Assignment 8

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Problem 1:

(a) False. If $2 = 3$, then $1 = 2$. That is, if two numbers are equal, they all must be equal; there is no possibility that $2 = 3$ while $1 \neq 2$.

(b) True. $2 = 3$ iff $1 = 2$, because if one number is equal to another number, all numbers must be equal. This means a necessary condition is that if one of the two conditions is true, so must both be.

(c)

(d)

(e) True. If a graph has a clique of more than one vertex, then it also has a clique that is one vertex smaller than that. But this is not necessarily true in reverse: a graph with a clique of k vertices does not necessarily have a clique of $k + 1$ vertices.

(f) False. If a graph has a clique of k vertices, it does not necessarily have a clique of $k + 1$ vertices. But this is true in reverse, as we have seen above: if we have a clique of $k + 1$ vertices, as long as $k > 1$, this graph also has a clique of k vertices.

(g) True. If c logically follows from b, and the conclusion that $b \Rightarrow c$ follows logically from a, then both b and c must logically follow from a. But this is not necessarily true the other way: if b and c follow from a, it may not be the case that c also follows from b, let alone that the conclusion that $b \Rightarrow c$ follows from a.

Problem 2:

(a) This is **not** a mapping reduction. $1, 2$, and 3 are all mapped to 1, where a function that maps reducibility should map each number uniquely to one other number. This is why cardinality is so important.

(b) This is a mapping reduction. 1 is mapped to both 1 and 4. A function that maps reducibility should produce each number once, but since our sets $A, B = \{1, 2, 3\}$, this is inconsequential: all numbers mapped after 3 are irrelevant.

(c) In order to reduce A_{TM} to A_{bt}, all we need to do is map all inputs from A_{TM} into A_{bt}. We can do this by sequentially putting $A_{TM}[i]$ into $A_{bt}[i]$, which basically means that every single ends up $A_{TM}[i]$ getting mapped to ε, which is what A_{bt} is all the time.

Problem 3: