UNRAVELLING BUENO DE MESQUITA'S GROUP DECISION MODEL

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Keywords: Multi-agent theory, Group decision making, Expected utility, Alliance, Risk, Influence.

Abstract: The development of societies of human and machine agents should benefit from an understanding of human group decision processes. Political Scientist and Professor, Bruce Bueno De Mesquita has made significant claims for the predictive accuracy of his computational model of group decision making, receiving much popular press including newspaper articles, books and a television documentary entitled "The New Nostradamus". Despite these and many journal and conference publications related to the topic, no clear elicitation of the model exists in the open literature. We expose and present the model by careful navigation of the literature and illustrate the soundness of our interpretation by replicating De Mesquita's own results. We also discuss concerns regarding model sensitivity and convergence.

1 INTRODUCTION

There is little doubt that some of the greatest social challenges for the future of mankind include terrorism, war, climate change, poverty, and economics. So, the pursuit of an integrated theory capable of explanation and prediction of group decision outcomes is a worthy endeavour. Such efforts, often classed under the realm of computational political science, aim to form testable yet tractable models for human agency (Kollman et al, 2010). Bueno De Mesquita (herein abbreviated to BDM) has laid claims to such an achievement. An example prediction was "…the ability to dominate Iran's politics resides with Khamenei and Rafsanjani. And between these two – though the contest is close – the advantage seems to lie with Khamenei." (BDM, 1984, p. 233)

The accuracy of this prediction is demonstrated by the fact that Khamenei succeeded Khomeinei as Supreme Leader of Iran in June 1989 and Rafsanjani became the fourth president of Iran in August 1989.

BDM's model of group decision making considers conflict and agreement, and is based on expected utility theory. BDM (1997) states:

> The model itself depicts a game in which actors simultaneously make proposals, and exert influence on one another. They evaluate options and build coalitions by shifting positions on the issue in question. The above steps are repeated sequentially until the issue is resolved. (p. 238)

A New York Times article by Thomson (2009) gives some insight into why the model has never been fully disclosed:

> …Bueno de Mesquita does not publish the actual computer code of his model. (Bueno de Mesquita cannot do so because his former firm owns the actual code, but he counters that he has outlined the math behind his model in enough academic papers and books for anyone to replicate something close to his work.)

At first BDM (1997) appears to offer the most promise in elucidating the model, however first impressions prove misleading. Significant errors and obfuscations become apparent to anyone who tries to replicate the model and results from this and later works. In the following, we carefully navigate and interpret earlier works to derive a working model and agent software that reproduces his published results to an adequate level of accuracy.

2 EVOLUTION OF THE MODEL

BDM's predictions depend on two parts. First, his

Scholz J., Calbert G. and Smith G..
UNRAVELLING BUENO DE MESQUITA'S GROUP DECISION MODEL .
DOI: 10.5220/0003121500180030
In *Proceedings of the 3rd International Conference on Agents and Artificial Intelligence* (ICAART-2011), pages 18-30
ISBN: 978-989-8425-40-9

method of data collection and interpretation from human subjects; second, the computational model which he applies to that data.

The first part is significant, but has not been described by BDM in the open literature and so evades current interpretation or analysis. The second part, the computational model we examine further.

The model deals with a single 'issue' decomposed into a metric scale, with 'position' values (x) corresponding to states of the issue. BDM illustrates, "The term x_i represents each nation's preferred date, measured in years, by which emission standards should be applied to medium-sized automobiles as revealed at the outset of discussions on the issue." (BDM, 1994, p.77). We will continue with this example later in the results section. A number of 'actors' ($i=1,2,\ldots,n$) exist, each of which hold a single 'position' (x_i) with regard to the issue, represented by their assignment to a location.

Each 'actor' is also considered to possess some 'capability' (c_i) with respect to the 'issue'. 'Capability' is sometimes interchangeably referred to as 'power' or 'resources' by BDM. Like 'position', 'capability' is given a value on a metric scale. This value represents an actor's level of influence with regard to the issue.

Lastly, each 'actor' is also considered to possess some 'salience' (s_i) with respect to the 'issue'. 'Salience' is sometimes interchangeably referred to as 'importance', 'priority', 'attention' by BDM. Like 'position' and 'capability', 'salience' is given a value on a metric scale. This value represents an actor's level of energy with regard to the issue.

Table 1 from BDM (1994, p. 78) illustrates.

Table 1: Example input data for the computational model. The issue is 'the date (years) of introduction of emission standards for medium-sized automobiles'.

Actor (i)	**Capability (c_i)**	**Position (x_i)**	**Salience (s_i)**
Netherlands	0.08	4	80
Belgium	0.08	7	40
Luxembourg	0.03	4	20
Germany	0.16	4	80
France	0.16	10	60
Italy	0.16	10	60
UK	0.16	10	90
Ireland	0.05	7	10
Denmark	0.05	4	100
Greece	0.08	7	70

BDM's model decomposes the social fabric into pairwise 'contests' between actors with support or otherwise of third-party alliances. Based on actor i's perception of expected utility, actor i considers whether or not to challenge each other actor j, in an attempt to convince them to adopt i's position. The expected utility includes an assessment of the level of third-party support for actor i's challenge. If actor i's expected utility of challenging actor j versus not challenging is greater than zero, actor i will challenge actor j, otherwise it will not. This model of mind or agency is confrontational and wholly self-interested.

Not surprisingly, BDM has adapted the model over the years. So it is necessary to clearly identify which version we are using when considering its form and results.

BDM (1980) provides the earliest form, which is repeated in BDM (1981). The notation is later revised in BDM (1985), and includes a modification to include a risk exponent; however the basic expected utility calculations remain the same from 1980 to 1985. We are readily familiar with the expected value of a random variable Z, with various states Z_w each with probability P_w of occurring as:

$$E(z) = \sum_{w} P_w Z_w$$

Expected utility follows the same structure in that the utilities of different contest outcomes are estimated along with the associated probabilities.

An apparent motivation for BDM's expected utility model was predicting the outbreak of war as per BDM (1981). It is thus not surprising to find a confrontational mentality to the basic form of the model. BDM considers an actor i to choose to 'challenge' a rival or opponent actor j. Thereby the expected utility for i to challenge j is:

$$E^i(U)_c = P_i U_{si} + (1 - P_i) U_{fi}$$

Where U_{si} refers to the utility for actor i if it succeeds and U_{fi} is the utility for actor i if it fails.

