① 17/09/2023

PHI 455: PHILOSOPHICAL LOGIC.

1.1. R = Rain P = Precipitation $\boxed{\text{FALSE}}$

1. $\Box(R \to P)$
2. $\Diamond P$ $/ \Diamond P$
3. $\neg \Diamond P$ 2,3 are contradictory

$\times$

1.2. $\Box(\alpha \wedge \beta) \vdash \Box\alpha \vee \Box\beta$ $\boxed{\text{FALSE !}}$

$\Box(\alpha \wedge \beta)$
$\neg(\Box\alpha \vee \Box\beta)$
$\Diamond\neg\alpha$
$\Diamond\neg\beta$
$\downarrow$
$0r1$
$\alpha, 1$
$0r2$
$\neg\beta, 2$ Branch open (Counter example)
$\checkmark$

1.3. In S_4 $\Box\Box P \equiv \Box P$; $\Diamond\Diamond P \equiv \Diamond P$ $\boxed{\text{TRUE}}$

$\Diamond\Box\Diamond\Box P \equiv \Diamond\Box P$ $\Box\Diamond\Box\Diamond P \equiv \Box\Diamond P$

$\underline{\Box\Diamond\Box\Diamond\Box\Diamond}\,\alpha \equiv \underline{\Box\Box\Diamond}\,\alpha.$

$\equiv \Box\Diamond\alpha.$

1.4. $\Diamond P \to \Box P$ Reflexivity ? $\boxed{(\text{FALSE})}$

$\neg(\Diamond P \to \Box P)$ | 0
$\Diamond P$
$\neg\Box P$
$\Diamond\neg P$
$\downarrow$
$0r1$
$\neg P, 1$
$0r2$
$P, 2$
$\checkmark$

5: $\Box\Diamond\Box\Diamond\alpha \leftrightarrow \Box\Diamond\alpha$ $\boxed{\text{TRUE}}$

$\Box\Diamond\Box\Diamond\alpha$ reduces to $\Box\Diamond\alpha$. (LHS) which is same as RHS

It holds in S4 $\Rightarrow$ Reflexive Transitivity on 'R'

6 $\Box\phi \to \Diamond\Diamond\phi$ $\boxed{\text{TRUE}}$

1	$\neg(\Box\phi \to \Diamond\Diamond\phi)$	
2	$\Box\phi$	0
3	$\neg\Diamond\Diamond\phi$	
4.	$\Box\neg\Diamond\phi$	
5	$\Box\Box\neg\phi$	
6	$\Box\neg\phi$	means $\Box$T Rule (Reflexive, 5
7.	$\neg\phi$	$\Box$T Rule again, 6
8	ϕ	$\Box$T Rule (Reflex) on 2

✓

Using $\Box$T Rule means $\Rightarrow$ It holds Reflexive frames

$\boxed{\text{FALSE}}$

in S5 (means $\Box$T Rule can be used)

7

1	$\Box(\phi \to \psi)$	0
2	$\Box(T \to \phi)$	
3	$\Diamond\neg\psi$	

All branches closes = inconsistency

4. $\boxed{0 r 1}$

5 $\neg\psi, 1$ 1, 4

5

$\neg\phi, 1$ $\psi, 1$

×

$\neg T$ $\phi, 1$

$\bot$ ×

1.8 Evil in the world $= E$ $\boxed{\text{FALSE}}$

There is God $= G$.

