Introduction to Statistical Learning

INF 552, Machine Learning for Data Informatics

University of Southern California

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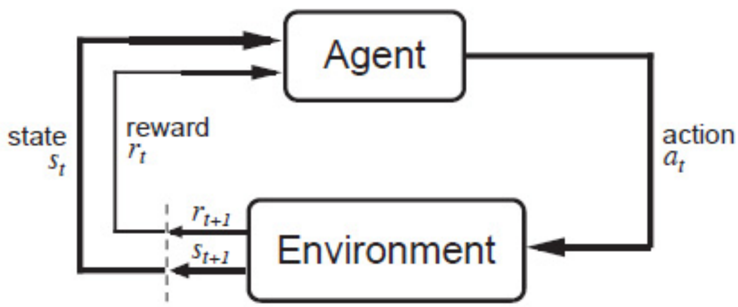
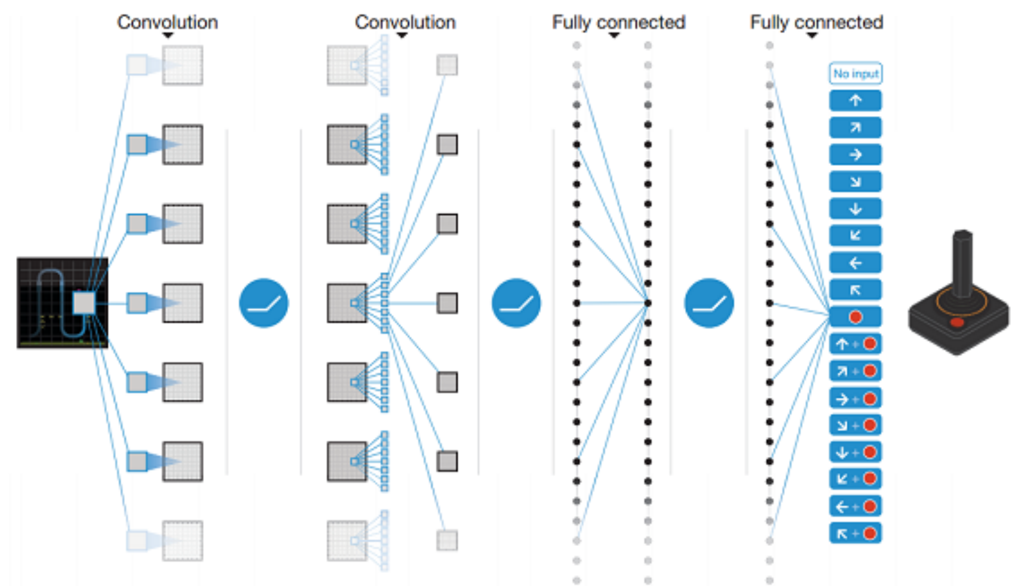
Lesson 13
Reinforcement Learning



AlphaGo



Overview

- Supervised Learning: Immediate feedback (labels provided for every input).

- Unsupervised Learning: No feedback (no labels provided).

- Reinforcement Learning: Delayed scalar feedback (a number called reward).

