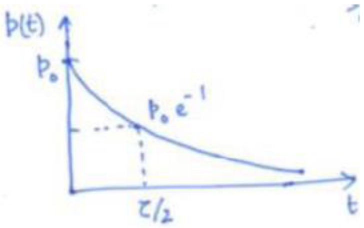
5) a) $I(t) = \frac{Blv(t)}{R}$, flowing counterclockwise as seen from +z direction

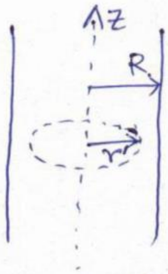
b) $x(t) = v_0\tau(1 - e^{-\frac{t}{\tau}})$

c) $P = \frac{(Blv_0)^2}{R} e^{-2t/\tau}$



6)

a)



$\vec{B}(t) = B_0 \cos\omega t \, \hat{z}$

$\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt} \Rightarrow E.\, 2\pi r' = -\pi r'^2 \dot{B}$

$\Rightarrow \vec{E} = \frac{1}{2} B_0 \omega r' \sin\omega t \, \hat{\varphi}$

$\Rightarrow \vec{J} = \sigma\vec{E} = \frac{1}{2}\sigma B_0 \omega r' \sin\omega t \, \hat{\varphi}$

b) The induced current will also produce a magnetic field.

Magnetic field inside a loop of radius r is the magnetic field due to solenoidal induced current outside the loop.

Each solenoid carries surface current $\vec{K}(\vec{r}') = \vec{J}\, dr'$

$\Rightarrow d\vec{B} = \mu_0 |\vec{K}| \, \hat{z}$

Total magnetic field ~~through the loop of~~ at radius $r (< R)$

$\vec{B}_{ind} = \hat{z}\int_r^R \mu_0 |\vec{J}|\, dr' = \hat{z}\int_r^R \mu_0 \frac{1}{2}\sigma B_0 \omega r' \sin\omega t \, dr'$

$= \frac{1}{2}\mu_0 \sigma B_0 \omega \left(\frac{R^2 - r^2}{2}\right) \sin\omega t \, \hat{z}$

Total flux through the loop of radius r

$\Phi(r) = \int_0^r 2\pi r' dr' \, B_{ind}(r') = 2\pi \int_0^r \frac{1}{2}\mu_0 \sigma B_0 \omega \sin\omega t \left(\frac{R^2 - r'^2}{2}\right) r' dr'$

c) $E_{cor} \cdot 2\pi r = -\frac{d\Phi(r)}{dt} \Rightarrow \vec{E}_{cor} = -\frac{\mu_0 \sigma B_0 \omega^2}{8} r\left(R^2 - \frac{r^2}{2}\right)\cos\omega t \, \hat{\varphi}$

$\vec{J}_{cor} = \sigma \vec{E}_{cor} = -\mu_0 B_0 \frac{\sigma^2 \omega^2}{8} \cos\omega t \; r\left(R^2 - \frac{r^2}{2}\right)\hat{\varphi}$

7) a) $\vec{E} = v_0 B_0 \hat{y}$

b) $\sigma = \sigma_0 \cos\theta$, where $\sigma_0 = -3\epsilon_0 v_0 B_0$

c) $\vec{p} = -4\pi\epsilon_0 R^3 v_0 B_0 \hat{y}$

d) potential difference $2v_0 B_0 R$

8) a) $\vec{E}(t) = \frac{\sigma(t)}{\epsilon_0} = \frac{Q(t)}{\epsilon_0 \pi R^2}$

For S_1, $B \cdot 2\pi r = \mu_0 \epsilon_0 \frac{I_c}{\epsilon_0 \pi R^2} \pi r^2 \quad \left(\because \frac{dE}{dt} = \frac{1}{\epsilon_0 \pi R^2}\frac{dQ}{dt} = \frac{I_c}{\epsilon_0 \pi R^2}\right)$

$\Rightarrow |\vec{B}| = \frac{\mu_0 I_c r}{2\pi R^2}$

For S_2, $B \cdot 2\pi r = -\mu_0 I_c + \mu_0 \epsilon_0 \frac{I_c}{\epsilon_0 \pi R^2} \pi (R^2 - r^2)$

$\Rightarrow |\vec{B}| = \frac{\mu_0 I_c r}{2\pi R^2}$

b Here $\vec{E} \perp$ to area vector $\Rightarrow$ no contribution due to displacement current.

$B \cdot 2\pi r = \mu_0 I_c + \mu_0 I_p \quad I_p \rightarrow$ current enclosed by S_3 on the plate

$\Rightarrow \frac{\mu_0 I_c r}{2\pi R^2} \cdot 2\pi r = \mu_0 (I_c + I_p) \Rightarrow I_p = -I_c\left(1 - \frac{r^2}{R^2}\right)$

$I_p = k_p(r) \cdot 2\pi r \Rightarrow \boxed{k_p(r) = \frac{-I_c}{2\pi r}\left(1 - \frac{r^2}{R^2}\right)}$