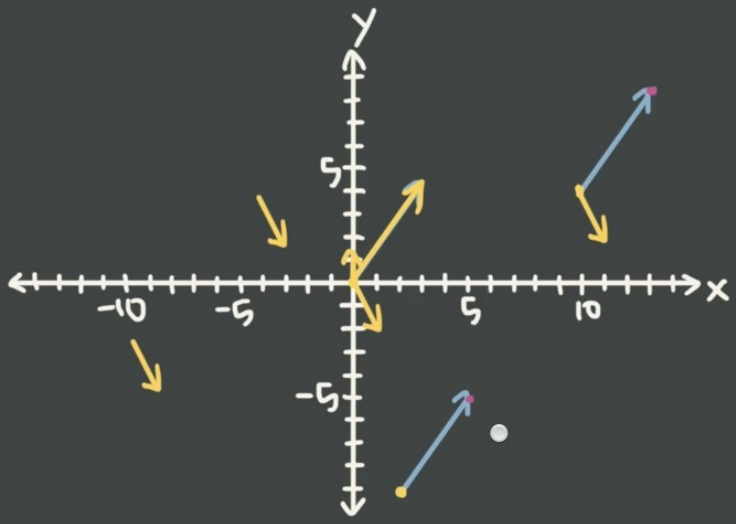
Vectors

1. direction
2. magnitude (length)



$\vec{a} = (3, 4)$

$\vec{a} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

$\vec{a} = \begin{bmatrix} 3 & 4 \end{bmatrix}$

$K = \begin{bmatrix} 1 & 0 & 0 & 3 \\ -2 & 1 & 1 & 4 \end{bmatrix}$

Col. vectors

$k_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ $k_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $k_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$k_4 = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$ $\mathbb{R}^2$

Row vectors

$k_1 = \begin{bmatrix} 1 & 0 & 0 & 3 \end{bmatrix}$ $\mathbb{R}^4$

$k_2 = \begin{bmatrix} -2 & 1 & 1 & 4 \end{bmatrix}$

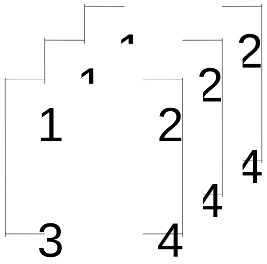
Tensors (Multilinear Map)

- Vectors = 1st order Tensor $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$ (1D arrays)

- Matrices = 2nd order Tensor $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ (2D arrays)

$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ **3x3 matrix Represented as 2D array data structure.**

- 3rd order Tensor , is a stack of matrices (3D arrays)



→ **2 x 2 x 3** Tensor (I ,j, k indices)

- Tensor may have n order.

vector operations

$$\begin{bmatrix}-1\\2\end{bmatrix}+\begin{bmatrix}3\\4\end{bmatrix}=\begin{bmatrix}2\\6\end{bmatrix} \qquad \begin{bmatrix}-1\\2\end{bmatrix}-\begin{bmatrix}3\\4\end{bmatrix}=\begin{bmatrix}-4\\-2\end{bmatrix}$$

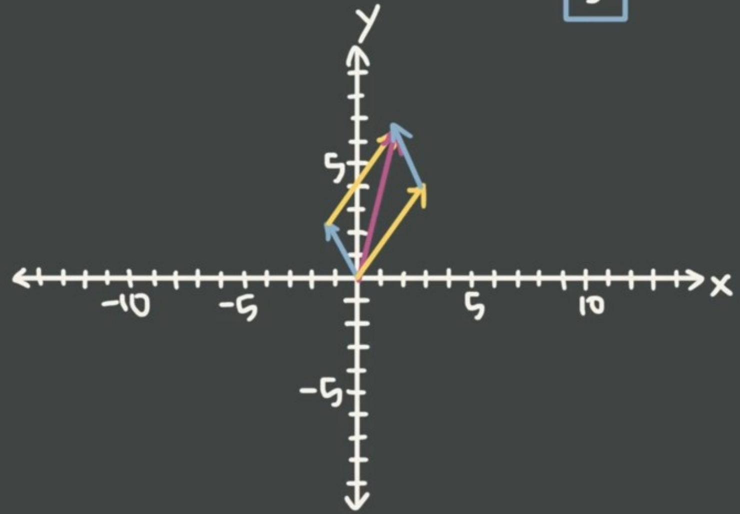
$$\vec{a}=(-1,2)=A$$
$$\vec{b}=(3,4)=B$$

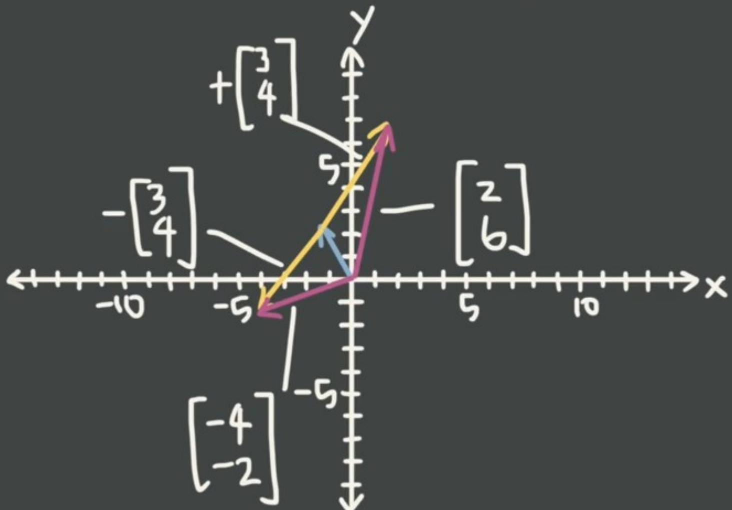
$$\vec{a}\cdot\vec{b}=(-1)(3)+(2)(4)$$
$$=-3+8$$
$$=\boxed{5}$$

$1\times2 \qquad 2\times1$

$$AB=\begin{bmatrix}-1 & 2\end{bmatrix}\begin{bmatrix}3\\4\end{bmatrix}$$
$$=(-1)(3)+(2)(4)$$
$$=-3+8=\boxed{5}$$

$$BA=\begin{bmatrix}3 & 4\end{bmatrix}\begin{bmatrix}-1\\2\end{bmatrix}$$
$$=(3)(-1)+(4)(2)$$
$$=-3+8$$
$$=\boxed{5}$$





Dot products and cross products

$\vec{a} \cdot \vec{b}$

Dot product:
How much $\vec{a}$ and $\vec{b}$
point in the same direction

$\vec{a} \times \vec{b}$

cross product:
The length shows how
much $\vec{a}$ and $\vec{b}$ point
in different directions

