Harvard University
Computer Science 121

Problem Set 4

Due Friday, October 15, 2010 at 1:20 PM.
Submit a single PDF (lastname+ps4.pdf) of your solutions to cs121+ps4@seas.harvard.edu
Late problem sets may be turned in until Monday, October 18, 2010 at 1:20 PM with a 20% penalty.
See syllabus for collaboration policy.

Name

Problem set by !!! Your Name Here !!!

with collaborator !!! Collaborators' names here !!!!

PROBLEM 1 (4+4+4 points)

Are the following languages context-free? Prove or disprove. *When giving a formal construction, no correctness proof is required: just an explanation.*
(A) $L = \{a^n b^m : n < m < 3n\}$
(B) $\{a^{n^2} : n \in \mathbb{N}\}$ over $\Sigma = \{a\}$
(C) $\{a^i b^j c^k : i, j, k \in \mathbb{N}, \text{ and if } i = 1 \text{ then } j \geq k\}$ over $\Sigma = \{a, b, c\}$

PROBLEM 2 (6 points)

Draw the state diagram for a PDA for the language of all strings with twice as many as as bs over the alphabet $\{a, b\}$. Use the state diagram notation for PDAs given in Sipser.

PROBLEM 3 (2+8 points)

(A) Give an English language description of the language generated by the following grammar:

$$S \to aB \mid bA \mid \varepsilon$$

$$A \to aS \mid bAA$$

$$B \to bS \mid aBB$$

(B) Now, prove that the grammar does indeed produce the language that you describe.

PROBLEM 4 (12 points)

Unlike the regular languages, the class of context-free languages is not closed under complement. In Example 2.38, Sipser proves that the language $L = \{ww : w \in \{a, b\}^*\}$ is not context-free. Here, you get to look at the complement of L:

Show that the language $\overline{L} = \overline{\{ww : w \in \{a, b\}^*\}}$ is context-free by giving a grammar that generates it. In a sentence or two, justify the correctness of your grammar.

(Hint: argue first that $\overline{L}$ is the set of all strings of the form $xaybz$ or $xbyaz$, where $|x|+|z| = |y|$, along with all strings of odd length.)

PROBLEM 5 (Challenge!! 3 points)

Show that every context-free language over a unary alphabet is regular.