CS 121 2010 Midterm solutions

1. Explain whether these statements are true always, sometimes but not other times, or never.

1A. If L is an infinite language, then L is the complement of some finite language.

Sometimes true. The complement of $L = \{\epsilon\}$ is an infinite language, but so is the complement of $L = \{a^n \mid n \text{ is even }\}$.

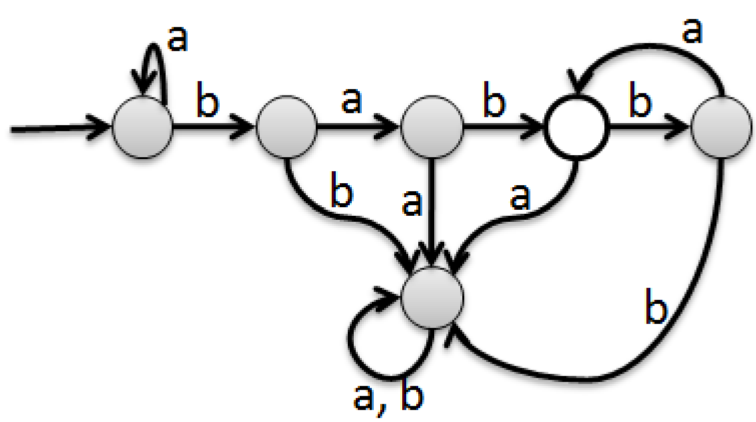
1B. If L is a regular language, then there are infinitely many different DFAs that accept L.

Always true. Simply add states which self-loop for any character in the alphabet to the DFA.

1C. If L is a context-free language, then the complement of L is context-free.

Sometimes true. Regular languages are always context-free and are closed under complement, but context-free languages are not.

2. Draw the state diagram for a DFA that recognizes the language $a^*bab(ba)^*$



3. Write a regular expression for $L = \{w \in \{a,b\}^* \ : \ w \text{ does not contain exactly two } b's\}$.

$$a^* \cup a^*ba^* \cup \Sigma^*b\Sigma^*b\Sigma^*b\Sigma^*$$

4. For any language L, let $\text{COPY}(L) = \{w^n \ : \ w \in L, n \geq 0\}$.

4a. Write a regular expression for $\text{COPY}(\{a, b\})$

$$a^* \cup b^*$$

4b. Show that if L is finite, then $\text{COPY}(L)$ is regular.

$$\forall s_i \in L, \bigcup_i s_i^*$$

Which is a finite union over regular expressions, itself a regular expression, defining $\text{COPY}(L)$ by definition. Since regular expressions generate regular languages $\text{COPY}(L)$ is regular.

4c. Show that if L is infinite, then $\text{COPY}(L)$ need not be regular.

Consider $L = \{a^n b^n \mid n \geq 0\}$. $\text{COPY}(L) = \{(a^n b^n)^k \mid n, k \geq 0\}$. Assume it is regular and let p be its pumping length. Consider the string $a^p b^p$ which is in $\text{COPY}(L)$. Pumping gives us strings of the form $a^{p+k|y|}b^p$ which are not in $\text{COPY}(L)$.

5. Are the following languages regular, context-free but not regular, or not context-free? Explain.

5a. $\{a^i b^j c^k \ : \ i + j + k \leq 10\}$

Regular and context-free since this is a finite language (no strings of length greater than 10 may be generated, and infinite languages over fixed alphabets require arbitrarily long strings) and all finite languages are regular which implies context-free.

5b. $\{a^i b^j c^k \ : \ j = 2i \wedge k = 3i\}$

Neither regular nor context-free. Suppose it is context-free and let p be its pumping length. Consider the string $a^p b^{2p} c^{3p}$ which is clearly in the language. Pumping this string can only increase the number of two of three characters in the string since $|vxy| \leq p$, which is not long enough to span all b's to reach from a's to c's. Increasing the count of only two characters in the string would violate the length constraints, however. Since it's not context-free it's also not regular.

6. Write a context-free grammar for the language $\{uc^n \ : \ u \in \{a, b\}^*, n \geq 0, |u| = n\}$.

$$S \to aSc \mid bSc \mid \epsilon$$