Test Name : 103AA mid-sem exam (27/12/2020)

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Test Start Time	Marks Scored	Total Questions
2020-12-27 09:30:34	103.0 / 120.0	24

Attempted Questions	Correct Questions	Incorrect Questions
24	21	3

Skipped Questions	Pending Evaluation
0	0

Status of application for viewing evaluated answers	Actions
Not Applied	Apply for viewing evaluated answers

List of Sections

Section - 1

Marks per question : 2 | Marks Scored : 26.0

Q No.	Q. Type	Status	Marks	Comment
1	Multiple Choice - Single Answer	✗	0.0	-

Hide Answer

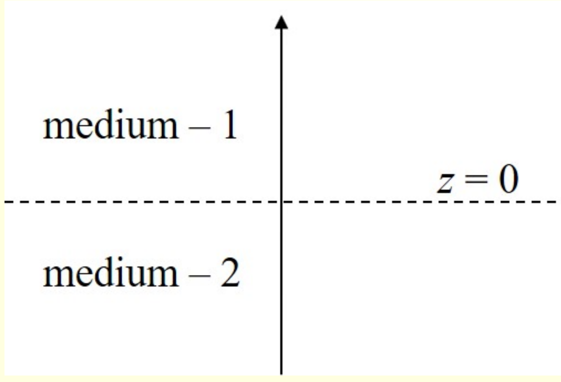
What is the value of the surface integral $\oint_S \frac{\vec{r}}{r^3} \cdot d\vec{a}$ for $r > 0$?

- 4π
- $\frac{\pi}{4}$
- 0
- 2π

Q No.	Q. Type	Status	Marks	Comment
2	Multiple Choice - Single Answer	✓	2.0	-

Hide Answer

Two infinitely large linear dielectric media (medium-1 and medium-2 with dielectric constant 2 and 5, respectively) meet at the $z = 0$ plane as shown in the figure. A uniform electric field exists everywhere. For $z \geq 0$, the electric field is given by $\vec{E}_1 = 2\hat{x} - 3\hat{y} + 5\hat{z}$. There are no free charges at the interface separating the two media. The electric displacement vector in the medium-2 is given by



- $\vec{D}_2 = \epsilon_0 \left[10\hat{x} - 15\hat{y} + 10\hat{z}\right]$
- $\vec{D}_2 = \epsilon_0 \left[4\hat{x} + 6\hat{y} + 10\hat{z}\right]$
- $\vec{D}_2 = \epsilon_0 \left[10\hat{x} + 15\hat{y} + 10\hat{z}\right]$
- $\vec{D}_2 = \epsilon_0 \left[4\hat{x} - 6\hat{y} + 10\hat{z}\right]$

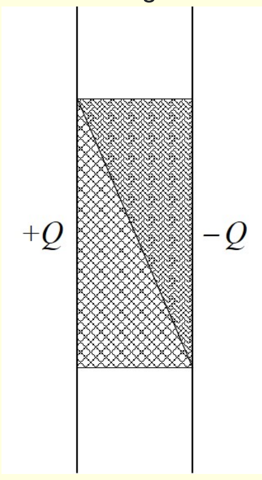
3 Multiple Choice - Single Answer ✔ 2.0 - Hide Answer

Among the given four forces in Cartesian and Spherical polar coordinate system,
(i) $\vec{F}_1 = K\exp\left(\frac{-r^2}{R^2}\right)\hat{r}$
(ii) $\vec{F}_2 = K\left(x^3\hat{y} - y^3\hat{z}\right)$
(iii) $\vec{F}_3 = K\left(x^3\hat{x} + y^3\hat{y}\right)$
(iv) $\vec{F}_4 = K\left(\frac{\hat{\phi}}{r}\right)$
where K is a constant, identify the correct option

- (i) and (ii) are conservative but (iii) and (iv) are not
- (i) and (iii) are conservative but (ii) and (iv) are not
- (iii) and (iv) are conservative but (i) and (ii)are not
- (ii) and (iii) are conservative but (i) and (iv) are not

4 Multiple Choice - Single Answer ✖ 0.0 - Hide Answer

The space between two plates of a capacitor carrying charges $+Q$ and $-Q$ is filled with two different dielectric materials, as shown in the figure. Across the interface of the two dielectric materials, which one of the following statements is correct?



