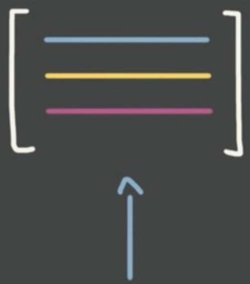
Transposes



The matrix A

Its transpose A^T

Transposes and their determinants

$$A=\begin{bmatrix}2 & 1\\ -3 & 0\end{bmatrix}\quad 2\times 2 \qquad B=\begin{bmatrix}3 & -4 & 1\\ 0 & 0 & 6\\ -1 & 2 & 2\end{bmatrix}\quad 3\times 3 \qquad C=\begin{bmatrix}1 & 1\\ 2 & 0\\ 4 & 0\\ -1 & 2\end{bmatrix}\quad 4\times 2$$

$$A^T=\begin{bmatrix}2 & -3\\ 1 & 0\end{bmatrix}\quad 2\times 2 \qquad B^T=\begin{bmatrix}3 & 0 & -1\\ -4 & 0 & 2\\ 1 & 6 & 2\end{bmatrix}\quad 3\times 3 \qquad C^T=\begin{bmatrix}1 & 2 & 4 & -1\\ 1 & 0 & 0 & 2\end{bmatrix}\quad 2\times 4$$

$$|A|=\begin{vmatrix}2 & 1\\ -3 & 0\end{vmatrix}=0+3=3$$

$$|B|=\begin{vmatrix}3 & -4 & 1\\ 0 & 0 & 6\\ -1 & 2 & 2\end{vmatrix}=-0\cancel{\begin{vmatrix}-4 & 1\\ 2 & 2\end{vmatrix}}+0\cancel{\begin{vmatrix}3 & 1\\ -1 & 2\end{vmatrix}}-6\begin{vmatrix}3 & -4\\ -1 & 2\end{vmatrix}$$

$$=-6(6-4)=-6(2)=-12$$

$$|A^T|=\begin{vmatrix}2 & -3\\ 1 & 0\end{vmatrix}=0+3=3$$

$$|B^T|=\begin{vmatrix}3 & 0 & -1\\ -4 & 0 & 2\\ 1 & 6 & 2\end{vmatrix}=-0\cancel{\begin{vmatrix}-4 & 2\\ 1 & 2\end{vmatrix}}+0\cancel{\begin{vmatrix}3 & -1\\ 1 & 2\end{vmatrix}}-6\begin{vmatrix}3 & -1\\ -4 & 2\end{vmatrix}$$

$$=-6(6-4)=$$

1x 8:50 / 9:14

Transposes of products, sums, and inverses

Products

AB $\qquad (ABC)^T = C^T B^T A^T$

$(AB)^T = B^T A^T$

$A = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 3 & 0 \\ 1 & 1 \end{bmatrix}$

$AB = \begin{bmatrix} 6+(-1) & 0+(-1) \\ 3+0 & 0+0 \end{bmatrix}$

$= \begin{bmatrix} 5 & -1 \\ 3 & 0 \end{bmatrix}$

$(AB)^T = \begin{bmatrix} 5 & 3 \\ -1 & 0 \end{bmatrix}$

$$B^T A^T = \overset{B^T}{\begin{bmatrix} 3 & 1 \\ 0 & 1 \end{bmatrix}} \overset{A^T}{\begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}}$$

$B^T A^T = \begin{bmatrix} 5 & 3 \\ -1 & 0 \end{bmatrix}$

Sums

$(A+B)^T = A^T + B^T$

$A^T + B^T = \begin{bmatrix} 5 & 2 \\ -1 & 1 \end{bmatrix}$

$(A+B)^T = \begin{bmatrix} 5 & -1 \\ 2 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 5 & 2 \\ -1 & 1 \end{bmatrix}$

Inverses

$(A^T)^{-1} = (A^{-1})^T$

$(A^T)^{-1} = \begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix} \qquad (A^{-1})^T = \begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}$

$\left[\begin{array}{cc|cc} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 2 \end{array}\right]$

