5	0.5	0.000000
17	5	1	0.008400
18	5	2	0.160200
19	5	3	0.369700
20	inf	3.0	0.369700

For $d = 10$:

index	d	P in L_P	$\mathbf{Pr}(< 1)$
21	10	0.5	0.000000
22	10	1	0.000000
23	10	2	0.002400
24	10	3	0.035600
25	inf	3.0	0.035600

For $d = 50$:

index	d	P in L_P	$\mathbf{Pr}(< 1)$
26	50	0.5	0.000000
27	50	1	0.000000
28	50	2	0.000000
29	50	3	0.000000
30	inf	3.0	0.000000