Indian Institute of Technology Kanpur

MTH-101AA
QUIZ I, 11-12-2020, 6:00-6:10PM

(1) For $n \geq 1$, let $x_n = 1 + \frac{1}{3} + \frac{1}{5} + \cdots + \frac{1}{2n-1}$. Does the sequence (x_n) converge? Justify your answer. (Do not use statements of the problems appeared in the Assignments or Practice Problems for justifications). [5]

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(2) Let $f : \mathbb{R} \to \mathbb{R}$ satisfy $f(x + y) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$. If f is continuous at $x_0 = 1$, show that f is continuous at $y_0 = 2$. [5]

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(3) Let $f : [0,1] \to \mathbb{R}$ be continuous and (x_n) be a sequence in $[0,1]$. Suppose

$$\lim_{n\to\infty} \frac{f(x_1) + f(x_2) + \cdots + f(x_n)}{n} = \alpha$$

for some $\alpha \in \mathbb{R}$. Show that there exists $x_0 \in [0,1]$ such that $f(x_0) = \alpha$.

[5]