Applied Economic Forecasting

5. Time Series Decomposition

Spring 2020

Section 1

Time series components

Time series patterns

Recall

- **Trend** pattern exists when there is a long-term increase or decrease in the data.
- **Cyclic** pattern exists when data exhibit rises and falls that are *not of fixed period* (duration usually of at least 2 years).
- **Seasonal** pattern exists when a series is influenced by seasonal factors (e.g., the quarter of the year, the month, or day of the week).
- **Irregular** pattern consists of unpredictable or random fluctuations.

Time series patterns

- Trend component (T) represents the underlying growth (or decline) in a time series. The trend may be produced, for example, by consistent population change, inflation, technological change, and productivity increases. The trend component is denoted by T.
- Cyclical component (T)* a series of wavelike fluctuations or cycles of more than one year's duration. Changing economic conditions generally produce cycles. C denotes the cyclical component. In practice, cycles are often difficult to identify and are frequently regarded as part of the trend. In this case, the underlying general growth (or decline) component is called the trend-cycle and denoted by T.

Time series patterns

- Seasonal component (S) typically found in quarterly, monthly, or weekly data. Seasonal variation refers to a more or less stable pattern of change that appears annually and repeats itself year after year. Seasonal patterns occur because of the influence of the weather or because of calendar-related events such as school vacations and national holidays. S denotes the seasonal component.
- Remainder (Irregular) component (R) consists of unpredictable or random fluctuations. These fluctuations are the result of a myriad of events that individually may not be particularly important but whose combined effect could be large. R denotes the irregular component.

Time series decomposition

$$y_t = f(S_t, T_t, R_t)$$

where
$y_t =$ data at period t
$T_t =$ trend-cycle component at period t
$S_t =$ seasonal component at period t
$R_t =$ remainder component at period t (Irregular Component)

Additive decomposition: $y_t = S_t + T_t + R_t$.

Multiplicative decomposition: $y_t = S_t \times T_t \times R_t$.

Mixed components model: $Y_t = T_t \times S_t + I_t$

Time series decomposition

- Additive model appropriate if magnitude of seasonal fluctuations does not vary with level.
- If seasonal are proportional to level of series, then multiplicative model appropriate.
- Multiplicative decomposition more prevalent with economic series
- Alternative: use a Box-Cox transformation, and then use additive decomposition.
- Logs turn multiplicative relationship into an additive relationship:

$$y_t = S_t \times T_t \times E_t \quad \Rightarrow \quad \log y_t = \log S_t + \log T_t + \log R_t.$$

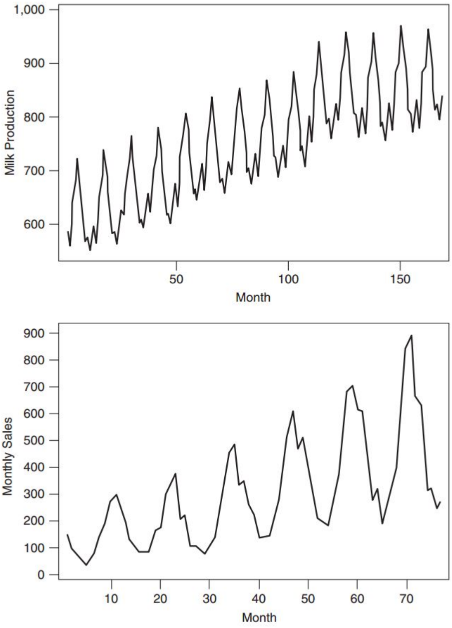


FIGURE 5-1 Time Series with Constant Variability (Top) and a Time Series with Variability Increasing with Level (Bottom)

Time Trend

- Trend can be linear or nonlinear
- If the trend is roughly linear, like a straight line, then

