Homework 6

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1 Analysis of the region of absolute stability for the 2-stage, 2th-order Runge-Kutta method

$$\left\{\begin{array}{l} u^{(1)} = u^{(0)} + \Delta t L(u^{(0)}) \\ u^{(2)} = \frac{1}{2}u^{(0)} + \frac{1}{2}u^{(1)} + \frac{1}{2}\Delta t L(u^{(1)}) \\ u_n = u^{(0)}, \quad u_{n+1} = u^{(2)} \end{array}\right.$$

Apply the Heun's method above to the solution of the ODE $u'(t) = \lambda u(t)$, we have

$$L(u^{(0)}) = \lambda u^{(0)}, \quad L(u^{(1)}) = \lambda u^{(1)}$$

$$u^{(1)} = u^{(0)} + \lambda \Delta t u^{(0)}$$

$$\begin{aligned} u_{n+1} &= u^{(2)} \\ &= \frac{1}{2}u^{(0)} + \frac{1}{2}u^{(1)} + \frac{1}{2}\lambda \Delta t u^{(1)} \\ &= \frac{1}{2}u^{(0)} + \frac{1}{2}(1 + \lambda \Delta t)(u^{(0)} + \lambda \Delta t u^{(0)}) \\ &= (1 + \lambda \Delta t + \frac{1}{2}(\lambda \Delta t)^2)u^{(0)}. \end{aligned}$$

Let z be $\lambda \Delta t$. The region of absolute stability is $\{z \in \mathbb{C} | \ |1 + z + \frac{1}{2}z^2| < 1\}$ and the interval of absolute stability is $(-2, 0)$.