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where $\left(I + ckD_h^2\right)$ *is a symmetric operator on* $L(I_h)$*. Thus,*

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From stability, we obtain the convergence result in the same way as earlier. Let $e_n^j = u_n^j - u(nh, jk)$, *we have*

$$\frac{e_n^{j+1} - e_n^j}{k} = c\,\frac{e_{n+1}^j - 2e_n^j + e_{n-1}^j}{h^2} - \ell_n^j, \qquad 0 < n < N, j = 0,1,\dots,M-1,$$

$$e_0^j = e_N^j = 0, \qquad j = 0,1,\dots,M-1,$$

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The stability result then yields

$$\|e^j\|_h \le \left(\max_m |1 - ck\lambda_m|\right)^j \|e^0\|_h + Mk \max_j \|\ell^j\|_h \le T \max_j \|\ell^j\|_h .$$