- $\vec{E}$ is continuous and $\vec{D}$ is discontinuous
- $\vec{E}$ and $\vec{D}$ are discontinuous
- $\vec{E}$ and $\vec{D}$ are continuous
- $\vec{E}$ is discontinuous and $\vec{D}$ is continuous

5 Multiple Choice - Single Answer ✔ 2.0 - Hide Answer

What is $\nabla \cdot \vec{E}$ for $s > 0$, where $\vec{E}$ is the electric field due to an infinite line charge of uniform charge density λ?

- $\frac{1}{4\pi\epsilon_0}\frac{\lambda}{s^2}$
- $\frac{\lambda}{\epsilon_0 s^2}$
- $\frac{\lambda}{s^2}$

0

6	Multiple Choice - Single Answer	✔	2.0	-	Hide Answer

If a vector $\vec{A} = y\hat{x} + x\hat{y} + \frac{x^2}{\sqrt{x^2+y^2}}\hat{z}$ is transformed from Cartesian (x, y, z) to Cylindrical (s, ϕ, z) coordinates, it becomes

- $\vec{A} = 2s\sin\phi\cos\phi\hat{s} + s(\cos^2\phi + \sin^2\phi)\hat{\phi} + s\cos^2\phi\hat{z}$
- $\vec{A} = s\cos^2\phi\hat{s} - s\sin^2\phi\hat{\phi} + s\cos^2\phi\hat{z}$
- $\vec{A} = s\cos^2\phi\hat{s} + s\sin^2\phi\hat{\phi} + s\cos^2\phi\hat{z}$
- $\vec{A} = 2s\sin\phi\cos\phi\hat{s} + s(\cos^2\phi - \sin^2\phi)\hat{\phi} + s\cos^2\phi\hat{z}$
- $\vec{A} = s\sin\phi\cos\phi\hat{s} + s(\cos^2\phi - \sin^2\phi)\hat{\phi} + s\cos^2\phi\hat{z}$
- $\vec{A} = 2s\cos^2\phi\hat{s} - s\sin^2\phi\hat{\phi} + s\cos^2\phi\hat{z}$

7	Multiple Choice - Single Answer	✔	2.0	-	Hide Answer

An infinite conducting slab, carrying a uniform charge density σ, is kept in a horizontal plane. Another infinite slab of thickness t, made of a linear dielectric material of dielectric constant k, is kept above the conducting slab. The bound charge density on the upper surface of the dielectric slab is

- 0
- $\frac{\sigma}{2k}$
- -$\frac{\sigma}{2k}$
- $\frac{\sigma(k-1)}{k}$
- $\frac{\sigma(k-2)}{k}$
- $-\sigma$

8	Multiple Choice - Single Answer	✔	2.0	-	Hide Answer

What is the value of the integral $\oint_{\text{all space}} (r^2 + \vec{r}\cdot\vec{a} + a^2)\delta(\vec{r} - \vec{a})d\tau$, where $\vec{a}$ is a constant vector of magnitude a?

