Exercise 1. If A, B are two local subrings of a given field, with respective maximal ideals $\mathfrak{m}_A$ and $\mathfrak{m}_B$, we say that B *dominates* A if $A \subset B$ and $\mathfrak{m}_A \subset \mathfrak{m}_B$.

Let R be a local domain with fraction field K, and X a scheme. Show that to give a morphism $\operatorname{Spec} R \to X$ is equivalent to giving:

- a pair of points z, y such that y is in the (reduced) closure Z of $\{z\}$,
- and an inclusion $k(z) \subset K$ such that R dominates $\mathcal{O}_{Z,y}$ (as subrings of K).

Exercise 2. Show that a morphism $X \to S$ of schemes is separated if and only if the subset $\Delta_{X/S}(X)$ is closed in $X \times_S X$.

Exercise 3. (i) Let $R \to S$ be an injective ring morphism. Show that every minimal prime of R is in the image of $\operatorname{Spec} S \to \operatorname{Spec} R$.

(ii) Let $f \colon X \to Y$ be a quasi-compact morphism of schemes. Assume that the closure of every point of $f(X)$ is contained in $f(X)$. Show that $f(X)$ is closed in Y.

Exercise 4. Prove the valuative criterion of separatedness. You may use without proof the following result:

Let A be a local domain with fraction field K. Then there is a valuation ring R of K which dominates A.