$$\hat{T}_t = b_0 + b_1 t \tag{1}$$

where $\hat{T}_t$ is the predicted value for the trend at time t

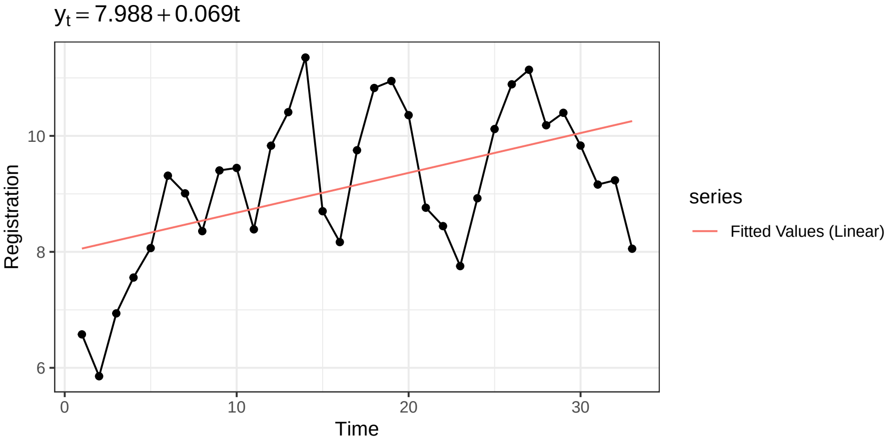
t represents time taking $t = 1, 2, 3, ..., T$

b_1 is the average decrease or increase in T of each period in time

Linear Trends

```r
register <- ts(readxl::read_xls("Tab5-1.xls"))
fitlinear <- tslm(register ~ trend)
fit.line <- fitted(fitlinear)
coef.fit1 <- round(coef(fitlinear), 3)
autoplot(register) + autolayer(fit.line, series = "Fitted Values (Linear)")
    geom_point() + labs(y = "Registration") + ggtitle(bquote(y[t] ==
    .(coef.fit1[1]) + .(coef.fit1[2]) * t ~ .(coef.fit1[3]) *
    t^2)) + theme_bw()
```

Linear Trends




```
##                    ME   RMSE  MAE    MPE   MAPE  MASE ACF1
## Training set  0 1.157 0.99 -1.765 11.274 1.197 0.57
```

Additional Trend Curves

- What if the trend is not linear?

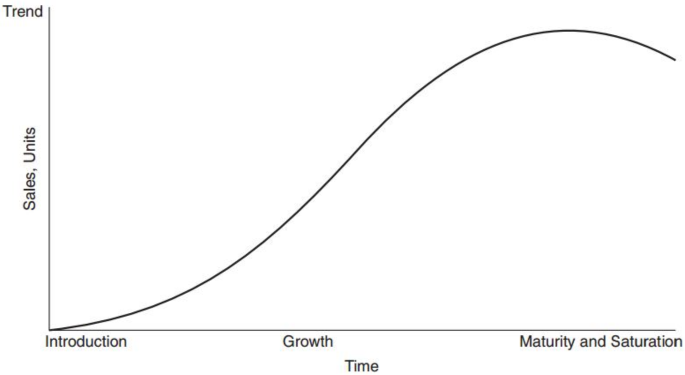
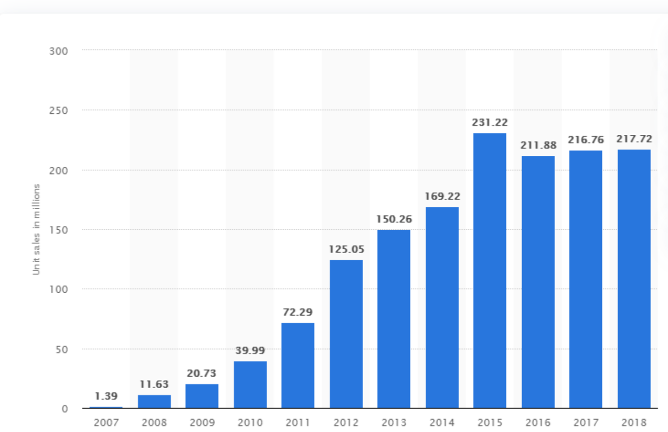


FIGURE 5-4 **Life Cycle of a Typical New Product**

The graph below presents an example of a non-linear trend in terms of Apple iPhone sales.

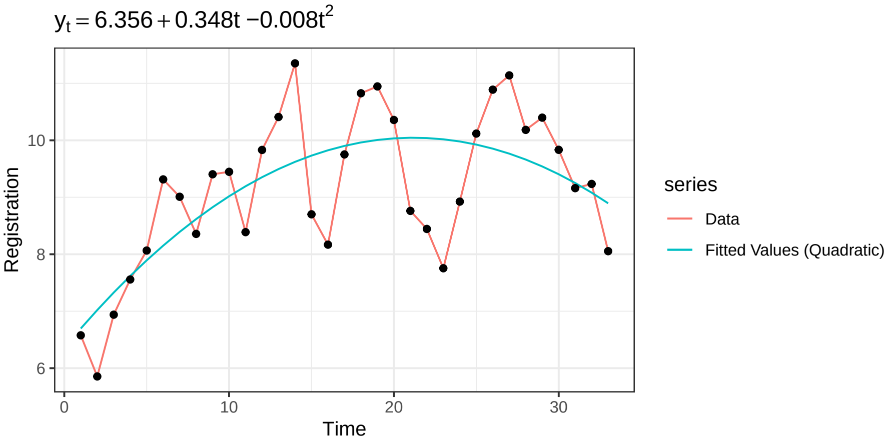


Quadratic Trend

- Consider nonlinear trend functions, such as polynomial, exponential, etc.
- For instance, a quadratic trend (polynomial of order 2):

$$\hat{T}_t = b_0 + b_1 t + b_t t^2$$

```
fitquad <- tslm(register ~ trend + I(trend^2))
fit.quad <- fitted(fitquad)
coef.fit2 <- round(coef(fitquad), 3)
autoplot(register) + autolayer(fit.quad, series = "Fitted Values (Quadratic
    geom_point() + labs(y = "Registration") + ggtitle(bquote(y[t] ==
    .(coef.fit2[1]) + .(coef.fit2[2]) * t ~ .(coef.fit2[3]) *
    t^2)) + theme_bw()
```

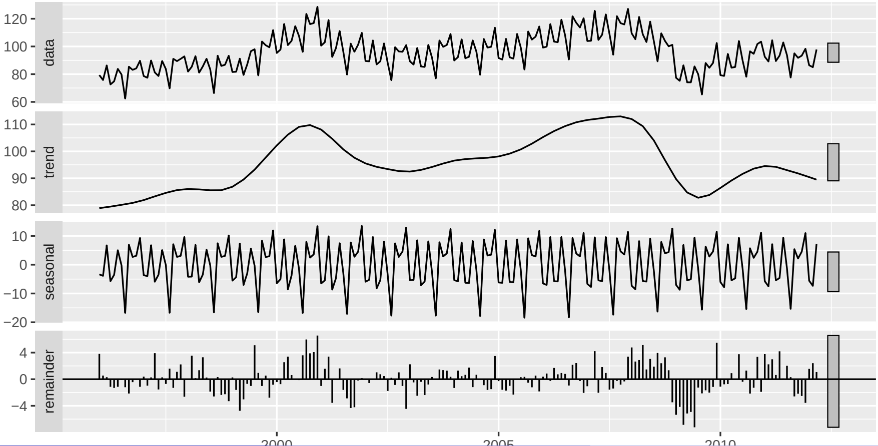



```
##                ME  RMSE   MAE    MPE  MAPE  MASE  ACF1
## Training set  0 0.945 0.774 -1.112 8.617 0.936 0.485
```

Seasonal and Trend decomposition using Loess (STL) Method

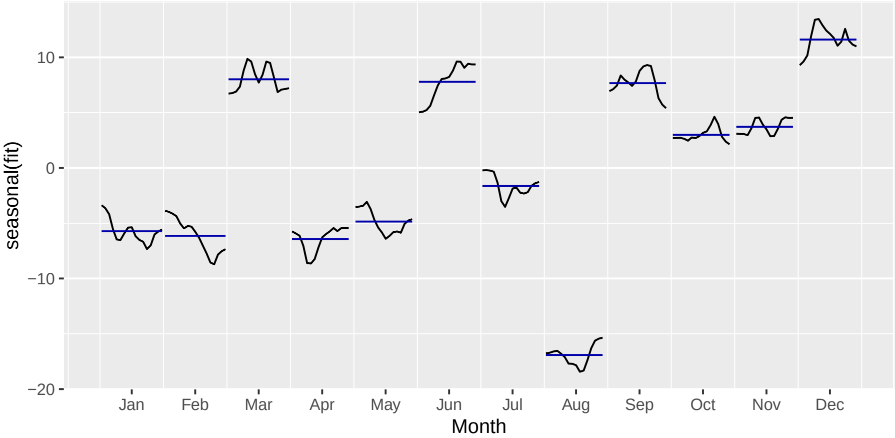
USing the "Euro electrical equipment" data

```r
fit <- stl(elecequip, s.window=7)
autoplot(fit) + xlab("Year")
```



