PS1

March 25, 2024

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0.0.1 Q6

[1]:
```
import numpy as np
import matplotlib.pyplot as plt

np.random.seed(42)
```

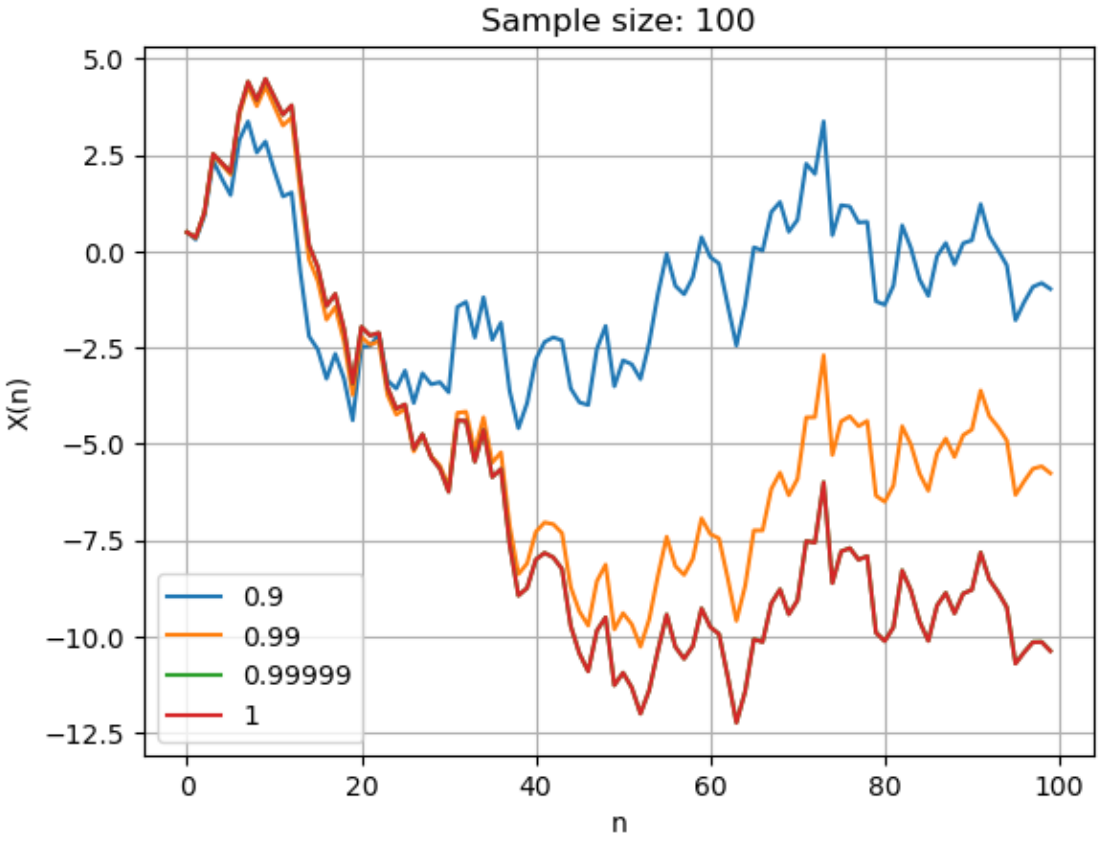
[2]:
```
def return_AR(phi,noise):
    x = np.zeros(len(noise))
    x[0] = noise[0]
    for i in range(1,len(noise)):
        x[i] = phi*x[i-1] + noise[i]
    return x
```

[3]:
```
sample_sizes = [100,1000,10000]
phis = [0.9,0.99,0.99999,1]
num_sims = 5
noises = np.random.normal(0,1,max(sample_sizes))
theoretical_stdevs = [np.sqrt(1/(1-phi**2)) if phi < 1 else "infinity" for phi␣
 ↪in phis]
print("Theoretical Stdevs for all phis are: ", theoretical_stdevs)
print("Theoretical means for all phis are: ", 0)

for sim in range(1,num_sims+1):
    print("Simulation Number: ", str(sim))
    for i in range(len(sample_sizes)):
        AR_processes = [{str(phi):return_AR(phi,noises[:sample_sizes[i]])} for␣
 ↪phi in phis] ## Sample noise from same generation instance
        [print("Mean for phi: ", str(list(process.keys())[0]), " is:", np.
 ↪mean(list(process.values())[0])) for process in AR_processes]
        [print("Standard Deviation for phi: ", str(list(process.keys())[0]), "␣
 ↪is:", np.sqrt(np.var(list(process.values())[0]))) for process in␣
 ↪AR_processes]
        [plt.plot(list(AR_process.values())[0],label = list(AR_process.
 ↪keys())[0]) for AR_process in AR_processes]
        plt.title("Sample size: " + str(sample_sizes[i]))
```

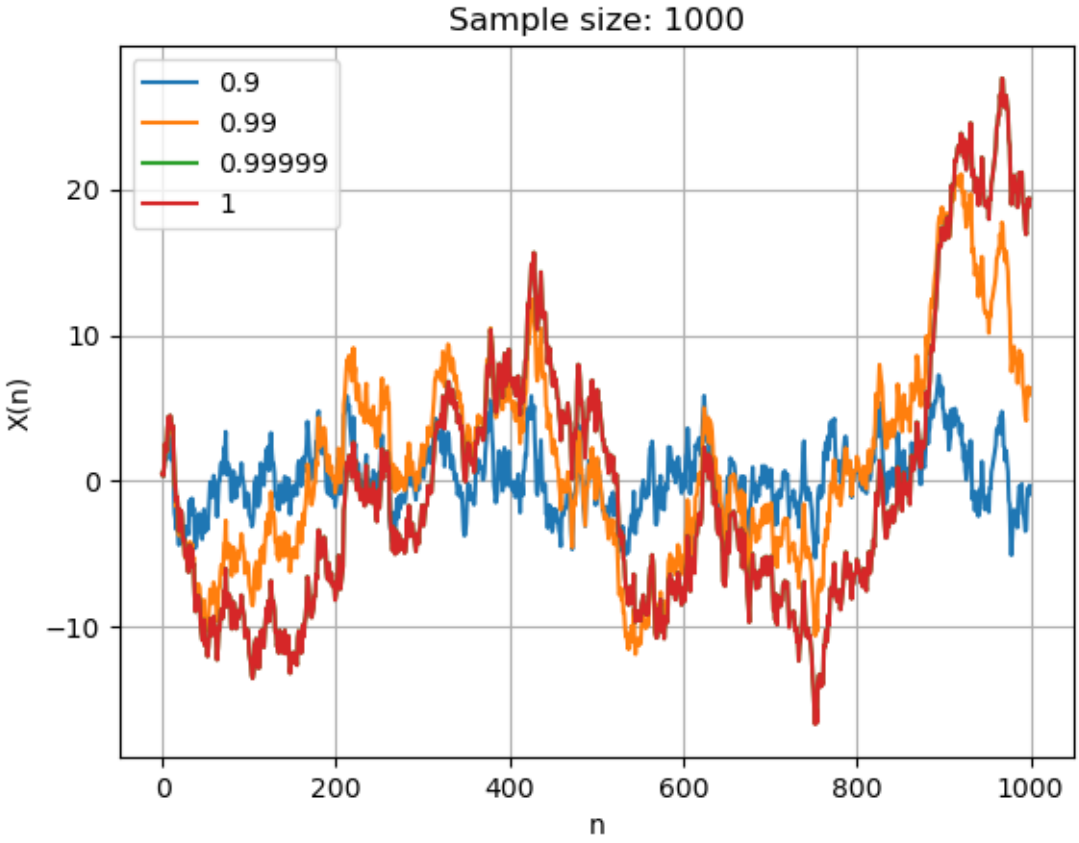
```
        plt.grid()
        plt.xlabel("n")
        plt.ylabel("X(n)")
        plt.legend()
        plt.show()
```

```
Theoretical Stdevs for all phis are:  [2.294157338705618, 7.088812050083353,
223.60735676962474, 'infinity']
Theoretical means for all phis are:  0
Simulation Number:  1
Mean for phi:  0.9  is: -0.9506127760995247
Mean for phi:  0.99  is: -4.678205540082792
Mean for phi:  0.99999  is: -6.4030794715341095
Mean for phi:  1  is: -6.405182462570341
Standard Deviation for phi:  0.9  is: 1.9719929533794034
Standard Deviation for phi:  0.99  is: 3.6178616637309866
Standard Deviation for phi:  0.99999  is: 4.619264221325336
Standard Deviation for phi:  1  is: 4.620719807498387
```

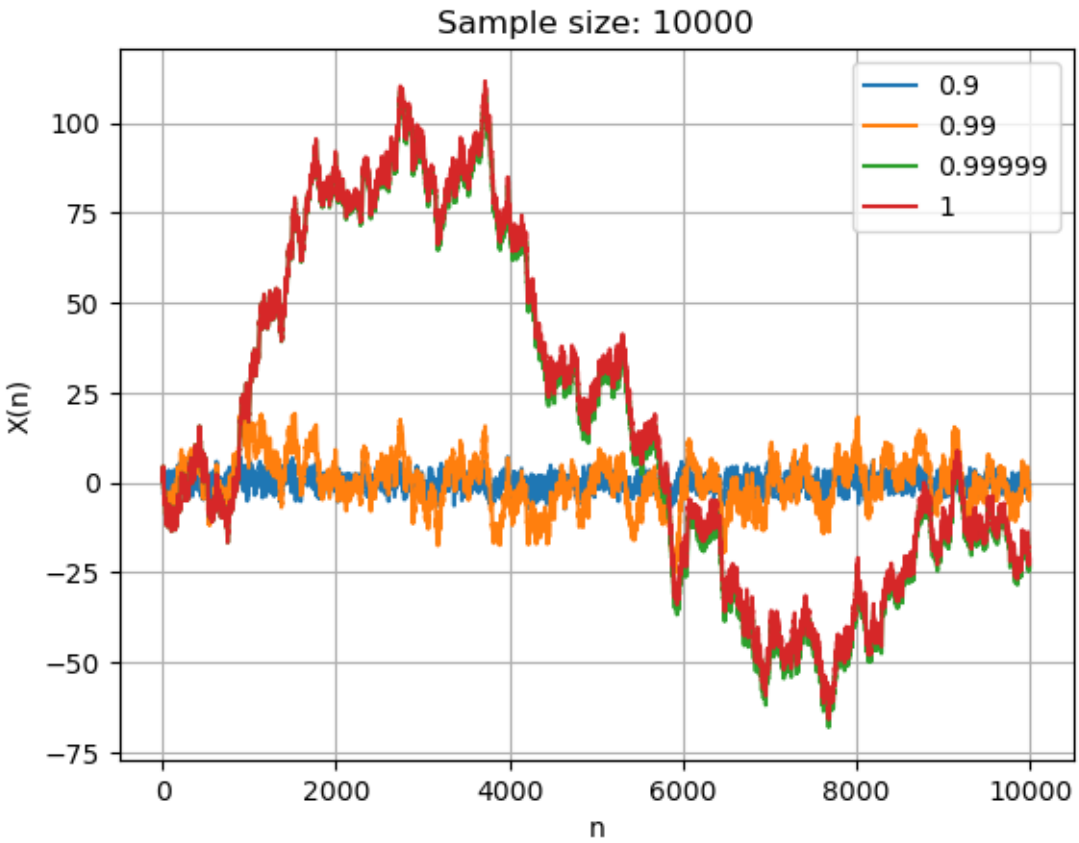



```
Mean for phi:  0.9  is: 0.19626089593974705
Mean for phi:  0.99  is: 1.3054089833638602
```

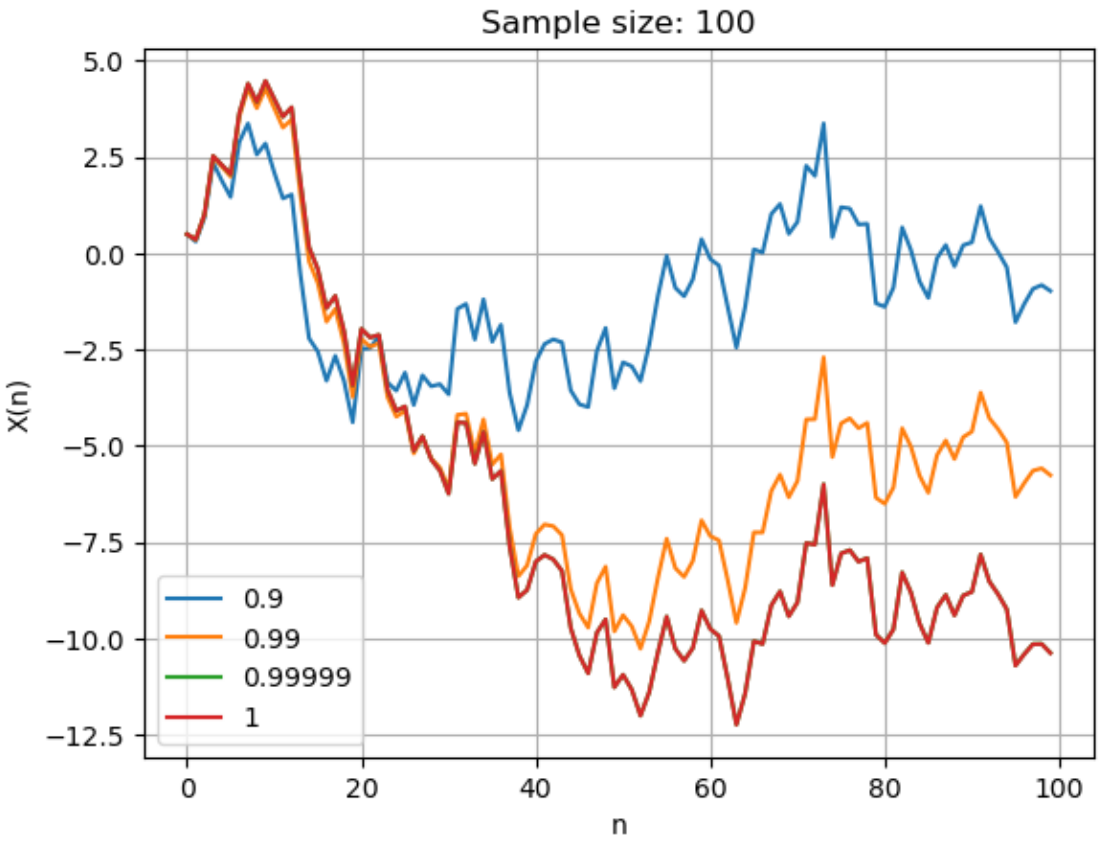
```
Mean for phi:  0.99999  is: -0.3356176531689023
Mean for phi:  1  is: -0.3496529446010139
Standard Deviation for phi:  0.9  is: 2.241191499090931
Standard Deviation for phi:  0.99  is: 6.936922270741513
Standard Deviation for phi:  0.99999  is: 9.555666184408318
Standard Deviation for phi:  1  is: 9.555186861076256
```



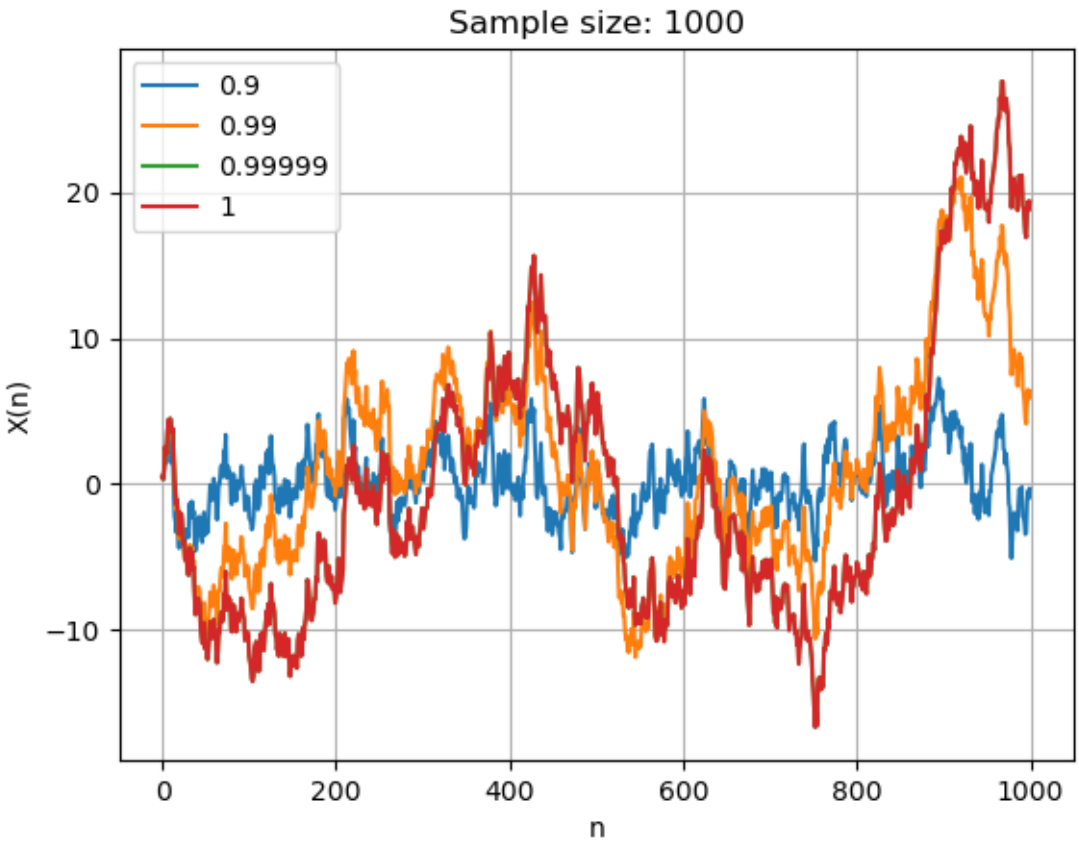

```
Mean for phi:  0.9  is: -0.02058758310025478
Mean for phi:  0.99  is: -0.18511365795688745
Mean for phi:  0.99999  is: 16.25316827068876
Mean for phi:  1  is: 17.95867388467473
Standard Deviation for phi:  0.9  is: 2.2209210699962973
Standard Deviation for phi:  0.99  is: 7.0346243168797615
Standard Deviation for phi:  0.99999  is: 47.88434322736055
Standard Deviation for phi:  1  is: 47.5894609364
```



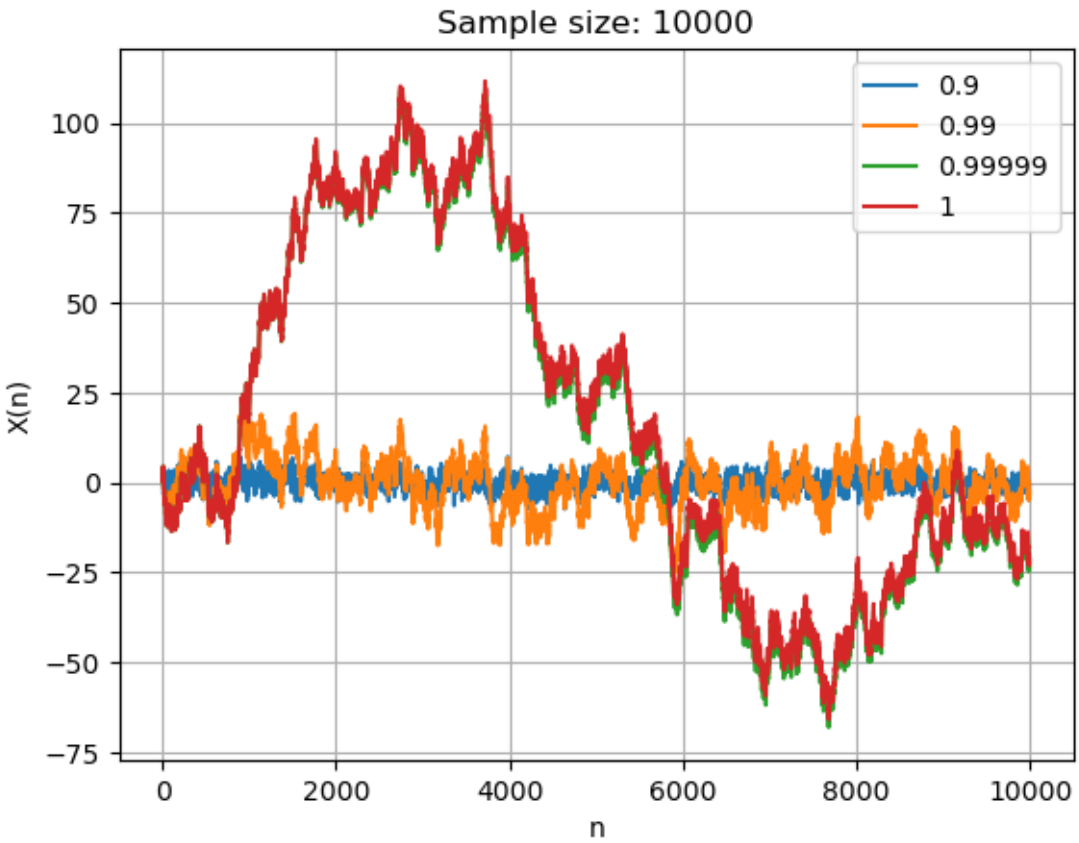

```
Simulation Number:  2
Mean for phi:  0.9  is: -0.9506127760995247
Mean for phi:  0.99  is: -4.678205540082792
Mean for phi:  0.99999  is: -6.4030794715341095
Mean for phi:  1  is: -6.405182462570341
Standard Deviation for phi:  0.9  is: 1.9719929533794034
Standard Deviation for phi:  0.99  is: 3.6178616637309866
Standard Deviation for phi:  0.99999  is: 4.619264221325336
Standard Deviation for phi:  1  is: 4.620719807498387
```



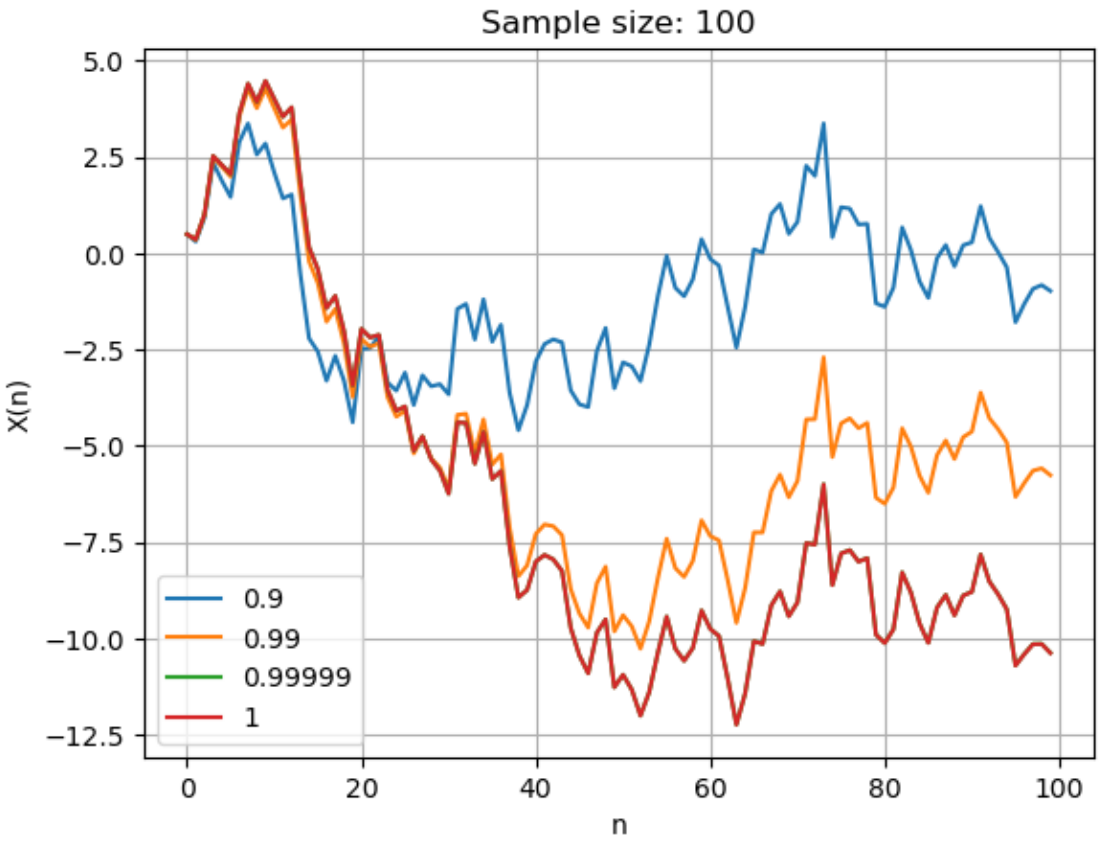

```
Mean for phi:  0.9  is: 0.19626089593974705
Mean for phi:  0.99  is: 1.3054089833638602
Mean for phi:  0.99999  is: -0.3356176531689023
Mean for phi:  1  is: -0.3496529446010139
Standard Deviation for phi:  0.9  is: 2.241191499090931
Standard Deviation for phi:  0.99  is: 6.936922270741513
Standard Deviation for phi:  0.99999  is: 9.555666184408318
Standard Deviation for phi:  1  is: 9.555186861076256
```



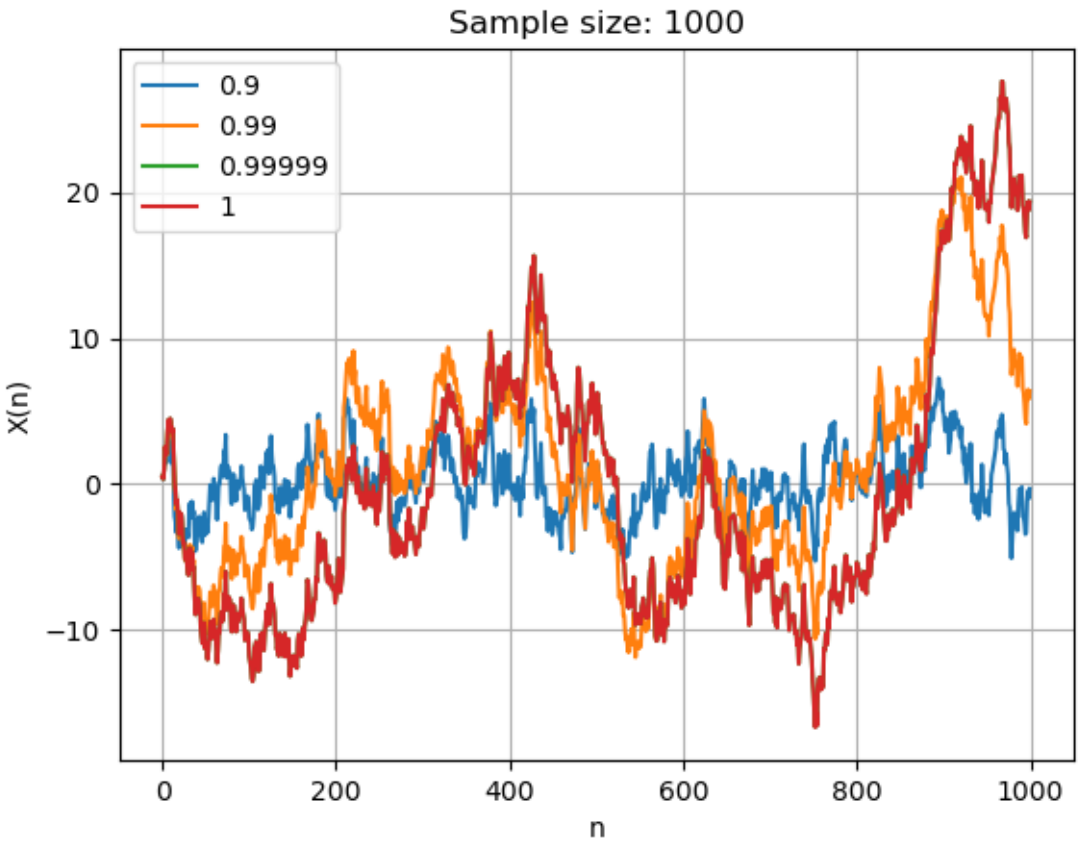

```
Mean for phi:  0.9  is: -0.02058758310025478
Mean for phi:  0.99  is: -0.18511365795688745
Mean for phi:  0.99999  is: 16.25316827068876
Mean for phi:  1  is: 17.95867388467473
Standard Deviation for phi:  0.9  is: 2.2209210699962973
Standard Deviation for phi:  0.99  is: 7.0346243168797615
Standard Deviation for phi:  0.99999  is: 47.88434322736055
Standard Deviation for phi:  1  is: 47.5894609364
```



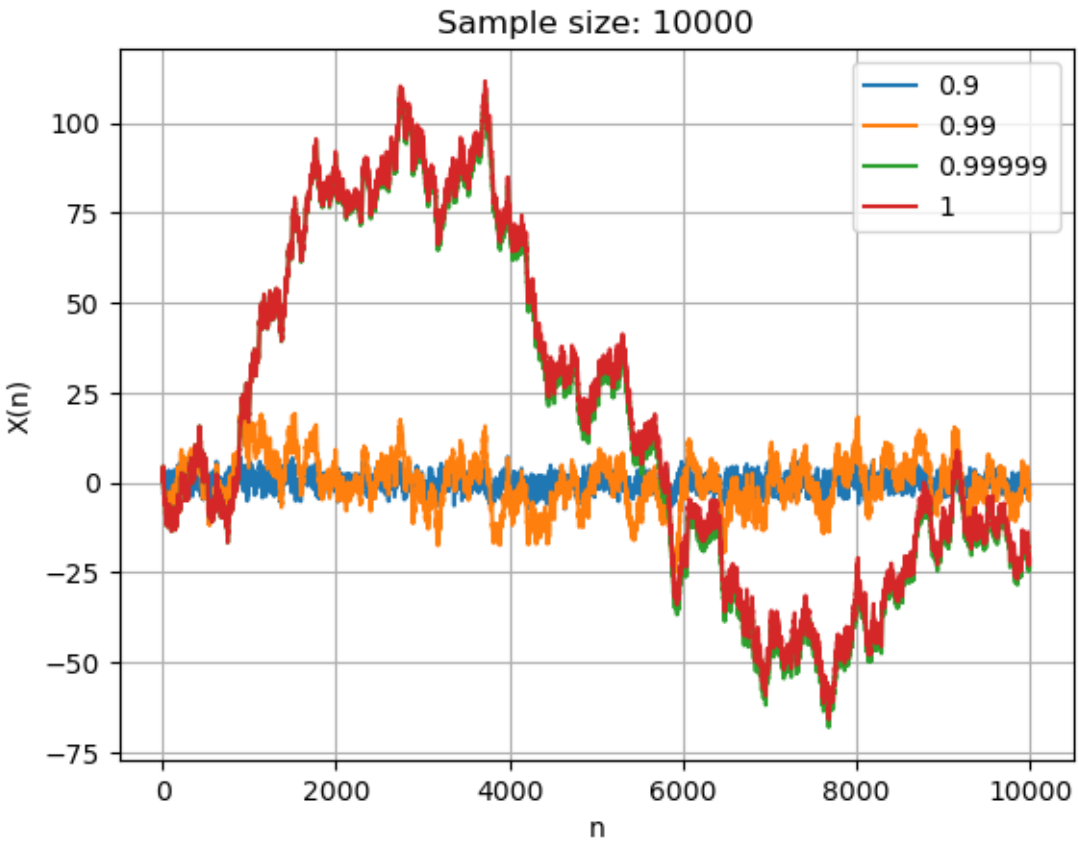

```
Simulation Number:  3
Mean for phi:  0.9  is: -0.9506127760995247
Mean for phi:  0.99  is: -4.678205540082792
Mean for phi:  0.99999  is: -6.4030794715341095
Mean for phi:  1  is: -6.405182462570341
Standard Deviation for phi:  0.9  is: 1.9719929533794034
Standard Deviation for phi:  0.99  is: 3.6178616637309866
Standard Deviation for phi:  0.99999  is: 4.619264221325336
Standard Deviation for phi:  1  is: 4.620719807498387
```



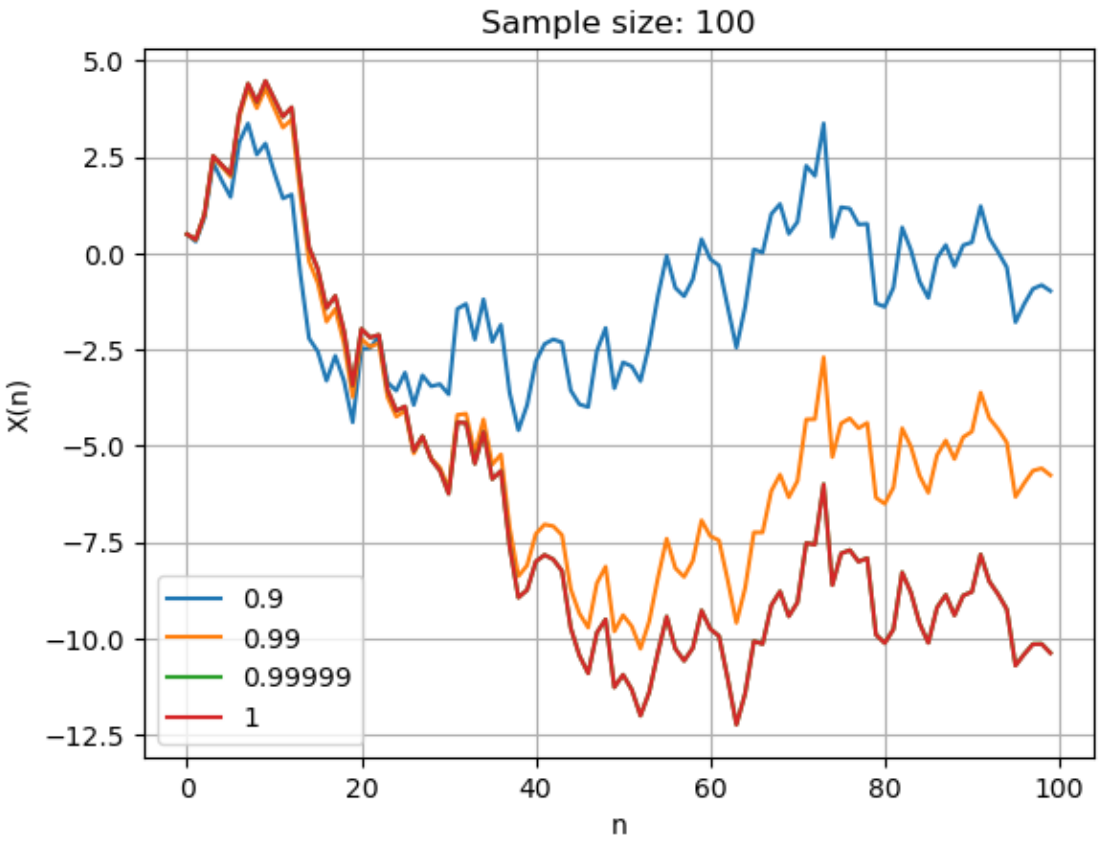

```
Mean for phi:  0.9  is: 0.19626089593974705
Mean for phi:  0.99  is: 1.3054089833638602
Mean for phi:  0.99999  is: -0.3356176531689023
Mean for phi:  1  is: -0.3496529446010139
Standard Deviation for phi:  0.9  is: 2.241191499090931
Standard Deviation for phi:  0.99  is: 6.936922270741513
Standard Deviation for phi:  0.99999  is: 9.555666184408318
Standard Deviation for phi:  1  is: 9.555186861076256
```



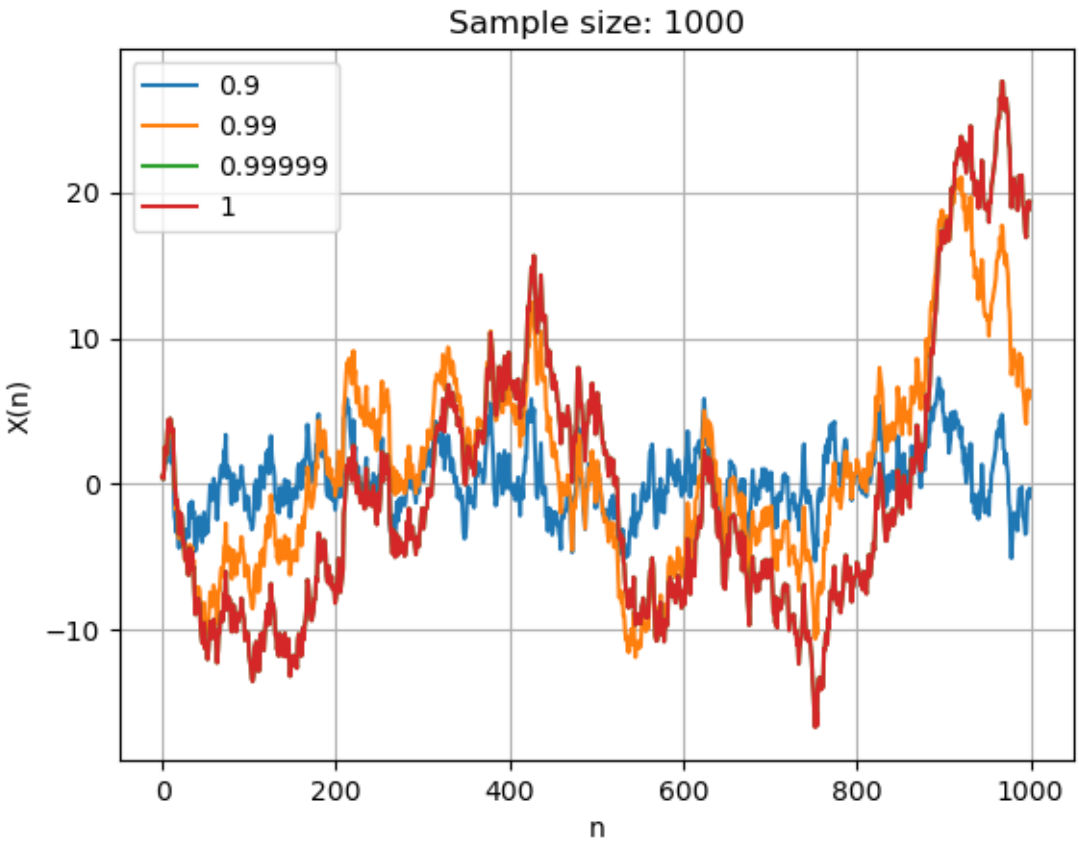

```
Mean for phi:  0.9  is: -0.02058758310025478
Mean for phi:  0.99  is: -0.18511365795688745
Mean for phi:  0.99999  is: 16.25316827068876
Mean for phi:  1  is: 17.95867388467473
Standard Deviation for phi:  0.9  is: 2.2209210699962973
Standard Deviation for phi:  0.99  is: 7.0346243168797615
Standard Deviation for phi:  0.99999  is: 47.88434322736055
Standard Deviation for phi:  1  is: 47.5894609364
```



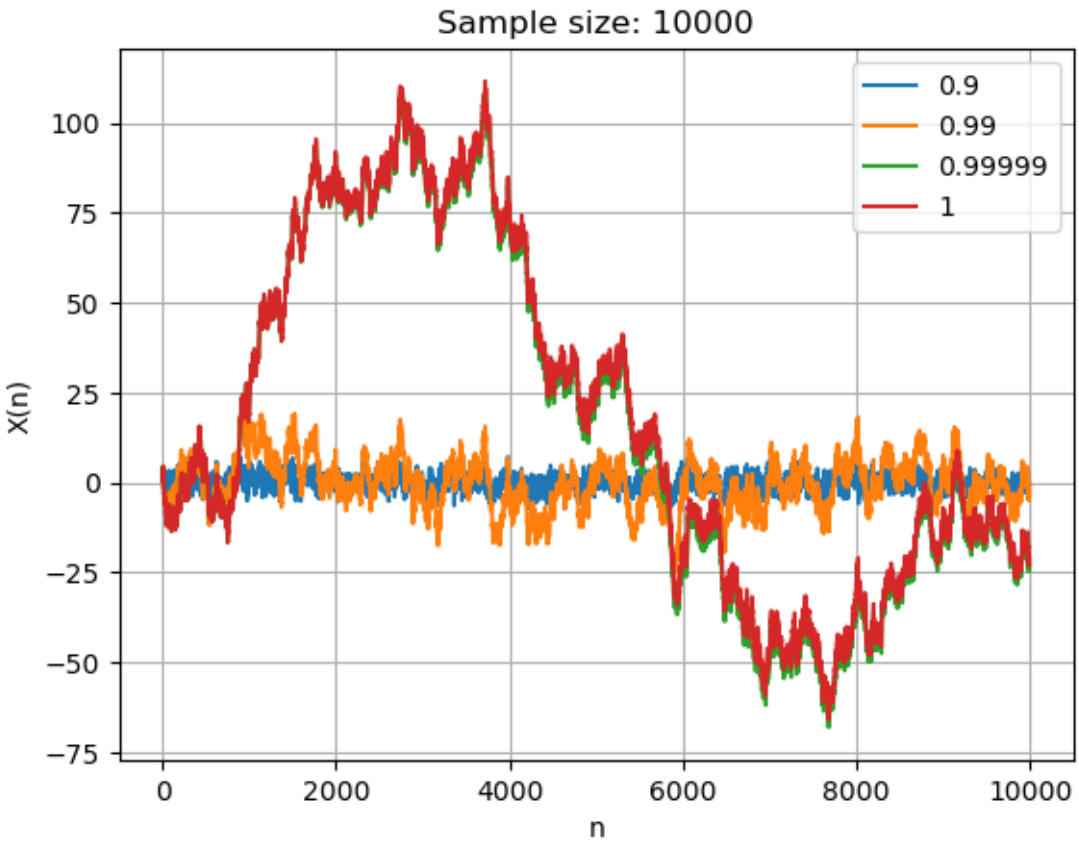

```
Simulation Number:  4
Mean for phi:  0.9  is: -0.9506127760995247
Mean for phi:  0.99  is: -4.678205540082792
Mean for phi:  0.99999  is: -6.4030794715341095
Mean for phi:  1  is: -6.405182462570341
Standard Deviation for phi:  0.9  is: 1.9719929533794034
Standard Deviation for phi:  0.99  is: 3.6178616637309866
Standard Deviation for phi:  0.99999  is: 4.619264221325336
Standard Deviation for phi:  1  is: 4.620719807498387
```