- 0
- 4π
- a^2
- $3a^2$

9	Multiple Choice -	✔	2.0	-	Hide Answer

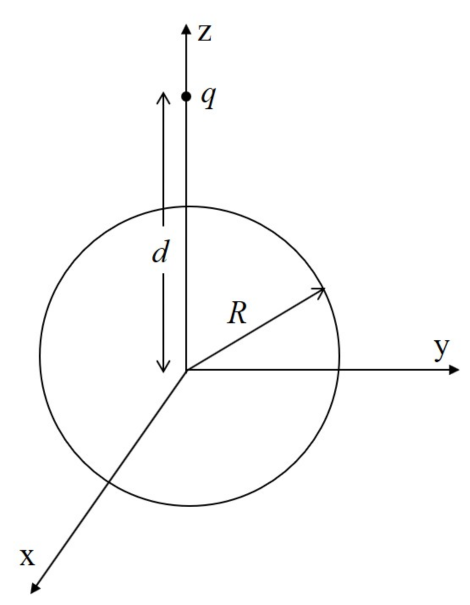
Single

The unit vector, which points from a point $z = h$ on the z axis toward a point $(s, \phi, 0)$ in cylindrical coordinates, is

- $\frac{(\hat{s}-\hat{z})}{\sqrt{2}}$
- $\frac{s\hat{s}-h\hat{z}}{\sqrt{s^2+h^2}}$
- $\frac{(\hat{s}+\hat{z})}{\sqrt{2}}$
- $\frac{s\hat{s}+h\hat{z}}{\sqrt{s^2-h^2}}$

10	Multiple Choice - Single Answer	✔	2.0	-	Hide Answer

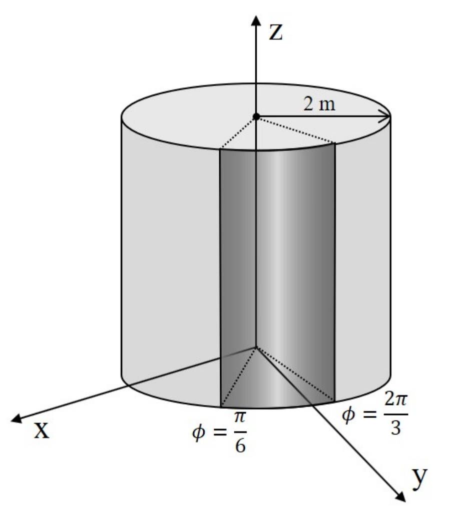
A point charge q is kept at a distance d on z-axis outside a spherical surface of radius R (see figure). The average potential over the surface of the sphere is



- $V_{\text{avg}} = \frac{1}{4\pi\epsilon_0}\frac{q}{R}$
- $V_{\text{avg}} = \frac{1}{4\pi\epsilon_0}\frac{q}{d}$
- $V_{\text{avg}} = \frac{V_{\text{centre}}}{2}$
- $V_{\text{avg}} = -V_{\text{centre}}$

11	Multiple Choice - Single Answer	✔	2.0	-	Hide Answer

For a cylinder of radius 2 m and height 5 m, what is the area of the shaded curved surface between, $\phi = \frac{\pi}{6}$ and $\phi = \frac{2\pi}{3}$ (see figure)?



- $5\pi\ \text{m}^2$

$\frac{5\pi}{6}\ \mathrm{m}^2$

$\frac{2\pi}{3}\ \mathrm{m}^2$

$2\pi\ \mathrm{m}^2$

12	Multiple Choice - Single Answer	✔	2.0	-	Hide Answer

For a scalar function φ satisfying the Laplace equation, $\vec{\nabla}\varphi$ has

- non-zero curl and zero divergence
- non-zero curl and non-zero divergence
- zero curl and non-zero divergence
- zero curl and zero divergence

13	Multiple Choice - Single Answer	✔	2.0	-	Hide Answer

The gradient of the scalar function $\phi = \ln|\vec{r}|$

- $\frac{\vec{r}}{r^2}$
- $\vec{r}$
- $\frac{\vec{r}}{r^3}$
- $\hat{r}$

14	Multiple Choice - Single Answer	✔	2.0	-	Hide Answer

The axis of an infinitely long thin cylindrical shell coincides with the z -axis. It carries a surface charge density $\sigma_0 \cos\phi$, where σ_0 is a constant. The magnitude of the electric field inside the cylinder is

- $\frac{\sigma_0}{2\epsilon_0}$
- 0
- $\frac{\sigma_0}{3\epsilon_0}$
- $\frac{\sigma_0}{4\epsilon_0}$

15	Multiple Choice - Multiple Answers	✔	2.0	-	Hide Answer

Identify the correct statement(s):
van der Waals interaction

- energy decays with separation as r^{-n}, where $6 < n < 12$

- [x] is a universal interaction, which acts between every two objects
- [x] is weak compared to Coulomb force
- [ ] is always repulsive
- [ ] acts between a charge and a dipole
- [x] is always attractive

Section - 2 Marks per question : 2 **Marks Scored : 8.0**

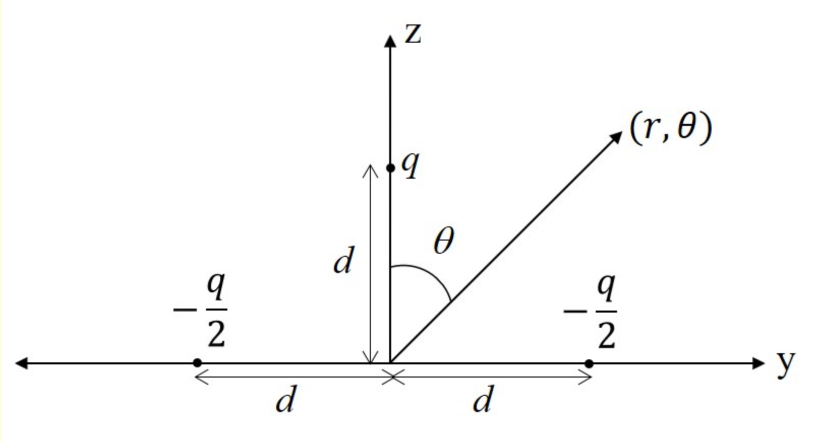
Q No.	Q. Type	Status	Marks	Comment	
1	Fill in the Blanks	✔	2.0	-	Hide Answer

A point charge is placed between two semi-infinite grounded conducting plates which are inclined at an angle of 30° with respect to each other. The number of image charges is 11

Q No.	Q. Type	Status	Marks	Comment	
2	Fill in the Blanks	✔	2.0	-	Hide Answer

A system of three charges is shown in the figure. For $r = 10$ m, $\theta = 60^\circ$, $q = 10^{-6}$ Coulomb, and $d = 10^{-3}$ m, the electric dipole potential in volts (rounded off to three decimal places) at a point (r, θ) is 0.045

[Use: $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \ \frac{\mathrm{Nm}^2}{\mathrm{C}^2}$]



Q No.	Q. Type	Status	Marks	Comment	
3	Fill in the Blanks	✔	2.0	-	Hide Answer

A spherical conductor of radius a is placed in a uniform external electric field $\vec{E} = E_0\hat{z}$. The potential at a point $P(r, \theta)$ for $r > a$, is given by $V(r, \theta) = \text{constant} - E_0 r\cos\theta + \frac{E_0 a^3}{r^2}\cos\theta$, where r is the distance of P from the centre O of the sphere and θ is the angle OP makes with the z-axis.The charge density on the sphere at $\theta = 30^\circ$ is given by $n\epsilon_0 E_0$. The value of n (rounded off to three decimal places) is 2.598

Q No.	Q. Type	Status	Marks	Comment	
4	Fill in the Blanks	✖	0.0	-	Hide Answer

Charge density in a one dimensional space is given by $\rho = Q\left[\delta(x - x_0) - \delta(x + x_0)\right]$. The electric field due to this charge distribution at point $(2x_0, 0, 0)$ is given as $\vec{E}(\vec{x}) = n\frac{Q}{\pi\epsilon_0 x_0^2}\hat{x}$. The value of n (rounded off to three decimal places) is 0.250 Expected Solutions: **0.125**

