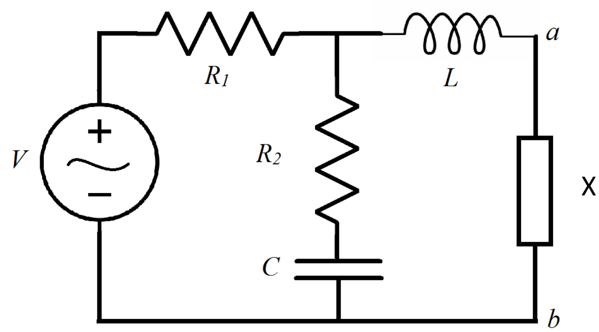
Major Quiz-2

Duration: 45 min. Max Marks: 15

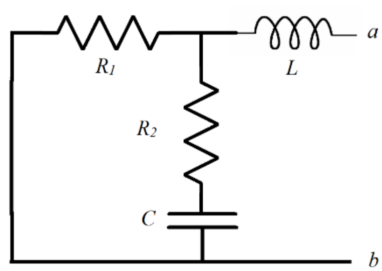
1. Consider the circuit alongside with $V = 10\angle 0^0 V$ and the impedances corresponding to the elements given as: $R_1 = 4\Omega$, $R_2 = 8\Omega$, $Z_C = -j6\Omega$ and $Z_L = j5\Omega$.



(a) Calculate the Thevenin equivalent as observed by the load, X. [3]

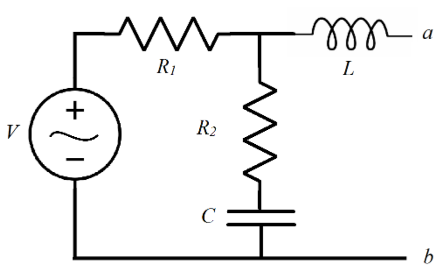
Solution:

Short voltage source to find Z_{Th}: ***[1.5 marks]***



$$Z_{Th} = j5 + 4||(8 - j6) = j5 + \frac{4(8 - j6)}{4 + 8 - j6}$$
$$= \mathbf{2.933 + j4.467\ \Omega = 5.344\angle 56.71^0 \Omega}$$

To find V_{Th}: ***[1.5 marks]***



$$V_{ab} = V_{Th} = \frac{8 - j6}{4 + 8 - j6} 10V = \mathbf{49.193 - j8.944V}$$
$$= \mathbf{7.454\angle{-10.3^0} V}$$

(b) Determine the load impedance, Z_X, that maximizes the average power drawn from the circuit. What is the maximum average power drawn by the load? [3]

Solution:

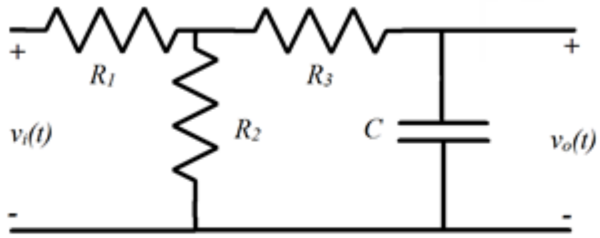
Load impedance for maximum average power: ***[1 mark]***

$$\mathbf{Z_L = Z_{Th}^* = 2.933 - j4.467\ \Omega}$$

Maximum average power: ***[1 mark]***

$$P_{max} = \frac{|V_{Th}|^2}{8Re(Z_{Th})} = \frac{(7.454)^2}{8(2.933)} = \mathbf{2.368\ W}$$

2. Consider the circuit given below, with $R_1 = 2k\Omega$, $R_2 = 2k\Omega$, $R_3 = 1k\Omega$ and $C = 10\mu F$.



(a) Calculate the transfer function, $H = \frac{v_o}{v_i}$ [3]

Solution: Using potential divider ***[2 marks for H(w)]***

$$v_0 = \frac{1/j\omega C}{R_3 + 1/j\omega C} \times \frac{R_2||(R_3 + \frac{1}{j\omega C})}{R_1 + R_2||(R_3 + \frac{1}{j\omega C})} v_i = \frac{v_i}{2(1 + 2000 j\omega C)}$$

$$H(\omega) = \frac{v_o}{v_i} = \frac{\mathbf{0.5}}{\mathbf{1} + j\frac{\boldsymbol{\omega}}{\boldsymbol{\omega_0}}};$$

$$\boldsymbol{\omega_0} = \frac{\mathbf{1}}{\mathbf{2000C}} = \mathbf{50}\, \boldsymbol{rad/s}$$

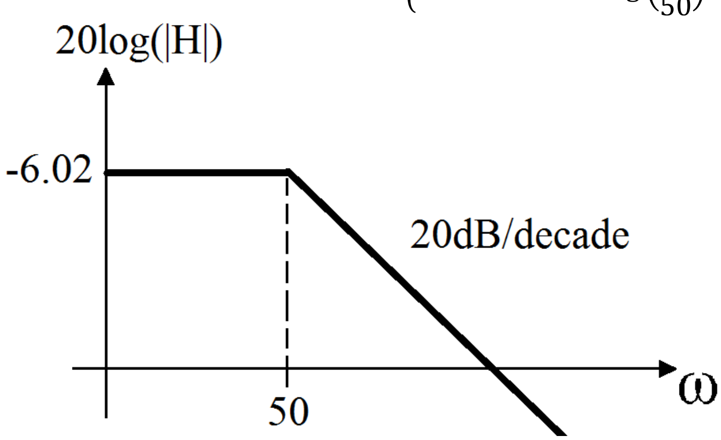
[1 mark for ω_0]

(b) Plot the Bode magnitude plot for the transfer function derived in (a). What sort of a filter does this circuit act as? [3]

Solution:

$$20log(|H|) = 20\log(0.5) - 20\log\left(\sqrt{1 + \left(\frac{\omega}{50}\right)^2}\right) dB$$

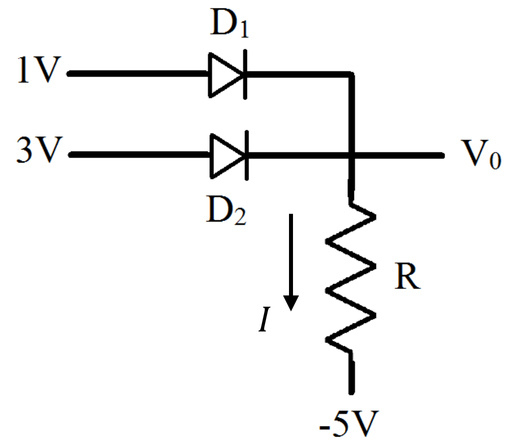
$$20log(|H|) = \begin{cases} -6.02dB; for\ \omega \ll 50\ rad/s \\ -6.02dB - 20\log\left(\frac{\omega}{50}\right) dB\ ; for\ \omega \gg 50\ rad/s \end{cases}$$



[2 marks for plot]

Represents a **low-pass** filter ***[1 mark]***

3. For the circuit shown below, calculate I and V_0, if $R = 1k\Omega$. Assume that the diodes are ideal. [3]



Solution:
Case 1: if both diodes are on:

In this case, $V_0 = 1V$ and $V_0 = 3V$; which is not possible.

⇒ Initial assumption is incorrect

Case 2: if both diodes are cut-off:
In this case, $V_0 = -5V$ which is less than both 1V and 3V implying that both diodes must be on.

⇒ Contradiction ⇒ Initial assumption is incorrect

Case 3: if D_1 is on while D_2 is off:
$V_0 = 1V$ ⇒voltage across D_2 2V (forward bias)⇒ D_2 is on; which contradicts the initial assumption

⇒ Initial assumption is incorrect

Case 4: if D_2 is on and D_1 is off: ***[1 mark]***
$V_0 = 3V$ voltage across D_1 is -2V ⇒D_1 is off which satisfies the initial condition.

Therefore,
$\boldsymbol{I = \frac{(3-(-5))V}{1k\Omega} = 8mA}$ **and** $\boldsymbol{V_0 = 3V}$ ***[1 mark+1 mark]***