# WHAT IS PHILOSOPHICAL LOGIC?

--- July 31, 2023

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# PROVERBS

1. He who knows not knows not that he knows not: he is a fool - shun him.
2. He who knows not and knows he knows not: he is simple - teach him.
3. He who knows and knows not he knows: he is asleep - wake him.
4. He who knows and knows he knows: he is wise - follow him....Arabian Proverb.

**Lao Tzu**

To attain knowledge add things every day.
To attain wisdom delete things every day.
......Lao Tzu (604 BC - 531 BC) Chinese Taoist Philosopher

# PROVERBS

1. (S1)He who knows not and knows not that (s)he knows not, is a fool ... shun him/her $\neg K_a p \wedge \neg K_a \neg K_a p$.
2. (S2)He who knows not and knows that (s)he knows not, is ignorant ... teach him/her $\neg K_a p \wedge K_a \neg K_a p$.
3. (S3)He who knows and knows not that (s)he knows, is asleep ... wake him/her $K_a p \wedge \neg K_a \neg K_a p$.
4. (S4)He who knows and knows that (s)he knows, is a wise (wo)man ... follow him/her$K_a p \wedge K_a K_a p$.

# LOFTI ZADEH

`Classical logic is like a person who comes to a party dressed in a black suit, a white, starched shirt, a black tie, shiny shoes, and so forth.`
`And Fuzzy Logic is like a person dressed informally, in jeans, tee shirt, and sneakers. In the past, this informal dress won't have been acceptable. Today, it's the other way.`
Lotfi A. Zadeh Communications of the Association for Computing Machinery(ACM), Volume 27, 1984

# WHAT IS LOGIC?

- Logic is the study of right reason or valid inferences, and attending fallacies, formal, informal.
- Logic is a way to think so that we can come to correct conclusions by understanding implications and mistakes people often make in thinking.
- Logic is the science of valid processes of reasoning. In Mathematical Logic, we investigate these processes by mathematical methods.

# WHAT IS PHILOSOPHICAL LOGIC?

## Philosophical Logic

1. Wikipedia: Philosophical logic refers to those areas of philosophy in which recognized methods of logic have traditionally been used to solve or advance the discussion of philosophical problems (Meaning, truth, Identity, Paradoxes).
2. philosophical logic means both (a) the philosophical investigation of the basic notions of logic and (b) the deployment of logic to help with philosophical problems (truth, meaning, possibility, paradox).
3. Alternative logic, Deviant Logic, Non-Classical Logic.
4. Philosophical logic as understood here is the part of logic dealing with what classical logic leaves out, or allegedly gets wrong.
5. Extensions, deviations of First order logic (Propositional and predicate logic).

# WHAT IS PHILOSOPHICAL LOGIC?

- Philosophical logic is philosophy that is logic and logic that is philosophy. It is where logic and philosophy come together become one.
- Logic is the theory of consequence relations and valid inferences.
- A part of Logic dealing with what classical logic leaves out.
- Non-classical Logic, Non-standard logics, Deviant Logics
- Limitations of classical Logic: Explanation of conditionals.
- Classical Logic (First order Logic) is created for the purposes of mathematical reasoning,
- Philosophical logic develops formal systems and structures to be applied to the analysis of concepts and arguments that are central to philosophical inquiry.

# PHILOSOPHICAL CONCEPTS AND VARIOUS LOGICS

- Necessity and possibility: Modal Logic
- Knowledge and Belief: Epistemic Logic
- Obligation: Deontic Logic
- Time: Temporal Logic
- Reasoning: Non monotonic Logic and Probabilistic Logic.
- Many other Logics (Handbook of Philosophical Logic)

### Extensions and Alternatives

- Intutionistic Logic: Particular perspectives on nature and judgment of truth.
- Many valued logic: Logics that avoid conclusions of fatalism and determinism.
- Vagueness: Fuzzy logic
- Philosophical concerns on logic itself: Relevant logic (critique of classical consequence relation).

# SYLLABUS

- Basic concepts: Propositional and Predicate logic.
- Modal Logic, Epistemic Logic
- Application: Conditionals (counterfactuals)
- Many valued and Fuzzy Logic

# SYLLABUS

1. Brief overview of first order logic, Problem of Logical Consequence,
2. Extensions of First order Logic: Basic concepts of Normal Modal Logic, Epistemic Logic, Problem of Counterfactuals.
3. Deviant Logics: Many- valued Logic-1: Three valued Logics, Many Valued Logic-2: Many valued logic and Degrees of truth, Basic concepts of Fuzzy Logic.

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# EVALUATION

Midsemester exam: 30%
End Semester Exam 40%
Term Paper 20%
Attendance: 10%

# SOME EXAMPLES OF NON-CLASSICAL LOGIC

## Examples of Non-Classical Logic

1. Modal logic extends classical logic with non-truth-functional (modal) operators.
2. Paraconsistent logic (e.g., dialetheism and relevance logic) rejects the law of noncontradiction;
3. Relevance logic, linear logic, and non-monotonic logic reject monotonicity of entailment.
4. Fuzzy logic rejects the law of the excluded middle and allows as a truth value any real number between 0 and 1. Intuitionistic logic rejects the law of the excluded middle, double negative elimination, and the De Morgan's laws.
5. Others:Temporal Logic, Quantum Logic etc.

## Classical Logic

1. Logic: A systematic study of argumentation, principles of valid reasoning.
2. Classical Logic (First order Logic: CL): Propositional and Predicate Logic: Good starting point of study of reasoning.
3. Most appropriate for mathematical reasoning, it is bivalent (only two truth values), based on material implication.
4. CL is not appropriate for formalizing human reasoning.

# CONTD...

1. Classical logic fails to provide satisfactory account of the following: conditionals, arguments involving possibility, necessity, logic of knowledge and belief, vagueness etc.
2. Non-classical logics are developed to overcome several defects of clasical logic.
3. Classical logics obeys transitivity, property of bivalence, and monotonic. But common sense reasoning is non-monotonic.

# TWO REASONS FOR DOING NON-CLASSICAL LOGIC

1. Classical logic is of no help to represent intensional concepts like modality and time. But, these notions are pervasive in common sense reasoning.
2. Human knowledge may be incomplete, and inconsistent. Classical Logic cannot express incomplete and inconsistent information.
3. It Fails to explain vagueness, which is part and parceal of our life.

# NON-CLASSICAL LOGIC:

## Complementary Logics

1. Modal Logics: Tesnse Logic, Epistemic Logic, doxastic logic, Deontic logic, Dynamic Logic, Conditional Logic, Intensional Logic
2. Modal Logic: Normal and non normal Modal Logic

## Deviant Logic

Intutionistic logic, para-consistent Logic, Many-valued Logic (Fuzzy Logic)

# BASIC PRINCIPLES OF LOGIC

1. Law of Identity: P is P
2. Law of Excluded Middle, Principle of Bivalence: $P \vee \neg P$.
3. Law of non- contradiction. $\neg(P \wedge \neg P)$. P cannot be both P and $\neg P$ at the same time and the same sense.
   A contradiction occurs when one statement excludes the possibility of another and yet both are claimed to be true. Truth is not self-contradictory
4. Three laws: Foundation for mathematical, physical, and rational thinking

Are all these laws complete?

A variety of arguments can easily be produced to show that these laws are incomplete; i.e., they do not specify all reality, for parts of reality can be shown to contradict one or more of Aristotle's laws.

# HIRACLITUS(500BC): THE PROBLEM OF CHANGE

Heraclitus pointed out that, for a thing to change, it must turn into something else, and then asked how a thing could be something other than itself?
You cannot step in to the same river twice.

1. if Aristotle's laws are taken to be all the fundamental laws of logic, then logically there can be no change whatsoever, because change negates all three laws. I.e., either change does not exist or it is totally illogical.
2. Since all measurements, detections, thoughts, and perceptions are simply changes, then it follows that these operations logically cannot exist.

http://www.cheniere.org/books/aids/appendixIII.htm

# RESOLUTION OF MOTION PARADOX

1. Aristotle's three laws must specify or apply to only that which is not changing, since change violates or negates all three laws.
2. If change is to logically exist, there must exist at least a fourth law of logic, one which applies to change;
3. This fourth law must contain the negations of each of the first three laws, since change negates them;
4. To be consistent, in any particular logical case, either the three laws explicitly apply or the fourth law explicitly applies (i.e., either change explicitly exists in that particular case or it does not)
5. Since all four laws must apply at all times, when the fourth law applies explicitly, the three laws must be implicit.

# Examples where classical Logic fails:

# MODAL SENTENCES

## Examples

1. If it is necessarily the case that 2 is the smallest prime number, then 2 is the smallest prime number.
2. It is known that Ravi is richer than Ramesh, then Ravi is richer than Ramesh.
3. If it is morally obligatory that you love your neighbour, then you love love your neighbour. Flase.

