Eg $(a \cup b)^* \stackrel{?}{=} (a^* b^*)^*$

Yes ✓

Q- re in disjunctive normal form (dnf)

$\alpha = \alpha_1 \cup \alpha_2 \; - - \; \alpha_k$

↑ no union further

$\beta \cup \gamma$ = ✓

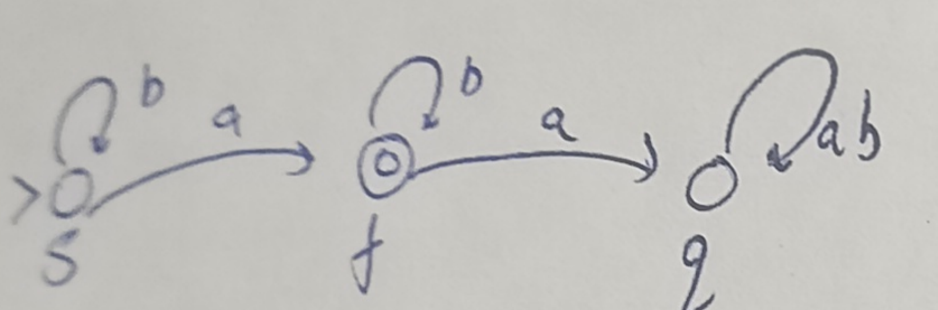
$\beta\gamma = (\beta_1 \cup \beta_2 - - \cup \beta_k)(\gamma_1 \cup \gamma_2 - - \cup \gamma_m) \Rightarrow \beta_1\gamma_1 \cup \beta_1\gamma_2 - -$

↑ no union :- dnf

Note Ex→ $L((\alpha_1 \cup \alpha_2)\beta) = L(\alpha_1\beta \cup \alpha_2\beta)$

DFA $\Sigma = \{ab\}$

Eg→ $\{w \in \Sigma^* : w$ ends with $a\} = L(b^* a\, b^*)$



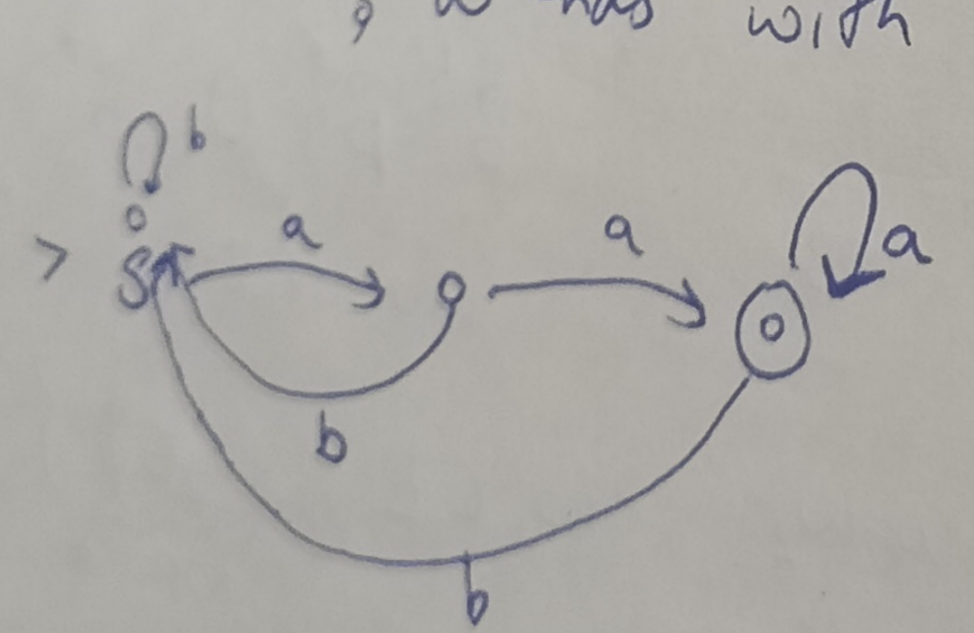
if we give baa, $(s, baa) \vdash_M (s, aa) \vdash_M (f, a)$