Transpose and Inverse Properties

$A \cdot A^{-1} = A^{-1} \cdot A = I$

$(A^{-1})^{-1} = A$

$(A^T)^T = A$

$(A^T)^{-1} = (A^{-1})^T$

$(AB)^T = B^T \cdot A^T$

$(AB)^{-1} = B^{-1} \cdot A^{-1}$

$(k.A)^{-1} = 1/k \cdot A^{-1}$

$(k.A)^T = k \cdot A^T$

$\det(A^T) = \det(A)$

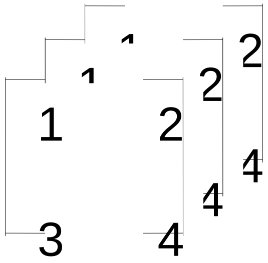
$\det(A^{-1}) = 1/\det(A)$

Tensors (Multilinear Map)

- Vectors = 1st order Tensor $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$ (1D arrays)
- Matrices = 2nd order Tensor $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ (2D arrays)

$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ **3x3 matrix Represented as 2D array data structure.**

- 3rd order Tensor , is a stack of matrices (3D arrays)



$\longrightarrow$ **2 x 2 x 3** Tensor (I ,j, k indices)

- Tensor may have n order.

144. Null and column spaces of the transpose

Null and column spaces of the transpose

$m \times n$

$$A = \begin{bmatrix} 2 & 1 & 1 & 0 \\ -1 & 0 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 & 2 \\ 2 & 1 & 1 & 0 \end{bmatrix} = \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \begin{bmatrix} \boxed{\begin{matrix}1\\0\end{matrix}} & \boxed{\begin{matrix}0\\1\end{matrix}} & \boxed{\begin{matrix}-1\\3\end{matrix}} & \boxed{\begin{matrix}2\\-4\end{matrix}} \end{bmatrix}$$

Column space $C(A)$ $\mathbb{R}^m = \mathbb{R}^2$ $\text{Dim} = 2$ $\text{span}\left(\begin{bmatrix} 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}\right)$

Null space $N(A)$ $\mathbb{R}^n = \mathbb{R}^4$ $\text{Dim} = n - r = 4 - 2 = 2$ $\text{span}\left(\begin{bmatrix} 1 \\ -3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 4 \\ 0 \\ 1 \end{bmatrix}\right)$

$A\vec{x} = \vec{0}$

Rowspace $C(A^T)$ $\mathbb{R}^n = \mathbb{R}^4$ $\text{Dim} = 2$ $\text{span}\left(\begin{bmatrix} 2 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \\ -2 \end{bmatrix}\right)$

$$A^T = \begin{bmatrix} 2 & -1 \\ 1 & 0 \\ 1 & 1 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \\ 0 & 1 \\ 0 & -2 \end{bmatrix}$$

Left null space $N(A^T)$ $\mathbb{R}^m = \mathbb{R}^2$ $\text{Dim} = m - r = 2 - 2 = 0$ $\text{span}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}\right) = \vec{0} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 0 \\ 2 & -1 \\ 1 & 1 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & -1 \\ 0 & -2 \end{bmatrix} = \begin{array}{cc} x_1 & x_2 \end{array} \begin{bmatrix} \boxed{\begin{matrix}1\\0\\0\\0\end{matrix}} & \boxed{\begin{matrix}0\\1\\0\\0\end{matrix}} \end{bmatrix}$$

$$(A^T\vec{x})^T = (\vec{0})^T$$
$$\vec{x}^T (A^T)^T = (\vec{0})^T$$
$$\vec{x}^T A = \vec{0}^T$$

$x_1 = 0$
$x_2 = 0$ $= \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

The product of a matrix and its transpose

$$A^TA = n \times n$$

$$n \times \boxed{m \quad m} \times n$$

$$A = \begin{bmatrix} 2 & 1 \\ 0 & -3 \\ 1 & 0 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 2 & 0 & 1 \\ 1 & -3 & 0 \end{bmatrix}$$

$$A^TA = \underset{2 \times 3}{\begin{bmatrix} 2 & 0 & 1 \\ 1 & -3 & 0 \end{bmatrix}} \underset{3 \times 2}{\begin{bmatrix} 2 & 1 \\ 0 & -3 \\ 1 & 0 \end{bmatrix}}$$

$$= \begin{bmatrix} 4+0+1 & 2+0+0 \\ 2+0+0 & 1+9+0 \end{bmatrix}$$

$$\boxed{A^TA = \begin{bmatrix} 5 & 2 \\ 2 & 10 \end{bmatrix}}$$

$$= \begin{bmatrix} 1 & \frac{2}{5} \\ 2 & 10 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & \frac{2}{5} \\ 0 & \frac{46}{5} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & \frac{2}{5} \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$10 - \frac{4}{5}$$

