Department of Mathematics and Statistics,
Indian Institute of Technology Kanpur
MTH 101A, 2021-2022, I Semester

Instructors:

Dr. Ajay Singh Thakur, asthakur@iitk.ac.in
Dr. Debasis Sen, debasis@iitk.ac.in
Dr. Santosha Kumar Pattanayak, santosha@iitk.ac.in
Dr. Prosenjit Roy, prosenjit@iitk.ac.in

Tutors:

Section A1: Dr. Shailesh Trivedi, shailtr@iitk.ac.in
Section A2: Dr. Ramesh Prasad Panda, rppanda@iitk.ac.in
Section A3: Dr. Purbita Jana, purbita@iitk.ac.in
Section B1: Dr. Satyajit Guin, sguin@iitk.ac.in
Section B2: Dr. Ashis Mandal, amandal@iitk.ac.in
Section B3: Dr. Sasmita Patnaik, sasmita@iitk.ac.in
Section C1: Dr. Choiti Bandyopadhyay, choitib@iitk.ac.in
Section C2: Dr. Ravitheja Vangala, rvangala@iitk.ac.in
Section C3: Dr. Kapil Kant, kapilk@iitk.ac.in
Section D1: Dr. Suraj Singh Khurana, 2907suraj@gmail.com
Section D2: Dr. Rama Rawat, rrawat@iitk.ac.in
Section D3: Dr. Abhijit Pal, abhipal@iitk.ac.in

Text-book: Thomas' Calculus, (12th edition) by George B. Thomas Jr., Maurice D. Weir & Joel R. Hass.

Reference-book: Introduction to Real Analysis (4th edition), by Robert G. Bartle & Donald R. Sherbert.

Course web site : https://sites.google.com/view/mth101-2021/home

Videos of Lectures: Videos of the pre-recorded lectures will be uploaded periodically on the Hello IITK platform at https://hello.iitk.ac.in/course/mth101a2122. Students can login to the Hello IITK portal using their IITK login id and password to view these videos.

Course materials: The course plan and all the necessary and relevant information will be available on the course website. Further, the lecture notes of each of the lectures, the assignments and a collection of practice problems with hints/solutions are also available on the website. Some proofs in the lecture notes and some problems in the practice problems are marked (*). These have been provided to increase your understanding of the subject. However, these proofs will not be asked in the exams and quizzes.

Tutorial and discussion hours: Each student will be allotted a tutorial section. Each tutorial section will be assigned a tutor. There will be a tutorial hour and a discussion hour every week. The tutor will conduct the tutorial and the discussion hour through Zoom. The tutors will provide the zoom meeting details. The students can discuss the assignment problems in the tutorial hour with the tutors. The practice problems available on the course website are usually not discussed in the tutorial classes; however, the students can discuss these problems with the tutors during discussion hours. The students can clear any doubt on the material covered in the lectures with

the tutors during the discussion hour. Students can also use the Hello IITK forum to discuss their questions/doubts with fellow students.

Course Evaluation: There will be two quizzes, one Mid-Semester exam and one End-Semester Exam. Each quiz will be worth 20 marks, the mid-semester exam will be for 60 marks, and the End-semester Examination will be for 100 marks. Grading will be relative. Tentative dates for the exams are as follow:

Quiz 1: 24th December, 2021
Mid-Sem Exam: January 10-15, 2022
Quiz 2: 18th February, 2022
End-Sem Exam: March 7-12, 2022.

Mode of conduction of Exams and Quizzes: The exams and the quizzes will be conducted online on the Gradescope platform at `https://www.gradescope.com/`. Students will be able to access the question papers and upload their answers sheets on the Gradescope Platform. **No email submission will be accepted.**

Make-up exam: There will be no makeup exam for Quizzes and Mid-Sem. Makeup exam for the End-Sem exam is as per the institute policy.

Course Plan:

Here the numbers (10.1) and others refer to the section of the text book Thomas' Calculus.

Lecture 1: Real number system: Completeness property, density of rationals (irrationals) in $\mathbb{R}$.

