**Quiz-I**
**MTH-204, MTH-204A**
**ABSTRACT ALGEBRA**
**Fall-2014**
**Date: 1st September 2014**

Time Allowed: 45 mins Max. Marks: 15

1. Suppose a group contains elements of orders 1 through 9. What is the minimum possible order of the group ? [2]

2. Suppose $|G| = 21$, and $G$ has precisely one subgroup of order 3, and one subgroup of order 7. Show that $G$ is cyclic ? [3]

3. Let $G = \langle a \rangle$ be the cyclic group generated by $a$ and order of $a$ is 24. How many left cosets of $\langle a^{10} \rangle$ in $G$ are there ? List all these cosets. [3]

4. Is $U_8$ isomorphic to $Z_4$ ? Justify your answer. [2]

5. Let $R^+$ be the multiplicative group of positive real numbers. Show that the map $x \mapsto \sqrt[3]{x}$ is an automorphism of $R^+$. [3]

6. Let $S_{15}$ be the group of permutations on 15 elements $\{1, 2, 3, \cdots, 15\}$. Prove that $S_{15}$ has elements of order 56 but does not have any elements of orders 49 or 50. [2]