Assignment 11

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The assertion that $\log(n!) \in \Omega(n \log n)$ is true essentially because the square of half of n will always be less than $n!$. The assertion that is provided statement taking the "hint" in a very literal sense.

To prove this, we are essentially showing that $\exists c : \log(n!) \geq c \cdot (n \log n)$ where $n >$ some n_0. This is more intimidating than it sounds. Another way of writing it is that $n \geq p^p, \forall n : n \geq n_0, n = 2p$. To illustrate the given claim, we start with some $n_0 = 6$:

$$\begin{aligned} n_0! &\geq p^p \\ 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 &\geq 3^3 \\ 720 &\geq 27 \end{aligned}$$

This is the basis. The intuition is that this will always be the case, for any n. But is it true?

Yes it is. First, consider that $n!$ is always a series of n terms, while p^p is a series of $n/2$ terms (*e.g.* for $n = 6, n! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$, while $p^p = 3 \cdot 3 \cdot 3$). More importantly, however, is that $n/2$ of these terms are *always* bigger than all the terms in p^p. So in the example above, $p^p = 3 \cdot 3 \cdot 3$, and the first 3 terms of $n!$ are $6 \cdot 5 \cdot 4$. So, even if we just had the first $n/2$ terms of $n!$, it would *still* be larger than $p^p, \forall p$.

Just like before, transforming this logic into the form of the assertion is trivial. Both are monotonically increasing, so the inequality will remain intact by log-ing both sides:

$$\begin{aligned} n! &\geq p^p &&, \forall n > n_0, n = 2p \\ \log(n!) &\geq p \log p &&, \forall n > n_0, n = 2p \end{aligned}$$

This is the definition of $\log(n!) \in \Omega(n \log n)$. Therefore $\log(n!) \in \Theta(n \log n)$.