$\vec{u}$ $\hat{u}$

✓. direction

×. length = 1

$\hat{i}$ $\hat{j}$ $\hat{k}$ = standard basis

i j k

$\|\vec{v}\| = \sqrt{4^2+(-3)^2}$

$= \sqrt{16+9}$

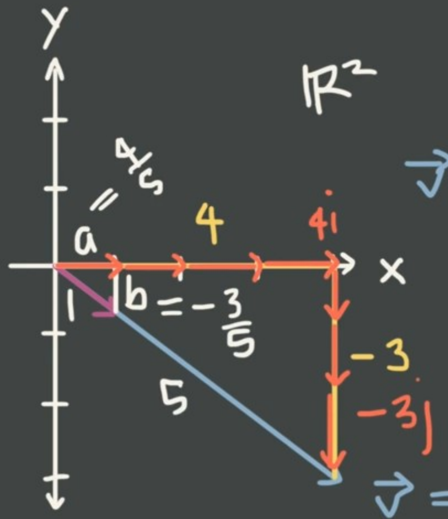
$= \sqrt{25}$

$= 5$

$\vec{v} = (4,-3)$

$\vec{v} = \begin{bmatrix}4\\-3\end{bmatrix} = [4 \;\; -3]$

$\vec{v} = 4i - 3j$



$\vec{v} = 4i - 3j$

$\vec{v} = (4, -3)$

$\vec{u} = \left(\frac{4}{5}, -\frac{3}{5}\right)$

$\frac{4}{5} = \frac{a}{1}$ $\qquad -\frac{3}{5} = \frac{b}{1}$

$a = \frac{4}{5}$ $\qquad b = -\frac{3}{5}$

$\mathbb{R}^2$ $i = (1,0)$

$j = (0,1)$

$\vec{u} = \frac{1}{\|\vec{v}\|}\vec{v}$

$= \frac{1}{5}\begin{bmatrix}4\\-3\end{bmatrix}$

$\vec{u} = \begin{bmatrix}\frac{4}{5}\\-\frac{3}{5}\end{bmatrix}$

$\vec{v} = i + 2j + 3k$ $\qquad \mathbb{R}^3$

$\vec{v} = (1,2,3)$

$\|\vec{v}\| = \sqrt{1^2+2^2+3^2}$

$= \sqrt{1+4+9}$

$= \sqrt{14}$

$\hat{u} = \frac{1}{\sqrt{14}}\begin{bmatrix}1\\2\\3\end{bmatrix}$

$= \begin{bmatrix}1/\sqrt{14}\\2/\sqrt{14}\\3/\sqrt{14}\end{bmatrix}$

$\mathbb{R}^3$ $i = (1,0,0)$

$j = (0,1,0)$

$k = (0,0,1)$

Linear combinations and span

↳ sum of scaled vectors

$\vec{v} = 4i - 3j$

$= 2\vec{a} + 4\vec{b} - \vec{c}$

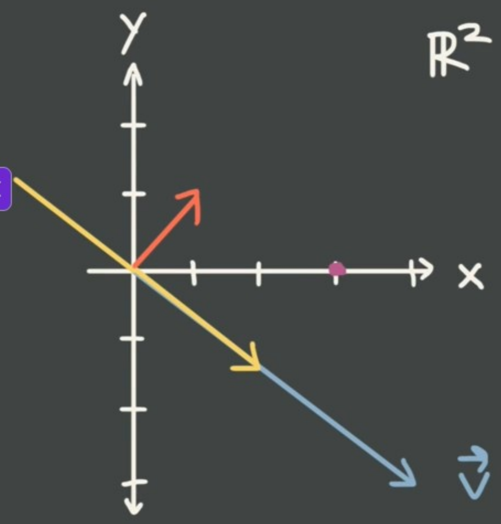
↳ all the linear combinations

$\mathbb{R}^2 = \text{span}(\hat{i}, \hat{j})$

$\vec{a} = (7, -2) = 7i - 2j$

$\vec{b} = (-2, 9) = -2i + 9j$

$\vec{c} = (0, 4) = 4j$



$\vec{v} = (4, -3)$

$\vec{v} = 4i - 3j$

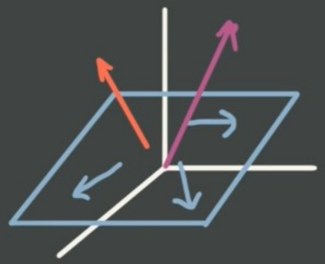
$\vec{v} = 4\begin{bmatrix}1\\0\end{bmatrix} - 3\begin{bmatrix}0\\1\end{bmatrix}$

$\mathbb{R}^3 = \text{span}(\hat{i}, \hat{j}, \hat{k})$

$\vec{d} = (0, 17, -4) = 17j - 4k$

$\mathbb{R}^2$: 2, L.I., in $\mathbb{R}_2$

$\mathbb{R}^3$: 3, L.I., in $\mathbb{R}_3$



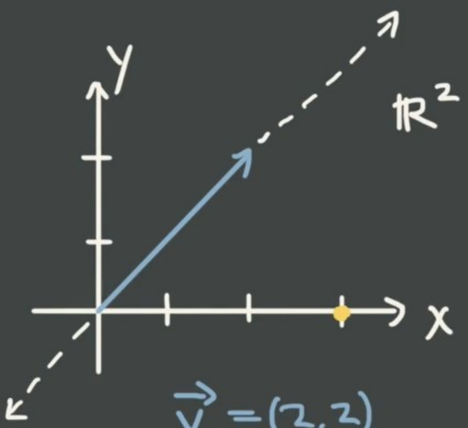
2x 13:28 / 13:28

Linear independence in $\mathbb{R}^2$

$\mathbb{R}^2$: 2, L.I., in $\mathbb{R}^2$

$\mathbb{R}^3$: 3, L.I., in $\mathbb{R}^3$

$\mathbb{R}^n$: n, L.I., in $\mathbb{R}^n$



$\vec{v} = (2,2)$

$\vec{w} = (4,4)$

$$\underbrace{\begin{bmatrix}1\\1\end{bmatrix} \quad \begin{bmatrix}0\\6\end{bmatrix}}_{\text{L.I.}}$$

L.D.

