Harvard University
Computer Science 121

Problem Set 1

Due Friday, September 24, 2010 at 1:20 PM.
Submit a single PDF (lastname+ps1.pdf) of your solutions to cs121+ps1@seas.harvard.edu
Late problem sets may be turned in until Monday, September 27, 2010 at 1:20 PM with a 20% penalty.
See syllabus for collaboration policy.

Name

Problem set by !!! Your Name Here !!!

with collaborator !!! Collaborators' names here !!!!

Notes

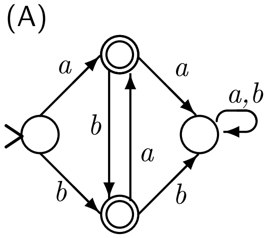
DFAs and NFAs are tough to draw in LaTeX, we recommend using an image editor like Paint, PowerPoint, drawing by hand and scanning in, etc., saving the image to a file and importing the file. The TEX file of this problem set contains the necessary code to import graphics where needed.

When not stated, assume $\Sigma = \{a, b\}$

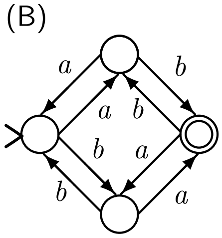
PROBLEM 1 (3+3 points)

Describe informally the language represented by each of the deterministic finite automata below.

(A)



(B)



PROBLEM 2 (4+4 points)

For the following two languages L draw a DFA that recognizes L and give its 5-tuple representation (from Sipser pg. 35) over the alphabet $\{0, 1\}$:

(A) L is the set of all strings containing no consecutive 1s and no consecutive 0s.
(B) L is the set of all strings starting with 00 and ending with 00.

PROBLEM 3 (4+8 points)

(A) Draw an NFA that recognizes the language of all strings with the substring aba.

(B) Convert your NFA from part (A) to a DFA using the subset construction.

PROBLEM 4 (4+4+4 points)

Are the following statements true or false? Justify your answers with a proof or counterexample.

(A) For any languages L_1 and L_2, $(L_1 \cap L_2)^* = L_1^* \cap L_2^*$.

(B) For any languages L_1 and L_2, $(L_1 \cup L_2)^* = L_1^* \cup L_2^*$.

(C) If L is a regular language, then the language of all the strings in L which do not contain ab is regular. (Hint: Regular languages are closed under intersection.)

PROBLEM 5 (5+5 points)

An NFA M contains a *cycle* if there is a state q and a string x such that if M is in state q and reads string x, M can return to state q. Prove or disprove the following statements:

(A) If M recognizes an infinite language, then M has a cycle.
(B) If M has a cycle, then M recognizes an infinite language.

PROBLEM 6 (5+5 points)

(A) For every $n \geq 6$ divisible by 3, prove that there is an undirected graph with exactly n nodes, each of which has degree 4.

(B) Prove that there is no undirected graph with any *odd* number of nodes with the property that every node has degree 3. (Hint: Every edge connects to two nodes.)

PROBLEM 7 (Challenge!!! 1 points)

Consider a regular language L. Prove that the language consisting of all strings which are the first half of a string in L is also regular. More formally, prove that the language HALF(L) = $\{w \mid \exists w' \in \Sigma^* \text{ s.t. } |w| = |w'| \text{ and } ww' \in L\}$ is regular. You may ignore odd length strings.