MTH 424 - PARTIAL DIFFERENTIAL EQUSTION

IIT KANPUR

Instructor: Indranil Chowdhury

Odd Semester, 2024-25

Assignment 1

---

1. Find the order of the following PDEs and clasify them (linear, semilinear, quasiliner or fullynonlinear):
   (i) $u_t - u_{xxt} + uu_x = 0$.
   (ii) $u_x + e^y u_y = 0$.
   (iii) $\displaystyle\sum_{i,j=1}^{n} a_{i,j}(x)\frac{\partial^2 u}{\partial x_i \partial x_j}(x) + \sum_{i=1}^{n} b_i(x)\frac{\partial u}{\partial x_i}(x) + c(x)u(x) + d(x) = 0$.
   (iv) $u_t + \frac{1}{2}(u^2)_x = f(x)$.
   (v) $u_t + \frac{1}{2}(u_x)^2 = f(x)$.
   (vi) $u_t + \sup_{\alpha}\left\{f^\alpha(t,x) + b^\alpha(t,x)\cdot\nabla u(t,x) + \frac{1}{2}\mathrm{trace}[A^\alpha(x)D^2u](t,x)\right\} = 0$, where A^α are symetric matrices and D^2u is the Hessian matrix.
   (vii) $(\sin u_{xx})u_x + y^2 u_y = u_x u_y$.

2. (i) Solve $u_y = 0$ in $\mathbb{R}^2$ with the auxiliary condition $u(x,0) = x^2$.
   (ii) Check what will happen solving $u_y = 0$ in $\mathbb{R}^2$ with the auxiliary condition $u(0,y) = y^2$?
   (iii) For which auxiliary condition, the similar phenomenon to (ii) occur while solving $au_x + bu_y = 0$ in $\mathbb{R}^2$ where $a, b \neq 0$? Justify.

3. Use the method of characteristics to solve for $u(x,y)$ where
$$\begin{cases} xu_x + u_y = 0 \\ u(x,0) = \frac{1}{x^2+1}. \end{cases}$$
   Plot the 'profiles' (values of $u(x,y)$) for $y = 0, 1, 2, 3, 4$.

4. Consider the PDE $xu_x + yu_y = 2u$ in the region $x > 0, y > 0$. Determine the characteristic curve. Solve the PDE on $x^2 + y^2 > 1$ when $u(x,y) = 1$ on $x^2 + y^2 = 1$.

5. Consider the following PDEs, sketch the characteristic curves, the initial curve and solve
   (a) $(x+2)u_x + 2yu_y = au$ with $u(-1,y) = \sqrt{y}$.
   (b) $x^2 u_x - y^2 = 0$ with $u(1,y) = F(y)$.

6. Use the method of characteristics to solve for $u(x,y)$ where
$$\begin{cases} uu_x + u_y = 0 \\ u(x,0) = \frac{1}{x^2+1}. \end{cases}$$
   Plot the 'profiles' (values of $u(x,y)$) for $y = 0, 0.5, 1$.