

The formula, with $h = 0.3$, $x_0 = 1.6$, and $s = -\frac{1}{3}$, becomes

$$\begin{aligned} f(1.5) &\approx P_4 \left(1.6 + \left(-\frac{1}{3} \right) (0.3) \right) \\ &= 0.4554022 + \left(-\frac{1}{3} \right) \left(\frac{0.3}{2} \right) ((-0.5489460) + (-0.5786120)) \\ &\quad + \left(-\frac{1}{3} \right)^2 (0.3)^2 (-0.0494433) \\ &\quad + \frac{1}{2} \left(-\frac{1}{3} \right) \left(\left(-\frac{1}{3} \right)^2 - 1 \right) (0.3)^3 (0.0658784 + 0.0680685) \\ &\quad + \left(-\frac{1}{3} \right)^2 \left(\left(-\frac{1}{3} \right)^2 - 1 \right) (0.3)^4 (0.0018251) = 0.5118200. \end{aligned}$$

Most texts on numerical analysis written before the wide-spread use of computers have extensive treatments of divided-difference methods. If a more comprehensive treatment of this subject is needed, the book by Hildebrand [Hild] is a particularly good reference.

EXERCISE SET 3.3

1. Use Eq. (3.10) or Algorithm 3.2 to construct interpolating polynomials of degree one, two, and three for the following data. Approximate the specified value using each of the polynomials.
 - a. $f(8.4)$ if $f(8.1) = 16.94410$, $f(8.3) = 17.56492$, $f(8.6) = 18.50515$, $f(8.7) = 18.82091$
 - b. $f(0.9)$ if $f(0.6) = -0.17694460$, $f(0.7) = 0.01375227$, $f(0.8) = 0.22363362$, $f(1.0) = 0.65809197$
2. Use Eq. (3.10) or Algorithm 3.2 to construct interpolating polynomials of degree one, two, and three for the following data. Approximate the specified value using each of the polynomials.
 - a. $f(0.43)$ if $f(0) = 1$, $f(0.25) = 1.64872$, $f(0.5) = 2.71828$, $f(0.75) = 4.48169$
 - b. $f(0)$ if $f(-0.5) = 1.93750$, $f(-0.25) = 1.33203$, $f(0.25) = 0.800781$, $f(0.5) = 0.687500$
3. Use Newton the forward-difference formula to construct interpolating polynomials of degree one, two, and three for the following data. Approximate the specified value using each of the polynomials.
 - a. $f(-\frac{1}{3})$ if $f(-0.75) = -0.07181250$, $f(-0.5) = -0.02475000$, $f(-0.25) = 0.33493750$, $f(0) = 1.10100000$
 - b. $f(0.25)$ if $f(0.1) = -0.62049958$, $f(0.2) = -0.28398668$, $f(0.3) = 0.00660095$, $f(0.4) = 0.24842440$
4. Use the Newton forward-difference formula to construct interpolating polynomials of degree one, two, and three for the following data. Approximate the specified value using each of the polynomials.
 - a. $f(0.43)$ if $f(0) = 1$, $f(0.25) = 1.64872$, $f(0.5) = 2.71828$, $f(0.75) = 4.48169$
 - b. $f(0.18)$ if $f(0.1) = -0.29004986$, $f(0.2) = -0.56079734$, $f(0.3) = -0.81401972$, $f(0.4) = -1.0526302$
5. Use the Newton backward-difference formula to construct interpolating polynomials of degree one, two, and three for the following data. Approximate the specified value using each of the polynomials.
 - a. $f(-1/3)$ if $f(-0.75) = -0.07181250$, $f(-0.5) = -0.02475000$, $f(-0.25) = 0.33493750$, $f(0) = 1.10100000$
 - b. $f(0.25)$ if $f(0.1) = -0.62049958$, $f(0.2) = -0.28398668$, $f(0.3) = 0.00660095$, $f(0.4) = 0.24842440$