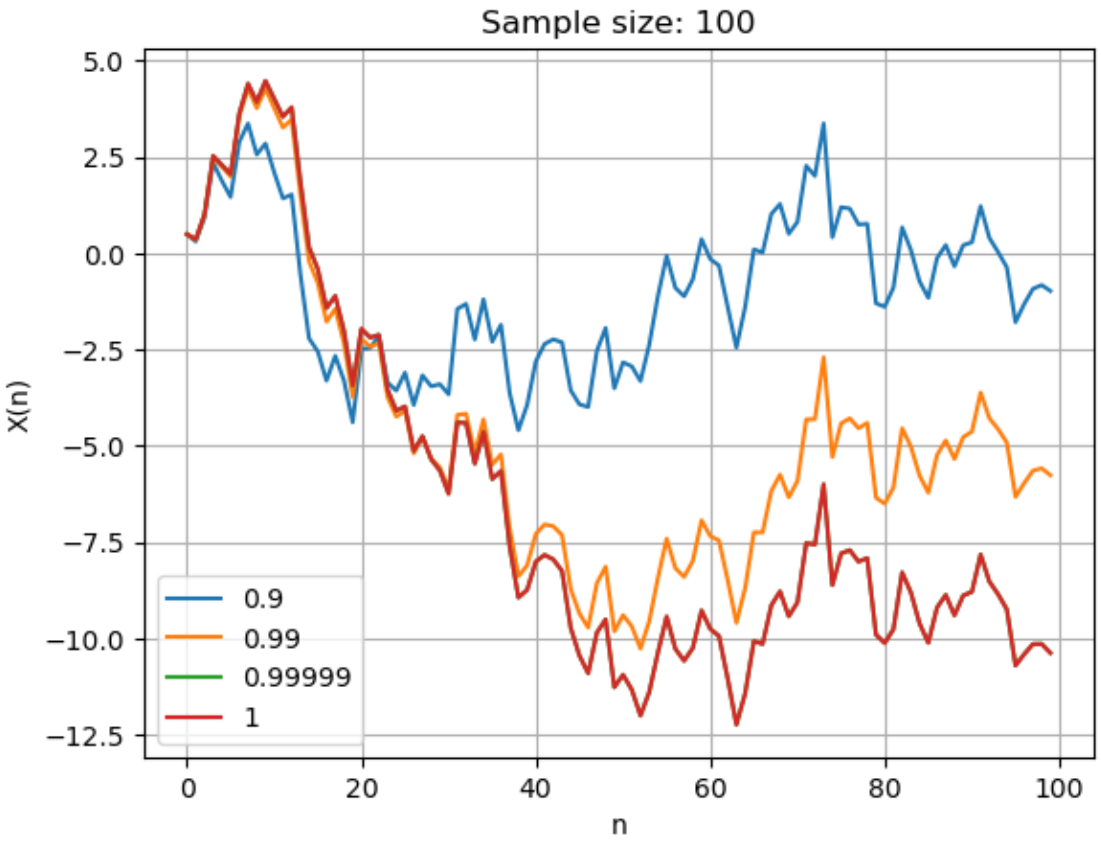

```
Mean for phi:  0.9  is: 0.19626089593974705
Mean for phi:  0.99  is: 1.3054089833638602
Mean for phi:  0.99999  is: -0.3356176531689023
Mean for phi:  1  is: -0.3496529446010139
Standard Deviation for phi:  0.9  is: 2.241191499090931
Standard Deviation for phi:  0.99  is: 6.936922270741513
Standard Deviation for phi:  0.99999  is: 9.555666184408318
Standard Deviation for phi:  1  is: 9.555186861076256
```



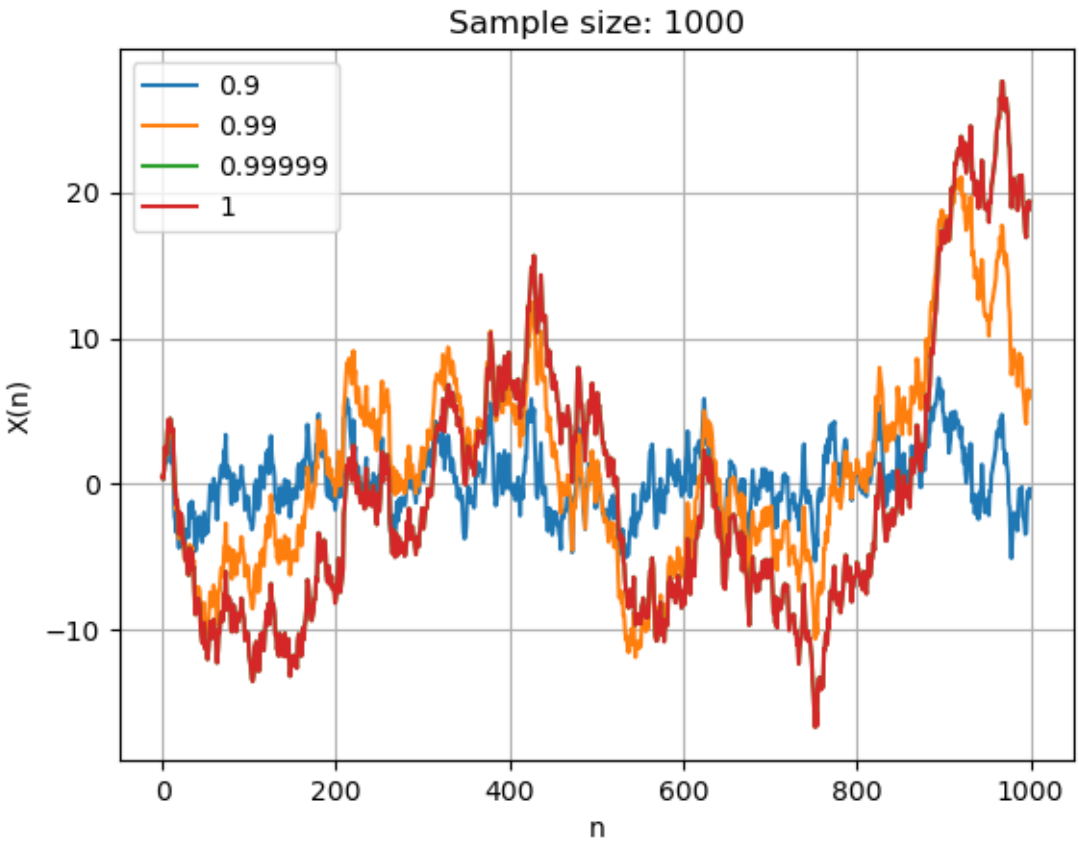

```
Mean for phi:  0.9  is: -0.02058758310025478
Mean for phi:  0.99  is: -0.18511365795688745
Mean for phi:  0.99999  is: 16.25316827068876
Mean for phi:  1  is: 17.95867388467473
Standard Deviation for phi:  0.9  is: 2.2209210699962973
Standard Deviation for phi:  0.99  is: 7.0346243168797615
Standard Deviation for phi:  0.99999  is: 47.88434322736055
Standard Deviation for phi:  1  is: 47.5894609364
```




```
Simulation Number:  5
Mean for phi:  0.9  is: -0.9506127760995247
Mean for phi:  0.99  is: -4.678205540082792
Mean for phi:  0.99999  is: -6.4030794715341095
Mean for phi:  1  is: -6.405182462570341
Standard Deviation for phi:  0.9  is: 1.9719929533794034
Standard Deviation for phi:  0.99  is: 3.6178616637309866
Standard Deviation for phi:  0.99999  is: 4.619264221325336
Standard Deviation for phi:  1  is: 4.620719807498387
```




```
Mean for phi:  0.9  is: 0.19626089593974705
Mean for phi:  0.99  is: 1.3054089833638602
Mean for phi:  0.99999  is: -0.3356176531689023
Mean for phi:  1  is: -0.3496529446010139
Standard Deviation for phi:  0.9  is: 2.241191499090931
Standard Deviation for phi:  0.99  is: 6.936922270741513
Standard Deviation for phi:  0.99999  is: 9.555666184408318
Standard Deviation for phi:  1  is: 9.555186861076256
```

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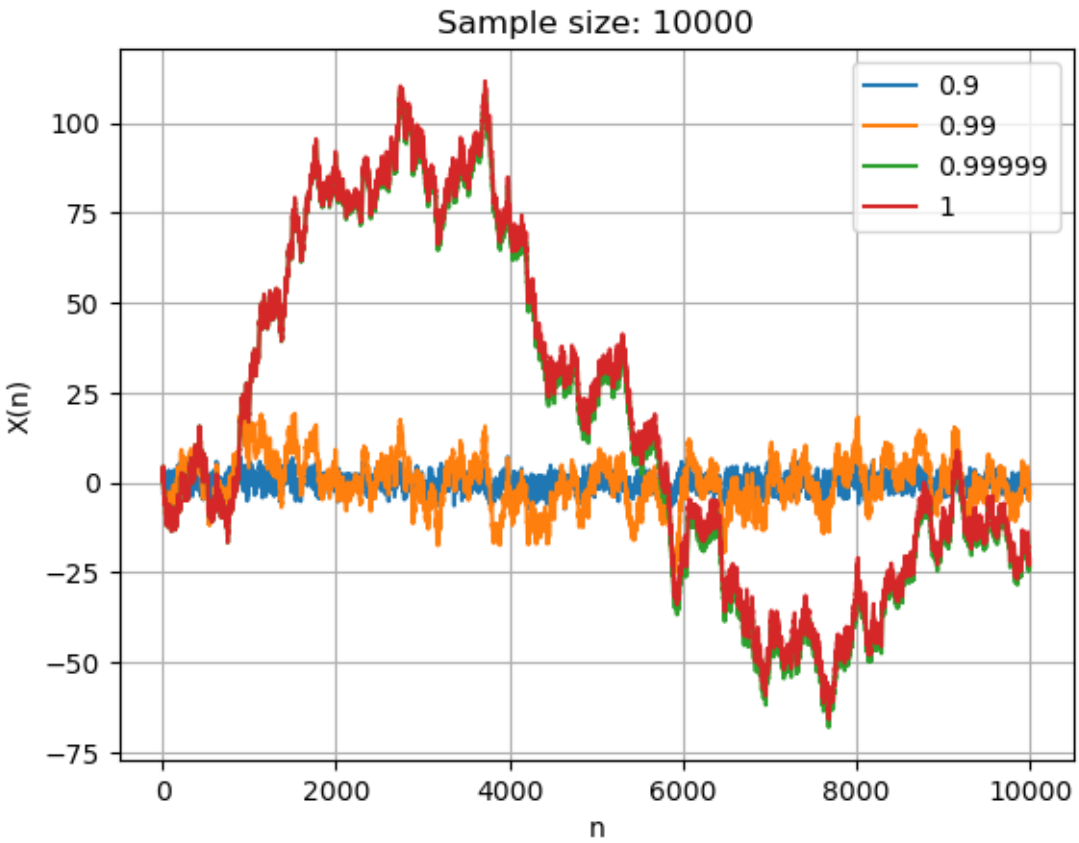
Mean for phi:  0.99  is: -0.18511365795688745
Mean for phi:  0.99999  is: 16.25316827068876
Mean for phi:  1  is: 17.95867388467473
Standard Deviation for phi:  0.9  is: 2.2209210699962973
Standard Deviation for phi:  0.99  is: 7.0346243168797615
Standard Deviation for phi:  0.99999  is: 47.88434322736055
Standard Deviation for phi:  1  is: 47.5894609364
```



0.0.2 Observations for the AR(1) process:

Clearly, we can see that as $\phi \to 1$, the process starts to become unstable leading to erratic values. The plots and sample statistics are also evidence for this as the mean and variance have comparibly smaller magnitude for ϕ values away from 1 and increases drastically as ϕ approaches 1. Another key observation is that for higher sample size, we can see that smaller ϕ yield sample mean which is close to population mean implying Law of large numbers. However that is not the case for standard deviation as we see very high standard deviations which can be attributed to the fact that the stationarity itself is fragile for higher ϕ

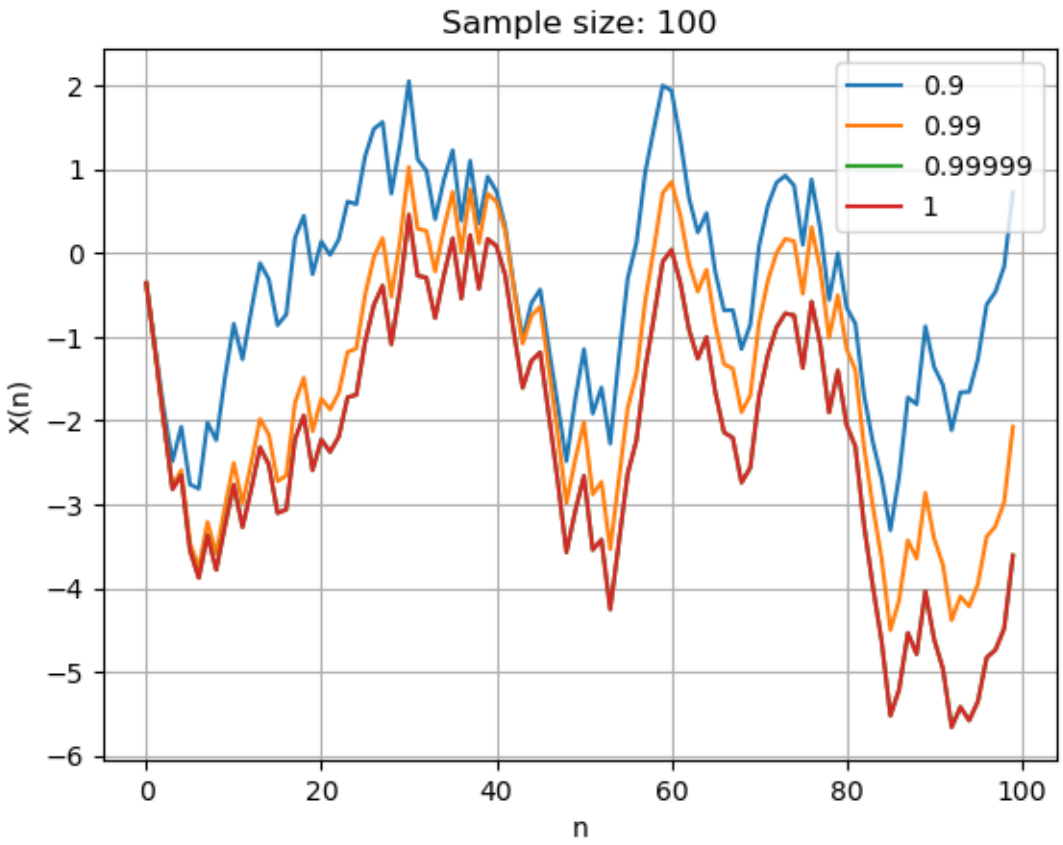
0.0.3 Q7

a) Uniform distribution

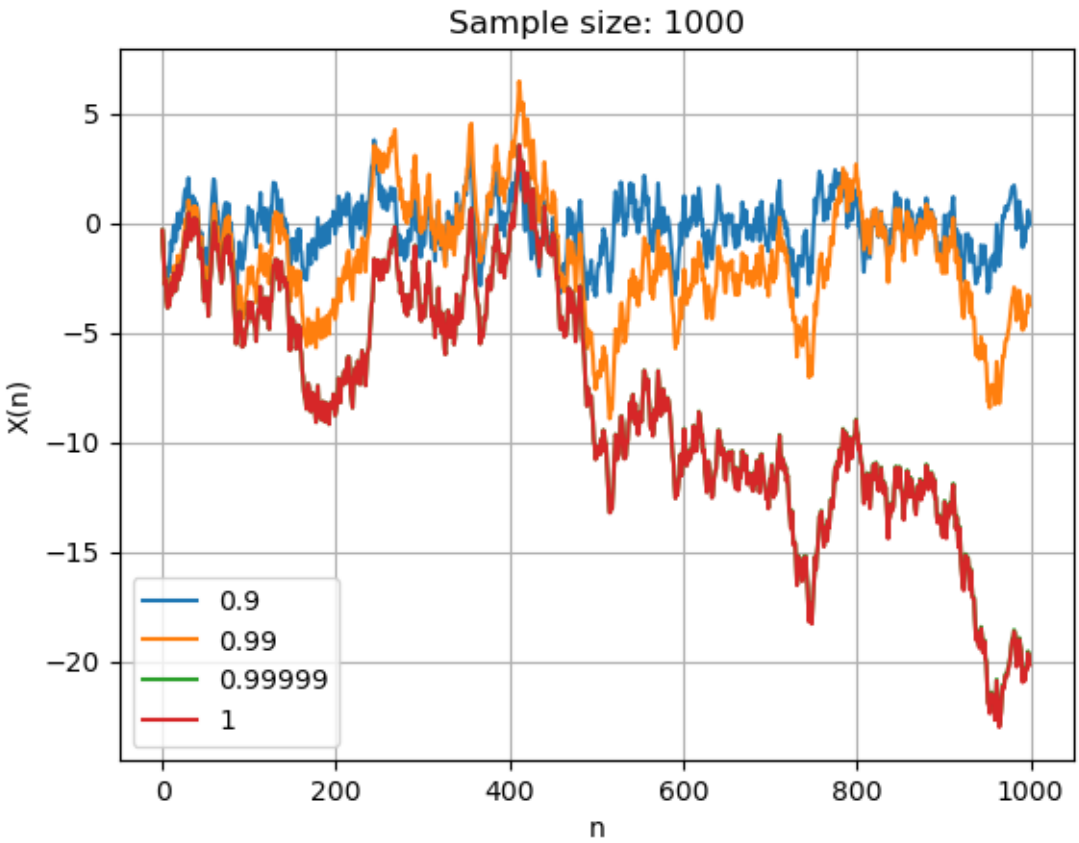
[4]:
```
sample_sizes = [100,1000,10000]
phis = [0.9,0.99,0.99999,1]
noises = np.random.uniform(-1,1,max(sample_sizes))
for sim in range(1,num_sims+1):
    print("Simulation Number: ", str(sim))
    for i in range(len(sample_sizes)):
```

```
        AR_processes = [{str(phi):return_AR(phi,noises[:sample_sizes[i]])} for␣
↪phi in phis]
        [print("Mean for phi: ", str(list(process.keys())[0]), " is:", np.
↪mean(list(process.values())[0])) for process in AR_processes]
        [print("Standard Deviation for phi: ", str(list(process.keys())[0]), "␣
↪is:", np.sqrt(np.var(list(process.values())[0]))) for process in␣
↪AR_processes]
        [plt.plot(list(AR_process.values())[0],label = list(AR_process.
↪keys())[0]) for AR_process in AR_processes]
        plt.title("Sample size: " + str(sample_sizes[i]))
        plt.grid()
        plt.xlabel("n")
        plt.ylabel("X(n)")
        plt.legend()
        plt.show()
```

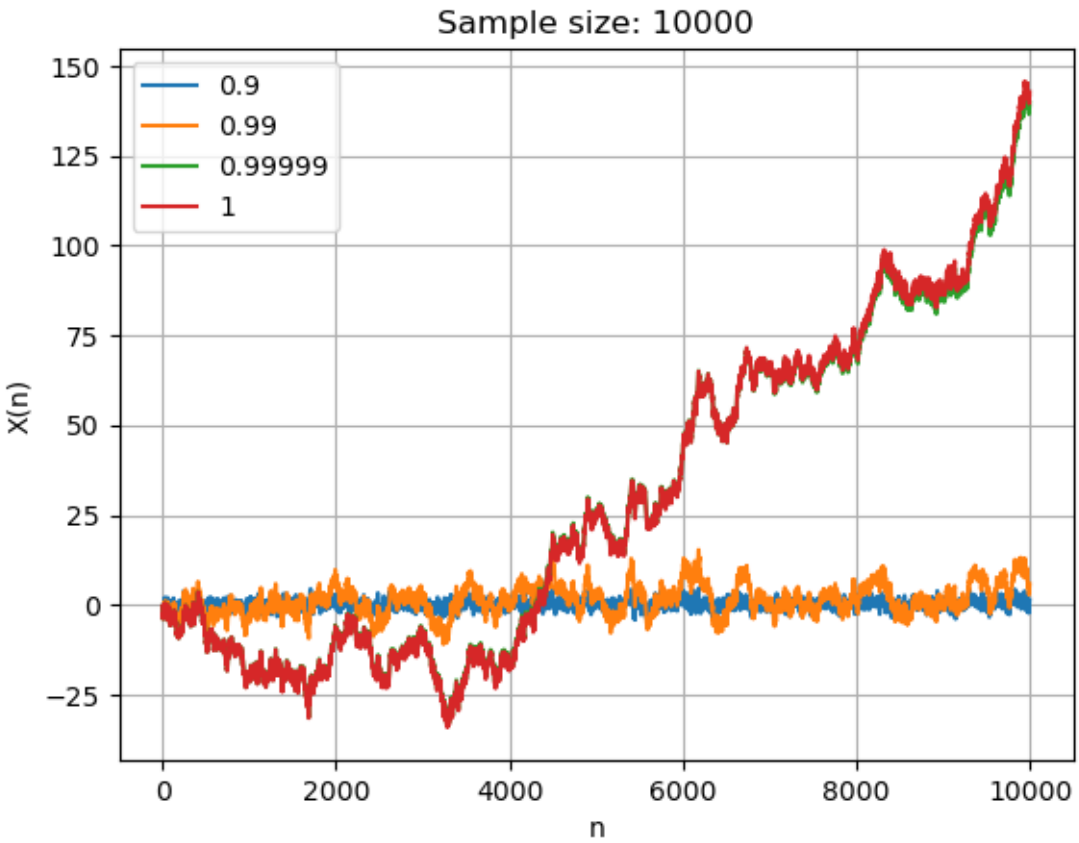
```
Simulation Number:  1
Mean for phi:  0.9  is: -0.4264800009600329
Mean for phi:  0.99  is: -1.5602867950325998
Mean for phi:  0.99999  is: -2.2555720368945367
Mean for phi:  1  is: -2.256517075619546
Standard Deviation for phi:  0.9  is: 1.255811317835891
Standard Deviation for phi:  0.99  is: 1.4873068387928094
Standard Deviation for phi:  0.99999  is: 1.595782879192064
Standard Deviation for phi:  1  is: 1.5960173390938241
```



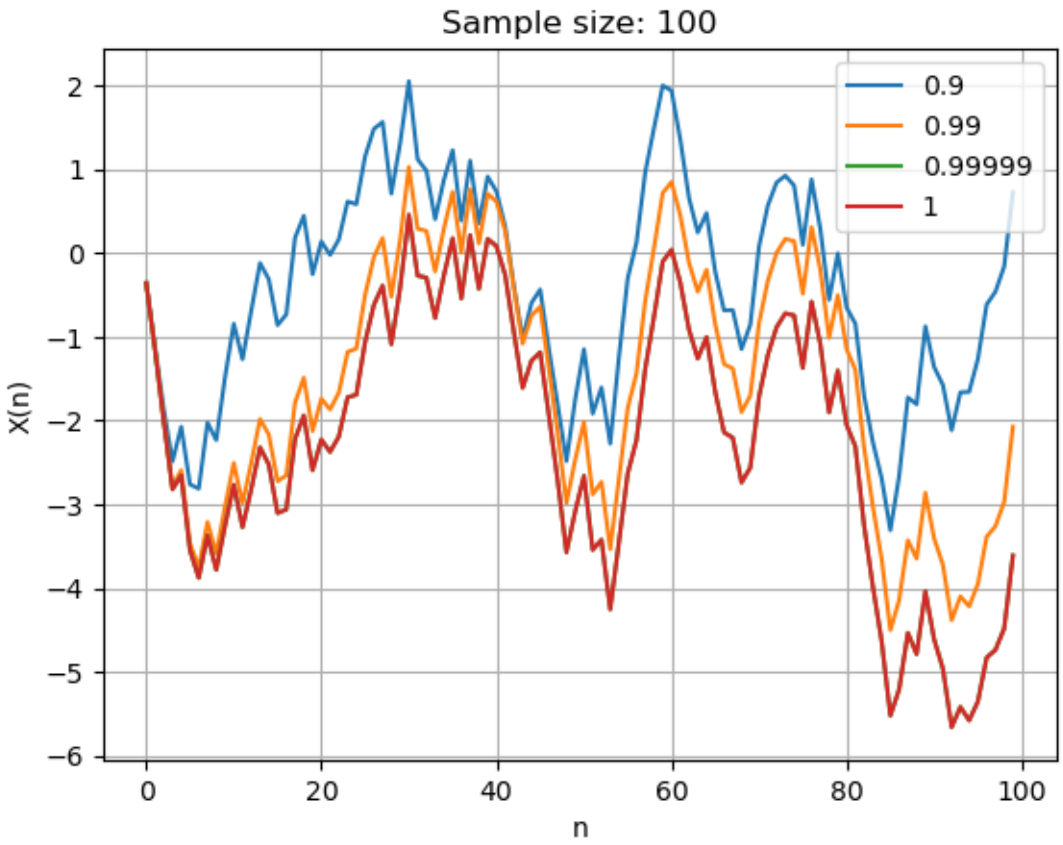