BDM (1985, p. 158) extends this with a third term relating to the third-party contribution to i's expected utility (using BDM's notation):

$$E^i(U_{ij})_c = P_i U^i_{si} + (1 - P_i) U^i_{fi} + \sum_{k \neq i,j} (P_{ik} + P_{jk} - 1)(U^i_{ki} - U^i_{kj}) \quad (1)$$

If actor i does not challenge j, i stays at the same position and j may either remain where it is (status quo) or j may move to a different position. If j moves, the utility of the outcome may prove either better for i or worse for i. The expected utility for i not challenging is then, BDM (1985, p.158):

$$E^i\left(U_{ij}\right)_{nc} = Q^i_{qi}U^i_{qi} + \left(1-Q^i_{qi}\right)\left(Q^i_{bi}U^i_{bi} + \left(1-Q^i_{bi}\right)U^i_{wi}\right) \quad (2)$$

Where Q_q refers to the probability of status quo and Q_b refers to a switch (value either 1 or 0) depending on whether the outcome was better or worse. These and other issues will be explained fully later.

The full form of the expected utility difference combines (1) and (2):

$$E^i\left(U_{ij}\right) = E^i\left(U_{ij}\right)_c - E^i\left(U_{ij}\right)_{nc} \quad (3)$$

In BDM and Lalman (1986) a problem with the following term in (1) is identified:

$$E^i\left(U_{ij}\right)_m = \sum_{k\neq i,j}(P_{ik}+P_{jk}-1)(U^i_{ki}-U^i_{kj}) \quad (4)$$

The problem is described in BDM and Lalman (1986):

> Because of the manner in which third parties are treated in earlier studies (Bueno De Mesquita, 1981, 1985), the operational estimate of expected utility values for any decision maker could vary between (2N-2) and –(2N-2), where N is the total number of nations in the relevant international system. Variations in the size of the international community, then, affected the possible range of values in the expected utility models set out earlier. This is a serious shortcoming in that it makes comparison of a single nation's utility scores in different years difficult. … The new formulation fixes the range of values, irrespective of system size, in a theoretically meaningful way. (p. 1119)

BDM's proposed solution involves removing the term (4) from (1) and incorporating a more complex form of calculation of the probability P_i. In BDM (1985, p.161) the probability P_i refers to "P_i = i's probability of succeeding in a bilateral contest with j".

From 1986 onwards, the definition of the probability is changed to account for *multilateral* contributions to the contest between i and j. The new form is denoted P^i_i . This will be defined later.

The form of the expected utilities as stated in BDM and Lalman (1986, p. 1118) are:

$$E^i\left(U_{ij}\right) = E^i\left(U_{ij}\right)_c - E^i\left(U_{ij}\right)_{nc} \quad (5)$$

$$E^i\left(U_{ij}\right)_c = s_j\left(P^i_iU^i_{si} + \left(1-P^i_i\right)U^i_{fi}\right) + \left(1-s_j\right)U^i_{si} \quad (6)$$

$$E^i\left(U_{ij}\right)_{nc} = QU^i_{sq} + (1-Q)\left(TU^i_{bi} + (1-T)U^i_{wi}\right) \quad (7)$$

The notable change is the inclusion of salience s_j and an extra term in the expected utility for challenge. Otherwise, notation changes are minimal.

This latter structure remains throughout BDM (1994, 1997, and 2002). In BDM (2009b) a new structure of model is announced, however we do not consider this new model further. As a result of the multi-lateral scaling issue with the model in its pre-1986 form, we focus on the model structure and results for 1986 and later, using the form from equations 5, 6 and 7.

3 UTILITIES

BDM (1997, p.242-243) uses equations (5), (6) and (7) though with different notation. The probability of status quo might be determined in a number of ways, however, a value of Q=1.0 is assumed in BDM (1985, p.161), corresponding to a stoic opposition and a value of Q=0.5 is assumed in both BDM and Lalman (1986, p.1122) and BDM (2009a, p.5), corresponding to a maximally uncertain outcome of whether the actor j will move or stay in position. No explicit value for Q is specified in other papers.

3.1 Base Utilities

To find the expected utility, we need to calculate the basic utilities:

$$U^i_{si}, U^i_{fi}, U^i_{bi}, U^i_{wi}, U^i_{sq} \quad (8)$$

These utilities are a function of the policy position of actors, x_i and x_j. BDM (1997, p.264) tells us that "… i's utility for x_k, u^ix_k, is a decreasing function of the distance between the proposal and i's preferred resolution, so that

$$u^ix_k = f\left(-\left|x_k - x^*_I\right|\right). \text{ "} \quad (9)$$

The notation $u^i x_j$ is not the same as that used in (8), so some transformation is probably required. Thus we interpret (9) as the *general* class of model only. It is worth pointing out that a *specific* class is stated in BDM (1997, p.245):

$$u^i x_j = 1 - \left| x_i - x_j^* \right|^{r_i} \tag{10}$$

However, (10) is *inconsistent* with the more detailed earlier explanations, as the following will now reveal.

One clue to the utility calculations is given in BDM (1994):

> Should i succeed, then i will derive the utility associated with convincing j to switch from its current policy stance to that supported by i. This is denoted by $u^i \Delta x_j^+ \mid d$, which equals $u^i (x_i - x_j)$. Should i fail, then it confronts the prospect of having to abandon its objectives in favour of those pursued by j, denoted by $u^i \Delta x_j^- \mid d = u^i (x_j - x_i)$. (p. 84)

Once again, BDM introduces additional notation $u^i (x_i - x_j)$ and $u^i (x_j - x_i)$ which remains undefined. However, BDM (1985, p.158) gives utility for i's success which is of the form of a difference between positions i and j:

$$U_{si}^{i} = 2 - 4\left[\frac{2 - \left(U_{ii}^{i} - U_{ij}^{i}\right)}{4}\right]^{r_i} \tag{11}$$

Also, the utility for i's failure, which is of the form of a difference between positions j and i:

$$U_{fi}^{i} = 2 - 4\left[\frac{2 - \left(U_{ij}^{i} - U_{ii}^{i}\right)}{4}\right]^{r_i} \tag{12}$$

Noting also in BDM (1985):

> The reason for the transformations by 2's and 4's is to preserve the original scale of numbers while avoiding the generation of imaginary numbers. Because r_i can be less than 1.0, the absence of transformations would mean that for negative values of, for instance U_{fi}, no real root would exist. This problem is eliminated with the introduction of these transformations. (p. 158)

U_{ii}^{i} and U_{ij}^{i} are defined by BDM (1985):

> With U_{ii}^{i} being equal to the value i attaches to his own policy portfolio (Both U_{ii} and U_{jj} are assumed to equal 1.0, with U_{ij} and U_{ji} ranging between possible values of 1.0 and -1.0), and with U_{ij}^{i} being equal to the value i attaches to j's policies as a function of their similarity to the policies of i. (p. 158)

Thus,

$$U_{ii}^{i} = U_{jj}^{i} = U_{ii}^{j} = U_{jj}^{j} = 1 \tag{13}$$

And to satisfy the stated range requirement *we* propose,

$$U_{ij}^{i} = U_{ji}^{i} = 1 - 2\left|\frac{x_i - x_j}{x_{\max} - x_{\min}}\right| \tag{14}$$

Equation (14) is consistent with the statement and equation at (9). Note that $-1 \le U_{ii}^{i} \le 1$. Where, $x_{\max} - x_{\min}$ is the range of positions. Note the maximum value of +1 occurs when policy positions of i and j coincide and is at its minimum of -1 when the positions are maximally separated.

