ID2222: Data Mining

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Mining Data Streams (Part 1)

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Outline

- Part 1 (this lecture)
  - The stream data model
  - Sampling data in a stream
  - Filtering streams
- Part 2 (next lecture: counting algorithms on streams)
  - Counting distinct elements
  - Computing moments
  - Queries over a (long) sliding window: Counting ones
  - Decaying Windows: Counting itemsets

Data Streams – Infinite Data

In many data mining situations, we do not know the entire data set in advance

Stream Management is important when the input rate is controlled **externally:**

- Google search queries
- Twitter or Facebook status updates

We can think of the **data** as **infinite** and **non-stationary** (the distribution changes over time)

J. Leskovec, A. Rajaraman, J. Ullman: Mining of Massive Datasets, http://www.mmds.org

The Stream Model

Input **elements** enter at a rapid rate, at one or more input ports (i.e., ***streams***)

- **We call elements of the stream tuples**

The system cannot store the entire stream accessibly

Q: How do you make critical calculations about the stream using a limited amount of (secondary) memory?

SGD – Example of a Streaming Algorithm

Stochastic Gradient Descent (SGD) is an example of a stream algorithm

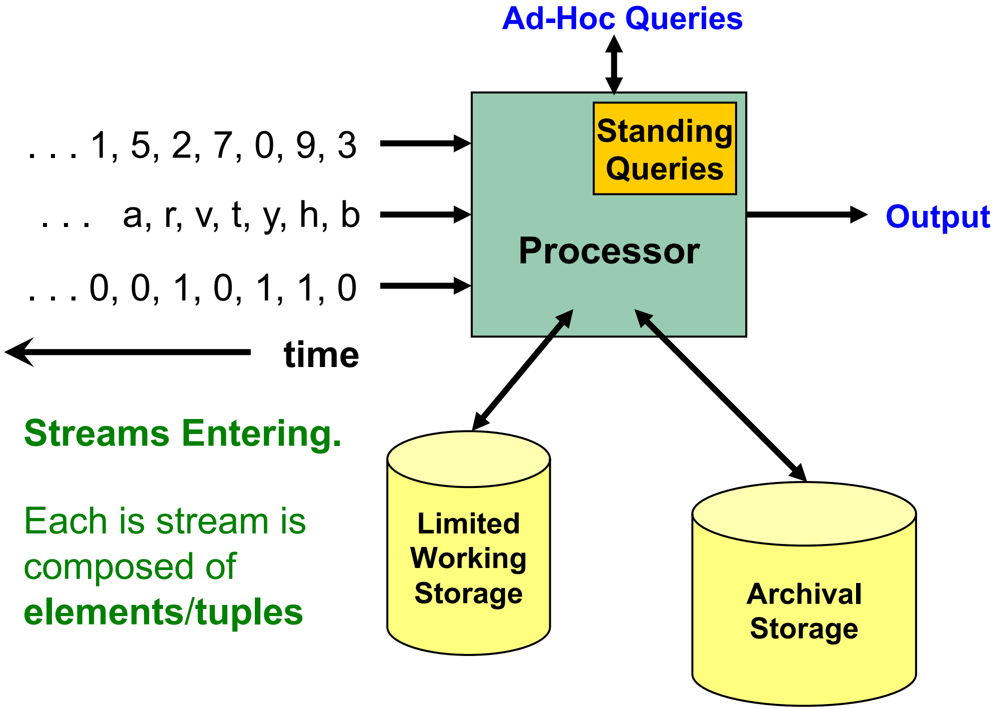
In Machine Learning we call this: Online Learning

- Allows for modeling problems where we have a continuous stream of data
- We want an algorithm to learn from it and slowly adapt to the changes in data

Idea: Do slow updates to the model

- **SGD** (SVM, Perceptron) makes small updates
- **So:** First train the classifier on training data.
- **Then:** For every example from the stream, we slightly update the model (using small learning rate)

General Stream Processing Model



Streams Entering.

Each is stream is composed of **elements/tuples**

Two forms of query

1. ***Ad-hoc queries***: Normal queries asked one time about streams.
   - **Example**: What is the maximum value seen so far in stream *S*?
2. ***Standing queries***: Queries that are, in principle, asked about the stream at all times.
   - **Example**: Report each new maximum value ever seen in stream S.

Problems on Data Streams (1)

Types of queries one wants on answer on a data stream:

(to be considered in this lecture)

- **Sampling data from a stream**
  - Construct a random sample
- **Filtering a data stream**
  - Select elements with property ***x*** from the stream

J. Leskovec, A. Rajaraman, J. Ullman: Mining of Massive Datasets, http://www.mmds.org

Problems on Data Streams (2)

Types of queries one wants on answer on a data stream: (to be considered in next lecture)

- **Queries over sliding windows**
  - Number of items of type ***x*** in the last ***k*** elements of the stream
- **Counting distinct elements**
  - Number of distinct elements in the last ***k*** elements of the stream
- **Estimating moments**
  - Estimate avg./std. dev. of last ***k*** elements
- **Finding frequent elements**

Applications (1)

Mining query streams

- Google wants to know what queries are more frequent today than yesterday

Mining click streams

- Yahoo! wants to know which of its pages are getting an unusual number of hits in the past hour
  - Often caused by annoyed users clicking on a broken page.

Mining social network news feeds

- E.g., look for trending topics on Twitter, Facebook

Applications (2)

Sensor Networks

- Many sensors feeding into a central controller

Telephone call records

- Data feeds into customer bills as well as settlements between telephone companies

IP packets monitored at a switch

- Gather information for optimal routing
- Detect denial-of-service attacks

Sampling from a Data Stream: Sampling a fixed proportion

As the stream grows the sample also gets bigger

Sampling from a Data Stream

Since **we can not store the entire stream**, one obvious approach is to store a **sample**

Two different problems:

- **(1)** Sample a **fixed proportion** of elements in the stream (say 1 in 10)
- **(2)** Maintain a **random sample of fixed size** over a potentially infinite stream
  - At any "time" ***k*** we would like a random sample of ***s*** elements
    - **What is the property of the sample we want to maintain?** For all time steps ***k***, each of ***k*** elements seen so far has equal probability of being sampled

Sampling a Fixed Proportion

Problem 1: Sampling fixed proportion

Scenario: Search engine query stream

- **Stream of tuples:** (user, query, time)
- **Answer questions such as: How often did a user run the same query in a single days**
- Have space to store **1/10th** of query stream

Naïve solution:

- Generate a random integer in **[0..9]** for each query
- Store the query if the integer is **0**, otherwise discard

Problem with Naïve Approach

Simple question: What fraction of queries by an average search engine user are duplicates?

- Suppose each user issues x queries once and d queries twice, i.e., in total of $(x + 2d)$ queries
  - **Correct answer is** $\boxed{d/(x+d)}$
- **Proposed solution: Let's keep 10% of the queries**
  - Sample will contain $x/10$ of the singleton queries and $2d/10$ of the duplicate queries at least once
  - But only $d/100$ pairs of duplicates: $d/100 = 1/10 \cdot 1/10 \cdot d = 0{,}01d$
  - Of d "duplicates" *18d/100* appear exactly once: $((1/10 \cdot 9/10)+(9/10 \cdot 1/10)) \cdot d = 0{,}18d$
- **So the sample-based answer is** $\boxed{\dfrac{0{,}01d}{0{,}1x+0{,}01d+0{,}18d} = \dfrac{d}{10x+19d}}$

Solution: Sample Users

- Pick **1/10th** of **users** and take all their searches in the sample

- Use a hash function that hashes the user name or user id uniformly into 10 buckets

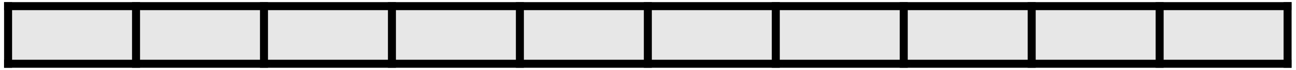
Generalized Solution

Stream of tuples with keys:

- Key is some subset of each tuple's components
  - e.g., tuple is (user, search, time); key is **user**
- Choice of key depends on application

To get a sample of *a/b* fraction of the stream:

- Hash each tuple's key uniformly into ***b*** buckets
- Pick the tuple if its hash value is at most ***a***



Hash table with ***b*** buckets, pick the tuple if its hash value is at most ***a***.
How to generate a 30% sample?
Hash into *b*=10 buckets, take the tuple if it hashes to one of the first *a*=3 buckets

Sampling from a Data Stream: Sampling a fixed-size sample

As the stream grows, the sample is of a fixed size

Maintaining a Fixed-size Sample

Problem 2: Fixed-size sample

Suppose we need to maintain a random sample *S* of size exactly *s* tuples

- E.g., main memory size constraint

Why? Don't know length of stream in advance

Suppose at time *n* we have seen *n* items

- **Each item is in the sample *S* with equal prob. *s/n***

Maintaining a Fixed-size Sample (cont)

How to think about the problem: say the sample size *s* = 2

Stream: $\underbrace{\underbrace{a\ x\ c\ y\ z}\ k\ c}\ d\ e\ g\ldots$

At ***n*= 5,** each of the first 5 tuples is included in the sample ***S*** with equal probability.

At **n= 7,** each of the first 7 tuples is included in the sample ***S*** with equal probability.

Impractical solution would be to store all the *n* tuples seen so far and out of them pick *s* at random

Solution: Fixed Size Sample

Algorithm (a.k.a. Reservoir Sampling)

- Store all the first s elements of the stream to S
- Suppose we have seen n-1 elements, and now the n^{th} element arrives ($n > s$)
  - With probability s/n, keep the n^{th} element, else discard it
  - If we picked the n^{th} element, then it replaces one of the s elements in the sample S, picked uniformly at random

Claim: This algorithm maintains a sample S with the desired property:

- After n elements, the sample contains each element seen so far with probability s/n

Proof: By Induction

We prove this by induction:

- Assume that after ***n*** elements, the sample contains each element seen so far with probability ***s/n***
- We need to show that after seeing element ***n+1*** the sample maintains the property
  - Sample contains each element seen so far with probability ***s/(n+1)***

Base case:

- After we see ***n=s*** elements the sample ***S*** has the desired property
  - Each out of ***n=s*** elements is in the sample with probability ***s/s = 1***

Proof: By Induction (cont)

Inductive hypothesis: After n elements, the sample S contains each element seen so far with probability s/n

Now element $n+1$ arrives

Inductive step: For elements already in S, probability that the algorithm keeps it in S:

$$\underbrace{\left(1-\frac{s}{n+1}\right)}_{\text{Element } n+1 \text{ discarded}}+\underbrace{\left(\frac{s}{n+1}\right)}_{\text{Element } n+1 \text{ not discarded}}\underbrace{\left(\frac{s-1}{s}\right)}_{\text{Element in the sample not picked}}=\frac{n}{n+1}$$

- So, at time n, tuples in S were there with probability s/n
- At time $n \to n+1$, tuple stayed in S with prob. $n/(n+1)$
- So prob. tuple is in S at time $n+1$: $(s/n)\cdot(n/n+1)=\dfrac{s}{n+1}$

Filtering Data Streams

Filtering Data Streams

Another common process on streams is ***selection***, or ***filtering***.

- Each element of data stream is a tuple
- Tuples in the stream that meet a criterion are accepted.
    - Accepted tuples are passed to another process as a stream
    - Non-accepted tuples are dropped.

Filtering Data Streams

Each element of data stream is a tuple

Given a list of keys ***S***

Determine which tuples of stream are in *S*

Obvious solution: Store keys in a hash table

- But suppose we **do not have enough memory** to store all of ***S*** in a hash table
  - E.g., we might be processing millions of filters on the same stream

Applications

Example: Email spam filtering

- We know 1 billion “good” email addresses
- If an email comes from one of these, it is **NOT** spam

Publish-subscribe systems

- You are collecting lots of messages (news articles)
- People express interest in certain sets of keywords
- Determine whether each message matches user’s interest

First Cut Solution (1)

Given a set of keys *S* that we want to filter

Create a **bit array *B*** of ***n*** bits, initially all ***0*s**

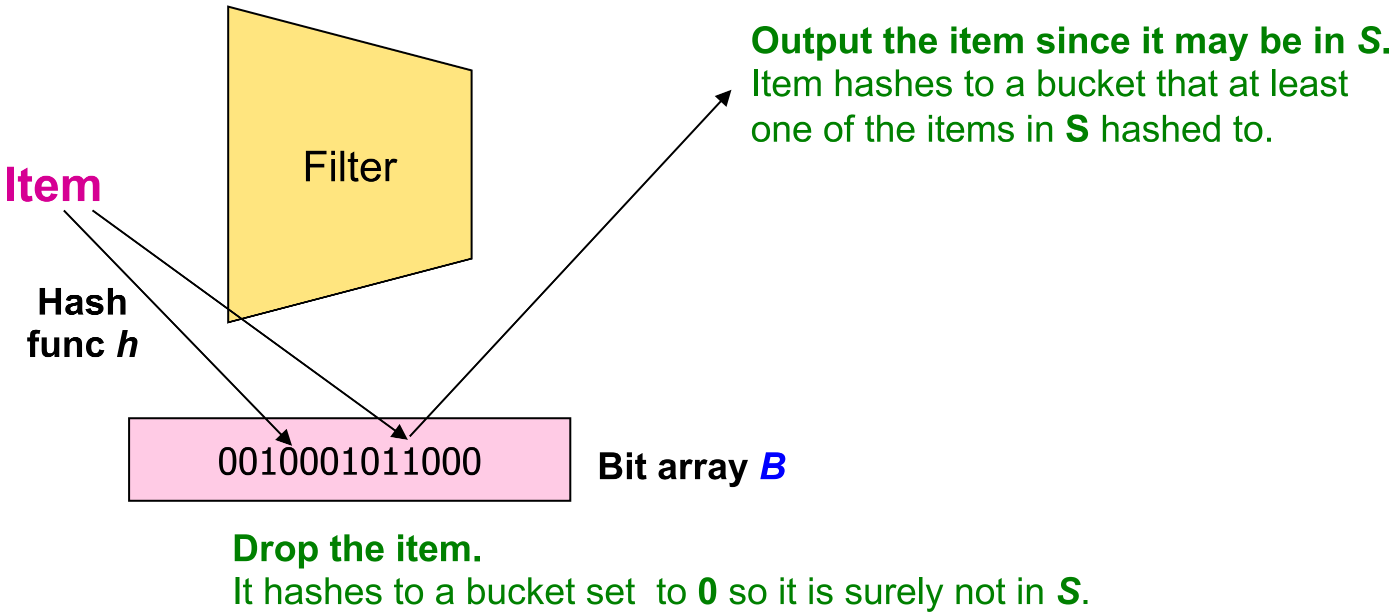
Choose a **hash function *h*** with range **[0,*n*)**

Hash each member of $s \in S$ to one of ***n*** buckets, and set a corresponding bit to **1**, i.e., $B[h(s)]=1$

Hash each element ***a*** of the stream and output only those that hash to a bucket with its corresponding bit in B that was set to **1**

- **Output *a* if $B[h(a)] == 1$**

First Cut Solution (2)



Creates false positives but no false negatives

– If the item is in ***S*** we surely output it, if not we may still output it

J. Leskovec, A. Rajaraman, J. Ullman: Mining of Massive Datasets, http://www.mmds.org

First Cut Solution (3)

- $|S|$ = 1 billion email addresses
  $|B|$= 1GB = 8 billion bits

If the email address is in ***S***, then it surely hashes to a bucket that has the bit set to 1, so it always gets through (***no false negatives***)

Approximately **1/8** of the bits are set to **1**, so about **1/8th** of the addresses not in ***S*** get through to the output (***false positives***)

- Actually, less than **1/8th**, because more than one address might hash to the same bit

Bloom Filter

A Bloom filter consists of:

1. An bit-array ***B*** of ***n*** bits, initially all 0's.
2. A collection of hash functions $h_1, h_2, \ldots, h_k$.
   Each hash function maps "key" values to ***n*** buckets, corresponding to the ***n*** bits of the bit-array.
3. A set ***S*** of ***m*** key values

The Bloom filter allows through all stream elements whose keys are in ***S***

- It rejects stream elements whose keys are not in ***S***

Bloom Filter (cont)

Initialization:

- Set ***B*** to all **0s**
- Hash each element $s \in S$ using each hash function h_i, set $B[h_i(s)] = 1$ (for each $i = 1,.., k$)

Run-time:

- When a stream element with key x arrives
  - If $B[h_i(x)] = 1$ for all $i = 1,..., k$ then declare that x is in S
    - That is, x hashes to a bucket set to **1** for every hash function $h_i(x)$
  - Otherwise discard the element x

Analysis of Bloom Filtering: Throwing Darts

Analysis for the number of false positives

(assuming one hash function $k = 1$)

Consider: If we throw $\boldsymbol{m}$ darts into $\boldsymbol{n}$ equally likely targets,
what is the probability that a target gets at least one dart?

In our case:

- **Targets** = bits/buckets
- **Darts** = hash values of items

Analysis: Throwing Darts (cont)

Assume, we have ***x*** targets and ***y*** darts

What is the probability that a target gets at least one dart?

The probability that a given dart will not hit a given target ***x*** is

$$(x-1)/x$$

The probability that none of the ***y*** darts will hit a given target ***x*** is

$$\left(\frac{x-1}{x}\right)^y = \left(1-\frac{1}{x}\right)^{x\left(\frac{y}{x}\right)} \approx e^{-y/x}$$

– using approximation $(1-\epsilon)^{1/\epsilon} = 1/e$

The probability that a target gets at least one dart: $1 - e^{-y/x}$

Bloom Filter – Analysis

Bloom filter: bit-array **B** of ***n*** bits; ***k*** hash functions; ***m*** keys

What fraction of the bit vector B are 1s?

- Throwing ***k·m*** darts at ***n*** targets
- So fraction of **1**s is $(1 - e^{-km/n})$

But we have ***k*** independent hash functions and
we only let the element ***x*** to pass through **if all *k*** hash functions hash element ***x*** to a bucket of value **1**

So, false **positive probability** = $(1 - e^{-km/n})^k$

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Bloom Filter – Analysis (cont)

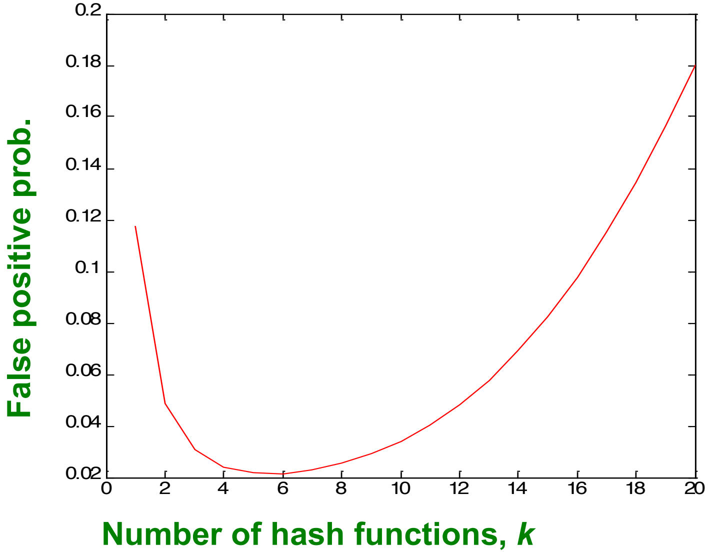
***m* = 1 billion, *n* = 8 billion**

- **k = 1**: $(1 - e^{-1/8})$ = **0.1175**
- **k = 2**: $(1 - e^{-1/4})^2$ = **0.0493**

What happens as we keep increasing *k*?

"Optimal" value of ***k***: ***n/m* ln(2)**

- **In our case:** Optimal **k = 8 ln(2) = 5.54 ≈ 6**
  - **Error at k = 6**: $(1 - e^{-1/6})^2$ = **0.0235**



Bloom Filter: Wrap-up

Bloom filters guarantee no false negatives, and use limited memory

- Great for pre-processing before more expensive checks

Suitable for hardware implementation

- Hash function computations can be parallelized

Options: Is it better to have **one** big **B** or ***k*** small **B**s?

- **It is the same:** $(1 - e^{-km/n})^k$ vs. $(1 - e^{-m/(n/k)})^k$
- But keeping **1 big B** is simpler