```
Mean for phi:  0.9  is: -0.20104745875185367
Mean for phi:  0.99  is: -1.6442552594189224
Mean for phi:  0.99999  is: -8.149636929171326
Mean for phi:  1  is: -8.176964051913451
Standard Deviation for phi:  0.9  is: 1.2458186962205509
Standard Deviation for phi:  0.99  is: 2.655584971340514
Standard Deviation for phi:  0.99999  is: 5.599434263862098
Standard Deviation for phi:  1  is: 5.618892246448196
```



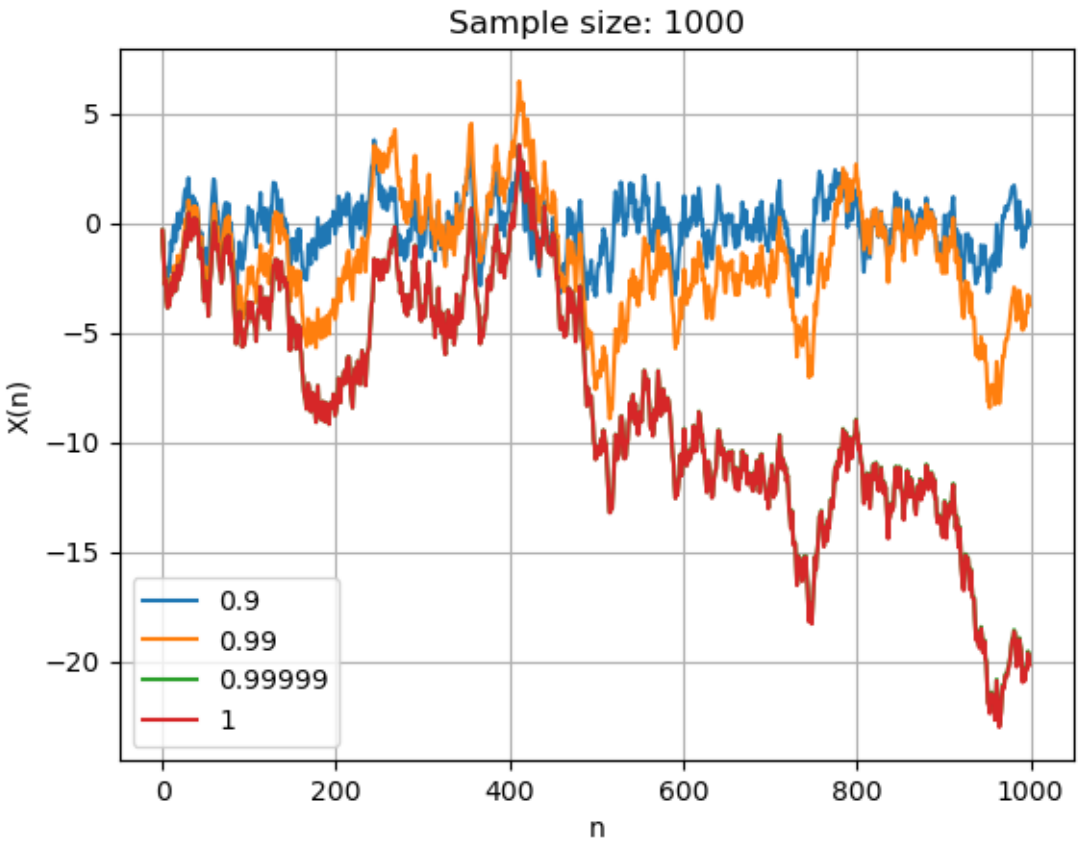

```
Mean for phi:  0.9  is: 0.14145507658295453
Mean for phi:  0.99  is: 1.3704445130771157
Mean for phi:  0.99999  is: 29.414436791455326
Mean for phi:  1  is: 29.664768147578467
Standard Deviation for phi:  0.9  is: 1.3422200587772295
Standard Deviation for phi:  0.99  is: 4.134148538531254
Standard Deviation for phi:  0.99999  is: 45.259411606268756
Standard Deviation for phi:  1  is: 46.05458886641797
```



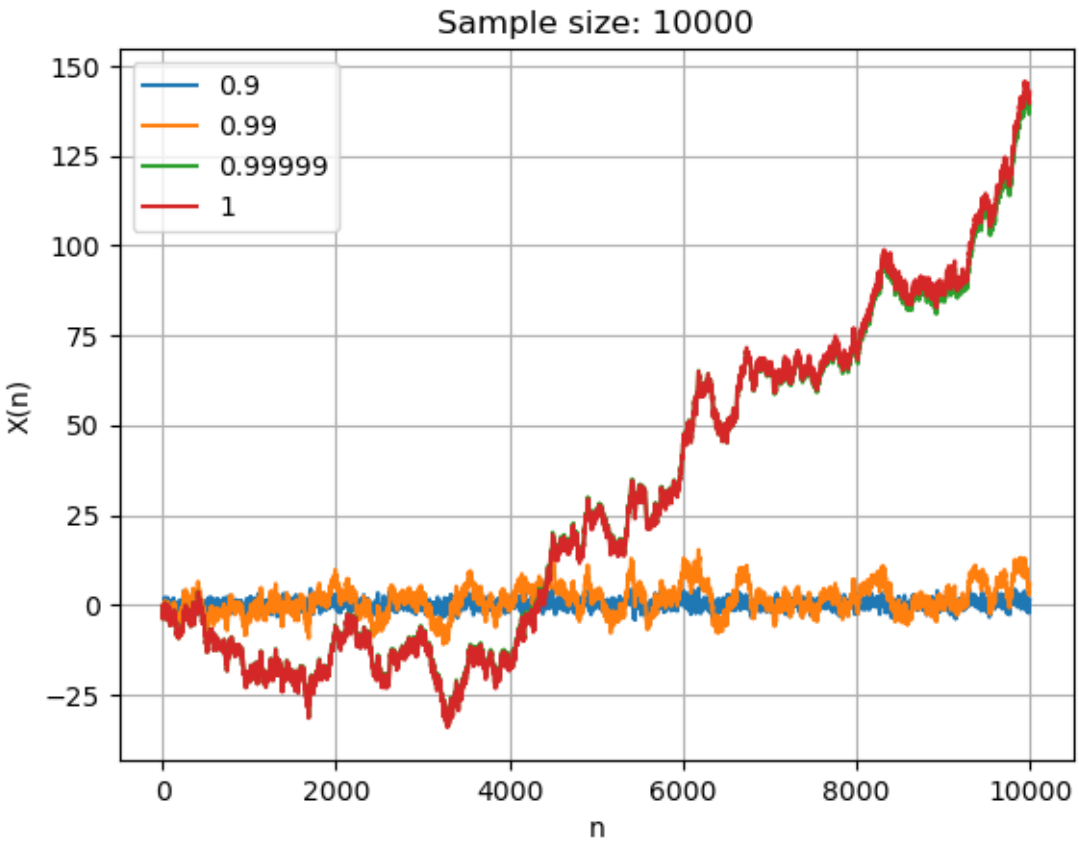

```
Simulation Number:  2
Mean for phi:  0.9  is: -0.4264800009600329
Mean for phi:  0.99  is: -1.5602867950325998
Mean for phi:  0.99999  is: -2.2555720368945367
Mean for phi:  1  is: -2.256517075619546
Standard Deviation for phi:  0.9  is: 1.255811317835891
Standard Deviation for phi:  0.99  is: 1.4873068387928094
Standard Deviation for phi:  0.99999  is: 1.595782879192064
Standard Deviation for phi:  1  is: 1.5960173390938241
```



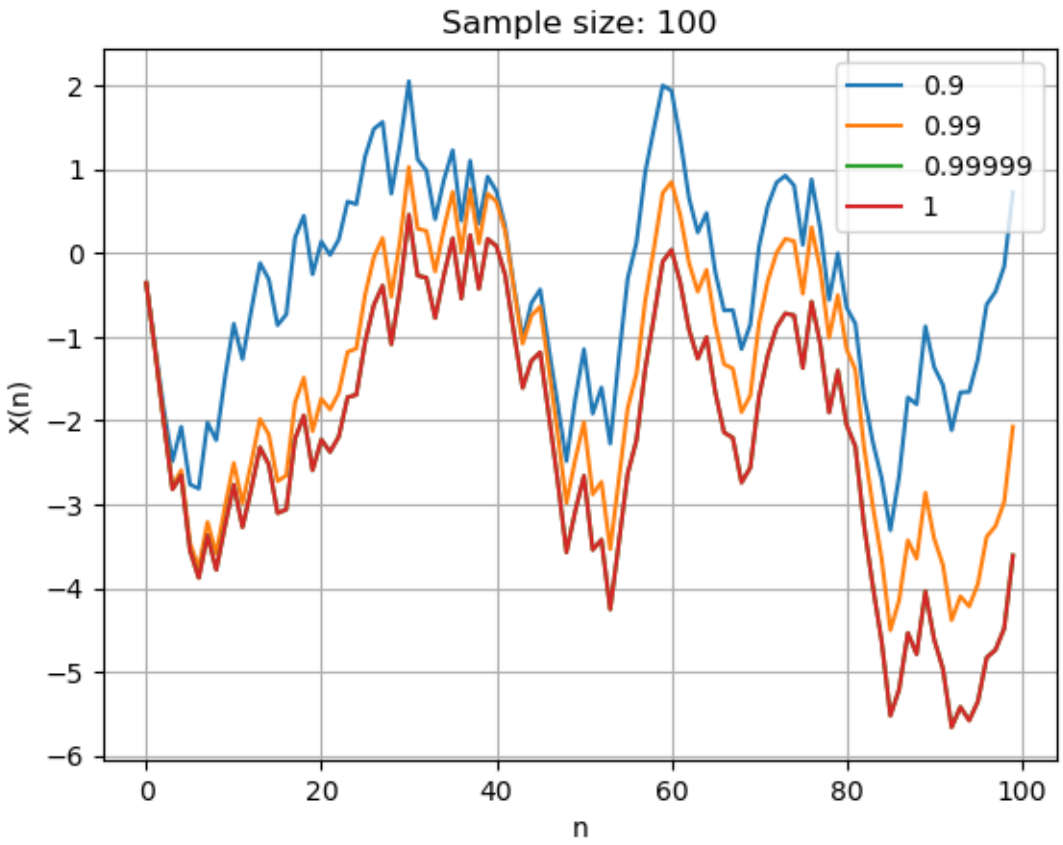

```
Mean for phi:  0.9  is: -0.20104745875185367
Mean for phi:  0.99  is: -1.6442552594189224
Mean for phi:  0.99999  is: -8.149636929171326
Mean for phi:  1  is: -8.176964051913451
Standard Deviation for phi:  0.9  is: 1.2458186962205509
Standard Deviation for phi:  0.99  is: 2.655584971340514
Standard Deviation for phi:  0.99999  is: 5.599434263862098
Standard Deviation for phi:  1  is: 5.618892246448196
```



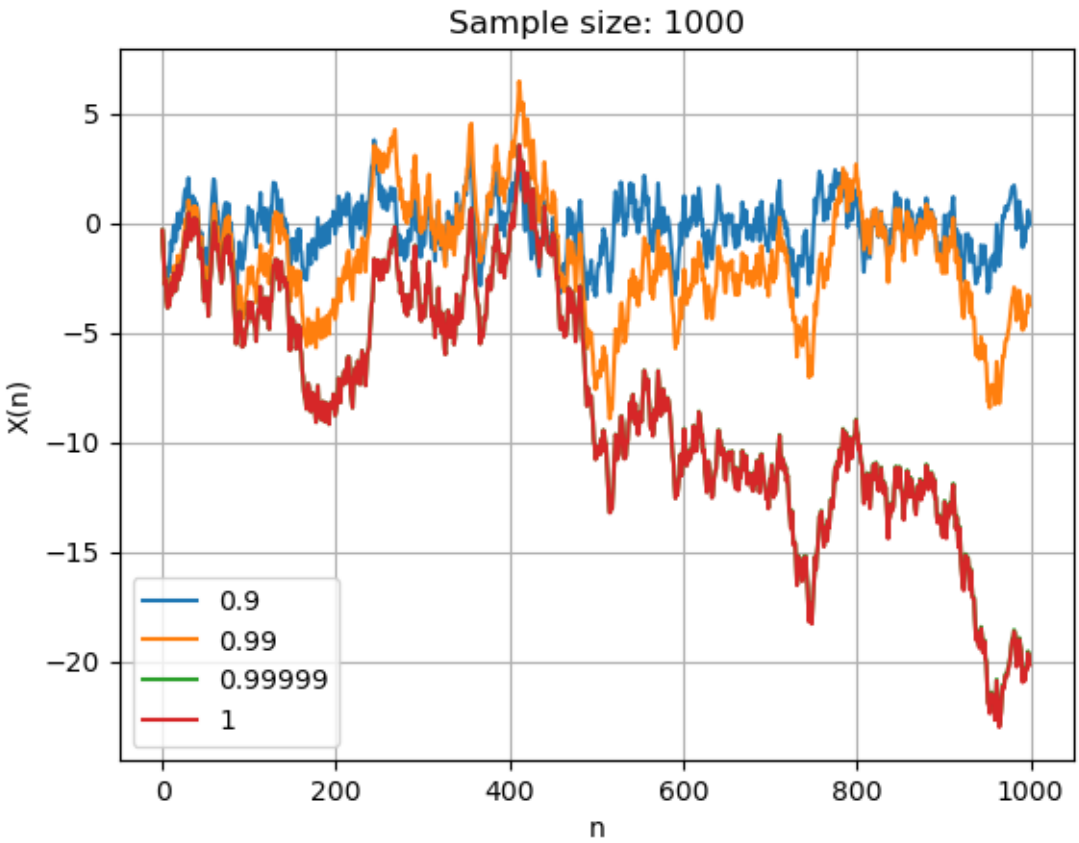

```
Mean for phi:  0.9  is: 0.14145507658295453
Mean for phi:  0.99  is: 1.3704445130771157
Mean for phi:  0.99999  is: 29.414436791455326
Mean for phi:  1  is: 29.664768147578467
Standard Deviation for phi:  0.9  is: 1.3422200587772295
Standard Deviation for phi:  0.99  is: 4.134148538531254
Standard Deviation for phi:  0.99999  is: 45.259411606268756
Standard Deviation for phi:  1  is: 46.05458886641797
```



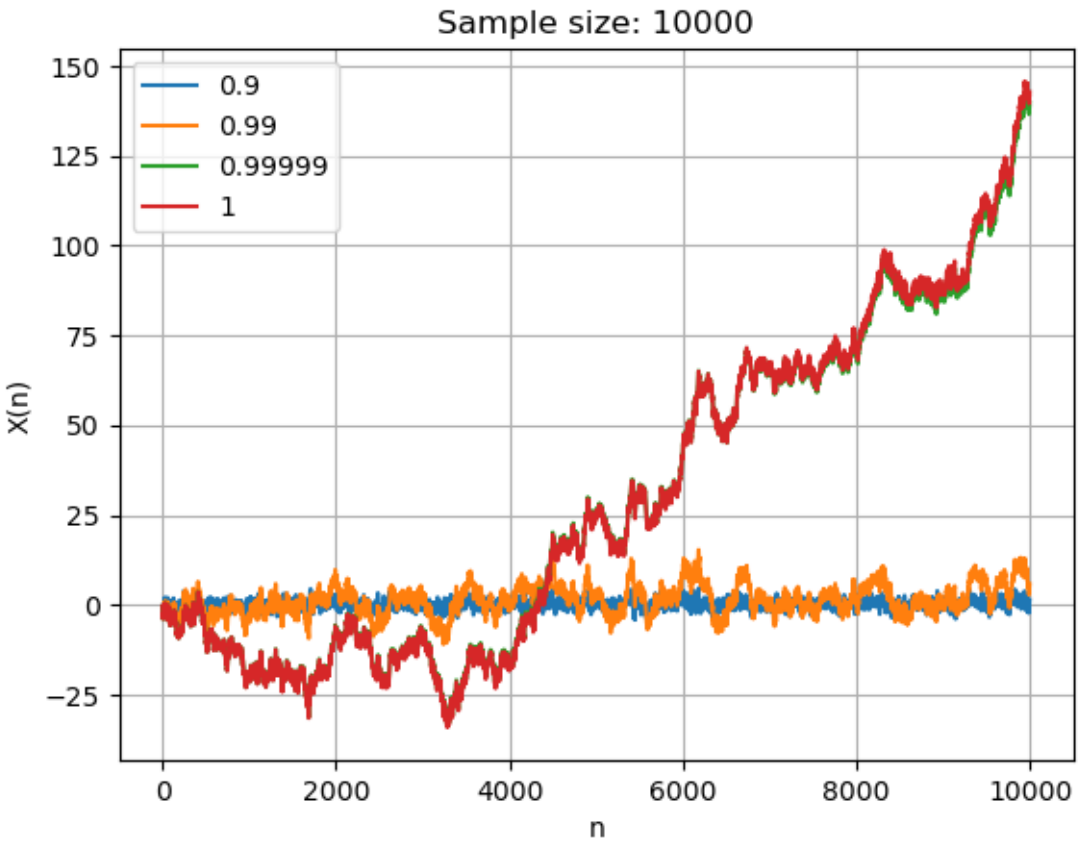

```
Simulation Number:  3
Mean for phi:  0.9  is: -0.4264800009600329
Mean for phi:  0.99  is: -1.5602867950325998
Mean for phi:  0.99999  is: -2.2555720368945367
Mean for phi:  1  is: -2.256517075619546
Standard Deviation for phi:  0.9  is: 1.255811317835891
Standard Deviation for phi:  0.99  is: 1.4873068387928094
Standard Deviation for phi:  0.99999  is: 1.595782879192064
Standard Deviation for phi:  1  is: 1.5960173390938241
```



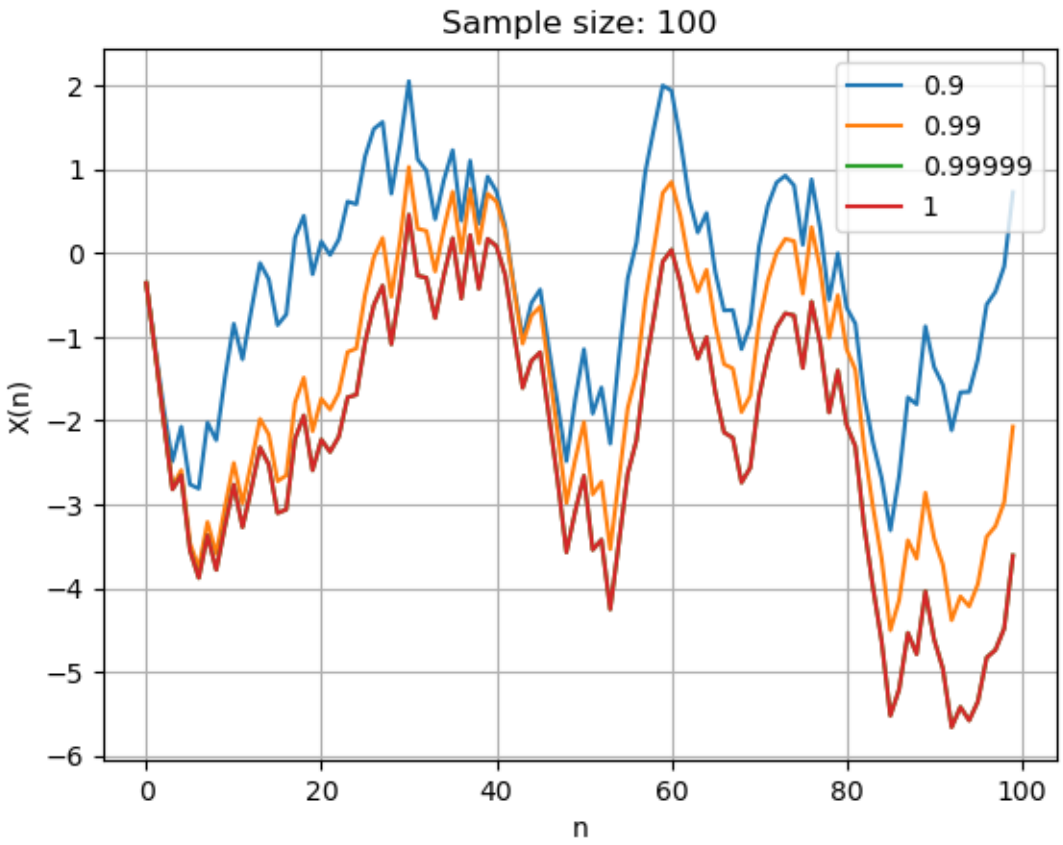