Summarising, so far we have now accounted for (8) parts a and b, which simplify to:

$$U_{si}^{i} = 2 - 4\left[0.5 - 0.5\left|\frac{x_i - x_j}{x_{\max} - x_{\min}}\right|\right]^{r_i} \tag{15}$$

$$U_{fi}^{i} = 2 - 4\left[0.5 + 0.5\left|\frac{x_i - x_j}{x_{\max} - x_{\min}}\right|\right]^{r_i} \tag{16}$$

Note the ranges $2 - 4(0.5)^{r_i} \le U_{si}^{i} \le 2$ and $-2 \le U_{fi}^{i} \le 2 - 4(0.5)^{r_i}$ are consistent with the diagram in BDM (1985, p.159).

BDM (1985) does not explicitly define U_{bi}^{i} or U_{wi}^{i}, however, we are given in BDM (1985, p.158):

$$U_{qi}^{i} = 2 - 4\left[\left(2 - \left[\left(U_{ii}^{i} - U_{ij}^{i}\right)_{tn} - \left(U_{ii}^{i} - U_{ij}^{i}\right)_{t0}\right]\right)/4\right]^{r_i} \tag{17}$$

The subscripts *t0* and *tn* are not defined, but in BDM (1981, p.48) these correspond to before and after *j*'s *policy change*, respectively. The utility subscript *q* is usually signifies status quo, but we believe this is an error in (17) and should instead refer to *j* making a policy change (or move) which gains or betters the situation for *i*. We are led to believe this by BDM (1981, p.47-48):

> $\left(U^i_{ii} - U^i_{ij}\right)_{t0}$ =*i*'s perception of what may be gained by succeeding in a bilateral conflict with *j*… $\left(U^i_{ij} - U^i_{ii}\right)_{t0}$ = *i*'s perception of what may be lost by failing in a bilateral conflict with *j*…

So we adopt the subscript, *b* indicating 'better' consistent with (8). Furthermore, we note a problem with scaling for (17), as it will become undefined (negative number raised to a power less than 1.0). Thus, corrections are also required to realign scaling. The result is as follows:

$$U^i_{bi} = 2 - 4\left[\frac{4 - \left(U^i_{ii} - U^i_{ij}\right)_{tn} - \left(U^i_{ii} - U^i_{ij}\right)_{t0}}{8}\right]^{r_i} \qquad (18)$$

Similarly we expect that *j*'s movement may potentially result in a worse condition for *i*:

$$U^i_{wi} = 2 - 4\left[\frac{4 - \left(U^i_{ij} - U^i_{ii}\right)_{tn} - \left(U^i_{ij} - U^i_{ii}\right)_{t0}}{8}\right]^{r_i} \qquad (19)$$

Note this adjusted scaling ensures $2 - 4(0.5)^{r_i} \le U_{bi} \le 2$ and $-2 \le U_{wi} \le 2 - 4(0.5)^{r_i}$.

We are given a clue that some relation to the median voter position is important by "$[u^i \Delta x^+_j \mid \bar{d}$ *and* $u^i \Delta x^-_j \mid \bar{d}]$ are approximated by comparing the value actor *i* attaches to the current median voter prediction to the value *i* attaches to the median anticipated if *i* accepts *j*'s preferred outcome." (BDM 1997, p. 248)

BDM (2009a) expresses this most clearly:

> … they are anticipated to move towards the median voter position if they make an uncoerced move. This means that if B lies on the opposite side of the median voter from A, then A anticipates that if B moves (probability=0.5), then B will move in such a way as to come closer to the policy outcome A supports and so A's welfare will improve without A having to exert any effort. If B lies between the median voter position and A, then whether A's welfare improves or worsens depends on how far B is expected to move compared to A. The same is true if A lies between B and the median. (p. 6)

We interpret this to mean that for no challenge (uncoerced), and B (or *j*) moves, that A (or *i*) expects B (or *j*) will move to the median position. The cases are illustrated in figures 1 to 4.

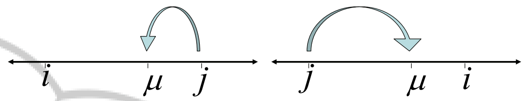

Figure 1: Case 1: μ between *i* and *j* – utility for *i* gets better as a result of *j* moving.

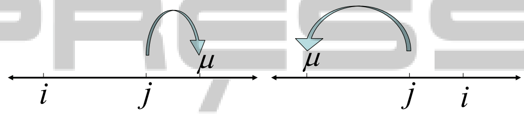

Figure 2: Case 2: *j* between *i* and μ utility for *i* gets worse as a result of *j* moving.

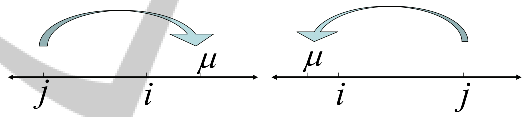

Figure 3: Case 3A: *i* between *j* and μ utility for *i* gets better as a result of *j* moving.

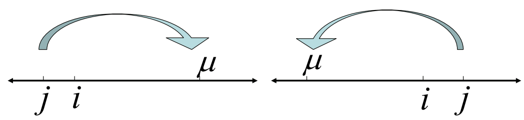

Figure 4: Case 3B: *i* between *j* and μ utility for *i* gets worse as a result of *j* moving.

