DD2434/FDD3434 Machine Learning, Advanced Course Module 1 Exercise

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1 Bayesian statistics – Theory

When conducting probabilistic modeling, we usually specify a model for how the data was generated. We denote the parameters of this model as Θ. In Bayesian statistics, we assume a prior distribution $p(\Theta)$ and infer the posterior $p(\Theta|X)$ through Bayes' theorem:

$$p(\Theta|X) = \frac{p(X|\Theta)p(\Theta)}{p(X)} \tag{1}$$

where $p(X|\Theta)$ is referred to as the likelihood function, $p(\Theta)$ the prior and $p(X)$ the evidence or marginal likelihood.
In some cases, we don't need to compute $p(X)$ in order to derive the posterior, instead we can apply the method of "identifying the distribution" that Jens describes in Video Lecture 1.3. Use this method in the exercises below.

Conjugate priors – Exercises

1.1 Beta-Binomial

Let $X = (X_1, .., X_N)$ be i.i.d. where $X_n|\theta, m \sim Binomial(m, \theta)$ and $\theta \sim Beta(\alpha, \beta)$. Show that the posterior $p(\theta|X, m)$ follows a Beta-distribution, i.e., that the Beta is conjugate prior to the Binomial with known m. What are the parameters of the posterior? Compare with the Wikipedia Conjugate prior table.

1.2 Poisson-Gamma

Let $D = (d_1, ..., d_N)$ be i.i.d. with $d_n|\Lambda \sim Poisson(\Lambda)$ and $\Lambda \sim Gamma(\alpha, \beta)$. Show that the posterior $p(\Lambda|D)$ follows a Gamma-distribution, i.e. that the Gamma is conjugate prior to the Poisson distribution. What are the parameters of the posterior? Compare with the Wikipedia Conjugate prior table.

1.3 Normal-NormalGamma (Hard)

Let $X = (X_1, ..., X_N)$ be i.i.d. with $X_n|\mu, \tau \sim Normal(\mu, \frac{1}{\tau})$ and $(\mu, \tau) \sim NormalGamma(\mu_0, \lambda, \alpha, \beta)$. Show that the posterior $p(\mu, \tau|X)$ follows a NormalGamma-distribution, i.e. that the NormalGamma is conjugate prior to the Normal distribution with unknown mean and precision. What are the parameters of the posterior? Compare with the Wikipedia Conjugate prior table.