1. Consider the following program:

```
1    for i from 1 to 12:
2        MakeSet(i)
3    Union(2, 10)
4    Union(7, 5)
5    Union(6, 1)
6    Union(3, 4)
7    Union(5, 11)
8    Union(7, 8)
9    Union(7, 3)
10   Union(12, 2)
11   Union(9, 6)
12   print(Find(6))
13   print(Find(3))
14   print(Find(11))
15   print(Find(9))
```

Assume that the disjoint sets data structure is implemented as an array $smallest[1\ldots 12]$: $smallest[i]$ is equal to the smallest element in the set containing i.

What is the output of the following program? As an answer, enter four integers separated by spaces.

1 3 3 1

✓ **Correct**

---

2. Consider the program:

```
1    for i from 1 to 12:
2        MakeSet(i)
3    Union(2, 10)
4    Union(7, 5)
5    Union(6, 1)
6    Union(3, 4)
7    Union(5, 11)
8    Union(7, 8)
9    Union(7, 3)
10   Union(12, 2)
11   Union(9, 6)
```

Assume that the disjoint sets data structure is implemented as disjoint trees with union by rank heuristic.

Compute the product of the heights of the resulting trees after executing the code. For example, for a forest consisting of four trees of height 1, 2, 3, 1 the answer would be 6. (Recall that the height of a tree is the number of edges on a longest path from the root to a leaf. In particular, the height of a tree consisting of just one node is equal to 0.)

2

✓ **Correct**
   Right! There will be 3 trees of height 1, 1, and 2.

3. Consider the following program:

```
1    for i from 1 to n:
2        MakeSet(i)
3    for i from 1 to n-1:
4        Union(i, i+1)
```

Assume that the disjoint sets data structure is implemented as disjoint trees with union by rank heuristic.

What is the number of trees in the forest and the maximum height of a tree in this forest after executing this code? (Recall that the height of a tree is the number of edges on a longest path from the root to a leaf. In particular, the height of a tree consisting of just one node is equal to 0.)

○ One tree of height 1.

○ $\log_2 n$ trees, the maximum height is 1.

○ Two trees, both of height 1.

○ n trees, the maximum height is 1.

◉ One tree of height $\log_2 n$.

○ $n/2$ trees, the maximum height is 2.

⊗ **Incorrect**

---

4. Consider the following program:

```
1    for i from 1 to 60:
2        MakeSet(i)
3    for i from 1 to 30:
4        Union(i, 2*i)
5    for i from 1 to 20:
6        Union(i, 3*i)
7    for i from 1 to 12:
8        Union(i, 5*i)
9    for i from 1 to 60:
10       Find(i)
11
```

Assume that the disjoint sets data structure is implemented as disjoint trees with union by rank heuristic and with path compression heuristic.

Compute the maximum height of a tree in the resulting forest. (Recall that the height of a tree is the number of edges on a longest path from the root to a leaf. In particular, the height of a tree consisting of just one node is equal to 0.)

```
1
```

✓ **Correct**

There is at least one tree of height 1 in the forest. Also, all trees have height at most 1, since the last for-loop calls Find() for all 60 elements. Since path compression is used, each non-root node will be attached directly to the corresponding root in this loop, and hence all the trees will have height at most 1.