MTH 101-Calculus

Spring-2021

Assignment-11: Double and Triple Integrals

1. Evaluate the following integrals:

   i) $\int_0^1 \int_0^{\sqrt{1-x^2}} \sqrt{1-y^2}\,dydx$ *ii)* $\iint_{0\,x}^{\pi\,\pi} \frac{\sin y}{y}\,dydx$ *iii)* $\iint_{0\,y}^{1\,1} x^2 \exp^{xy}\,dxdy.$

2. Evaluate $\iint_R xdxdy$ where R is the region $1 \leq x(1-y) \leq 2$ and $1 \leq xy \leq 2$.

3. Using double integral, find the area enclosed by the curve $r = sin3\theta$ given in polar cordinates.

4. Compute $\lim_{a \longrightarrow \infty} \int\int_{D(a)} \exp^{-(x^2+y^2)}\,dxdy$, where

   i) $D(a) = \{(x,y) : x^2 + y^2 \leq a^2\}$ and *ii)* $D(a) = \{(x,y) : 0 \leq x \leq a,\ 0 \leq y \leq a\}$.

   Hence prove that (i) $\int_0^\infty e^{-x^2}dx = \frac{\sqrt{\pi}}{2}$ (ii) $\int_0^\infty x^2 e^{-x^2}dx = \frac{\sqrt{\pi}}{4}$.

5. Find the volume of the solid which is common to the cylinder $x^2 + y^2 = 1$ and $x^2 + z^2 = 1$.

6. Evaluate the integral $\iiint_W \frac{dzdydx}{\sqrt{1+x^2+y^2+z^2}}$; where W is the ball $x^2 + y^2 + z^2 \leq 1$.