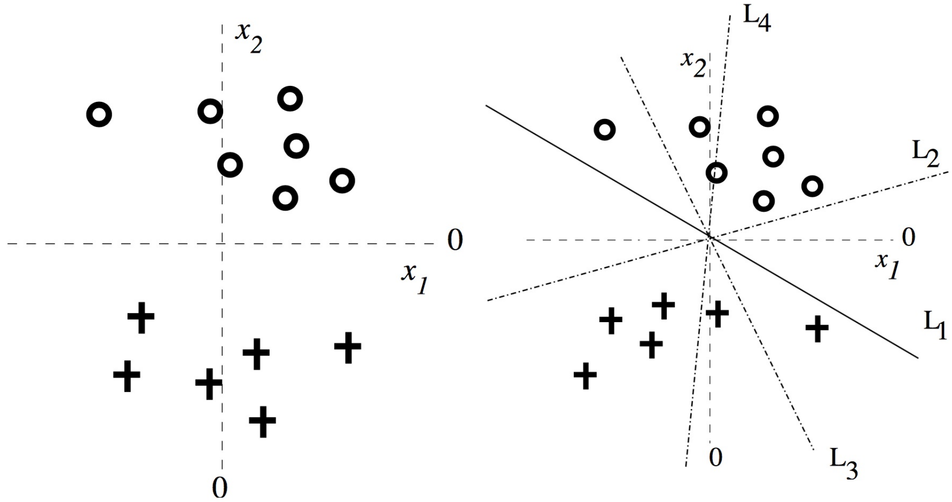
1. Assume the binary classification task depicted in Figure **??**, which we attempt to solve with the simple linear logistic regression model

$$\widehat{\Pr}(Y = 1|X = x) = \hat{p}(x) = g(\beta_1 x_1 + \beta_2 x_2) = \frac{1}{1 + \exp(\beta_1 x_1 + \beta_2 x_2)}$$

for simplicity we do not use the parameter $\beta_0$. The training data is linearly separable, and the line $\mathrm{L}_1$ is the result of logistic regression, with zero training error..




Assume that we would like to find the classifier by *maximizing* the following regularized objective function, in which *only* $\beta_2$ is regularized.

$$\prod_{i=1}^{n} [p(x_i)]^{y_i} [1 - p(x_i)]^{1-y_i} - \lambda \beta_2^2 = L(\beta_1, \beta_2) - \lambda \beta_2^2$$

(a) Assume that $\lambda$ is large. Which of the four lines $\mathrm{L}_2, \mathrm{L}_3$ or $\mathrm{L}_4$ determine whether it can result from regularizing $\beta_2$. Explain very briefly your reasons.

(b) If we change the form of regularization to one-norm (absolute value) and also regularize $\beta_2$ we get the following penalized log-likelihood

$$\prod_{i=1}^{n} [p(x_i)]^{y_i} [1 - p(x_i)]^{1-y_i} - \lambda(|\beta_1| + |\beta_2|) = L(\beta_1, \beta_2) - \lambda(|\beta_1| + |\beta_2|)$$

As we increase the regularization parameter $\lambda$ which of the following scenarios is expected to be observed? Explain why.

i. First $\beta_1$ will become 0, then $\beta_2$.
ii. $\beta_1$ and $\beta_2$ will become zero simultaneously.
iii. First $\beta_2$ will become 0, then $\beta_1$.
iv. None of the weights will become exactly zero, only smaller as $\lambda$ increases

**Solution**:[1]

(a) When we regularize $\beta_2$, the resulting boundary can rely less on the value of $x_2$ and therefore becomes more vertical.