no accepted

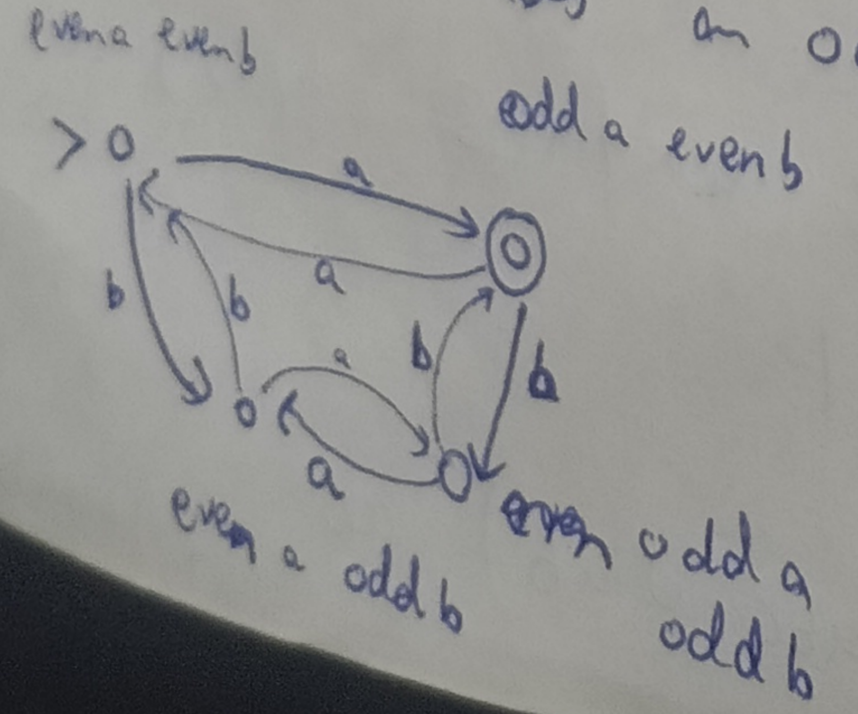
$\vdash_M$

$(q, e) \notin F$

Eg 2) $\{w \in \Sigma^* ; w$ ends with $aa\}$

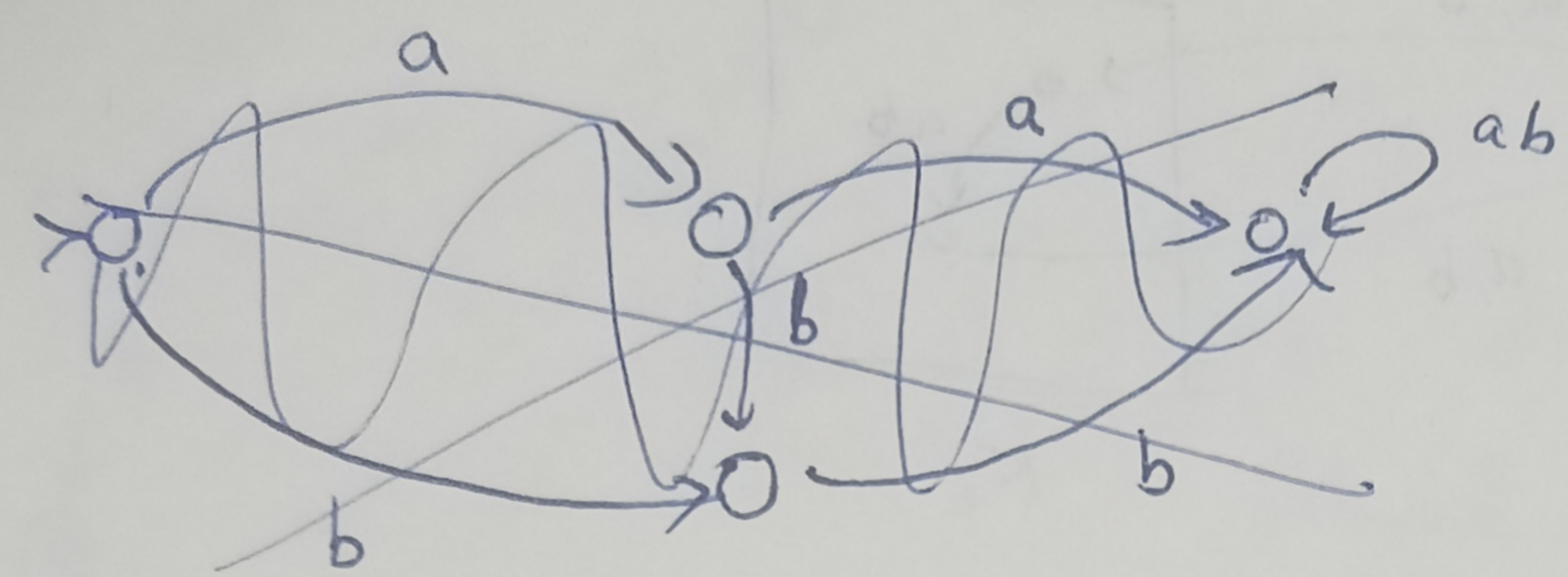


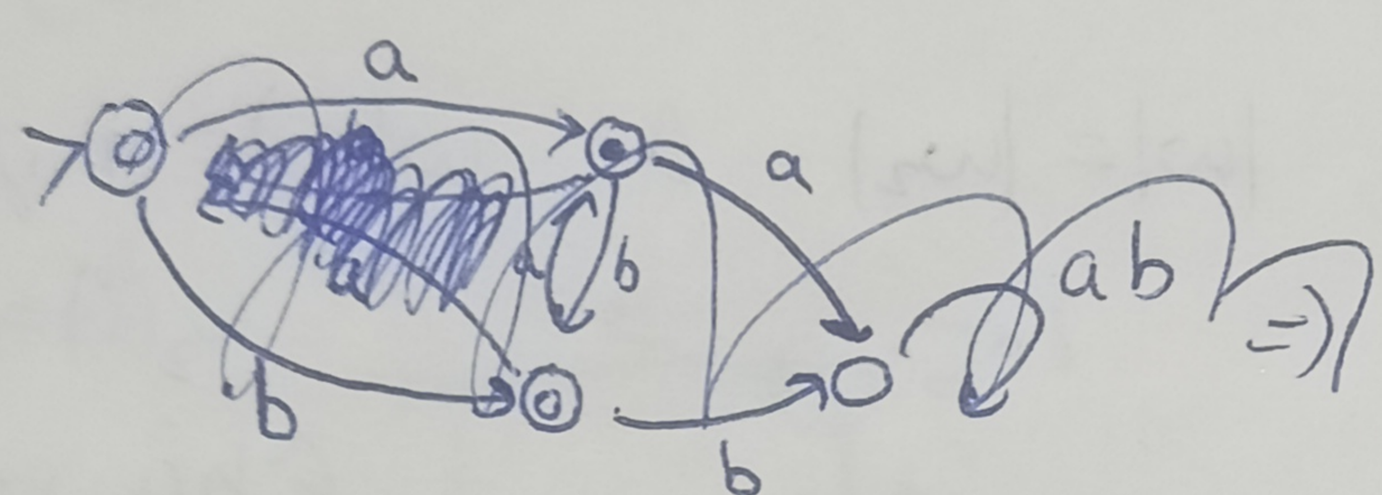
Eg 3) $w \in \Sigma^*$: w has an odd no of a's 4 even no. of b's$\}$