```
Mean for phi:  0.9  is: -0.20104745875185367
Mean for phi:  0.99  is: -1.6442552594189224
Mean for phi:  0.99999  is: -8.149636929171326
Mean for phi:  1  is: -8.176964051913451
Standard Deviation for phi:  0.9  is: 1.2458186962205509
Standard Deviation for phi:  0.99  is: 2.655584971340514
Standard Deviation for phi:  0.99999  is: 5.599434263862098
Standard Deviation for phi:  1  is: 5.618892246448196
```



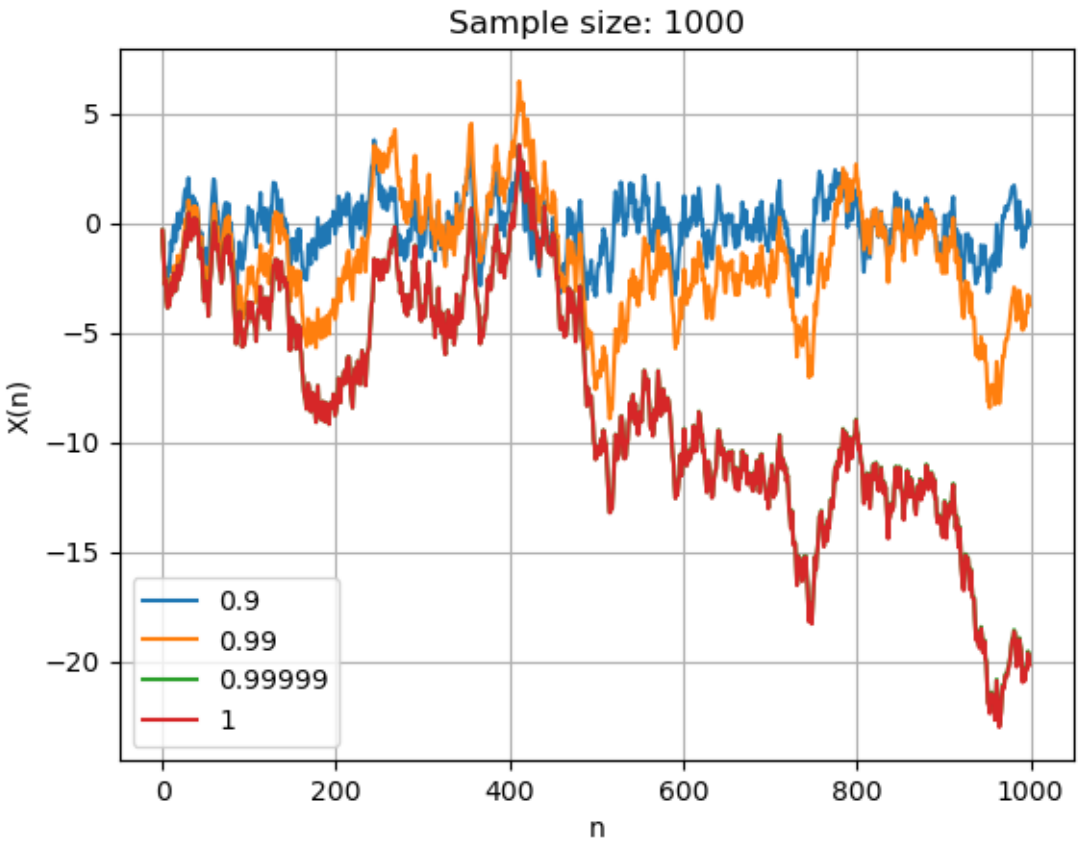

```
Mean for phi:  0.9  is: 0.14145507658295453
Mean for phi:  0.99  is: 1.3704445130771157
Mean for phi:  0.99999  is: 29.414436791455326
Mean for phi:  1  is: 29.664768147578467
Standard Deviation for phi:  0.9  is: 1.3422200587772295
Standard Deviation for phi:  0.99  is: 4.134148538531254
Standard Deviation for phi:  0.99999  is: 45.259411606268756
Standard Deviation for phi:  1  is: 46.05458886641797
```



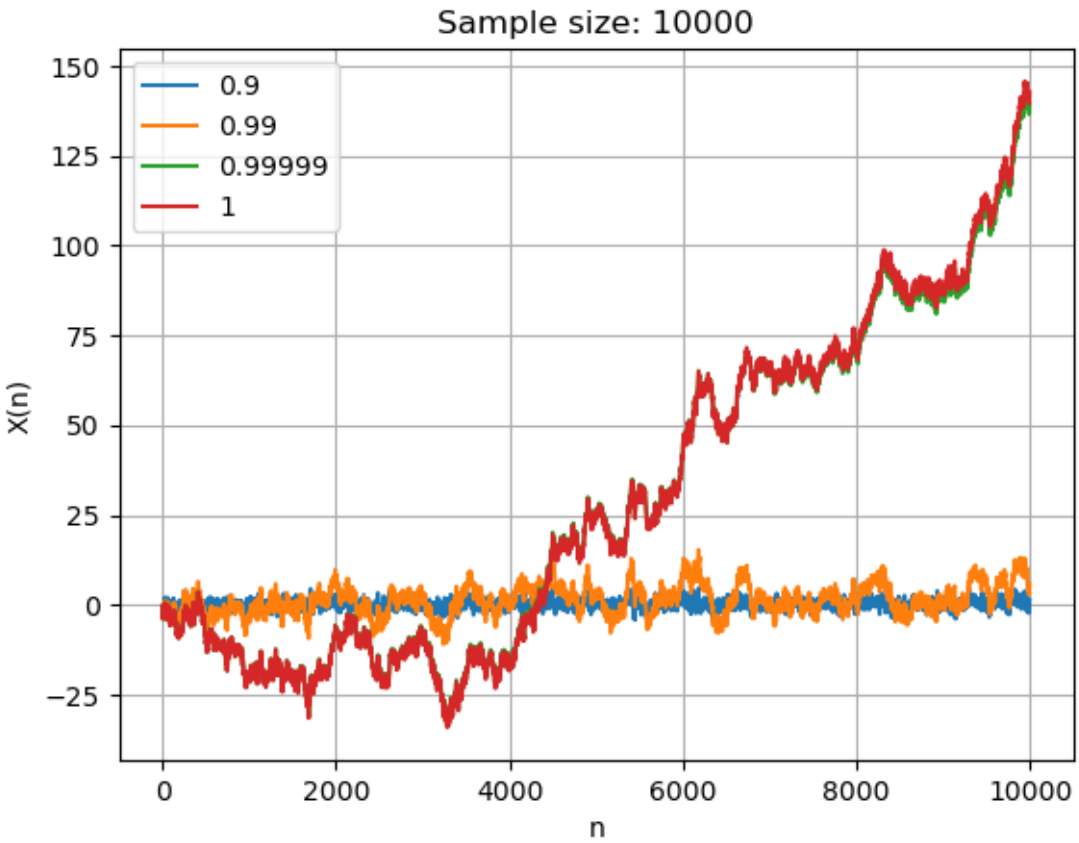

```
Simulation Number:  4
Mean for phi:  0.9  is: -0.4264800009600329
Mean for phi:  0.99  is: -1.5602867950325998
Mean for phi:  0.99999  is: -2.2555720368945367
Mean for phi:  1  is: -2.256517075619546
Standard Deviation for phi:  0.9  is: 1.255811317835891
Standard Deviation for phi:  0.99  is: 1.4873068387928094
Standard Deviation for phi:  0.99999  is: 1.595782879192064
Standard Deviation for phi:  1  is: 1.5960173390938241
```



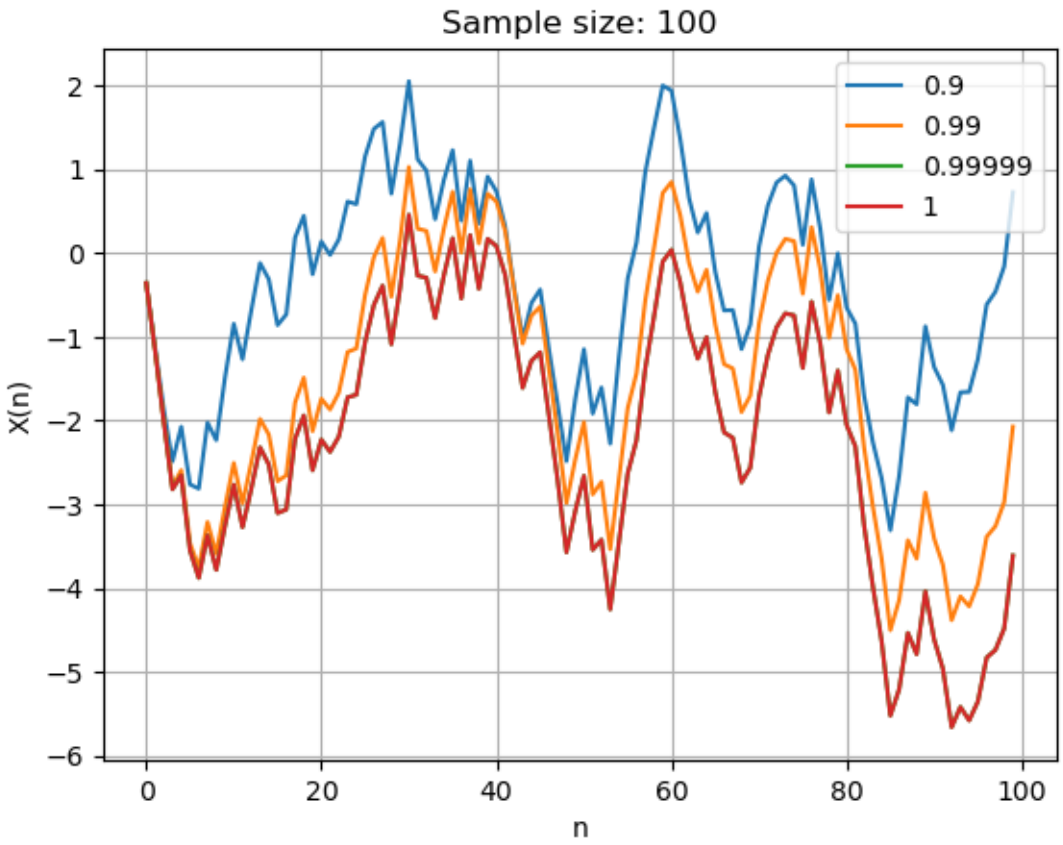

```
Mean for phi:  0.9  is: -0.20104745875185367
Mean for phi:  0.99  is: -1.6442552594189224
Mean for phi:  0.99999  is: -8.149636929171326
Mean for phi:  1  is: -8.176964051913451
Standard Deviation for phi:  0.9  is: 1.2458186962205509
Standard Deviation for phi:  0.99  is: 2.655584971340514
Standard Deviation for phi:  0.99999  is: 5.599434263862098
Standard Deviation for phi:  1  is: 5.618892246448196
```



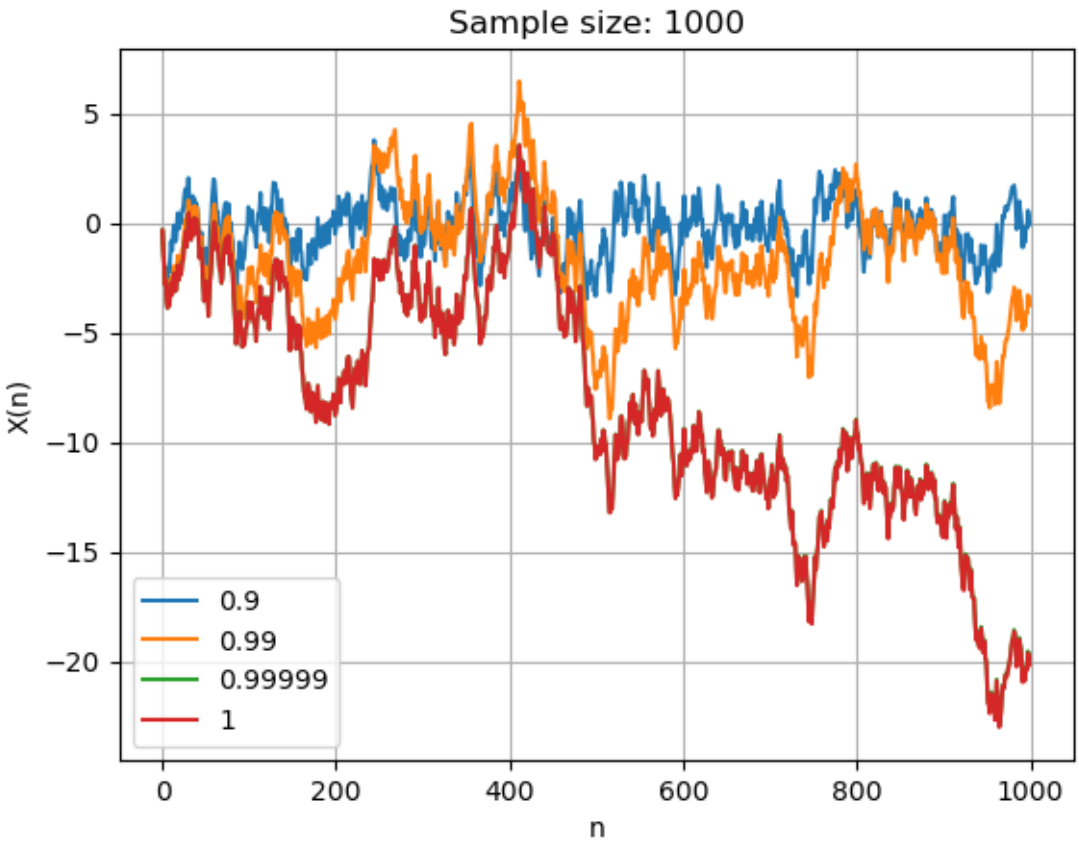

```
Mean for phi:  0.9  is: 0.14145507658295453
Mean for phi:  0.99  is: 1.3704445130771157
Mean for phi:  0.99999  is: 29.414436791455326
Mean for phi:  1  is: 29.664768147578467
Standard Deviation for phi:  0.9  is: 1.3422200587772295
Standard Deviation for phi:  0.99  is: 4.134148538531254
Standard Deviation for phi:  0.99999  is: 45.259411606268756
Standard Deviation for phi:  1  is: 46.05458886641797
```



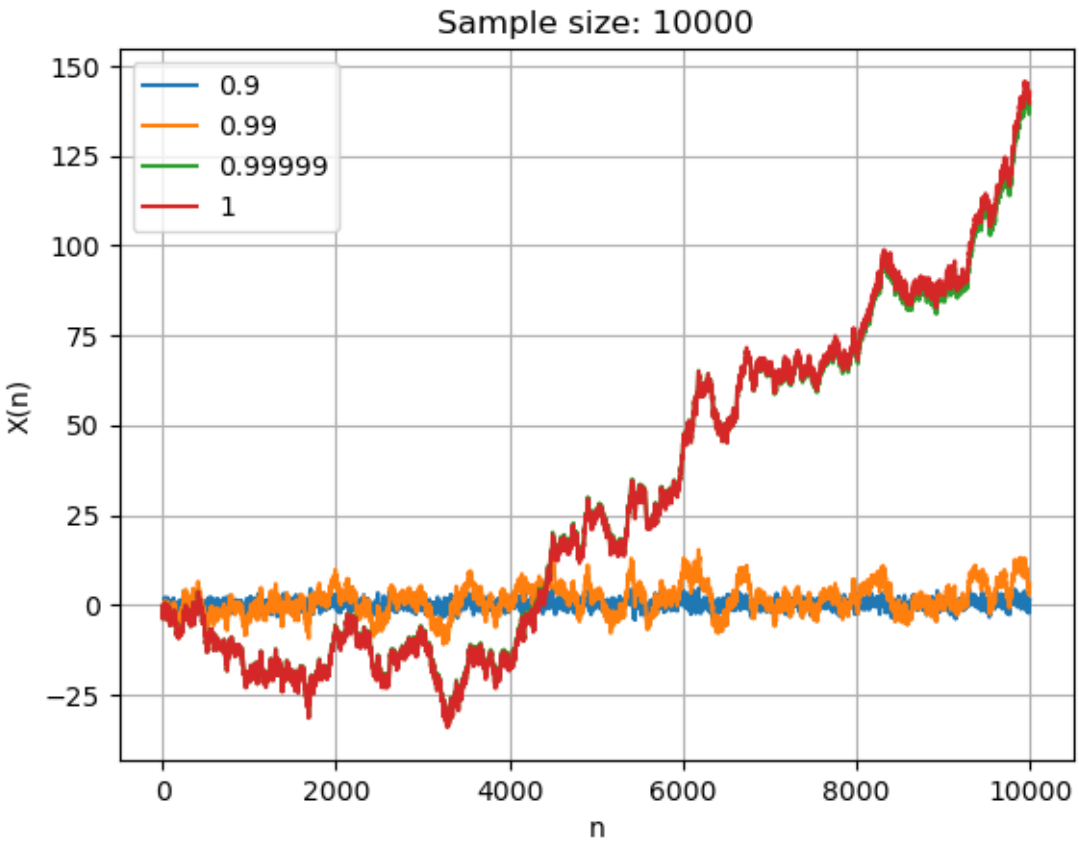

```
Simulation Number:  5
Mean for phi:  0.9  is: -0.4264800009600329
Mean for phi:  0.99  is: -1.5602867950325998
Mean for phi:  0.99999  is: -2.2555720368945367
Mean for phi:  1  is: -2.256517075619546
Standard Deviation for phi:  0.9  is: 1.255811317835891
Standard Deviation for phi:  0.99  is: 1.4873068387928094
Standard Deviation for phi:  0.99999  is: 1.595782879192064
Standard Deviation for phi:  1  is: 1.5960173390938241
```




```
Mean for phi:  0.9  is: -0.20104745875185367
Mean for phi:  0.99  is: -1.6442552594189224
Mean for phi:  0.99999  is: -8.149636929171326
Mean for phi:  1  is: -8.176964051913451
Standard Deviation for phi:  0.9  is: 1.2458186962205509
Standard Deviation for phi:  0.99  is: 2.655584971340514
Standard Deviation for phi:  0.99999  is: 5.599434263862098
Standard Deviation for phi:  1  is: 5.618892246448196
```




```
Mean for phi:  0.9  is: 0.14145507658295453
Mean for phi:  0.99  is: 1.3704445130771157
Mean for phi:  0.99999  is: 29.414436791455326
Mean for phi:  1  is: 29.664768147578467
Standard Deviation for phi:  0.9  is: 1.3422200587772295
Standard Deviation for phi:  0.99  is: 4.134148538531254
Standard Deviation for phi:  0.99999  is: 45.259411606268756
Standard Deviation for phi:  1  is: 46.05458886641797
```



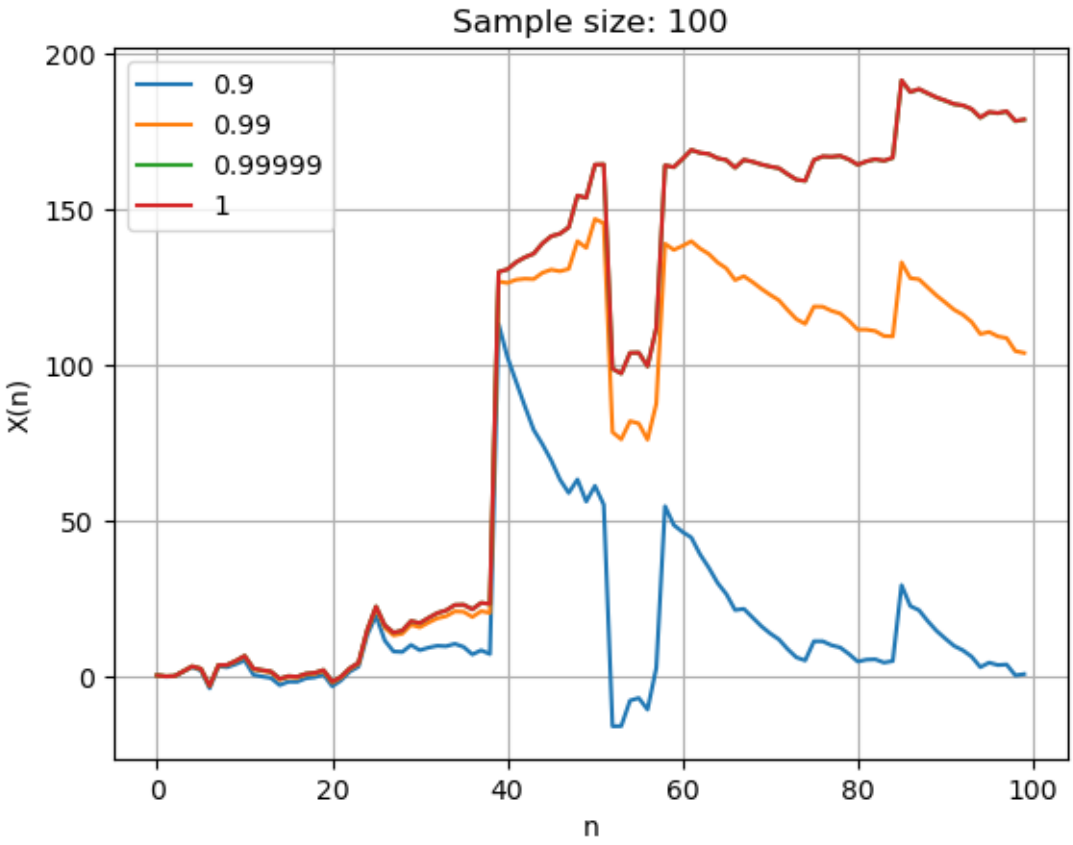
Can see similar behavior to that of normal distribution in terms of process stability. As $\phi \to 1$ we can see that the process starts becoming very unstable

b) Cauchy distribution

[5]:
```
sample_sizes = [100,1000,10000]
phis = [0.9,0.99,0.99999,1]
#noises = [np.random.standard_cauchy(size) for size in sample_sizes]
noises = np.random.standard_cauchy(max(sample_sizes))
for sim in range(1,num_sims+1):
    print("Simulation Number: ", str(sim))
    for i in range(len(sample_sizes)):
        AR_processes = [{str(phi):return_AR(phi,noises[:sample_sizes[i]])} for␣
↪phi in phis]
        [print("Mean for phi: ", str(list(process.keys())[0]), " is:", np.
↪mean(list(process.values())[0])) for process in AR_processes]
        [print("Standard Deviation for phi: ", str(list(process.keys())[0]), "␣
↪is:", np.sqrt(np.var(list(process.values())[0]))) for process in␣
↪AR_processes]
        [plt.plot(list(AR_process.values())[0],label = list(AR_process.
↪keys())[0]) for AR_process in AR_processes]
```

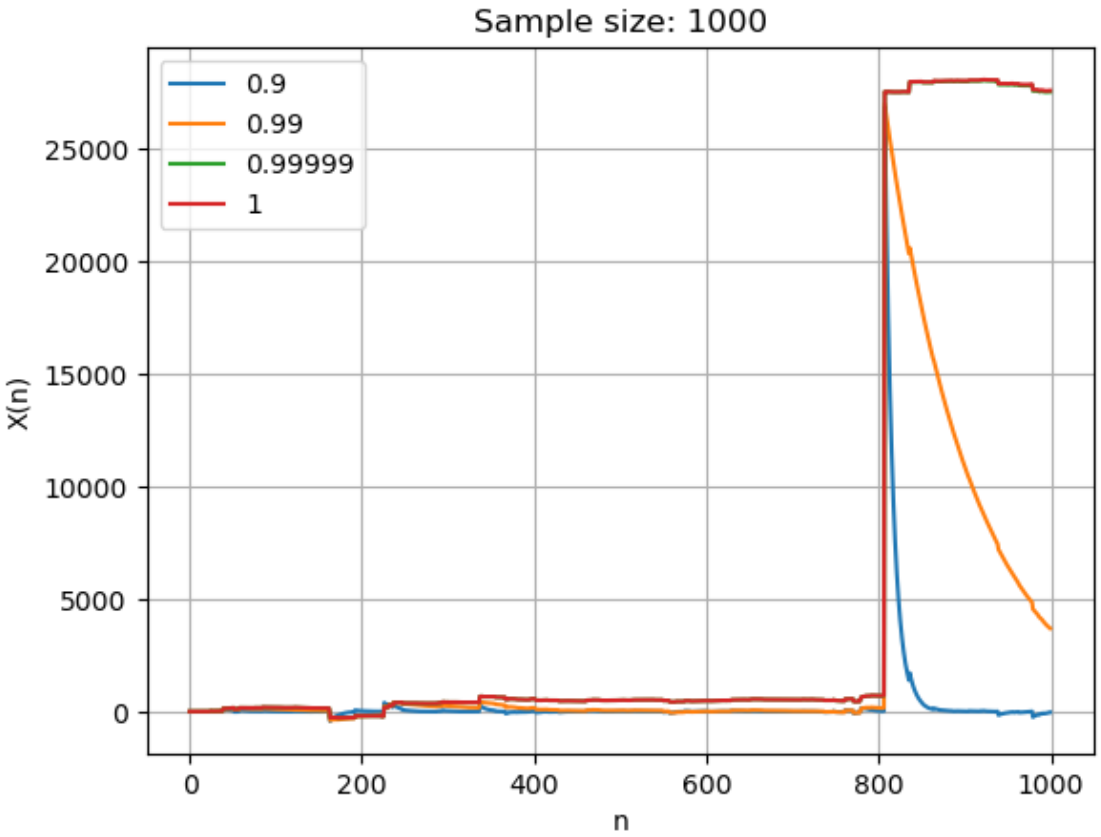
```
        plt.title("Sample size: " + str(sample_sizes[i]))
        plt.grid()
        plt.xlabel("n")
        plt.ylabel("X(n)")
        plt.legend()
        plt.show()
```

```
Simulation Number:  1
Mean for phi:  0.9  is: 17.79959283522824
Mean for phi:  0.99  is: 75.92370342084295
Mean for phi:  0.99999  is: 100.28797990208271
Mean for phi:  1  is: 100.31752966465282
Standard Deviation for phi:  0.9  is: 26.117090594292453
Standard Deviation for phi:  0.99  is: 56.134663906764345
Standard Deviation for phi:  0.99999  is: 75.81692677960868
Standard Deviation for phi:  1  is: 75.84312784889579
```

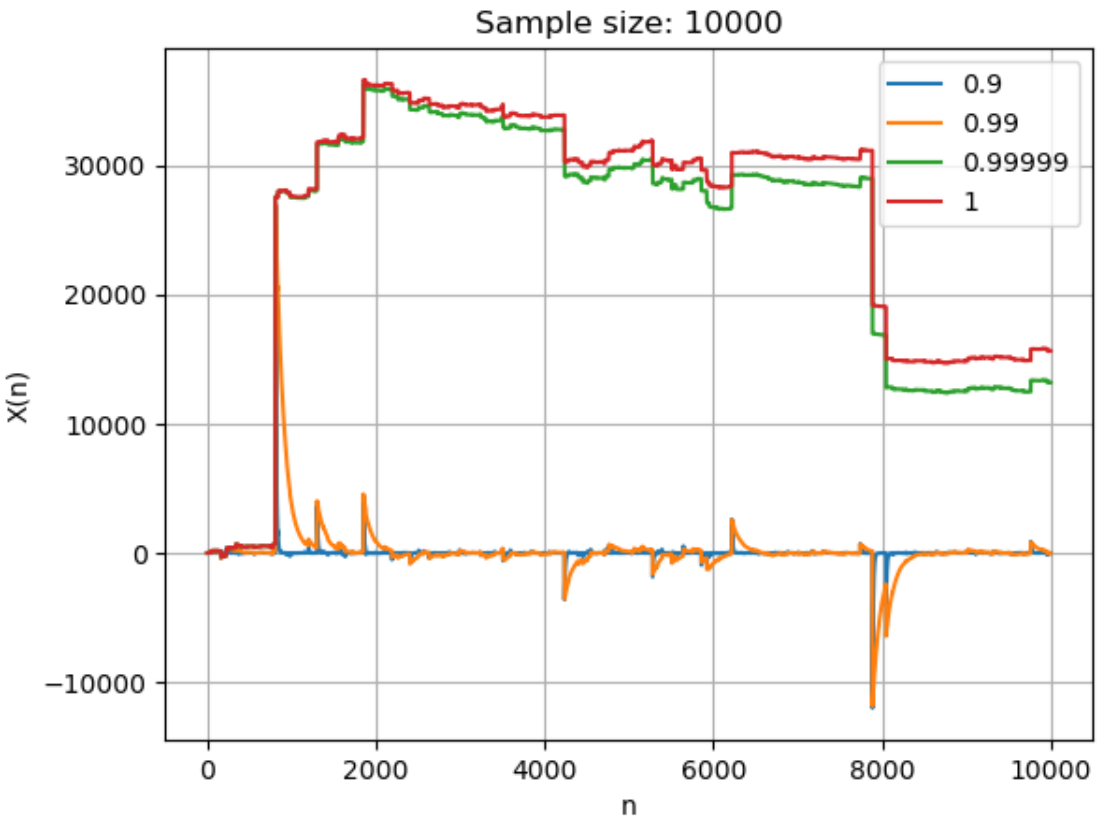



```
Mean for phi:  0.9  is: 276.2580920086971
Mean for phi:  0.99  is: 2393.6230161832964
Mean for phi:  0.99999  is: 5667.47793469148
Mean for phi:  1  is: 5674.047082045982
```

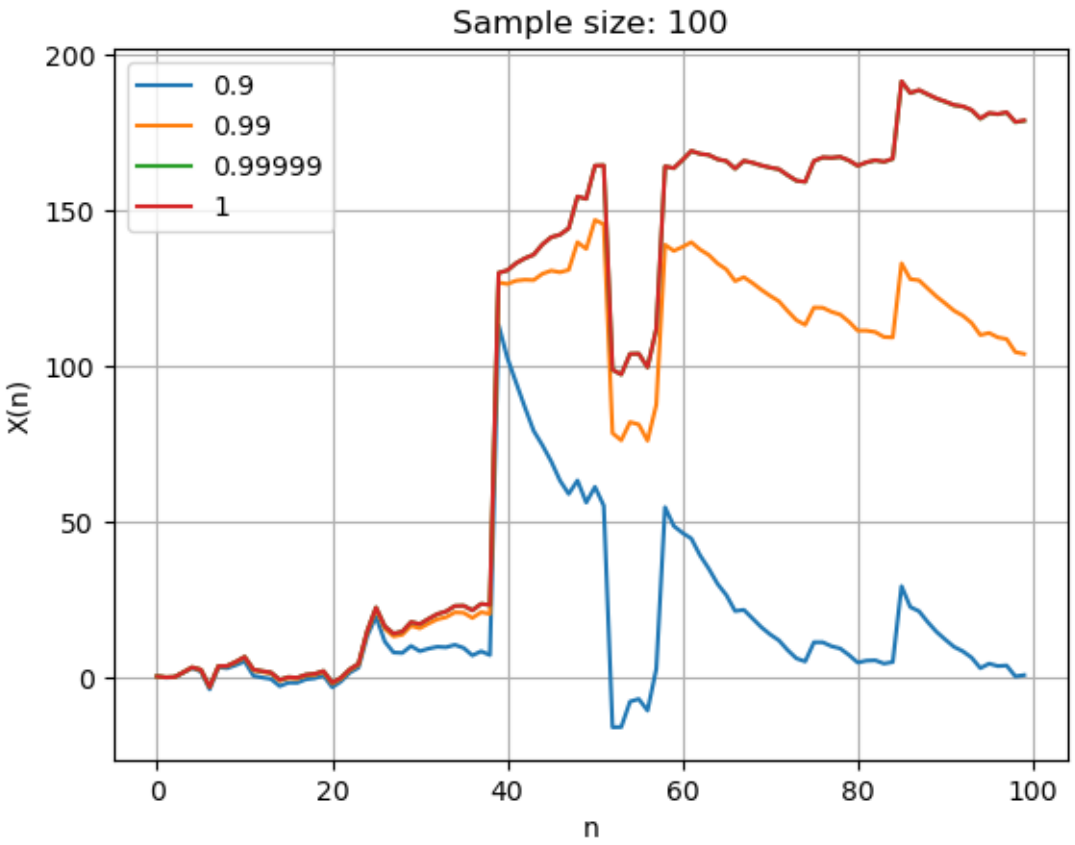
```
Standard Deviation for phi:  0.9  is: 1930.5180483742406
Standard Deviation for phi:  0.99  is: 5563.754092219061
Standard Deviation for phi:  0.99999  is: 10848.417365980167
Standard Deviation for phi:  1  is: 10859.72275407653
```



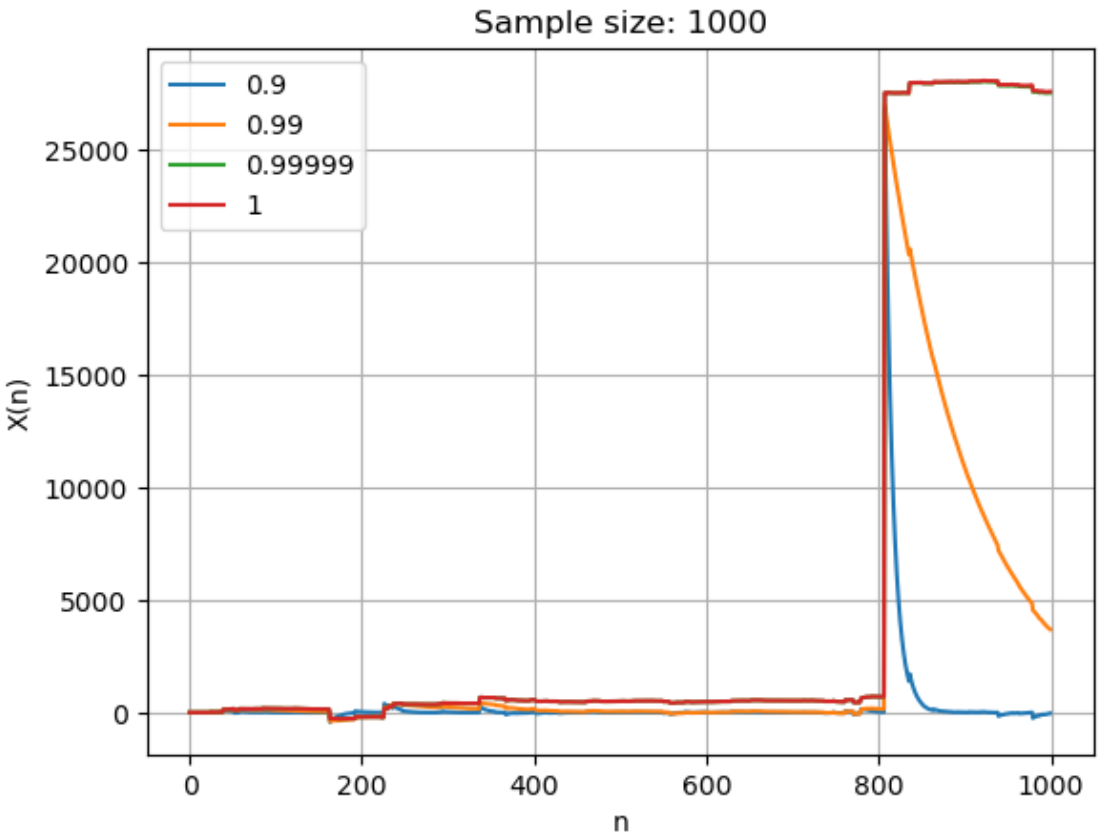

```
Mean for phi:  0.9  is: 15.633946613246959
Mean for phi:  0.99  is: 156.57356853854597
Mean for phi:  0.99999  is: 24514.860632418204
Mean for phi:  1  is: 25797.287727638268
Standard Deviation for phi:  0.9  is: 705.7571758912204
Standard Deviation for phi:  0.99  is: 2220.264004435358
Standard Deviation for phi:  0.99999  is: 10353.793733007407
Standard Deviation for phi:  1  is: 10250.783616513101
```



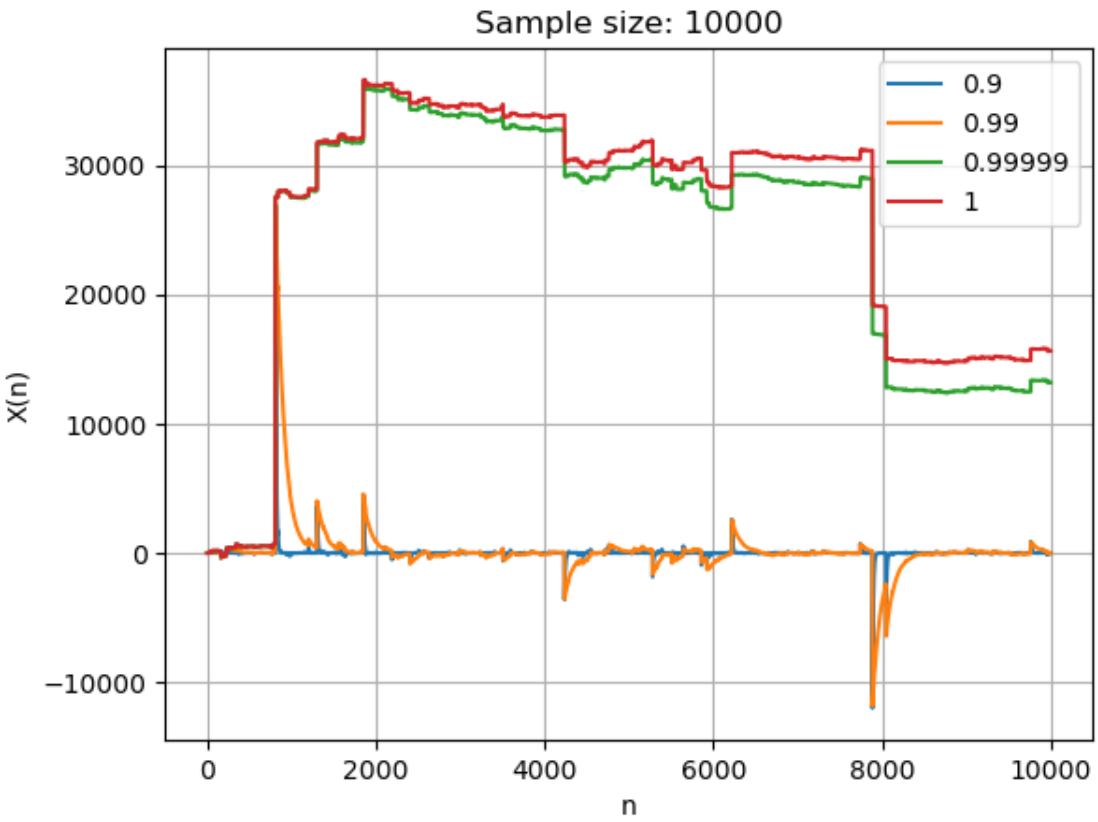

```
Simulation Number:  2
Mean for phi:  0.9  is: 17.79959283522824
Mean for phi:  0.99  is: 75.92370342084295
Mean for phi:  0.99999  is: 100.28797990208271
Mean for phi:  1  is: 100.31752966465282
Standard Deviation for phi:  0.9  is: 26.117090594292453
Standard Deviation for phi:  0.99  is: 56.134663906764345
Standard Deviation for phi:  0.99999  is: 75.81692677960868
Standard Deviation for phi:  1  is: 75.84312784889579
```



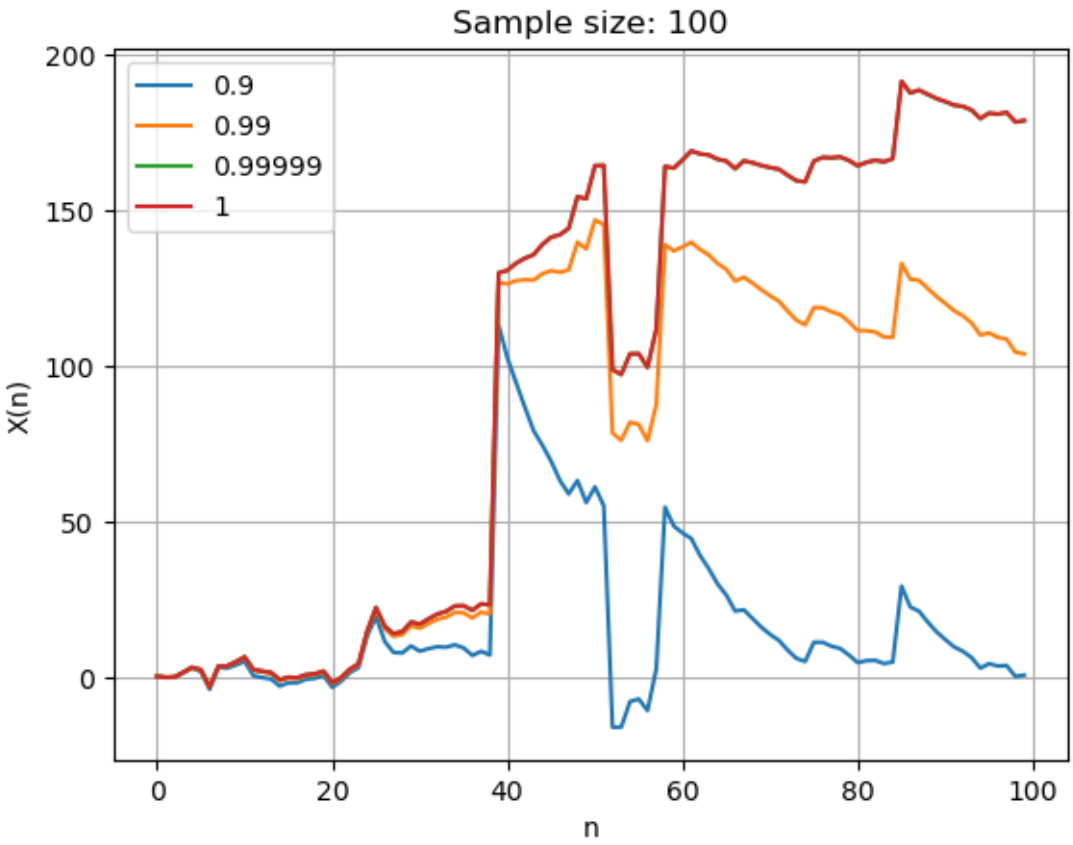

