MTH101A: 2021 - 2022

Mid-Semester Exam: Question 4
Time: 11:30 am - 12:00 pm

Q1. (a) Define a function f on $[0,1]$ by

$$f(x) = \begin{cases} x^2 \text{ if } x \text{ is rational} \\ x^3 \text{ if } x \text{ is irrational.} \end{cases}$$

Is f Riemann integrable on $[0,1]$? If yes, then find $\int_0^1 f(x)dx$. (You may use the formula: $\sum_{k=0}^{n} k^3 = (\frac{n(n+1)}{2})^2$). **[8 marks]**

(b) Evaluate $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{k}{n^2+k^2}$ using definite integrals.

[7 marks]