Determinants

square matrices   Determinants

$$\begin{bmatrix} 3 & -1 \\ 0 & 2 \end{bmatrix} \longrightarrow \begin{vmatrix} 3 & -1 \\ 0 & 2 \end{vmatrix}$$

$$\begin{bmatrix} 1 & 0 & -1 \\ -2 & 3 & 0 \\ 1 & 1 & -1 \end{bmatrix} \longrightarrow \begin{vmatrix} 1 & 0 & -1 \\ -2 & 3 & 0 \\ 1 & 1 & -1 \end{vmatrix}$$

1. The Geometry of 2D Space

Imagine a unit square on a grid (a 1×1 square). When you apply a matrix transformation to that square, it usually turns into a tilted parallelogram.

The Determinant Value: The area of that new parallelogram is the determinant.

Example: If your determinant is 3, the transformation has tripled the area of any shape in that space. If it's 0.5, everything has been squished to half its original size.

2. What about 3D and beyond?

In three dimensions, the logic stays the same, but the "size" changes:

Instead of area, the determinant represents the change in volume of a parallelepiped (a 3D slanted box).

In n dimensions, it represents the change in hyper-volume.

3. The "Sign" of the Determinant

Sometimes you'll get a negative determinant (e.g., -2). Since you can't have "negative area," what does that mean?

Orientation Flip: A negative determinant means the transformation flipped space over.

Think of it like looking at a piece of paper in a mirror. In 2D, if the order of your axes (the x and y vectors) switches from counter-clockwise to clockwise, the determinant becomes negative.

Determinants

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$B = \begin{bmatrix} -2 & 3 & 1 & 4 \\ 1 & 1 & 7 & 2 \\ 0 & 0 & -1 & 2 \\ -3 & 2 & 1 & 1 \end{bmatrix}$$

$$\mathrm{Det}(A) = |A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$|B| = \quad -5 \qquad -2\begin{vmatrix} -2 & 3 & 1 \\ 1 & 1 & 7 \\ -3 & 2 & 1 \end{vmatrix}$$

$\mathrm{Det}(A) = 0$: Singular

$\mathrm{Det}(A) \neq 0$: Invertible

$$+1\begin{vmatrix} 1 & 1 \\ -3 & 2 \end{vmatrix} -7\begin{vmatrix} -2 & 3 \\ -3 & 2 \end{vmatrix} +1\begin{vmatrix} -2 & 3 \\ 1 & 1 \end{vmatrix}$$

$$1(5) - 7(5) + 1(-5)$$

$$\cancel{5} - 35 - \cancel{5}$$

udemy

Determinants

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$Det(A) \neq 0$: Invertible

$$B = \begin{bmatrix} -2 & 3 & 1 & 4 \\ 1 & 1 & 7 & 2 \\ 0 & 0 & -1 & 2 \\ -3 & 2 & 1 & 1 \end{bmatrix}$$

$$|B| = \quad -5 \quad -2(-35)$$

$$|B| = -5 + 70$$
$$= 65$$

Rule of Sarrus

$$\begin{vmatrix} -2 & 3 & 1 & -2 & 3 \\ 1 & 1 & 7 & 1 & 1 \\ 0 & 0 & -1 & 0 & 0 \end{vmatrix}$$

$$(-2)(1)(-1) + (3)(7)(0) + (1)(1)(0) \quad -(3)(1)(-1) - (-2)(7)(0) - (1)(1)(0)$$

$$2 + 0 + 0 - (-3) - 0 - 0$$

Cramer's rule for solving systems

$$9x + 10y = 34$$
$$-6x - 5y = -26$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ -2 \end{bmatrix}$$

$$x = \frac{D_x}{D} \qquad y = \frac{D_y}{D}$$

$$\begin{array}{cc} x & y \end{array}$$
$$\begin{bmatrix} 9 & 10 \\ -6 & -5 \end{bmatrix} \qquad \begin{bmatrix} 34 \\ -26 \end{bmatrix}$$

