Royal Institute of Technology

DGM CONDITIONAL INDEPENDENCE

CONDITIONAL INDEPENDENCE

- X and Y are conditionally independent given Z iff

$$p(X, Y | Z) = p(X | Z)p(Y | Z)$$

- Implies

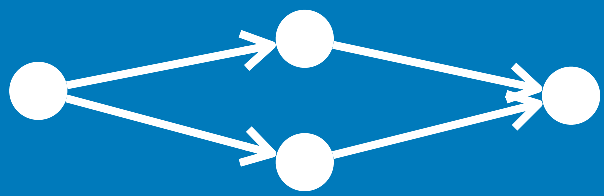
$$p(X | Y, Z) = p(X, Y | Z)/p(Y | Z) = p(X | Z)$$

- Denoted

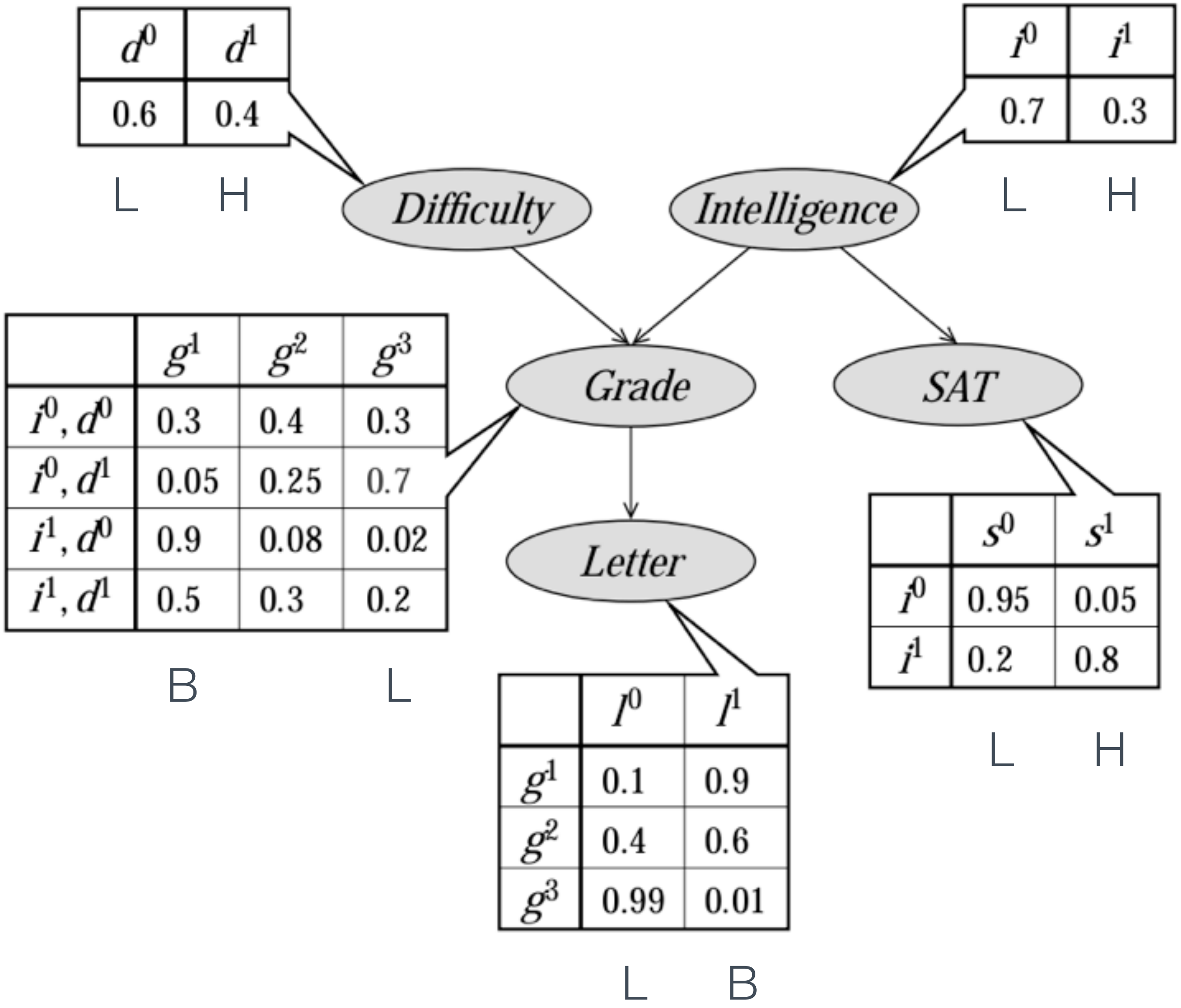
$$X \perp Y | Z$$

WHEN ARE X AND Y CONDITIONALLY INDEPENDENT?

