1. Alice (a mathematician specialized in mathematical logic) says that every recursive ordinal is constructive. Bob wants to refute Alice's claim by giving a counterexample. Which of the following could he provide? (*Note that in order to answer this question you do not need to know what an ordinal is, or what it means for it to be constructive or recursive.*).

1 / 1 point

- (●) A non-constructive ordinal that is recursive
- ( ) A non-constructive ordinal that is not recursive
- ( ) A constructive ordinal that is not recursive

Correct

Correct! Indeed, this is not possible if every recursive ordinal (including this one) is constructive, because it cannot be constructive and non-constructive at the same time.

2. Which of the following statements are equivalent? Group the equivalent statements:

1/1 point

1. All recursive ordinals are constructive
2. All constructive ordinals are recursive
3. All non-recursive ordinals are non-constructive
4. All non-constructive ordinals are non-recursive
5. Some constructive ordinals are recursive
6. Some recursive ordinals are constructive
7. Some non-recursive ordinals are non-constructive

- ○ Two groups: 1, 4 and 6; 2, 3, 5 and 7.
- ○ One group: 5, 6 and 7.
- ◉ Three groups: 1 and 4; 2 and 3; 5 and 6.
- ○ Two groups: 1 and 2, 3 and 4.

Correct

Correct!

If all recursive ordinals are constructive, then any non-constructive ordinal cannot be recursive, and thus all non-constructive ordinals are non-recursive. Vice versa, if all non-constructive ordinals are non-recursive, than any recursive ordinal cannot be non-constructive, and so all recursive ordinals are constructive.

The second pair is symmetric.

5 and 6 both mean that there exists at least one ordinal that is both constructive and recursive, so they are equivalent.

Statements with "Some" are all existential, while statements with "All" are universal, so they cannot be equivalent.

5 and 7 are not equivalent, for example, because it could be that 5 is true, and in fact all ordinals are constructive, some of them are also recursive (thus some constructive ordinals are recursive), but no non-recursive ordinals are non-constructive as there are no non-constructive ordinals at all, and so 7 is false. 6 and 7 are not equivalent because 5 and 6 are equivalent, but 5 and 7 are not equivalent as we've just shown.

3. Alice claims that every student in a group knows both French and German. Which of the following sentences asserts that Alice's statement is wrong (no more, no less)?

1 / 1 point

- [ ] There is a student who does not know French and does not know German
- [ ] Every student who knows French does not know German
- [ ] All students know at most one of these two languages
- [x] There is a student who does not know French or does not know German (or both)

Correct

Correct! If Alice is wrong, then not all students know both languages, so there must be at least one student who doesn't know both languages, so there must be at least one student who doesn't know French or doesn't know German or both.

In other words, to negate a universal quantifier, we replace it with an existential quantifier and negate everything inside, and to negate logical AND we replace it with OR and negate both sides.

4. Alice claims that every student in a group knows French, German, or both. Which of the following sentences asserts that Alice's statement is wrong (no more, no less)?

1 / 1 point

- ○ All students know at most one of these two languages
- ◉ There is a student who does not know French and does not know German
- ○ There is a student who does not know French or does not know German (or both)
- ○ Every student who knows French does not know German

Correct

Correct! Indeed, if there is such a student, then Alice is wrong that all the students in the group know at least one of the two languages.

In other words, to negate a universal quantifier, we replace it with an existential quantifier and negate everything inside, and to negate logical OR we replace it with logical AND and negate both sides.

5. Let x be a number. Consider the following five statements: $x > 10, x > 20, x > 30, x > 40, x > 50$. Three of these statements are true and two are false. Which ones are true?

1 / 1 point

- [x] $x > 10$

Correct
If any of the statements is true, then this one is also true, because $x > 10$ follows from all the other statements.

- [x] $x > 20$

Correct
If at least two statements are true, then this one is also true, because $x > 20$ follows from all the other statements but for $x > 10$.

- [x] $x > 30$

Correct
If three statements are true, then this one is also true, because $x > 30$ follows from $x > 40$ and $x > 50$, and there are only two other statements.

- [ ] $x > 40$

- [ ] $x > 50$

6. Consider the following statements about a number x:

1 / 1 point

- x is a multiple of 2 (x/2 is integer);
- x is a multiple of 3;
- x is a multiple of 6.

Is it possible that for some integer x

- [x] None of them are true?

Correct
Indeed, for $x = 1$ none of them are true.

- [x] Exactly one of them is true?

Correct
Indeed, for $x = 2$ only the statement that x is a multiple of 2 is true, and both other statements are false.

- [ ] Exactly two of them are true?

- [x] All three statements are true?

Correct
Indeed, for $x = 6$ all three statements are true.

7. Alice says that in every region there is a town where all inhabitants are happy. Bob wants to say that Alice is wrong. Which of the following sentences should Bob say?

1 / 1 point

- (selected) There is a region where in all towns at least one inhabitant is unhappy.
- In every region there is a town where at least one inhabitant is unhappy.
- In every region in all towns all inhabitants are happy.
- There is a region where there is a town where all inhabitants are happy.

Correct

Correct! Recall that to negate a universal quantifier, we need to replace it with existential and negate the statement inside, and vice versa, to negate an existential quantifier, we need to replace it with universal and negate the statement inside. The initial statement is a universal quantifier with an existential quantifier inside with a universal quantifier inside. We replace this sequence "universal, existential, universal" with "existential, universal, existential" and negate "happy" with "unhappy".