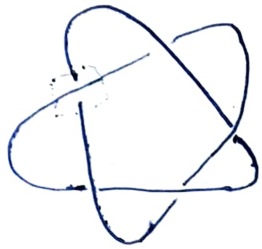
Assignment I - 210429 - Havi Bohra

1) Unknotting no. of 5_1



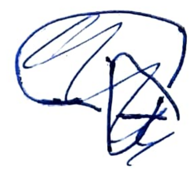
5_1

→ 1st unknotting

↓ 2nd unknotting

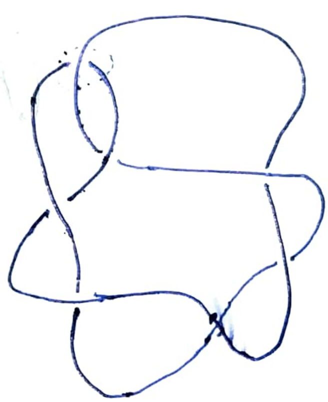
O_1

≡

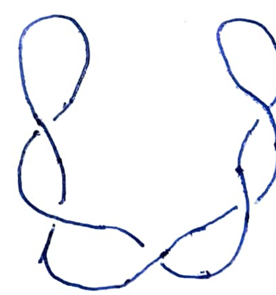


$\Rightarrow$ Unknotting no. of $5_1 = 2$

• Unknotting no. of 7_2



→ 1st unknotting



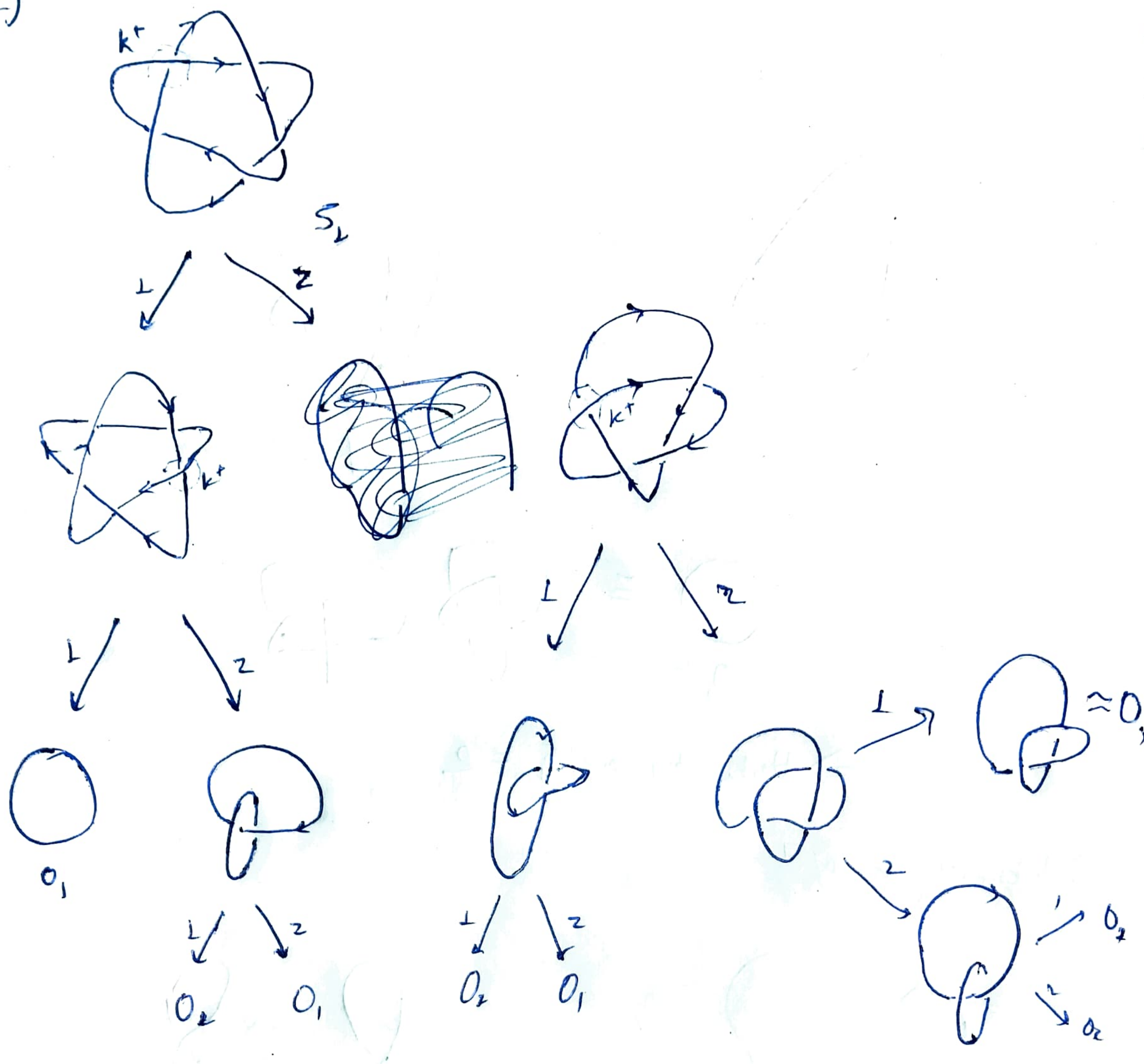
Untwist it

|||

O_1

$\Rightarrow$ Unknotting no. of $7_2 = 1$

2)



$$\nabla_{k^+}^{\bullet} - \nabla_{k^-}^{\bullet} = 2\nabla_{k^0}^{\bullet}$$

$$\Rightarrow \nabla_{k^+}^{\bullet} = \nabla_{k^-}^{\bullet} + 2\nabla_{k^0}^{\bullet} \quad — (1)$$

$$\Rightarrow \nabla_{k^+}^{\bullet} = \nabla_{k^-} + 2\nabla_{k^0}$$

$$= \nabla_{0_1} + 2\nabla_{k^0} = 1 + 2\nabla_{k^0} \quad — (2)$$

$$\nabla_{k^+} = \nabla_{k^-} + 2\nabla_{k^0}$$

$$= 0 + 2 = 2 \quad — (3)$$

from (2)

$$\nabla_{k^+} = 1 + z(z) = 1 + z^2$$

from (1)

$$\nabla_{k^+} = (1 + z^2) + 2(\nabla_{k^0}) \quad — (4)$$

$$\nabla_{k^+} = \nabla_{k^-} + z\nabla_{k^0} \quad — (5)$$

$$= 0 + z = z$$

from (5)

$$\nabla_{k^+} = z + z\nabla_{k^0}$$

$$\nabla_{n^+} = \nabla_{k^-} + z\nabla_{k^0}$$

$$= 1 + z(z \times 1) = 1 + z^2$$

$$\nabla_{k^+} = z + z(1 + z^2)$$

$$= z + z + z^3 = 2z + z^3$$

$$\nabla_{k^+} = 1 + z^2 + z(2z + z^3)$$

$$= 1 + z^2 + 2z^2 + z^4 = 1 + 3z^2 + z^4$$

$$\Rightarrow z = \left(\sqrt{t} - \frac{1}{\sqrt{t}}\right) \Rightarrow z^2 = \left(t + \frac{1}{t} - 2\right)$$

$$z^4 = \left(t^2 + \frac{1}{t^2} + 4 + 2 - \frac{4}{t} - 4t\right)$$

$$\Rightarrow 1 + 3t + \frac{3}{t} - \cancel{6} + t^2 + \frac{1}{t^2} + \cancel{6} - \frac{4}{t} - 4t$$

$$\Rightarrow \left(1 - t - \frac{1}{t} + t^2 + \frac{1}{t^2}\right)$$

Jones's

$$V_{k^+} = t^2(V_{k^-}) + \underbrace{t(t^{1/2} - t^{-1/2})}_{a} (V_{ko})$$

$$= t^2 V_{k^-} + (t^{3/2} - t^{1/2}) V_{ko}$$

$$V_{k^+} = t^2 V_{k^-} + at V_{ko} \quad — ①$$

$$V_{k^+} = t^2 (V_{0_1}) + at(V_{ko})$$

$$= t^2 + \cancel{at} V_{ko} \quad — ②$$

$$V_{k^+} = t^2 (V_{0_2}) + at (V_{0_1})$$

$$= t^2 (V_{0_2}) + at \quad — ③$$

from 2:

$$V_{k^+} = t^2 + at(t^2 (V_{0_2}) + at)$$

$$= t^2 + at^3 (V_{0_2}) + = t^2 - at^3 (t^{1/2} + t^{-1/2})$$

$$= t^2 - at^{7/2} - at^{5/2}$$

from 1:

$$V_{k^+} = (t^2)(t^2 - at^{7/2} - at^{5/2}) + at(V_{ko})$$

$$V_{k^+} = t^4 - at^{11/2} - at^{9/2} + at(V_{ko}) \quad — ④$$

$$V_{k^+} = t^2 (V_{k^-}) + at(V_{ko}) \quad — ⑤$$

$$V_{k^+} = t^2 (V_{k^-}) + at(V_{ko})$$

$$= t^2 (V_{0_2}) + at(V_{0_1})$$

$$= -t^2 (t^{1/2} + t^{-1/2}) + at(1)$$

$$= -t^{5/2} - t^{3/2} + at \quad — ⑥$$

from 5:

$$V_{k^+} = t^2(-t^{5/2} - t^{3/2} + at) + at(V_{k^+})$$

$$= -t^{9/2} - t^{7/2} + at^3 + at(V_{k^+}) \quad — ⑦$$

$$V_{k^+} = t^2 (V_{k^-}) + at(V_{ko})$$

$$= t^2 (V_{0_1}) + at(t^2 (V_{0_2}) + at) = t^2 + at^3 (V_{0_2}) + a^2 t^2$$

~~from 5.~~

~~$V_{k^+} = t$~~

from 5:

