Barrier Mediated Predator-Prey Dynamics under the Periodic Boundary Condition

Theoretical Physics II: Soft Matter

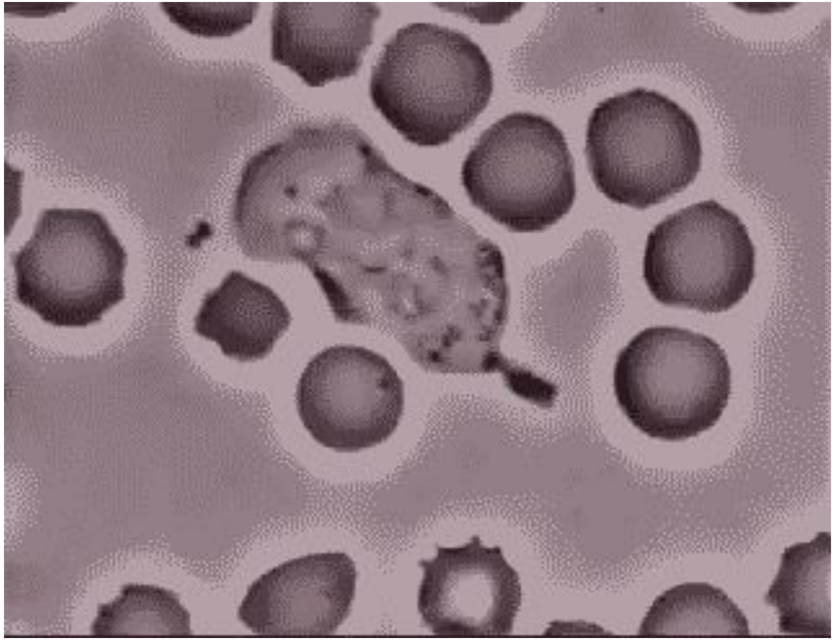
By Jessica Wonges

Outline

Predator-Prey Dynamics

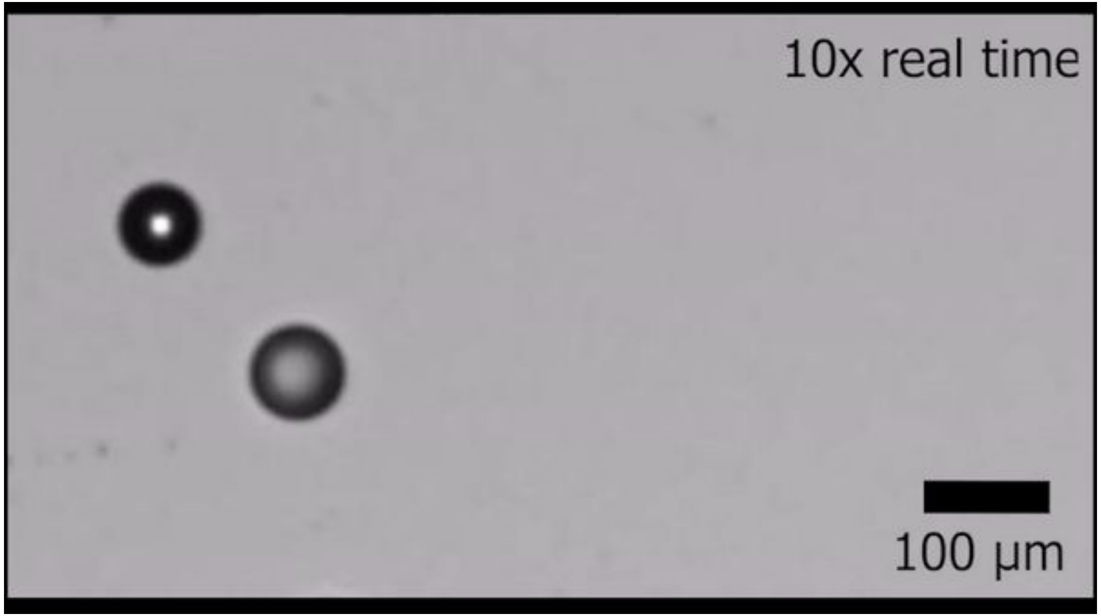
Image source: Pixabay

Predator-Prey Dynamics



Neutrophil Chases and Eats a Staphylococcus aureus Bacterium, from 16mm movie by David Rogers, Vanderbilt University, in the 1950s

Predator-Prey Dynamics



Meredith, C. H., Moerman, P. G., Groenewold, J., Chiu, Y. J., Kegel, W. K., van Blaaderen, A., & Zarzar, L. D. (2020). Predator–prey interactions between droplets driven by non-reciprocal oil exchange. Nature Chemistry, 12(12), 1136-1142.