- L2 here seems to be more horizontal than the unregularized solution so it cannot come as a result of penalizing $\beta_2$.
- When $\beta_2$ is small relative to $\beta_1^2$ (as evidenced by high slope), and even though it would assign a rather low log-probability to the observed labels, it could be forced by a large regularization parameter $\lambda$. So L3 can arise as a result.
- For very large $\lambda$, we obtain a separator that is very close to vertical (with negative slope) and in the limit, entirely vertical (line $x_1 = 0$ or the $x_2$ axis). $L_4$ here is reflected across the $x_2$ axis and has a positive slope, and therefore represents a poorer solution than its counterpart on the other side. For moderate regularization we have to get the best solution that we can construct while keeping $\beta_2$ small. $L_4$ is not the best and thus cannot come as a result of regularizing $\beta_2$.

(b) The data can be classified with zero training error and therefore also with high log-probability by looking at the value of $x_2$ alone, i.e. making $\beta_1 = 0$. Initially we might prefer to have a non-zero value for $\beta_1$ but it will go to zero rather quickly as we increase regularization. Note that we pay a regularization penalty for a non-zero value of $\beta_1$ and if it does not help classification why should the penalty be paid? The $\mathscr{L}_1$ regularization ensures that $\beta_1$ will indeed go to exactly zero. As $\lambda$ increases further, even $\beta_2$ will eventually become zero. We pay higher and higher cost for setting $\beta_2$ to a non-zero value. Eventually this cost overwhelms the gain from the log-probability of labels that we can achieve with a non-zero $\beta_2$. Note that when $\beta_1 = \beta_2 = 0$, the log-probability of labels is a finite value $n \log(0.5)$.

[1]Important Note: Posting the course material to online forums or sharing it with other students is strictly prohibited. Instances will be reported to USC officials as academic dishonesty for disciplinary action.

2. A statistician is working on the amount of funding that companies obtain on a crwod-sourcing website and has developed the following model. She used 26 companies to obtain the model

$$\hat{y} = b_0 + b_1 x_1 + b_2 x_2 + b_3 x_3 + b_4 x_4 + b_5 x_5$$
$$\hat{y} = 964.8 + 700.2 x_1 + 317.5 x_2 - 200.2 x_3 + 15.3 x_4 + 17.1 x_5$$

The standard errors are:

$$s_{b_1} = 12.$$
$$s_{b_2} = 22.5$$
$$s_{b_3} = 101.8$$
$$s_{b_4} = 45.3$$
$$s_{b_5} = 2.3$$

- $\hat{y}$: the amount of funding obtained by a company in 1000 dollars
- $x_1$: the average annual salary of the founders
- $x_2$: the number of employees the startup hired
- $x_3$: a dummy vaiable that is 1 when the company's field is information technology and 0 otherwise
- $x_4$: the age of the company
- $x_5$ is a dummy variable taking value 1 if the founders had previous failures and 0 otherwise

(a) Interpret the estimated coefficients $b_0 = 964.8$ and $b_3 = -200.2$ (10 pts)

(b) Test, at the 2% level, the null hypothesis that the true coefficient on the dummy variable $x_5$ is 0 against the alternative that it is not 0. (10 pts)

(c) Find and interpret a 99.8% confidence interval for the parameter $\beta_4$. (10 pts)

(d) If for the model, SSR=18147.5 (Regression Sum of Squares) and SSE = 17136.5 (Residual Sum of Squares), test the hypothesis that all the coefficients of the model are 0 (test overal significance of the model) using $\alpha = 5\%$. (10 pts).

**Solutions:**

(a) $b_0$ is the amount of the dependent variable that was not explaned by the independent variables. $b_3$ means that if the field of the company is information technology, on average, the funding tat it wil receive frm the website will decrease by 200.2 units (1000 dollars)

(b) $\mathbf{H_0} : \beta_5 = 0,\ \mathbf{H_1} : \beta_5 \neq 0,\ t_{b_5} = \frac{b_5 - 0}{s_{b_5}} = \frac{17.1}{2.3} = 7.44$