$V_{k^+} = -t^{9/2} \cdot t^{7/2} + at^3 + at\,(t^2 - at^{7/2} - at^{9/2} + a^2t^2)$

$= -t^{9/2} - t^{7/2} + at^3 - a^2t^{9/2} - a^2t^{7/2} + a^2t^3$

from 4:

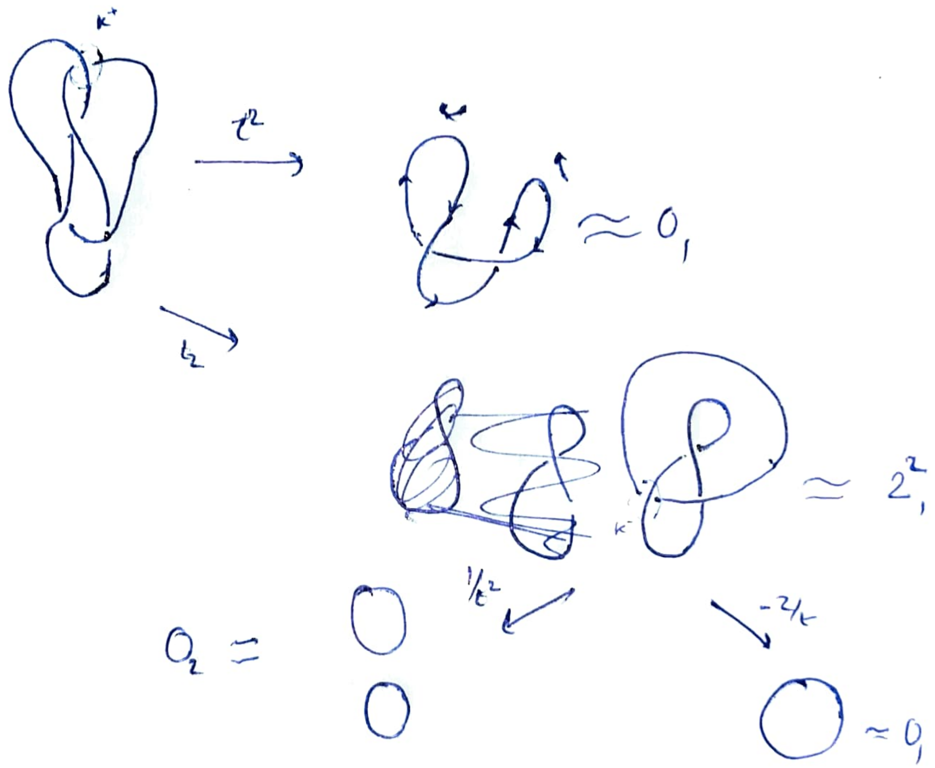
$V_{k^+} = t^4 - at^{11/2} - at^{9/2} + at(-t^{9/2} - t^{7/2}) + \ldots)$

$= t^4 - at^{11/2} - at^{9/2} - at^{11/2} - at^{9/2} + 2a^2t^4$

$- a^3t^{11/2} - a^3t^{9/2} + a^{11}t^4$

$= -t^7 + t^6 - t^5 + t^4 + t^2$

3) Jones polynomial of 4_1



$V_{4_1} = t^2 V_{0_1} + \frac{1}{t} z V_{O_2} + -z^2 V_{0_1}$

$= t^2 + \frac{1}{t}\left(\sqrt{t} - \frac{1}{\sqrt{t}}\right)\left(-\sqrt{t} - \frac{1}{\sqrt{t}}\right) - \left(t + \frac{1}{t} - 2\right)$

$= t^2 - \frac{1}{t}\left(t - \frac{1}{t}\right) - t - \frac{1}{t} + 2 = \frac{1}{t^2}\left(1 - t + t^2 - t^3 + t^4\right)$

Jones poly. of 5_1 (ref. friends)

for knot 5_2

if $L_+ = 5_2 \Rightarrow L_- = 5_2$ & $L_0 = 3_1$

$t^{-1} V(5_1) - tV(5_1) = \left(\sqrt{t} - \frac{1}{\sqrt{t}}\right)(t + t^3 - t^4)$

$\Rightarrow V(5_1) = t^4 - t^3 + t^2 - t + 1$