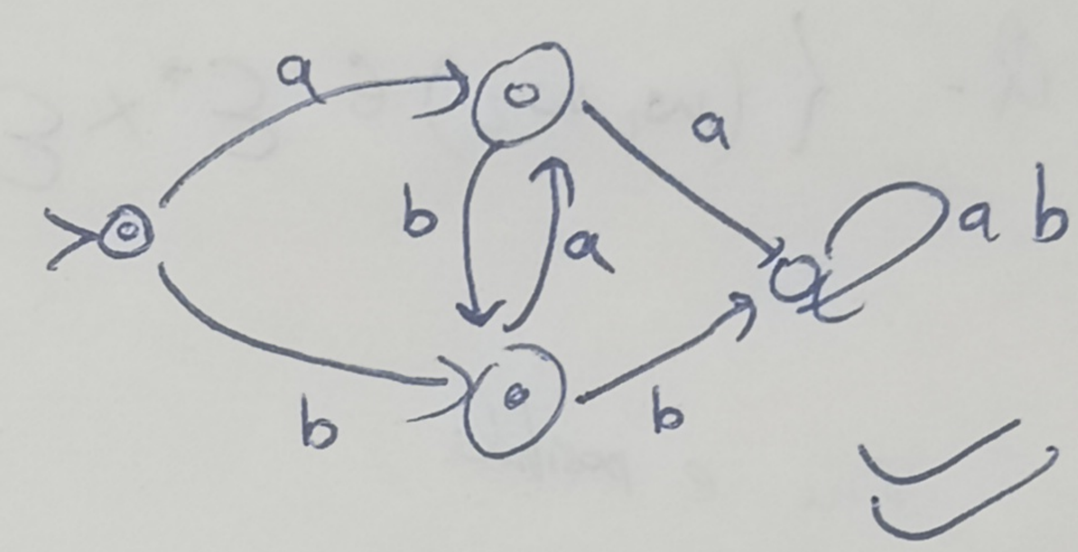


Lec 3

Q- $\{w \in \Sigma^* : w$ has neither aa or bb as a substring$\}$

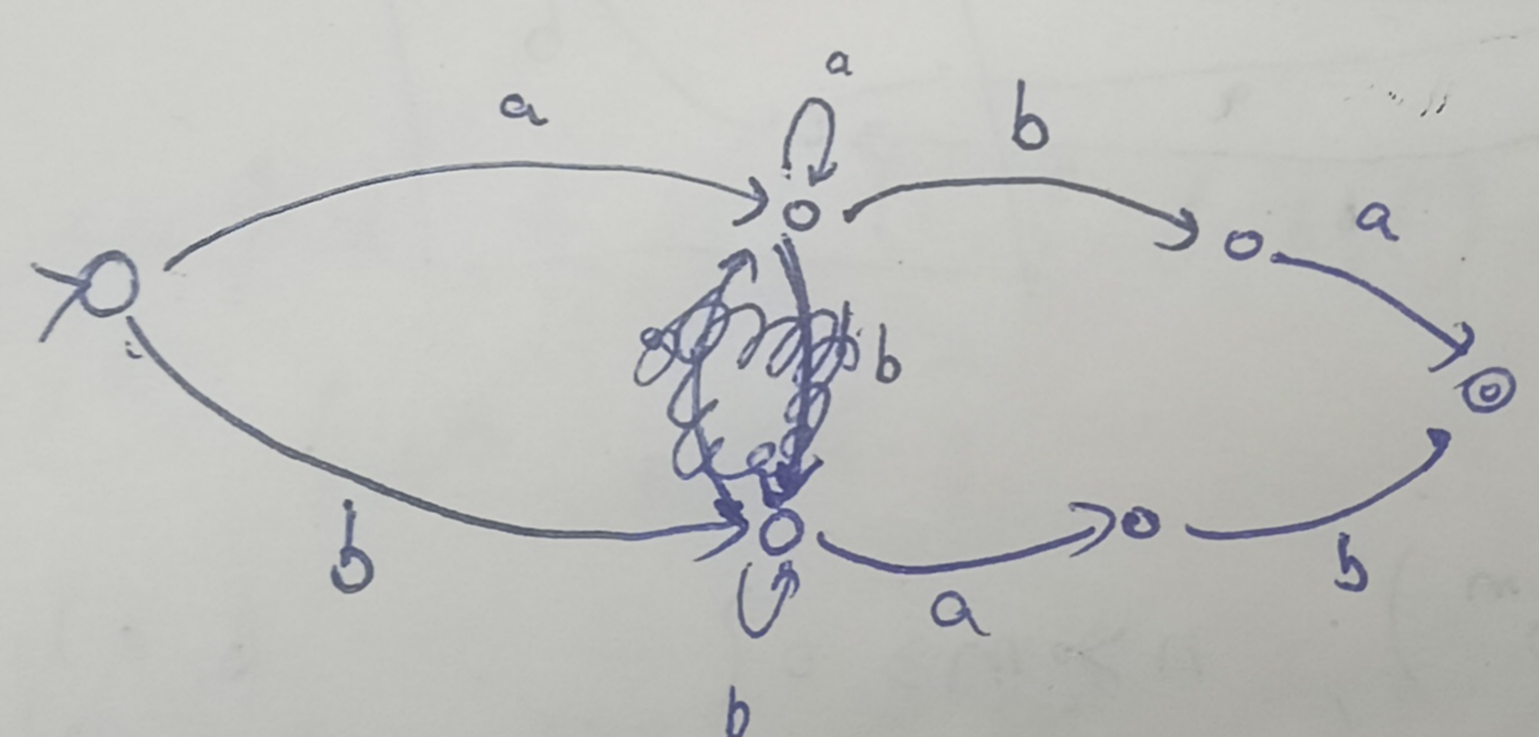






Q→ $\{w \in \Sigma^* : w$ has both ab & ba as substring$\}$

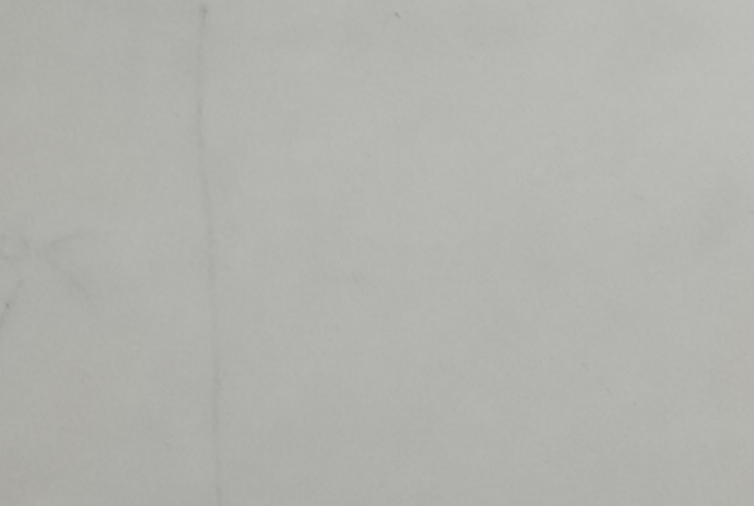
minimum string aba
abab



Q- Deterministic 2state FA

$K = K_1 \cup K_2$

$$(s,(w_1,w_2)) \vdash_M^* (f,(e,e))$$



let $q \in K_1$

$$(q,(w_1,w_2)) \vdash_M (p,(w_1',w_2'))$$

Eg → $\{ w w^R : w \in \{a,b\}^* \}$

Suppose L is regular.

Take $L \cap L(\underbrace{a^*b}_{string}\ \underbrace{ba^*}_{reverse}) = \{ a^n b b a^n : n \geq 0 \}$

*

Q Every subset of a regular language is regular. (F)

Claim → There is a regular language for which there is a subset that is not regular.

Pf Take $a^* b^*$ as r.l & $a^n b^n$ as subset

NP. $\boxed{}$

Note → Is Σ^* regular Pf $\Sigma = \{a,b\} \Rightarrow \{a,b\}^* = (a \cup b)^*$

Q → Every regular language has a regular proper subset. (F)

for every r.l ∴ ϕ is a proper regular subset

but for ϕ to be r.l ∴ there exist no proper r.s.

Q → If L is regular, then so is $\underbrace{\{xy : x \in L \text{ \& } y \notin L\}}_{LL^c}$

∴ L^c is regular & concatenation of 2 r = r

Q → $\{x y x^k : x, y \in \Sigma^*\}$ is regular

$k=4$

~~$h: \Sigma \to \Delta^*$~~

~~end to $h: \Sigma^* \to \Delta^*$~~

~~$w \in \Sigma^*$~~ $h(e) = e$

h

$\Delta^*)$: regular

$h^{-1}(L)$ is also regular

$L) := \{w \in \Sigma^* : h(w) \in L\}$

$\Sigma^*)$

M : $L = L(M)$ → given

M' : $L(M') = h^{-1}(L)$

a $(K, \Sigma, \delta', s, F)$

q $(s, w) \vdash^*_{M'} (t, e)$ iff $(s', h(w)) \vdash^*_{M} (t', e)$

Pumping Lemma -

L: infinite regular language over Σ

There are strings $x, y, z \in \Sigma^*$, st. $y \neq e$ and $xy^n z \in L$ for all $n \geq 0$

Alternate version → L: regular

Then there exist $m \geq 1$ st for all $w \in L$ with $|w| \geq m$, $w = xyz$ where $y \neq e$ and $|xy| \leq m$ & $xy^i z \in L$ for all $i \geq 0$

eg → $L := \{a^n b^n : n \geq 0\}$

Suppose L is regular

$i>0$ $w = a^m b^m = xyz \Rightarrow x = a^l$ $y = a^i$, $z = a^{m-(i+l)} b^m$

Suppose take xz, By PL $xz \in L$ {ie $= a^k b^k$ for some $k>0$}

However $xz = a^{m-i} b^m$ & $m-i < m$

contradiction since $i>0$ here $=0$

∴ L is not regular.

Eg. $\{a^n b a^m b a^{n+m} : n, m \geq 1\}$

$L/L' = \{w \in \Sigma^* : wx \in L \text{ for some } x \in L'\}$

eg $\Rightarrow$ $L/\Sigma^* = \{w \in \Sigma^* : wx \in L \text{ for some } x \in \Sigma^*\} \rightarrow$ prefix

Q. given $\begin{cases} \Sigma, \Delta \text{ , alphabet} \\ h : \Sigma \longrightarrow \Delta^* \end{cases}$

Extend h to a form $h^* : \Sigma^* \longrightarrow \Delta^*$ (homomorphism)

$w \in \Sigma^*$ $h^*(e) = e$

$w = u\sigma$ $\quad h^*(u\sigma) = h^*(u)\, h(\sigma)$

$\sigma \in \Sigma$

Sol. i) given $L (\subseteq \Sigma^*)$: regular, To show $h(L)$ is also regular, where $h(L) = \{h(w) : w \in L\}$

pf: L is regular $\Rightarrow$ There is a r.e. α st $L = L(\alpha)$

$\therefore$ To find r.e β for $h(L)$ ie $L(\beta) = h(L)$

($h(L)$ is $h(w)$ $\therefore$ image of w where w is generated by L which in turn by α $\therefore$ $h(\alpha)$ but α is re.)

Modify

'$h(\alpha)$' : If $\sigma \in \Sigma$ is in α, replace each occurence by $h(\sigma)$

eg $\alpha = a \cup b^*$

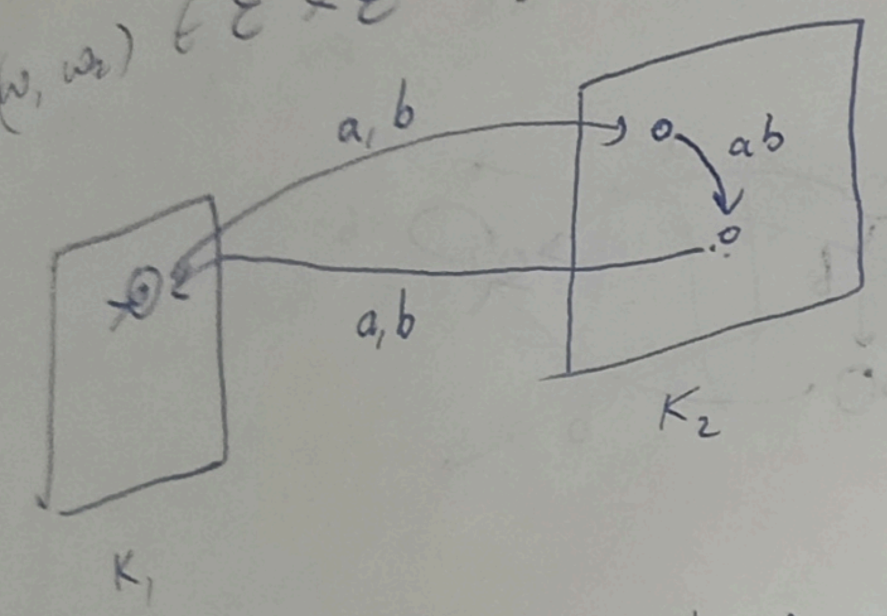
$h(\alpha) = h(a) \cup h(b)^*$

<u>Claim</u> $\rightarrow$ $L(h(\alpha)) = h(L(\alpha))$

Induction on no. of connectives in α

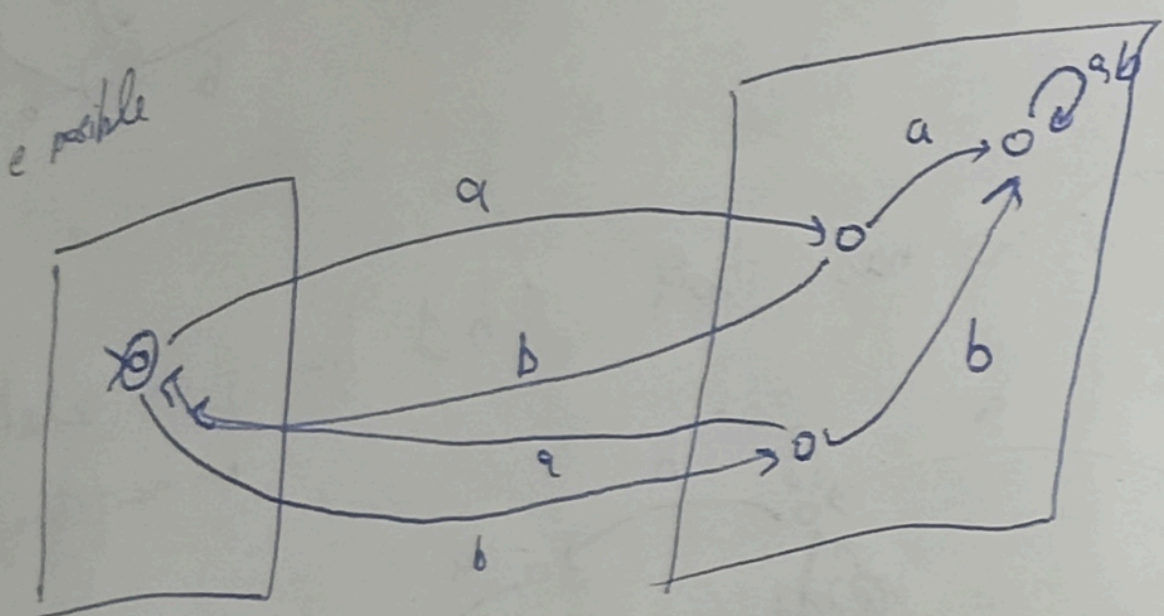
$\alpha = \phi$ $:=$ $L = \phi$ $\Rightarrow$ $h(L) = \phi$, $h(\alpha) = \phi$