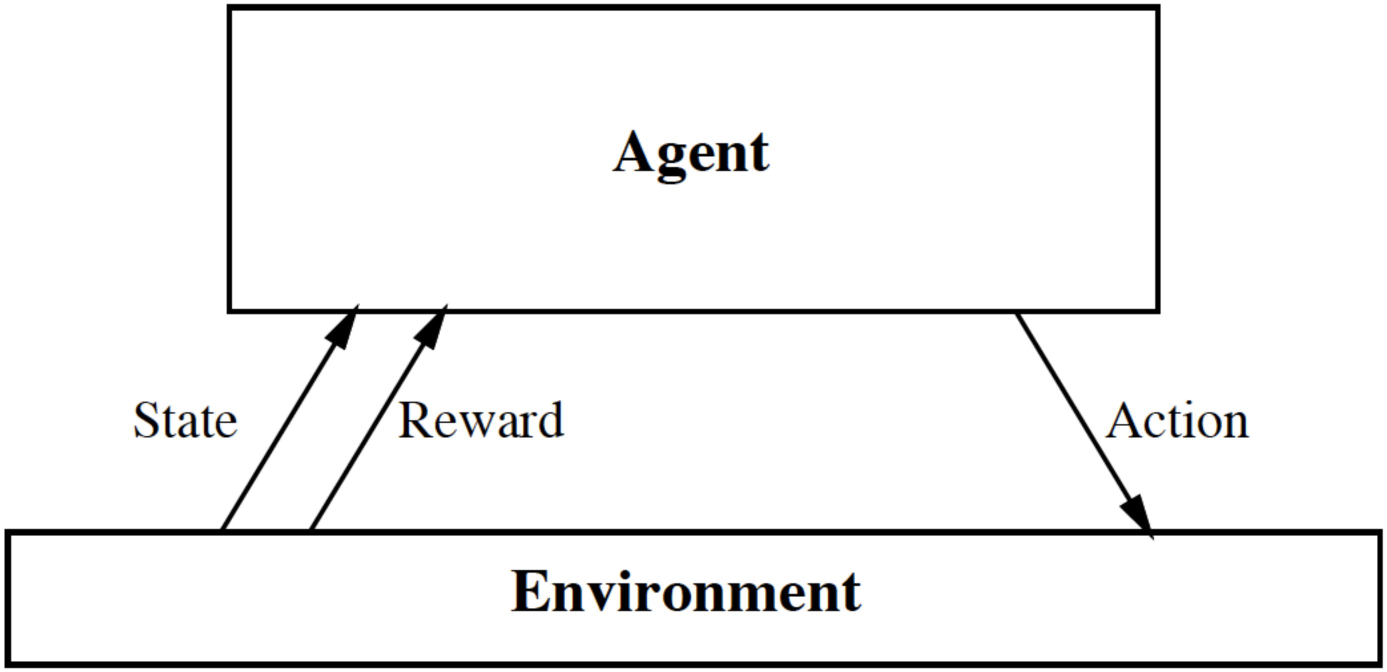
Overview

• RL deals with agents that must sense & act upon their environment.
•This combines classical agent-based AI and machine learning techniques.
It is a very comprehensive problem setting.

Overview

- Examples:
  - A robot cleaning my room and recharging its battery
  - Robot-soccer
  - How to invest in shares
  - Modeling the economy through rational agents
  - Learning how to fly a helicopter
  - Scheduling planes to their destinations
  - and so on

The Big Picture



$$s_0 \xrightarrow[r_0]{a_0} s_1 \xrightarrow[r_1]{a_1} s_2 \xrightarrow[r_2]{a_2} \ldots$$

Your action influences the state of the world which determines its reward

Complications

- The outcome of your actions may be uncertain

- You may not be able to perfectly sense the state of the world

- The reward may be stochastic.

- Reward is delayed (i.e. finding food in a maze)

Complications

- You may have no clue (model) of how rewards are being paid off.

- The world may change while you try to learn it

- How much time do you need to explore uncharted territory before you exploit what you have learned?

The Task

- To learn an optimal *policy* that maps states of the world to actions of the agent.
  I.e., if this patch of room is dirty, I clean it. If my battery is empty, I recharge it.

Action Space

$$\pi : S \rightarrow A$$

State of Space

- What is it that the agent tries to optimize?
  Answer: the total future discounted reward:

The Task

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- What is it that the agent tries to optimize?

Answer: the total future discounted reward:

$$V^{\pi}(s_t) = r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \ldots$$

$$= \sum_{i=0}^{\infty} \gamma^i r_{t+i} \qquad 0 \leq \gamma < 1$$

Note: immediate reward is worth more than future reward.

What would happen to mouse in a maze with gamma = 0 ?

Value Function

- Let's say we have access to the optimal value function that computes the total future discounted reward $V^*(s)$

- What would be the optimal policy $\pi^*(s)$?

