Exercise 1. (i) Let A be a commutative ring. Describe the set of scheme morphisms $\operatorname{Spec} A \to (\mathbb{A}^n - 0)$ in terms of elements of A.

(ii) Let X be a scheme. Describe the set of scheme morphisms $X \to (\mathbb{A}^n - 0)$ in terms of global sections of $\mathcal{O}_X$.

Exercise 2. Let X be a scheme of finite type over a field k.

(i) Show that a point of X is closed if and only if its residue field is a finite extension of k.

(ii) Show that closed points are dense in X.

(iii) Give an example of a scheme where closed points are not dense.

Exercise 3. Let k be an algebraically closed field.

(i) Let S be an $\mathbb{N}$-graded ring, generated by elements of degree one. Describe the set of closed points of $\operatorname{Proj} S$ in terms of ideals of S.

(ii) Let $(x_0, \dots, x_n) \in k^{n+1} - 0$. Find a homogeneous prime ideal $\mathfrak{p}$ of $k[T_0, \dots, T_n]$ such that $V(\mathfrak{p})(k) \subset \mathbb{A}^{n+1}(k)$ is identified with

$$\{(\lambda x_0, \dots, \lambda x_n) | \lambda \in k\} \subset k^{n+1}.$$

(iii) Let $\mathfrak{p}$ be a closed point of $\mathbb{P}^n_k$. We view $\mathfrak{p}$ as an ideal of $k[T_0, \dots, T_n]$. Show that we can find $(x_0, \dots, x_n) \in k^{n+1} - 0$ such that

$$V(\mathfrak{p})(k) = \{(\lambda x_0, \dots, \lambda x_n) | \lambda \in k\} \subset \mathbb{A}^{n+1}(k).$$

(iv) Deduce a bijection between $\mathbb{P}^n(k)$ and the set of lines in k^{n+1} containing 0.