$$\frac{50}{5} - \frac{4}{5} = \frac{46}{5}$$

$A = LU$ factorization

$$A = \begin{bmatrix} 2 & 1 \\ 8 & 1 \end{bmatrix} = \begin{bmatrix} \boxed{1} & 0 \\ 4 & \boxed{1} \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 0 & -3 \end{bmatrix}$$

$$\begin{matrix} 8 & 1 \\ 2 & 1 \end{matrix} \qquad A \quad = \quad L \qquad U$$

$$\cancel{E^{-1}_{2,1}}\, \cancel{E_{2,1}}\, A \qquad E^{-1}_{2,1}\, U$$

$$\begin{bmatrix} 1 & 0 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 8 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 0 & -3 \end{bmatrix}$$

$$R_2 - 4R_1 \rightarrow R_2$$

$$A = E^{-1}_{2,1}\, U$$

$$A = L \, U$$

$$\begin{bmatrix} 2 & 1 \\ 8 & 1 \end{bmatrix} = \begin{bmatrix} \boxed{1} & 0 \\ 4 & \boxed{1} \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 0 & -3 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 8 & 1 \end{bmatrix}$$

$A = LU$ factorization

$\cancel{E_{2,1}^{-1}}\ \cancel{E_{3,1}^{-1}}\ \cancel{E_{3,2}^{-1}}\ \cancel{E_{3,2}}$ $\qquad \cancel{E_{3,1}}$ $\qquad \cancel{E_{2,1}}$

$$\boxed{A = \underline{E_{2,1}^{-1} E_{3,1}^{-1} E_{3,2}^{-1}} U}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 3 & 3 \\ 8 & 14 & 12 \\ 4 & 4 & 5 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 3 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$R_2 - \boxed{4} R_1 \rightarrow R_2$

