Exercise 1. (i) Let X be a noetherian scheme, and $\mathcal{F}$ a coherent sheaf of $\mathcal{O}_X$-modules. Let $\mathcal{F}_\alpha$, for $\alpha \in A$, be a collection of sheaves of $\mathcal{O}_X$-modules and $\bigoplus_{\alpha\in A} \mathcal{F}_\alpha \to \mathcal{F}$ a surjective morphism (i.e. surjective on stalks). Show that there is a finite subset B of A such that the induced morphism $\bigoplus_{\beta\in B} \mathcal{F}_\beta \to \mathcal{F}$ is surjective.

(ii) Let X be an affine scheme, and $\mathcal{G}$ a quasi-coherent sheaf of $\mathcal{O}_X$-modules. Show that the natural morphism $\bigoplus_{\alpha\in A} \mathcal{G}_\alpha \to \mathcal{G}$ is surjective, where $\mathcal{G}_\alpha$ runs over the coherent sheaves of $\mathcal{O}_X$-submodules of $\mathcal{G}$.

(iii) Let X be an affine noetherian scheme and U an open of X. Let $\mathcal{F}$ be a coherent sheaf of $\mathcal{O}_U$-modules. Show that $\mathcal{F}$ is the restriction to U of some coherent sheaf of $\mathcal{O}_X$-modules. (Hint: Let $j\colon U \to X$ be the open immersion. Apply (ii) with $\mathcal{G} = j_*\mathcal{F}$. Observe that the morphism $\mathcal{F} \to j^*j_*\mathcal{F}$ is an isomorphism, and use (i).)

Exercise 2. Let S be a graded ring, generated as an S_0-algebra by S_1. Let $d \geq 1$ be an integer, and consider the graded ring R such that $R_n = S_{nd}$ (with ring structure by that of S). Show that there is an isomorphism $\varphi\colon \operatorname{Proj}(S) \to \operatorname{Proj}(R)$ such that $\varphi^*\mathcal{O}_{\operatorname{Proj}(R)}(1) \simeq \mathcal{O}_{\operatorname{Proj}(S)}(d)$.