- Answer: we choose the action that maximizes:

$$\pi^*(s) = \underset{a}{\operatorname{argmax}} \left[r(s,a) + \gamma V^*(\delta(s,a)) \right]$$

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Value Function

- We assume that we know what the reward will be if we perform action "*a*" in state "*s*":

$$r(s,a)$$

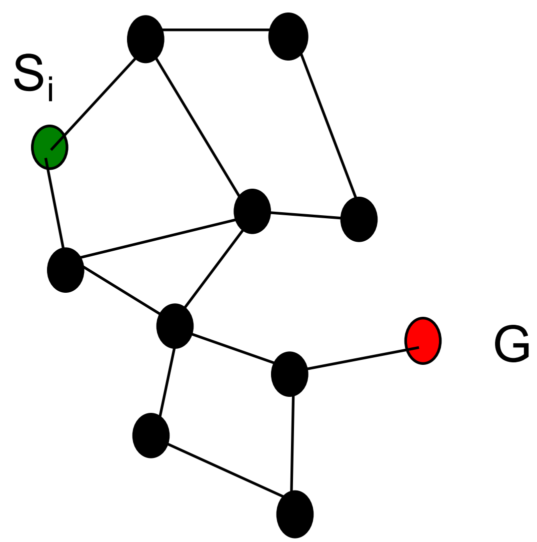
- We also assume we know what the next state of the world will be if we perform action "*a*" in state "*s*":

$$s_{t+1} = \delta(s_t, a)$$

Example I

- Consider some complicated graph, and we would like to find the shortest path from a node S_i to a goal node G.

- Traversing an edge will cost you “length edge” dollars.

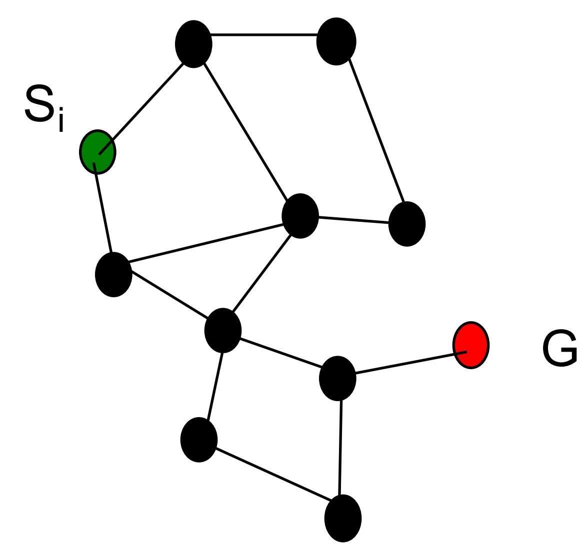


Example I

• The value function encodes the total remaining distance to the goal node from any node s, i.e.

$V(s)$ = "*1 / distance*" to goal from s.

• If you know $V(s)$, the problem is trivial. You simply choose the node that has highest $V(s)$.

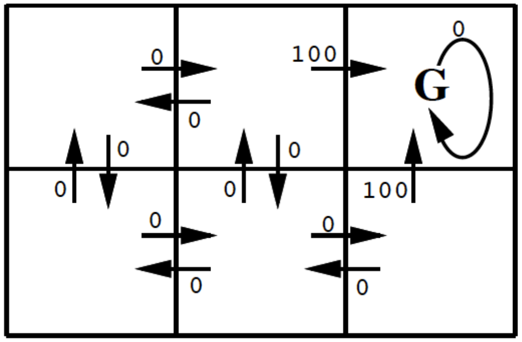


Example II

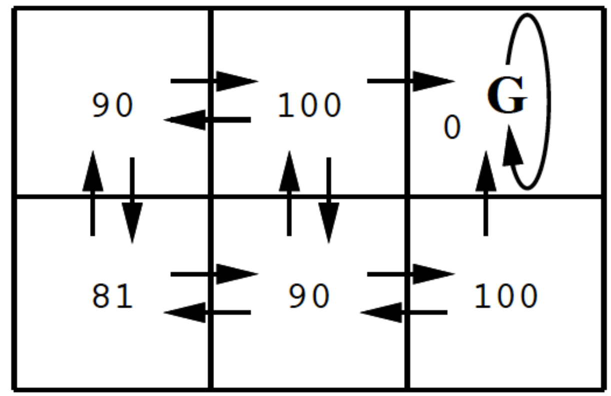
- A simple deterministic world.
- Each grid square represents a distinct state, each arrow a distinct action.
- The immediate reward function, r(s, a) gives reward 100 for actions entering the goal state G, and zero otherwise. Values of $V^*(s)$ follow from $r(s, a)$, and the discount factor $\gamma = 0.9$.
- An optimal policy is also shown.