$$D = \begin{vmatrix} 9 & 10 \\ -6 & -5 \end{vmatrix} = -45 - (-60) = -45 + 60 = 15$$

$$D_x = \begin{vmatrix} 34 & 10 \\ -26 & -5 \end{vmatrix} = -170 - (-260) = -170 + 260 = 90$$

$$D_y = \begin{vmatrix} 9 & 34 \\ -6 & -26 \end{vmatrix} = -234 - (-204) = -234 + 204 = -30$$

$$x = \frac{90}{15} = 6$$

$$y = \frac{-30}{15} = -2$$

$$D_x = 12 \qquad D = 12$$

$$3x - 2y + 7z = -41$$
$$-2x + y - 5z = 26$$
$$x + 5y - 4z = 57$$

$$\begin{bmatrix} 3 & -2 & 7 \\ -2 & 1 & -5 \\ 1 & 5 & -4 \end{bmatrix}$$

$$D_y = \begin{vmatrix} 3 & -41 & 7 \\ -2 & 26 & -5 \\ 1 & 57 & -4 \end{vmatrix} = 3(181) + 41($$

$$= 3\begin{vmatrix} 26 & -5 \\ 57 & -4 \end{vmatrix} - (-41)\begin{vmatrix} -2 & -5 \\ 1 & -4 \end{vmatrix} + 7\begin{vmatrix} -2 & 26 \\ 1 & 57 \end{vmatrix}$$

$$= 3(-104 + 285) + 41(8 + 5) + 7(-114 - 26)$$

udemy

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$$x = \frac{90}{15} = 6$$

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$$D_x = 12 \qquad D_y = 96 \qquad D_z = -48 \qquad D = 12$$

$$3x - 2y + 7z = -41$$
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$$x + 5y - 4z = 57$$

$$\begin{bmatrix} 3 & -2 & 7 \\ -2 & 1 & -5 \\ 1 & 5 & -4 \end{bmatrix}$$

$$x = \frac{D_x}{D} = \frac{12}{12} = 1$$

$$y = \frac{D_y}{D} = \frac{96}{12} = 8$$

$$z = \frac{D_z}{D} = \frac{-48}{12} = -4$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 8 \\ -4 \end{bmatrix}$$

Modifying determinants

Scalar

$$A=\begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad |A|$$

$$A=\begin{bmatrix} ka & kb \\ kc & kd \end{bmatrix} \quad k^2|A|$$

Swapped row

$$A=\begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix} \quad |A| \qquad \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \quad |A|$$

$$\begin{bmatrix} 2 & 2 \\ 3 & -1 \end{bmatrix} \quad -|A| \qquad \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \quad -|A|$$

1 swap: -1

2 swaps: $(-1)(-1)=1$

3 swaps: $(-1)^3=-1$

$|A|=-|A|$

$x=-x$

$0=0$

Sum of rows

$$\begin{bmatrix} 2 & 1 \\ 3 & 0 \end{bmatrix} \quad \begin{bmatrix} 2 & 1 \\ 0 & 4 \end{bmatrix} \quad \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$$

$$|A| + |B| = |C|$$

Row operations

$R_1 - R_3 \rightarrow R_3$

$$A=\begin{bmatrix} 3 & 1 & 0 \\ 2 & 2 & 6 \\ 3 & 1 & 0 \end{bmatrix} \qquad \begin{bmatrix} 3 & 1 & 0 \\ 2 & 2 & 6 \\ 0 & 0 & 0 \end{bmatrix} \quad |A|$$

