Department of Humanities and Social Sciences
Indian Institute of Technology Kanpur

Answer all questions. Write answers in a separate booklet.
Max. Marks: 30 Time: 2hrs

> Logical investigations can obviously be a useful tool for philosophy. They must, however, be informed by a sensitivity to the philosophical significance of the formalism and by a generous admixture of common sense, as well as a thorough understanding both of the basic concepts and of the technical details of the formal material used. It should not be supposed that the formalism can grind out philosophical results in a manner beyond the capacity of ordinary philosophical reasoning. There is no mathematical substitute for philosophy.(Saul Kripke (1940-2022))

1. **Answer whether the following statements are True or False. Justify your answer preferably with a short note or a remark.**[1] (10)

    1. In K, the following argument is valid: If there is rain, there should also be precipitation. There is a possibility of precipitation. Therefore, There is a possibility of precipitation.
    2. The following wff in K, $\Box\alpha \vee \Box\beta$ is a logical consequence of $\Box(\alpha \wedge \beta)$.
    3. The following well formed formulas are logically equivalent: $\Box\Diamond\Box\Diamond\Box\Diamond\alpha \leftrightarrow \Box\Diamond\alpha$ in $S4$ ($\leftrightarrow$ is same as $\equiv$)
    4. The wff $\Diamond P \rightarrow \Box P$, (i.e., It is possible then it is nessaey) holds in an axiomatic system that respects reflexivity.
    5. The well-formed formula $\Box\Diamond\Box\Diamond\alpha \leftrightarrow \Box\Diamond\alpha$ holds in a Kripke model where the accessibility relation is both relflexive and transitive.
    6. $\Box\Psi \vdash \Diamond\Diamond\Psi$ is valid relative to the set of models whose accessibility relation is constrained by a Reflexive relation between the possible worlds.
    7. Normal Modal Logic $S5$ considers the following group of well-formed formulas to be inconsistent. $\{\Box(\Phi \rightarrow \Psi), \Box(\top \rightarrow \Phi), \Diamond\neg\Psi\}$.
    8. In propositional Modal Logic K, the sentence "There is evil in the world (E)" is logically incompatible with "There is a God (G)," is more accurately translated as $\Diamond(G \wedge \neg E)$.
    9. Necessarily, water is either H_2O (p) or it might be XYZ (q) is better translated as $\Box(p \vee \Diamond q)$.
    10. The following argument is valid in $S5$. *If Plato is right, then it is necessary that ideas are superior to material things. It is possible that ideas are not superior to material things. Therefore, Plato is not right [You may use propositional variables: P, S].*

2. Answer the questions below. Where the inference is invalid, create counterexamples using Kripke structures ($< W, R, V >$). (20)

    1. Show whether the following wffs are valid or invalid in $\{S4, S5\}$ using semantic tableaux method for normal modal logic. Where the tableau does not close, use it to define a counter model. [6M]

(a) $((\Box \Diamond p \wedge \Box \Diamond q) \rightarrow \Diamond (p \wedge q))$

(b) $\Diamond \Diamond \Box \Box p \leftrightarrow \Diamond \Diamond \Box p$

2. In Shakespeare's The Merchant of Venice, Portia screened her suitors by asking them to determine which of three caskets concealed her portrait, while inscriptions on the caskets contained riddles that assessed the moral character of her potential partners. What is the Name of this Book? is a classic. Raymond Smullyan, a logician, imagined several generations of clever Portias presenting their suitors with caskets inscribed with statements that, when pieced together, constituted a logic puzzle whose solution would lead to the portrait. The objective is to choose the correct casket based on the clues provided by the casket inscriptions. Portia invites the user to choose the casket concealing the portrait based on knowing the overall number of true statements on the caskets. **Of all the statments on the caskets, only one is true.** Sove this puzzle using semantic tableaux method [4M].

   (a) Casket 1: The portrait is not in Casket 3.

   (b) Casket2: This casket has the portriat.

   (c) Casket 3: The portrait is in casket 2.

3. Provide an explanation of the paradox of material implication and a discussion of its occurrences. Explain how, according to C. I. Lewis, preserving the distinction between intensional and extensional disjunction would assist in resolving the paradox of material implication. Do you agree with Lewis's argument? Demonstrate that the subsequent instance of material implication does not apply to K: $\nvdash \Box(p \rightarrow \Box(q \rightarrow p))$ [4M]

4. Construct Kripke diagram and show whether the following formulas hold with respect to the possible worlds mentioned below: In the Kripke model $\{W, R, V\}$, Let $W = \{w1, w2, w3, w4\}$, $R = \{(w2, w1), (w1, w4), (w2, w2), (w3, w2), (w4, w3), (w4, w4)\}$ and V is a valuation function defined as follows: $M, w1 \models p, q$ and $M, w2 \models p \neg q$ $M, w3 \models \neg p$, $\neg q$ and $M\, w4 \models \neg p$, q. Evaluate the following formulas:[6M]

   (a) $p \wedge \rightarrow \Box\Box\Box\neg p$

   (b) $\neg p \wedge q \rightarrow \Diamond p$

   (c) $\Box \Diamond \Box(p \wedge q) \rightarrow \Diamond\Box\Diamond(p \vee q)$

Notes

[1]

Material Implication: $p \rightarrow q = (\neg p \vee q) = \neg(p \wedge \neg q)$

Strict Implication $p \dashrightarrow q = \Box(p \rightarrow q)$

D1: $p \leftrightarrow q \downarrow ((p \wedge q) \vee (\neg p \wedge \neg q))$

D2: $\neg(p \leftrightarrow q) \downarrow ((p \wedge \neg q) \vee (\neg p \wedge q))$

Duality: $\neg \Box p, i \downarrow \Diamond \neg p, i$

Duality: $\neg \Diamond p, i \downarrow \Box \neg p, i.$

$\Box D$ **rule:** $\Box p, i \downarrow \Diamond p, j$, where $i\, R\, j$.

$\Box T$ **rule:** $\Box p, i \downarrow p, i$, where $i\, R\, i$.

$\Box\phi$**:** $M, w \models \Box\phi$ iff $M, u \models \phi$ for all worlds u with $w\, R\, u$.

$\Diamond\phi$**:** $M, w \models \Diamond\phi$ iff $M \models \phi$ for some world u with $w\, R\, u$.

$\Diamond S5$ **rule** $\Diamond p, i \downarrow \Box p, j$ and j is new.

$\Box$ **rule:** $\Box p, i$ [i R j] $\downarrow p, j$.

$\Diamond$ **Rule:** $\Diamond p, i \downarrow i\, R\, j\, (p, j)$ where j is NEW

Constraints on R**:** K: No constraints on R; T: Reflexive: B: Reflexive,Symmetric S4: Reflexive, Transitive; S5: reflexive, transitive, symmetric frames.

Reflexive: $i\ i$); Transitivity $(i\, R\, j)$ and $j\, R\, k$, then $(i\, R\, K)$