**MTH101A: 2021 - 2022**

# End-Semester Exam: Question 6
# Time: 11.55 am - 12.30 pm

**Q6.** (a) Determine the convergence/ divergence of the improper integral

$$\int_1^{10} \frac{t^3 \sin t}{e^t \ln t} dt.$$

[6]

(b) Consider the region bounded by the curves $y = \sqrt{16 - x^2}$, $y = \sqrt{9 - x^2}$ and $y = 0$. Using Pappus theorem, find the volume of the solid obtained by revolving the above region around the line $y = -2$. [5]

(c) Prove that

$$\left|\sin\left(||X||^2\right) - \sin\left(||Y||^2\right)\right| \leq 2||X - Y||,$$

for all $X, Y \in \mathbb{R}^2$ such that $||X||, ||Y|| \leq 1$.

[6]