MTH101A: 2021 - 2022

End-Semester Exam: Question 2
Time: 9:35 am - 10.10 am

Q2. (a) Let $f : [a, b] \longrightarrow \mathbb{R}$ be a continuous function with $f(x) \geq 0$ for all $x \in [a, b]$. Show that

$$\lim_{n\to\infty} \left(\int_a^b f(x)^n dx \right)^{\frac{1}{n}} = \sup\{f(x) : x \in [a, b]\}.$$

[10]

(b) Let $f : (-1, 1) \to \mathbb{R}$ be a differentiable function and $f'(x)$ is continuous. If $f(0) = 0$ and $f'(x) \geq 2|x|$ for all $x \in (-1, 1)$ then show that $|f(x)| \geq x^2$ for all $x \in (-1, 1)$. [7]