Exercise 1. (i) Let X be a variety and U a dense open subset of X. Show that the morphism $\mathcal{O}(X) \to \mathcal{O}(U)$ is injective.

(ii) Let X be a variety and U, V two dense open subsets of X such that $X = U \cup V$. Show that $\mathcal{O}(X) = \mathcal{O}(U) \cap \mathcal{O}(V) \subset \mathcal{O}(U \cap V)$.

(iii) Use the two open subsets of $\mathbb{P}^1$

$$\Omega_i = \mathbb{P}^1 - Z_h(X_i) = \{(x_0 : x_1) | x_i \neq 0\}$$

for $i = 0, 1$ to compute $\mathcal{O}(\mathbb{P}^1)$.

(iv) Is $\mathbb{P}^1$ affine?

Exercise 2. Let $f \in k[X_1, \cdots, X_n]$, and $U = \mathbb{A}^n - Z(f)$. Let $Y = Z(X_{n+1} \cdot f - 1) \subset \mathbb{A}^{n+1}$. Show that the morphism $\mathbb{A}^{n+1} \to \mathbb{A}^n$ given by $(x_1, \cdots, x_{n+1}) \mapsto (x_1, \cdots, x_n)$ induces an isomorphism of varieties $Z \to U$.

Exercise 3. We say that a subset E of $\mathbb{A}^n$ is stable under the action of $k^\times$ when for all $\lambda \in k^\times$

$$(x_1, \cdots, x_n) \in E \Longrightarrow (\lambda x_1, \cdots, \lambda x_n) \in E.$$

Let Y be a closed subset of $\mathbb{A}^n$ stable under the action of $k^\times$. Show that each irreducible component of Y is stable under the action of $k^\times$.

Exercise 4. (Time permitting, the solution will be explained on Nov 23th, otherwise on Nov 30th.) Let X be a quasi-projective variety.

(i) Show that the set of morphisms $X \to \mathbb{A}^n$ may be identified with the product of n copies of $\mathcal{O}(X)$.

(ii) Deduce that for any affine variety Y, the set of morphisms of varieties $X \to Y$ may be identified with the set of k-algebra morphisms $\mathcal{O}(Y) \to \mathcal{O}(X)$.

(iii) Give a counterexample with Y non-affine.