END-SEMESTER EXAMINATION
MTH-204, MTH-204A
ABSTRACT ALGEBRA
Fall-2014
Date: 24th November 2014

Time Allowed: 3 hrs   Max. Marks: 40

1. State PRECISELY the following theorems. [6]

a. Lagrange'e theorem.
b. Sylow's theorems.
c. Fundamental theorem of finite abelian groups.
d. First isomorphism theorem of Rings
e. Chinese reminder theorem.

2. Find all the subgroups of D_4 and determine which are normal. [4]

3. Give an example of a group G and a proper subgroup H such that $G = \bigcup_{g \in G} gHg^{-1}$. Justify. [4]

4. Prove that every finite group having more than two elements has a nontrivial automorphism. [5]

5. For each list of groups a, b and c below, decide which of the groups within each list are isomorphic, if any: [5]

a. $\mathbb{Z}_3 \times \mathbb{Z}_3 \times \mathbb{Z}_2 \times \mathbb{Z}_2$, $\mathbb{Z}_9 \times \mathbb{Z}_2 \times \mathbb{Z}_2$, $\mathbb{Z}_{18} \times \mathbb{Z}_2$ and $\mathbb{Z}_6 \times \mathbb{Z}_6$.

b. $S_4, A_4 \times \mathbb{Z}_2, D_{12}$ and $Q_8 \times \mathbb{Z}_3$.

c. $(\mathbb{Q}, +), (\mathbb{R}, +)$ and $(\mathbb{R}_+, \times)$, where $\mathbb{R}_+$ is the set of all positive real numbers.

6. Let G be a p-group, where p is a prime. Prove that $Z(G)$ has more than one element. [4]

7. Show that if a group G has a conjugacy class with two elements then G is not simple. [4]

8. Let R be a ring in which every element x satisfies $x^n = x$ for some n (depending on x). Show that every prime ideal in R is maximal. [4]

9. State and prove the prime avoidance theorem. [5]

10. Let R be a UFD. Prove that $x \in R$ is prime iff x is irreducible. [4]