Example II

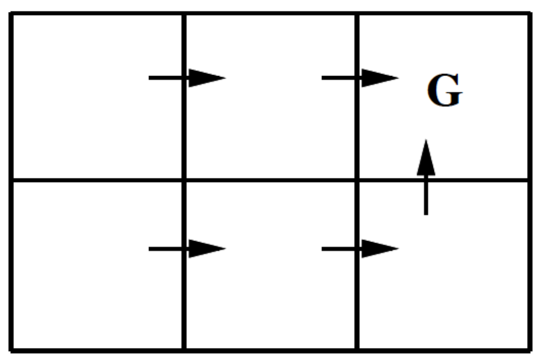
Find your way to the goal.



$r(s,a)$ (immediate reward) values



$V^*(s)$ values



One optimal policy

$$V^{\pi}(s_t) = r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \ldots$$

$$= \sum_{i=0}^{\infty} \gamma^i r_{t+i} \qquad 0 \le \gamma < 1$$

Q-Function

- One approach to RL is then to try to estimate $V^*(s)$.

Bellman Equation:

$$V^*(s) \leftarrow \max_a \left[r(s,a) + \gamma V^*(\delta(s,a)) \right]$$

- However, this approach requires you to know $r(s,a)$ and $\delta(s,a)$.

- This is unrealistic in many real problems. What is the reward if a robot is exploring mars and decides to take a right turn?

Q-Function

- Fortunately we can circumvent this problem by exploring and experiencing how the world reacts to our actions. We need to *learn* r & δ.

Q-Function

- We want a function that directly learns good state-action pairs, i.e. what action should I take in this state. We call this $Q(s,a)$.

Q-Function

- Let us define the evaluation function $Q(s, a)$ so that its value is the maximum discounted cumulative reward that can be achieved starting from state s and applying action a as the first action.
- In other words, the value of Q is the reward received immediately upon executing action a from state s, plus the value (discounted by γ) of following the optimal policy thereafter.

$$Q(s,a) \equiv r(s,a) + \gamma V^{*}(\delta(s,a))$$

Q-Function

- Why is this rewrite important? Because it shows that if the agent learns the Q function instead of the V^* function, it will be able to select optimal actions even when it has no knowledge of the functions r and δ.

Q-Function

- It need only consider each available action a in its current state s and choose the action that maximizes *Q*(*s*, *a*).

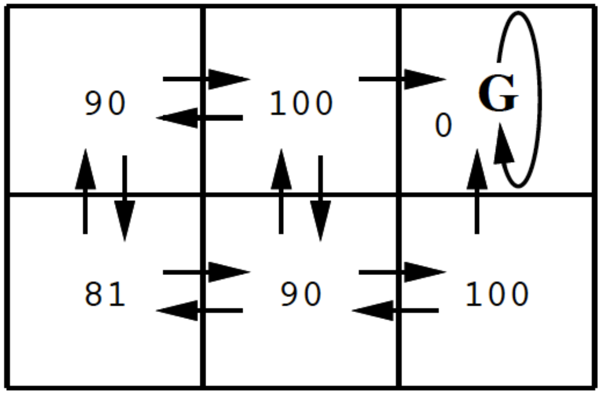
$$\pi^*(s) = \underset{a}{\operatorname{argmax}}\, Q(s,a)$$

