MTH 101-Calculus

Spring-2021

Assignment 8-Solutions:Vectors, Curves, Surfaces, Vector Functions

1. (a) If the three planes intersect at a single point then the determinant of the coefficients will be nonzero and hence $a \neq -4$.

   (b) If the three planes intersect in a line, then the plane P_3 must pass through the line of intersection of the planes P_1 and P_2.
   Hence, clearly $a = -4$. There exists $\alpha, \beta \in \mathbb{R}$ such that $\alpha(x - y + z - 1) + \beta(x - 4y - 2z + 10) = 2x - 3y + z + b$. Therefore, $b = \frac{5}{3}$.

   (c) $a = -4$ and $b \neq \frac{5}{3}$.

2. Any point on the curve is of the form (x_0, y_0, h). The equation of a line passing through (x_0, y_0, h) and $(0, -a, 0)$ is
   $\frac{x-0}{x_0} = \frac{y+a}{y_0+a} = \frac{z-0}{h-0}$. We get $x_0 = \frac{hx}{z}$ and $y_0 = \frac{h(y+a)}{z} - a$.
   Since (x_0, y_0, h) lies on the curve, we get the equation of the cone to be $h^2x^2 = 2z[h(y+a) - az]$.

3. $c(t) = (\sin t + 2)i + (\cos t - 1)j + (t + 1)k$. $\cos\theta = \frac{c(t).c'(t)}{\|c(t)\|\|c'(t)\|}$

4. $\|c(t)\| = \sqrt{\sin^2 t^2 + \cos^2 t^2 + 25} = \sqrt{26}$. Easy to see $c(t).c'(t) = 0$. $\|c'(t)\| = 2t$, thereby showing that the velocity vector is not of constant magnitude.

5. $s(t) = \int\limits_0^t \sqrt{25\cos^2 u + 25\sin^2 u + 144}\ du = 13t$.
   Since the arc length is given to be 26π, we get $t = 2\pi$. The coordinates of the required point are $(0, 5, 24\pi)$.

6. (a) $s(t) = \int\limits_0^t \sqrt{u^2 + u^4}du = (1 + t^2)^{\frac{3}{2}} - 1$. Hence $t = \sqrt{(3s+1)^{\frac{2}{3}} - 1}$. Substitute t in the equation.

   (b) $s(t) = 2t \Rightarrow t = \frac{s}{2}$.

7. Let $f(x) = ax^2$. Then $f'(x) = 2ax$ and $f''(x) = 2a$. Use the formula for the curvature $\kappa = \dfrac{|f''(x)|}{[1 + f'(x)^2]^{3/2}}$ to get the result.