$$2\vec{v} = 2\begin{bmatrix}2\\2\end{bmatrix} = \begin{bmatrix}4\\4\end{bmatrix} = \vec{w}$$

$$2\vec{v} = \vec{w}$$

$$\vec{v} = \frac{1}{2}\vec{w}$$

$$c_1\begin{bmatrix}2\\2\end{bmatrix}$$

$$c_1\begin{bmatrix}1\\1\end{bmatrix} + c_2\begin{bmatrix}0\\6\end{bmatrix} + c_3\begin{bmatrix}2\\3\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

$$c_1 = -2c_3$$
$$c_1 = -6c_2 - 3c_3$$
$$-6c_2 - 3c_3 = -2c_3$$
$$-6c_2 = c_3$$
$$c_1 + 2c_3 = 0$$
$$c_1 + 6c_2 + 3c_3 = 0$$

$$c_1\begin{bmatrix}1\\1\end{bmatrix} + c_2\begin{bmatrix}0\\6\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

$$c_1 + 0 = 0 \Rightarrow \boxed{c_1 = 0}$$
$$c_1 + 6c_2 = 0$$
$$0 + 6c_2 = 0$$
$$6c_2 = 0$$
$$\boxed{c_2 = 0}$$

$$c_1\begin{bmatrix}2\\2\end{bmatrix} + c_2\begin{bmatrix}4\\4\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

$$2c_1 + 4c_2 = 0 \rightarrow c_1 + 2c_2 = 0$$
$$2c_1 + 4c_2 = 0 \qquad c_1 = -2c_2$$

$c_2 = 1 \quad c_1 = -2$

$c_2 = 2 \quad c_1 = -4$

Linear independence in $\mathbb{R}^3$

- $\mathbb{R}^2$: 2, L.I., in $\mathbb{R}^2$
- $\mathbb{R}^3$: 3, L.I., in $\mathbb{R}^3$
- $\mathbb{R}^n$: n, L.I., in $\mathbb{R}^n$

$$\left.\begin{bmatrix}1\\1\\1\end{bmatrix} \quad \begin{bmatrix}0\\-3\\2\end{bmatrix} \quad \begin{bmatrix}1\\0\\3\end{bmatrix}\right\}\text{L.I.}$$

$$c_1\begin{bmatrix}1\\1\\1\end{bmatrix} + c_2\begin{bmatrix}0\\-3\\2\end{bmatrix} + c_3\begin{bmatrix}1\\0\\3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\begin{array}{c} \begin{array}{ccc} c_1 & c_2 & c_3 \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 1 & -3 & 0 & 0 \\ 1 & 2 & 3 & 0 \end{array}\right] \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & -3 & -1 & 0 \\ 0 & 2 & 2 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 1/3 & 0 \\ 0 & 0 & 4/3 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 1/3 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$\begin{array}{c} \begin{array}{ccc} c_1 & c_2 & c_3 \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \end{array}$$

$$\boxed{\begin{array}{l} c_1 = 0 \\ c_2 = 0 \\ c_3 = 0 \end{array}}$$

Linear subspaces

$\mathbb{R}^2$: all 2D vectors $\vec{v} = (x, y)$

$\mathbb{R}^3$: all 3D vectors $\vec{v} = (x, y, z)$

$\vdots$

$\mathbb{R}^n$: all nD vectors $\vec{v} = (v_1, v_2, v_3, \dots v_n)$

subspace of $\mathbb{R}^2$ → $V = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \in \mathbb{R}^2 \mid x + y = 0 \right\}$

$x = -y \quad y = -x$

$= \left\{ \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \dots \right\}$

Subspace

✓1. Include zero vector

✓2. closed under scalar multiplication

✓3. closed under addition

$V = \{ v_1, v_2, v_3, v_4 \dots \}$

$0 = c\,v_3 \qquad v_2 + v_4$

✓Scalar multiplication

$V = \begin{bmatrix} x \\ -x \end{bmatrix} \quad \begin{bmatrix} -y \\ y \end{bmatrix}$

$c \begin{bmatrix} x \\ -x \end{bmatrix} = \begin{bmatrix} cx \\ -cx \end{bmatrix} \qquad cx + (-cx) = 0$

$cx - cx = 0$

$0 = 0$ ✓

$c \begin{bmatrix} -y \\ y \end{bmatrix} = \begin{bmatrix} -cy \\ cy \end{bmatrix} \qquad -cy + cy = 0$

$0 = 0$ ✓

✓Addition

$\begin{bmatrix} x \\ -x \end{bmatrix} + \begin{bmatrix} -y \\ y \end{bmatrix} = \begin{bmatrix} x - y \\ -x + y \end{bmatrix} = \begin{bmatrix} x - y \\ y - x \end{bmatrix}$

$x - \cancel{y} + \cancel{y} - x = 0 \qquad 0 = 0$ ✓

Spans as subspaces

1. Closed under scalar multiplication
2. Closed under addition

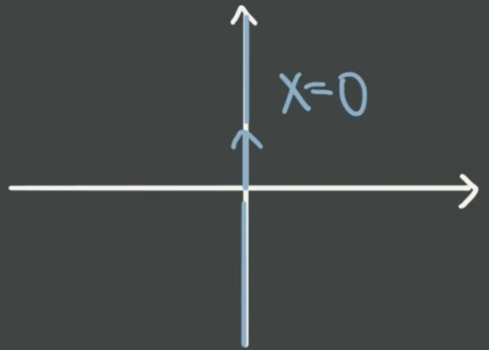
$\mathbb{R}^2$	$\mathbb{R}^3$
$\mathbb{R}^2$	$\mathbb{R}^3$ V_3
$\vec{0} = (0,0)$	$\vec{0} = (0,0,0)$
Any line through $(0,0)$ V_2	Any plane through $(0,0,0)$ V_1

$$V_1 = \text{Span}\left(\begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}0\\2\\3\end{bmatrix}\right) \rightarrow \text{plane}$$

$$c_1\begin{bmatrix}1\\1\\1\end{bmatrix} + c_2\begin{bmatrix}0\\2\\3\end{bmatrix}$$

$$V_2 = \text{span}\left(\begin{bmatrix}0\\3\end{bmatrix}\right)$$

$$V_3 = \text{Span}\left(\begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}0\\\cancel{0}\\2\end{bmatrix}, \begin{bmatrix}-1\\0\\3\end{bmatrix}\right)$$



Basis

Subspace:

vector set that is

1. closed under multiplication
2. closed under addition

Basis:

1. span the subspace ✓
2. Linearly independent ✓

$\mathbb{R}^2$: 2, L.I., in $\mathbb{R}^2$

$\mathbb{R}^3$: 3, L.I., in $\mathbb{R}^3$

$V = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 6 \end{bmatrix} \right\}$

$\begin{bmatrix} 3 \\ 4 \end{bmatrix}$

$V = \left\{ \begin{bmatrix} 2 \\ -3 \end{bmatrix} \begin{bmatrix} 5 \\ 1 \end{bmatrix} \right\}$ Basis for $\mathbb{R}^2$?

