Necessary and Sufficient Conditions for Macrorealism from Quantum Mechanics

Lucas Clemente and Johannes Kofler

Max-Planck-Institute of Quantum Optics, Garching, Germany

Abstract

Macroscopic realism, the classical world view that macroscopic objects exist independently of and are not influenced by measurements, is usually tested using Leggett-Garg inequalities. Recently, another necessary condition called no-signaling in time (NSIT) has been proposed as a witness for non-classical behavior. Here, we show that a combination of NSIT conditions is not only necessary but also sufficient for a macrorealistic description of a physical system. Any violation of macroscopic realism must therefore be witnessed by a suitable NSIT condition. Subsequently, we derive an operational formulation for NSIT in terms of positive-operator valued measurements and the system Hamiltonian. We argue that this leads to a suitable definition of "classical" measurements and Hamiltonians, and apply our formalism to some generic coarse-grained quantum measurements.

Reference: arXiv:1501.07517

Macrorealism (MR)

Is Quantum mechanics valid on the macro-level?

- Superconducting devices
- Quantum optomechanics
- Large superpositions
- Cats

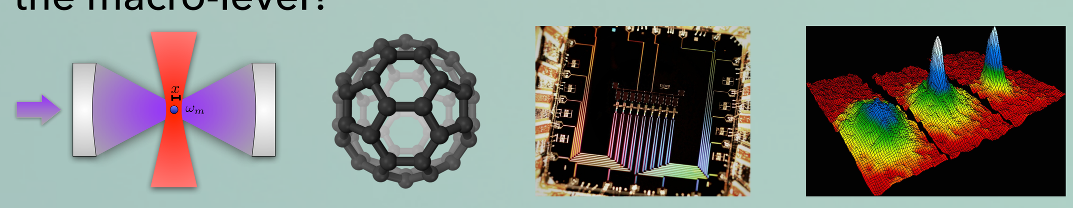


Photo: Julian Kelly @Martinis Group

Macrorealism (Leggett, Garg, PRL 1985)

The world view that macroscopic properties are independent of measurements.

- **Macrorealism per se** A macroscopic object is always in a definite macrostate.
- **Non-invasive measurements** It is possible to determine the macrostate of an object without changing it or its subsequent evolution.
- Freedom of choice
- Arrow of time

Leggett-Garg Inequalities

A simple experiment

- Macro-observable $Q = \pm 1$
- Measurements at 3 times with outcomes Q_0, Q_1, Q_2
- Correlations $C_{ij} = \langle Q_i Q_j \rangle$

Macrorealism ⇒ joint probability distribution

$$C_{01} + C_{12} - C_{02} \le 1$$

Leggett-Garg Inequality

Necessary condition for MR

QM: qubit 1.5, max 3

Tested in various microscopic systems (Superconducting qubits, NV centers, NMR, photons, …)

No-Signaling in Time (NSIT)

In general, joint probability distributions depend on performed measurements:

$$P_{12}(Q_1, Q_2) \neq P_{012}(Q_1, Q_2) \equiv \sum_{Q_0'} P_{012}(Q_0', Q_1, Q_2)$$

subscripts denote measurement times

$$P_2(Q_2) = P_{12}(Q_2) = \sum_{Q_1'} P_{12}(Q_1', Q_2)$$ here: $\mathrm{NSIT}_{(1)2}$

No-signaling in time (Kofler, Brukner, PRA 2013)

There are more NSITs for different combinations of measurement times, see figure.

The arrow of time (AoT) is a trivial special case:

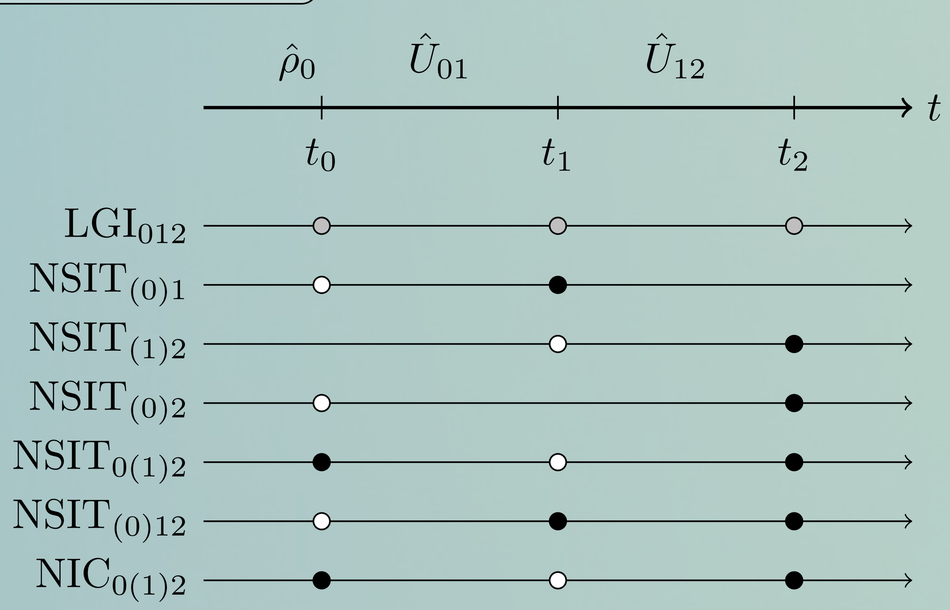
$\mathrm{NSIT}_{0(1)} : P_0 = P_{01}$

$\mathrm{NSIT}_{0(2)} : P_0 = P_{02}$

$\mathrm{NSIT}_{1(2)} : P_1 = P_{12}$

$\mathrm{NSIT}_{01(2)} : P_{01} = P_{012}$

Note that $\mathrm{NSIT}_{(1)2}$ only requires two measurements, where LGI requires three.