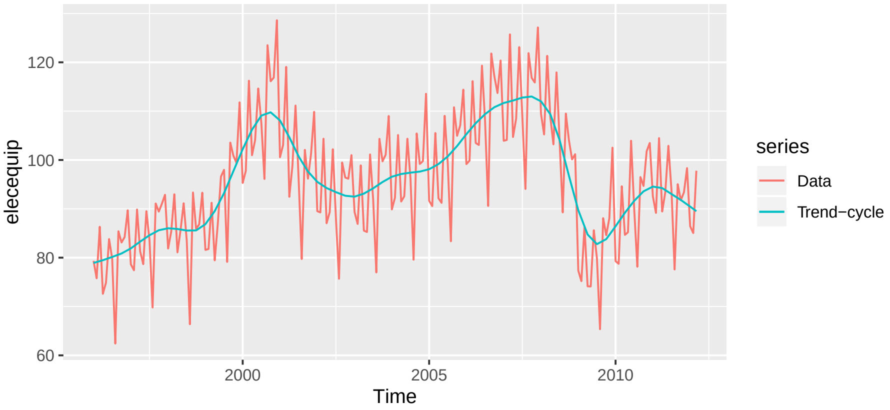
Euro electrical equipment

```r
ggsubseriesplot(seasonal(fit))
```



Euro electrical equipment

```
autoplot(elecequip, series="Data") +
  autolayer(trendcycle(fit), series="Trend-cycle")
```



Helper functions

- `seasonal()` extracts the seasonal component
- `trendcycle()` extracts the trend-cycle component
- `remainder()` extracts the remainder component.
- `seasadj()` returns the seasonally adjusted series.

Your turn

Repeat the decomposition using

```
elecequip %>%
  stl(s.window=7, t.window=11) %>%
  autoplot()
```

What happens as you change `s.window` and `t.window`?

Section 2

Seasonal adjustment

Seasonal adjustment

- Useful by-product of decomposition: an easy way to calculate seasonally adjusted data.
- Additive decomposition: seasonally adjusted data given by

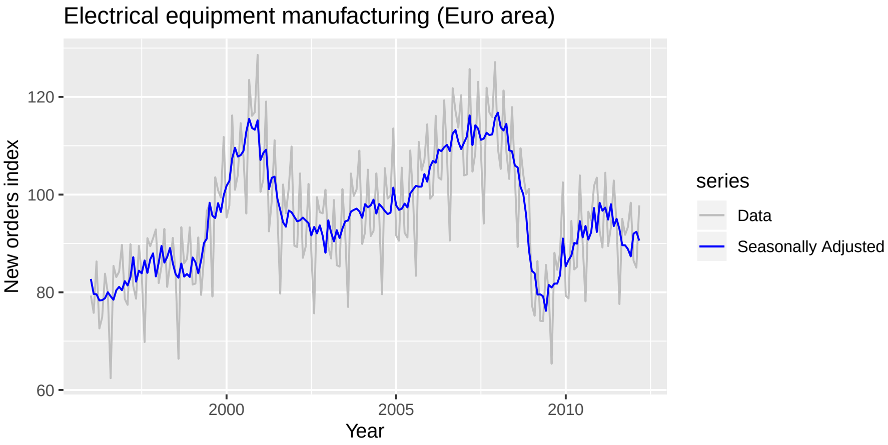
$$y_t - S_t = T_t + R_t$$

- Multiplicative decomposition: seasonally adjusted data given by

$$y_t / S_t = T_t \times R_t$$

Euro electrical equipment

```
fit <- stl(elecequip, s.window=7)
autoplot(elecequip, series="Data") +
  autolayer(seasadj(fit), series="Seasonally Adjusted")
```



Seasonal adjustment

- We use estimates of S based on past values to seasonally adjust a current value.
- Seasonally adjusted series reflect **remainders** as well as **trend**. Therefore they are not "smooth"" and "downturns"" or "upturns" can be misleading.
- It is better to use the trend-cycle component to look for turning points.

History of time series decomposition

- Classical method originated in 1920s.
- Census II method introduced in 1957. Basis for X-11 method and variants (including X-12-ARIMA, X-13-ARIMA)
- STL method introduced in 1983
- TRAMO/SEATS introduced in 1990s.

