**Exercise 1.** Let $K = \mathbb{Q}(\sqrt{d})$ where $d \in \mathbb{Z} \smallsetminus \{0, 1\}$ is square-free.

(i) Let $q \in \mathbb{N} \smallsetminus \{0\}$. Show that $\mathcal{O}_K$ admits a nonzero principal ideal $I$ such that $\mathrm{N}(I) = q$ if and only if there exist $a, b \in \mathbb{Z}$ such that

$$|a^2 - db^2| = \begin{cases} q & \text{if } d = 2, 3 \mod 4, \\ 4q & \text{if } d = 1 \mod 4. \end{cases}$$

(ii) If $d \in \{7, -11\}$, show that $\mathcal{O}_K$ is principal.

(iii) If $d = -6$, show that the ideal class group $\mathcal{C}(\mathcal{O}_K)$ is isomorphic to $\mathbb{Z}/2$.