

## Assignment 28

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We'll examine the simplest algorithm I could think of. Given  $r = a^{b^c} \pmod p$ : obviously  $b^c$  is  $c$  multiplications of  $b$  (e.g.,  $b^3 = b \cdot b \cdot b$ ). So right there, in our simplest possible algorithm, we have  $c$  operations. We take the result of this, let's call it  $j$ , and then we perform  $j$  multiplications of  $a$ . So if  $j = 3$ , then we multiply  $a$  3 times:  $a \cdot a \cdot a$ . Our result is then modded by  $p$ , giving us  $r = a^j \pmod p$ . We can

The running time should be  $O(n^2 \cdot n)$ : multiplication of the grade-school variety requires  $O(n^2)$  time and we're doing it a variable number (say  $t$ ) times. Since we know that we do first  $c$  multiplications, and then  $j$  multiplications, and since we know that  $c, j, a < p$ , we know that in the worst case this is bounded by the length of  $p$ .

If we dealt with arbitrary-length integers, we may be in trouble. Since we're not, it should take  $O(n^3)$ .