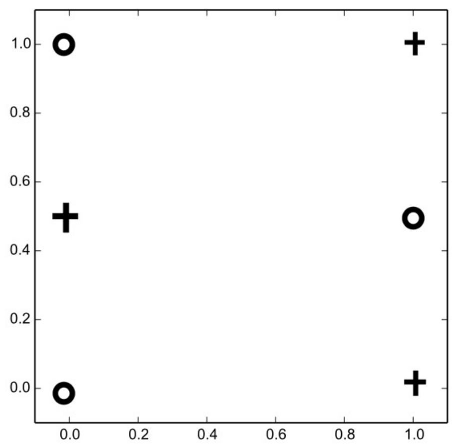
The rejection region is $t > t_{n-K-1,\alpha/2} = t_{26-5-1,.01}$ or $t < -t_{n-K-1,\alpha/2} = -t_{26-5-1,.01}$ .

But from the table, $t_{26-5-1,.01} = 2.528$, therefore, we reject the null hypothsi that $\beta_5 = 0$.

(c) $t_{n-K-1,\alpha/2} = t_{26-5-1,0.001} =$, so the Confidence interval for $\beta_4$ is: $[b_4 - t_{n-K-1,\alpha/2} s_{b_4}, b_4 + t_{n-K-1,\alpha/2} s_{b_1}] = [15.3-(3.552)(45.3), 15.3+(3.552)(45.3)] = [-145.61, 176.21]$.

(d) $F = \frac{SSR/K}{SSE/(n-K-1)} = \frac{18147.5/5}{17136.5/(26-5-1)} = 4.2$. The rejection region is $F > F_{K,n-K-1,\alpha} = F_{5,20,0.05} = 2.7109$, which means that we reject the null hypothesis that all coefficients are 0.

3. For the two dimensional training data shown below, determine whether or not each of the classification methods below, when trained appropriately, will have zero errors on the training set. In each case, briefly justify your answer. Moreover, provide a reasonable confusion matrix for each case.




(a) Logistic Regression

(b) SVM with Linear Kernel

(c) SM with RBF Kernel

(d) Decision Tree

(e) 3-Nearest-Neighbor Classifier (with Euclidean Distance).

**Solution:**

(a) Logistic Regression and Linear SVM: linear decision boundaries,hence no.

(b) SVM with RBF kernel: yes.

(c) 3-NN: the 3 nearest neighbors of any point in our training set are 1 of the same class and 2 of the opposite class, hence 3-NN will be systematically wrong.

(d) DT: yes, one can partition the space with lines orthogonal to the axes so that every sample ends up in a different region.

For methods with zero training error, the Confusion Matrix would be:

| | Class o | Class + |
|---|---|---|
| Class o | 3 | 0 |
| Class + | 0 | 3 |

For KNN:

| | Class o | Class + |
|---|---|---|
| Class o | 0 | 3 |
| Class + | 3 | 0 |

For SVM, LR:

| | Class o | Class + |
|---|---|---|
| Class o | 2 | 1 |
| Class + | 1 | 2 |

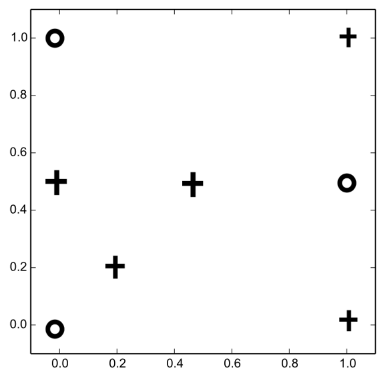
4. For a numeric input, instead of a binary split in a decision tree, one can use a ternary split with two thresholds and three branches as $X_j < s_1, s_1 \leq X_j < s_2, X_j \geq s_2$.

   (a) Propose a modification of the tree learning method to adjust the two thresholds, $s_1$ and $s_2$.

   (b) What are the advantages and the disadvantages of such a node over a binary node?

   (c) How would you choose between a binary and a trenary decision tree for a given data set?