NSITs are Neccessary and Sufficient for MR

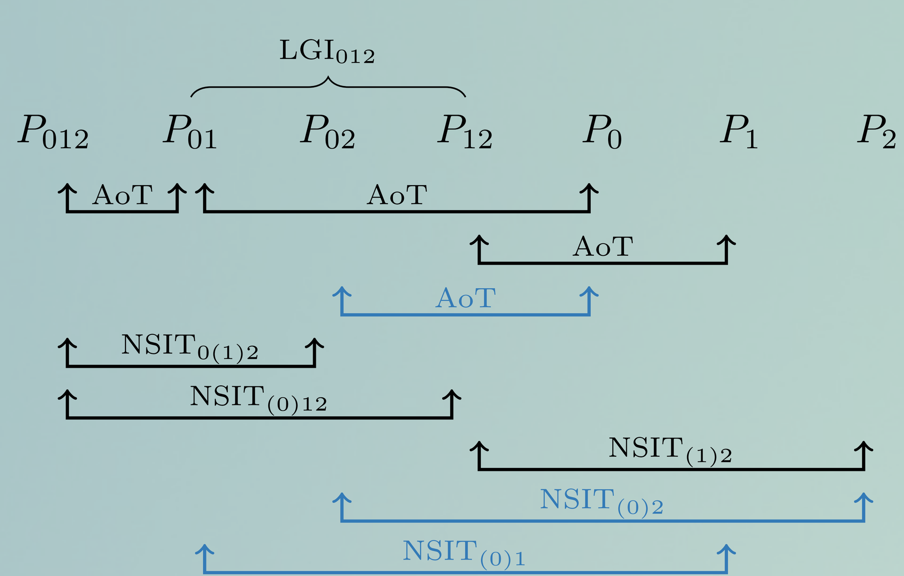
NSITs are sufficient for LGI: (Maroney, Timpson, arXiv:1412.6139):

$$\mathrm{NSIT}_{0(1)2} \wedge \mathrm{NSIT}_{(0)12} \wedge \mathrm{AoT} \Rightarrow \mathrm{LGI}_{012}$$

NSITs in the right combination are **necessary and sufficient** for MR:

$$\mathrm{NSIT}_{(1)2} \wedge \mathrm{NSIT}_{0(1)2} \wedge \mathrm{NSIT}_{(0)12} \wedge \mathrm{AoT} \Leftrightarrow \mathrm{MR}_{012}$$

There are cases where violations of MR can be detected by NSIT, but not by LGI.

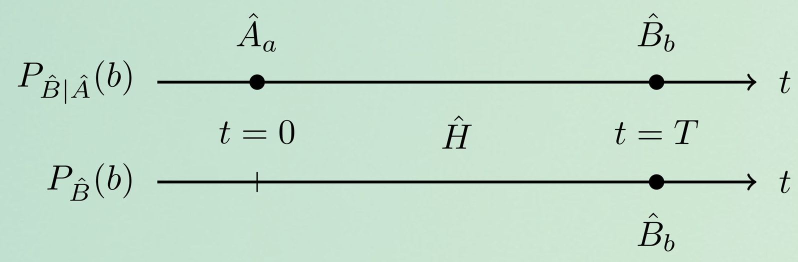


NSIT for Quantum Measurements

Consider POVM measurements on a system:

$$P_{\hat B}(b) = \mathrm{tr}(\hat B_b \hat U_T \hat\rho_0 \hat U_T^\dagger \hat B_b^\dagger)$$

$$P_{\hat B|\hat A}(b) = \int \mathrm{d}a\ \mathrm{tr}(\hat B_b \hat U_T \hat A_a \hat\rho_0 \hat A_a^\dagger \hat U_T^\dagger \hat B_b^\dagger)$$



$$\mathrm{NSIT}_{(0)T} \Leftrightarrow P_{\hat B}(b) = P_{\hat B|\hat A}(b)$$

Hermitian $\hat A, \hat B$ and $T = 0$

$$\forall \hat\rho_0 : \mathrm{NSIT}_{(0)0} \Leftrightarrow \int \mathrm{d}a [\hat A_a \hat B_b, \hat B_b \hat A_a] = 0$$

For projectors:

$$\forall \hat\rho_0 : \mathrm{NSIT}_{(0)0} \Leftrightarrow [\hat A_a, \hat B_b] = 0$$

Hermitian $\hat A, \hat B$ and $\tilde B_b^T \equiv \hat U_T^\dagger \hat B_b \hat U_T$

$$\forall \hat\rho_0 : \mathrm{NSIT}_{(0)T} \Leftrightarrow \int \mathrm{d}a [\hat A_a \tilde B_b^T, \tilde B_b^T \hat A_a] = 0$$

For projectors:

$$\forall \hat\rho_0 : \mathrm{NSIT}_{(0)T} \Leftrightarrow [\hat A_a, \tilde B_b^T] = 0$$

For obtaining non-classical behavior, we need a non-classical resource: Non-commuting measurements or Hamiltonians effecting non-commutation.

