Exercise 1. Let A be a commutative unital ring, and $\mathfrak{p}$ a prime ideal of A. Show that the set $\{\mathfrak{p}\}$ is closed in $\operatorname{Spec} A$ if and only if the ideal $\mathfrak{p}$ is maximal in A.

Exercise 2. Let $(X, \mathcal{O}_X)$ be a scheme and U an open subset of X. Show that $(U, \mathcal{O}_X|_U)$ is a scheme.

Exercise 3. A topological space X is called *quasi-compact* if for every family $\{U_i, i \in I\}$ of open subsets of X such that $X = \bigcup_{i \in I} U_i$ we may find a finite subset $J \subset I$ such that $X = \bigcup_{i \in J} U_i$. Show that $\operatorname{Spec} A$ is quasi-compact, when A is commutative unital ring.

Exercise 4. Let k be a field, and $A = k[X_i | i \in \mathbb{N}]$ the polynomial ring in a countable infinite set of variables. Let $I \subset A$ be the ideal generated by the variables X_i for $i \in \mathbb{N}$. Show that the topological space $\operatorname{Spec} A - V(I)$ is not quasi-compact. [Hint: Look at the chain of closed subsets $\cdots \subset V(X_1, \ldots, X_{s+1}) \subset V(X_1, \ldots, X_s) \subset \ldots$]

Exercise 5. Let $(X, \mathcal{O}_X)$ be a scheme and $e \in \mathcal{O}_X(X)$ be such that $e^2 = e$. Let $f = 1 - e$. Show that there exists open subsets X_e and X_f of X such that $X = X_e \cup X_f$ and $X_e \cap X_f = \varnothing$, and such that $e|_{X_e} \in \mathcal{O}_X(X_e)^\times$ and $f|_{X_f} \in \mathcal{O}_X(X_f)^\times$. (For a ring R we denote by $R^\times$ its set of invertible elements.)