**Exercise 1.** Let $C, D$ be two chain complexes of $R$-modules. Their tensor product $C \otimes_R D$ is defined as follows. We let

$$(C \otimes_R D)_n = \bigoplus_{i \in \mathbb{Z}} C_i \otimes_R D_{n-i}$$

and for $x \in C_i$ and $y \in D_{n-i}$, the differential is given by

$$d_n^{C \otimes_R D}(x \otimes y) = d_i^C(x) \otimes y + (-1)^i x \otimes d_{n-i}^D(y).$$

(i) Show that $(C \otimes_R D, d^{C \otimes_R D})$ defines a chain complex.

(ii) Show that the complexes $C \otimes_R D$ and $D \otimes_R C$ are isomorphic.

**Exercise 2.** Let $f \colon B \to C$ be a morphism of chain complexes. We let

$$\mathit{cone}(f)_n = B_{n-1} \oplus C_n$$

and define a morphism $d_n \colon \mathit{cone}(f)_n \to \mathit{cone}(f)_{n-1}$ by

$$d_n(b, c) = (-d_{n-1}^B(b), d_n^C(c) - f_{n-1}(b)).$$

(i) Show that $(\mathit{cone}(f), d)$ defines a chain complex.

(ii) Show that we have an exact sequence of complexes

$$0 \to C \to \mathit{cone}(f) \to B[-1] \to 0,$$

where $B[-1]$ is the complex defined by $B[-1]_n = B_{n-1}$ and $d_n^{B[-1]} = -d_{n-1}^B$.

(iii) Deduce that we have a long exact sequence

$$\cdots \to H_{n+1}(\mathit{cone}(f)) \to H_n(B) \xrightarrow{\delta} H_n(C) \to H_n(\mathit{cone}(f)) \to \ldots$$

(iv) Show that the morphism $\delta \colon H_n(B) \to H_n(C)$ may be chosen to coincide with the morphism induced by $f$.

(v) Deduce that $f$ is a quasi-isomorphism if and only if $\mathit{cone}(f)$ is exact.