Q No.	Q. Type	Status	Marks	Comment	
5	Fill in the Blanks	✔	2.0	-	Hide Answer

A Hydrogen atom of radius $1\ \mathrm{nm}$ is kept in an uniform external electric field of magnitude $10^3\ \mathrm{V/m}$. Assuming that the shape of the atom remains spherical, its electronic polarizability is $10^n\ \mathrm{Fm^2}$. The value of n is -37

[Use: $\frac{1}{4\pi\epsilon_0} = 10^{10}\ \frac{\mathrm{Nm^2}}{\mathrm{C^2}}$

Section - 3 Marks per question : 20 | Marks Scored : 69.0

Q No.	Q. Type	Status	Marks	Comment	
1	File Upload	✔	14.0	-	Hide Answer

(a) Consider two concentric rings of inner radius a with linear charge density $-\lambda$ and outer radius b with linear charge density $+\lambda$ in $x-y$ plane with their centre coinciding with the origin as shown in the adjoining figure.

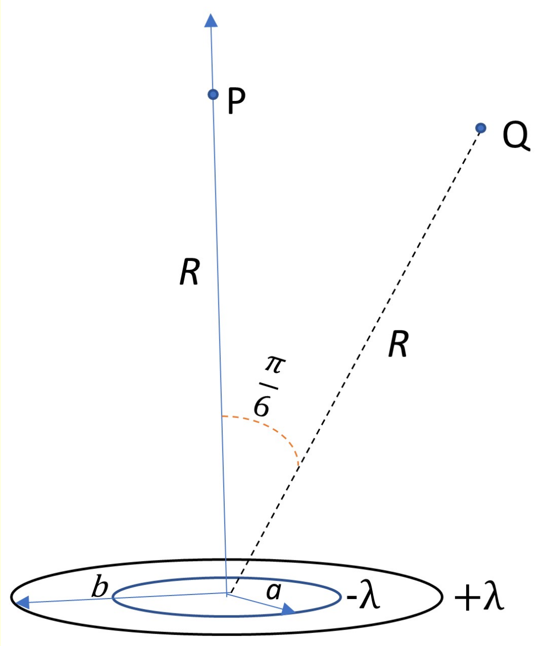
i) Write down the exact potential of the charge configuration at a point P which is at a distance R on the z-axis.

ii) Use the above result to write down the potential for the point P (considering R to be large as compared to the dimensions of the charge configuration) in terms of contributions from monopole, dipole and quadrupole terms.

iii) Calculate explicitly the dipole moment of the charge configuration.

iv) For a point Q whose spherical polar co-ordinates are $(R, \frac{\pi}{6}, \frac{\pi}{2})$, find for which terms there is a change in the potential expansion as compared to that of point P, and by what factor considering R to be large.

Marks: 1, 3, 1, 1

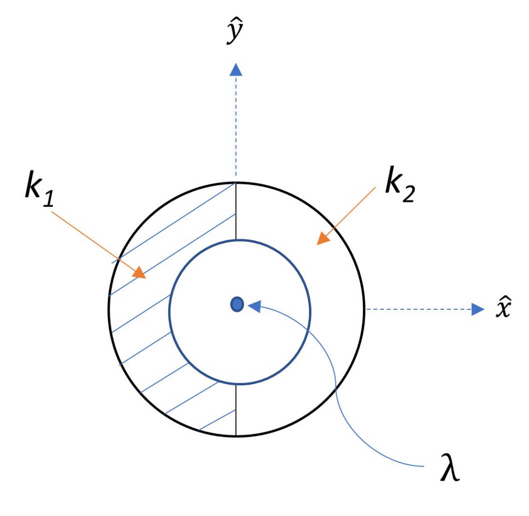


(b) The two halves of a long circular **cylindrical shell** (inner radius a, and outer radius b) is made of two different materials of dielectric constant k_1 and k_2 (relative permittivity) as shown in the figure of its cross-section. At the center of the cylinder a line charge $+\lambda$ runs all along the axis.