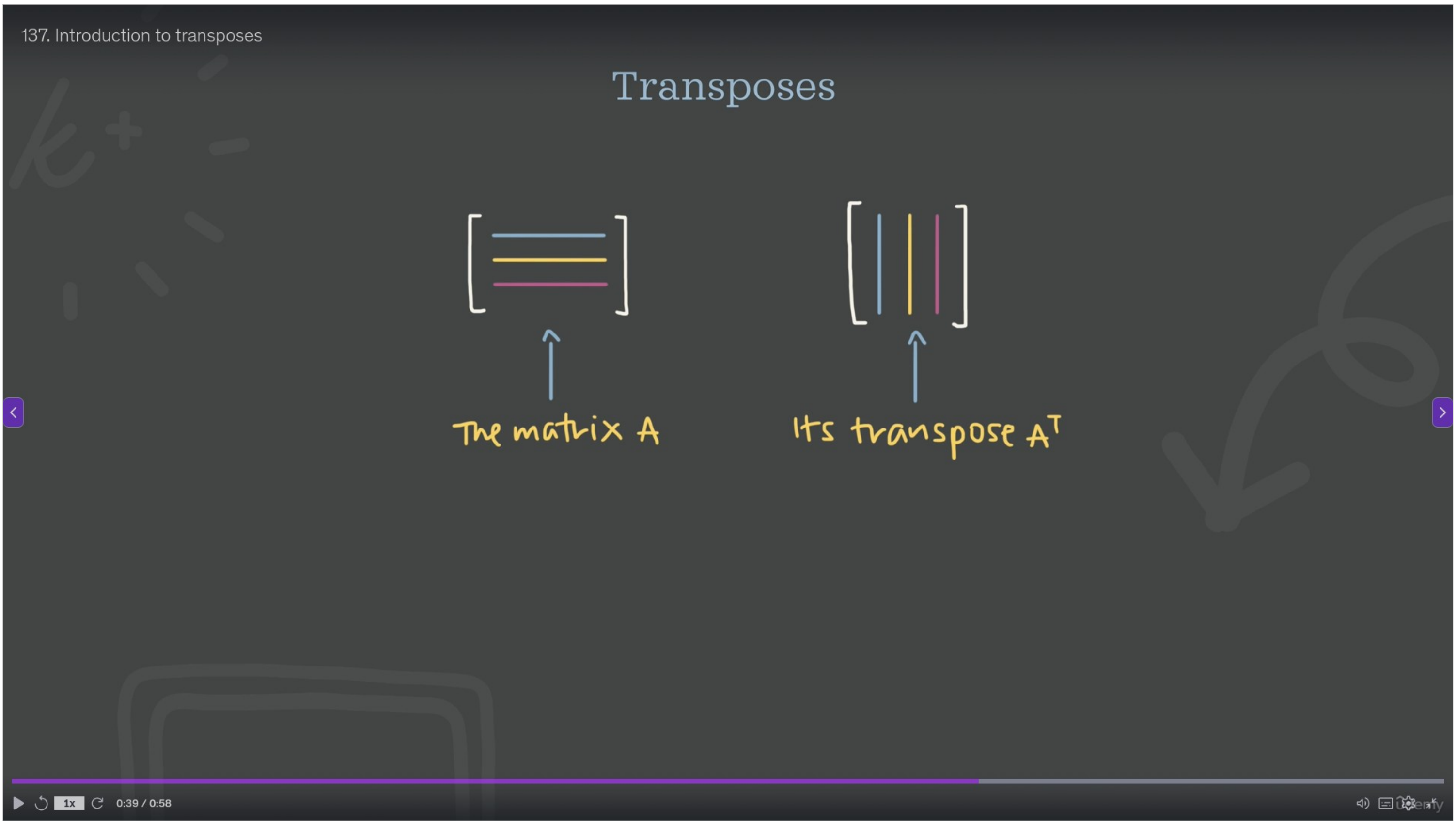
$R_3 - \boxed{2} R_1 \rightarrow R_3$

$R_3 + (\boxed{1} R_2) \rightarrow R_3$

$$\begin{array}{ccccc} A & = & I \rightarrow L & & A \rightarrow U \\ \begin{bmatrix} 2 & 3 & 3 \\ 8 & 14 & 12 \\ 4 & 4 & 5 \end{bmatrix} & = & \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 2 & -1 & 1 \end{bmatrix} & & \begin{bmatrix} 2 & 3 & 3 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix} \end{array}$$

$$\underset{A}{\begin{bmatrix} 2 & 3 & 3 \\ 8 & 14 & 12 \\ 4 & 4 & 5 \end{bmatrix}} = \overbrace{\begin{bmatrix} 1 & 0 & 0 \\ \boxed{4} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \boxed{2} & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \boxed{-1} & 1 \end{bmatrix}}^{L} \underset{U}{\begin{bmatrix} 2 & 3 & 3 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix}}$$

$$\begin{array}{ccccc} A & = & L & D & U \\ \begin{bmatrix} 2 & 3 & 3 \\ 8 & 14 & 12 \\ 4 & 4 & 5 \end{bmatrix} & = & \begin{bmatrix} \boxed{1} & 0 & 0 \\ 4 & \boxed{1} & 0 \\ 2 & -1 & \boxed{1} \end{bmatrix} & \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix} & \begin{bmatrix} \boxed{1} & 3/2 & 3/2 \\ 0 & \boxed{1} & 0 \\ 0 & 0 & \boxed{1} \end{bmatrix} \end{array}$$


137. Introduction to transposes
Transposes
The matrix A
Its transpose A^T
0:39 / 0:58

140. Transposes and their determinants

$$A=\begin{bmatrix}2&1\\-3&0\end{bmatrix}\ 2\times2 \qquad B=\begin{bmatrix}3&-4&1\\0&0&6\\-1&2&2\end{bmatrix}\ 3\times3 \qquad C=\begin{bmatrix}1&1\\2&0\\4&0\\-1&2\end{bmatrix}\ 4\times2$$

$$A^T=\begin{bmatrix}2&-3\\1&0\end{bmatrix}\ 2\times2 \qquad B^T=\begin{bmatrix}3&0&-1\\-4&0&2\\1&6&2\end{bmatrix}\ 3\times3 \qquad C^T=\begin{bmatrix}1&2&4&-1\\1&0&0&2\end{bmatrix}\ 2\times4$$

