CS648A: Randomized Algorithms
Last worked out problem

Randomized algorithm are **not** designed using random ideas

(**Random bit-fixing protocol does NOT work**)

Recall the bit fixing protocol we discussed for permutation routing on n-hypercube. We showed that this protocol will take $\Omega(2^n/n)$ rounds for the transpose permutation : $ab \rightarrow ba$, where a and b are any two binary strings of length $n/2$ each. Now suppose some one suggests the following randomized algorithm based on any arbitrary random thought: Instead of fixing the bits from left to right, a packet fixes its bits in a uniformly random order independent of other packets. Unfortunately, this randomized protocol will be useless: It can be shown that it will also take $2^{\Omega(n)}/n$ rounds for the transpose permutation. We provide a sketch of it as follows.

We shall bound the packets that pass through 0^n. For this we focus on all those packets that originate from address $a0^{n/2}$ where a is a binary string of length $n/2$.

1. Let P be the set of packets with source address $a0^{n/2}$ for any binary string a of length $n/2$. What is $|P|$?

   Answer:

2. Let P_k be the subset of P consisting of those packets whose source address has exactly k 1's. What is $|P_k|$?

   Answer:

3. Consider any packet $p \in P_k$. State the necessary and sufficient condition for the packet p to pass though 0^n while executing the randomized protocol described above.

   Answer:

4. What is the probability that a packet $p \in P_k$ passes though 0^n ?

   Answer:

5. What is the expected number of packets from the set P that will pass though 0^n ?

   Answer:

6. Show that the expected number of packets passing through 0^n will be $2^{\Omega(n)}$.

   Answer: