MTH101A: 2021 - 2022

End-Semester Exam: Question 5
Time: 11.20 am - 11.55 am

Q5. (a) Let T be the region in $\mathbb{R}^2$ bounded by the curves $y = 4 - x^2$, $x = 0$ and $y = 0$. Draw the region T by indicating the coordinates of intersecting points of the three curves above, taken two at a time. Evaluate the following double integral:

$$\iint_T \frac{xe^{2y}}{4-y}\, dxdy.$$

[7]

(b) Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by

$$f(x,y) = \begin{cases} \frac{x^2 y\sqrt{|y|}}{x^4+y^2} & \text{if } (x,y) \neq (0,0), \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$

Is f continuous at $(0,0)$? Do the directional derivatives of f at $(0,0)$ exist along any direction? Is f differentiable at $(0,0)$? Justfy your answers. [10]