Ideal Predator-prey Dynamics

- One-dimensional system of two particles moving in environments that are modelled as potential landscapes

Overdamped Newtonian Equation

$$m\frac{d\mathbf{u}(t)}{dt} = -\gamma\mathbf{u}(t) + \gamma v_0\mathbf{e} + \mathbf{F}(\mathbf{r}, t)$$

↓

Equation of Motion

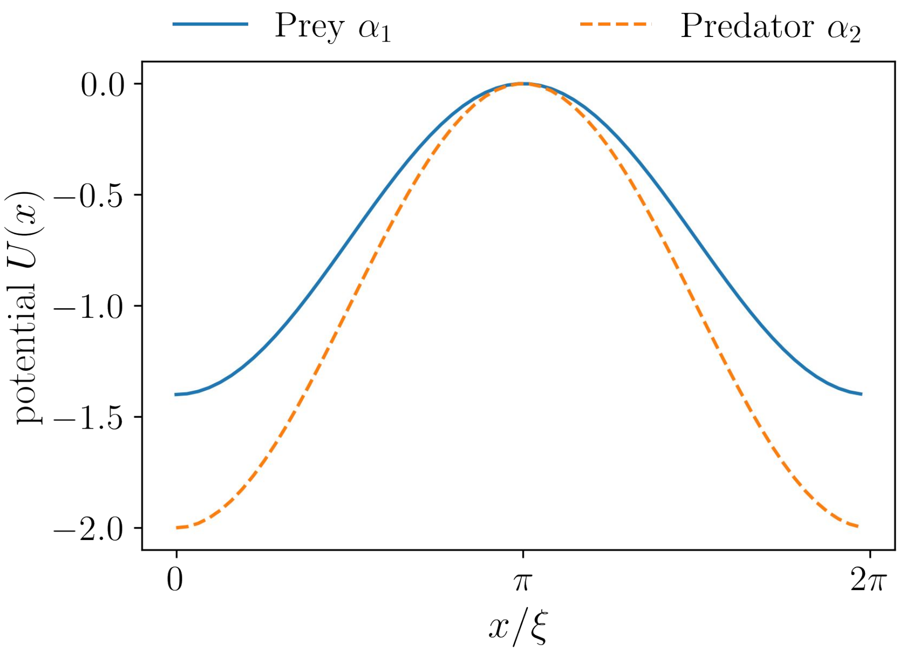
$$\mathbf{u}(t) = v_0\mathbf{e} + \frac{1}{\gamma}\mathbf{F}(\mathbf{r}, t)$$

Periodic Boundary Condition

- Closest observation of a circular barrier

$$U_i(x) = -\alpha_i \cdot (1 + \cos x_i)$$

$$U(0) = U(2\pi)$$



Ideal Predator-prey Dynamics

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Equation of Motion

$$\mathbf{u}(t) = v_0\mathbf{e} + \frac{1}{\gamma}\mathbf{F}(\mathbf{r}, t)$$

$$\Downarrow$$

Prey	$\dot{x}_1 = v_1 - \alpha_1 \sin x_1$
Predator	$\dot{x}_2 = v_2 - \alpha_2 \sin x_2$

Parameters	
Self-propulsion velocity	v_1, v_2
Coupling constant	α_1, α_2

Analytical Solution

- Non-linear differential equation

$$x_1(t) = 2 \cdot \arctan\left(\frac{\alpha_1 - \sqrt{v_1^2 - \alpha_1^2} \cdot \tan\left(-\frac{1}{2} t \sqrt{v_1^2 - \alpha_1^2} - \arctan\frac{-\alpha_1\sqrt{v_1^2-\alpha_1^2} + v_1\sqrt{v_1^2-\alpha_1^2} \cdot \tan\left(\frac{x_1}{2}\right)}{v_1^2-\alpha_1^2}\right)}{v_1}\right)$$

$$x_2(t) = 2 \cdot \arctan\left(\frac{\alpha_2 - \sqrt{v_2^2 - \alpha_2^2} \cdot \tan\left(-\frac{1}{2} t \sqrt{v_2^2 - \alpha_2^2} - \arctan\frac{-\alpha_2\sqrt{v_2^2-\alpha_2^2} + v_2\sqrt{v_2^2-\alpha_2^2} \cdot \tan\left(\frac{x_2}{2}\right)}{v_2^2-\alpha_2^2}\right)}{v_2}\right)$$

Catching Time and Position

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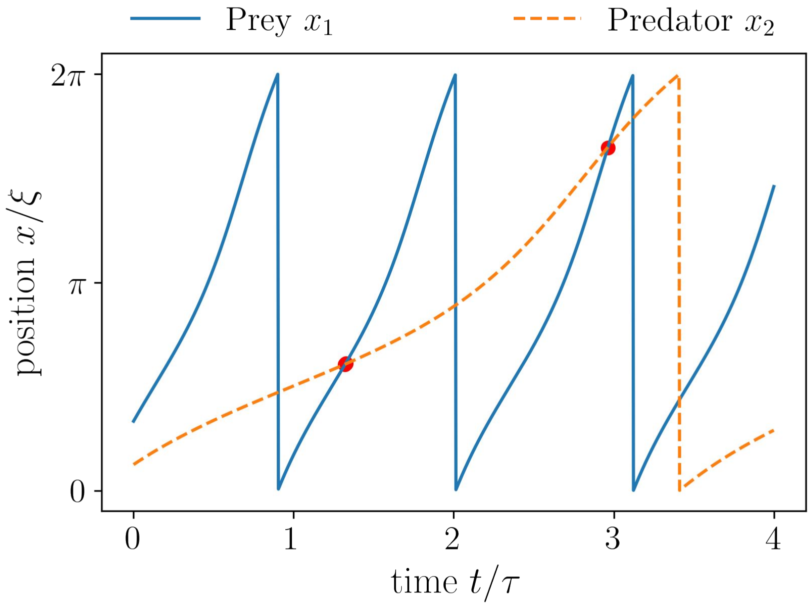
Catching Time t^*