$|A|$

9:41 / 9:52

Upper and lower triangular matrices

UTF : $|A| = -3$

$$A = \begin{bmatrix} 1 & 0 & 3 & 1 \\ 0 & 2 & -1 & 0 \\ 1 & -3 & 2 & 2 \\ -1 & 0 & 1 & -2 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 0 & 3 & 1 \\ 0 & 2 & -1 & 0 \\ 1 & -3 & 2 & 2 \\ -1 & 0 & 1 & -2 \end{vmatrix}$$

$$|A| = \begin{vmatrix} 1 & 0 & 3 & 1 \\ 0 & 2 & -1 & 0 \\ 0 & -3 & -1 & 1 \\ 0 & 0 & 4 & -1 \end{vmatrix}$$

$$\frac{1}{2}|A| = \begin{vmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & -\frac{1}{2} & 0 \\ 0 & -3 & -1 & 1 \\ 0 & 0 & 4 & -1 \end{vmatrix}$$

$$\frac{1}{2}|A| = \begin{vmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & -\frac{5}{2} & 1 \\ 0 & 0 & 4 & -1 \end{vmatrix}$$

$$\left(-\frac{2}{5}\right)\left(\frac{1}{2}\right)|A| = \begin{vmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & -\frac{2}{5} \\ 0 & 0 & 4 & -1 \end{vmatrix}$$

$$-\frac{1}{5}|A| = \begin{vmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & -\frac{2}{5} \\ 0 & 0 & 0 & \frac{3}{5} \end{vmatrix}$$

$$-\frac{1}{5}|A| = \frac{3}{5}$$

$$|A| = -3$$

upper and lower triangular matrices

UTF : $|A| = -3$ LTF : $|A| = -3$

$$A = \begin{bmatrix} 1 & 0 & 3 & 1 \\ 0 & 2 & -1 & 0 \\ 1 & -3 & 2 & 2 \\ -1 & 0 & 1 & -2 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 0 & 3 & 1 \\ 0 & 2 & -1 & 0 \\ 1 & -3 & 2 & 2 \\ -1 & 0 & 1 & -2 \end{vmatrix}$$

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$$-\frac{1}{2}|A| = \begin{vmatrix} 1 & 0 & 3 & 1 \\ 0 & 2 & -1 & 0 \\ 0 & -3 & 3 & 0 \\ \frac{1}{2} & 0 & -\frac{1}{2} & 1 \end{vmatrix}$$

$$-\frac{1}{2}|A| = \begin{vmatrix} \frac{1}{2} & 0 & \frac{7}{2} & 0 \\ 0 & 2 & -1 & 0 \\ 0 & -3 & 3 & 0 \\ \frac{1}{2} & 0 & -\frac{1}{2} & 1 \end{vmatrix}$$

$$-\frac{1}{6}|A| = \begin{vmatrix} \frac{1}{2} & 0 & \frac{7}{2} & 0 \\ 0 & 2 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ \frac{1}{2} & 0 & -\frac{1}{2} & 1 \end{vmatrix}$$

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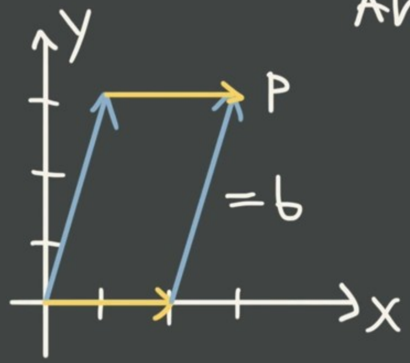
$$-\frac{1}{6}|A| = \frac{1}{2}$$

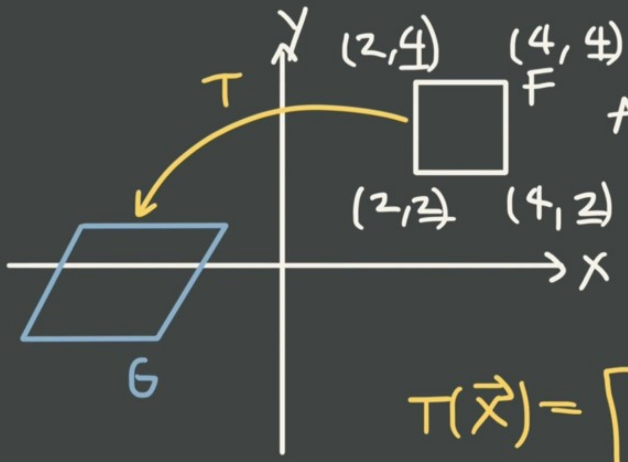
$$|A| = -\frac{6}{2} = -3$$

Using determinants to find area

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix} \qquad |A|$$



$$Area_P = |Det(A)|$$
$$= |0 - 6|$$
$$= |-6|$$
$$= 6$$



$$Area_F = 2 \cdot 2$$
$$= 4$$

$$T(\vec{x}) = \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix} \vec{x}$$

