

$$j_t(k) = p(x_{1:t-1}, z_t = k)$$

$$= \sum_{k' \in [k]} p(x_{1:t-1}, z_t = k, z_{t-1} = k')$$

$$= \sum p(x_{1:t-2}, z_{t-1} = k') p(x_{t-1}, z_t = k | z_{t-1} = k')$$

$$= \sum j_{t-1}(k') p(x_{t-1} | z_{t-1} = k') p(z_t = k | z_{t-1} = k')$$