For any of these cases then we expect,

$$\left(U^i_{ij}\right)_{t0} = 1 - 2\left|\frac{x_i - x_j}{x_{\max} - x_{\min}}\right| \qquad (20)$$

$$\left(U^i_{ij}\right)_{tn} = 1 - 2\left|\frac{x_i - \mu}{x_{\max} - x_{\min}}\right| \qquad (21)$$

Substituting (20) and (21) into (18) yields:

$$U^i_{bi} = 2 - 4\left[0.5 - 0.25\frac{\left(|x_i - \mu| + |x_i - x_j|\right)}{|x_{\max} - x_{\min}|}\right]^{r_i} \qquad (22)$$

Substituting (20) and (21) into (19) yields:

$$U^i_{wi} = 2 - 4\left[0.5 + 0.25\frac{\left(|x_i - \mu| + |x_i - x_j|\right)}{|x_{\max} - x_{\min}|}\right]^{r_i} \qquad (23)$$

We note further, that these utilities must be applied in the appropriate cases. Thus if case 1 is true, then the probability of *i*'s utility improving is 1.0 and by implication the probability of *i*'s utility worsening is 0.0. We believe this defines the probability T in equation (7), though no known publication by the author states this explicitly. So, cases 1 and 3A correspond to *T=1* and cases 2 and 3B correspond to *T=0*. This helps explain the description in BDM (1986, p. 1122).

Lastly, for the situation of no change in policy (status quo) i does not challenge and j does not move, BDM (1985, p.158) defines:

$$U^i_{sq} = 2 - 4\left[\frac{(2-0)}{4}\right]^{r_i} = 2 - 4(0.5)^{r_i} \qquad (24)$$

We expect this corresponds to (8) part *e*.
To determine the calculations for actor *j*, BDM (1985) notes:

> Of course, the U^j_s, U^j_f and U^j_q terms (with appropriate superscripts) are defined analogously. These terms vary as a function of whose estimate of expected utility is being calculated (i.e., who is the superscripted actor) by varying the risk exponent, so that for expected utility equations with a j superscript, *j*'s risk taking propensity is used to estimate what *j* perceives to be the value of success, failure, or no challenge for i in accordance with the equations delineated below. (p. 158)

Thus, the two main equations become:

$$\begin{aligned} E^i\left(U_{ij}\right) = {} & s_j\left(P^i_i U^i_{si} + \left(1 - P^i_i\right)U^i_{fi}\right) + \left(1 - s_j\right)U^i_{si} \\ & - QU^i_{sq} - (1-Q)\left(TU^i_{bi} + (1-T)U^i_{wi}\right) \end{aligned} \qquad (25a)$$

$$\begin{aligned} E^j\left(U_{ji}\right) = {} & s_j\left(P^j_j U^j_{sj} + \left(1 - P^j_j\right)U^j_{fj}\right) + \left(1 - s_j\right)U^j_{sj} \\ & - QU^j_{sq} - (1-Q)\left(TU^j_{bj} + (1-T)U^j_{wj}\right) \end{aligned} \qquad (25b)$$

3.2 The Median Voter Position

In the previous section, it became clear that the "median voter position" must be determined in order to calculate the utility terms in (22) and (23).

BDM defines comparative *votes* in direct proportion to utility difference, capability and salience. The votes 'cast' by agent i in *comparing* positions x_j and x_k is given as BDM (1997, p.239):

$$v^{jk}_i \mid x_j, x_k = c_i s_i \left(u^i x_j - u^i x_k\right) \qquad (26)$$

We emphasise, that these votes cast to k may indeed be negative, if for example, agent *i* prefers *j* to k. To map this notation to that used previously, we interpret:

$$u^i x_j = U_{ij} \quad \& \quad u^i x_k = U_{ik} \qquad (27)$$

Using (14) we get:

$$v^{jk}_i \mid x_j, x_k = 2c_i s_i \left(\frac{|x_i - x_k| - |x_i - x_j|}{|x_{\max} - x_{\min}|}\right) \qquad (28)$$

According to BDM (1997):

> The prospect that a proposal will succeed is assumed to depend on how much support can be mustered in its favour as compared with the feasible alternatives. This is calculated as the sum of "votes" across all actors in comparison between x_j and x_k. (p. 240)

$$v^{jk} = \sum_{i=1}^{n} v^{jk}_i \qquad (29)$$

In general, this pairwise determination is termed a Condorcet Method of voting. A Condorcet winner is the candidate whom voters prefer to each other candidate, when compared to them one at a time.

Black's Median Voter theory now comes into play, so "the decision adopted by the committee becomes determinant as soon as the position of the one optimum – which we can refer to conveniently enough as the median optimum – is given." (Black, 1948).

That is, in a majority election where a voter's attitude is represented as a point in one dimension, if all voters vote for a candidate closest to their own preference and there are only two candidates, then if the candidates want to maximise their votes they should commit to the policy attitude preferred by the median voter.

The median voter's ideal attitude is always a Condorcet winner (Congleton, 2003). Thus the median voter attitude index and the number of votes at the median attitude may be determined.

4 ALLIANCE PROBABILITY

The probabilities of equations (25) are determined by the bilateral alliances. BDM determines these probabilities by combining across all pairs, an assessment of ‘who is with me’ (positive valued vote) versus ‘who is against me’ (negative valued vote) and normalising. BDM (1997, p.244) states the estimator as:

$$P^i = \sum_{k|u^k x_i > u^k x_j} v_k^{ij} \Bigg/ \sum_{k=1}^{n} \left| v_k^{ij} \right| \tag{30}$$

When more agents are ‘for’ than ‘against’, this raises the probability of winning the bilateral contest. As per previous derivation of votes, substitute and expand (31):

$$P_i^i = \frac{\sum\limits_{k \ if \ \arg>0} c_k s_k \left(|x_k - x_j| - |x_k - x_i| \right)}{\sum\limits_{k=1}^{n} c_k s_k \left| \left(|x_k - x_j| - |x_k - x_i| \right) \right|} \tag{31}$$

5 RISK PROPENSITY

As seen in the previous section, utility calculations involve a risk exponent. This risk exponent is in turn derived from the expected utility. BDM (1985, p.157) is first to describe the basis for risk calculation,

> I define each nation’s security level as $\sum_{j \neq i} E(U_{ji})$. The greater the sum, the more utility *i* believes its adversaries expect to derive from challenging *i*. … as this sum decreases, *i*’s relative security increases, so that *i* is assumed to have adopted safe policies ...

BDM (1985, p.157) goes on to define:

$$R_i = \frac{2\sum E(U_{ji}) - \sum E(U_{ji})_{\max} - \sum E(U_{ji})_{\min}}{\sum E(U_{ji})_{\max} - \sum E(U_{ji})_{\min}} \tag{32}$$

Note that BDM (1997, p.247) reverses the subscripts of the above, which is inconsistent with his conceptual basis of security. Further, BDM (1997,p.247) provides an inconsistent transformation formula which would not accommodate the range $-1 \leq R_i \leq +1$. Thus, we choose the earlier conversion formula from BDM (1985, p.157):

$$r_i = \frac{1 - R_i/3}{1 + R_i/3} \tag{33}$$

The purpose of the formula according to BDM (1985, p.157) is to ensure that r_i ranges between 0.5 and 2, noting that the divisor of 3 appears arbitrary, but effects curvature. Equation (33) is illustrated in figure 5.

