MTH636A (2023-24, EVEN SEMESTER)

PROBLEM SET 4

1. Describe the following game with incomplete information as an extensive-form game. There are two players $N = \{\mathrm{I}, \mathrm{II}\}$. Each player has three types, $T_{\mathrm{I}} = \{\mathrm{I}_1, \mathrm{I}_2, \mathrm{I}_3\}$ and $T_{\mathrm{II}} = \{\mathrm{II}_1, \mathrm{II}_2, \mathrm{II}_3\}$, with common prior:

$$\rho(\mathrm{I}_k, \mathrm{II}_l) = \frac{k(k+l)}{78}, \ \ 1 \leq k, l \leq 3.$$

   The number of possible actions available to each type is given by the index of that type: the set of actions of Player I of type I_k contains k actions $\{1, 2, 3\}$; the set of actions of Player II of type II_l contains l actions $\{1, 2, 3\}$. When the type vector is $(\mathrm{I}_k, \mathrm{II}_l)$, and the vector of actions chosen is $(a_{\mathrm{I}}, a_{\mathrm{II}})$, the payoffs to the players are given by

$$u_{\mathrm{I}}(\mathrm{I}_k, \mathrm{II}_l, a_{\mathrm{I}}, a_{\mathrm{II}}) = (k+l)(a_{\mathrm{I}} - a_{\mathrm{II}}),$$

$$u_{\mathrm{II}}(\mathrm{I}_k, \mathrm{II}_l, a_{\mathrm{I}}, a_{\mathrm{II}}) = (k+l)a_{\mathrm{I}}a_{\mathrm{II}}.$$

   For each player, and each of his types, write down the conditional probability that the player ascribes to each of the types of the other player, given his own type.

2. Find a Bayesian equilibrium in the following game with incomplete information:

   - $N = \{\mathrm{I}, \mathrm{II}\}$,
   - $T_{\mathrm{I}} = \{\mathrm{I}_1, \mathrm{I}_2\}$ and $T_{\mathrm{II}} = \{\mathrm{II}_1\}$,
   - $p(\mathrm{I}_1, \mathrm{II}_1) = \frac{1}{3}$, $p(\mathrm{I}_2, \mathrm{II}_1) = \frac{2}{3}$,
   - Every player has two possible actions, and state games are given by the following matrices:

		Player II	
		L	R
Player I	T	$(2,10)$	$(0,3)$
	B	$(0,4)$	$(1,0)$

Table 1: State game for $t = (\mathrm{I}_1, \mathrm{II}_1)$

		Player II	
		L	R
Player I	T	$(0,3)$	$(3,1)$
	B	$(2,0)$	$(0,1)$

Table 2: State game for $t = (\mathrm{I}_2, \mathrm{II}_1)$

3. **Signaling games** This exercise illustrates that a college education serves as a form of signaling to potential employers, in addition to expanding the knowledge of students. A young person entering the job market may be talented or untalented. Suppose that one-quarter of high school graduates are talented, and the rest untalented. A recent high school graduate, who knows whether or not he is talented, has the option of spending a year traveling overseas or enrolling at college (we will assume that he or she cannot do both) before applying for a job. An employer seeking to fill a job opening cannot know whether or not a job applicant is talented; all he knows is that the applicant either went to college or traveled overseas. The payoff an employer gets from hiring a worker depends solely on the talents of the hired worker (and not on his educational level), while the payoff to the youth depends on what he chose to do after high school, on his talents (because talented students enjoy their studies at college more than untalented students), and on whether or not he gets a job. These payoffs are described in the following tables (where the employer is the row player and the youth is the column player, so that a payoff vector of (x, y) represents a payoff of x to the employer and y to the youth).

		Youth	
		Travel	Study
Employer	Hire	$(0,6)$	$(0,2)$
	Not hire	$(3,3)$	$(3,-3)$

Table 3: If youth is talented

   (a) Depict this situation as a Harsanyi game with incomplete information.

   (b) List the pure strategies of the two players.

		Youth	
		Travel	Study
Employer	Hire	$(8, 6)$	$(8, 4)$
	Not hire	$(3, 3)$	$(3, 1)$

Table 4: If youth is not talented

(c) Find two Bayesian equilibria in pure strategies.