$$V^*(s) = \max_{a} Q(s,a)$$

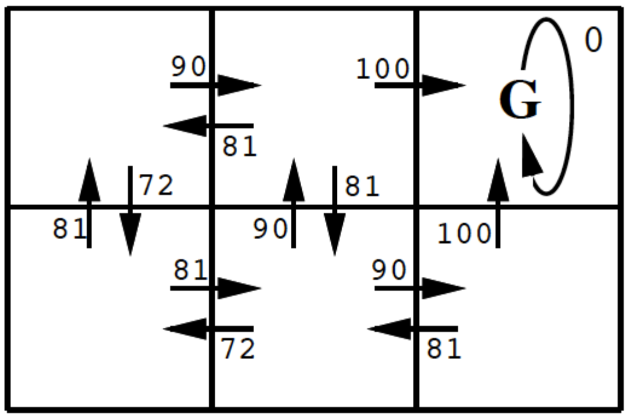
Example

- To illustrate, the figure in the next slide shows the Q values for every state and action in the simple grid world.
- The *Q* value for each state-action transition equals the *r* value for this transition plus the *V** value for the resulting state discounted by γ.
- The optimal policy shown in the figure corresponds to selecting actions with maximal *Q* values.

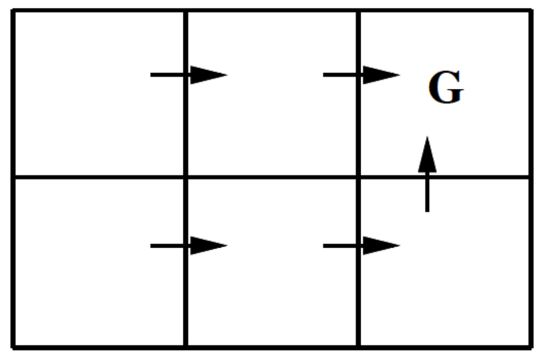
Example II



$V^*(s)$ values



$Q(s,a)$ values



One optimal policy

Check that

$$\pi^*(s) = \arg\max_a Q(s,a)$$

$$V^*(s) = \max_a Q(s,a)$$

Q-Learning

- Learning the *Q* function corresponds to learning the optimal policy. How can *Q* be learned?
- The key problem is finding a reliable way to estimate training values for *Q*, given only a sequence of immediate rewards *r* spread out over time.

Q-Learning

- This can be accomplished through iterative approximation.
- To see how, notice the close relationship between Q and V^*,

$$V^*(s) = \max_{a'} Q(s, a')$$

Q-Learning

which allows rewriting the *Q* function as:

$$Q(s,a) \equiv r(s,a) + \gamma V^*(\delta(s,a))$$
$$= r(s,a) + \gamma \max_{a'} Q(\delta(s,a),a')$$

This still depends on *r*(*s*,a) and *δ*(*s*,*a*); however,

Q-Learning

$$Q(s,a) \equiv r(s,a) + \gamma V^*(\delta(s,a))$$
$$= r(s,a) + \gamma \max_{a'} Q(\delta(s,a),a')$$

this recursive definition of Q provides the basis for algorithms that iteratively approximate Q.

$\hat{Q}$ refers to the learner's estimate, or hypothesis, of the actual Q function.

Q-Learning

$$Q(s,a) \equiv r(s,a) + \gamma V^*(\delta(s,a))$$
$$= r(s,a) + \gamma \max_{a'} Q(\delta(s,a),a')$$

- Imagine the robot is exploring its environment, trying new actions as it goes.
- At every step it receives some reward "*r*", and it observes the environment change into a new state *s'* for action *a*.
- How can we use these observations, (*s*,*a*,*s'*,*r*) to learn a model?

Q-Learning

- The learner represents its hypothesis $\hat{Q}$ by a large table with a separate entry for each state-action pair.
- The table entry for the pair (s, a) stores the value for $\hat{Q}(s,a)$, learner's current hypothesis about the actual but unknown value $Q(s,a)$.
- The table can be initially filled with random values (though it is easier to understand the algorithm if one assumes initial values of zero).

Q-Learning

- The agent repeatedly observes its current state s, chooses some action a, executes this action, then observes the resulting reward $r = r(s, a)$ and the new state $s' = \delta(s, a)$.

