Indian Institute of Technology Kanpur
Department of Mathematics and Statistics
MTH 636M: Game Theory
Quiz 3, Date: April 12, 2024, Friday

Timing: 04:00 PM to 05:15 PM

- Answer all the questions. The exam is for 20 marks.

- Try not to use any result not done in the class. However, if you use any such result, clearly state and prove it.

- Write your name, roll no., program name, and seat number clearly on the top of your answer sheet.

- For prove or disprove type questions, clearly state whether it's a prove or a disprove and then provide the arguments.

- One A4 sheet with ONLY necessary definitions and results are allowed during the exam. Use of a calculator, mobile, and smart watch is strictly prohibited.

- Be precise in writing the answers. Unnecessary arguments would lead to a deduction in marks.

$$* * * * *$$

1. Find all the Nash Equilibria of the following game: (**5 marks**)

Player 2

		a	b
Player 1	A	$(1,1)$	$(0,1)$
	B	$(0,1)$	$(2,1)$

Answer: (A, a) and (B, b) are two pure strategy NE. For mixed strategy NE, let $(q, 1-q)$ be the strategy of Player 2 such that Player 1 is indifferent over her two actions A and B. This means

$$q = 2(1-q) \implies q = \frac{2}{3}.$$

Further, Player 2 is indifferent over her two actions for any strategy of Player 1, we have the following mixed equilibria

$$\left\{ \left(p\,[A], 1 - p\,[B] \right), \left(\frac{1}{2}\,[a], \frac{1}{2}\,[b] \right) \mid p \in [0, 1] \right\}.$$

2. Establish whether there exists a two-player game in extensive form with perfect information, and possible outcomes I (Player I wins), II (Player II wins), and D (a draw), whose strategic-form description is

Player II

	s_{II}^1	s_{II}^2	s_{II}^3	s_{II}^4
s_{I}^1	D	I	II	I
s_{I}^2	I	II	I	D
s_{I}^3	I	I	II	II

Player I is labelled on the left with rows s_{I}^1, s_{I}^2, s_{I}^3.

If the answer is yes, describe the game. If not, explain why not. (**5 marks**)

Answer: Note that the game satisfies all the conditions stated in the theorem related to chess. Therefore, the game either has winning strategy for one of the players or both the players have a strategy that assures at least a draw. But from the strategic form we can say that none of two conditions are satisfied here. Thus, there is no extensive form game with perfect information satisfying the conditions.

3. Prove or disprove: There is a 2×2 game with infinitely many Nash Equilibria such that there exist two pure-strategy profiles s and s' with $u_i(s) \neq u_i(s')$ for some $i \in \{1, 2\}$ where u_i denotes the utility of Player i. (**5 marks**)

Answer: *(Disprove)* see Problem 1.

4. Determine all mixed strategies that are outcome equivalent with the behavioral strategy represented in the following one-player extensive form structure:
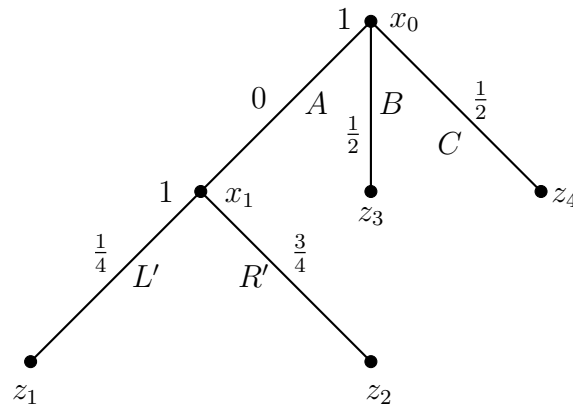
(**5 marks**)

Answer: Note that the pure strategies of the game are as following

$$\| \quad AL' \quad AR' \quad BL' \quad BR' \quad CL' \quad CR' \quad \|$$

From the behavioral strategy, it is clear that z_1 and z_2 will never reach in the game implying strategies AL' and AR' will get zero probability. Further, nodes z_3 and z_4 will reach with probability $\frac{1}{2}$ each. Therefore, the total probability of BL' and BR' is $\frac{1}{2}$ and total probability of CL' and CR' is $\frac{1}{2}$. Therefore, the set of all mixed strategies that are outcome equivalent to the given behavioral strategy is

AL'	AR'	BL'	BR'	CL'	CR'
0	0	p	$\frac{1}{2} - p$	q	$\frac{1}{2} - q$

where $0 \leq p \leq \frac{1}{2}$ and $0 \leq q \leq \frac{1}{2}$.