# PARADOX OF MATERIAL IMPLICATION

The Paradoxes of Material Implication concern some logical consequences(entailments) which are valid according to the principles of propositional logic but which contradict our universal linguistic intuitions.

# INSTANCES OF PARADOX OF MATERIAL IMPLICATION:

1. $p \vdash q \rightarrow p$.
2. $\neg q \models p \rightarrow q$
3. $p \rightarrow s \models (p \wedge q) \rightarrow s$
4. $(p \wedge \neg p) \rightarrow q$
5. $\models p \rightarrow (q \vee q)$
6. $\models p \rightarrow (q \rightarrow p)$
7. $p \wedge q \models p \rightarrow q$
8. $\models (p \rightarrow q) \vee (q \rightarrow p)$

# EXAMPLES:

Example 1

$\neg$p: There is no oil in my coffee.
q: I like it.

Example 2

1. p: I will play football tomorrow.
2. q: I break my leg today.

# PARADOX OF MATERIAL IMPLICATION

$P \to Q =_{\text{Def}} \neg P \vee Q$

1. $P \models Q \to P$: I am alive to if I am dead, I am alive
2. $P \models \neg P \to Q$. I'm alive to if I'm not alive, I'm famous.
3. $(P \wedge Q) \to R \models (P \to R) \wedge (Q \to R)$.

# IRRELEVANCE: BAD TASTE IN REASONING

1. if the earth is flat, then today is Thursday.
2. If today is Thursday, then 2+2=4.
3. Therefore If the earth is flat, then 2+2 = 4.

1. $(p \wedge \neg p) \rightarrow q$.
2. $(p \wedge \neg p) \rightarrow (q \vee \neg q)$.
3. All are classically true, but

# ARISTOTLE'S SEA BATTLE ARGUMENT

A general is contemplating whether or not to give an order to attack. The general reasons as follows:

1. If I give the order to attack, then, necessarily, there will be a sea battle tomorrow
2. If not, then, necessarily, there will not be one.
3. Now, I give the order or I do not.
4. Hence, either it is necessary that there is a sea battle tomorrow or it is necessary that none occurs.

The conclusion is that either it is inevitable that there is a sea battle tomorrow or it is inevitable that there is no battle. So, why should the general bother giving the order?

# FUTURE CONTINGENT SENTENCES

1. There will be a sea-battle tomorrow
2. There will not be a sea-battle tomorrow

According to the Law of the Excluded Middle, it seems that exactly one of these must be true and the other false.

But if (1) is now true, then there must be a sea-battle tomorrow, and there cannot fail to be a sea-battle tomorrow. The result, according to this puzzle, is that nothing is possible except what actually happens: there are no unactualized possibilities.

If it was true 10,000 years ago that there would be a sea-battle tomorrow, then its truth was a fact about the past; if the past is now unchangeable, then so is the truth value of that past utterance.

Thus it is necessarily true that there will be a sea-battle tomorrow.

# SEA BATTLE ARGUMENT

# SORITES PARADOX

Consider a heap of grains of sand. Take away one solitary grain from the heap--you still have a heap, for one grain of sand is not enough to make the transition from heap to non-heap. So, if a pile of 10,000 grains of sand makes a heap (call this statement h10,000) then a pile of 9,999 grains also makes a heap.

1. A heap of sand is comprised of a large collection of grains. (Premise 1)
2. A heap of sand minus one grain is still a heap. (Premise 2)

The premises seem true.
The argument seems valid.
The conclusion seems false

# SORITES PARADOX

The paradox is tricky for philosophers because they must explain why one of the two premises, or the conclusion, is wrong even though they appear to be self evident.

# POSSIBLE SOLUTIONS

1. Deny that the problem is legitimately set up. That is, hold that logic does not apply to vague expressions.
2. Accept that logic does legitimately apply here but hold that this particular argument is invalid.
3. Accept both that logic applies in such cases and that the argument is valid, but deny one of the premises.
4. Accept the argument and the premises, and hence embrace the conclusion also.

# LIARS PARADOX

The liar paradox is an ancient conundrum of logic. It was originally cast in the form of a fable.

In ancient times, all the inhabitants of Crete were incapable of making a true statement. Epimenides, who lived in Crete, made the following statement: All Cretans are liars. Is Epimenides lying?