National Statistics Offices

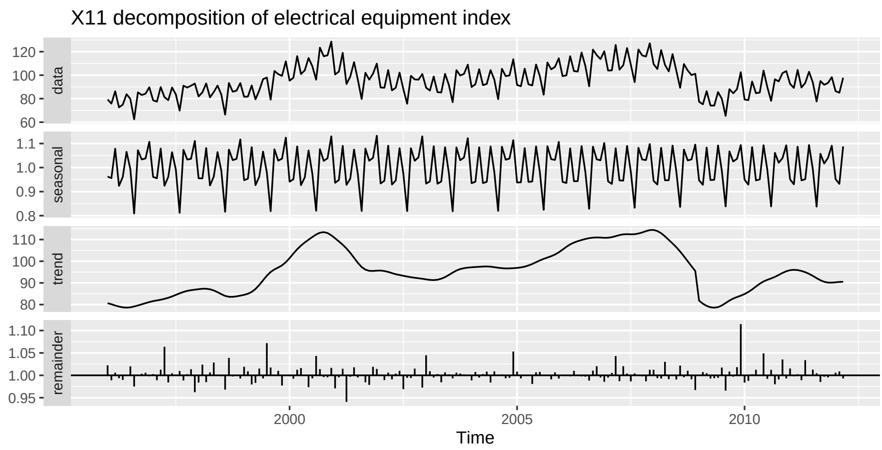
- ABS uses X-12-ARIMA
- US Census Bureau uses X-13-ARIMA-SEATS
- Statistics Canada uses X-12-ARIMA
- ONS (UK) uses X-12-ARIMA
- EuroStat use X-13-ARIMA-SEATS

Section 3

X-11 decomposition

X-11 decomposition

```
library(seasonal)
fit <- seas(elecequip, x11="")
autoplot(fit)
```



(Dis)advantages of X-11

Advantages

- Relatively robust to outliers
- Completely automated choices for trend and seasonal changes
- Very widely tested on economic data over a long period of time.

Disadvantages

- No prediction/confidence intervals
- Ad hoc method with no underlying model
- Only developed for quarterly and monthly data

Extensions: X-12-ARIMA and X-13-ARIMA

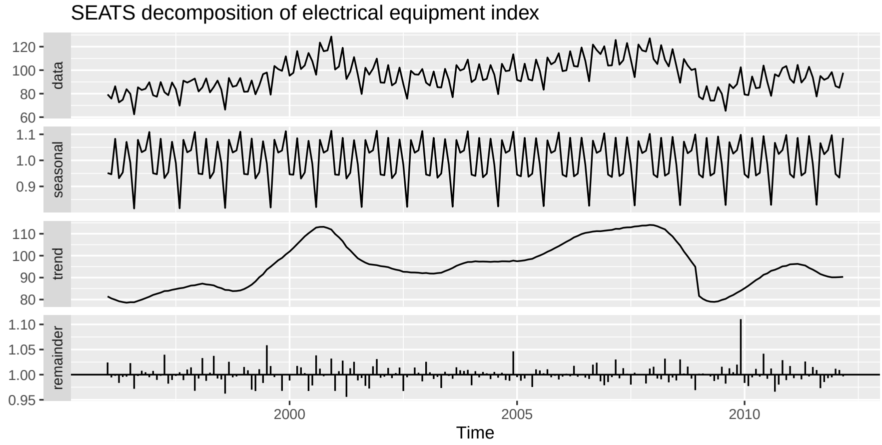
- The X-11, X-12-ARIMA and X-13-ARIMA methods are based on Census II decomposition.
- These allow adjustments for trading days and other explanatory variables.
- Known outliers can be omitted.
- Level shifts and ramp effects can be modelled.
- Missing values estimated and replaced.
- Holiday factors (e.g., Easter, Labour Day) can be estimated.

