Harvard University
Computer Science 121

Section 4 Notes
Week of 10.11.10

1 Closure Properties of CFLs

Are the CFLs closed under:

- Union?
- Concatenation?
- Kleene star?
- Complement?
- Intersection?
- Intersection with a *regular* language?

2 Sample languages

Are the following languages context-free?

- $\{www : w \in \Sigma^*\}$
- $\{w \in \{(,)\}^* : w$ is not properly parenthesized$\}$

3 PDAs

- Show that $L = \{a^n b^{2n}\}$ is context-free by giving a PDA that accepts it. Draw the state diagram and write the 6-tuple $(Q, \Sigma, \Lambda, \delta, q_0, F)$.[1]

[1] Λ, the stack alphabet, is a capital Lambda.

$M = \{Q, \Sigma, \Lambda, \delta, q_0, F\}$ where:

- Draw a PDA that recognizes $\{a^i b^j c^k | i, j, k \geq 0 \text{ and } i = j \text{ or } i = k\}$.
- Draw a PDA that recognizes $\{ww^R | w \in \{0,1\}^*\}$.

4 The Pumping Lemma for Context-Free Languages

For all context-free languages L, **there exists** a "pumping length" p such that **for all** strings $s \in L$ of length at least p, **there exist** strings u, v, x, y, z such that:

- $s = uvxyz$
- $|vy| > 0$
- $|vxy| \leq p$
- **For all** $i \geq 0$, $uv^i xy^i z \in L$

In other words: in any context-free language, every sufficiently long string can be "pumped" somehow. In other words: push-down automata can only recall one thing at a time.

- Show that the language $L = \{a^i b^j c^k | 0 \leq i \leq j \leq k\}$ is not context free.

5 "Almost all" languages are neither regular nor context-free.

- What do we mean?
- Why is this true?

[1] By Σ_ε, we mean $\Sigma \cup \{\varepsilon\}$.