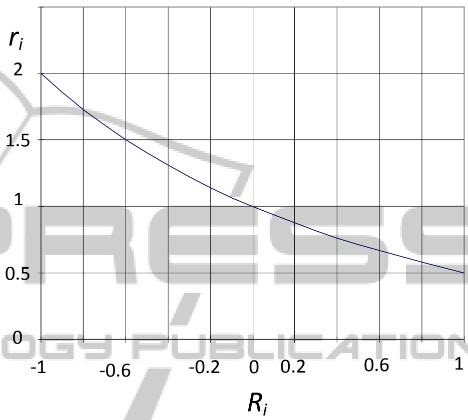

Figure 5: Scaling conversion formula.

The following equation (34), expresses (32) more precisely:

$$R_i = \frac{2\sum\limits_{j=1, j\neq i}^{n} E^i(U_{ji}) - \max\limits_i \left\{ \sum\limits_{j=1, j\neq i}^{n} E^i(U_{ji}) \right\} - \min\limits_i \left\{ \sum\limits_{j=1, j\neq i}^{n} E^i(U_{ji}) \right\}}{\max\limits_i \left\{ \sum\limits_{j=1, j\neq i}^{n} E^i(U_{ji}) \right\} - \min\limits_i \left\{ \sum\limits_{j=1, j\neq i}^{n} E^i(U_{ji}) \right\}} \tag{34}$$

We still need to know, however, how to calculate the expected utilities in (25), which use a modified notation. BDM (1985) describes:

> Thus, the risk terms are calculated by manipulating the alliance portfolios used as the policy indicator through simulation to locate the best and worse portfolios for any given nation, where the best and worst are defined in terms of the sum of expected utilities of all others vis-à-vis the nation in question under the assumption that utilities are strictly a function of similarities in alliance commitments. (Note: That is, temporarily applying the expected utility equations (without risk or uncertainty taken into account) as developed in *The War Trap*, I identify the worst and best case alliance strategy for each nation each year, using the original, linear utility functions to define the

> range of possible expected gains or losses for each nation. These, then, are utilized to measure risk propensities and thereby to introduce curvature into the utility functions.) (p. 167-168)

This implies a process to first determine the expected utilities of equations (25) using r_i=1, then apply (34) and (33) to estimate r_i and lastly apply the r_i estimates to re-estimate the expected utilities of equations (25).

6 DECISION

6.1 Offer Categories

The expected utilities $E^i(U_{ij})$ and $E^j(U_{ji})$ are used to classify the 'offers' between all actor pairs into categories according to potential outcomes as illustrated in figure 6. An actor may expect to conflict, compromise, capitulate, or stalemate with another. Unfortunately no single publication by BDM explains how to quantify these.

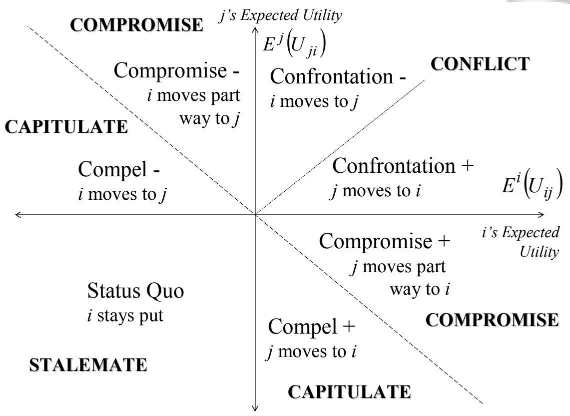

Figure 6: Classifying the outcome of challenges according to i's viewpoint.

Conflict. Actors i and j conflict if $E^i(U_{ij})>0$ and $E^j(U_{ji})>0$. So "If both i and j believe that they have the upper hand in the relationship, then conflict is likely and that conflict has an uncertain outcome." BDM (1997, p.244)

BDM (1984, p. 230) labels for the "Confrontation-" octant, "Challenger Favored" and for the "Confrontation+" octant "Favoring Focal Group". We interpret this to mean i moves to j and j moves to i respectively, as shown in figure 6.

Compromise. Actor i has the upper hand if $E^i(U_{ij})>0, E^j(U_{ji})<0$ and $|E^i(U_{ij})|>|E^j(U_{ji})|$. Actor j has the upper hand if $E^i(U_{ij})<0, E^j(U_{ji})<0$ and $|E^i(U_{ij})|<|E^j(U_{ji})|$. BDM (1997) describes:

> … both players agree that i has the upper hand. In this instance, j is expected to be willing to offer concessions to i, although the concessions are not likely to be as large as what i would like. The likely resolution of their exchange is a compromise reflecting the weighted average of i's expectation and j's. (p. 243-244)

However, the weighted average is not clear. BDM (1994) states (presumably with regard to i having the 'upper hand'):

> … the concession is assumed to equal the distance on R_a between x_i and x_j multiplied by the ratio of the absolute value of j's expected utility to i's expected utility. This treats the compromise as the weighted average of the perceived enforceability of the demand… (p. 96)

We might interpret this literally as:

$$\hat{x}=(x_i-x_j)\left|\frac{E^j(U_{ji})}{E^i(U_{ij})}\right| \tag{35}$$

Noting this relates only to the octant labeled "Compromise +" in figure 6. Considering the boundary conditions in this octant, if $|E^i(U_{ij})|>>|E^j(U_{ji})|$ then $\hat{x}\to 0$ and actor j does not move from x_j and if $|E^i(U_{ij})|\to|E^j(U_{ji})|$ then $\hat{x}\to 1$ and actor j moves from x_j to x_i. For the octant labelled "Compromise -" we use:

$$\hat{x}=(x_i-x_j)\left|\frac{E^i(U_{ij})}{E^j(U_{ji})}\right| \tag{36}$$

Acquiescence and Stalemate. The states for acquiescence and stalemate are illustrated in figure 6 and require no further explanation.