$$\begin{vmatrix} -1 & 2 \\ 0 & 1 \end{vmatrix} = -1 - 0 = -1$$

$$Area_G = |Area_F \cdot Det(T)|$$
$$= |4 \cdot (-1)|$$
$$= |-4| = 4$$

Determinants

square matrices → Determinants

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$$+1\begin{vmatrix} 1 & 1 \\ -3 & 2 \end{vmatrix} - 7\begin{vmatrix} -2 & 3 \\ -3 & 2 \end{vmatrix} + 1\begin{vmatrix} -2 & 3 \\ 1 & 1 \end{vmatrix}$$

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$$\cancel{5} - 35 - \cancel{5}$$

127. Determinants

Determinants

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Rule of Sarrus

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$$(-2)(1)(-1) \quad -(3)(1)(-1)$$
$$+(3)(7)(0) \quad -(-2)(7)(0)$$
$$+(1)(1)(0) \quad -(1)(1)(0)$$

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20:02 / 20:22

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$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 8 \\ -4 \end{bmatrix}$$

131. Modifying determinants

Scalar

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $\quad |A|$

$A = \begin{bmatrix} ka & kb \\ kc & kd \end{bmatrix}$ $\quad k^2|A|$

Swapped row

$A = \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix}$ $\quad |A|$ $\qquad \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$ $\quad |A|$

$\begin{bmatrix} 2 & 2 \\ 3 & -1 \end{bmatrix}$ $\quad -|A|$ $\qquad \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$ $\quad -|A|$

1 swap: -1

2 swaps: $(-1)(-1) = 1$

3 swaps: $(-1)^3 = -1$

$|A| = -|A|$

$x = -x$

$0 = 0$

Sum of rows

$\begin{bmatrix} 2 & 1 \\ 3 & 0 \end{bmatrix} \quad \begin{bmatrix} 2 & 1 \\ 0 & 4 \end{bmatrix} \quad \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$

$|A| + |B| = |C|$

Row operations $\quad R_1 - R_3 \rightarrow R_3$

$A = \begin{bmatrix} 3 & 1 & 0 \\ 2 & 2 & 6 \\ 3 & 1 & 0 \end{bmatrix}$ $\qquad \begin{bmatrix} 3 & 1 & 0 \\ 2 & 2 & 6 \\ 0 & 0 & 0 \end{bmatrix}$ $\quad |A|$

$|A|$

2x 9:41 / 9:52

upper and lower triangular matrices

UTF : $|A| = -3$

$$A = \begin{bmatrix} 1 & 0 & 3 & 1 \\ 0 & 2 & -1 & 0 \\ 1 & -3 & 2 & 2 \\ -1 & 0 & 1 & -2 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 0 & 3 & 1 \\ 0 & 2 & -1 & 0 \\ 1 & -3 & 2 & 2 \\ -1 & 0 & 1 & -2 \end{vmatrix}$$

$$|A| = \begin{vmatrix} 1 & 0 & 3 & 1 \\ 0 & 2 & -1 & 0 \\ 0 & -3 & -1 & 1 \\ 0 & 0 & 4 & -1 \end{vmatrix}$$

$$\frac{1}{2}|A| = \begin{vmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & -\frac{1}{2} & 0 \\ 0 & -3 & -1 & 1 \\ 0 & 0 & 4 & -1 \end{vmatrix}$$

$$\frac{1}{2}|A| = \begin{vmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & -\frac{5}{2} & 1 \\ 0 & 0 & 4 & -1 \end{vmatrix}$$

$$\left(-\frac{\cancel{2}}{5}\right)\left(\frac{1}{\cancel{2}}\right)|A| = \begin{vmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & -\frac{2}{5} \\ 0 & 0 & 4 & -1 \end{vmatrix}$$