$c_1 \begin{bmatrix} 2 \\ -3 \end{bmatrix} + c_2 \begin{bmatrix} 5 \\ 1 \end{bmatrix} = \cancel{\begin{bmatrix} x \\ y \end{bmatrix}} \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$\begin{array}{cc} c_1 & c_2 \end{array}$$

$$\left[\begin{array}{cc|c} 2 & 5 & x \\ -3 & 1 & y \end{array}\right] \qquad \left[\begin{array}{cc|c} 1 & 5/2 & x/2 \\ 0 & 1 & \frac{2}{17}y + \frac{3}{17}x \end{array}\right]$$

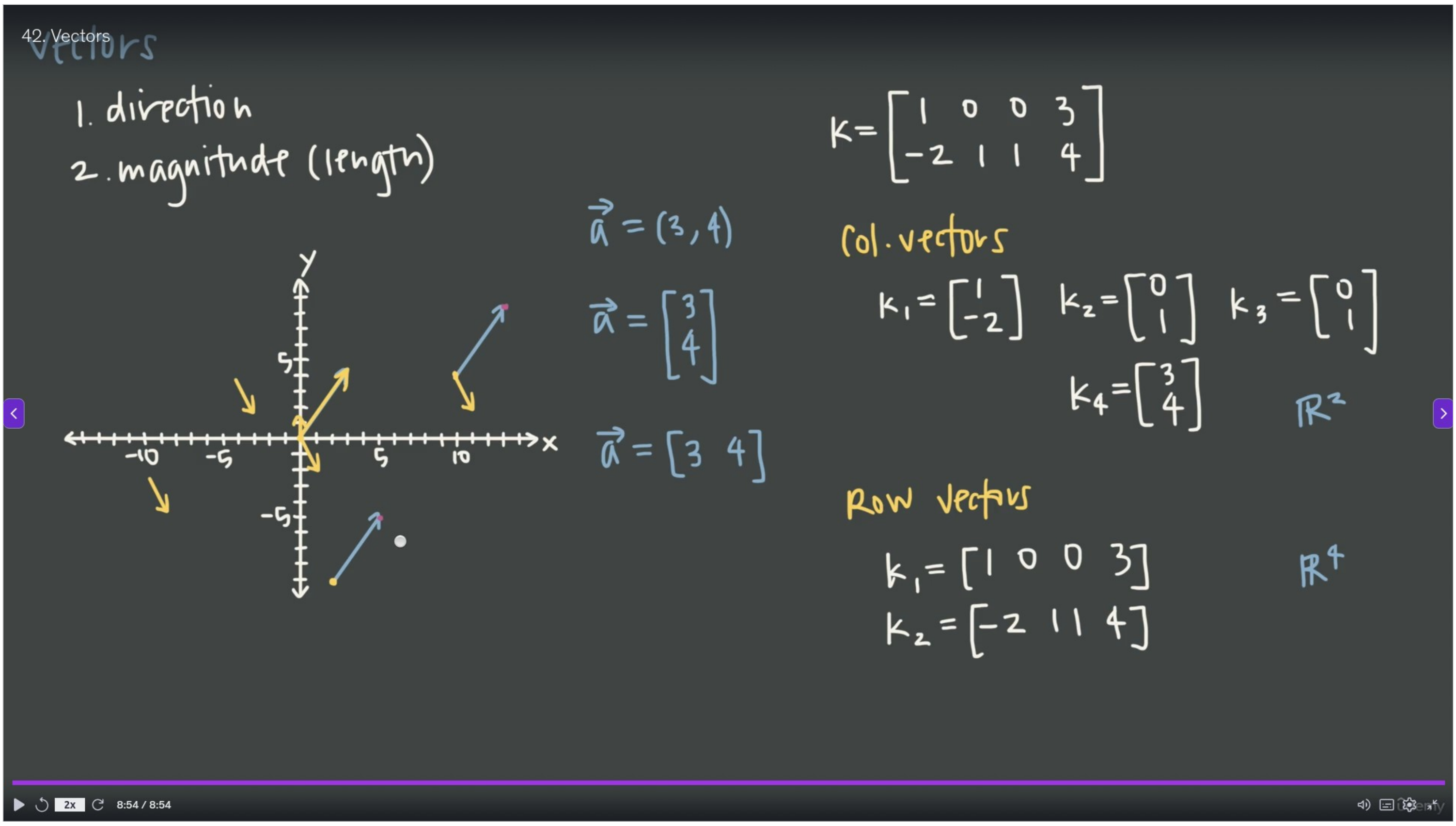
$$\left[\begin{array}{cc|c} 1 & 5/2 & x/2 \\ 0 & 17/2 & y + 3x/2 \end{array}\right] \qquad \begin{array}{c} \begin{array}{cc} c_1 & c_2 \end{array} \\ \left[\begin{array}{cc|c} 1 & 0 & \frac{1}{17}x - \frac{5}{17}y \\ 0 & 1 & \frac{2}{17}y + \frac{3}{17}x \end{array}\right] \end{array}$$

$c_1 = \frac{1}{17}x - \frac{5}{17}y \rightarrow c_1 = 0$

$c_2 = \frac{2}{17}y + \frac{3}{17}x \rightarrow c_2 = 0$

$V = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix} \right\}$

$i = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad j = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad k = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$


42. Vectors
Vectors
1. direction
2. magnitude (length)
y
x
5
-5
-10
10
-5
5
$\vec{a} = (3, 4)$
$\vec{a} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$
$\vec{a} = \begin{bmatrix} 3 & 4 \end{bmatrix}$
$K = \begin{bmatrix} 1 & 0 & 0 & 3 \\ -2 & 1 & 1 & 4 \end{bmatrix}$
Col. vectors
$k_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ $k_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $k_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$
$k_4 = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$
$\mathbb{R}^2$
Row vectors
$k_1 = \begin{bmatrix} 1 & 0 & 0 & 3 \end{bmatrix}$
$k_2 = \begin{bmatrix} -2 & 1 & 1 & 4 \end{bmatrix}$
$\mathbb{R}^4$
2x
8:54 / 8:54

Tensors (Multilinear Map)

- Vectors = 1^{st} order Tensor $\begin{bmatrix}1\\3\end{bmatrix}$ (1D arrays)
- Matrices = 2^{nd} order Tensor $\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$ (2D arrays)

$\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\\7 & 8 & 9\end{bmatrix}$ **3x3 matrix Represented as 2D array data structure.**

- 3^{rd} order Tensor , is a stack of matrices (3D arrays)

$\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}\begin{bmatrix}1 & 2\\ & 4\end{bmatrix}\begin{bmatrix}1 & 2\\ & 4\end{bmatrix}$ $\longrightarrow$ **2 x 2 x 3** Tensor (I ,j, k indices)

- Tensor may have n order.