```
Mean for phi:  0.9  is: 276.2580920086971
Mean for phi:  0.99  is: 2393.6230161832964
Mean for phi:  0.99999  is: 5667.47793469148
Mean for phi:  1  is: 5674.047082045982
Standard Deviation for phi:  0.9  is: 1930.5180483742406
Standard Deviation for phi:  0.99  is: 5563.754092219061
Standard Deviation for phi:  0.99999  is: 10848.417365980167
Standard Deviation for phi:  1  is: 10859.72275407653
```



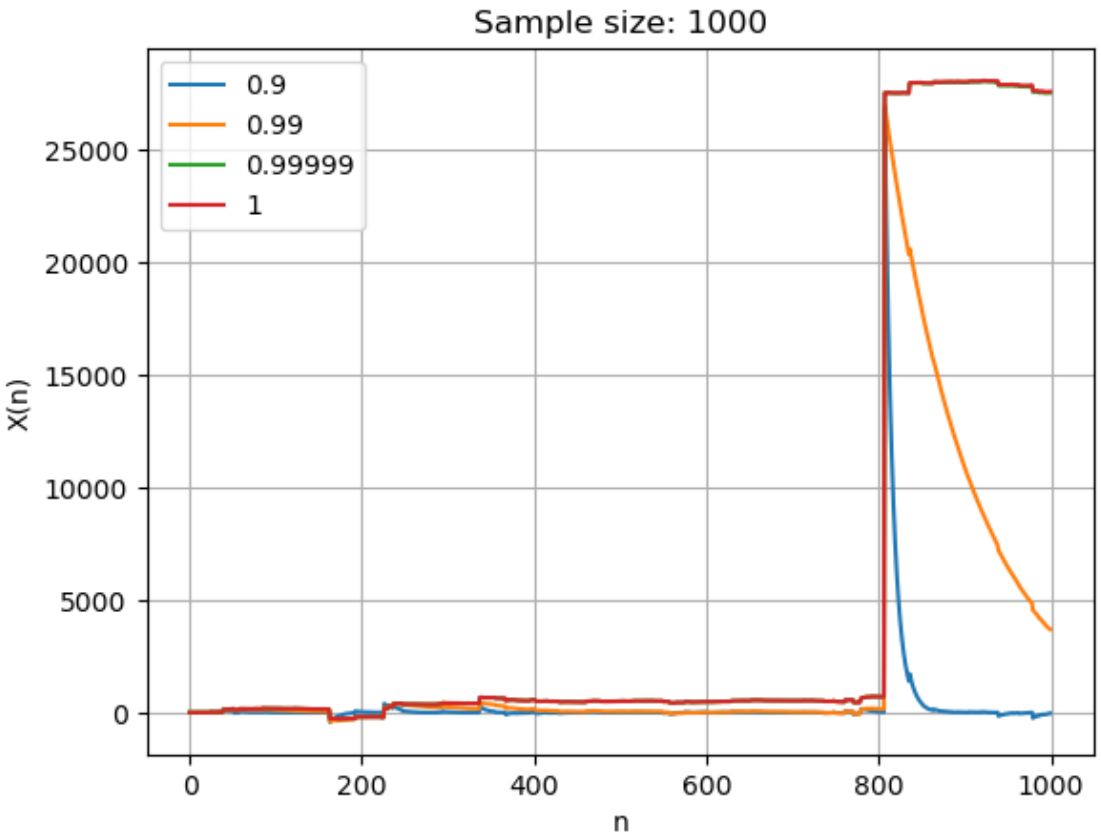

```
Mean for phi:  0.9  is: 15.633946613246959
Mean for phi:  0.99  is: 156.57356853854597
Mean for phi:  0.99999  is: 24514.860632418204
Mean for phi:  1  is: 25797.287727638268
Standard Deviation for phi:  0.9  is: 705.7571758912204
Standard Deviation for phi:  0.99  is: 2220.264004435358
Standard Deviation for phi:  0.99999  is: 10353.793733007407
Standard Deviation for phi:  1  is: 10250.783616513101
```



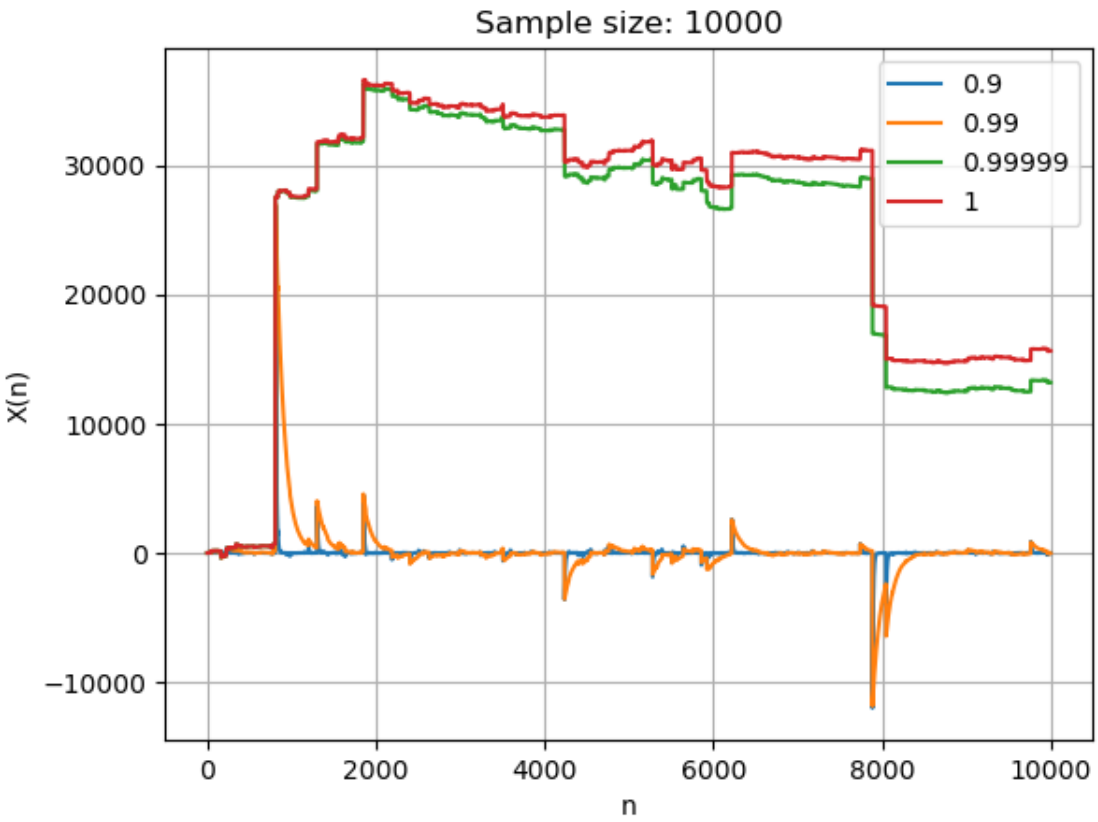

```
Simulation Number:  3
Mean for phi:  0.9  is: 17.79959283522824
Mean for phi:  0.99  is: 75.92370342084295
Mean for phi:  0.99999  is: 100.28797990208271
Mean for phi:  1  is: 100.31752966465282
Standard Deviation for phi:  0.9  is: 26.117090594292453
Standard Deviation for phi:  0.99  is: 56.134663906764345
Standard Deviation for phi:  0.99999  is: 75.81692677960868
Standard Deviation for phi:  1  is: 75.84312784889579
```



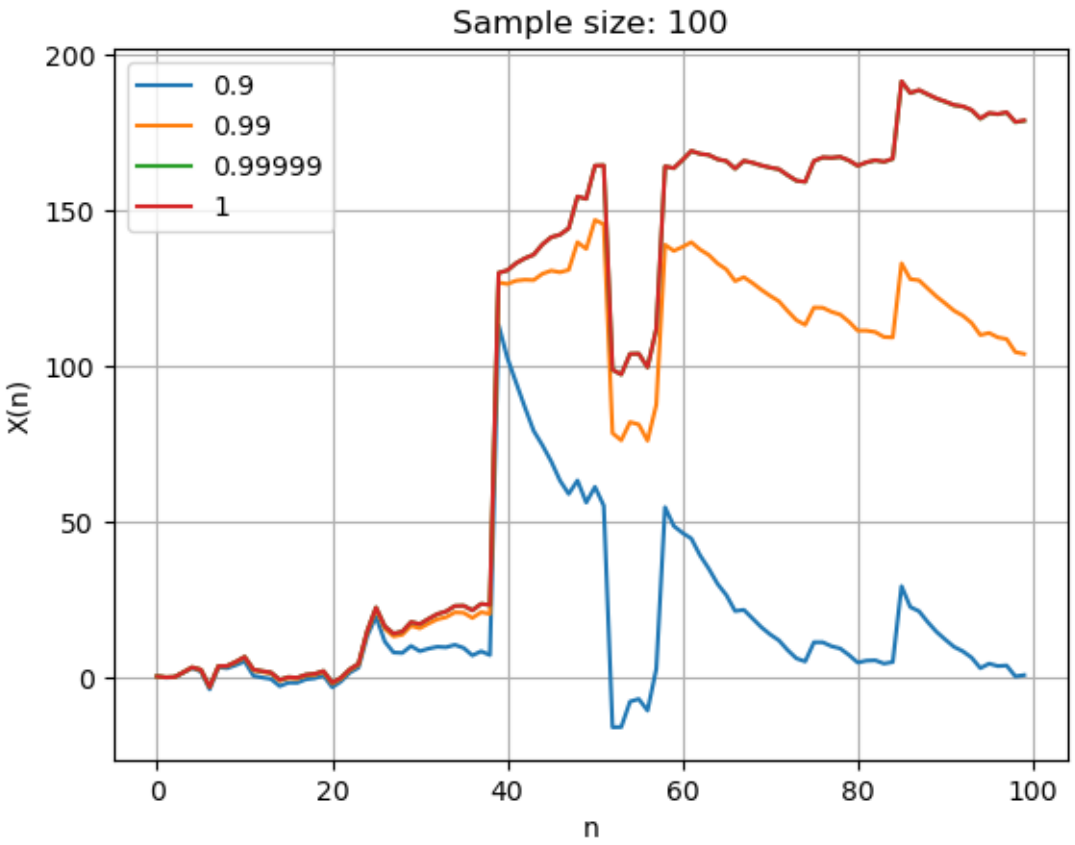

```
Mean for phi:  0.9  is: 276.2580920086971
Mean for phi:  0.99  is: 2393.6230161832964
Mean for phi:  0.99999  is: 5667.47793469148
Mean for phi:  1  is: 5674.047082045982
Standard Deviation for phi:  0.9  is: 1930.5180483742406
Standard Deviation for phi:  0.99  is: 5563.754092219061
Standard Deviation for phi:  0.99999  is: 10848.417365980167
Standard Deviation for phi:  1  is: 10859.72275407653
```



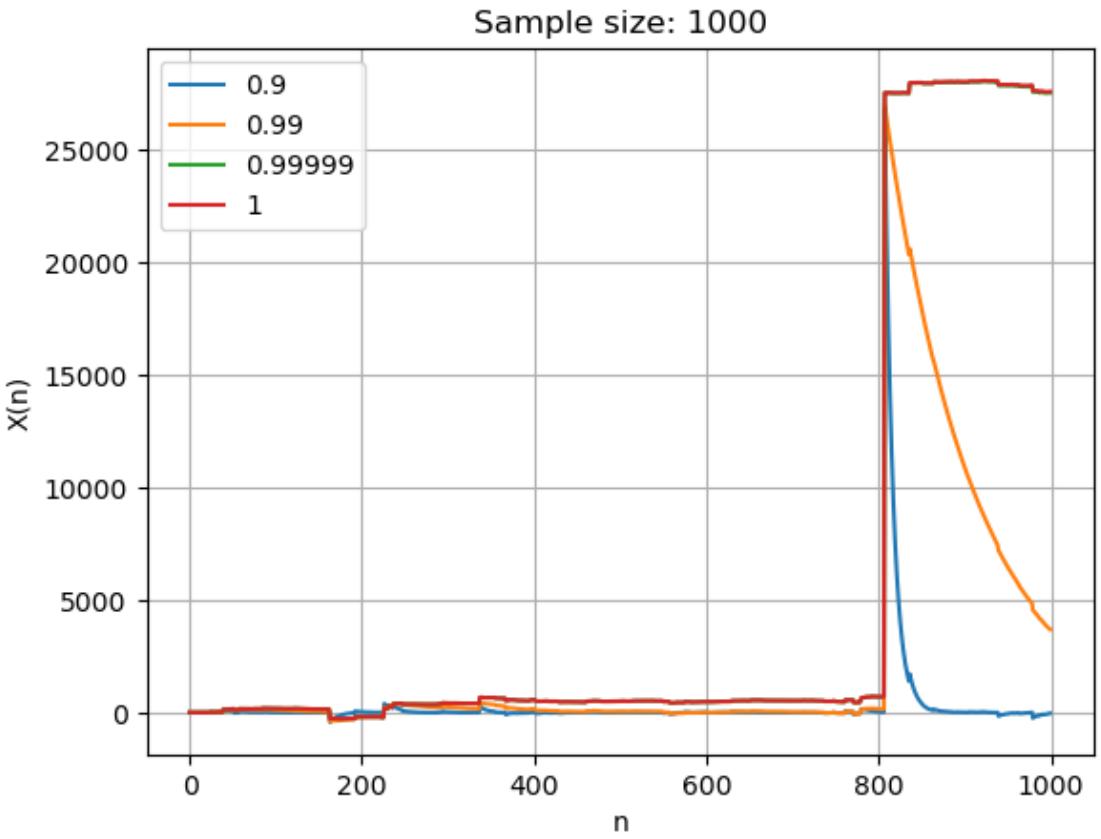

```
Mean for phi:  0.9  is: 15.633946613246959
Mean for phi:  0.99  is: 156.57356853854597
Mean for phi:  0.99999  is: 24514.860632418204
Mean for phi:  1  is: 25797.287727638268
Standard Deviation for phi:  0.9  is: 705.7571758912204
Standard Deviation for phi:  0.99  is: 2220.264004435358
Standard Deviation for phi:  0.99999  is: 10353.793733007407
Standard Deviation for phi:  1  is: 10250.783616513101
```



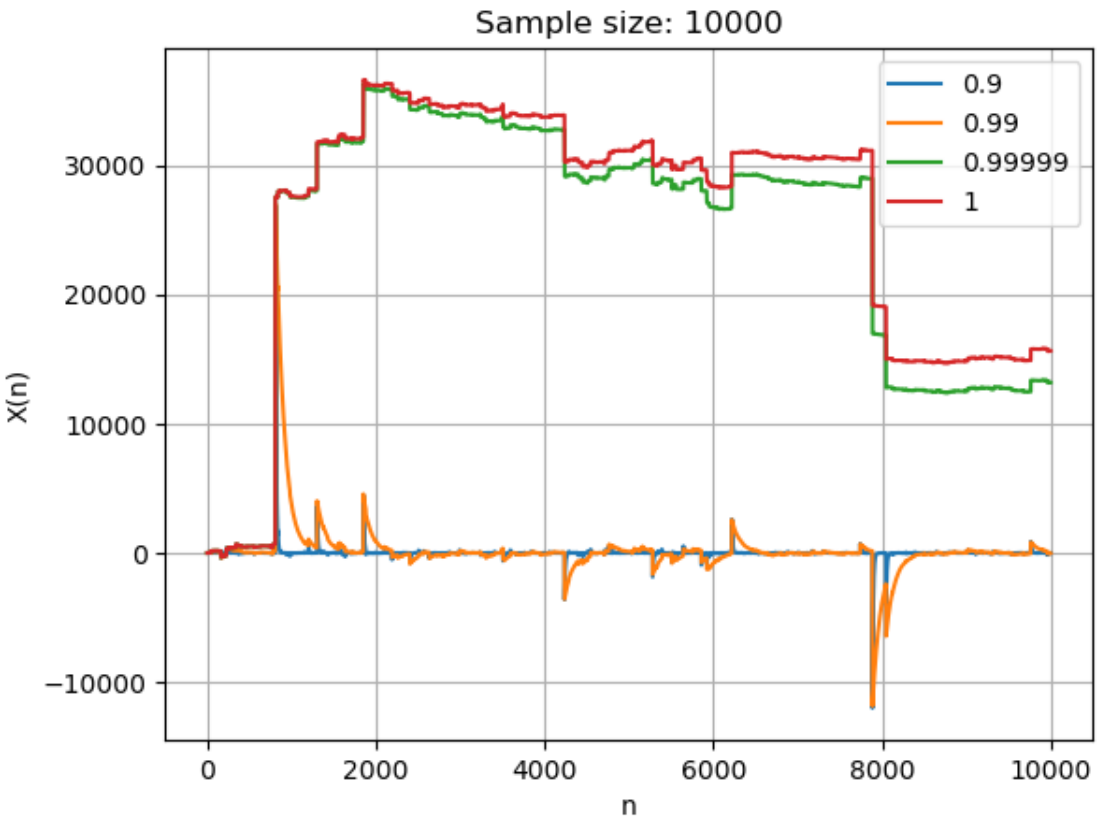

```
Simulation Number:  4
Mean for phi:  0.9  is: 17.79959283522824
Mean for phi:  0.99  is: 75.92370342084295
Mean for phi:  0.99999  is: 100.28797990208271
Mean for phi:  1  is: 100.31752966465282
Standard Deviation for phi:  0.9  is: 26.117090594292453
Standard Deviation for phi:  0.99  is: 56.134663906764345
Standard Deviation for phi:  0.99999  is: 75.81692677960868
Standard Deviation for phi:  1  is: 75.84312784889579
```



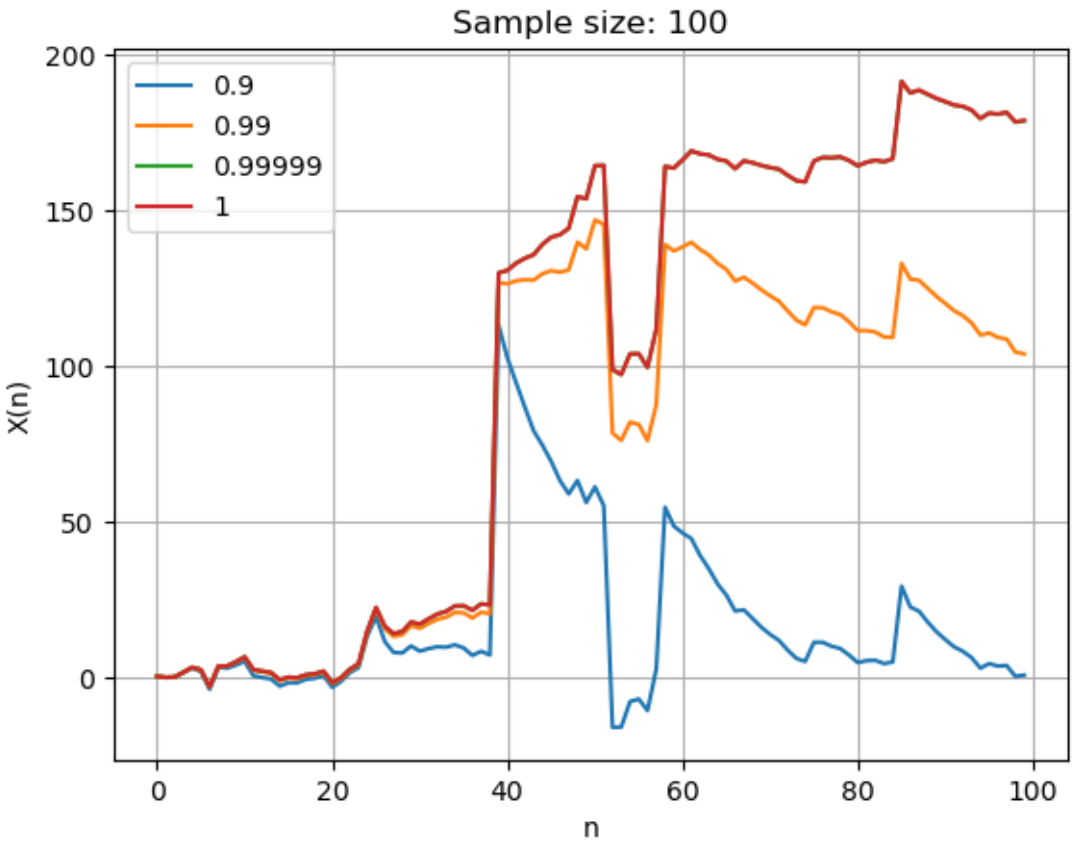

```
Mean for phi:  0.9  is: 276.2580920086971
Mean for phi:  0.99  is: 2393.6230161832964
Mean for phi:  0.99999  is: 5667.47793469148
Mean for phi:  1  is: 5674.047082045982
Standard Deviation for phi:  0.9  is: 1930.5180483742406
Standard Deviation for phi:  0.99  is: 5563.754092219061
Standard Deviation for phi:  0.99999  is: 10848.417365980167
Standard Deviation for phi:  1  is: 10859.72275407653
```



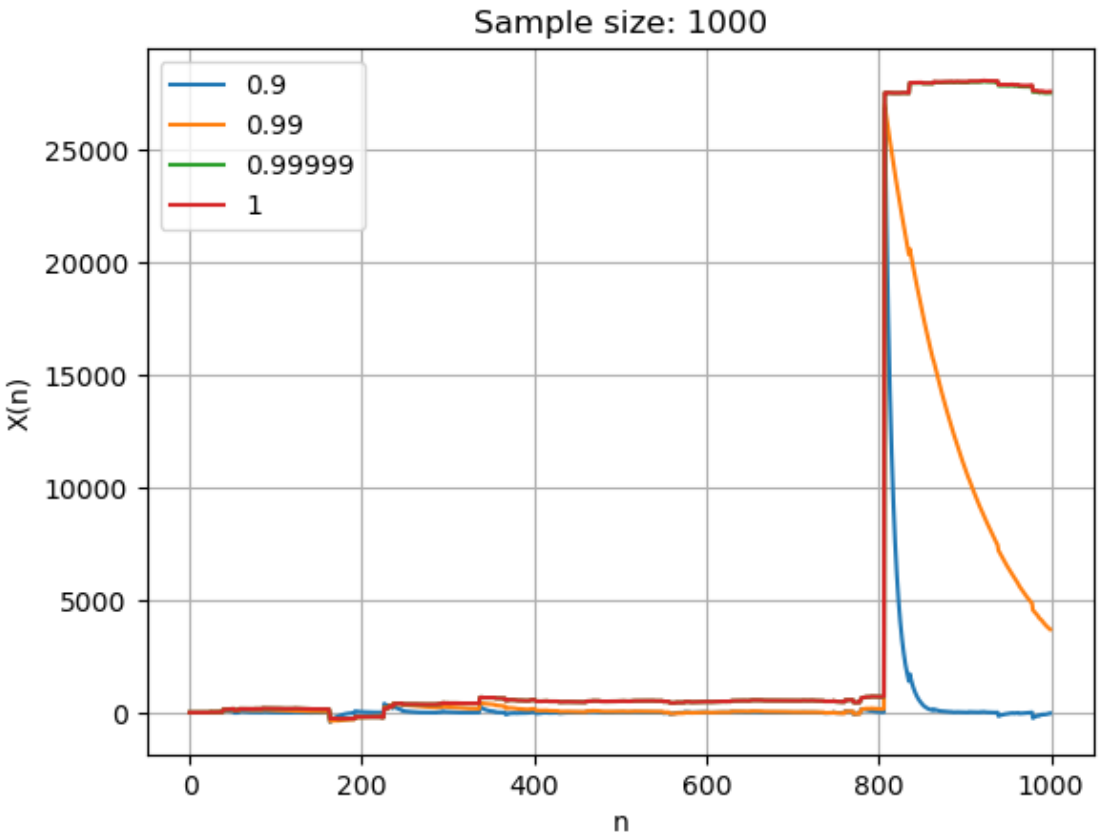