$$p(x_1, \ldots, x_N) = \prod_{n=1}^{N} p(x_n | \boldsymbol{x}_{\mathrm{pa}(x_n)})$$



STUDENT EXAMPLE



d^0	d^1
0.6	0.4

i^0	i^1
0.7	0.3

	g^1	g^2	g^3
i^0, d^0	0.3	0.4	0.3
i^0, d^1	0.05	0.25	0.7
i^1, d^0	0.9	0.08	0.02
i^1, d^1	0.5	0.3	0.2

	s^0	s^1
i^0	0.95	0.05
i^1	0.2	0.8

	l^0	l^1
g^1	0.1	0.9
g^2	0.4	0.6
g^3	0.99	0.01

B - better

H - higher

L - less

INDEPENDENCE I-MAP

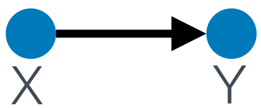
- I(G) (conditional) independences implied by G (not yet defined)
- I(P) (conditional) independences in the distribution P
- G is an I-map for P if I(G) ⊆ I(P)

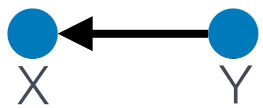
p

X	Y	$P(X,Y)$
x^0	y^0	0.08
x^0	y^1	0.32
x^1	y^0	0.12
x^1	y^1	0.48

q

X	Y	$P(X,Y)$
x^0	y^0	0.4
x^0	y^1	0.3
x^1	y^0	0.2
x^1	y^1	0.1





INDEPENDENCE I-MAP

- I(G) independences implied by G (not yet defined)
- I(P) independences in the distribution P
- G I-map for P in I(G) ⊆ I(P)

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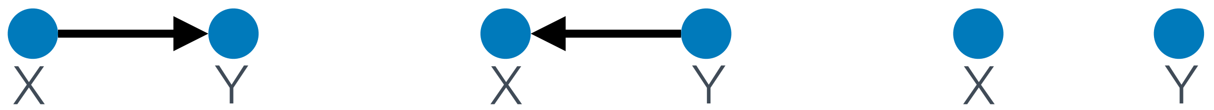

- p: X and Y ind. ex. p(X=1) = 0.48 + 0.12 =0.6, p(Y=1) = 0.8, and p(X=1,Y=1) = 0.48
- q: X and Y are dependent

INDEPENDENCE I-MAP

- ★ I(G) independences implied by G (not yet defined)
- ★ I(P) independences in the distribution P
- ★ G I-map for P in I(G) ⊆ I(P)

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x^0	y^1	0.3
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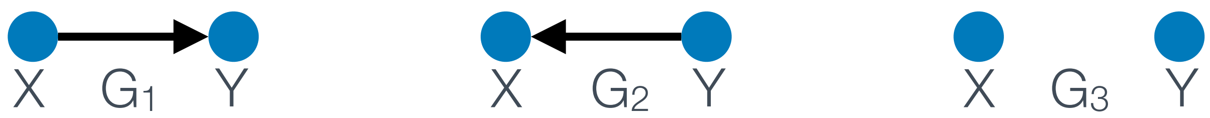
x^1	y^1	0.1

