Exercise 1. Let k be a field. Show that $\mathbb{A}^1_k \to \operatorname{Spec} k$ is not proper.

Exercise 2. Let $Z \xrightarrow{g} Y \xrightarrow{f} X$ be morphisms of schemes. Assume that $f \circ g$ is proper and that f is separated.

(i) Show that g is proper.

(ii) Assume that g is surjective and that f is of finite type. Show that f is proper.

Exercise 3. Let k be a field, and A a commutative k-algebra. We assume that $p\colon \operatorname{Spec} A \to \operatorname{Spec} k$ is proper.

(i) Assume that $\dim_k A = \infty$. Show that p factors through a dominant morphism $\operatorname{Spec} A \to \mathbb{A}^1_k$.

(ii) Deduce that $\dim_k A < \infty$.

(iii) Deduce that a proper affine morphism of schemes is quasi-finite.

Exercise 4. Let S be a graded ring.

(i) Let I be a homogeneous ideal. Show that there is a closed immersion $\operatorname{Proj}(S/I) \to \operatorname{Proj}(S)$.

(ii) Let Z be a reduced closed subscheme of $\operatorname{Proj} S$. Show that there is a homogeneous ideal J of S such that $Z = \operatorname{Proj}(S/J)$ in $\operatorname{Proj}(S)$.

(iii) Find a graded ring S and homogeneous ideals $I \neq I'$ such that $\operatorname{Proj}(S/I) = \operatorname{Proj}(S/I')$ as closed subschemes of $\operatorname{Proj}(S)$.