Department of Humanities and Social Sciences
Indian Institute of Technology Kanpur

Answer all questions. Write your answers in a separate booklet.
Max. Marks: 40 Time: 3 hours

> The common knowledge assumption underlies all of game theory and much of economic theory. Whatever be the model under discussion, whether complete or incomplete information, consistent or inconsistent, repeated or one shot, cooperative or non cooperative, the model itself must be assumed common knowledge; otherwise the model is insufficiently specified and the analysis incoherent.
> ..... Robert Aumann

1. Conditional Logic: $\{C, C+, C1, C2, S\}$ systems (Graham Priest: Conditional Logic, pp)
2. Many-valued Logic(K3(strong, weak),L3,B3(external,internal),LP, RM3), Sorites Paradox

1. **Answer whether the following statements are True or False. Justify it preferably with a short note while making your rough work reflected in the answer sheet**. (15)

   1. In any three valued logic, we say that a set of formulas (Γ) entails a formula P, whenever all of the formulas in Γ are true, P is true as well. It means, there is no truth value assignment on which all the formulas in Γ have the value T, while P has the value F or N. So If $\Gamma \models_{K_3^s} P$, then $\Gamma \models P$.
   2. The well formed formula $[p \rightarrow (q \rightarrow p)] \rightarrow p \rightarrow q$ is a tautology in three valued logic (LP).
   3. $p > q \models_C \neg q > \neg p$ is invalid in minimal conditional logic C
   4. When interpreted in three valued Luckasewicz logic (L_3), the truth value of the following modal logic formula $\Box P \wedge \Diamond(p \vee \Diamond \neg p)$ is $1/2$, when p takes the value $1/2$.
   5. Necessity of a proposition is determined in terms of conditional as follows: $\Box A =_{\text{Definition}} = \neg(A > \neg A)$, where $\Box$ represents necessity, $>$ represents conditional connective.

6. In Stalnaker's conditional logic (c2), the following assumption is dropped. *For every world w and proposition A of a conditional $A > B$ there is at most one A world minimally different from w.*False
7. The following argument is valid in Stalnaker's conditional logic (C2):*If this match were struck, it would light. If this match were soaked in water overnight and this match were struck, it would light.*
8. The well formed formula $p \rightarrow (q \rightarrow p)$ is not a tautology in Kleene's three valued Logic (K_3).
9. If there is a convention among a group that ϕ, then everyone knows ϕ , everyone knows that everyone knows ϕ, everyone knows that everyone knows that everyone knows ϕ, and so on ad- infinitum. In such a case, we say that the group has distributive knowledge of ϕ.
10. The following argument is valid in the conditional logic system C. *If I were to kick the door, my foot would hurt. If my foot were to hurt, I would be sad. So, if I were to kick the door, I would be sad.*
11. In Stalnaker's conditional logic (C2), the following conditional is accepted to be true. *If $E = m * c^3$, then there would be more energy in the universe.*
12. The conditional sentence, *If The Indian army general in the year 2070 will be a female then the Indian army general in the year is a female* is evaluated to be true in the Luckasewicz three valued logical system.
13. We will say that a formula is a *quasi tautology* if it is never false. Based on this definition, $A \wedge \neg A \rightarrow B$ is a quasi tautology in Bochvar's internal three valued logical system (B_3^I).

2. Answer the following questions in 50-100 words. Each question carries 2 marks (10)

   1. Analyse the following sentences in any one of three valued logic LPS, L_3)
      (a) This sentence is false.
      (b) Thiggledy piggledy and grass is green.

3. An important property of Luckasewicz three valued semantics is that some classically inconsistent formulas are no more contradictory in $L3$. One of such formulas:$p \leftrightarrow \neg p$, is connected with the Russell paradox since the equivalence $Z \in Z \equiv Z \in \neg Z$. Why Russell's paradox ceases to be paradox in the three valued logic?

[20] Answer the following questions. Each question carries 4 marks

1. Using truth tables of three valued logic, determine which of the following formulas are tautologies and valid in L3 (note that they are all classical tautologies). Also, discuss what happens when the number of designated values are taken to be 1, i?

   (a) $\neg P \rightarrow (P \rightarrow Q)$

(b) $P \rightarrow (Q \rightarrow \neg Q)$

(c) $\forall_{LP}\ p, p \rightarrow\ q \not\forall\ q$

2. Show that the following wffs are valid or invalid in $C+$, S [conditions (a-b)], but hold in S(a-e)[1].

(a) $p > q, \neg(p > \neg r) \models (p \wedge r) > q$

(b) $(p > q) \vee (p > \neg q)$

3. Show that the following wffs are valid or invalid in C1, C2

(a) $P \wedge Q \vdash P > Q$

(b) $(\phi > \psi \wedge \gamma) \rightarrow (\phi > \psi) \wedge (\phi > \gamma)$

[1]

(a) $[A]$ is a set of worlds in which A is true. $f_A(w) = \{x \in W : wR_Ax\}$ and $f_A(w) \subseteq [A]$.

(b) If $w \in [A]$, then $w \in f_A(w)$.

(c) If $[A] \neq \emptyset$, then $f_A(w) \neq \emptyset$.

(d) If $f_A(w) \subseteq [B]$ and $f_B(w) \subseteq [A]$ then $f_A(w) = f_B(w)$

(e) If $f_A(w) \bigcap [B] \neq \emptyset$, then $f_{A \bigcap B}(w) \subseteq f_A(w)$.

(f) **C2:** If $x \in f_A(w)$ and $y \in f_A(w)$, then $x = y$, where f_A is a singleton.

(g) **Lewis (VC):** (a-e)+ **P**: If $[A] = [B]$, then $f_A(w) = f_B(w)$