
MTH 424 - PARTIAL DIFFERENTIAL EQUATION

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Assignment 2

1. Consider the following PDEs, verify the transversality condition and solve
 - (a) $u_t + (x \cos t)u_x = 0$ with $u(x, 0) = \frac{1}{x^2+1}$, $x \in \mathbb{R}$ and $t > 0$.
 - (b) $uu_x + u_y = 1$ with $u(x, x) = x/2$, $x \in (0, 1]$.
 - (c) $u_{x_1} + 2u_{x_2} + u_{x_3} + x_2u_{x_4} = x_1 + x_2$ with $u(x_1, x_2, x_3, 0) = x_2 + x_3$.
2. Use the method of characteristic to solve $u_x^2 + u_y = 0$ for $x \in \mathbb{R}$ and $y \geq 0$ with $u(x, 0) = x^2$.
3. Sketch the characteristic curve of $uu_x + u_y = 0$ with the following initial conditions:

(a)

$$u(x, 0) = \begin{cases} 0 & \text{if } x < 0 \\ 1 & \text{if } x \geq 0. \end{cases}$$

(b)

$$u(x, 0) = \begin{cases} 1 & \text{if } x < 0 \\ 0 & \text{if } x \geq 0. \end{cases}$$

(c) $u(x, 0)$ is a smooth and increasing function with

$$u(x, 0) = \begin{cases} 0 & \text{if } x < 0 \\ 1 & \text{if } x \geq 1. \end{cases}$$

4. Consider the equation

$$u_{xx} + 4u_{xy} + u_x = 0.$$

- (a) Bring the equation to a canonical form.
 - (b) Find the general solution $u(x, y)$ and check by substituting back into the equation that your solution is indeed correct.
 - (c)* Find a specific solution satisfying $u(x, 8x) = 0$, $u_x(x, 8x) = 4e^{-2x}$.
5. Consider the equation

$$u_{xx} - 6u_{xy} + 9u_{yy} = xy^2.$$

Find a coordinates system (ξ, η) in which the equation transformed to the canonical form.

6. Consider the equation

$$u_{xx} + yu_{yy} = 0.$$

Find the canonical forms of the equation for the domain where the equation is hyperbolic, and for the domain where it is elliptic.