Question: Prove that the result of iterated elimination of strictly dominated strategies (that is, the set of strategies remaining after the elimination process has been completed) is independent of the order of elimination.

Proof. Let $G = \langle N, \{S_i\}_{i\in N}, \{u_i\}_{i\in N}\rangle$ be a strategic form game where the notations have there usual meanings. Assume for contradiction that using iterated elimination of strictly dominated strategies completely we obtain two different reduced games $\hat{G} = \langle N, \{\hat{S}_i\}_{i\in N}, \{u_i\}_{i\in N}\rangle$ and $\hat{\hat{G}} = \langle N, \{\hat{\hat{S}}_i\}_{i\in N}, \{u_i\}_{i\in N}\rangle$ (this means there exists $j \in N$ such that $\hat{S}_j \neq \hat{\hat{S}}_j$). Let $k \in N$ be an agent such that $\hat{S}_k \setminus \hat{\hat{S}}_k \neq \emptyset$ and $s_k \in \hat{S}_k \setminus \hat{\hat{S}}_k$ be the first strategy in the set $\cup_{i\in N}\hat{S}_i \setminus \hat{\hat{S}}_i$ that was eliminated at some stage while obtaining $\hat{\hat{G}}$. Since s_k was eliminated while obtaining $\hat{\hat{G}}$, there must be a strategy $t_k \in S_k$ such that t_k strictly dominated s_k at that stage. We distinguish two cases:

Case 1: $t_k \in \hat{S}_k$.

Since $\hat{G}$ cannot be further reduced, there exists a strategy-profile $r_{-k} \in \hat{S}_{-k}$ of the other players such that

$$u_k(s_k, r_{-k}) \geq u_k(t_k, r_{-k}).$$

Since s_k was eliminated by t_k at some stage of obtaining $\hat{\hat{G}}$, it must be the case that at least some component of r_{-k} were not present at that stage. Let r_l be a component of r_{-k} that was not present at that stage. This means $r_l \notin \hat{\hat{S}}_l$ and it was eliminated before s_k was eliminated. But as $r_l \in \hat{S}_l$, this contradicts the fact that s_k is the first strategy in the set $\cup_{i\in N}\hat{S}_i \setminus \hat{\hat{S}}_i$ that was eliminated at some stage while obtaining $\hat{\hat{G}}$.

Case 2: $t_k \notin \hat{S}_k$.

Complete the proof... ■