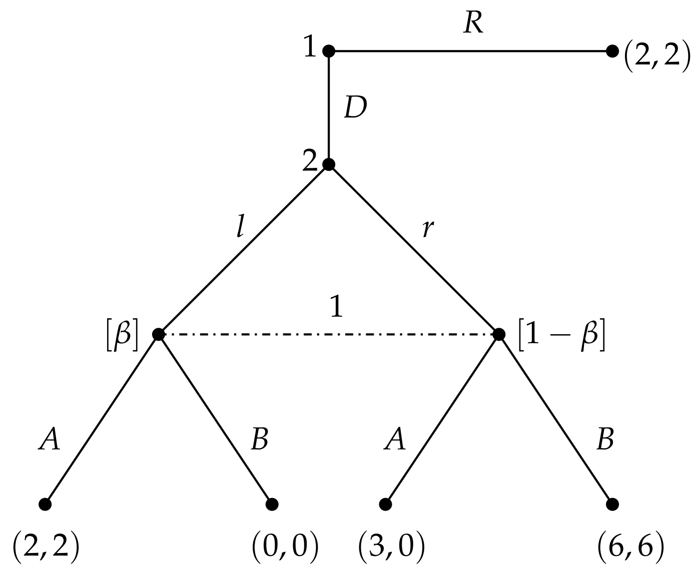
Question: Compute all Nash, subgame perfect, Perfect Bayesian, and sequential equilibria in the following game.



Answer: First consider the subgame starting from the information set of Player 2. The corresponding normal form of this game is the following

1 \ 2	l	r
A	$(2,2)$	$(3,0)$
B	$(0,0)$	$(6,6)$

The NEs of this game are (A,l), (B,r), and $([\frac{3}{4}\ (A), \frac{1}{4}\ (B)], [\frac{3}{5}\ (l), \frac{2}{5}\ (r)])$. Firstly, If (A,l) is played in this subgame, Player 1 will be indifferent between playing R and D in his first information set. So, we get the following SPNEs, $([\alpha\ (R), (1-\alpha)\ (D)]A, l)$ where $\alpha \in [0,1]$. Secondly, if (B,r) is played in this subgame, Player 1 will prefer playing D over R. So, we have (DB, r) as an SPNE. Finally, if $([\frac{3}{4}\ (A), \frac{1}{4}\ (B)], [\frac{3}{5}\ (l), \frac{2}{5}\ (r)])$ is played in this subgame, Player 1 will prefer playing D over R. Thus, we have the following SPNE $(D\ [\frac{3}{4}\ (A), \frac{1}{4}\ (B)], [\frac{3}{5}\ (l), \frac{2}{5}\ (r)])$.

Coming to perfect Bayesian equilibrium if $\beta > \frac{3}{5}$, $b_1(A) = 1$ and if $\beta < \frac{3}{5}$, $b_1(A) = 0$, and at $\beta = \frac{3}{5}$, $b_1(A) \in [0,1]$. For Player 2, if Player 1 plays A, optimal strategy is to play l, and if Player 1 plays B, optimal strategy is to play r. Note that $\beta = b_2(l)$. Thus, $\beta = \frac{3}{5}$ implies that Player 2 is indifferent between l and r, which in turn implies $2b_1(A) = 6b_1(B)$.

Now, for Player 1 in his first information set, if Player 2 chooses l and Player 1 chooses A, will be indifferent over R and D. This gives us one perfect Bayesian equilibrium,

$$b_1(R) \in [0,1], b_2(l) = 1, b_1(A) = 1, \text{ and } \beta = 1. \tag{1}$$

Similarly, if Player 2 chooses r and Player 1 chooses B, Player 1 will select $b_1(D) = 1$. Thus, there is another perfect Bayesian equilibrium,

$$b_1(R) = 0, b_2(l) = 0, b_1(B) = 1, \text{ and } \beta = 0. \tag{2}$$

Lastly, if $b_2(l) = \frac{3}{5}$ and $b_1(A) = \frac{3}{4}$ (as $2b_1(A) = 6b_1(B)$), Player 1 will select $b_1(D) = 1$ as the utility by playing D is

$$\frac{3}{5} * \frac{3}{4} * 2 + \frac{2}{5} * \frac{3}{4} * 3 + \frac{2}{5} * \frac{1}{4} * 6 = \frac{48}{20} > 2.$$

This gives us the third perfect Bayesian equilibrium,

$$b_1(D) = 1, b_2(l) = \frac{3}{5}, b_1(A) = \frac{3}{4}, \text{ and } \beta = \frac{3}{5}. \tag{3}$$

For sequential equilibria, first consider the perfect Bayesian equilibria in (1). If $b_1(R) \in (0,1)$, consider the sequence of Bayesian consistent assessments $(b^m, \beta^m)_{m \in \mathbb{N}}$ for where $b_1^m(R) = b_1(R)$, $b_2^m(l) = 1 - \frac{1}{m}$, $b_1^m(A) = 1 - \frac{1}{m}$, and $\beta^m = 1 - \frac{1}{m}$. If $b_1(R) = 1$, consider $b_1^m(R) = 1 - \frac{1}{m}$ and $b_2^m(l)$ and $b_1^m(A)$ are same as before. Finally, if $b_1(R) = 0$, take $b_1^m(R) = \frac{1}{m}$ and $b_2^m(l)$ and $b_1^m(A)$ are same as before. Note that $\lim_{m \to \infty}(b^m, \beta^m) = (b, \beta)$ and $(b^m, \beta^m)_{m \in \mathbb{N}}$ is Bayesian Consistent for all $m \in \mathbb{N}$. Thus, all perfect Bayesian equilibria in (1) are sequential equilibria. Similarly, try to construct a sequence of Bayesian consistent assessments for the equilibria in (2) and (3).