Find $\vec{D}$, $\vec{E}$, $\vec{P}$ and all bound charges for $0 < s \leq b$, where s is the radial distance from the axis in cylindrical coordinates.

Marks: 14

IMPORTANT: Each of your answers must be put inside a box, clearly specifying the region for which it is valid.



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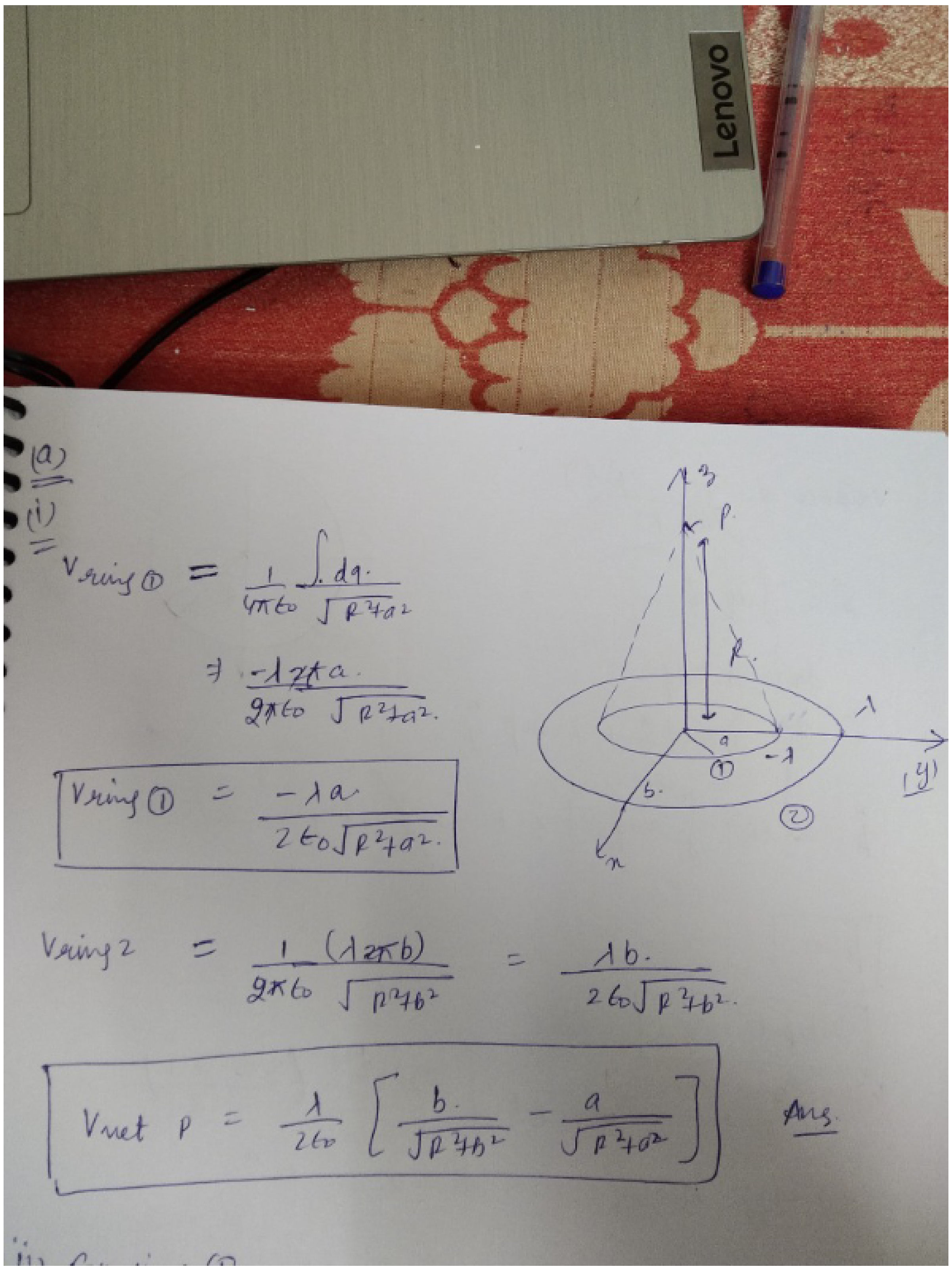

