MTH 101-Calculus

Spring-2021

Assignment 8-Vectors, Curves, Surfaces, Vector Functions

1. Consider the planes $x - y + z = 1,\ x + ay - 2z + 10 = 0$ and $2x - 3y + z + b = 0$, where a and b are parameters. Determine the values of a and b such that the three planes
   (a) intersect at a single point,
   (b) intersect in a line,
   (c) intersect (taken two at a time) in three distinct parallel lines.
2. Determine the equation of a cone with vertex $(0, -a, 0)$ generated by a line passing through the curve $x^2 = 2y,\ z = h$.
3. The velocity of a particle moving in space is $\frac{d}{dt}c(t) = (\cos t)\vec{i} - (\sin t)\vec{j} + \vec{k}$. Find the particle's position as a function of t if $c(0) = 2\vec{i} + \vec{k}$. Also find the angle between its position vector and the velocity vector.
4. Show that $c(t) = \sin t^2 \vec{i} + \cos t^2 \vec{j} + 5\vec{k}$ has constant magnitude and is orthogonal to its derivative. Is the velocity vector of constant magnitude?
5. Find the point on the curve $c(t) = (5\sin t)\vec{i} + (5\cos t)\vec{j} + 12t\vec{k}$ at a distance 26π units along the curve from $(0, 5, 0)$ in the direction of increasing arc length.
6. Reparametrize the curves
   (a) $c(t) = \frac{t^2}{2}\vec{i} + \frac{t^3}{3}\vec{k},\ \ 0 \le t \le 2,$
   (b) $c(t) = 2\cos t\vec{i} + 2\sin t\vec{j},\ \ 0 \le t \le 2\pi$

   in terms of arc length.
7. Show that the parabola $y = ax^2,\ a \neq 0$ has its largest curvature at its vertex and has no minimum curvature.