Problem Set-7
MTH-204, 204A
Abstract Algebra

1. Describe a Sylow p-subgroup of $GL_n(\mathbb{Z}_p)$.

2. Prove that a group of order 72 is not simple.

3. Prove that a group of order 255 is cyclic.

4. A group of order 30 has a normal subgroup of order 5.

5. Determine all the possible abelian groups, up to isomorphism, of order 40500.

6. Find all the composition series for $S_3 \times \mathbb{Z}_2$.

7. Let G be a group and N be a normal subgroup of G. If G/N and N both have composition series, show that G has a composition series.

8. Let G be a nilpotent group and H be a proper subgroup of G. Then show that $H \neq N_G(H)$.

9. Show that a finite group is nilpotent if and only if it is the direct product of its Sylow subgroups.

10. Let G be a finite nilpotent group. Then show that for every positive divisor m of $|G|$, G has a subgroup of order m.

11. Show that the Dihedral group D_n is solvable for all n and it is nilpotent if and only if $n = 2^k$ for some k.