vector operations

$$\begin{bmatrix}-1\\2\end{bmatrix}+\begin{bmatrix}3\\4\end{bmatrix}=\begin{bmatrix}2\\6\end{bmatrix} \qquad \begin{bmatrix}-1\\2\end{bmatrix}-\begin{bmatrix}3\\4\end{bmatrix}=\begin{bmatrix}-4\\-2\end{bmatrix}$$

$\vec{a}=(-1,2)=A$

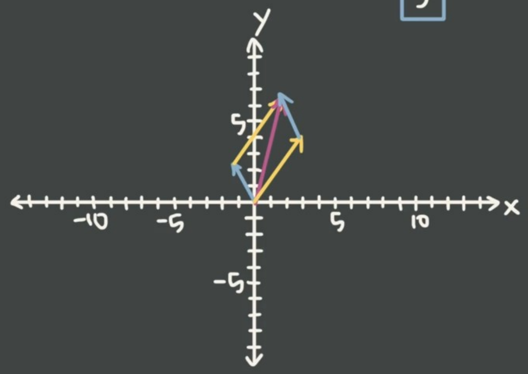
$\vec{b}=(3,4)=B$

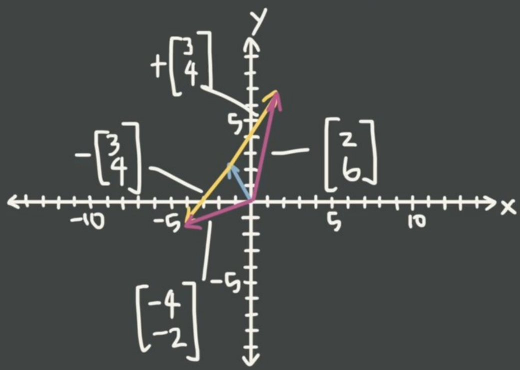
$$\vec{a}\cdot\vec{b}=(-1)(3)+(2)(4)$$
$$=-3+8$$
$$=\boxed{5}$$