$$|A|=\begin{vmatrix}2&1\\-3&0\end{vmatrix}=0+3=3$$

$$|B|=\begin{vmatrix}3&-4&1\\0&0&6\\-1&2&2\end{vmatrix}=-0\cancel{\begin{vmatrix}-4&1\\2&2\end{vmatrix}}+0\cancel{\begin{vmatrix}3&1\\-1&2\end{vmatrix}}-6\begin{vmatrix}3&-4\\-1&2\end{vmatrix}=-6(6-4)=-6(2)=-12$$

$$|A^T|=\begin{vmatrix}2&-3\\1&0\end{vmatrix}=0+3=3$$

$$|B^T|=\begin{vmatrix}3&0&-1\\-4&0&2\\1&6&2\end{vmatrix}=-0\cancel{\begin{vmatrix}-4&2\\1&2\end{vmatrix}}+0\cancel{\begin{vmatrix}3&-1\\1&2\end{vmatrix}}-6\begin{vmatrix}3&-1\\-4&2\end{vmatrix}$$

$$=-6(6-4)=$$

1x 8:50 / 9:14

Transposes of products, sums, and inverses

Products

$AB \qquad (ABC)^T = C^T B^T A^T$

$(AB)^T = B^T A^T$

$A = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 3 & 0 \\ 1 & 1 \end{bmatrix}$

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$$= \begin{bmatrix} 5 & -1 \\ 3 & 0 \end{bmatrix}$$

$$(AB)^T = \begin{bmatrix} 5 & 3 \\ -1 & 0 \end{bmatrix}$$

$$B^T A^T = \underset{B^T}{\begin{bmatrix} 3 & 1 \\ 0 & 1 \end{bmatrix}} \underset{A^T}{\begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}}$$

$$B^T A^T = \begin{bmatrix} 5 & 3 \\ -1 & 0 \end{bmatrix}$$

Sums

$(A+B)^T = A^T + B^T$

$$A^T + B^T = \begin{bmatrix} 5 & 2 \\ -1 & 1 \end{bmatrix}$$

$$(A+B)^T = \begin{bmatrix} 5 & -1 \\ 2 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 5 & 2 \\ -1 & 1 \end{bmatrix}$$

Inverses

$(A^T)^{-1} = (A^{-1})^T$

$$(A^T)^{-1} = \begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix} \qquad (A^{-1})^T = \begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}$$

$$\left[\begin{array}{cc|cc} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 2 \end{array}\right]$$

Transpose and Inverse Properties

$A \,.\, A^{-1} = A^{-1} \,.\, A = I$

$(A^{-1})^{-1} = A$

$(A^{T})^{T} = A$

$(A^{T})^{-1} = (A^{-1})^{T}$

$(AB)^{T} = B^{T} \,.\, A^{T}$

$(AB)^{-1} = B^{-1} \,.\, A^{-1}$

$(k.A)^{-1} = 1/k \,.\, A^{-1}$

$(k.A)^{T} = k \,.\, A^{T}$

$\det(A^{T}) = \det(A)$

$\det(A^{-1}) = 1/\det(A)$

Tensors (Multilinear Map)

- Vectors = 1^{st} order Tensor $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$ (1D arrays)
- Matrices = 2^{nd} order Tensor $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ (2D arrays)