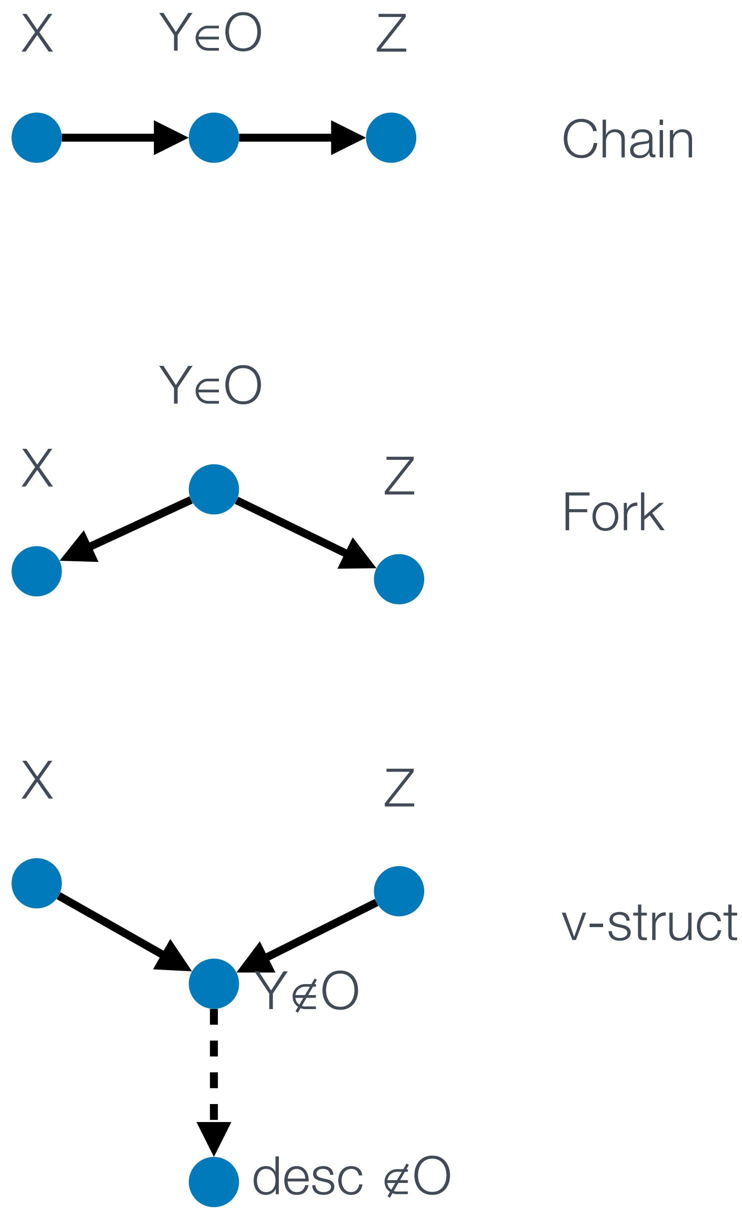


- ★ All three graphs are I-maps for p
- ★ G_1 and G_2 are I-maps for q, but G_3 is not

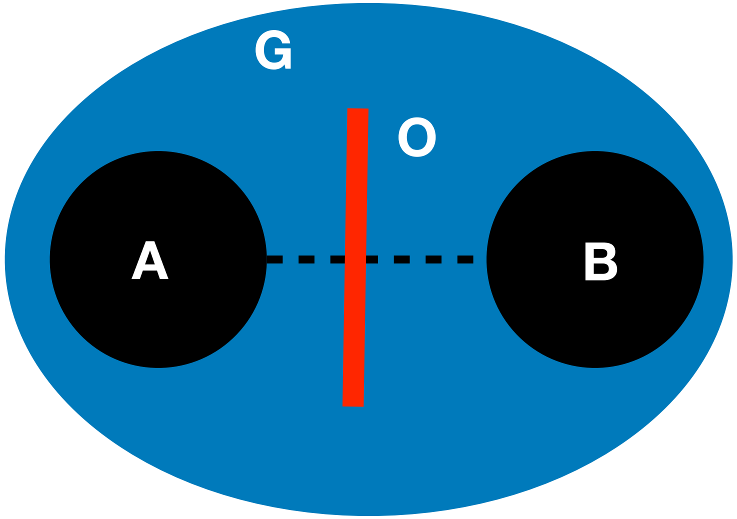
D-SEPARATION

★ A path is d-separated by O if it has

- a chain X → Y → Z where Y ∈ O
- a fork X ← Y → Z where Y ∈ O
- a v-structure X → Y ← Z where (Y ∪ desc(Y)) ∩ O = ∅



D-SEPARATION SETS AND CI OF DAGS



★ A is d-separated from B given O if every undirected path between A and B is d-separated by O

★ Cond. ind rel. in DAG G,

$$\boldsymbol{x}_A \perp_G \boldsymbol{x}_B | \boldsymbol{x}_O$$

$$\Updownarrow$$

A is d-separated from B given O

SOUNDNESS AND COMPLETENESS

- I(G) conditional independence relations implied by d-sep in G
- I(p) conditional independence relations satisfied by p

- Theorem

  A distribution P can be factorised over G iff I(G) ⊆ I(p)

- “=“ not possible to achieve, ex. clique and independent distribution