Q. $\{(w_1, w_2) \in \Sigma^* \times \Sigma^* : |w_2| = 2|w_1|\}$



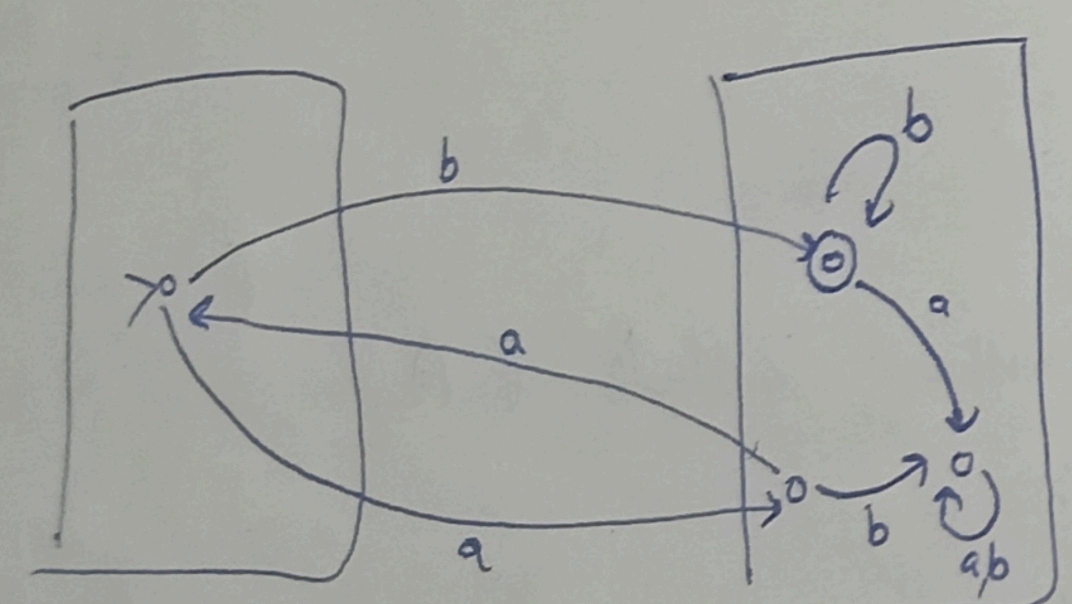
Q. $\{(w_1, w_2) \in \Sigma^* \times \Sigma^* : |w_1| = |w_2|$ a. $w_1(i) = a$ iff $w_2(i) = b$ and vice versa$)$

since e possible



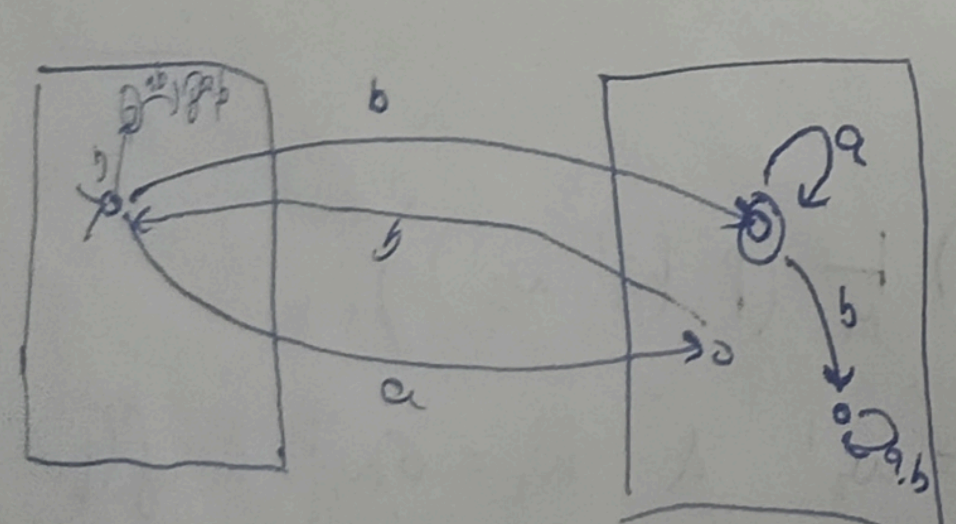
Q. $\{(a^n b, a^n b^m), n \geq m \geq 0\}$

(b, e)
(b, b^m)
$(a^n b, a^n)$ ...



Q. $\{(a^n b, a^n b^n)$ $n, m \geq 0\}$

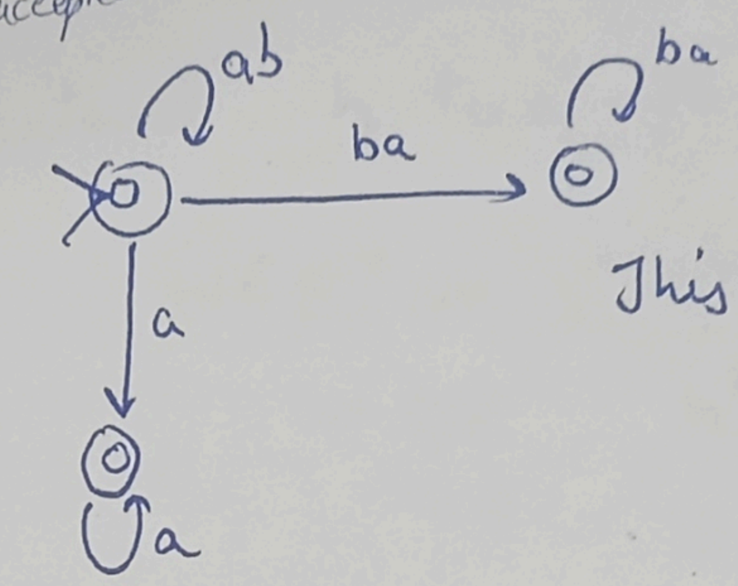
(b, e)
(b, a^n)
$(a^n b, b^n)$
$(a^n b, a^m b^n)$



NFA

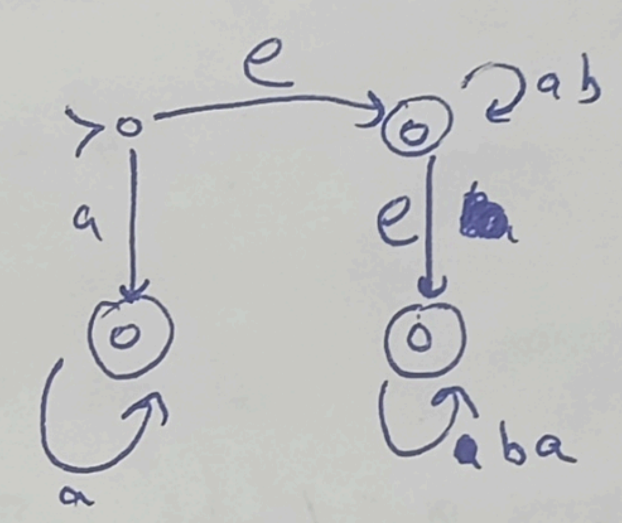
Q. $(ab)^*(ba)^* \cup aa^*$

e accepted



This is wrong since aba can be formed which is NP

Correct



Q. L accepted by $M \in (K, \Sigma, \delta, s, F)$

$L^R = \{w^R : w \in L\}$

Since nothing given we will do NFA

$(s, w) \vdash_M^* (f, e)$ iff $(f, w^R) \vdash_{M'}^* (s, e)$ {from down claim}

$\Rightarrow (s', w^R) \vdash (f, w^R)$

$\therefore$ Create a new start state

$(s', w^R) \vdash_M^* (s, e)$

$(s', w) \vdash_{M'} (f, w^R) \vdash_M^* (s, e)$

formally $K' = K \cup (\{s'\})$

s', $F' = \{s\}$

$\Delta' = \{(s', e, f) : f \in F\} \cup \{(p, \sigma, q) \ \sigma \in \Sigma \ (\delta(q, \sigma) = p)\}$

Claim $\rightarrow$ $(q, w) \vdash^* (p, e)$ iff $(p, w^R) \vdash^* (q, e)$ where $q, p \in K$

Week-5

Myhill Nerode Thm →

Standard Automation for any reg. L.

L = reg lag.
To find a dfa M st $L(M) = L$ & $|K|$ is 'minimum'.

$\Rightarrow$ M': $L(M') = L$; $|K'| \geq |K|$

Reachibility

(I) Reachable states p

$(s, x) \vdash^* (p, e)$ $\quad \exists\, x \in \Sigma^*$

$\sim_M$ on Σ^* $\quad$ $x \sim_M y$ iff there is a $q \in K$ st $(s, x) \vdash^* (q, e)$ & $(s, y) \vdash^* (q, e)$

q is unique!

Let $p \neq q$ $\quad$ for dfa $\quad s \to p$, $s \to q$

E_q ? E_q

but for dfa unique

$E_p = E_q \Rightarrow \boxed{p = q}$

$\therefore$ q is unique.

Observations

- $\sim_M$: equivalence class : $\Sigma^*/\sim_M$
- Any equivalence class of $\Sigma^*/\sim_M$ is uniquely determined by a state q, so denote it by E_q ie $\Sigma^*/\sim_M = \{E_q : q \in K\}$
- $|K| = |\Sigma^*/\sim_M|$

prop → If $x \sim_M y$ then for all $z \in \Sigma^*$, $xz \in L(M)$ iff $yz \in L(M)$

• $x \approx_L y$ iff for any $z \in \Sigma^*$, $xz \in L$ iff $yz \in L$

$$x \sim_M y \Rightarrow x \approx_{L(M)} y$$

$$\sim_M \subseteq \approx_{L(M)} \subseteq \Sigma^* \times \Sigma^*$$

$$|K| = |\Sigma^*/\sim_M| \geq |\Sigma^*/\approx_{L(M)}|$$

$\approx_{L(M)}$

Can we have a dfa M' st. $|K'| = |\Sigma^*/\approx_{L(M)}|$ & $M' \equiv M$

Let $|K''| < |\Sigma^*/\approx_{L(M)}|$ & $M'' \equiv M$

$$|K''| \geq |\Sigma^*/_{L(M'')}| = |\Sigma^*/\approx_{L(M)}| = |K'|$$

contradiction

Myhill Nerode Thm (1957) → $L (\subseteq \Sigma^*)$: regular

There exist a dfa M st $|K| = |\Sigma^*/\approx_L|$ & $L(M) = L$

Pf → $K = \{[x]_{\approx_L} : x \in \Sigma^*\}$, show finite since regular

• $s = [e]_{\approx_L}$

• $F = \{[x]_{\approx_L} : x \in L\}$

• $\delta([x]_{\approx_L}, a) = [xa]_{\approx_L}$

$a \in \Sigma$

if $[y]_{\approx_L}, a \rightarrow [ya]_{\approx_L}$ also

$\in L(M)$ iff

$yz \in L$

& $M' \equiv M$

& $L(M) = L$

ular

$([x]_{\approx_L}, y) \vdash_M^* ([xy]_{\approx_L}, e)$

$(s, w) \vdash^* (f, e)$ iff $w \in L$

$([e], w) \vdash^* ([w], e)$

Corollary → L is regular iff $|\Sigma^*/\approx_L|$ is finite

($\Rightarrow$) L reg $\Rightarrow$ M st $L(M) = L$ $\quad \therefore |K| \geq |\Sigma^*/\approx_{L(M)}|$

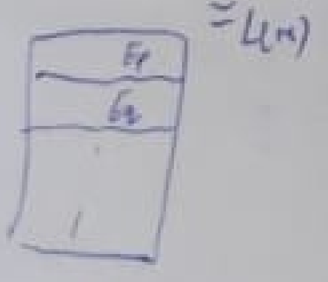
$\therefore$ finite $\quad = |\Sigma^*/\approx_L|$ → finite

($\Leftarrow$) Previous Thm uses regularity to find finiteness so take finite $|\Sigma^*/\approx_L|$ & ~~...~~ construct a reg. language

L: M = dfa $\Rightarrow$ $L(M) = L$ $\therefore \Rightarrow$ L = regular

$\equiv$ on K, $q \equiv p$ iff for all $z \in \Sigma^*$, $(q, z) \vdash^* (f, e)$ iff $(p, z) \vdash^* (f', e)$, $f, f' \in F$