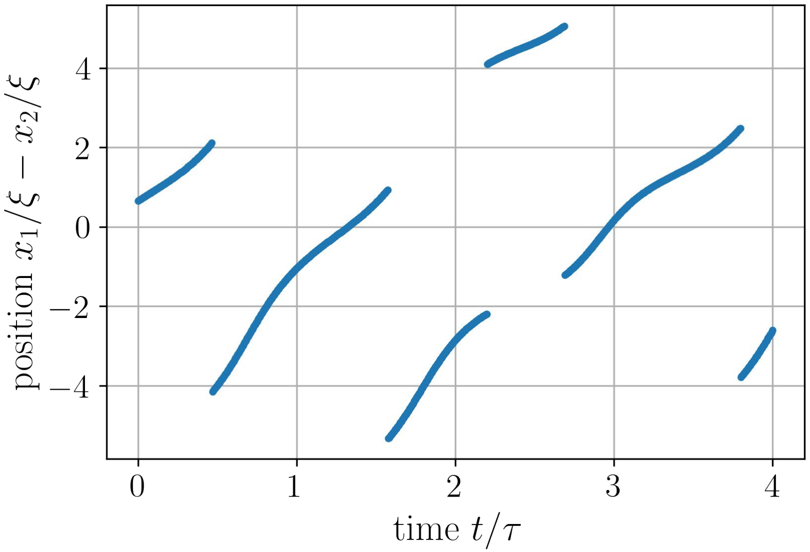
$$x_1(t^*) = x_2(t^*)$$

Catching Position x^*

$$x^* = x_{1,2}(t^*)$$

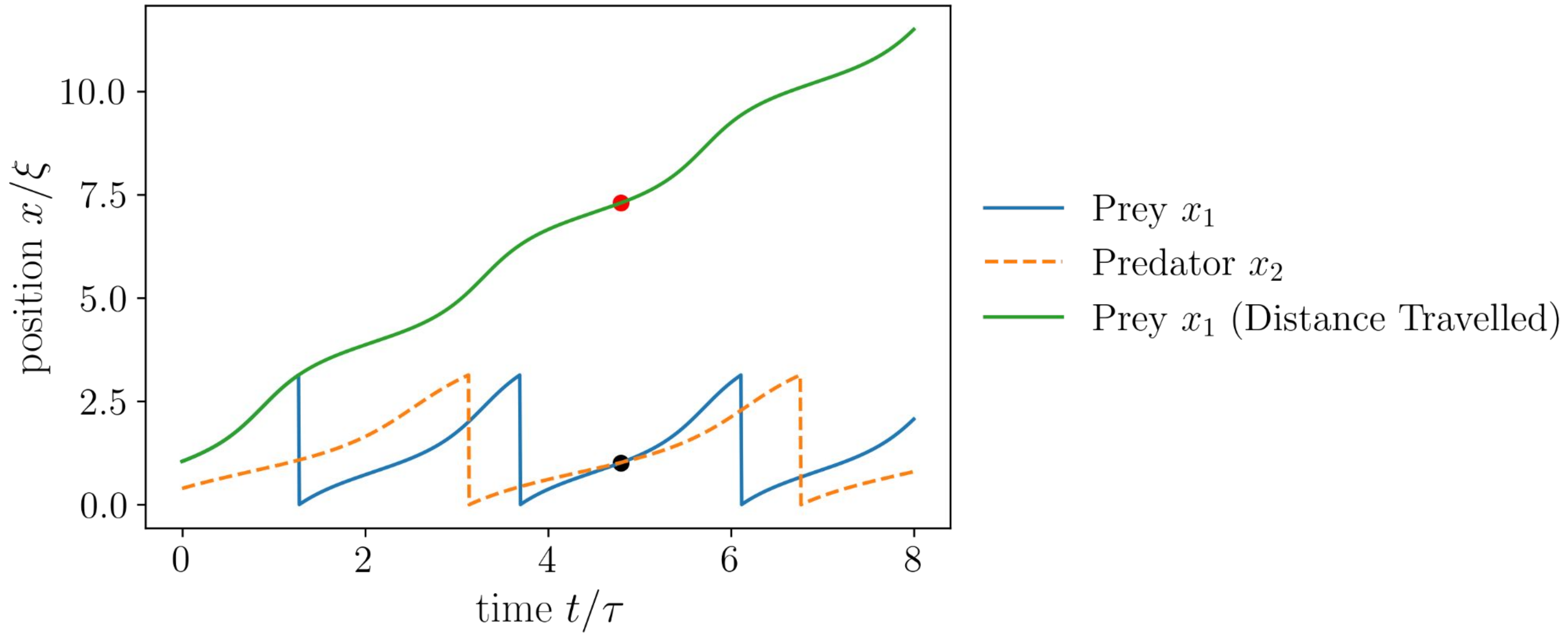
Catching Time: Problems?





Catching Time t^*	$\lvert x_1(t^*) - x_2(t^*) \rvert \approx 0$

Catching Revolutions



Periodic Predator-Prey Results

Prey

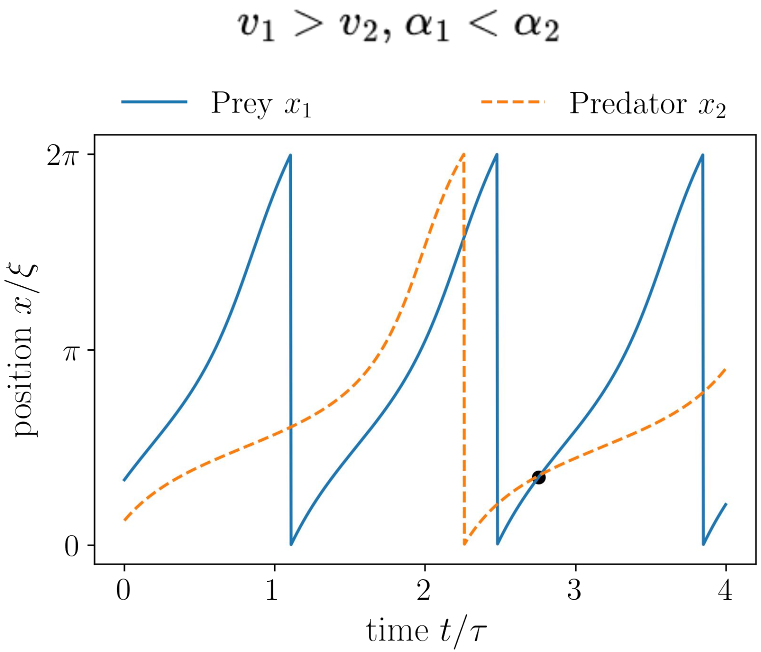
$$x_1(t) = 2 \cdot \arctan\left(\frac{\alpha_1 - \sqrt{v_1^2 - \alpha_1^2} \cdot \tan\left(-\frac{1}{2}t\sqrt{v_1^2 - \alpha_1^2} - \arctan\frac{-\alpha_1\sqrt{v_1^2-\alpha_1^2}+v_1\sqrt{v_1^2-\alpha_1^2}\cdot\tan\left(\frac{x_1}{2}\right)}{v_1^2-\alpha_1^2}\right)}{v_1}\right)$$

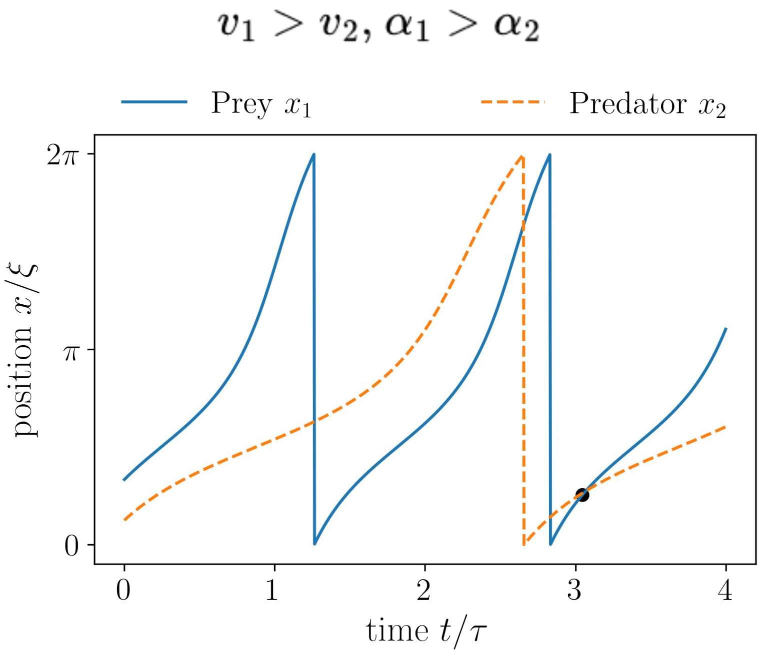
Predator

$$x_2(t) = 2 \cdot \arctan\left(\frac{\alpha_2 - \sqrt{v_2^2 - \alpha_2^2} \cdot \tan\left(-\frac{1}{2}t\sqrt{v_2^2 - \alpha_2^2} - \arctan\frac{-\alpha_2\sqrt{v_2^2-\alpha_2^2}+v_2\sqrt{v_2^2-\alpha_2^2}\cdot\tan\left(\frac{x_2}{2}\right)}{v_2^2-\alpha_2^2}\right)}{v_2}\right)$$

- Self-propulsion speed of both prey and predator must be greater than their coupling constant, for both to cross the potential barrier
- Algorithm can only process real-valued solutions
- Initial conditions: $x_1(0) = \pi\xi/3,\ x_2(0) = \pi\xi/8$

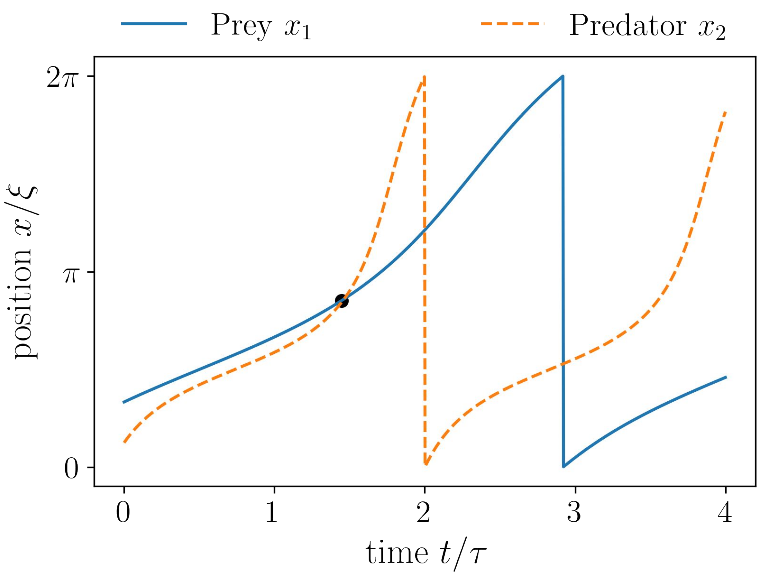
Results: Trajectories



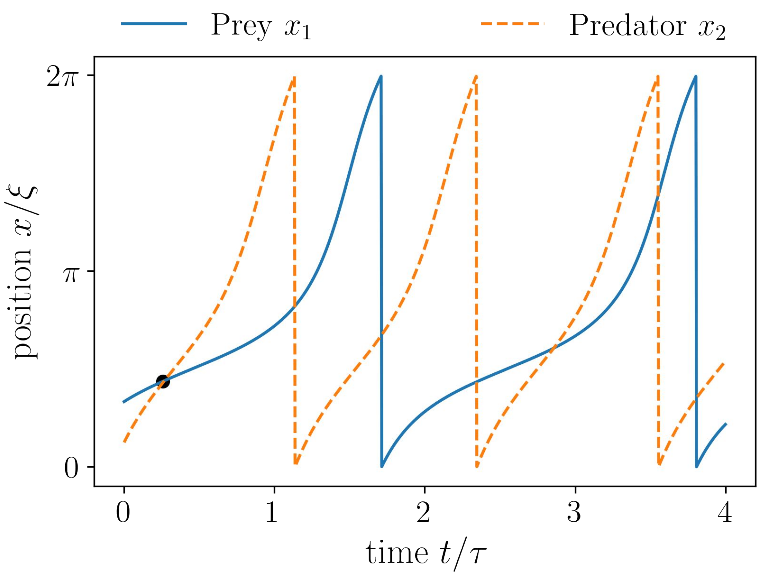


Results: Trajectories

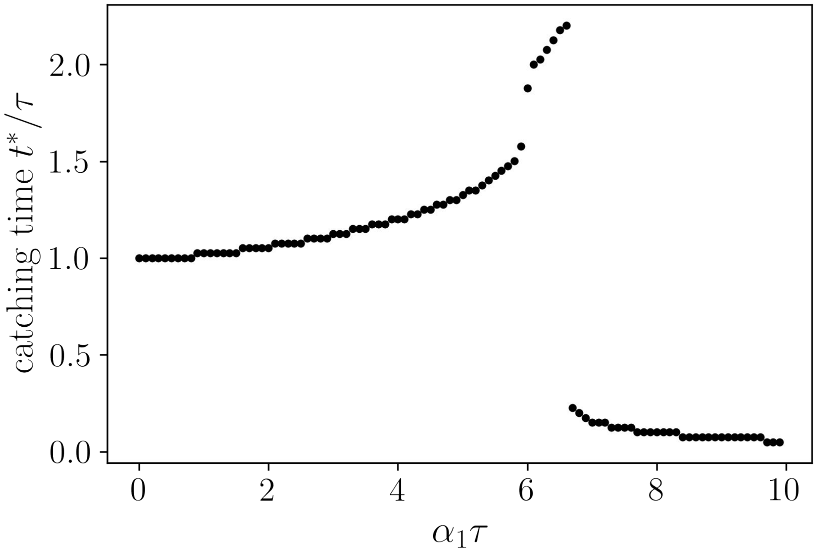
$v_1 < v_2, \alpha_1 < \alpha_2$



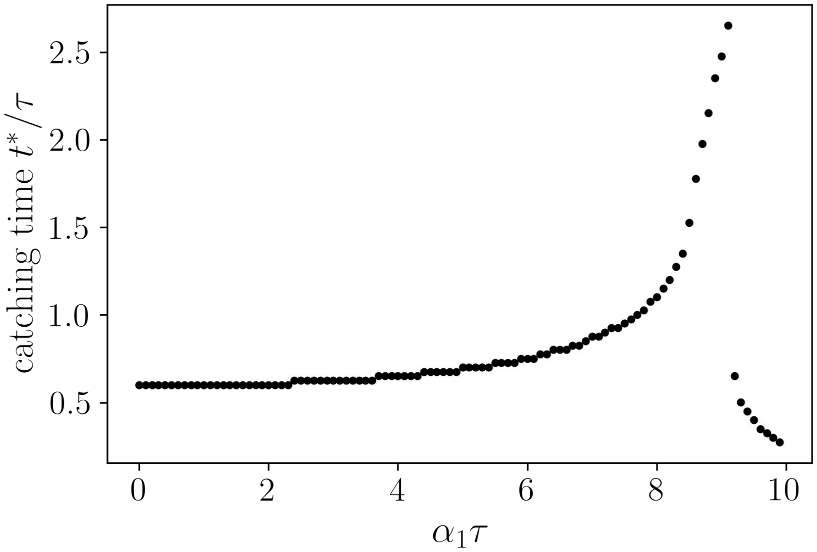