Compatibility $\Diamond(p \wedge q)$

In Compatibility $\neg\Diamond(p \wedge q)$

The Correction Version should be

$$\neg\Diamond(G \wedge E)$$

$\boxed{\text{TRUE}}$

1.9. ~~Exp~~

Necessarily is operated over the entire disjunction.

it is the 'Comma' (,) after necessary creating confusion

and that is the beauty of 'Vagueness'

1. $\Box(p \rightarrow s)$ $\Box$ should be over entire 'Conditional'

1.10 2. $\Diamond\neg s$ / $\neg p$ (no modality in the Conclusion) $\boxed{\text{FALSE}}$

3. $\neg s, 0$ Negation of Conclusion

4. $0r1$ from 2 It is invalid in 'K'

5. $\neg s, 2$

$\neg p, 2$ $s, 2$ from 1

✓ ✗

✓ $K \rightarrow T \rightarrow D \rightarrow S_4 \rightarrow S_5$

$\vdash_{S5(?)}$

2.a. $[(\Box\Diamond p \wedge \Box\Diamond q) \rightarrow \Diamond(p \wedge q)]$

In S_4, S_5

$S_4 \rightarrow S_5$

1 $\neg[\Box\Diamond p \wedge \Box\Diamond q) \rightarrow \Diamond(p \wedge q)]$

2 $\Box\Diamond p \wedge \Box\Diamond q$

3 $\neg\Diamond(p \wedge q)$

4 $\Box\neg(p \wedge q)$ 3, simplification

5 $\Box\Diamond p$ 2, simp

6 $\Box\Diamond q$

~~7~~ ~~$\neg(p \wedge q)$~~, ~~0~~ ~~$\Box T$, 3~~

for S_4 S_5 we can use $\Box T$ Rule.

7 $\Diamond p$, 0 $\Box T$, 5

8 $\Diamond q$, 0 $\Box T$, 6

$\downarrow$

0r1

p, 1 from 7

$\downarrow$

0r2 from 8.

q, 2

$\neg p$, 2 $\quad$ $\neg q$, 2 from 4

X (under $\neg q$, 2)

$\neg p$, 1 $\quad$ $\neg q$, 1

X $\quad$ ✓

It is invalid in S_4, S_5

2.b: Discussed in the Class

- write equivalances in S_4, S_5
- what formulas are allowed (for ex: in S_5 ($- \Box, \Diamond$) holds in S_4, S_5 (By Definition) -

2.2. Pohait is in Casket 1. p
Pohait is in Casket 2 q
Pohait is in Casket 3 r

Let Caskets be A, B, C.

1. $A \leftrightarrow \neg r$
2. $B \leftrightarrow q$
3. $C \leftrightarrow q$

only one is true
$(A \land \neg B \land \neg C) \lor (\neg A \land B \land \neg C) \lor$
$(\neg A \land \neg B \land C)$

2.3 Definition $p \to q = \Box(p \to q) = \neg \Diamond (p \land \neg q)$

$p \to (q \to p)$ is Paradox of material implication

$X = \Box(p \to \Box(q \to p))$

1. $\neg X = \neg \Box (p \to \Box(q \to p))$
2. $\Diamond \neg (p \to \Box(q \to p))$
$\downarrow$
3. $0 r 1$