## Essence of Liar's Paradox

This sentence is False

Any attempt to assign a truth value to the statement in (1) leads to a vicious cycle of contradiction (sometimes called a vicious circle. If the statement is true, then it is false. But if it is false, then it must be true. Thus we have a vicious cycle of contradictory statements

# SELF REFERENTIAL STATEMENTS

1. Epimenides: All Cretans are liars.
2. This sentence is false.
   Is the sentence true or false?
3. Card Paradox:
   3.1 The sentence on the other side of the card is true
   3.2 the sentence of the other side of card is false.

# THE LIARS PARADOX

Let S be the statement S is false.Other way of saying is: This statement is false.

1. S is the statement s is false (from Definition).
2. Exactly one of the following is true:
   2.1 S is true
   2.2 S is false.
3. If S is true, then S is false (result of 1).
4. S is not true (2,3)
5. If S is false then S is true
6. S is not false. (2,5 MP)

Together with 4,6 we can say that S is neither true nor false.

# RUSSELL'S PARADOX

Consider set of all those sets which are not members of itself.

$r = \{x \colon x \notin x\}$

1. Case1: If r is a member of itself, then it is one of the sets that is not not a member of itself, so r is not a member of itself.
2. Case 2: If r is not a member of itself, then it is one of the sets in r, and hence it is a member of itself.
3. $r \in r \, \text{iff} \, r \notin r$

# FREGE'S LETTER TO RUSSELL:

A scientist can hardly meet with anything more undesirable than to have the foundation give way just as the work is finished. I was put in this position by a letter from Mr. Bertrand Russell when the work was nearly through the press.

# BARBERS PARADOX:

Suppose there is a town with just one male barber; and that every man in the town keeps himself clean-shaven: some by shaving themselves, some by attending the barber. It seems reasonable to imagine that the barber obeys the following rule: He shaves all and only those men in town who do not shave themselves.
Does the barber shave himself?

1. If the barber does not shave himself, he must abide by the rule and shave himself.
2. If he does shave himself, according to the rule he will not shave himself.

# OMNIPOTENCE PARADOX

1. An omnipotent being can do absolutley anything.
2. So can an omnipotent being create a stone that is too heavy for him to lift?
3. If he can create it, then there is one thing he can't do: lift the stone. If he can't create it, then there is one thing he can't do: create such a stone.
4. Either way, there is something he can't do, which contradicts the assumption that he is omnipotent. This is a paradox.
5. It has been taken by some to show that there cannot be an omnipotent being, and by others to show merely that the concept of omnipotence is misunderstood (It does not involve the power to bring about logical impossibilities).

# OMNIPOTENCE PARADOX

1. If God can make a rock He cannot lift, then God is not omnipotent.
2. If God cannot make a rock He cannot lift, then God is not omnipotent.
3. Either God can make a rock He cannot lift, or God cannot make a rock He cannot lift.
4. Therefore, 4, God is not omnipotent. (From 1,2,3. Constructive Dilemma).

# Appendix

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# DEDUCTION AND INDUCTION

Example 3 Deduction

1. All men are mortal. Socrates is a man. Therefore, Socrates is a mortal.
2. The book is on the table. The table is on the floor. Therefore, the book is above the floor.

Example 4 Induction

1. It has rained everyday on January 1st in Kanpur for the past several years. Therefore, it will rain next year on that day as well
2. Every crow I have observed is black in color. Therefore, all crows are black.

# EXAMPLE

There was a robbery in which a lot of goods were stolen. The robber (s) left in a truck. It is known that :

1. Nobody else could have been involved other than A, B and C.
2. C never commits a crime without A's participation.
3. B does not know how to drive.

Is A innocent or guilty?

# EXAMPLE

There was a robbery in which a lot of goods were stolen. The robber (s) left in a truck. It is known that :

1. Nobody else could have been involved other than A, B and C. $(A \vee B \vee C)$.
2. C never commits a crime without A' s participation. $(C \rightarrow A)$
3. B does not know how to drive $(B \rightarrow [(B \wedge A) \vee (B \wedge C)]$.

Is A innocent or guilty?

A is Guilty

# KNIGHTS AND KNAVES

Suppose, A and B say the following:

1. A: All of us are knaves.
2. B: Exactly one of us is a knave.

Can it be determined what B is? Can it be determined what C is?

# LADY OR TIGER:

There is a lady or Tiger; both Tigers, or Both Ladies.
One of the sign boards is true and the other is false.

1. Room A: In this Room there is a Lady, and in the other room there is a Tiger
2. Room B: In one of these rooms there is a Lady and in one of these room there is a tiger.

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