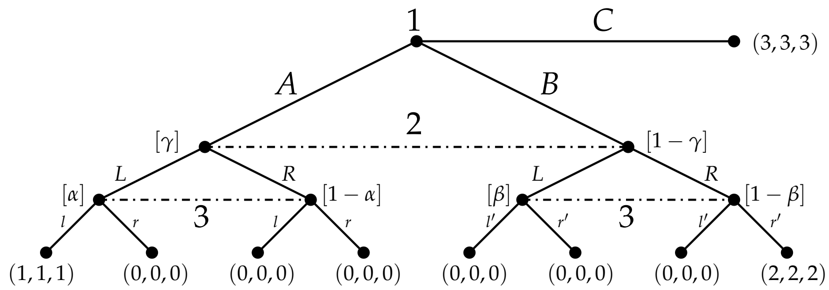
Question: Find all the perfect Bayesian equilibria for the following game.



Answer: Observe that Player 1 will always play C as she will get maximum pay-off by playing that action. This means for every perfect Bayesian equilibrium, there is no subgame where the three imperfect information sets are reached with a positive probability implying α, β, and γ are "free".

Considering Player 3, if $\alpha > 0$, Player 3's optimal action is l and if $\alpha = 0$, she is indifferent between l and r. Similarly, if $\beta < 1$, Player 3's optimal action is r' and if $\beta = 1$, she is indifferent between l' and r'. We distinguish four cases based on the possible values of α and β.

Case 1: $\alpha > 0$ and $\beta < 1$.

This indicates that Player 3 will play $b_3(l) = 1$ and $b_3(r') = 1$. Accordingly, Player 2 will prefer playing L over R if $\gamma > 2(1-\gamma) \implies \gamma > \frac{2}{3}$ and R over L if $\gamma < \frac{2}{3}$. If $\gamma = \frac{2}{3}$, Player 2 will be indifferent between L and R. This gives us the following equilibria:

$$b_1(C) = 1, b_2(L) = 1, b_3(l) = 1, b_3(r') = 1, \alpha > 0, \beta < 1, \gamma > \frac{2}{3},$$

$$b_1(C) = 1, b_2(R) = 1, b_3(l) = 1, b_3(r') = 1, \alpha > 0, \beta < 1, \gamma < \frac{2}{3},$$

$$b_1(C) = 1, b_2(L) \in [0,1], b_3(l) = 1, b_3(r') = 1, \alpha > 0, \beta < 1, \gamma = \frac{2}{3}.$$

Case 2: $\alpha = 0$ and $\beta < 1$.

It follows that Player 3 will play $b_3(l) \in [0,1]$ and $b_3(r') = 1$. Accordingly, Player 2 will prefer playing L over R if $\gamma b_3(l) > 2(1-\gamma) \implies \gamma > \frac{2}{2+b_3(l)}$ and R over L if $\gamma < \frac{2}{2+b_3(l)}$. If $\gamma = \frac{2}{2+b_3(l)}$, Player 2 will be indifferent between L and R. Hence, the following equilibria:

$$b_1(C) = 1, b_2(L) = 1, b_3(l) \in [0,1], b_3(r') = 1, \alpha = 0, \beta < 1, \gamma > \frac{2}{2+b_3(l)},$$

$$b_1(C) = 1, b_2(R) = 1, b_3(l) \in [0, 1], b_3(r') = 1, \alpha = 0, \beta < 1, \gamma < \frac{2}{2 + b_3(l)},$$

$$b_1(C) = 1, b_2(L) \in [0, 1], b_3(l) \in [0, 1], b_3(r') = 1, \alpha = 0, \beta < 1, \gamma = \frac{2}{2 + b_3(l)}.$$

Solve the other two cases in a similar way.