MTH101A: 2021 - 2022

Mid-Semester Exam: Question 2

Time: 10:30 am - 11:00 am

Q1. (a) Let $f : \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} \cos(\frac{1}{x}) \text{ if } x \neq 0 \\ 0 \text{ if } x = 0 \end{cases} .$$

Using ϵ-δ definition of continuity, show that the function $f(x)$ is not continuous at 0. Find a Cauchy sequence (x_n) such that $f(x_n)$ is not a Cauchy sequence. **[8 marks]**

(b) Let $f : (a, b) \to \mathbb{R}$ be an infinitely differentiable function, and let $x_0 \in (a, b)$. Suppose there exist $k \geq 1$ such that $f''(x_0) = f^{(3)}(x_0) = \cdots = f^{(2k)} = 0$. If $f^{(2k+1)}(x_0) \neq 0$ then show that x_0 is a point of inflection for f. **[7 marks]**