Lecture 2: (10.1) Convergence of a sequence, Sandwich theorem, Monotone sequences.

Lecture 3: Cauchy criterion, Subsequence, Every bounded sequence has a convergent subsequence, convergence of a sequence satisfying Cauchy criterion.

Lecture 4: (2.1-2.5) Limits and Continuity of functions, Boundedness of a continuous function on [a,b].

Lecture 5: (2.5,3.2) Existence of maximum of a continuous function on [a,b], Intermediate value property, Differentiability.

Lecture 6: (4.1-4.3) Necessary condition for local maxima, Rolle's theorem and Mean value theorem.

Lecture 7, 8, 9: (4.3,4.4) Cauchy mean value theorem, L'Hospital rule, Increasing and decreasing functions, Convexity, Second derivative test for max and min, Point of inflection, curve sketching.

Lecture 10 (4.4,10.9) Curve sketching (contd.), Taylor's theorem with remainder.

Lecture 11: (10.2) Convergence of series, Geometric and Harmonic series, Absolute convergence.

Lecture 12: (10.4) Comparison test, Cauchy condensation test : $\sum a_n$ conv. $\Leftrightarrow \sum 2^k a_{2^k}$ conv. for $a_n \geq 0$ and $a_{n+1} \leq a_n$. Examples: $\sum \frac{1}{n^p}, \sum \frac{1}{n(logn)^p}$.

Lecture 13: (10.5,10.6) Ratio test, Root test, Examples, Leibniz's theorem.

Lecture 14: (10.7-10.9) Power series, Radius of convergence, Taylor series, Maclaurin series.

Lecture 15: (5.3) Introduction to Riemann integration, Integrability.

Lecture 16: (5.3) The integral existence theorem for continuous functions and monotone functions, Elementary properties of integral.

Lecture 17: (5.4) Fundamental Theorems of calculus, Riemann Sum.

Lecture 18: (8.7) Improper integral of first & second kind, Comparison test, Absolute convergence.

Lecture 19: (5.6,11.3) Applications of definite integral: Area between two curves.

Lecture 20: (11.5,6.1) Polar coordinates, Graphs of polar coordinates, Area between two curves when their equations are given in polar coordinates, Volumes by slicing.

Lecture 21: (6.2,6.3) Volumes by Shell and Washer methods, Length of a curve.

Lecture 22: (6.4,6.6) Area of surface of revolution, Pappus's Theorem.

Lecture 23: (12.1-12.6) Review of vector algebra, Equations of lines and planes.

Lecture 24: (13.1,13.3) Continuity and Differentiability of vector functions, Arc length for space curves, Unit tangent vector.

Lecture 25: (13.4) Unit normal and Curvature to plane and space curves, Binormal.

Lecture 26: (14.1-14.3) Functions of several variables, Continuity, Partial derivatives, differentiability.

Lecture 27: (14.4) Differentiability $\Rightarrow$ Continuity, Increment theorem, Chain rule.

Lecture 28: (14.5,14.6) Gradient, Directional derivatives, Tangent plane and Normal line.

Lecture 29: (14.3,14.9, Appendix 9) Mixed derivative theorem, Mean value theorem (MVT), Extended MVT, Hessian.

Lecture 30: (14.7) Necessary and sufficient conditions for Maxima, Minima and Saddle point.

Lecture 31: (14.8) The method of Lagrange multipliers.

Lecture 32: (15.1,15.2) Double integral, Fubini's theorem, Volumes and Areas.

Lecture 33: (15.3,15.4,15.8) Change of variable in a double integral, special case: Polar coordinates, Triple integral, Applications.

Lecture 34: (15.5,15.8,16.5) Change of variables in a triple integral, Special cases : Cylindrical and Spherical coordinates, Surface area.

Lecture 35: (16.5,16.1,16.3) Surface area (contd.), Surface integrals, Line integrals.

Lecture 36: (16.4) Green's Theorem.

Lecture 37: (16.2) Vector fields, Divergence and Curl of a vector field.

Lecture 38: (16.7) Stokes' Theorem.

Lecture 39: (16.8) The divergence theorem.