   (d) Perform two iterations of the trenary tree algoritm on the tree shown in below and draw the corresponding tree



**Solution**:

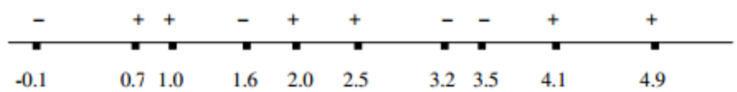
(a) For the numeric attributes, instead of one split threshold, we need to try all possible pairs of split thresholds and choose the best. When there are two splits, there are three children, and in calculating the entropy/ Gini index after the splits, we need to sum up over the three sets corresponding to the instances taking the three branches.

(b) The computational complexity of finding the best pair is higher and each node stores two thresholds instead of one and has three branches instead of two. The advantage is that one ternary node splits an input into three, whereas this requires two successive binary nodes.

(c) Which one is better depends on the data at hand; if we have hypotheses that require bounded intervals (e.g., rectangles), a ternary node may be advantageous. One has to choose between binary and trenary nodes using *Cross Validation*.

(d) Left to the students.

5. Consider the following dataset with one real-valued input and one binary output (+ or -). The following questions assume that we are using $k$- nearest-neighbor learning with Euclidean distance to predict $Y$ for an input $X$. What is the leave-one-out cross-validation error of 1-NN and 3-NN on this dataset, and decide which $k$ is better.



**Solution**: For each $X_i$, consider the majority vote of 1 nearest neighbors.

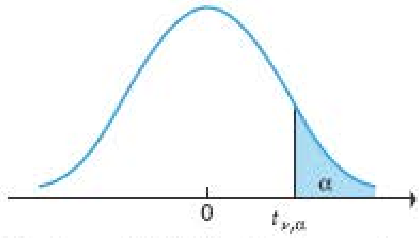
$E_{LOOCV} = 0.4$

For each $X_i$, consider the majority vote of 3 nearest neighbors.

$E_{LOOCV} = 0.8$

The 1-NN method is better

## Upper Critical Values of Student's $t$ Distribution with $\nu$ Degrees of Freedom




For selected probabilities, $\alpha$, the table shows the values $t_{\nu,\alpha}$ such that $P(t_\nu > t_{\nu,\alpha}) = \alpha$, where $t_\nu$ is a Student's $t$ random variable with $\nu$ degress of freedom. For example, the probability is .10 that a Student's $t$ random variable with 10 degrees of freedom exceeds 1.372.

| | Probability of Exceeding the Critical Value | | | | | |
|---|---|---|---|---|---|---|
| $\nu$ | 0.10 | 0.05 | 0.025 | 0.01 | 0.005 | 0.001 |
| 1 | 3.078 | 6.314 | 12.706 | 31.821 | 63.657 | 318.313 |
| 2 | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 | 22.327 |
| 3 | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 | 10.215 |
| 4 | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 | 7.173 |
| 5 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 | 5.893 |
| 6 | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 | 5.208 |
| 7 | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 | 4.782 |
| 8 | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 | 4.499 |
| 9 | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 | 4.296 |
| 10 | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 | 4.143 |
| 11 | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 | 4.024 |
| 12 | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 | 3.929 |
| 13 | 1.350 | 1.771 | 2.160 | 2.650 | 3.012 | 3.852 |
| 14 | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 | 3.787 |
| 15 | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 | 3.733 |
| 16 | 1.337 | 1.746 | 2.120 | 2.583 | 2.921 | 3.686 |
| 17 | 1.333 | 1.740 | 2.110 | 2.567 | 2.898 | 3.646 |
| 18 | 1.330 | 1.734 | 2.101 | 2.552 | 2.878 | 3.610 |
| 19 | 1.328 | 1.729 | 2.093 | 2.539 | 2.861 | 3.579 |
| 20 | 1.325 | 1.725 | 2.086 | 2.528 | 2.845 | 3.552 |
| 21 | 1.323 | 1.721 | 2.080 | 2.518 | 2.831 | 3.527 |
| 22 | 1.321 | 1.717 | 2.074 | 2.508 | 2.819 | 3.505 |
| 23 | 1.319 | 1.714 | 2.069 | 2.500 | 2.807 | 3.485 |
| 24 | 1.318 | 1.711 | 2.064 | 2.492 | 2.797 | 3.467 |
| 25 | 1.316 | 1.708 | 2.060 | 2.485 | 2.787 | 3.450 |
| 26 | 1.315 | 1.706 | 2.056 | 2.479 | 2.779 | 3.435 |
| 27 | 1.314 | 1.703 | 2.052 | 2.473 | 2.771 | 3.421 |
| 28 | 1.313 | 1.701 | 2.048 | 2.467 | 2.763 | 3.408 |
| 29 | 1.311 | 1.699 | 2.045 | 2.462 | 2.756 | 3.396 |
| 30 | 1.310 | 1.697 | 2.042 | 2.457 | 2.750 | 3.385 |
| 40 | 1.303 | 1.684 | 2.021 | 2.423 | 2.704 | 3.307 |
| 60 | 1.296 | 1.671 | 2.000 | 2.390 | 2.660 | 3.232 |
| 100 | 1.290 | 1.660 | 1.984 | 2.364 | 2.626 | 3.174 |
| $\infty$ | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 | 3.090 |
| $\nu$ | 0.10 | 0.05 | 0.025 | 0.01 | 0.005 | 0.001 |

