HW-2

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1) From plots, it can be observed that Chi-square is not sub gaussian except when $n \to \infty$, it will tend to gaussian.

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3) $$f(t) = \sup_{\lambda} \left(\lambda t - \log(E(e^{\lambda X}))\right)$$

X - centred.

$$X = \begin{cases} 1, & p \\ 0, & 1-p \end{cases}$$

$$\Rightarrow Y = X - p = \begin{cases} 1-p, & p \\ -p, & 1-p \end{cases}$$

$E(Y) = 0$

$$\Rightarrow E(e^{\lambda Y}) = e^{-\lambda p}(1-p+pe^{\lambda})$$

$$\Rightarrow f(t) = \sup_{\lambda}\big(\underbrace{\lambda t + \lambda p - \log(1-p+pe^{\lambda})}_{g(\lambda)}\big)$$

$g'(\lambda) = 0$

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$$\Rightarrow p+t - \frac{pe^{\lambda}}{1-p+pe^{\lambda}} = 0$$

$$\Rightarrow pe^{\lambda} = (p+t)(1-p) + (p+t)pe^{\lambda}$$

$$\Rightarrow e^{\lambda} = \frac{(p+t)(1-p)}{p(1-p-t)}$$

$$\Rightarrow \lambda = \log\left(\frac{(p+t)(1-p)}{p(1-p-t)}\right)$$

(wherever defined)

say $k = \dfrac{(p+t)(1-p)}{p(1-p-t)}$

$$\Rightarrow \lambda = \log k$$

$$\Rightarrow \cancel{f(t) = (t+p)\log k}$$

$$\Rightarrow f(t) = (p+t)\log k - \log(1-p+kp)$$

$$\Rightarrow \boxed{f(t) = (p+t)\log k - \log((k-1)p+1) \quad \text{where } k = \frac{(p+t)(1-p)}{p(1-(p+t))}}$$

$\forall t$ wherever $f(t)$ is defined

4) **Benett's Inequality :-**

$X_1, X_2, \ldots, X_n$ - centred

$X_i \leq b, \quad b > 0$

$$\vartheta = \sum_{i=1}^{n} X_i^2 \quad , \quad S = \sum_{i=1}^{n} X_i$$

$$E(e^{\lambda S}) \leq e^{\frac{\vartheta}{b^2}\phi(\lambda b)}$$

$$P(S \geq t) = P(e^{\lambda S} \geq e^{\lambda t}) \leq \frac{E(e^{\lambda S})}{e^{\lambda t}}$$

$$\Rightarrow \boxed{P(S \geq t) \leq e^{-\lambda t + \frac{\vartheta}{b^2}\phi(\lambda b)}}$$

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$$f(\lambda) = \frac{v}{b^2}\phi(\lambda b) - \lambda t$$

$$f'(\lambda) = 0$$

$$\Rightarrow \frac{v}{b^{\cancel{2}}} \cdot \phi'(\lambda b) \cdot \cancel{b} = t$$

$$\Rightarrow \phi'(\lambda b) = \frac{bt}{v}$$

$\Rightarrow$

$$\phi(\lambda b) = e^{\lambda b} - \lambda b - 1$$

$$\phi'(\lambda b) = e^{\lambda b} - 1$$

$$\left(\frac{\partial \phi}{\partial(\lambda b)}\right)$$

$$\Rightarrow e^{\lambda b} = 1 + \frac{bt}{v}$$

$$\Rightarrow \lambda = \left(\frac{1}{b}\right) \log\left(1 + \frac{bt}{v}\right)$$

$$\cancel{\Rightarrow P(S \geq t) \leq e^{-\frac{t}{b}\log\left(1+\frac{bt}{v}\right)}}$$

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~~P(X>t)~~

$\Rightarrow P(S \geq t)$

$$\leq e^{-\lambda t + \frac{v}{b^2}\left(e^{\lambda b} - \lambda b - 1\right)}$$

$$= e^{-\frac{t}{b}\log\left(1+\frac{bt}{v}\right) + \frac{v}{b^2}\left(\frac{tb}{v} - \log\left(1+\frac{tb}{v}\right)\right)}$$

$$= e^{\underbrace{-\frac{t}{b}\log\left(1+\frac{tb}{v}\right) + \frac{t}{b} - \frac{v}{b^2}\log\left(1+\frac{bt}{v}\right)}_{h(t)}}$$

$$H(t) = \frac{v}{b^2}\left(\frac{tb}{v} - \log\left(1+\frac{tb}{v}\right) - \frac{tb}{v}\log\left(1+\frac{tb}{v}\right)\right)$$

$$= -\frac{v}{b^2}\left(\left(1+\frac{tb}{v}\right)\log\left(1+\frac{tb}{v}\right) - \frac{tb}{v}\right)$$

u

$$\Rightarrow \boxed{P(S \geq t) \leq e^{-\frac{v}{b^2}\left(h\left(\frac{bt}{v}\right)\right)}, \quad h(u) = (1+u)\log(1+u) - u.}$$