Department of Physics
IIT Kanpur, 2021-22 Ist Semester

PHY103AA (Physics - II) **Tutorial # 5** **04-01-2022**

Problems 5.1, 5.2, 5.3 and 5.4 will be discussed during the Tutorial Hour.

Problem 5.1: A thin insulating rod, running from $z = -a$ to $z = +a$, carries the a line charge $\lambda(z)$ as, (a) $\lambda(z) = k\cos(\pi z/2a)$, (b) $\lambda(z) = k\sin(\pi z/a)$ and (c) $\lambda(z) = k\cos(\pi z/a)$, where k is a constant. For each case, find the leading order term in the multipole expansion of the potential.

Problem 5.2: According to *Quantum Mechanics*, the electron cloud for a hydrogen atom in the ground state has a charge density

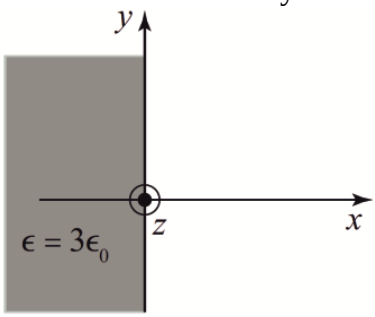
$$\rho(r) = \frac{q}{\pi a^3} e^{-2r/a}$$

where q is the electron charge and a is the Bohr radius. Find the atomic polarizability of such an atom.

Problem 5.3: Consider the situation shown in the figure. The region of space with $x < 0$ is uniformly filled with a dielectric of permittivity $\varepsilon = 3\varepsilon_0$ and has the electric field in the region given by $\vec{\mathrm{E}} = (1-x)\hat{\mathrm{x}} + 2\hat{\mathrm{y}} + 3\hat{\mathrm{z}}$. $x > 0$ region is the vacuum. Assume no free surface charge on the boundary at $x = 0$.

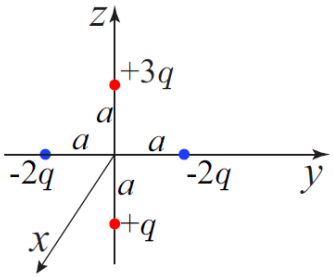
(i) Calculate the electric field $\vec{\mathrm{E}}(0^+, y, z)$ in the vacuum region at $x = 0^+$ (i.e., very close to the boundary at $x = 0$ in the vacuum region).

(ii) Calculate the polarization and find the resulting bound charge densities.



Problem 5.4: A point charge q is imbedded at the center of a sphere of linear dielectric material (with susceptibility $\chi_e$and radius R). Find the electric field, the polarization, and the bound charge densities. What is the total bound charge on the surface? Where is the compensating negative bound charge located?

Problem 5.5: For the charge distribution shown in figure, find the approximate potential at points far from the origin.



Problem 5.6: A charge $+Q$ is distributed uniformly along the z axis from $z = -a$ to $z = +a$. Show that the electric potential at a point r is given by

$$V(r,\theta) = \frac{Q}{4\pi\varepsilon_0}\frac{1}{r}\left[1 + \frac{1}{3}\left(\frac{a}{r}\right)^2 P_2(\cos\theta) + \frac{1}{5}\left(\frac{a}{r}\right)^4 P_4(\cos\theta) + \cdots\right] \qquad \text{for } r > a.$$

Problem 5.7: A spherical conductor of radius a carries total charge Q. It is surrounded by linear dielectric material of susceptibility χ_e out to radius b. Find the energy of this configuration.

Problem 5.8: A spherical shell with inner radius a and outer radius b is made of dielectric material with polarization $\mathbf{P} = \frac{k}{r}\hat{\mathbf{r}}$ where k is a constant r is the distance from the center (see figure). Assume no free charge anywhere.

(a) Calculate all the bound charges.

(b) Use Gauss's law for electric fields to calculate the electric field in all the three regions.

(c) Use Gauss's law for electric displacement to calculate the electric field in all the three regions.