Section 4

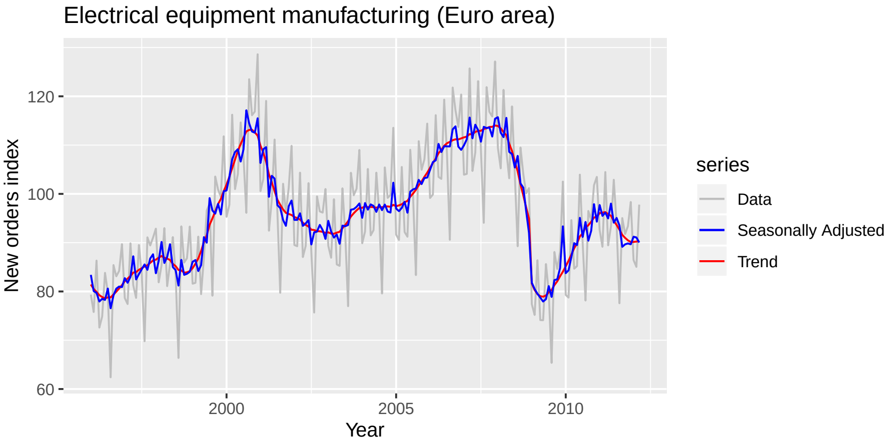
Seasonal Extraction in ARIMA Time Series (SEATS) decomposition

SEATS decomposition

```r
library(seasonal)
fit <- seas(elecequip)
autoplot(fit)
```

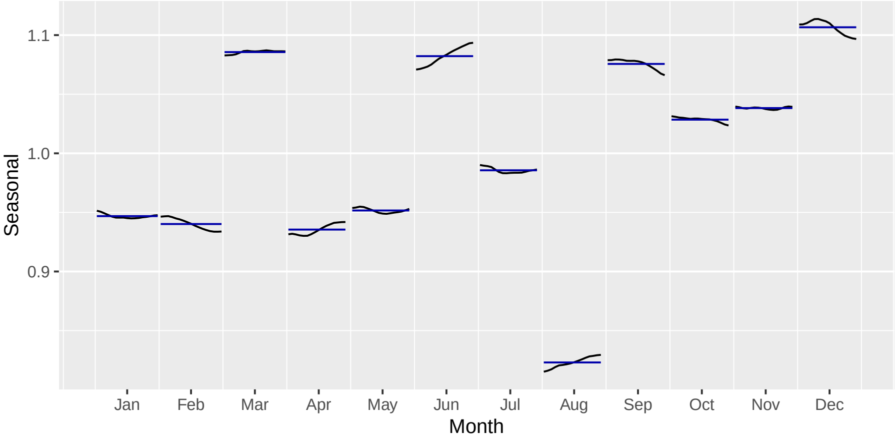


SEATS decomposition



SEATS decomposition

```r
ggsubseriesplot(seasonal(fit)) + ylab("Seasonal")
```



(Dis)advantages of SEATS

Advantages

- Model-based
- Smooth trend estimate
- Allows estimates at end points
- Allows changing seasonality
- Developed for economic data

Disadvantages

- Only developed for quarterly and monthly data

Section 5

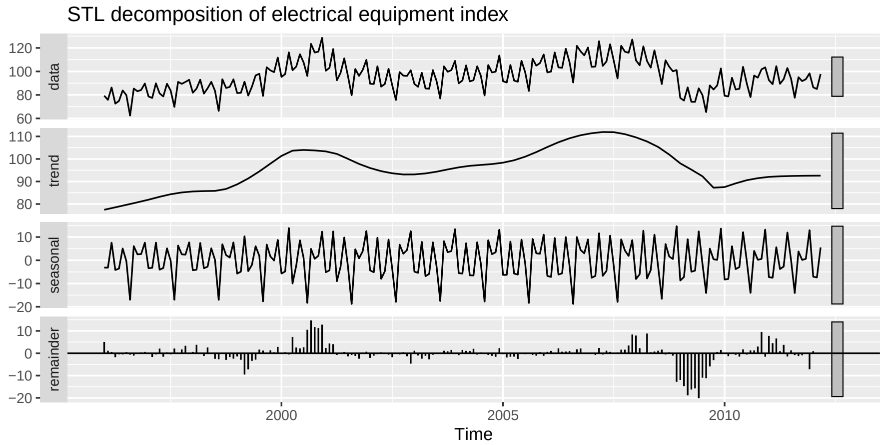
STL decomposition

STL decomposition

- STL: "Seasonal and Trend decomposition using Loess"
- Very versatile and robust.
- Unlike X-12-ARIMA, STL will handle any type of seasonality.
- Seasonal component allowed to change over time, and rate of change controlled by user.
- Smoothness of trend-cycle also controlled by user.
- Robust to outliers
- Not trading day or calendar adjustments.
- Only additive.
- Take logs to get multiplicative decomposition.
- Use Box-Cox transformations to get other decompositions.

