Optimization

Prob:3.14,3.15

N Chakradhar[1] Havish[2]

[1]EE16BTECH11022

[2]EE16BTECH11023

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Problem 3.14

- Solve

$$\min_{x} f(x) = 4x_1^2 + 2x_2^2 \tag{1}$$

with constraints

$$g_1(x) = 3x_1 + x_2 - 8 = 0 \tag{2}$$
$$g_2(x) = 15 - 2x_1 - 4x_2 \leq 0 \tag{3}$$

Solution

Considering the Lagrangian

$$L(x, \lambda, \mu) = f(x) + \lambda g_1(x) + \mu g_2(x) \tag{4}$$

$$= 4x_1^2 + 2x_2^2 + \lambda(3x_1 + x_2 - 8) + \mu(15 - 2x_1 - 4x_2) \tag{5}$$

$$\nabla L(x, \lambda, \mu) = \begin{pmatrix} 8x_1 + 3\lambda - 2\mu \\ 4x_2 + \lambda - 4\mu \\ 3x_1 + x_2 - 8 \\ -2x_1 - 4x_2 + 15 \end{pmatrix} = 0 \tag{6}$$

Solution

resulting in the matrix equation

$$\Rightarrow \begin{pmatrix} 8 & 0 & 3 & -2 \\ 0 & 4 & 1 & -4 \\ 3 & 1 & 0 & 0 \\ 2 & 4 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \lambda \\ \mu \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 8 \\ 15 \end{pmatrix} \tag{7}$$

$$\Rightarrow \begin{pmatrix} x_1 \\ x_2 \\ \lambda \\ \mu \end{pmatrix} = \begin{pmatrix} 1.7 \\ 2.9 \\ -3.12 \\ 2.12 \end{pmatrix} \tag{8}$$

Observations

- The solution (x_1, x_2) is a feasible local minima.
- The value of μ is positive.
- The feasible solution lies on the boundary of the inequality

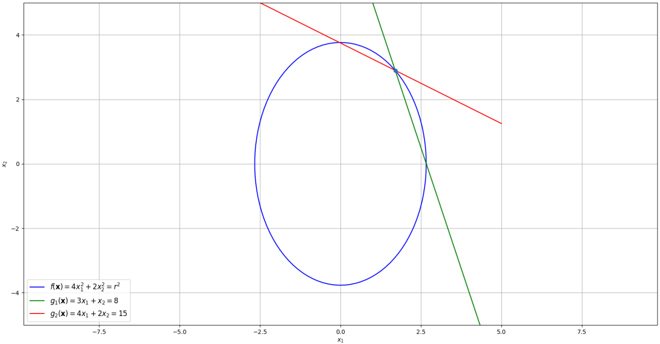

Figure: Correct solution is at intersection of the two lines $r = 5.33$

Problem 3.15

Steps to solve convex optimization problems using Lagrange Multipliers

$$x^* = \min_x f(x) \tag{9}$$

$$\text{subject to } h_i(x) = 0, \forall i = 1, .., m \tag{10}$$

$$\text{subject to } g_i(x) \leq 0, \forall i = 1, .., n \tag{11}$$

the optimal solution is obtained by considering the following

$$x^* = \min_x L(x, \vec{\lambda}, \vec{\mu}) \tag{12}$$

$$= \min_x f(x) + \sum_{i=1}^{m} \lambda_i h_i(x) + \sum_{i=1}^{n} \mu_i g_i(x) \tag{13}$$

Problem 3.15

$$\Rightarrow \nabla f(x) + \sum_{i=1}^{m} \lambda_i \nabla h_i(x) + \sum_{i=1}^{n} \mu_i \nabla g_i(x) = 0 \tag{14}$$

$$\tag{15}$$

The solution to the above equation should satisfy the below conditions to be feasible:

$$\mu_i g_i(x) = 0, \forall i = 1, .., m \tag{16}$$

$$\text{and } \mu_i \geq 0, \forall i = 1, .., m \tag{17}$$

It can be verified that the solution obtained in problem 3.14 satisfies the above conditions and hence it is a feasible solution.