6.2 Offer Selection

Given that each actor has chosen who to challenge and to remain silent for those not to be challenged, then each actor will have received 'challenge offers' from other actors. How does an actor come to a decision on it which challenge offer it should accept? BDM (1997) elucidates:

> Each player would like to choose the best offer made to it and each proposer enforces its bid to the extent that it can. Those better able to enforce their wishes than others can make their proposals stick. Given equally enforceable proposals, players move the least that they can. … When the players finish sorting out their choices among proposals, each shifts to the position contained in the proposal it accepted. (p. 251-252)

If all offers are equally enforceable, we would propose to order these according to an actor's preferred choice as follows, so that actor *i* moves 'the *least* that it can'. Thus, summarising the order of decision choice for actor *i* is as follows:

1. Actor *i* conflicts with actor *j* and actor *j* (or with some chance actor *i*) acquiesces.
2. Actor *i* compromises to actor *j*. Actor *i* loses some ground.
3. Actor *i* acquiesces to actor *j*. Actor *i* loses most ground.
4. Actor *i* stalemate with actor *j*. Actor *i* status quo.

Thus, for example if actor *i* is in conflict with several other actors, each of which have greater expected utility than *i*, then the agent will need to concede to the one that allows *i* to move the least.

If all offers are not equally enforceable, then we might expect an actor to be more likely to concede to the most powerful actor. Thus, in the prior example, actor *i* concedes to the actor with highest expected utility.

7 RESULTS

BDM (1994) provides an example. The data for this was introduced in table 1. BDM (1994) provides three graphs of results. These compare expected utility for Belgium versus the others, France versus the others and the Netherlands versus the others.

Figures 7 and 8 compare the result using our interpretation of the algorithm as given in section 6, compared directly with the results published in BDM (1994, p.91). No value for Q was given. We chose Q=1.0.

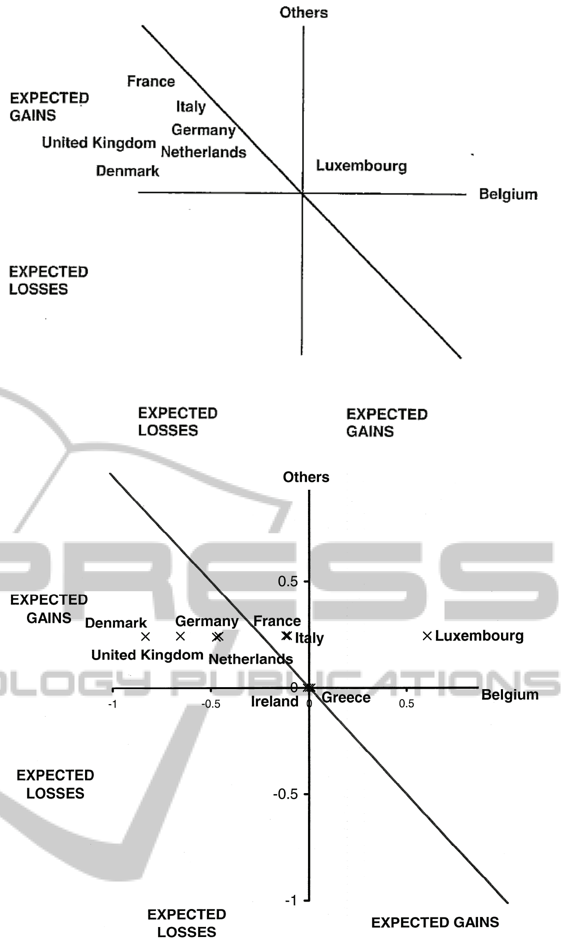

Figure 7: Comparison of results for BDM (1994) (top) and our interpretation (bottom), view from Belgium.

Note that some countries are not shown on BDM's graphs. In figure 7, our expected utility results for Ireland and Greece were (0,0) and in figure 8, UK and Italy were at (0,0).

As a result of the fact that BDM does not explicitly plot the point locus of the expected utilities, we can only reasonably assume the quadrants where the names are labelled corresponds to the location of each locus. The correspondence of our results to this level of accuracy (within a quadrant) is 100%. We note that if the expected utilities were derived randomly, the probability of getting any one of these points located in the correct quadrant is one in four. In order to get all nine results in the correct quadrants for any one graph of the two graphs above would constitute a probability of $(1/4)^9 \sim 4\text{x } 10^{-6}$. We therefore assert that BDM's results have effectively been reproduced.

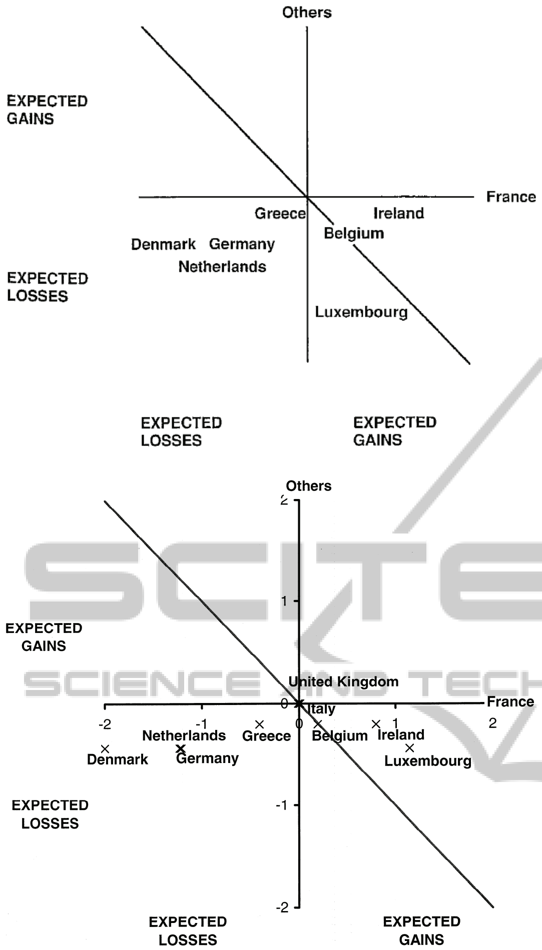

Figure 8: Comparison of results for BDM (1994) (top) and our interpretation (bottom), view from France.

We came across an issue with trying to reproduce the result given by BDM for the Netherlands as compared in figure 9. We assert that the result published by BDM was in error. Our result showing 'Others' as $E^j(U_{ji})$ against $E^i(U_{ji})$ for i=Luxembourg is compared with BDM's quoted result in figure 10. This shows 100% correspondence in terms of quadrant accuracy as for the previous two results.

BDM (1994) summarises the final result:

> The dominant outcome would be, as indicated above, a lag of 8.35 years. However, if the participants were prepared to bear the costs of slightly prolonged negotiations, then the model's predicted dominant outcome rises to 9.05 years and stabilizes at that point. … The actual resolution was for a delay of 8.833 years. (p.98)

We found the median voter position at the end of the first round to be 8.4 years. At the end of the second, third, fourth and fifth rounds the median voter position was for each 9.9 years.

