HW1 – Lambda Calculus

Problem 1. $(\lambda x. x)y$

$$(\lambda x. x)y = y$$

Problem 2. $(\lambda x. y)x$

$$(\lambda x. y)x = y$$

Problem 3. $(\lambda x. x\ y)(\lambda y. y\ z)$

$$(\lambda x. x\ y)(\lambda y. y\ z) = (\lambda x. x\ y)(\lambda a. a\ z) = (\lambda a. a\ z)y = y\ z$$

Problem 4. $(\lambda x. x\ y)(\lambda a. a\ b)p$

$$(\lambda x. x\ y)(\lambda a. a\ b)p = (\lambda a. a\ b)\ y\ p = y\ b\ p$$

Problem 5 $(\lambda x. x\ y)(\lambda a. b\ a)p$

$$(\lambda x. x\ y)(\lambda a. b\ a)p = (\lambda a. b\ a)y\ p = b\ y\ p$$

Problem 6 $(\lambda x. (\lambda y. x\ y))\ y$

$$\big(\lambda x. (\lambda y. x\ y)\big)y = \big(\lambda x. (\lambda z. x\ z)\big)y = \lambda z. y\ z$$

Problem 7 $(\lambda x. y\ x)y$

$$(\lambda x. y\ x)y = y\ y$$

Problem 8 $(\lambda x. \lambda y. x\ y\ z)(\lambda x. x\ y)z$

$$(\lambda x. \lambda y. x\ y\ z)(\lambda x. x\ y)z = (\lambda x. \lambda a. x\ a\ z)(\lambda x. x\ y)z = (\lambda x. x\ y)\ z\ z = z\ y\ z$$

Problem 9 $(\lambda x. y\ x)x$

$$(\lambda x. y\ x)x = (\lambda z. y\ z)x = y\ x$$

Problem 10 $(\lambda y. y\ x)(\lambda z. z\ y)$

$$(\lambda y. y\ x)(\lambda z. z\ y) = (\lambda a. a\ x)(\lambda z. z\ y) = (\lambda z. z\ y)\ x = x\ y$$