MTH 101-Calculus

Spring-2021

Assignment-10: Directional derivatives, Maxima, Minima, Lagrange Multipliers

1. Let $f(x, y) = \frac{1}{2}\Big(\big|\, |x| - |y| \,\big| - |x| - |y|\Big)$. Is f continuous at $(0, 0)$? Which directional derivatives of f exist at $(0, 0)$? Is f differentiable at $(0, 0)$?

2. Let $f(x, y) = \frac{x^2 y}{x^2 + y^2}$ for $(x, y) \neq (0, 0)$ and $f(0, 0) = 0$. Show that the directional derivative of f at $(0, 0)$ in all directions exist but f is not differentiable at $(0, 0)$.

3. Let $f(x, y) = x^2 e^y + \cos(xy)$. Find the directional derivative of f at $(1, 2)$ in the direction $(\frac{3}{5}, \frac{4}{5})$.

4. Find the equation of the surface generated by the normals to the surface $x + 2yz + xyz^2 = 0$ at all points on the z-axis.

5. Examine the following functions for local maxima, local minima and saddle points:

   $i)$ $4xy - x^4 - y^4$ $\quad$ $ii)$ $x^3 - 3xy^2$

6. Find the absolute maxima of $f(x, y) = xy$ on the unit disc $\{(x, y) : \; x^2 + y^2 \leq 1\}$.