Classicality

Idea: use the condition for NSIT to judge "classicality":

- **Classical measurements** fulfill $\mathrm{NSIT}_{(0)0}$ with each member of a given reference set.
- **Classical Hamiltonians** fulfill $\mathrm{NSIT}_{(0)T}$ with each pair of a given reference set.

An *a-priori* choice of the reference set captures the subjectiveness of classicality.

Natural candidates are coarse-grained versions of quantum measurements:

- Coarse-grained coherent state measurements
- Unsharp quadrature measurements
- Coarse-grained Fock measurements

Classicality of Quantum Measurements

Measure of overlap for probabilities

$$V = \int \mathrm{d}b\ \sqrt{P_{\hat B}(b) P_{\hat B|\hat A}(b)} \in [0,1]$$

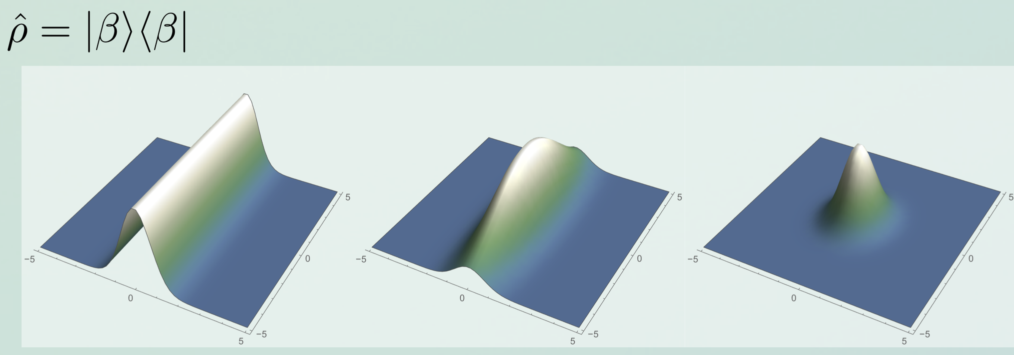
Coherent state POVMs:

$$\hat A_\alpha = |\alpha\rangle\langle\alpha|,\ \hat\rho = |\beta\rangle\langle\beta| : \ V = \frac{2\sqrt{2}}{3} \approx 0.943$$

Quadrature measurements of $\hat\rho = |\beta\rangle\langle\beta|$

$$\hat A_\Delta(x) = \frac{1}{(\Delta\pi)^{1/4}} e^{-\frac{1}{2\Delta}(x-\hat x)^2}$$

$$V = \left(\frac{8\Delta(1+2\Delta)}{(1+4\Delta)^2}\right)^{1/4}$$

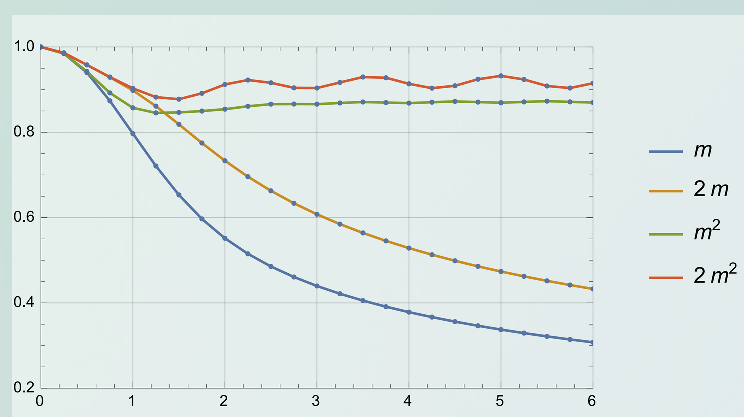


$\Delta \approx 0, V \approx 0.17$ $\Delta = 0.03, V \approx 0.67$ $\Delta = 1, V \approx 0.99$

Fock measurements of $\hat\rho = |\beta\rangle\langle\beta|$

$$\hat A_m = \sum_k \begin{cases} |k\rangle\langle k| & \text{if } b(m) \le k < b(m+1) \\ 0 & \text{else} \end{cases}$$

- sharp (a states per bin): $b(m) = am$
- coarse-grained (am states per bin): $b(m) = am^2$



References

- L. Clemente, J. Kofler. arXiv:1501.07517.
- J. Kofler, Č. Brukner. PRL 2007, PRL 2008, PRA 2013.
- A. J. Leggett, A. Garg. PRL 1985.
- O. J. E. Maroney, C. G. Timpson. arxiv:1412.6139.