$v_1 < v_2, \alpha_1 > \alpha_2$



Results: Catching Time

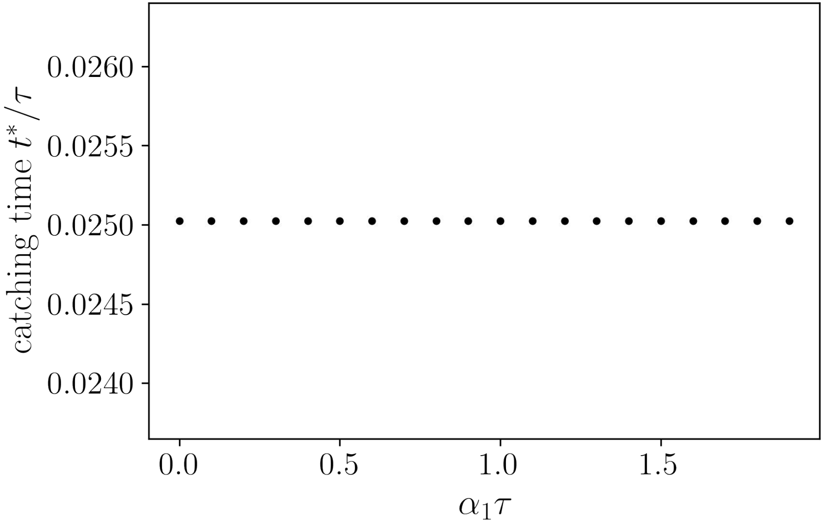


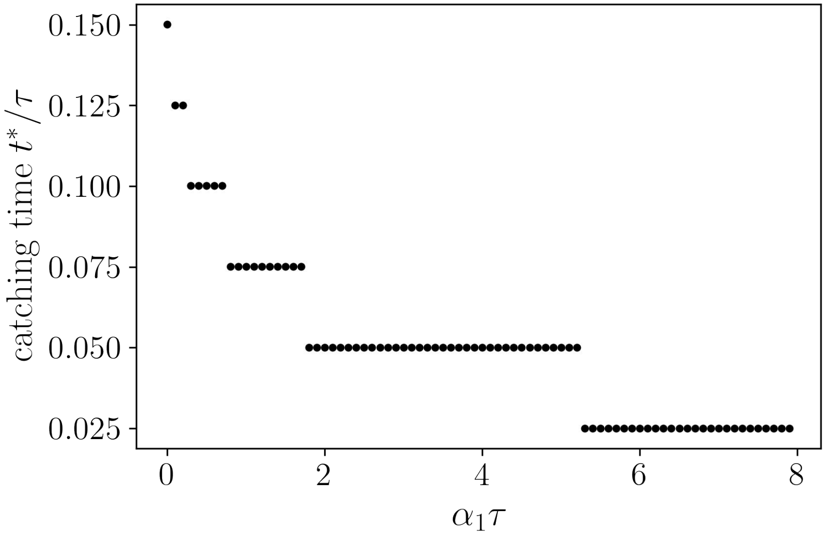
$$\alpha_1 = [1, 10], v_1 = 10$$
$$\alpha_2 = 1, \underline{v_2 = 7}$$