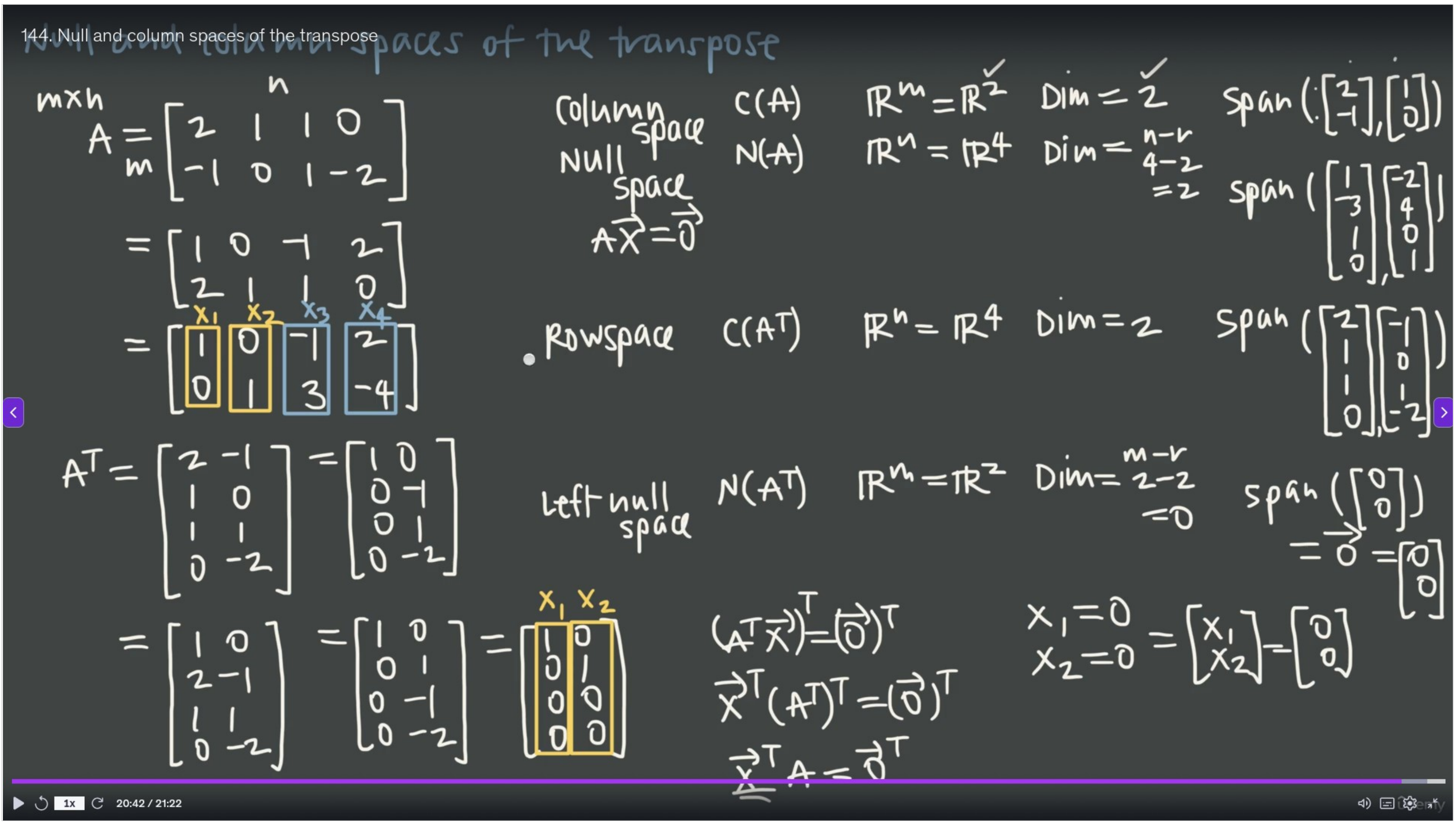
$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

3x3 matrix Represented as 2D array data structure.

- 3^{rd} order Tensor , is a stack of matrices (3D arrays)

$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ → **2 x 2 x 3** Tensor (I ,j, k indices)

- Tensor may have n order.


144. Null and column spaces of the transpose
Null and column spaces of the transpose
m×n
A = [2 1 1 0; −1 0 1 −2]
= [1 0 −1 2; 2 1 1 0]
= [1 0 −1 2; 0 1 3 −4]
x1 x2 x3 x4
Column space C(A) R^m = R^2 Dim = 2 Span([2; −1], [1; 0])
Null space N(A) R^n = R^4 Dim = n−r = 4−2 = 2 Span([1; −3; 1; 0], [−2; 4; 0; 1])
Ax = 0
Rowspace C(A^T) R^n = R^4 Dim = 2 Span([2; 1; 1; 0], [−1; 0; 1; −2])
A^T = [2 −1; 1 0; 1 1; 0 −2] = [1 0; 0 −1; 0 1; 0 −2]
= [1 0; 2 −1; 1 1; 0 −2] = [1 0; 0 1; 0 −1; 0 −2] = [1 0; 0 1; 0 0; 0 0]
x1 x2
Left null space N(A^T) R^m = R^2 Dim = m−r = 2−2 = 0 span([0; 0]) = 0 = [0; 0]
(A^T x)^T = (0)^T
x^T (A^T)^T = (0)^T
x^T A = 0^T
x1 = 0
x2 = 0 = [x1; x2] = [0; 0]
1x 20:42 / 21:22

