MTH101A: 2021 - 2022

End-Semester Exam: Question 1

Time: 9:00 am - 9.35 am

Q1. (a) Let (a_n) be a Cauchy sequence with $a_n \geq a > 0$ for all n. Using the definition of Cauchy sequence decide whether the sequence $(\frac{1}{a_n^2})$ is a Cauchy sequence or not. [5]

(b) Let $g : \mathbb{R} \to \mathbb{R}$ be the function defined by $g(x) = x^6 + 2e^{-x} + 8(x+1)^2 - 13\cos x$, for all $x \in \mathbb{R}$. Find the number of real roots of $g(x)$. [6]

(c) Find all values of $x \in \mathbb{R}$ for which the following series converge:

$$\sum_{n=1}^{\infty} \frac{(2x-3)^{2n+1}}{n^{5/2}}.$$

[6]