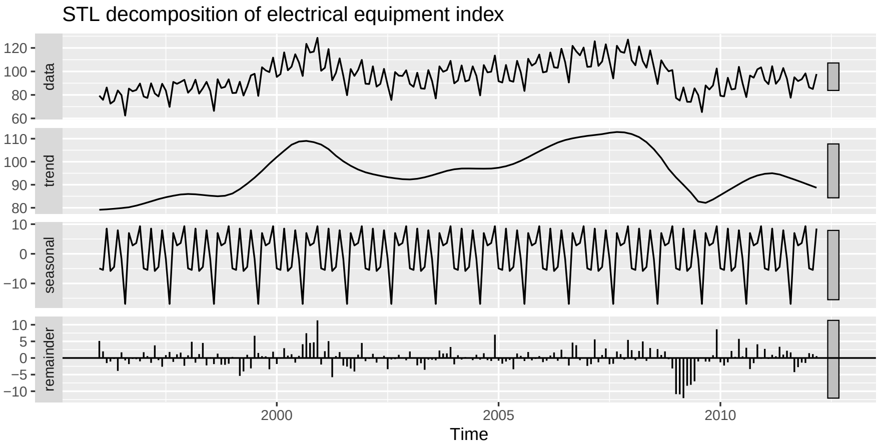
STL decomposition

```r
fit <- stl(elecequip, s.window=5, robust=TRUE)
autoplot(fit) +
  ggtitle("STL decomposition of electrical equipment index")
```



STL decomposition

```
fit <- stl(elecequip, s.window="periodic", robust=TRUE)
autoplot(fit) +
  ggtitle("STL decomposition of electrical equipment index")
```



STL decomposition

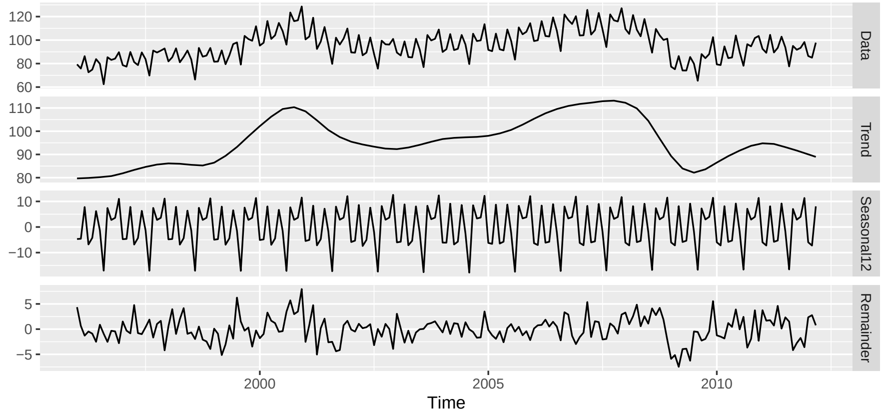
```
stl(elecequip,s.window=5)

stl(elecequip, t.window=15,
  s.window="periodic", robust=TRUE)
```

- `t.window` controls wiggliness of trend component.
- `s.window` controls variation on seasonal component.

STL decomposition

```
elecequip %>% mstl() %>% autoplot()
```



- `mstl()` chooses `s.window=13`
- Can include a `lambda` argument.

