MTH 101-Calculus

Spring-2021

Assignment-9:Functions of several variables (Continuity and Differentiability

1. Identify the points, if any, where the following functions fail to be continuous:

$$(i)\ \ f(x,y)=\begin{cases} xy & \text{if } xy\geq 0\\ -xy & \text{if } xy<0\end{cases}\qquad (ii)\ \ f(x,y)=\begin{cases} xy & \text{if } xy \text{ is rationnal}\\ -xy & \text{if } xy \text{ is irrational.}\end{cases}$$

2. Consider the function $f:\mathbb{R}^2\longrightarrow\mathbb{R}$ defined by

$$f(x,y)=\begin{cases} \frac{x^2y^2}{x^2y^2+(x-y)^2} & \text{if } (x,y)\neq(0,0)\\ 0 & \text{if}(x,y)=(0,0)\end{cases}$$

   Show that the function satisfy the following:

   (a) The iterated limits $\lim\limits_{x\longrightarrow 0}\left[\lim\limits_{y\longrightarrow 0} f(x,y)\right]$ and $\lim\limits_{y\longrightarrow 0}\left[\lim\limits_{x\longrightarrow 0} f(x,y)\right]$ exist and equals 0;

   (b) $\lim\limits_{(x,y)\longrightarrow(0,0)} f(x,y)$ does not exist;

   (c) $f(x,y)$ is not continuous at $(0,0)$;

   (d) the partial derivatives exist at $(0,0)$.

3. Let $f(x,y)=(x^2+y^2)\sin\frac{1}{x^2+y^2}$ if $(x,y)\neq(0,0)$ and 0, otherwise. Show that f is differentiable at every point of $\mathbb{R}^2$ but the partial derivatives are not continuous at $(0,0)$.

4. Let $f(x,y)=|xy|$ for all $(x,y)\in\mathbb{R}^2$. Show that

   (a) f is differentiable at $(0,0.)$

   (b) $f_x(0,y_0)$ does not exist if $y_0\neq 0$.

5. Suppose f is a function with $f_x(x,y)=f_y(x,y)=0$ for all (x,y). Then show that $f(x,y)=c$, a constant.