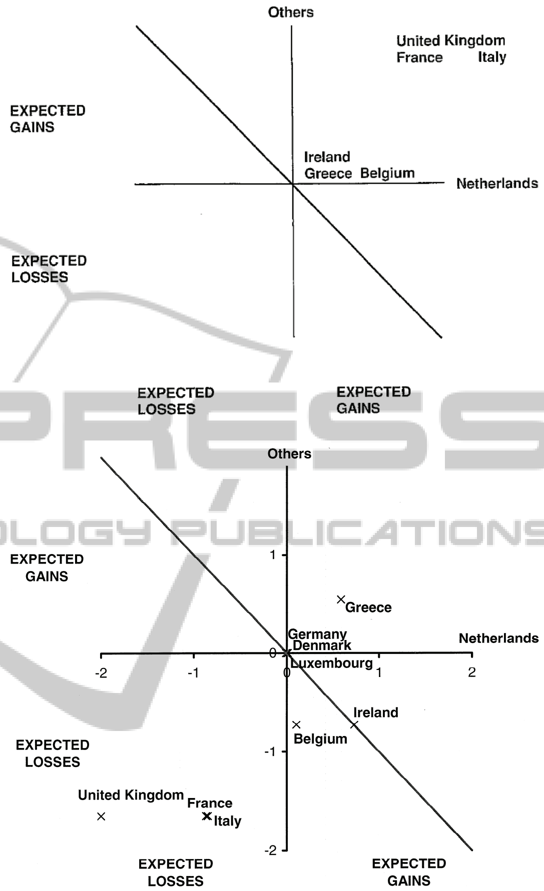

Figure 9: Comparison of results for BDM (1994) (top) and our interpretation (below), view from Netherlands.

8 DISCUSSION

As identified earlier, we chose Q=1.0 to reproduce the results above. We observed that a value of Q=0.5 produces very different results. The following figure 11 provides an illustrative example.

In figure 11, the positions of Greece and Belgium change entire quadrants if Q=1 or Q=0.5 is chosen.

Recalling that Q relates to the probability of a status quo and is an arbitrary parameter, it is not desirable for results to be so sensitive.

We examined the applicability of the interpretation to other examples from later papers. Despite the fact that BDM (1994) and BDM (1997) differ only in the detailed example used, it is perplexing that we were unable to reproduce the results from the 1997 paper. Indeed attempts to apply this algorithm (using either Q=0.5 or Q=1.0) to the 1997 "Sultan" problem yielded wildly different results to those published.

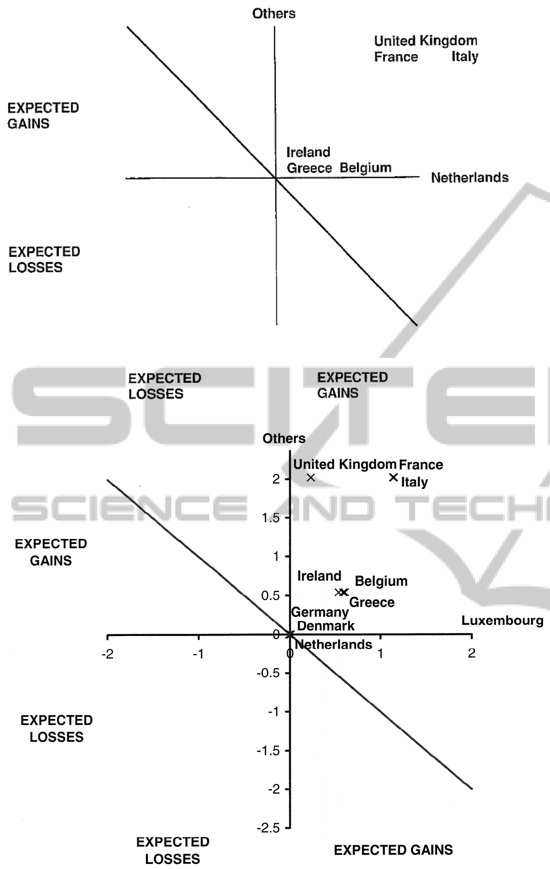

Figure 10: Comparison of results for BDM (1994) 'view from Netherlands' and our interpretation, view from Luxembourg.

Further insight on the evolution of the median voter position over rounds is also warranted. This is shown in table 2.

Table 2 shows that the median appears to stabilise, but then continues to change. Calculation of the *mean* voter position provides insight. The coarseness of the median voter position becomes evident. Indeed given the fact that the "compromise" state allows for intermediate valued positions (as per equation 36) it is surprising that BDM would want to continue with median over mean values. In general the results do not stabilise. There is no reason from examining the algorithm to expect that they should.

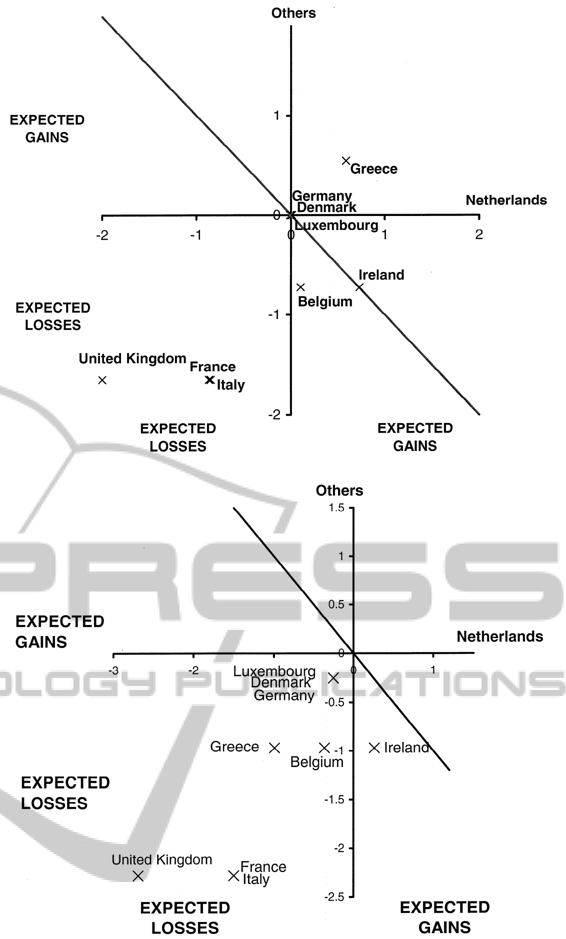

Figure 11: Example for Netherlands comparing results using Q=1.0 (top) and Q=0.5 (bottom).