Section 6

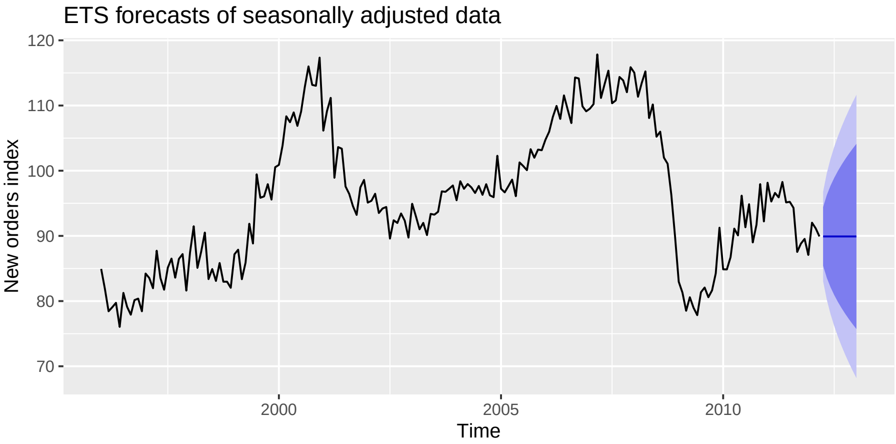
Forecasting and decomposition

Forecasting and decomposition

- Forecast seasonal component by repeating the last year
- Forecast seasonally adjusted data using non-seasonal time series method.
- Combine forecasts of seasonal component with forecasts of seasonally adjusted data to get forecasts of original data.
- Sometimes a decomposition is useful just for understanding the data before building a separate forecasting model.

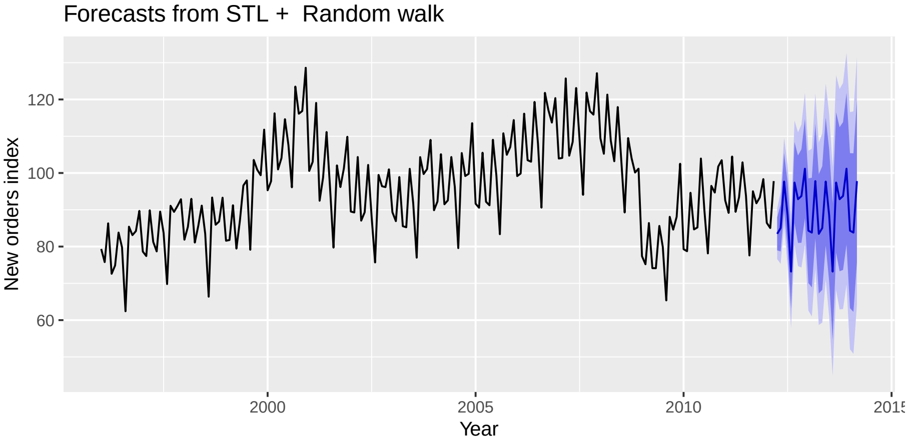
Electrical equipment

```
fit <- stl(elecequip, t.window=13, s.window="periodic")
fit %>% seasadj() %>% naive() %>%
  autoplot() + ylab("New orders index") +
  ggtitle("ETS forecasts of seasonally adjusted data")
```



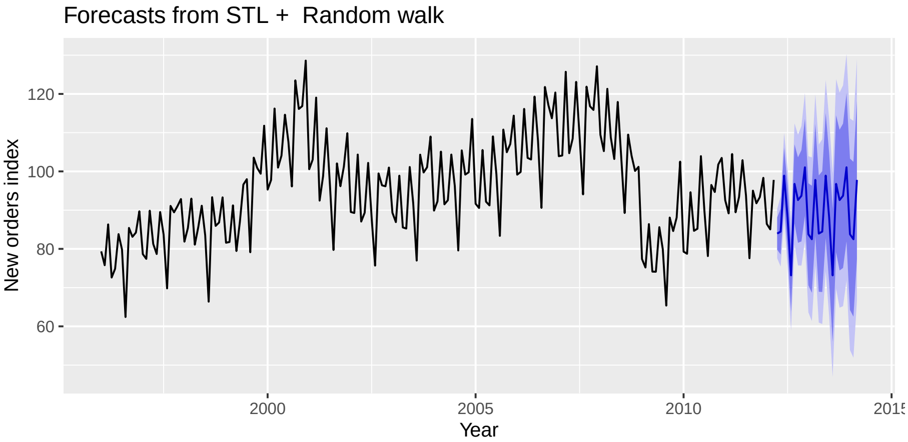
Electrical equipment

```
fit %>% forecast(method='naive') %>%
  autoplot() + ylab("New orders index") + xlab("Year")
```



Forecasting and decomposition

```
elecequip %>% stlf(method='naive') %>%
  autoplot() + ylab("New orders index") + xlab("Year")
```



Decomposition and prediction intervals

- It is common to take the prediction intervals from the seasonally adjusted forecasts and modify them with the seasonal component.
- This ignores the uncertainty in the seasonal component estimate.
- It also ignores the uncertainty in the future seasonal pattern.