$$-\frac{1}{5}|A| = \begin{vmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & -\frac{2}{5} \\ 0 & 0 & 0 & \frac{3}{5} \end{vmatrix}$$

$$-\frac{1}{5}|A| = \frac{3}{5}$$

$$|A| = -3$$

upper and lower triangular matrices

UTF: $|A| = -3$ LTF: $|A| = -3$

$$A = \begin{bmatrix} 1 & 0 & 3 & 1 \\ 0 & 2 & -1 & 0 \\ 1 & -3 & 2 & 2 \\ -1 & 0 & 1 & -2 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 0 & 3 & 1 \\ 0 & 2 & -1 & 0 \\ 1 & -3 & 2 & 2 \\ -1 & 0 & 1 & -2 \end{vmatrix}$$

$$|A| = \begin{vmatrix} 1 & 0 & 3 & 1 \\ 0 & 2 & -1 & 0 \\ 0 & -3 & 3 & 0 \\ -1 & 0 & 1 & -2 \end{vmatrix}$$

$$-\frac{1}{2}|A| = \begin{vmatrix} 1 & 0 & 3 & 1 \\ 0 & 2 & -1 & 0 \\ 0 & -3 & 3 & 0 \\ \frac{1}{2} & 0 & -\frac{1}{2} & 1 \end{vmatrix}$$

$$-\frac{1}{2}|A| = \begin{vmatrix} \frac{1}{2} & 0 & \frac{7}{2} & 0 \\ 0 & 2 & -1 & 0 \\ 0 & -3 & 3 & 0 \\ \frac{1}{2} & 0 & -\frac{1}{2} & 1 \end{vmatrix}$$

$$-\frac{1}{6}|A| = \begin{vmatrix} \frac{1}{2} & 0 & \frac{7}{2} & 0 \\ 0 & 2 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ \frac{1}{2} & 0 & -\frac{1}{2} & 1 \end{vmatrix}$$

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$$-\frac{1}{6}|A| = \begin{vmatrix} \frac{1}{2} & \frac{7}{2} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ \frac{1}{2} & 0 & -\frac{1}{2} & 1 \end{vmatrix}$$

$$-\frac{1}{6}|A| = \begin{vmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ \frac{1}{2} & 0 & -\frac{1}{2} & 1 \end{vmatrix}$$

$$-\frac{1}{6}|A| = \frac{1}{2}$$

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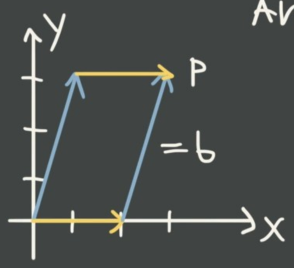
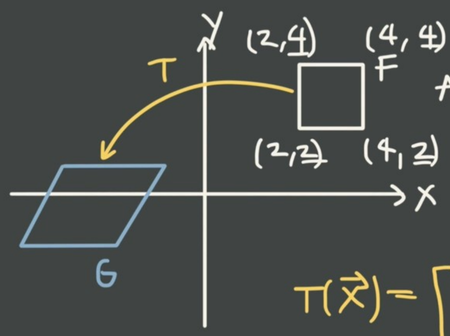
Using determinants to find area

$$A=\begin{bmatrix}1&2\\3&0\end{bmatrix} \qquad |A|$$



$$\text{Area}_P = |\text{Det}(A)|$$
$$=|0-6|$$
$$=|-6|$$
$$=6$$



$$\text{Area}_F = 2\cdot 2$$
$$=4$$

$$T(\vec{x}) = \begin{bmatrix}-1&2\\0&1\end{bmatrix}\vec{x}$$

$$\begin{vmatrix}-1&2\\0&1\end{vmatrix} = -1-0 = -1$$

$$\text{Area}_G = |\text{Area}_F \cdot \text{Det}(T)|$$
$$=|4\cdot(-1)|$$
$$=|-4| = 4$$