# Section 2 Handout
# Week of 9.27.10

September 27, 2010

## Outline

- Recap
- Regular Expressions
- Countability/uncountability

## 1 Recap of the course so far...

After covering fundamental mathematical preliminaries, we examined the *finite automaton*, a simple computational model with limited memory. We proved that DFAs, NFAs and regular expressions are equal in computing power and recognize the *regular languages*, which are closed under *union, concatenation, Kleene Star, intersection, difference, complement, reversal*. We used the concept of *countability* to prove the existence of *non-regular languages*. A good reminder: if you have questions on the problem set or these section notes, just send an email to *cs121@seas.harvard.edu*.

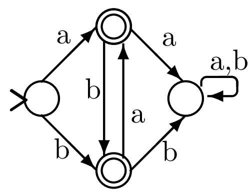
## 2 Regular expressions

**Exercise 2.1.** *Write a regular expression for the following languages $L$ over the alphabet $\Sigma = \{a, b\}$:*

1. $L = \{w \mid w$ *contains abb as a substring* $\}$
2. $L = \{w \mid w = a^n b^m$ *where* $n + m$ *is even* $\}$
3. $L = \{w \mid w = a^n b^m$ *where* $n \geq 4, m \leq 3\}$

Regular expressions are just as expressive as a DFA. As a refresher of this fact, let's use the GNFA construction to obtain a regular expression from the following DFA:

**Exercise 2.2.** *Find the regular expression that generates the same language as the following DFA:*




## 3 Countability/uncountability

**Things to note:** The differences between cardinalities: finite, countable, countably infinite, and uncountable (uncountably infinite). How do we show that a set is countable? How do we show that a set is uncountable? Understand diagonalization and constructing bijections.
**Example problems:**

**Exercise 3.1.** *Which of the following sets is countably infinite? Uncountably infinite?*

1. $\mathbb{N} \times \{0, 1\}$
2. $\mathcal{P}(\mathbb{N})$
3. *The set of all languages over* $\Sigma = \{a, b\}$

**Exercise 3.2.** *Here's a proposed way to make a 1-1 map from* $[0, 1) \in \mathbb{R}$ *to* $\mathbb{N}$*. For every* $x \in \mathbb{R}$*, we map it to the integer corresponding to the reversal of the decimal expansion of* $x$*. For example,* $0.25$ *maps to* $52$*,* $0.12345$ *maps to* $54321$*, and so forth. Obviously, to go back from the integer back to* $[0, 1)$*, we just reverse it again and prepend "0."*.
*Is this a valid construction? Why or why not?*

## 4 Extra Problems

Here are a couple of problems where you can further apply the material learned in section. If you want more practice doing induction proofs, do this problem by induction on the length of a regular expression:

**Exercise 4.1.** *For any language $L$, let $L^R = \{w^R | w \in L\}$, where $w^R$ is the* reversal *of string $w$. Prove that if $L$ is regular, so is $L^R$.*

And if you want something (significantly) more challenging, try this:

**Exercise 4.2.** *An arithmetic progression is a set $\{p+qn : n = 0, 1, 2, \ldots\}$ for some $p, q \in \mathbb{N}$. Show that for any $\Sigma$, if $L$ is a regular language and $L' = \{|w| : w \in L\}$, then $L'$ is a union of finitely many arithmetic progressions.*

**Exercise 4.3.** *A spy is located on a one-dimensional line. At time 0, the spy is at location $A$. With each time interval, the spy moves $B$ units to the right (if $B$ is negative, the spy is moving left). $A$ and $B$ are fixed integers, but they are unknown to you. You are to catch the spy. The means by which you can attempt to do that is: at each time interval (starting at time 0), you can choose a location on the line and ask whether or not the spy is currently at that location. That is, you will ask a question like "Is the spy currently at location 27?" and you will get a yes/no answer. Devise an algorithm that will eventually find the spy.*