```
Mean for phi:  0.9  is: 15.633946613246959
Mean for phi:  0.99  is: 156.57356853854597
Mean for phi:  0.99999  is: 24514.860632418204
Mean for phi:  1  is: 25797.287727638268
Standard Deviation for phi:  0.9  is: 705.7571758912204
Standard Deviation for phi:  0.99  is: 2220.264004435358
Standard Deviation for phi:  0.99999  is: 10353.793733007407
Standard Deviation for phi:  1  is: 10250.783616513101
```



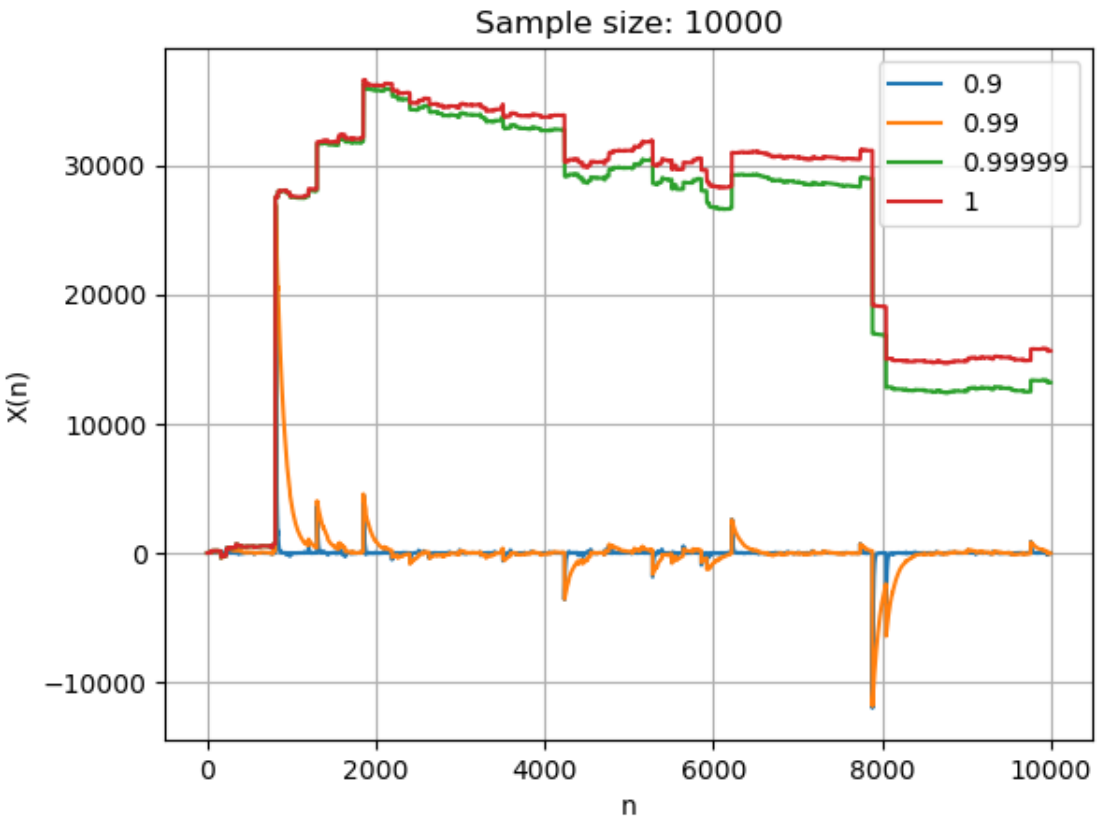

```
Simulation Number:  5
Mean for phi:  0.9  is: 17.79959283522824
Mean for phi:  0.99  is: 75.92370342084295
Mean for phi:  0.99999  is: 100.28797990208271
Mean for phi:  1  is: 100.31752966465282
Standard Deviation for phi:  0.9  is: 26.117090594292453
Standard Deviation for phi:  0.99  is: 56.134663906764345
Standard Deviation for phi:  0.99999  is: 75.81692677960868
Standard Deviation for phi:  1  is: 75.84312784889579
```




```
Mean for phi:  0.9  is: 276.2580920086971
Mean for phi:  0.99  is: 2393.6230161832964
Mean for phi:  0.99999  is: 5667.47793469148
Mean for phi:  1  is: 5674.047082045982
Standard Deviation for phi:  0.9  is: 1930.5180483742406
Standard Deviation for phi:  0.99  is: 5563.754092219061
Standard Deviation for phi:  0.99999  is: 10848.417365980167
Standard Deviation for phi:  1  is: 10859.72275407653
```




```
Mean for phi:  0.9  is: 15.633946613246959
Mean for phi:  0.99  is: 156.57356853854597
Mean for phi:  0.99999  is: 24514.860632418204
Mean for phi:  1  is: 25797.287727638268
Standard Deviation for phi:  0.9  is: 705.7571758912204
Standard Deviation for phi:  0.99  is: 2220.264004435358
Standard Deviation for phi:  0.99999  is: 10353.793733007407
Standard Deviation for phi:  1  is: 10250.783616513101
```



0.0.4 Observations for Cauchy distribution

Here the process is quite unstable for all values of specified ϕ's which is a contrast to the behavior we see when errors are normally/uniformly distributed. This is evident in the sample statistics as we see erratic nature of mean and standard deviations. This can be attributed to the fact that cauchy distribution itself has both first and second order statistics as undefined. However, we can see some semblance of a pattern repeating after around 400 samples.

0.0.5 Q8

[6]:
```
import pandas_datareader.data as web
from scipy.stats import chi2
import quandl
def get_data(symbol):
    df = web.DataReader(symbol, 'quandl',
                                start='1928-01-01',
                                end='2024-03-25',
                                api_key="JxbPCBkt_tN6rsETQCTf")
    df['Growth'] = (df['Value'] / df['Value'].shift(1)).fillna(1) # Assuming␣
 ↪fill with 1 for the first value
    return df
```

```
def acf_analysis(data):
    data = data - np.mean(data)
    AutoCorrel = np.correlate(data,data,mode = "full")/(len(data)*np.var(data))␣
 ↪### Standardize to have correls wrt 1
    return AutoCorrel[len(AutoCorrel) // 2 :]

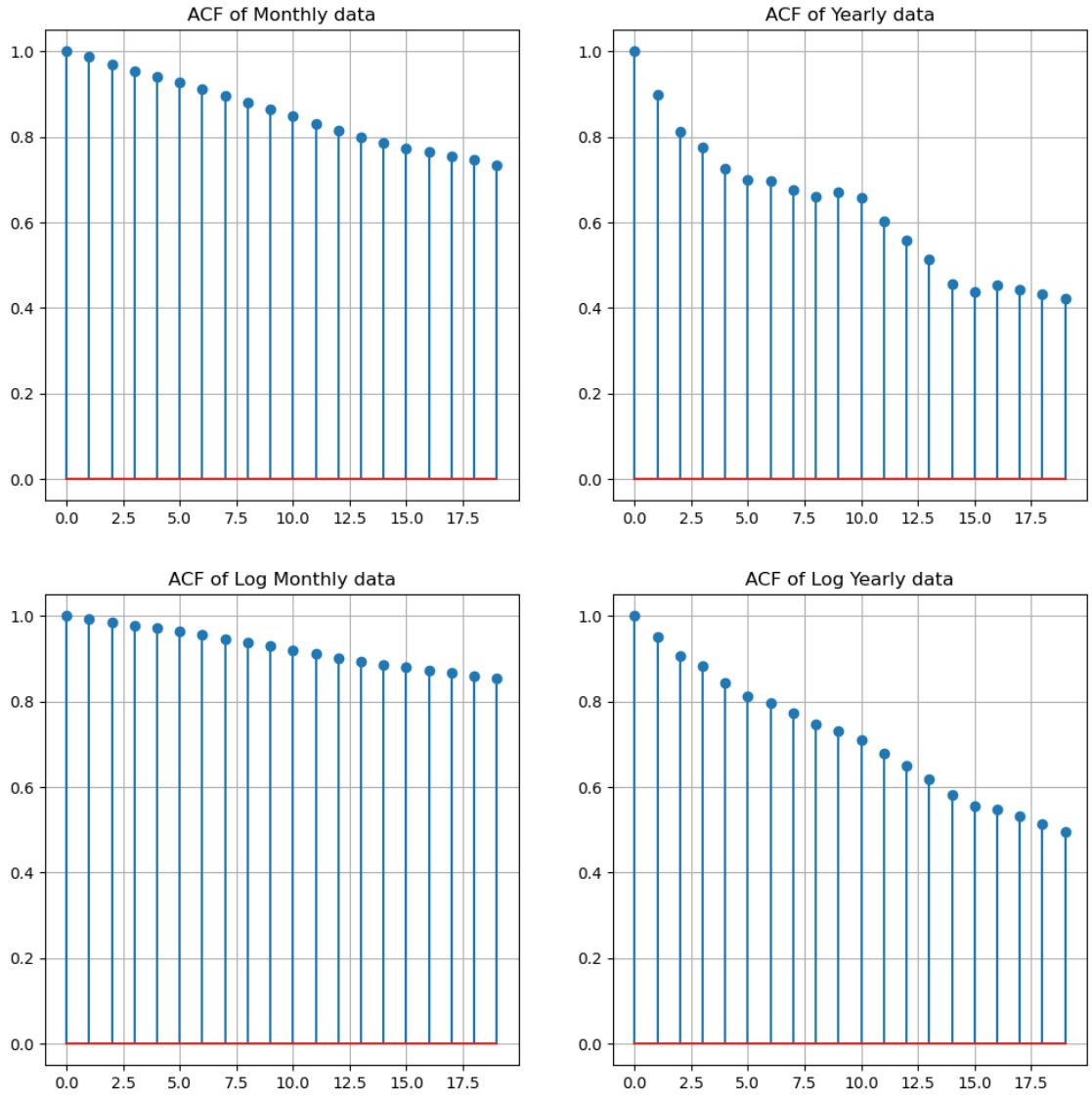
def Box_Ljung(ACFs, n, lag):
    Q_score = n * (n + 2) * np.sum((ACFs[1:(lag+1)]**2) / (n - np.arange(1,␣
 ↪lag+1)))
    p_value = 1.0 - chi2.cdf(Q_score, lag)
    return Q_score, p_value
```

[7]:
```
Monthly_data = get_data("MULTPL/SP500_DIV_YIELD_MONTH")
Monthly_data["Log_Vals"] = np.log(Monthly_data["Value"])
Monthly_data["Log_Growth"] = np.log(Monthly_data["Growth"])

Yearly_data = get_data("MULTPL/SP500_DIV_YIELD_YEAR")
Yearly_data["Log_Vals"] = np.log(Yearly_data["Value"])
Yearly_data["Log_Growth"] = np.log(Yearly_data["Growth"])
```

0.0.6 Analysis for the dividend yields

[8]:
```
Monthly_acf = acf_analysis(np.array(Monthly_data["Value"]))
Yearly_acf = acf_analysis(np.array(Yearly_data["Value"]))
Monthly_log_acf = acf_analysis(np.array(Monthly_data["Log_Vals"]))
Yearly_log_acf = acf_analysis(np.array(Yearly_data["Log_Vals"]))
max_lag = 20
fig, axes = plt.subplots(2, 2, figsize=(12, 12))
axes[0][0].set_title("ACF of Monthly data")
axes[0][0].stem(Monthly_acf[:max_lag])
axes[0][0].grid()
axes[0][1].set_title("ACF of Yearly data")
axes[0][1].stem(Yearly_acf[:max_lag])
axes[0][1].grid()
axes[1][0].set_title("ACF of Log Monthly data")
axes[1][0].stem(Monthly_log_acf[:max_lag])
axes[1][0].grid()
axes[1][1].set_title("ACF of Log Yearly data")
axes[1][1].stem(Yearly_log_acf[:max_lag])
axes[1][1].grid()
```



0.0.7 Box Test for dividend yields

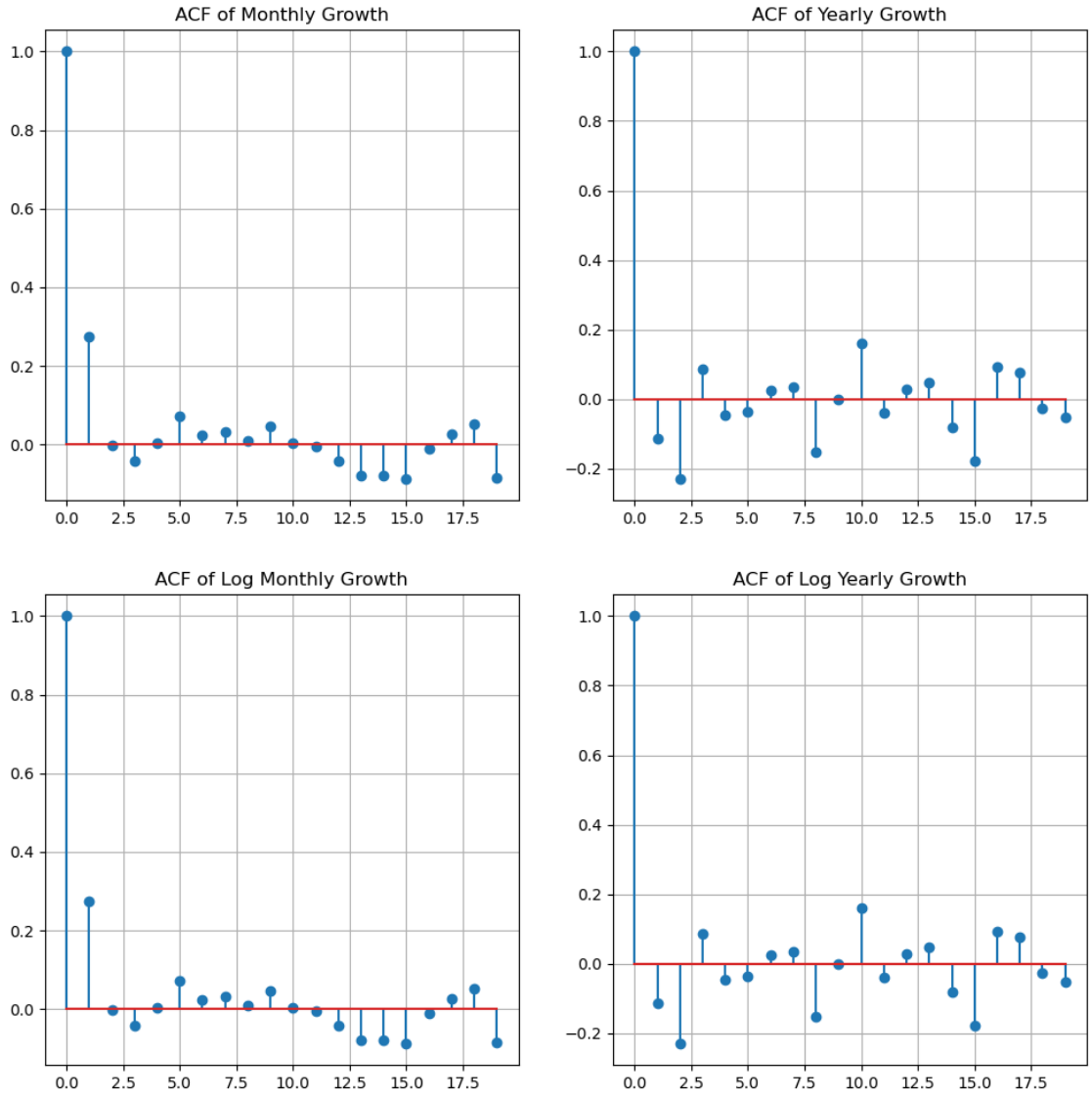
[9]:
```
Monthly_box = Box_Ljung(Monthly_acf, len(Monthly_data), 10)
Yearly_box = Box_Ljung(Yearly_acf, len(Yearly_data), 10)
Monthly_log_box = Box_Ljung(Monthly_log_acf, len(Monthly_data), 10)
Yearly_log_box = Box_Ljung(Yearly_log_acf, len(Yearly_data), 10)
print("BL statistic and p value for monthly dividend yields are: ",␣
 ↪str(Monthly_box[0]), ",", str(Monthly_box[1]))
print("BL statistic and p value for yearly dividend yields are: ",␣
 ↪str(Yearly_box[0]), ",", str(Yearly_box[1]))
print("BL statistic and p value for monthly log dividend yields are: ",␣
 ↪str(Monthly_log_box[0]), ",", str(Monthly_log_box[1]))
```

```
print("BL statistic and p value for yearly log dividend yields are: ",␣
 ↪str(Yearly_log_box[0]), ",", str(Yearly_log_box[1]))
```

```
BL statistic and p value for monthly dividend yields are:  10094.090418792812 ,
0.0
BL statistic and p value for yearly dividend yields are:  921.4648791842818 ,
0.0
BL statistic and p value for monthly log dividend yields are:
10975.558182504614 , 0.0
BL statistic and p value for yearly log dividend yields are:  1156.2487664458445
, 0.0
```

0.0.8 Growth analysis

[10]:
```
Monthly_acf = acf_analysis(np.array(Monthly_data["Growth"]))
Yearly_acf = acf_analysis(np.array(Yearly_data["Growth"]))
Monthly_log_acf = acf_analysis(np.array(Monthly_data["Log_Growth"]))
Yearly_log_acf = acf_analysis(np.array(Yearly_data["Log_Growth"]))
max_lag = 20
fig, axes = plt.subplots(2, 2, figsize=(12, 12))
axes[0][0].set_title("ACF of Monthly Growth")
axes[0][0].stem(Monthly_acf[:max_lag])
axes[0][0].grid()
axes[0][1].set_title("ACF of Yearly Growth")
axes[0][1].stem(Yearly_acf[:max_lag])
axes[0][1].grid()
axes[1][0].set_title("ACF of Log Monthly Growth")
axes[1][0].stem(Monthly_acf[:max_lag])
axes[1][0].grid()
axes[1][1].set_title("ACF of Log Yearly Growth")
axes[1][1].stem(Yearly_acf[:max_lag])
axes[1][1].grid()
```



[11]:
```
Monthly_box = Box_Ljung(Monthly_acf, len(Monthly_data), 10)
Yearly_box = Box_Ljung(Yearly_acf, len(Yearly_data), 10)
Monthly_log_box = Box_Ljung(Monthly_log_acf, len(Monthly_data), 10)
Yearly_log_box = Box_Ljung(Yearly_log_acf, len(Yearly_data), 10)
print("BL statistic and p value for monthly dividend yield growth are: ",␣
 ↪str(Monthly_box[0]), ",", str(Monthly_box[1]))
print("BL statistic and p value for yearly dividend yield growth are: ",␣
 ↪str(Yearly_box[0]), ",", str(Yearly_box[1]))
print("BL statistic and p value for monthly log dividend yield growth are: ",␣
 ↪str(Monthly_log_box[0]), ",", str(Monthly_log_box[1]))
print("BL statistic and p value for yearly log dividend yields are: ",␣
 ↪str(Yearly_log_box[0]), ",", str(Yearly_log_box[1]))
```

```
BL statistic and p value for monthly dividend yield growth are:
101.44272905932442 , 0.0
BL statistic and p value for yearly dividend yield growth are:
21.79890365003753 , 0.016162484512925457
BL statistic and p value for monthly log dividend yield growth are:
97.1823547767107 , 2.220446049250313e-16
BL statistic and p value for yearly log dividend yields are:  22.833924751517024
, 0.011376469972015801
```

0.0.9 Observations

Some key observations are as below: - At an initial look, we can see that using logarithms doesn't present us new information. This makes sense because dividends themselves are a percentage of the stock value implying we already have a ratio/normalization in the raw data - Looking at the test statistic, we can see that the test rejects the null for dividend yields time series. This makes sense because even the autocorrelation plot shows significantly high autocorrelation for higher lags - However, when we start looking at the growths time series which is basically the first difference of the original series, we can see that the acf plots show a weaker relationship as lag increases implying a removal of unit root - If we consider a statistical significance of 0.01 then we can start to not reject the null for yearly dividend yield growth rate based on its p-value implying that there is some evidence for higher lag autocorrelations to vanish - Another observation is on the statistic values. We can see that the yearly data has a smaller statistic value compared to the monthly data(both dividend and dividend growth). This can be due to the granularity of both the series as monthly data has closer samples than yearly data leading to a relatively high insample correlation