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2	File Upload	✔	20.0	-	Hide Answer

(a) Consider the function $\varphi(\mathbf{r}) = x^2 + y^2 + z^2$. Given $\mathbf{r}_2 = (10, 10, 10)$ and $\mathbf{r}_1 = (11, 11, 10)$. Find $\Delta\varphi = \varphi(\mathbf{r}_2) - \varphi(\mathbf{r}_1)$ in two ways:
(i) exactly by direct calculation.
(ii) approximately by using the gradient $\nabla\varphi$.
Marks: 5, 5

(b) Evaluate $\oint(xdy - ydx)$ over the unit circle centered at the origin in the $xy-$plane in two ways:
(i) by direct integration.
(ii) using Stokes' theorem.
Marks: 5, 5

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(a) (i) $\varphi(\vec{x}_2) = 3 \times 100 = 300$

$\varphi(\vec{x}_1) = 121 + 121 + 100 = 342.$

$$\boxed{\Delta\varphi = -42.}$$

(ii) $\vec{\nabla}\varphi = \hat{x}\frac{\partial\varphi}{\partial x} + \hat{y}\frac{\partial\varphi}{\partial y} + \hat{z}\frac{\partial\varphi}{\partial z}$

$\Rightarrow \hat{x}(2x) + \hat{y}(2y) + \hat{z}(2z).$

$(\vec{\nabla}\varphi) = 2(x\hat{x} + y\hat{y} + z\hat{z})$

$(\vec{\nabla}\varphi \;@\; x = \vec{x}_1) = 2(11\hat{x} + 11\hat{y} + 10\hat{z})$

$\vec{d} = (10\hat{i} + 10\hat{j} + 10\hat{k}) - (11\hat{i} + 11\hat{j} + 10\hat{k})$

$\Rightarrow (-\hat{i} - \hat{j})$

separation vector from $\vec{x}_1$ to $\vec{x}_2$

$\Delta\varphi = (\vec{\nabla}\varphi) \cdot (\vec{d})$

$\Rightarrow -22 - 22.$

$$\boxed{\Delta\varphi = -44.}$$

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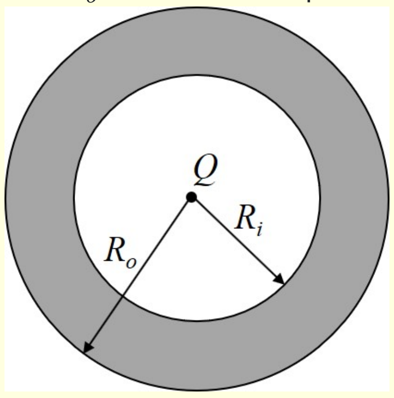
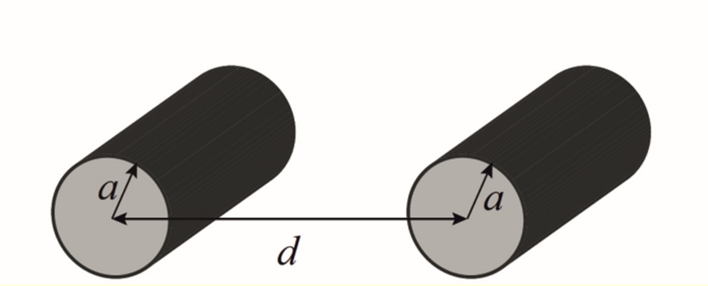
3	File Upload	✔	19.0	No comments	Hide Answer

