MTH636A (2023-24, Even Semester)

Problem Set 1

1. A Nash equilibrium s^* is *strict* if every deviation undertaken by a player yields a definite loss for that player, i.e., $u_i(s^*) > u_i(s_i, s^*_{-i}))$ for each player $i \in N$ and each strategy $s_i \in S_i \setminus \{s_i^*\}$.

   (a) Prove that if the process of iterative elimination of strictly dominated strategies results in a unique strategy vector s^*, then s^* is a strict Nash equilibrium.

   (b) Find a game that has at least one equilibrium, but in which iterative elimination of dominated strategies yields a game with no equilibria.

2. In the following three-player game, Player 1 chooses a row (A or B), Player 2 chooses a column (a or b), and Player 3 chooses a matrix (α, β, or γ). Find all the equilibria of this game. (Ab@, Aby, Ba@, Bay)

α	a	b
A	$0,0,5$	$0,0,0$
B	$2,0,0$	$0,0,0$

β	a	b
A	$1,2,3$	$0,0,0$
B	$0,0,0$	$1,2,3$

γ	a	b
A	$0,0,0$	$0,0,0$
B	$0,5,0$	$0,0,4$

3. Find an example of a game $G = \langle N, \{S_i\}_{i\in N}, \{u_i\}_{i\in N}\rangle$ such that the game $\hat{G}$ derived from G by elimination of one strategy in one player's strategy set has an equilibrium that is not an equilibrium in the game G. in the above Q, remove Y(gamma strategy)

4. Find an example of a strategic form game G and of an equilibrium s^* of that game such that for each player $i \in N$ the strategy s_i^* is dominated.
   [(1,1) (1,1) (1,1)]
   [(1,1) (1,1) (0,0)]
   [(1,1) (0,0) (1,1)]

A two-person zero-sum game (with finite number of pure strategies) is called a *matrix game* as it can be represented in a matrix. Formally, consider a two-player zero-sum game

$\langle \{1,2\}, S_1, S_2, u_1, u_2 \rangle$ where $u_1(s_1, s_2) = -u_2(s_1, s_2)$. Suppose $S_1 = \{s_1^1, \ldots, s_1^k\}$ and $S_2 = \{s_2^1, \ldots, s_2^l\}$. This game can be expressed in a matrix $A = ((a_{ij}))$ with dimensions $k \times l$ where $u_1(s_1^i, s_2^j) = a_{ij}$.

5. (a) Let A be an arbitrary $m \times n$ matrix game. Show that any two saddle points of the matrix A must have the same value. In other words, if (i, j) and (k, l) are two saddle points, show that $a_{ij} = a_{kl}$.

   both will have same utility

   u(s*) = v <= saddle point <=> Nash Eqm <=> max-min strategy profile

   (b) Let A be a matrix game where both the players have 4 pure strategies. Assume that $(1, 1)$ and $(4, 4)$ are saddle points. Show that A has at least two other saddle points. Trivial.

   (c) Give an example of a matrix game where both the players have 4 pure strategies such that there are exactly three saddle points.

   [1 1 1 1]
   [3 2 2 2]
   [1 1 1 1]
   [1 1 1 1]

6. Consider the following matrix game:

$$\begin{array}{cc} \begin{bmatrix} 3 & 2 & 0 \\ 1 & 2 & 2 \\ 0 & 2 & 4 \\ 0 & 3 & 1 \end{bmatrix} & \begin{array}{c} 0 \\ 1 \\ 0 \\ 0 \end{array} \\ \begin{array}{ccc} 3 & 3 & 4 \end{array} & \end{array}$$

$\underline{v_1} = 1$ (maxmin)

$\overline{v_2} = 3$ (minmax)

   (a) Determine all the max-min (pure) strategies of both the players. What can you conclude about the value of the game in mixed strategies? (value wil lie b/w 1 and 3 ?!!) ?

   (b) The value of the game is $\frac{12}{7}$. Use this to give an argument why player 2 will put zero probability on column 2 in any maxmin strategy. [1/5 1/5 6 ?)

7. Let A and B be two finite-dimensional matrices with positive entries. Show that the game

A	0
0	B

has no value in pure strategies. (Each 0 here represents a matrix of the proper dimensions, such that all of its entries are 0.)

(Hint: See saddle point)

8. **(Symmetric games:)** ~~A matrix game A is called *symmetric* if~~ A is a anti symmetric matrix. Prove that the value of a symmetric game is zero and that the sets of max-min strategies of players 1 and 2 coincide.

9. (**Equalizer Theorem:**) Let v be the value in mixed strategies of a $m \times n$-matrix game A, and suppose that $u(\hat{\sigma}_1, s_2^n) = v$ for every max-min strategy $\hat{\sigma}_1$ of player 1. Show that player 2 has a max-min strategy σ_2 with $\sigma_2(s_2^n) > 0$.