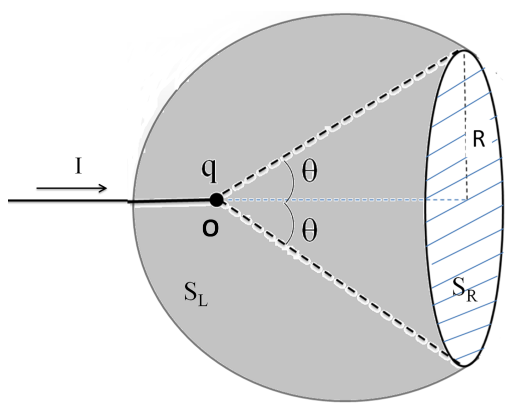
Problems $1-4$ will be discussed in the tutorial.

1. Two long coaxial cylindrical shells of radii a and b ($b > a$) are placed with their axis along the z-direction. A current I goes through the inner shell in the z-direction and returns through the outer shell, the current being uniformly distributed on the surface. If $I = I_0 \exp -t/\tau$, find the induced electric field everywhere.

2. Two large plates at $z = \pm\frac{d}{2}$ carry slow time-varying surface currents $K(t)\hat{x}$ and $-K(t)\hat{x}$, respectively. Find the magnetic field everywhere using quasi-static approximation. Find the induced electric field everywhere.

3. A parallel plate capacitor is made of two circular sheets of radius R with a separation $d \ll R$. The capacitor is getting charged at a very slow rate with charge $Q(t) = Q_0\{1 - \exp(-t/t_0)\}$
   (a) Plot the charge as a function of time.
   (b) Determine and plot the displacement current as a function of time
   (c) Determine the magnetic field in between the plates.
   (d) What is the origin of the magnetic field? When and where would the above analysis represent the true fields in between the plates?

4. A half-infinite straight wire carries current I from negative infinity to the origin **O** as shown by the solid straight line in the figure below. The termination of the wire leads to a build-up (increase) of charge q at the origin with time (so that $\frac{dq}{dt} = I$). Consider the circle shown in the figure below which has radius R and subtends an angle 2θ with respect to the charge. Compute the integral $\oint \vec{B}\cdot\vec{dl}$ around the circle for the two surfaces S_R and S_L as shown in the figure.

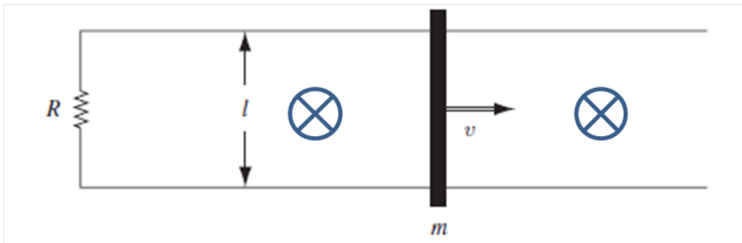


5. A conducting rod of mass m slides freely along a rail made of two parallel conducting wires (along $\hat{x}$) connected together by a resistance R. The two wires are separated along the y-direction by a distance l. The resistor R, the two parallel wires and the rod form a closed rectangular loop and the whole thing is immersed in a constant magnetic field $-B\hat{z}$ (pointing into the page, as shown in figure). The rod is given a initial velocity $v_0\hat{x}$ such that the area of the loop increases. Determine the following:

(a) Compute the electric current in the loop at any time t.

(b) Compute the rod's displacement $x(t)$ and velocity $v(t)$.

(c) Compute the power lost through Joule heating and plot it as a function of time. What is the total power dissipated?



6. A longitudinal alternating magnetic field $\vec{B}(t) = B_0 \cos \omega t\, \hat{z}$ is driven through the interior of an ohmic solid conducting tube (conductivity σ) with length L and radius $R \ll L$.

   (a) Find the low-frequency induced ('eddy') current density distribution $\vec{J}_{ind}(r', t)$ inside the tube, neglecting the effects of self-inductance. (Here r' is the radial distance from the center axis of the tube.)

   (b) Find the magnetic field $\vec{B}_{ind}$ produced by $\vec{J}_{ind}(r', t)$ inside the tube. (Hint: Note that the volume current in (a) is circumferential, similar to that in solenoids, and can be imagined as a collection of surface currents. Keep in mind that magnetic field is non-zero only inside the solenoid.)

   (c) Utilize (b) to find the correction to $\vec{J}_{ind}$ produced by self-induction, i.e., due to the time varying flux of $\vec{B}_{ind}$ inside the tube.

7. A metallic sphere of radius R is moving with a constant velocity $v_0\hat{x}$ in an uniform magnetic field $B_0\hat{z}$. Find

   (a) the electric field inside the sphere;

   (b) the induced charge density (surface or bulk) in the sphere;

   (c) the electric dipole moment of the charge distribution;

   (d) the potential difference between points $y = \pm R$ on the sphere.

8. A parallel plate capacitor with circular plates (radius R) is being charged by a constant current I_C as shown in the figure below. Assume that the electric field inside the capacitor is uniform and neglect fringing fields near the edges. The instantaneous total charge on the plates are $\pm Q(t)$ and the free space permittivity (permeability) is ϵ_0 (μ_0).

   (a) Compute the integral $\oint \vec{B} \cdot \vec{dl}$ and the corresponding magnetic field $\vec{B}$ for the closed loop C with radius $r < R$ for the two surfaces S_1 (shaded flat surface) and S_2 (unshaded surface which does not cut any capacitor plate) attached to C as shown in the figure, using the generalized form of Ampere's law (including the displacement current). Find the magnetic field. Show explicit calculations in both cases.

(b) Calculate the distribution of surface current density flowing in the capacitor plates by applying the same law on surface S_3 (unshaded surface which cuts the capacitor plate on the right) attached to C. [You may utilize the magnetic field value calculated in (b).]