CS648 : Randomized Algorithms
CSE, IIT Kanpur

Practice sheet
MOBD, Delay Sequences

1. **Power of Method of Bounded Difference**
   Recall that an instance of a random graph $G(n, p)$ for a given n and p is built as follows. For each pair $i, j \in V$, the edge (i, j) is added in the graph with probability p independent of other edges. Let X be the number of triangles in a random graph $G(n, 1/2)$.

   (a) What is expected value of X ?

   (b) What is variance of X ?

   (c) Use Chebyshev Inequality to derive a bound on $\mathbf{P}[|X - \mathbf{E}[X]| > 4n^2 \log n]$.

   (d) Use the method of bounded difference suitably to derive bound on $\mathbf{P}[|X - \mathbf{E}[X]| > 4n^2 \log n]$.

   (e) Draw inferences from (c) and (d).

2. **How well did you internalize the Delay Sequences ?**
   Suppose there is an undirected graph $G = (V, E)$ on n vertices where degree of each vertex is d. Each vertex hosts a counter and a coin. Each coin, when tossed, gives heads with probability p. Let $Count(v)$ denotes the value of the counter hosted at vertex v at any moment of time. $Count(v)$ is initialized to 0 in the beginning for each $v \in V$. In each round, each vertex $v \in V$ tosses its coin and increments its counter value if the following conditions holds true.

   - The outcome of coin tossed by the vertex v is heads.
   - There is no neighboring counter of v with counter value less than $Count(v) - b$.

   Assume that b and d are constants of integer values and $0 < p < 1$. Show that the all the counters will reach $\log n$ value with in $O(\log n)$ rounds with high probability.
   Note: The constant hidden in big-Oh notation of $O(\log n)$ may be a function of b, d, and p; it is perfectly fine.

3. **Alternate solution for a special case of Counter Problem**.
   Refer to the previous problem. If the graph is a complete graph on n vertices, provide an alternate analysis to show that each counter will reach value n in $O(n \log n)$ rounds with high probability. Can you show that this is also the lower bound asymptotically ?