$$\alpha_1 = [1, 10], v_1 = 10$$
$$\alpha_2 = 1, \underline{v_2 = 2}$$

Results: Catching Time

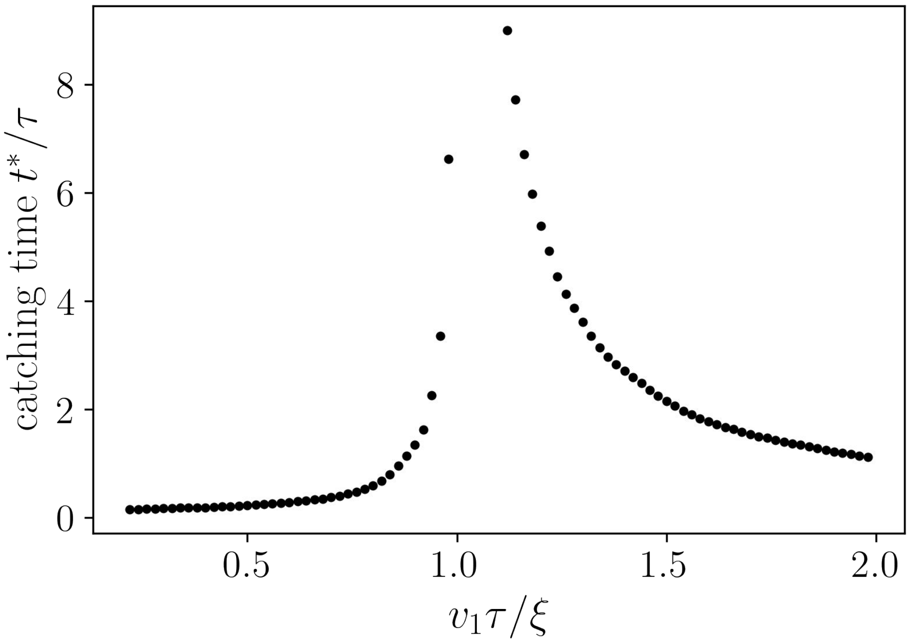




$\alpha_1 = [1, 10]$, $v_1 = 2$
$\alpha_2 = 1$, $v_2 = 10$

$\alpha_1 = [1, 10]$, $v_1 = 8$
$\alpha_2 = 1$, $v_2 = 10$

Results: Catching Time

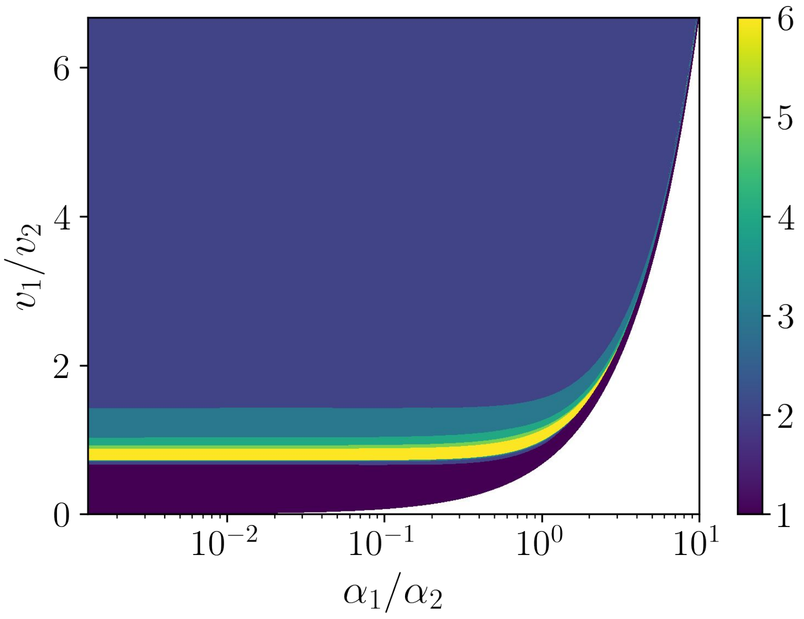


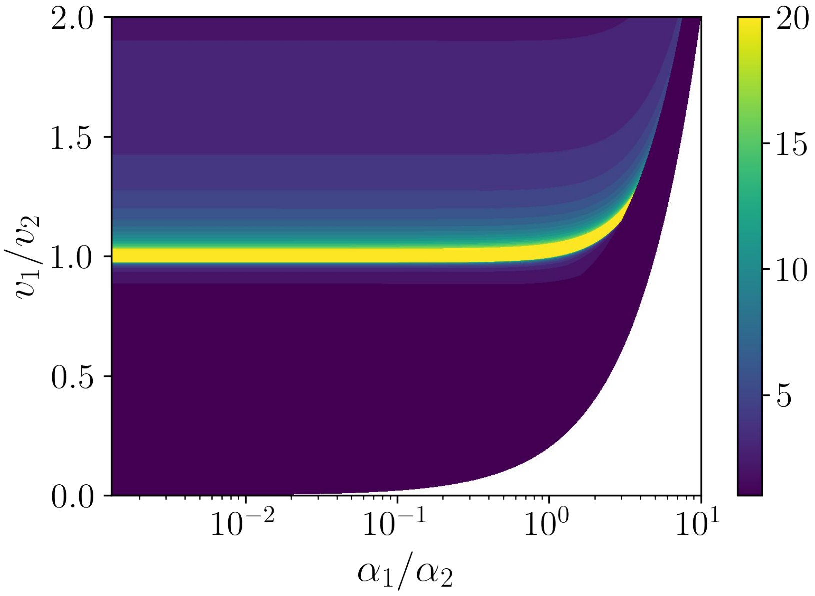
$$\alpha_1 = 1,\, v_1 = [1, 10]$$
$$\alpha_2 = 1,\, v_2 = 5$$

$$\frac{\overline{v_1}}{\overline{v_2}} \approx 1$$

Results: Catching Revolutions

- The colour bar shows the number of revolutions needed for catching to happen





Summary

- Function diagram that shows the number of revolutions need for catching
- Possible for more than one revolution for catching
- Behaviour of prey and predator depends on the type of potential barrier
- Escape is undefined