# F - Distribution ($\alpha = 0.05$ in the Right Tail)

| $df_2$ \ $df_1$ | Numerator Degrees of Freedom 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 161.45 | 199.50 | 215.71 | 224.58 | 230.16 | 233.99 | 236.77 | 238.88 | 240.54 |
| 2 | 18.513 | 19.000 | 19.164 | 19.247 | 19.296 | 19.330 | 19.353 | 19.371 | 19.385 |
| 3 | 10.128 | 9.5521 | 9.2766 | 9.1172 | 9.0135 | 8.9406 | 8.8867 | 8.8452 | 8.8123 |
| 4 | 7.7086 | 9.9443 | 6.5914 | 6.3882 | 6.2561 | 6.1631 | 6.0942 | 6.0410 | 6.9988 |
| 5 | 6.6079 | 5.7861 | 5.4095 | 5.1922 | 5.0503 | 4.9503 | 4.8759 | 4.8183 | 4.7725 |
| 6 | 5.9874 | 5.1433 | 4.7571 | 4.5337 | 4.3874 | 4.2839 | 4.2067 | 4.1468 | 4.0990 |
| 7 | 5.5914 | 4.7374 | 4.3468 | 4.1203 | 3.9715 | 3.8660 | 3.7870 | 3.7257 | 3.6767 |
| 8 | 5.3177 | 4.4590 | 4.0662 | 3.8379 | 3.6875 | 3.5806 | 3.5005 | 3.4381 | 3.3881 |
| 9 | 5.1174 | 4.2565 | 3.8625 | 3.6331 | 3.4817 | 3.3738 | 3.2927 | 3.2296 | 3.1789 |
| 10 | 4.9646 | 4.1028 | 3.7083 | 3.4780 | 3.3258 | 3.2172 | 3.1355 | 3.0717 | 3.0204 |
| 11 | 4.8443 | 3.9823 | 3.5874 | 3.3567 | 3.2039 | 3.0946 | 3.0123 | 2.9480 | 2.8962 |
| 12 | 4.7472 | 3.8853 | 3.4903 | 3.2592 | 3.1059 | 2.9961 | 2.9134 | 2.8486 | 2.7964 |
| 13 | 4.6672 | 3.8056 | 3.4105 | 3.1791 | 3.0254 | 2.9153 | 2.8321 | 2.7669 | 2.7144 |
| 14 | 4.6001 | 3.7389 | 3.3439 | 3.1122 | 2.9582 | 2.8477 | 2.7642 | 2.6987 | 2.6458 |
| 15 | 4.5431 | 3.6823 | 3.2874 | 3.0556 | 2.9013 | 2.7905 | 2.7066 | 2.6408 | 2.5876 |
| 16 | 4.4940 | 3.6337 | 3.2389 | 3.0069 | 2.8524 | 2.7413 | 2.6572 | 2.5911 | 2.5377 |
| 17 | 4.4513 | 3.5915 | 3.1968 | 2.9647 | 2.8100 | 2.6987 | 2.6143 | 2.5480 | 2.4943 |
| 18 | 4.4139 | 3.5546 | 3.1599 | 2.9277 | 2.7729 | 2.6613 | 2.5767 | 2.5102 | 2.4563 |
| 19 | 4.3807 | 3.5219 | 3.1274 | 2.8951 | 2.7401 | 2.6283 | 2.5435 | 2.4768 | 2.4227 |
| 20 | 4.3512 | 3.4928 | 3.0984 | 2.8661 | 2.7109 | 2.5990 | 2.5140 | 2.4471 | 2.3928 |