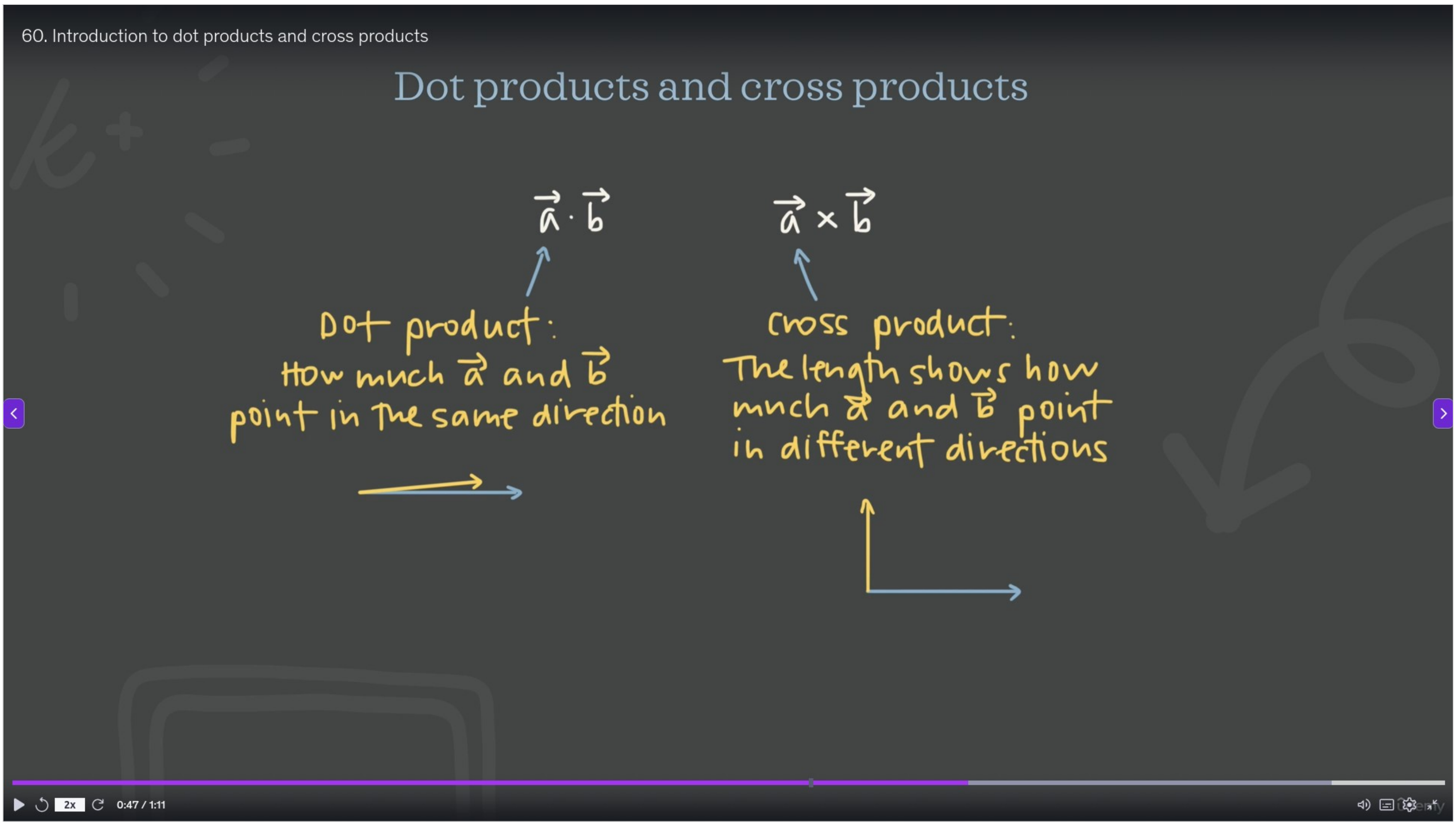
$1\times2 \qquad 2\times1$

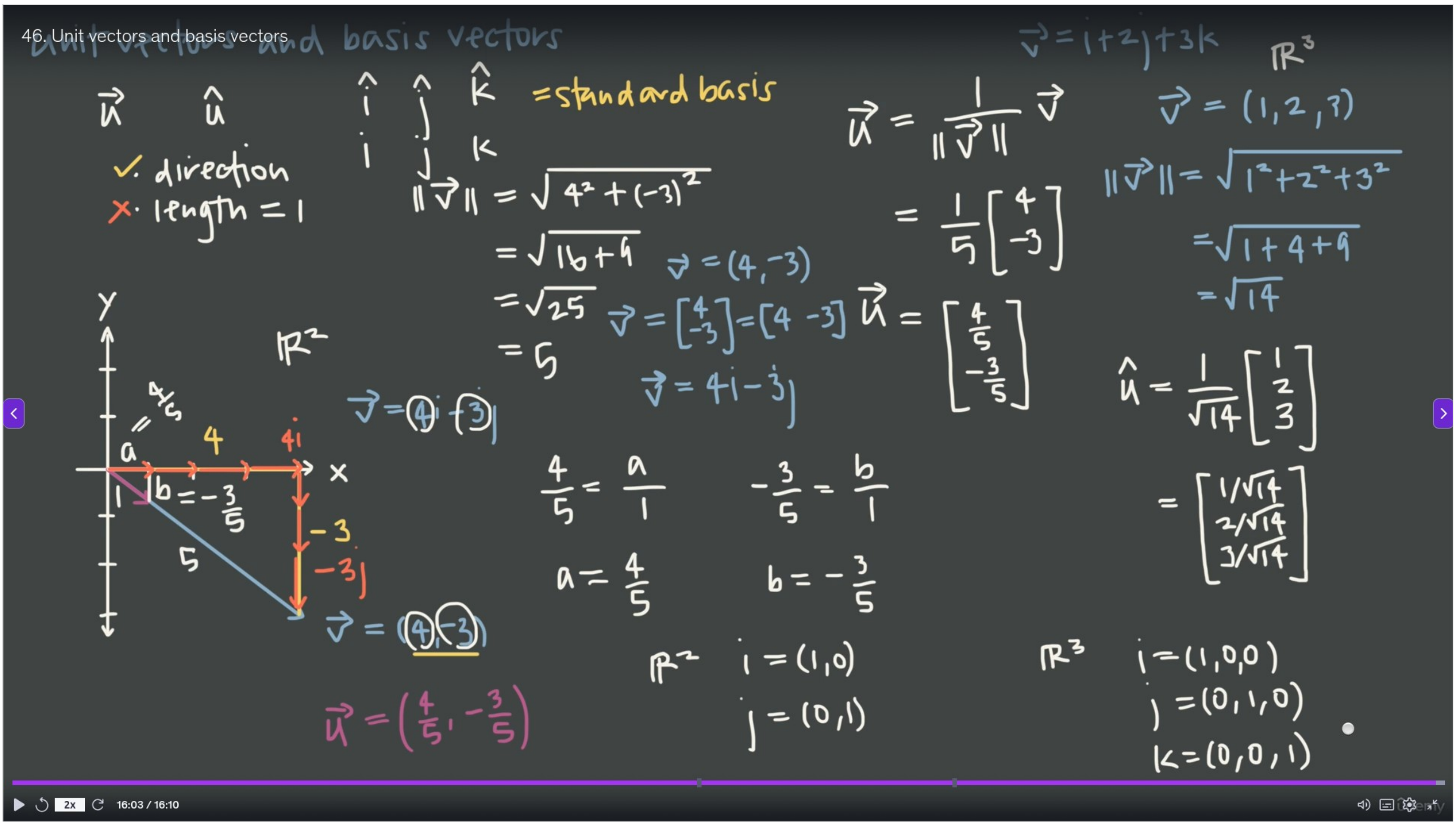
$$AB=\begin{bmatrix}-1 & 2\end{bmatrix}\begin{bmatrix}3\\4\end{bmatrix}$$
$$=(-1)(3)+(2)(4)$$
$$=-3+8=\boxed{5}$$

$$BA=\begin{bmatrix}3 & 4\end{bmatrix}\begin{bmatrix}-1\\2\end{bmatrix}$$
$$=(3)(-1)+(4)(2)$$
$$=-3+8$$
$$=\boxed{5}$$






60. Introduction to dot products and cross products
Dot products and cross products
$\vec{a} \cdot \vec{b}$
$\vec{a} \times \vec{b}$
Dot product:
How much $\vec{a}$ and $\vec{b}$ point in the same direction
Cross product:
The length shows how much $\vec{a}$ and $\vec{b}$ point in different directions
2x
0:47 / 1:11


46. Unit vectors and basis vectors
Unit vectors and basis vectors
û
✓. direction
×. length = 1
î ĵ k̂ = standard basis
i j k
‖v‖ = √(4² + (-3)²)
= √(16+9)
= √25
= 5
v = (4, -3)
v = [4 -3] = [4 -3]
v = 4i - 3j
v = 4i - 3j
u = 1/‖v‖ v
= 1/5 [4 -3]
u = [4/5 -3/5]
v = i + 2j + 3k
R³
v = (1, 2, 3)
‖v‖ = √(1² + 2² + 3²)
= √(1 + 4 + 9)
= √14
û = 1/√14 [1 2 3]
= [1/√14 2/√14 3/√14]
y
x
R²
a = 4/5
b = -3/5
4
4i
-3
-3j
5
1
v = (4, -3)
u = (4/5, -3/5)
4/5 = a/1
a = 4/5
-3/5 = b/1
b = -3/5
R² i = (1,0)
j = (0,1)
R³ i = (1,0,0)
j = (0,1,0)
k = (0,0,1)
2x
16:03 / 16:10

48. Linear combinations and span

Linear combinations and span

↳ sum of scaled vectors

$\vec{v} = 4i - 3j$

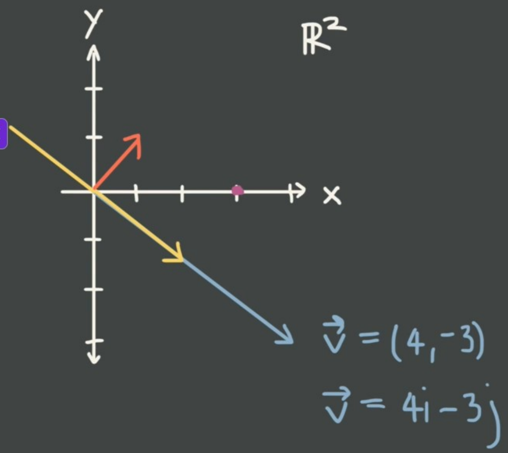
$= 2\vec{a} + 4\vec{b} - \vec{c}$

↳ all the linear combinations

$\mathbb{R}^2 = \text{span}(\hat{i}, \hat{j})$

$\vec{a} = (7,-2) = 7i - 2j$ $\quad \vec{c} = (0,4) = 4j$

$\vec{b} = (-2,9) = -2i + 9j$



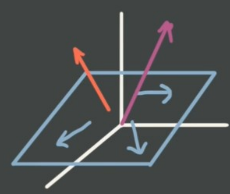
$\vec{v} = 4\begin{bmatrix}1\\0\end{bmatrix} - 3\begin{bmatrix}0\\1\end{bmatrix}$

