Harvard University Computer Science 121

Section 3 Handout Week of 10.4.10

Outline

Part 1 Countability

Part 2 Regular vs. non-regular languages

Part 3 Context Free Grammars

1 Countability

Consider a set S. It is each of the following if...

• Finite: if there exists a bijection from $\{1,\ldots,n\}$ to S for some $n\geq 0$

• Countably infinite: if there is a bijection $f: \mathbb{N} \to S$

• Countable: if it is either finite or countably infinite

• Uncountable: it is not countable

Exercise 1.1. Classify the following as countable or uncountable and provide proof:

- $\mathbb{N} \times \{0,1\}$
- $\mathcal{P}(\mathbb{N})$

2 Regular and Nonregular Languages

There are countably many regular expressions over a language, but there are uncountably many languages—so some of these languages *must not* be regular! But how do we find an explicitly non-regular language? We have two techniques: the pumping lemma and the closure properties of regular languages. You can use either of these techniques to prove (by contradiction) that a language is non-regular.

Pumping Lemma for regular languages:

If L is a regular language, then there exists a constant p > 0 such that for any string $w \in L$ with |w| > p, there exist strings $x, y, z \in \Sigma^*$, such that w = xyz, $|xy| \le p$, $y \ne \epsilon$, and $xy^nz \in L$ for all $n \ge 0$.

Closure Properties:

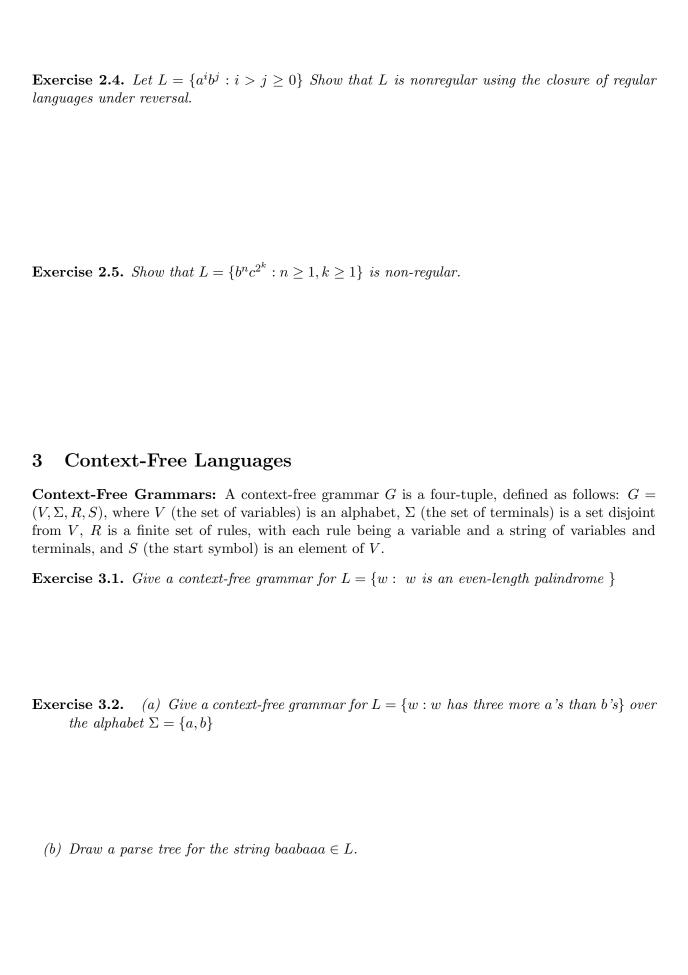
Recall from lecture (and from last week's section) that regular languages are closed under union, concatenation, Kleene Star, intersection, difference, complement, reversal.

Exercise 2.1. Which of the following are necessarily regular?

- A finite language.
- A union of finitely many regular languages.
- $\{x: x \in L_1 \text{ and } x \notin L_2\}$ where L_1 and L_2 are regular.
- A subset of a regular language.

Exercise 2.2. Show that $L = \{a^i b^j : 0 \le i < j\}$ is non-regular using the pumping lemma.

Exercise 2.3. Let $L = \{ww|w \in \Sigma^*\}$. Show that L is non-regular using the pumping lemma.



Exercise 3.3. Let $L=\{wy: w,y\in L(a^*\cup b^*) \text{ and } |w|=|y|\}.$ Is L regular? Is L context-free?