| 21 | 4.3248 | 3.4668 | 3.0725 | 2.8401 | 2.6848 | 2.5727 | 2.4876 | 2.4205 | 2.3660 |
| 22 | 4.3009 | 3.4434 | 3.0491 | 2.8167 | 2.6613 | 2.5491 | 2.4638 | 2.3965 | 2.3419 |
| 23 | 4.2793 | 3.4221 | 3.0280 | 2.7955 | 2.6400 | 2.5277 | 2.4422 | 2.3748 | 2.3201 |
| 24 | 4.2597 | 3.4028 | 3.0088 | 2.7763 | 2.6207 | 2.5082 | 2.4226 | 2.3551 | 2.3002 |
| 25 | 4.2417 | 3.3852 | 2.9912 | 2.7587 | 2.6030 | 2.4904 | 2.4047 | 2.3371 | 2.2821 |
| 26 | 4.2252 | 3.3690 | 2.9752 | 2.7426 | 2.5868 | 2.4741 | 2.3883 | 2.3205 | 2.2655 |
| 27 | 4.2100 | 3.3541 | 2.9604 | 2.7278 | 2.5719 | 2.4591 | 2.3732 | 2.3053 | 2.2501 |
| 28 | 4.1960 | 3.3404 | 2.9467 | 2.7141 | 2.5581 | 2.4453 | 2.3593 | 2.2913 | 2.2360 |
| 29 | 4.1830 | 3.3277 | 2.9340 | 2.7014 | 2.5454 | 2.4324 | 2.3463 | 2.2783 | 2.2229 |
| 30 | 4.1709 | 3.3158 | 2.9223 | 2.6896 | 2.5336 | 2.4205 | 2.3343 | 2.2662 | 2.2107 |
| 40 | 4.0847 | 3.2317 | 2.8387 | 2.6060 | 2.4495 | 2.3359 | 2.2490 | 2.1802 | 2.1240 |
| 60 | 4.0012 | 3.1504 | 2.7581 | 2.5252 | 2.3683 | 2.2541 | 2.1665 | 2.0970 | 2.0401 |
| 120 | 3.9201 | 3.0718 | 2.6802 | 2.4472 | 2.2899 | 2.1750 | 2.0868 | 2.0164 | 1.9588 |
| $\infty$ | 3.8415 | 2.9957 | 2.6049 | 2.3719 | 2.2141 | 2.0986 | 2.0096 | 1.9384 | 1.8799 |

Denominator Degrees of Freedom

Cumulative Distribution Function, $F(z)$, of the Standard Normal Distribution Table

| z | 0 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| 3.0 | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 |
| 3.1 | 0.9990 | 0.9991 | 0.9991 | 0.9991 | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9993 | 0.9993 |
| 3.2 | 0.9993 | 0.9993 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9995 | 0.9995 | 0.9995 |
| 3.3 | 0.9995 | 0.9995 | 0.9995 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9997 |
| z | 0 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |

Cumulative Distribution Function, $F(z)$, of the Standard Normal Distribution Table