$\equiv$: quotient of $\approx_{L(M)}$ by $\sim_M$



$q \equiv P$

$E_p, E_q \subseteq [w]_{\approx_{L(M)}}$

of $\quad w_1 \quad w_2$ $\quad$ To prove $w' = w''$

Take z, $w'z \in L(M)$ iff $w''z \in L(M)$

Since $s \xrightarrow{w'} (p, z) \rightarrow$ = & given $q \equiv P$ $\therefore$ (f, e)

$\xrightarrow{w''} (q, z) \rightarrow$ (f', e)

(a) $\equiv_n$ $|z| \le n$

- $\equiv \subseteq \equiv_n$, if $q \equiv p$, then $q \equiv_n p$ for any n
- $\equiv_{n+1} \subseteq \equiv_n$

$\therefore q \equiv_0 p$ $K/\equiv_0 = \{F, K \setminus F\}$ $\therefore$ only 2 states possible

$z = e$ $\therefore q = f$ & $p = f'$

since $(q, z) \vdash^*_M (f, e)$
$(p, z) \vdash^*_M (f', e)$

* $\exists n$ st $\equiv_n = \equiv_{n+1} = \equiv_{n+2} \dots$

at that n $\equiv_n = \equiv$

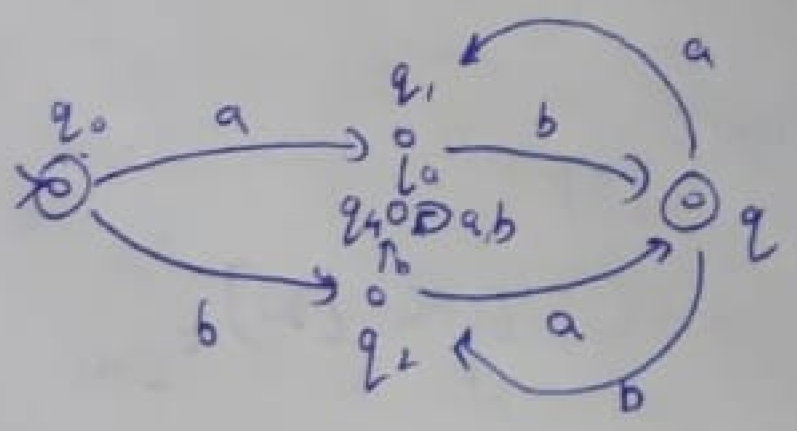
Prop → $q, p \in K$, $n \ge 1$

$q \equiv_n p$ iff 1) $q \equiv_{n-1} p$ and

2) for any $\sigma \in \Sigma$, $\delta(q, \sigma) \equiv_{n-1} \delta(p, \sigma)$

-x-

$(ab \cup ba)^*$ dfa →



not related

q_1	×			
q_2		×		
q_3	×	×	×	
q_4	×	×	×	×
	q_0	q_1	q_2	q_3

$\equiv_0$ $\{q_0, q_2\}$, $\{q_1, q_3, q_4\}$

$\equiv_1$ $\{q_0, q_2\}$, $\{q_1\}$, $\{q_3\}$, $\{q_4\}$

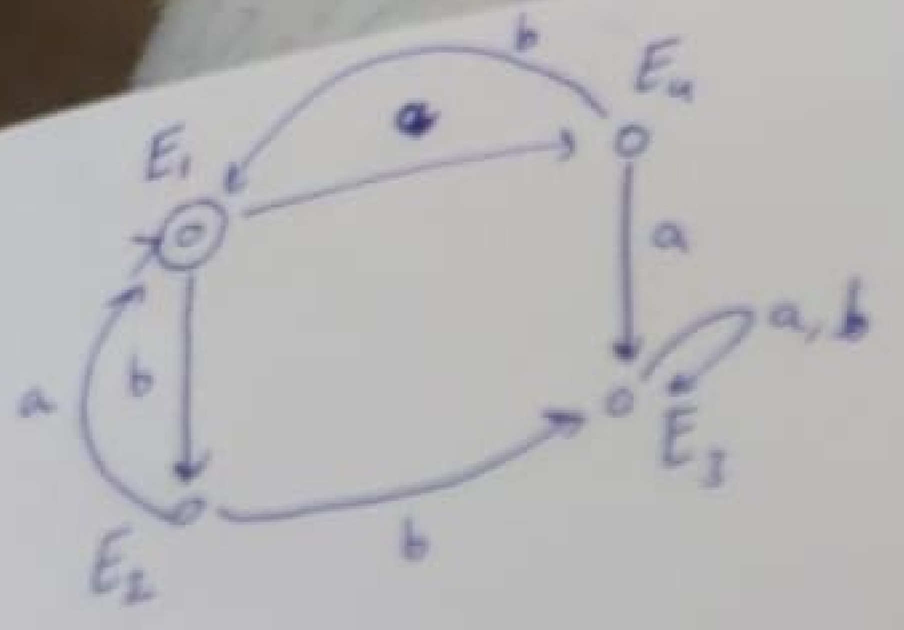
both q_0 & q_2 reading a & b reach same state $\therefore$ related

$\therefore E_1 = E_{q_0} \cup E_{q_2}$

$E_2 = E_{q_3}$, $E_3 = E_{q_4}$ & $E_4 = E_{q_1}$

but q_1 & q_3 reading a reach diff state $\therefore$ not related & lly q_4

$\therefore$ minimum 4 states



$$E_1 = E_{q_0} \cup E_{q_2}$$
$$= L\left((ab \cup ba)^*\right)$$

$$E_2 = E_{q_3} = L\left((ab \cup ba)^* b\right)$$