$\mathbb{R}^3 = \text{span}(\hat{i}, \hat{j}, \hat{k})$

$\vec{d} = (0,17,-4) = 17j - 4k$

$\mathbb{R}^2$: 2, L.I., in $\mathbb{R}_2$

$\mathbb{R}^3$: 3, L.I., in $\mathbb{R}_3$



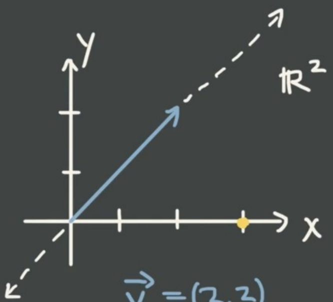
13:28 / 13:28

Linear independence in $\mathbb{R}^2$

$\mathbb{R}^2$: 2, L.I., in $\mathbb{R}^2$

$\mathbb{R}^3$: 3, L.I., in $\mathbb{R}^3$

$\mathbb{R}^n$: n, L.I., in $\mathbb{R}^n$



$\vec{v} = (2,2)$

$\vec{w} = (4,4)$

$$\underbrace{\begin{bmatrix}1\\1\end{bmatrix} \quad \begin{bmatrix}0\\6\end{bmatrix}}_{\text{L.I.}}$$

$$c_1\begin{bmatrix}1\\1\end{bmatrix} + c_2\begin{bmatrix}0\\6\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

$c_1 = -2c_3$

$c_1 = -6c_2 - 3c_3$

$-6c_2 - 3c_3 = -2c_3$

$-6c_2 = c_3$

$c_1 + 2c_3 = 0$

$c_1 + 6c_2 + 3c_3 = 0$

$c_1 + 0 = 0 \Rightarrow \boxed{c_1 = 0}$

$c_1 + 6c_2 = 0$

$0 + 6c_2 = 0$

$6c_2 = 0$

$\boxed{c_2 = 0}$

L.D.

$$2\vec{v} = 2\begin{bmatrix}2\\2\end{bmatrix} = \begin{bmatrix}4\\4\end{bmatrix} = \vec{w}$$

$2\vec{v} = \vec{w}$

$\vec{v} = \frac{1}{2}\vec{w}$

$$c_1\begin{bmatrix}2\\2\end{bmatrix} + c_2\begin{bmatrix}4\\4\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

$2c_1 + 4c_2 = 0 \rightarrow c_1 + 2c_2 = 0$

$2c_1 + 4c_2 = 0$

$c_1 = -2c_2$

$c_2 = 1 \quad c_1 = -2$

$c_2 = 2 \quad c_1 = -4$

$$c_1\begin{bmatrix}2\\2\end{bmatrix}$$

$$c_1\begin{bmatrix}1\\1\end{bmatrix} + c_2\begin{bmatrix}0\\6\end{bmatrix} + c_3\begin{bmatrix}2\\3\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

Linear independence in $\mathbb{R}^3$

- $\mathbb{R}^2$: 2, L.I., in $\mathbb{R}^2$
- $\mathbb{R}^3$: 3, L.I., in $\mathbb{R}^3$
- $\mathbb{R}^n$: n, L.I., in $\mathbb{R}^n$

$$\left.\begin{bmatrix}1\\1\\1\end{bmatrix} \quad \begin{bmatrix}0\\-3\\2\end{bmatrix} \quad \begin{bmatrix}1\\0\\3\end{bmatrix}\right\} \text{L.I.}$$

$$c_1\begin{bmatrix}1\\1\\1\end{bmatrix} + c_2\begin{bmatrix}0\\-3\\2\end{bmatrix} + c_3\begin{bmatrix}1\\0\\3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\begin{array}{c} \begin{array}{ccc} c_1 & c_2 & c_3 \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 1 & -3 & 0 & 0 \\ 1 & 2 & 3 & 0 \end{array}\right] \end{array}$$

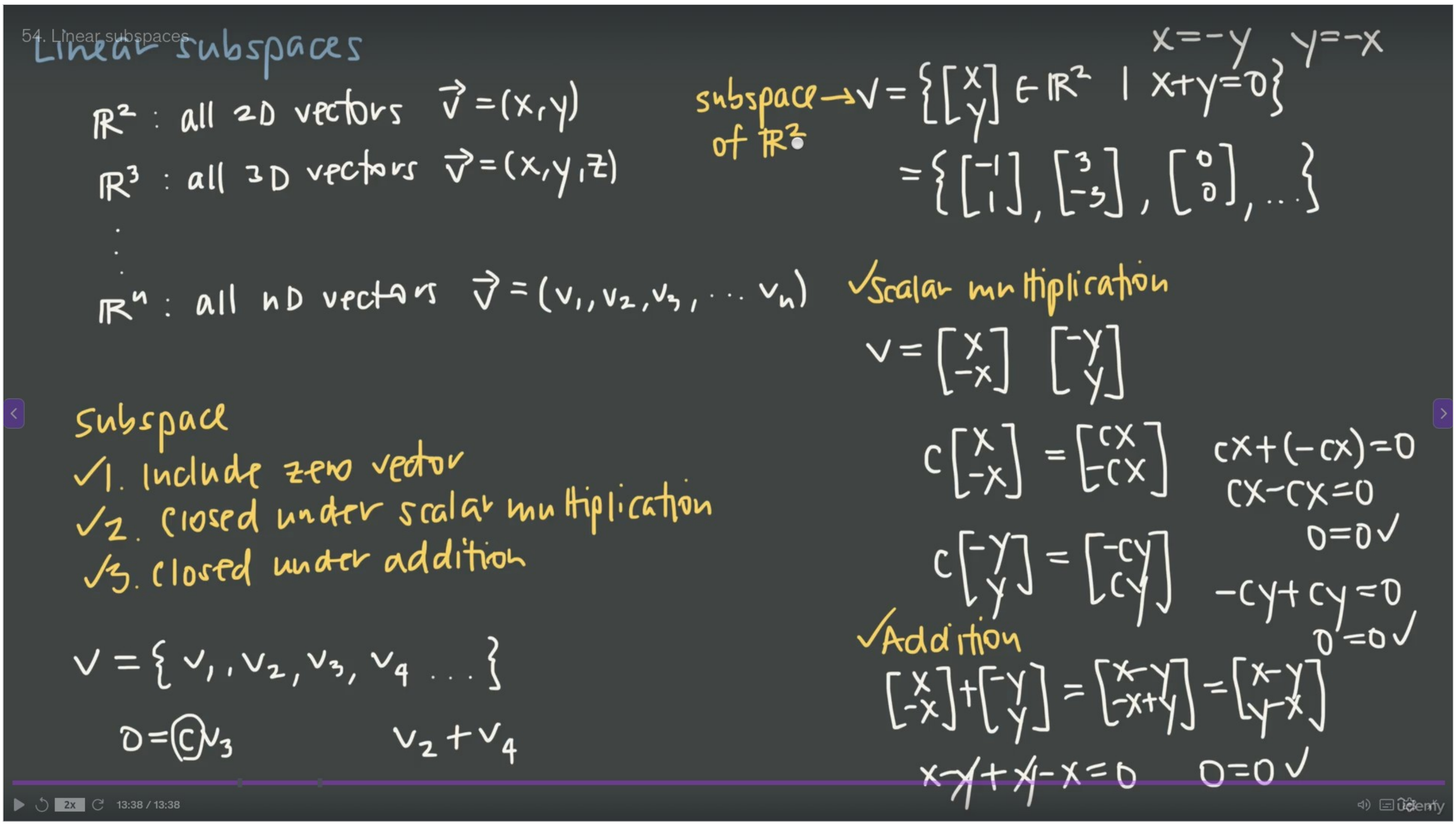
$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & -3 & -1 & 0 \\ 0 & 2 & 2 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 1/3 & 0 \\ 0 & 0 & 4/3 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 1/3 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$\begin{array}{c} \begin{array}{ccc} c_1 & c_2 & c_3 \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \end{array}$$