Q-Learning

- It then updates the table entry for $\hat{Q}(s,a)$ following each such transition, according to the rule:

$$\hat{Q}(s,a) \leftarrow r + \gamma \max_{a'} \hat{Q}(s',a')$$

$s' = s_{t+1}$

Q-Learning

$$\hat{Q}(s,a) \leftarrow r + \gamma \max_{a'} \hat{Q}(s',a') \qquad s' = s_{t+1}$$

- This equation continually estimates *Q* at state *s* consistent with an estimate of *Q* at state *s'*, one step in the future: temporal difference (TD) learning.

- Note that *s'* is closer to goal, and hence more "reliable", but still an estimate itself.

Q-Learning Summary

Q learning algorithm

For each s, a initialize the table entry $\hat{Q}(s,a)$ to zero.
Observe the current state s
Do forever:

- Select an action a and execute it
- Receive immediate reward r
- Observe the new state s'
- Update the table entry for $\hat{Q}(s,a)$ as follows:

$$\hat{Q}(s,a) \leftarrow r + \gamma \max_{a'} \hat{Q}(s',a')$$

- $s \leftarrow s'$

Q-Learning

$$\hat{Q}(s,a) \leftarrow r + \gamma \max_{a'} \hat{Q}(s',a')$$

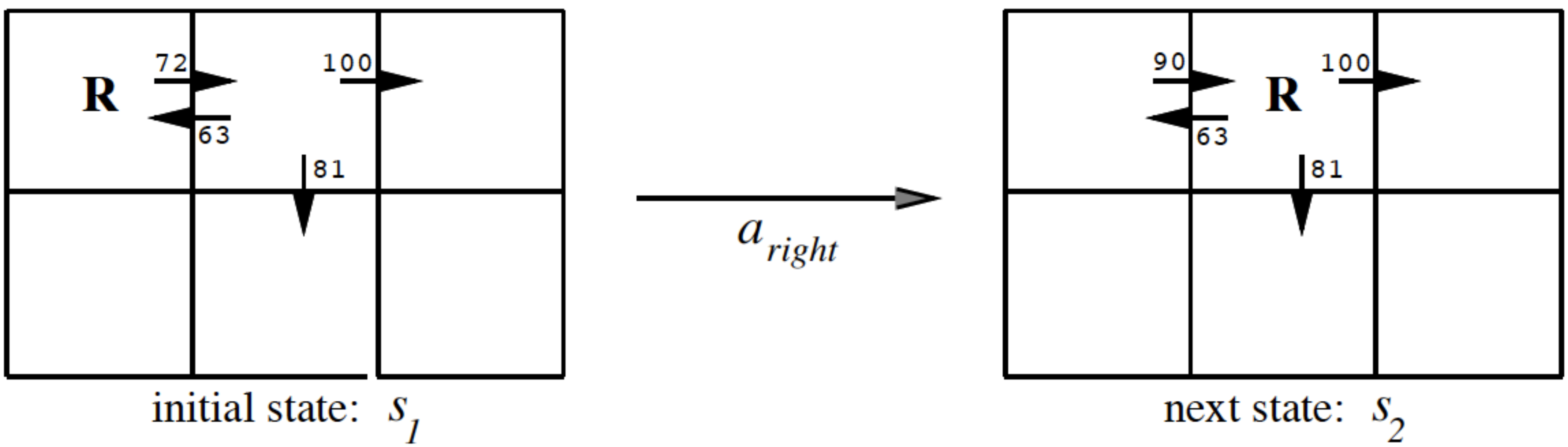
$s' = s_{t+1}$

- We do an update after each state-action pair. I.e., we are learning online!

- We are learning useful things about explored state-action pairs. These are typically most useful because they are likely to be encountered again.

- Under suitable conditions, these updates can actually be proved to converge to the real answer.

Example: Q-Learning



$$\hat{Q}(s_1, a_{right}) \leftarrow r + \gamma \max_{a'} \hat{Q}(s_2, a')$$

$$\leftarrow 0 + 0.9 \max\{66, 81, 100\}$$

$$\leftarrow 90$$

Q-learning propagates Q-estimates 1-step backwards