(a) A positive point charge Q is kept at the center of a spherical conducting shell of an inner radius R_i and an outer radius R_o. Determine and plot $\vec{E}$ and V as function of the radial distance r.
Mark: 12



(b) As shown in the figure, two long identical cylindrical conductors of radius a are parallel and separated by a distance d, where $d >> a$. Find the capacitance per unit length of the system.
Mark: 8



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(3) (a)

i) $\vec{E}_r$

$E\,4\pi r^2 = \left(\frac{Q}{\epsilon_0}\right)$ $(r<R_i)$

$\boxed{E = \frac{Q}{4\pi\epsilon_0 r^2}\hat{r} \quad (r<R_i)}$

$\boxed{E = 0. \quad \text{for } R_i<r<R_o.}$

$E\,4\pi r^2 = \left(\frac{Q}{\epsilon_0}\right)$

$\boxed{E = \frac{Q}{4\pi r^2\epsilon_0}\hat{r} \quad \text{for } R>R_o.}$

(ii) Potential

a) for $(r>R_o)$

$$V = -\int_{\infty}^{r} \frac{Q}{4\pi\epsilon_0\, r^2}\, dr.$$

$\boxed{V = \frac{Q}{4\pi\epsilon_0 r}.}$

$\cancel{V = \frac{Q}{4\pi\epsilon_0 R}}$

(b) for $R_i<r<R_o$

$V = \frac{Q}{4\pi\epsilon_0 R_o}.$ (constant)

(c) for $r<R_i$

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4	File Upload	✔	16.0	No comments	Hide Answer

(a) Let the electric field in some region be given as $\vec{E} = \frac{C}{\epsilon_0}z^2\hat{z}$ where C is a constant. For this electric field calculate the following:
(i) Charge density inside a sphere of radius R centred at the origin.
(ii)Total charge in the sphere.
(iii) Electric flux through the sphere.
Marks: 2, 5, 3

(b) An isolated soap bubble of radius 1 cm is charged to a potential of 100 V. Calculate its electrostatic energy?
Marks: 4

(c) Two large plane sheets having surface charge densities $+\sigma$ and $-\sigma$ intersect at 90°. Find the electric field everywhere and sketch the field lines.
Marks: 6

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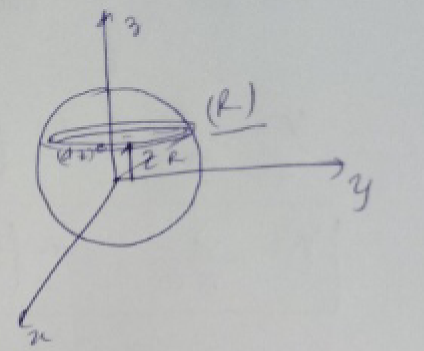
(a) $(\vec{\nabla}\cdot\vec{E}) = \left(\frac{\rho}{\epsilon_0}\right)$

$\frac{\rho}{\epsilon_0} = \left(\frac{c}{\epsilon_0}\right)(2z)$

$\boxed{\rho = 2cz.}$



(b) $dV_{element} = (\pi r^2\, dz)$ $r = \sqrt{R^2 - z^2}$

$= \pi(R^2 - z^2)\, dz.$

$\int dq = \int_{-R}^{R} 2cz\,\pi\,(R^2 - z^2)\, dz.$

$q = 2c\pi\left(R^2\frac{z^2}{2} - \frac{z^4}{4}\right)_{-R}^{R}$

$q = 2c\pi\left[\frac{R^2}{2}(0) - (0)\right]$

$\boxed{q = 0}$

(ii) $\cancel{\oint E}\ \phi_{sphere} = \left(\frac{q_{in}}{\epsilon_0}\right)$

since $q_{in} = 0$

$\boxed{\phi_{sphere} = 0.}$

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