$$\boxed{\begin{array}{l} c_1 = 0 \\ c_2 = 0 \\ c_3 = 0 \end{array}}$$


54. Linear subspaces
Linear subspaces
$\mathbb{R}^2$: all 2D vectors $\vec{v} = (x, y)$
$\mathbb{R}^3$: all 3D vectors $\vec{v} = (x, y, z)$
$\vdots$
$\mathbb{R}^n$: all nD vectors $\vec{v} = (v_1, v_2, v_3, \dots v_n)$
subspace of $\mathbb{R}^2$ → $V = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \in \mathbb{R}^2 \mid x + y = 0 \right\}$
$x = -y$ $y = -x$
$= \left\{ \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \dots \right\}$
Subspace
✓1. Include zero vector
✓2. Closed under scalar multiplication
✓3. Closed under addition
$V = \{ v_1, v_2, v_3, v_4 \dots \}$
$0 = c v_3$ $v_2 + v_4$
✓Scalar multiplication
$V = \begin{bmatrix} x \\ -x \end{bmatrix}$ $\begin{bmatrix} -y \\ y \end{bmatrix}$
$c \begin{bmatrix} x \\ -x \end{bmatrix} = \begin{bmatrix} cx \\ -cx \end{bmatrix}$ $cx + (-cx) = 0$
$cx - cx = 0$
$0 = 0$ ✓
$c \begin{bmatrix} -y \\ y \end{bmatrix} = \begin{bmatrix} -cy \\ cy \end{bmatrix}$ $-cy + cy = 0$
$0 = 0$ ✓
✓Addition
$\begin{bmatrix} x \\ -x \end{bmatrix} + \begin{bmatrix} -y \\ y \end{bmatrix} = \begin{bmatrix} x - y \\ -x + y \end{bmatrix} = \begin{bmatrix} x - y \\ y - x \end{bmatrix}$
$x - \cancel{y} + \cancel{y} - x = 0$ $0 = 0$ ✓
13:38 / 13:38
2x
udemy

56. Spans as subspaces

Spans as subspaces

1. Closed under scalar multiplication
2. Closed under addition

$\mathbb{R}^2$

$\mathbb{R}^2$

$\vec{0} = (0,0)$

Any line through $(0,0)$ V_2

$\mathbb{R}^3$

$\mathbb{R}^3$ V_3

$\vec{0} = (0,0,0)$

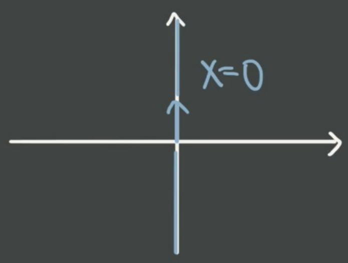
Any plane through $(0,0,0)$ V_1

$$V_1 = \text{Span}\left(\begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}0\\2\\3\end{bmatrix}\right) \to \text{plane}$$

$$c_1\begin{bmatrix}1\\1\\1\end{bmatrix} + c_2\begin{bmatrix}0\\2\\3\end{bmatrix}$$

$$V_2 = \text{span}\left(\begin{bmatrix}0\\3\end{bmatrix}\right)$$

$$V_3 = \text{Span}\left(\begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}0\\\cancel{0}\\2\end{bmatrix}, \begin{bmatrix}-1\\0\\3\end{bmatrix}\right)$$



2x 9:22 / 9:22

Basis

Subspace:

vector set that is

1. closed under multiplication
2. closed under addition

Basis:

1. span the subspace ✓
2. Linearly independent ✓

$$\left[\begin{array}{l} \mathbb{R}^2 :\ 2,\ \text{L.I.},\ \text{in } \mathbb{R}^2 \\ \mathbb{R}^3 :\ 3,\ \text{L.I.},\ \text{in } \mathbb{R}^3 \end{array}\right]$$

$$V = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 6 \end{bmatrix} \right\}$$

$$\begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$V = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix} \right\}$$

$$i = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad j = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad k = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$V = \left\{ \begin{bmatrix} 2 \\ -3 \end{bmatrix} \begin{bmatrix} 5 \\ 1 \end{bmatrix} \right\} \qquad \text{Basis for } \mathbb{R}^2 ?$$

$$c_1 \begin{bmatrix} 2 \\ -3 \end{bmatrix} + c_2 \begin{bmatrix} 5 \\ 1 \end{bmatrix} = \cancel{\begin{bmatrix} x \\ y \end{bmatrix}} \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{array}{cc} c_1 & c_2 \end{array}$$

$$\left[\begin{array}{cc|c} 2 & 5 & x \\ -3 & 1 & y \end{array}\right] \qquad \left[\begin{array}{cc|c} 1 & 5/2 & x/2 \\ 0 & 1 & \frac{2}{17}y + \frac{3}{17}x \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & 5/2 & x/2 \\ 0 & 17/2 & y + 3x/2 \end{array}\right] \qquad \begin{array}{cc} c_1 & c_2 \end{array} \left[\begin{array}{cc|c} 1 & 0 & \frac{1}{17}x - \frac{5}{17}y \\ 0 & 1 & \frac{2}{17}y + \frac{3}{17}x \end{array}\right]$$

$$c_1 = \frac{1}{17}x - \frac{5}{17}y \rightarrow c_1 = 0$$

$$c_2 = \frac{2}{17}y + \frac{3}{17}x \rightarrow c_2 = 0$$