Harvard University
Computer Science 121

Problem Set 8

Due Tuesday, November 23, 2010 at 1:20 PM.
Submit a single PDF (lastname+ps8.pdf) of your solutions to cs121+ps8@seas.harvard.edu
LATE PROBLEM SETS WILL NOT BE ACCEPTED.
See syllabus for collaboration policy.

Name

Problem set by !!! Your Name Here !!!

with collaborator !!! Collaborators' names here !!!!

PROBLEM 1 (3+3+3+3 points)

For each of the following statements, state whether it is true or false, and prove your assertion. For parts A-C, if the statement is true, you should state what the actual parameters n_0 and c are, and why.

(A) $4n^3 + 2n^2 = O(n^3 + n)$.
(B) $\log_2 n = \Theta(\log_{10} n^2)$.
(C) $2^{n/2} = \Omega(2^n)$.
(D) $n^{100001} = o(1.00001^n)$. (*Hint:* you may want to recall l'Hôpital's rule.)

PROBLEM 2 (10 points)

Throughout the class we have focused on computability properties of *languages*. But often we want to understand computational problems that are not YES/NO problems, for example computing a function. In this problem we will see how such problems can be translated into "equivalent" languages (so focusing on languages is not a big restriction).

Given any function $f : \Sigma^* \to \Delta^*$, show how to define a language L_f such that (a) any algorithm to compute f can be computably transformed into an algorithm that decides L_f, and (b) conversely, any algorithm that decides L_f can be transformed into an algorithm that computes f.

PROBLEM 3 (8+12 points)

Prove that the class P is closed under
(A) Concatenation.
(B) Kleene star. (*Hint:* Use dynamic programming. Look at the algorithm we gave in class for recognizing context-free languages via Chomsky Normal Form.)

PROBLEM 4 (10 points)

Show that there is a polynomial time algorithm which, given an NFA N and string w, determines whether N accepts w. Assume a multitape TM model of computation and analyze the degree of the polynomial as a function of both $|\langle N\rangle|$ and $|w|$.

Note that converting N to a DFA won't do the trick, because that step alone would be exponential in $|\langle N\rangle|$.

PROBLEM 5 (10 points)

Prove that $\mathrm{ALL}_{\mathrm{DFA}} = \{\langle D\rangle : D \text{ is a DFA and } L(D) = \Sigma^*\}$ is in P.

PROBLEM 6 (Challenge!! 3 points)

It is known (though not trivial) that testing whether a binary number represents a prime number is in P. However, it is currently unknown whether or not a number (given in binary) can be *factored* in polynomial time. Explicitly construct a TM M that factors numbers such that M runs in polynomial time iff there exists a TM that factors numbers in polynomial time. We want you to give us a correspondence between existence and construction. M should factor a number no matter what, but do it in polynomial time if in fact there exists *some* TM that factors numbers in polynomial time. M should take a binary number n as input and then halt with the (unique) prime factorization of n written on its tape.