The product of a matrix and its transpose

$$A^TA = n\times n$$

$$n\times \boxed{m \quad m}\times n$$

$$A = \begin{bmatrix} 2 & 1 \\ 0 & -3 \\ 1 & 0 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 2 & 0 & 1 \\ 1 & -3 & 0 \end{bmatrix}$$

$$A^TA = \underset{2\times 3}{\begin{bmatrix} 2 & 0 & 1 \\ 1 & -3 & 0 \end{bmatrix}} \underset{3\times 2}{\begin{bmatrix} 2 & 1 \\ 0 & -3 \\ 1 & 0 \end{bmatrix}}$$

$$= \begin{bmatrix} 4+0+1 & 2+0+0 \\ 2+0+0 & 1+9+0 \end{bmatrix}$$

$$\boxed{A^TA = \begin{bmatrix} 5 & 2 \\ 2 & 10 \end{bmatrix}}$$

$$= \begin{bmatrix} 1 & \frac{2}{5} \\ 2 & 10 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & \frac{2}{5} \\ 0 & \frac{46}{5} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & \frac{2}{5} \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$10 - \frac{4}{5}$$

$$\frac{50}{5} - \frac{4}{5} = \frac{46}{5}$$

$A = LU$ factorization

$$A = \begin{bmatrix} 2 & 1 \\ 8 & 1 \end{bmatrix} = \begin{bmatrix} \boxed{1} & 0 \\ 4 & \boxed{1} \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 0 & -3 \end{bmatrix}$$

$$\begin{matrix} 8 & 1 \\ 2 & 1 \end{matrix}$$

$$A = L \quad U$$

$$\cancel{E^{-1}_{2,1}}\ \cancel{E_{2,1}}\ A \qquad E^{-1}_{2,1}\ U$$

$$\begin{bmatrix} 1 & 0 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 8 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 0 & -3 \end{bmatrix}$$

$$R_2 - 4R_1 \rightarrow R_2$$

$$A = E^{-1}_{2,1}\ U$$

$$A = L \quad U$$

$$\begin{bmatrix} 2 & 1 \\ 8 & 1 \end{bmatrix} = \begin{bmatrix} \boxed{1} & 0 \\ 4 & \boxed{1} \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 0 & -3 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 8 & 1 \end{bmatrix}$$

$A = LU$ factorization

$\cancel{E_{2,1}^{-1}}\ \cancel{E_{3,1}^{-1}}\ \cancel{E_{3,2}^{-1}}\ \cancel{E_{3,2}}\qquad \cancel{E_{3,1}}\qquad \cancel{E_{2,1}}$

$$\boxed{A = E_{2,1}^{-1} E_{3,1}^{-1} E_{3,2}^{-1} U}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 3 & 3 \\ 8 & 14 & 12 \\ 4 & 4 & 5 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 3 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$R_2 - \boxed{4} R_1 \to R_2$

$R_3 - \boxed{2} R_1 \to R_3$

$R_3 + (\boxed{\ } R_2) \to R_3$

$$\begin{array}{ccccc} A & = & I \to L & & A \to U \end{array}$$

$$\begin{bmatrix} 2 & 3 & 3 \\ 8 & 14 & 12 \\ 4 & 4 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 2 & -1 & 1 \end{bmatrix}\begin{bmatrix} 2 & 3 & 3 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} 2 & 3 & 3 \\ 8 & 14 & 12 \\ 4 & 4 & 5 \end{bmatrix}}_{A} = \overbrace{\begin{bmatrix} 1 & 0 & 0 \\ \boxed{4} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \boxed{2} & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \boxed{-1} & 1 \end{bmatrix}}^{L}\underbrace{\begin{bmatrix} 2 & 3 & 3 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix}}_{U}$$

$$\begin{array}{ccccc} A & = & L & D & U \end{array}$$

$$\begin{bmatrix} 2 & 3 & 3 \\ 8 & 14 & 12 \\ 4 & 4 & 5 \end{bmatrix} = \begin{bmatrix} \boxed{1} & 0 & 0 \\ 4 & \boxed{1} & 0 \\ 2 & -1 & \boxed{1} \end{bmatrix}\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} \boxed{1} & 3/2 & 3/2 \\ 0 & \boxed{1} & 0 \\ 0 & 0 & \boxed{1} \end{bmatrix}$$