The letter k denotes a field.

Exercise 1. Let Y and Z be two closed subschemes of X. Show that $(Y \times_X Z)_{top} = Y_{top} \cap Z_{top}$ as subspaces of X_{top}.

Exercise 2. Let X a scheme of finite type over $\operatorname{Spec} k$. Assume that X is integral with function field K (the residue field $\kappa(\eta)$ at the generic point η). Show that $\dim X$ coincides with the transcendence degree of K over k.

Exercise 3. Let K/k be a field extension, and A a finitely generated k-algebra. Show that $\dim(A \otimes_k K) = \dim A$.

Exercise 4. Let A, B two finitely generated k-algebras. Show that $\dim(A \otimes_k B) = \dim A + \dim B$.

Exercise 5. Let Z be a closed subscheme of $\mathbb{A}^2_k$. Consider the two projections $p, q\colon \mathbb{A}^2_k = \mathbb{A}^1_k \times_{\operatorname{Spec} k} \mathbb{A}^1_k \to \mathbb{A}^1_k$. If $\dim Z = 1$, show that $p|_Z$ or $q|_Z$ is dominant (i.e. has dense image).

Exercise 6. Let S be a commutative $\mathbb{N}$-graded ring, and $f \in S_0$. Describe the open subscheme $D_h(f) \subset \operatorname{Proj}(S)$, and give an example where it is not affine.

Exercise 7 (Optional)**.** Assume that k is algebraically closed. Let X be an integral scheme of finite type over k. Denote by X_{Var} the set of closed points in X, with its induced topology. Show that $X \mapsto X_{Var}$ induces an equivalence of categories between:

- integral, quasi-projective schemes over $\operatorname{Spec} k$ and morphisms of schemes over $\operatorname{Spec} k$,
- and quasi-projective k-varieties and their morphisms.