Table 2: Evolution of the median and mean voter positions for 'the date of introduction of emission standards for medium-sized automobiles' problem in BDM (1994).

Round	1	2	3	4	5	6	7	8
Median	8.4	9.9	9.9	9.9	9.9	7.4	8.8	9.6
Mean	7.4	7.5	7.6	7.3	7.3	7.4	7.5	7.6

9 CONCLUSIONS

The algorithm outlined (and summarised in the Appendix) has for the first time exposed and provided independent means of replicating the results of BDM's computational model. This opens BDM's model, method and claims to scientific discussion.

The correctness of the interpretation was illustrated using the example from BDM (1994). We note the chance of replicating to this level of accuracy by random selection would be much less than one in one million. This fulfils Bueno De Mesquita's own prediction that enough material is available so that "anyone (may) replicate something close to his work"!

Concerns with regard to the model's sensitivity and convergence have been identified.

Given these concerns, we conclude that adoption of BDM's model for agent development would be premature at this time.

REFERENCES

Black, D. (1948). On the Rationale of Group Decision-making. *Political Economy* 56, No 1, Feb: 23-34.

Congleton, R. D. (2003). The Median Voter Model. *The Encyclopedia of Public Choice.* Kluwer Academic Press: 382-386.

Bueno De Mesquita, B. B. (1980). An Expected Utility Theory of International Conflict. *The American Political Science Review*, Vol. 74, No. 4 (Dec), pp. 917-931.

Bueno De Mesquita, B. B. (1981). *The War Trap.* Yale, CT: Yale University Press.

Bueno De Mesquita, B. B. (1984). Forecasting Policy Decisions: An Expected Utility Approach to Post-Khomeini Iran, *The American Political Science Review*, Vol. 17, No. 2, pp. 226-236.

Bueno De Mesquita, B. B. (1985). The War Trap Revisited: A Revised Expected Utility Model. *American Political Science Review* 79, No 1, 156-177.

Bueno De Mesquita, B.B. and Lalman, D. (1986). Reason and War, *The American Political Science Review*, Vol. 80, No. 4, (Dec), pp. 1113-1129.

Bueno De Mesquita, B. B. (1994). Political Forecasting: An Expected Utility Method, *European Community Decision Making*, Yale University Press, Stockman, F. (ed.), Chapter 4, pp 71-104.

Bueno De Mesquita, B. B. (1997). A Decision Making Model: Its Structure and Form. *International Interactions* 23: No 3-4 235-266.

Bueno De Mesquita, B. B. (2002). *Predicting Politics*, Ohio State University Press.

Bueno De Mesquita, B. B. (2009a). *A New Model for Predicting Policy Choices: Preliminary Tests*, 50th Meeting of the International Studies Association, New York, Feb 15-18.

Bueno De Mesquita, B. B. (2009b). *The Predictioneer's Game* (1st ed.). New York: Random House.

Kollman, K, Miller, J. H. and Page, S.E. eds (2010) *Computational models in political economy.* MIT Press.

APPENDIX

The following summarises the full procedure.

1. Given $i=1,2,\ldots,n$ actors, initial positions for each actor $x_i(t=0)$, c_i, s_i and number of rounds$=\tau$.
2. Let $r_i=1$
3. Calculate the pairwise votes:

$$v^{jk} = \sum_{i=1}^{n} c_i s_i \left(\frac{|x_i - x_k| - |x_i - x_j|}{|x_{\max} - x_{\min}|} \right)$$

Then find the maximum value which corresponds to the Condorcet winner position or median $= \mu$.

4. Calculate basic utilities,

$$U^i_{si} = 2 - 4\left[0.5 - 0.5\left|\frac{x_i - x_j}{x_{\max} - x_{\min}}\right|\right]^{r_i}$$

$$U^i_{fi} = 2 - 4\left[0.5 + 0.5\left|\frac{x_i - x_j}{x_{\max} - x_{\min}}\right|\right]^{r_i}$$

$$U^i_{bi} = 2 - 4\left[0.5 - 0.25\frac{\left(|x_i - \mu| + |x_i - x_j|\right)}{|x_{\max} - x_{\min}|}\right]^{r_i}$$

$$U^i_{wi} = 2 - 4\left[0.5 + 0.25\frac{\left(|x_i - \mu| + |x_i - x_j|\right)}{|x_{\max} - x_{\min}|}\right]^{r_i}$$

$$U^i_{sq} = 2 - 4(0.5)^{r_i}$$

5. Calculate probabilities:

$$P^i_i = \frac{\displaystyle\sum_{k \text{ if } \arg>0} c_k s_k \left(|x_k - x_j| - |x_k - x_i|\right)}{\displaystyle\sum_{k=1}^{n} c_k s_k \left|\left(|x_k - x_j| - |x_k - x_i|\right)\right|}$$

6. Let Q=0.5 (or 1.0).
7. Calculate:

$$E^i(U_{ij}) = s_j\left(P^i_i U^i_{si} + (1 - P^i_i)U^i_{fi}\right) + (1 - s_j)U^i_{si} - QU^i_{sq} - (1-Q)\left(TU^i_{bi} + (1-T)U^i_{wi}\right)$$

$$E^j(U_{ji}) = s_j\left(P^j_j U^j_{sj} + (1 - P^j_j)U^j_{fj}\right) + (1 - s_j)U^j_{sj} - QU^j_{sq} - (1-Q)\left(TU^j_{bj} + (1-T)U^j_{wj}\right)$$

If second pass (used the calculated values of r_i) then, go to step 11.

8. Calculate:

$$R_i = \frac{2\displaystyle\sum_{j=1, j\neq i}^{n} E^i(U_{ji}) - \max_i\left\{\sum_{j=1, j\neq i}^{n} E^i(U_{ji})\right\} - \min_i\left\{\sum_{j=1, j\neq i}^{n} E^i(U_{ji})\right\}}{\max_i\left\{\displaystyle\sum_{j=1, j\neq i}^{n} E^i(U_{ji})\right\} - \min_i\left\{\displaystyle\sum_{j=1, j\neq i}^{n} E^i(U_{ji})\right\}}$$

9. Calculate:

$$r_i = \frac{1 - R_i \ / 3}{1 + R_i \ / 3}$$

10. Go to step 4, using calculated values of r_i.
11. Determine new position decisions x, based on rules in section 5 for octant of E_{ij}(i) vs E_{ji}(j).
12. Increment the rounds, $t=t+1$
13. If $t=\tau$ then stop.