Poisson :-

$$P(X=x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

Say $x_1, \ldots, x_n$ are n such independent observations

$$\Rightarrow P(X=x_i) = \frac{\lambda^{x_i} e^{-\lambda}}{x_i!}$$

$$\Rightarrow L(x, \lambda) = \prod_{i=1}^{n} P(X=x_i)$$

$$\Rightarrow \log L(x, \lambda) = \sum_{i=1}^{n} \left(x_i \log \lambda - \lambda - \log(x_i!)\right)$$

$$\frac{\partial \log L(x,\lambda)}{\partial \lambda} = 0$$

$$\Rightarrow \frac{\sum_{i=1}^{n} x_i}{\lambda} = n$$

$$\Rightarrow \boxed{\lambda = \frac{\sum x_i}{n}}$$

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Binomial :-

x_i = no. of success in n trials.

Let us record N observations each observation involving ~~n~~ trials, n (fixed)

~~i.e, $x_i = \{0, 1\}$~~

$$\Rightarrow L(x_i, \theta) = \prod_{i=1}^{N} p^{x_i} (1-p)^{n-x_i} \cdot {}^{n}C_{x_i}$$

θ being p

${}^{n}C_{x_i}$ constant

$$\Rightarrow \log L(x_i, \theta) = \sum_{i=1}^{N} x_i \log p + (1-x_i) \log(1-p) + k$$

$$\Rightarrow \sum_{i=1}^{N} \frac{x_i}{p} - \sum_{i=1}^{N} \frac{n-x_i}{1-p} = 0$$

$$\Rightarrow \frac{\sum x_i}{p} - \frac{N}{1-p} + \frac{\sum x_i}{1-p} = 0$$

$$\Rightarrow \frac{Nn}{\cancel{1-p}} = \frac{\sum x_i}{p\cancel{(1-p)}}$$

$$\Rightarrow \boxed{p = \frac{\sum x_i}{N n}}$$

Exponential :-

$$f(x;\lambda) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0 & \text{else} \end{cases}$$

Say $x_i\ (>0)$ are some N observa-tions

$$\Rightarrow L(x_i, \lambda) = \prod_{i=1}^{N} \lambda e^{-\lambda x_i}$$

$$\Rightarrow \log L(x_i, \lambda) = \sum_{i=1}^{N} \log \lambda - \lambda x_i$$

$$\Rightarrow \cancel{\frac{1}{\lambda} - nx}$$

$$\sum_{i=1}^{N} \frac{1}{\lambda} - x_i = 0$$

$$\Rightarrow \frac{N}{\lambda} = \sum_{i=1}^{N} x_i$$

$$\Rightarrow \boxed{\lambda = \frac{N}{\sum_{i=1}^{N} x_i}}$$

Laplacian :

$$p_{\mu, b} = \frac{1}{2b} e^{\left(-\frac{|x-\mu|}{b}\right)}$$

$$L = \prod_{i=1}^{N} \frac{1}{2b} e^{-\frac{|x_i-\mu|}{b}}$$

$$\log L = \sum_{i=1}^{N} \left[\log\left(\frac{1}{2b}\right) - \frac{|x_i - \mu|}{b} \right]$$

$$= \sum_{i=1}^{N} -\log(2b) - \frac{|x_i - \mu|}{b}$$

$$\frac{\partial \log L}{\partial b} = 0$$

$$\Rightarrow \sum_{i=1}^{N} \frac{-1}{\cancel{2}b} + \frac{|x_i-\mu|}{b^2} = 0$$

$$\Rightarrow \frac{N}{\cancel{2}b} = \frac{\sum_{i=1}^{N} |x_i-\mu|}{b^2}$$

$$\Rightarrow \boxed{b = \frac{\left(\sum_{i=1}^{N} |x_i - \mu|\right)}{N}}$$

$$\frac{\partial L}{\partial \mu} = 0$$

$$\Rightarrow \left(\frac{1}{4}\right) \sum_{i=1}^{N} \frac{|x_i - \mu|}{x_i - \mu} = 0$$

$$\Rightarrow \sum_{i=1}^{N} \left(\frac{|x_i - \mu|}{x_i - \mu}\right) = 0$$

$$\Rightarrow \boxed{\sum_{i=1}^{N} \operatorname{sgn}(x_i - \mu) = 0}$$

say μ = median (x_i)

$\Rightarrow$ there will be half samples $< \mu$ & half $> \mu$ $\Rightarrow$ summation $= 0$

$$\boxed{\therefore\ \mu = \text{median}(x_i)}$$

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Gaussian :-

$$p = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$\Rightarrow L = \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x_i-\mu)^2}{2\sigma^2}}$$

$$\Rightarrow \log L = \sum_{i=1}^{N} \log\left(\frac{1}{\sqrt{2\pi}\sigma}\right) - \frac{(x_i-\mu)^2}{2\sigma^2}$$

$$\frac{\partial(\log L)}{\partial\mu} = 0$$

$$\Rightarrow \sum_{i=1}^{N} x_i - \mu = 0$$

$$\Rightarrow \boxed{\mu = \frac{\sum x_i}{N}}$$

$$\frac{\partial}{\partial\sigma}(\log L) = 0$$

$$\Rightarrow \sum_{i=1}^{N} -\frac{1}{\sigma} + \frac{(x_i-\mu)^2}{\sigma^3} = 0.$$

$$\Rightarrow \boxed{\sigma^2 = \sum (x_i-\mu)^2}$$

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$$\boxed{\therefore \mu = \frac{\sum x_i}{N} = \text{mean}} \text{ of data}$$

$$\cancel{\sigma = \sqrt{\frac{(x_i - \mu)^2}{}}}$$

$$\boxed{\text{variance} = \text{variance of data}}$$