$\neg(p \to \Box(q \to p)), 1$

$p, 1$ ✓

$\neg \Box (q \to p), 1$

$\Diamond (q \to p), 1$

$\downarrow$

$1 r 2$

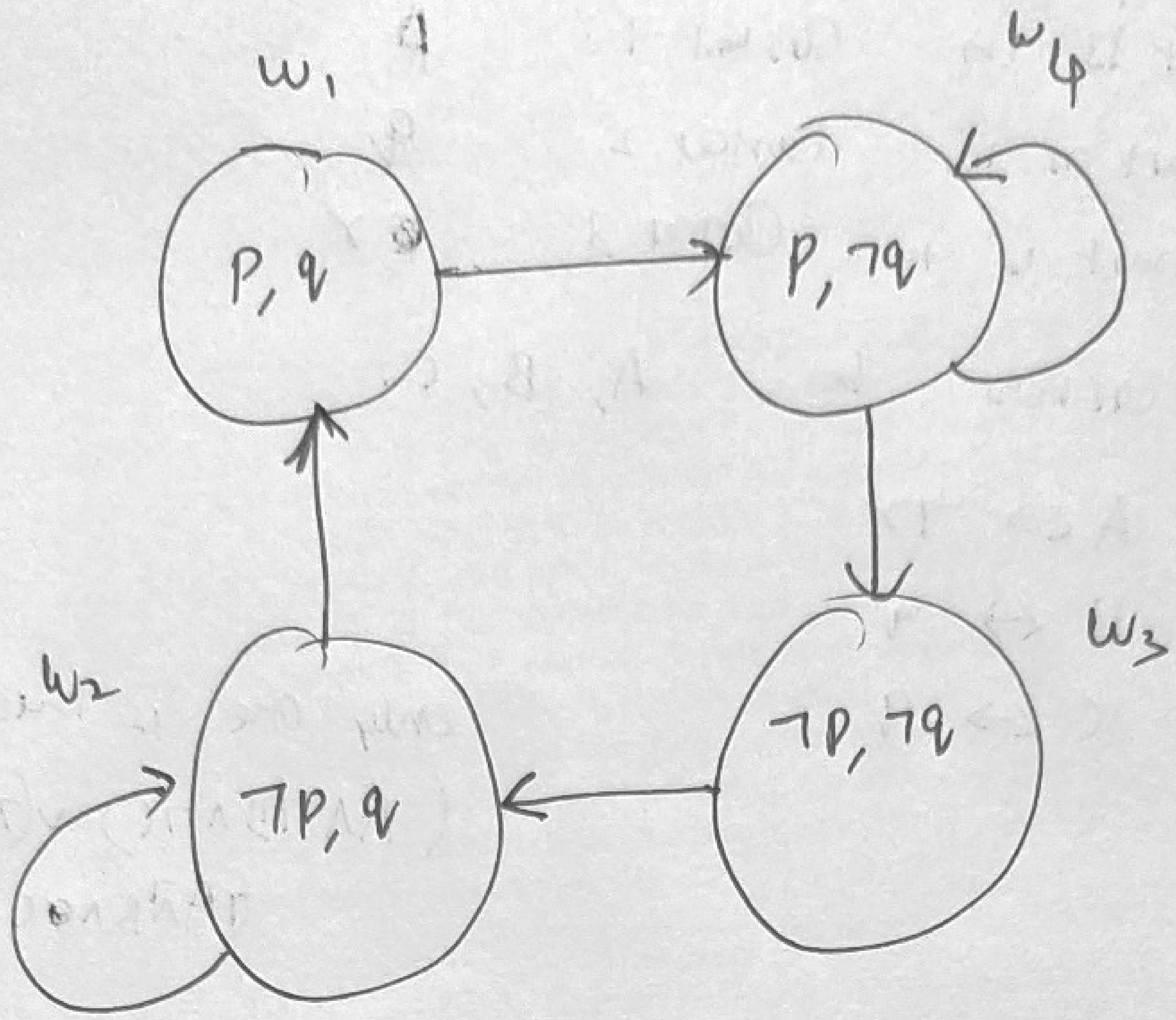
$q \to p, 2$

$\neg q, 2$ ✓ $\quad p, 2$ ✓

$\not\vdash_K \Box(p \to \Box(q \to p))$

K has - No Constraint is needed.

4



I solve 4a.

<u>Please try others</u> (Please see slides)
- if it does not match with Definition 'say' it is <u>false</u>.

⇒ * (It needs to be presented as below)
(there is a typo)

$\models (p \wedge q) \rightarrow \Box\Box\Box \neg q$

	w_1	w_2	w_3	w_4
$p \wedge q \rightarrow \Box\Box\Box \neg q$	X	✓	✓	✓

$(p \wedge q)$ (Antecedent is false)

$p \wedge q$ is true in w_1

$\Box\Box\Box \neg q$ w_1

$\Box\Box \neg q$ is true in w_4 $\quad \Box \neg q, w_4$

$\Box \neg q, w_3$

$\neg q$ is false (as q is true in w_2)

⇒ In w_2
p is false
that makes
$p \wedge q = F$
⇒ So entire
Conditional is ✓