When $\mathcal{F}$ is a presheaf on X, we denote by $\mathcal{F}_x$ the stalk at $x \in X$, and by $a(\mathcal{F})$ the sheaf associated with $\mathcal{F}$.

Exercise 1. Let $\pi\colon Y \to X$ be a local homeomorphism, and Γ_π its sheaf of sections. Show that for any $x \in X$ the natural map $(\Gamma_\pi)_x \to \pi^{-1}\{x\}$ is a bijection.

Exercise 2. Let $\mathcal{F}$ and $\mathcal{G}$ be two presheaves on a topological space X.

(i) Let $x \in X$. Show that the natural map $(\mathcal{F} \times \mathcal{G})_x \to \mathcal{F}_x \times \mathcal{G}_x$ is a bijection.

(ii) Deduce that the natural morphism $a(\mathcal{F} \times \mathcal{G}) \to a(\mathcal{F}) \times a(\mathcal{G})$ is an isomorphism.

Exercise 3. Let E be a set and X a topological space. The value of the *constant sheaf* $\underline{E}$ on an open subset U of X is the set of continuous maps $U \to E$, where E is endowed with the discrete topology. Show that $\underline{E}$ is isomorphic to the sheaf associated with the presheaf taking the value E on every open subset of X.

Exercise 4. Let $\mathcal{F}$ be a presheaf on a topological space X.

(i) Let $x \in X$ and $i\colon \{x\} \to X$ be the inclusion. Let E be a set, and $\underline{E}$ the constant sheaf on $\{x\}$ associated with E (i.e. $\underline{E}(\{x\}) = E$ and $\underline{E}(\varnothing) = \{*\}$). Show that the set of presheaf morphisms $\mathcal{F} \to i_*\underline{E}$ is in bijection with the set of maps $\mathcal{F}_x \to E$.

(ii) Let $j\colon U \to X$ be the inclusion of an open subset. Let $\mathcal{G}$ be a presheaf on U. Show that the set of presheaf morphisms $\mathcal{F} \to j_*\mathcal{G}$ and $\mathcal{F}|_U \to \mathcal{G}$ are in bijection.