Indian Institute of Technology Kanpur

Department of Mathematics and Statistics

MTH 636M: Game Theory

Quiz 3, Date: April 12, 2024, Friday

Timing: 06:15 PM to 07:30 PM

- Answer all the questions. The exam is for 20 marks.
- Try not to use any result not done in the class. However, if you use any such result, clearly state and prove it.
- Write your name, roll no., program name, and seat number clearly on the top of your answer sheet.
- For prove or disprove type questions, clearly state whether it's a prove or a disprove and then provide the arguments.
- One A4 sheet with ONLY necessary definitions and results are allowed during the exam. Use of a calculator, mobile, and smart watch is strictly prohibited.
- Be precise in writing the answers. Unnecessary arguments would lead to a deduction in marks.

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1. Find all the max-min strategies and the value of the following two-player zero sum game: (**5 marks**)

		Player 2	
		a	b
Player 1	A	1	1
	B	2	1

Answer: Do it yourself.

2. Prove or disprove: The value of the two-player zero-sum game given by the matrix A is 0. Then the value of the two-player zero-sum game given by the matrix -A is also 0. (**5 marks**)

Answer: (*Disprove*) Consider the following matrix game

$$
\begin{array}{cc|c|c|c|}
 & & \multicolumn{3}{c}{\text{Player 2}} \\
 & & a & b & c \\
\cline{3-5}
\text{Player 1} & A & 0 & 0 & 1 \\
\cline{3-5}
 & B & -1 & 1 & 0 \\
\cline{3-5}
\end{array}
$$

If Player 1 plays the strategy $[x\ (A), 1-x\ B]$, the expected utility of Player 1 if Player 2 plays a, b, and c are $x-1$, $1-x$, and x, respectively. This implies the value of the game is 0. For the matrix game where the matrix is the negative of the above matrix, the value is $-\frac{1}{2}$.

3. A two-player game $G = \langle \{1,2\}, (S_i)_{i\in\{1,2\}}, (u_i)_{i\in\{1,2\}} \rangle$ is called symmetric if

   (i) each player has the same set of strategies: $S_i = S_j$ for each $i, j \in N$, and

   (ii) $u_i(s_i, s_j) = u_j(s_j, s_i)$ for all $i, j \in \{1,2\}$, all $s_i \in S_i$, and all $s_j \in S_j$.

   Prove or disprove: In every two-player symmetric game where $|S_1| = |S_2| = 2$ there exists a symmetric equilibrium in mixed strategies: an equilibrium $\sigma = (\sigma_i)_{i\in\{1,2\}}$ satisfying $\sigma_1 = \sigma_2$ **(5 marks)**

   Answer: (*Prove*) Note that by the restriction of symmetricity, the matrix of the game will look like the following:

$$
\begin{array}{cc|c|c|}
 & & \multicolumn{2}{c}{\text{Player 2}} \\
 & & A & B \\
\cline{3-4}
\text{Player 1} & A & (x,x) & (z,w) \\
\cline{3-4}
 & B & (w,z) & (y,y) \\
\cline{3-4}
\end{array}
$$

   If either $x \geq w$ or $y \geq z$, there is a symmetric equilibrium. Suppose these two conditions are not true, i.e., $x < w$ and $y < z$. We show that there is a symmetric mixed NE. Consider a strategy profile $[\alpha\ (A), 1-\alpha\ (B)], [\beta\ (A), 1-\beta\ (B)]$ that is an NE. This means by the indifference principle:

$$\alpha x + (1-\alpha)z = \alpha w + (1-\alpha)y$$

$$\implies \alpha = \frac{z-y}{z-y+w-x} \in (0,1) \ \ (\text{ as } x < w \text{ and } y < z).$$

   Similarly,

$$\beta x + (1-\beta)z = \beta w + (1-\beta)y$$

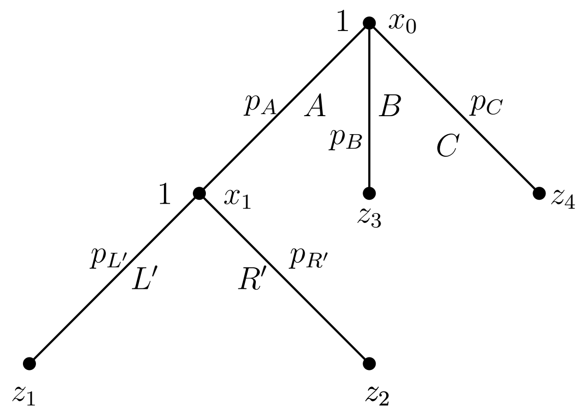
$$\implies \beta = \frac{z-y}{z-y+w-x} \in (0,1) \ \ (\text{ as } x < w \text{ and } y < z).$$

   As $\alpha = \beta$, we have a symmetric NE.

4. Determine all the behavioral strategies that are outcome equivalent with the following mixed strategy

AL'	AR'	BL'	BR'	CL'	CR'
0.5	0	0.2	0.1	0.1	0.1

in the following one-player extensive form structure:



(**5 marks**)

Answer: For the given mixed strategy σ, the outcome equivalent behavioral strategy is the following:

$$b_{x_0}(A) = \frac{\sigma(AL') + \sigma(AR')}{\sigma(AL') + \sigma(AR') + \sigma(BL') + \sigma(BR')\sigma(CL') + \sigma(CR')}$$
$$\implies b_{x_0}(A) = 0.5,$$

$$b_{x_0}(B) = \frac{\sigma(BL') + \sigma(BR')}{\sigma(AL') + \sigma(AR') + \sigma(BL') + \sigma(BR')\sigma(CL') + \sigma(CR')}$$
$$\implies b_{x_0}(B) = 0.3,$$

$$b_{x_0}(C) = \frac{\sigma(CL') + \sigma(CR')}{\sigma(AL') + \sigma(AR') + \sigma(BL') + \sigma(BR')\sigma(CL') + \sigma(CR')}$$
$$\implies b_{x_0}(C) = 0.2,$$

$$b_{x_1}(L') = \frac{\sigma(AL')}{\sigma(AL') + \sigma(AR')}$$
$$\implies b_{x_1}(L') = 1,$$

and

$$b_{x_1}(R') = \frac{\sigma(AR')}{\sigma(AL') + \sigma(AR')}$$
$$\implies b_{x_1}(R') = 0.$$