MID-SEMESTER EXAMINATION
MTH-204, MTH-204A
ABSTRACT ALGEBRA
Fall-2014
Date: 14th September 2014

Time Allowed: 2 hrs Max. Marks: 30

1. Give complete and precise definitions for the following. [5]

a. Group b. Quotient Group c. Isomorphism of Groups d. Direct Product of Groups e. Group action

2. Give an example of each of the following. [4]

a. A non-abelian group of order 10.

b. A group for which elements of finite order don't form a subgroup.

c. A group for which any two non-trivial subgroups have non-trivial intersection.

d. A non-identity automorphism of $\mathbb{Z}$.

3. Show that there is a homomorphism of the symmetric group S_3 onto $\mathbb{Z}_2$, but that there is no homomorphism from S_3 onto $\mathbb{Z}_3$. [4]

4. Show that a group cannot be written as the set-theoretic union of two proper subgroups. [3]

5. Find the centralizer of $A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ in $GL_2(\mathbb{R})$. [4]

6. List all the elements of the alternating group A_4 in disjoint cycle notation. [3]

7. State and prove the 1st isomorphism theorem of groups. [5]

8. Let m, n be two integers coprime to each other. Show that $\mathbb{Z}_m \times \mathbb{Z}_n \cong Z_{mn}$ by giving an isomorphism between them. [5]